Bose and Non-Fermi Liquid Metals

KITP Program on AdS/CMT November 1, 2011

Matthew PA Fisher

Desperately Seeking Non-Fermi Liquid phases of 2d *itinerant* electrons

- ***Parton*** approach to NFL's, Gutzwiller wavefunctions $c_\alpha = d_1 d_2 f_\alpha$
- Bose Metals – pathway to electron NFL's
- D-wave Bose-Metal in the continuum (AdS/CFT?)
- Lattice Hamiltonians and energetics (DMRG) for D-wave Bose-Metal and D-wave NFL

Collaborators

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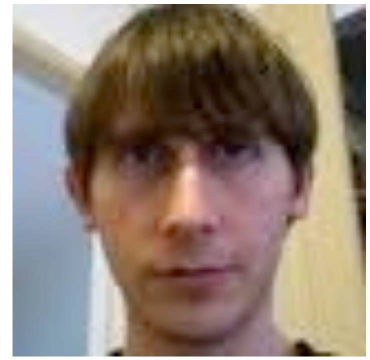

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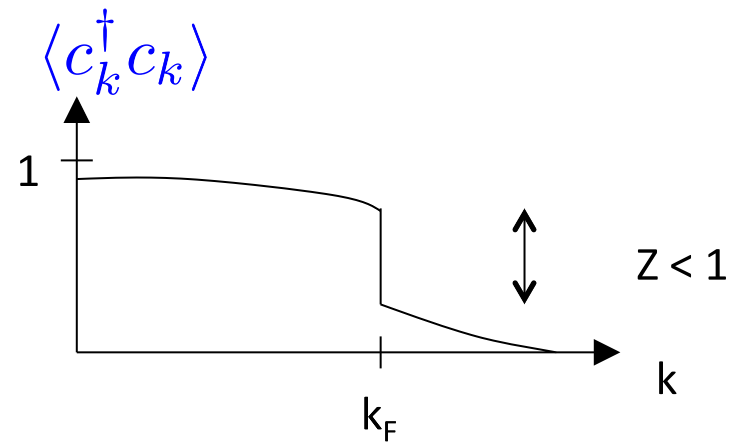
What is a “Non-Fermi-liquid metal”?

A NFL is ***not*** a Fermi Liquid

2D Fermi Liquid

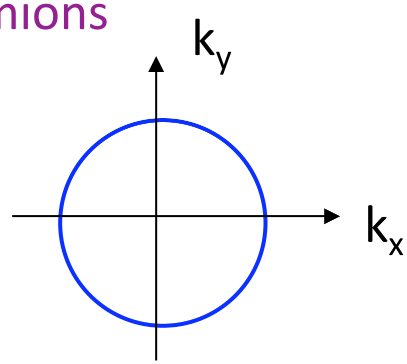
Momentum Distribution Function: $n_k = \langle c_k^\dagger c_k \rangle$

Jump discontinuity
defines location of Fermi surface



Luttingers Thm: Volume inside Fermi surface
set by total density of fermions

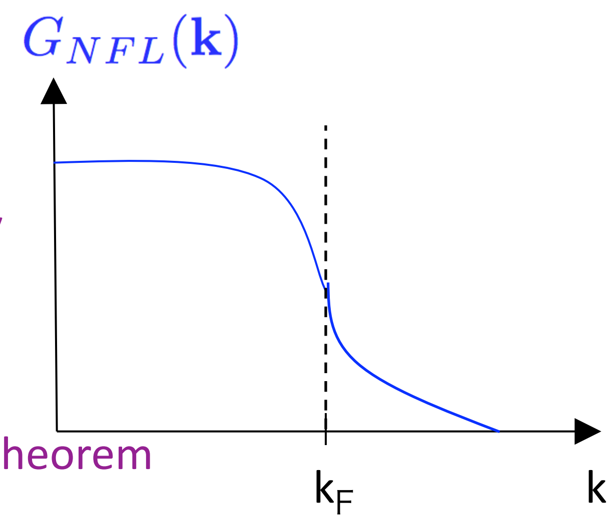
$$\rho = k_F^2/4\pi$$



2D Non-Fermi Liquid

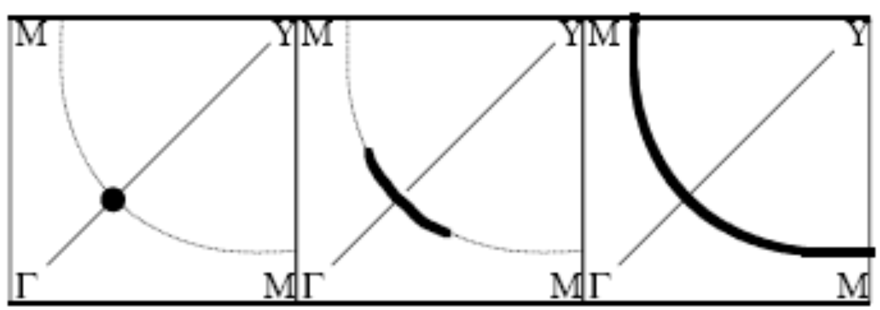
Various possibilities:

1) A (dominant) singular surface satisfying Luttinger's theorem but without a jump discontinuity



2) A (dominant) singular surface that violates Luttinger's theorem (eg. volume "x" rather than "1-x")

3) A singular "Fermi surface" with ``arc"



4.) Other….

How to access putative NFL metals?

One approach: Construct wavefunctions!

Venerable history in condensed matter physics:

BCS wavefunction

Laughlin wavefunction

Construct wavefunctions from partons

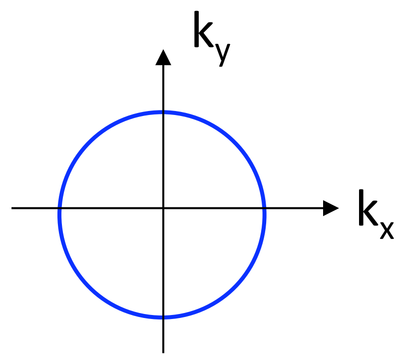
Wavefunction for 2D Free Fermi gas

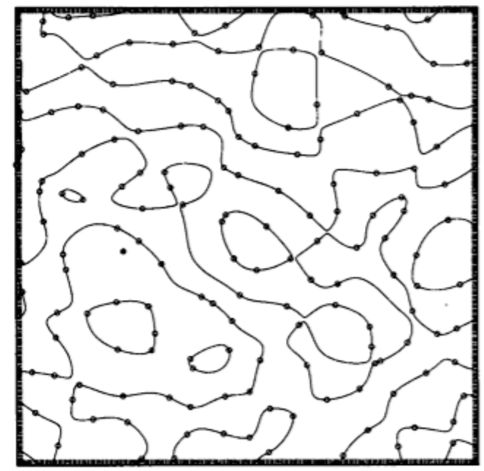
Free Fermion determinant:

$$\Psi_{FF}(\{\mathbf{r}_i\}) = det[e^{i\mathbf{k}_i\cdot\mathbf{r}_j}]$$

Real space ***"nodal structure"***
Define a "single particle function"

$$\Phi_{\mathbf{r}_2,\ldots,\mathbf{r}_N}(\mathbf{r}) \equiv \Psi(\mathbf{r},\mathbf{r}_2,\ldots,\mathbf{r}_N)$$





Nodal lines

Wavefunction for interacting Fermi liquid?

Retain sign (nodal) structure of free fermions, modify amplitude, keep particles apart with Jastrow factor

$$\Psi_{FL} = e^{-\sum_{i<j} u(\mathbf{r}_i-\mathbf{r}_j)}\psi_{FF}$$

with u(r) a variational function

Parton approach to NFL wavefunctions

Decompose electron:
spinless charge e boson,
s=1/2 neutral fermionic spinon

$$c_\sigma = b f_\sigma$$

Mean Field Theory

Treat "Spinons" and Bosons as Independent: $\mathcal{H} = \mathcal{H}_f + \mathcal{H}_b$

Wavefunctions $\psi_f(\mathbf{r_{i\uparrow}}, \mathbf{r_{i\downarrow}})$ $\psi_b(\mathbf{R_i})$

(enlarged Hilbert space - twice as many particles)

"Fix-up" Mean Field Theory

"Glue" together Fermion and Boson "partons"

$$\Psi \equiv \psi_f(\mathbf{r_{i\alpha}}) \times \psi_\mathbf{b}(\mathbf{R_i} \rightarrow \mathbf{r_{i\alpha}})$$

Project back into physical Hilbert space

Fermi and Non-Fermi Liquids via partons?

Spinons in a filled Fermi sea $$\psi_f = \det[e^{i\mathbf{k_i}\cdot\mathbf{r_{j\uparrow}})}]\det[e^{i\mathbf{k_i}\cdot\mathbf{r_{j\downarrow}})}]$$

Fermi Liquid: Bosons into Bose condensate

$$\Psi_{FL} = \psi_f \times \psi_b^{BEC}$$

$$\psi_b^{BEC} = e^{-\sum_{i<j} u(\mathbf{R_i}-\mathbf{R_j})}$$

$$c_\sigma = \langle b \rangle f_\sigma \sim (const) f_\sigma$$

Non-Fermi Liquid: Bosons into *uncondensed* fluid - a "Bose metal"

$$\Psi_{NFL} = \psi_f \times \psi_b^{BoseMetal}$$

NFL Metal: Product of Fermi sea and uncondensed "Bose-Metal"

2D Bose-Metal

- A ***stable T=0 liquid phase*** of bosons that is not a superfluid

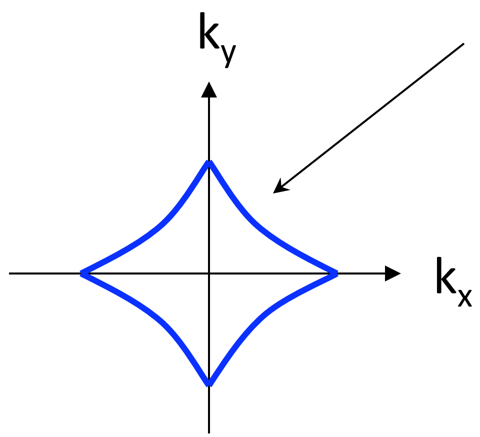
- Green's function has oscillatory power law decay

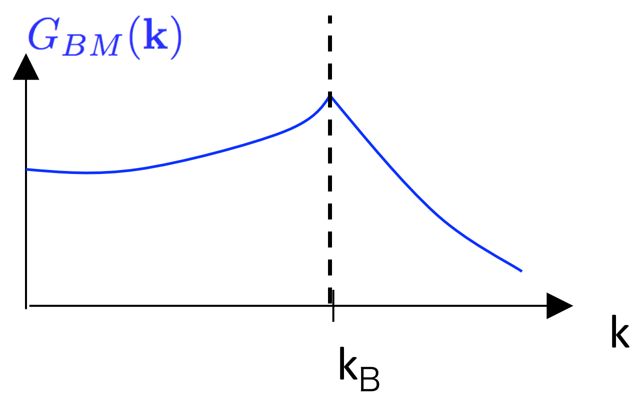
$$G_{BM}(\mathbf{r}) \sim \frac{\cos[\mathbf{k}_B(\hat{\mathbf{r}}) \cdot \mathbf{r}]}{|\mathbf{r}|^{\alpha(\hat{\mathbf{r}})}}$$

- Momentum distribution function singular on a ***"Bose surface"***

$$G_b(\mathbf{k}) = \langle b_{\mathbf{k}}^{\dagger} b_{\mathbf{k}} \rangle$$

$\mathbf{k}_B(\hat{\mathbf{r}})$





Angular dependent anomalous dimension $\alpha(\hat{\mathbf{r}})$

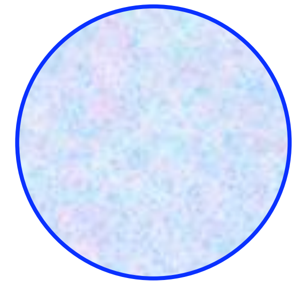
Aside: Partons in QHE and the "Composite Fermi liquid"

CFL wf for half-filled LL $\Psi_{CFL} = \prod_{i<j} (z_i - z_j)^2 \det[e^{i\vec{k}_i \cdot \vec{r}_j}]$

$c = bf$

Laughlin state, nu=1/2
Bosons, times Ferm sea

$$\Psi_{CFL} = \Phi_{\nu=1/2} \times \Psi_{Fermi-sea}$$



But CFL breaks T-reversal invariance

One particle around another - $\Phi_{\nu=1/2}(z) \sim (z - z_i)^2$

$d_{x^2-y^2} + i d_{xy}$ two-particle correlations

One Strategy to construct T-invariant NFL

(a) Construct boson wavefcn with d_{xy} 2-particle correlations $\Phi^{Bose}_{d_{xy}}$

(b) Multiply Boson wavefunction by filled Fermi sea

$$\Psi^{Metal}_{d_{xy}} = \Phi^{Bose}_{d_{xy}} \times \det[e^{i\vec{k}_i \cdot \vec{r}_{j\uparrow}}] \cdot \det[e^{i\vec{k}_i \cdot \vec{r}_{j\downarrow}}]$$

A "d-wave Metal":

Hint: Laughlin state (at nu=1/2) via partons

Decompose boson into product of 2 fermions

$$b = d_1 d_2 \qquad \psi_b = det_1 \times det_2$$

Put d_1 and d_2 into LLL determinants

$$\Psi_{\nu=1} = \det_{LLL} = \prod_{i<j}(z_i - z_j)$$

Gutzwiller wavefunction

$$\Psi_b = \det_1 \det_2 = (\det_{LLL})^2 = \prod_{i<j}(z_i - z_j)^2$$

Recover nu=1/2 Laughlin state for bosons!

CFL via all fermionic decomposition of the electron

$$c = bf = d_1 d_2 f$$

Product of three determinants

$$\Psi_{CFL} = \det_1 \det_2 \det_f = (\det_{LLL})^2 \det(e^{i\mathbf{k}_i \cdot \mathbf{r}_j})$$

$$\Psi_{CFL} = \prod_{i<j}(z_i - z_j)^2 \det[e^{i\vec{k}_i \cdot \vec{r}_j}]$$

Wavefunction for D-wave Bose Metal

Follow Laughlin (partons) $b = d_1 d_2$ $\Psi_{\nu=1/2} = [\Psi_{\nu=1}]^2$

$\Phi_{\nu=1}(z) \sim (z - z_i)$ p+ip 2-body

Try squaring a Fermi sea determinant

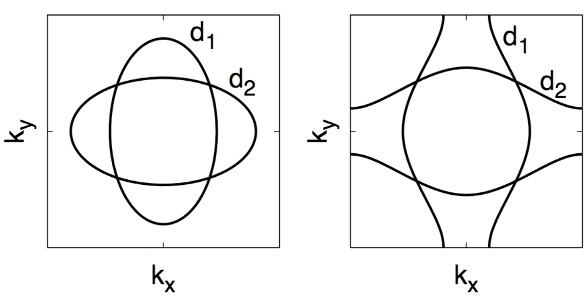
$$\Psi(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_N) = (\det e^{i\mathbf{k}_i \cdot \mathbf{r}_j})^2 \ , \quad \text{(S-type)}.$$

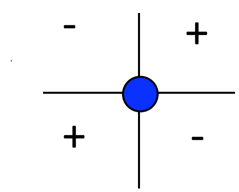
No! S-wave 2-particle correlations: wf has ODLRO

D-wave Bose-Metal

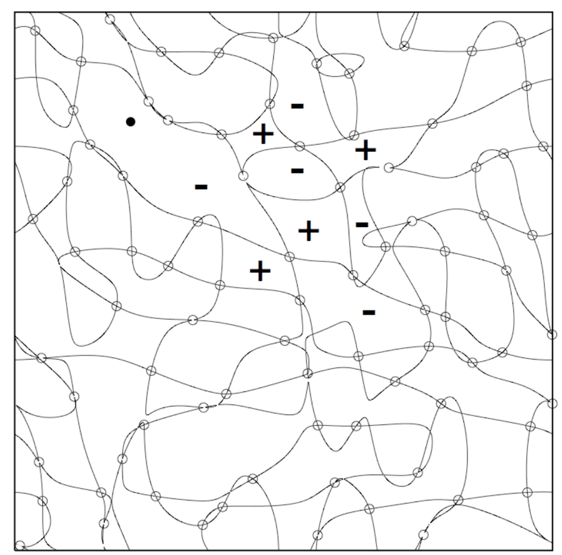
Product of 2 Fermi sea determinants, elongated in the x or y directions

$$\psi_{DBM} = det_x \times det_y$$





D_{xy} relative
2-particle correlations



Hamiltonians for 2d Bose-Metal?

Continuum Hamiltonians

(A) (Non-relativistic) Interacting Bosons at finite density $b_x \longrightarrow \psi(x)$

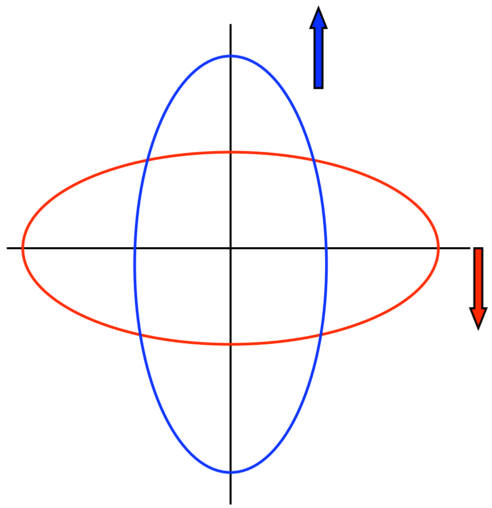
$$\mathcal{L} = \psi^*(\partial_\tau - \mu - \nabla^2/2m) + u|\psi|^4 + \ldots$$

Ground state wavefunction will be node-less, either a superfluid or a crystal

(B) Replace partons with fundamental Fermions:

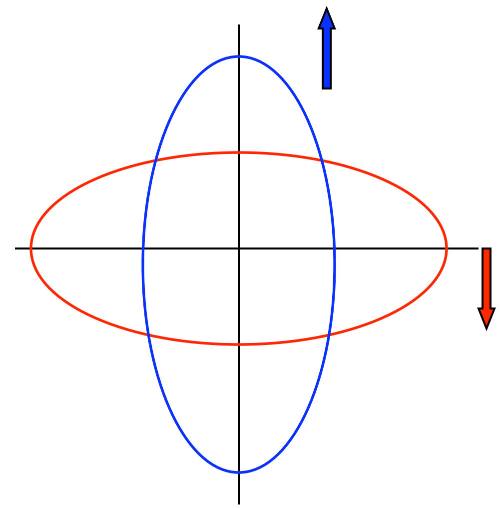
$$b = d_1 d_2 \longrightarrow b = c_\uparrow c_\downarrow$$

$$\mathcal{H} = c_\alpha^\dagger(\partial_\tau - \mu - \partial_x^2/2m_{x\alpha}) - U c_\uparrow^\dagger c_\uparrow c_\downarrow^\dagger c_\downarrow$$



Anisotropic Fermions with attractive interaction

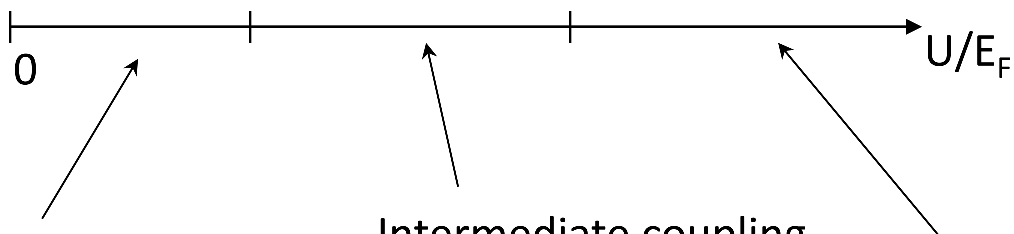
$$\mathcal{H} = c^\dagger_\alpha(\partial_\tau - \mu - \partial_x^2/2m_{x\alpha}) - U c^\dagger_\uparrow c_\uparrow c^\dagger_\downarrow c_\downarrow$$



Generates Frustration for Cooper pairing

Phase diagram??

?????



Metal
(no weak coupling
BCS instability)

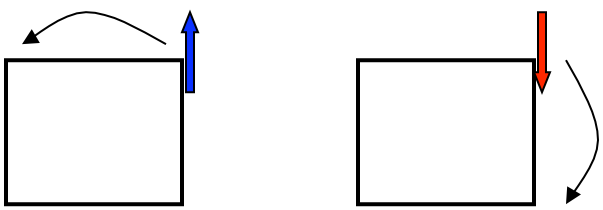
Intermediate coupling
phase??
Cooper-Pair Bose-Metal ??

Superconductor (paired superfluid)

Lattice Hamiltonian for Bose Metal?

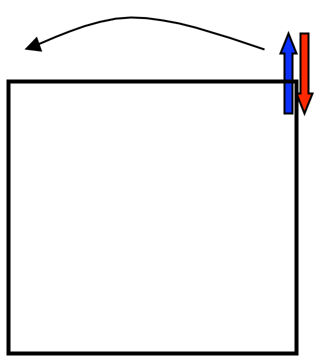
Attractive U Hubbard model for Fermions with ***anisotropic hopping***

$$\mathcal{H} = -\sum_{ij}(t^{\alpha}_{ij} c^{\dagger}_{i\alpha} c_{j\alpha} + h.c.) - U \sum_i n_{i\uparrow} n_{i\downarrow}$$



U>>t generate Boson (Cooper-pair) hopping model

$$b^{\dagger}_i = c^{\dagger}_{i\uparrow} c^{\dagger}_{i\downarrow}$$



Lattice Hamiltonian for Bose Metal?

Hint: Attractive U Hubbard model for Fermions on a ***lattice*** with ***anisotropic hopping***

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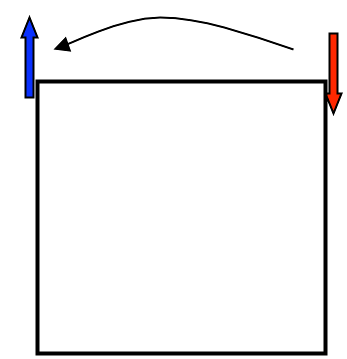
$$b_i^{\dagger} = c^{\dagger}_{i\uparrow} c^{\dagger}_{i\downarrow}$$



Lattice Hamiltonian for Bose Metal?

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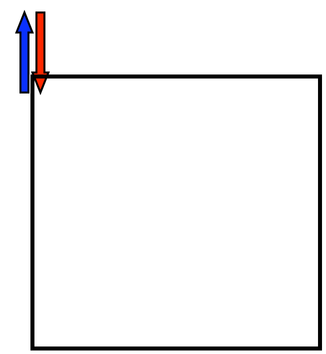

Lattice Hamiltonian for Bose Metal?

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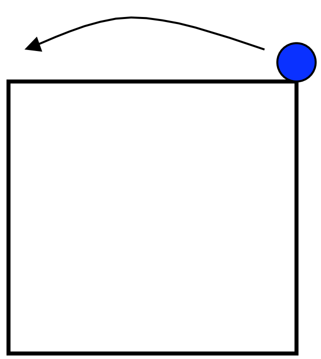
U>>t generate Boson (Cooper-pair) hopping model

$$b^{\dagger}_i = c^{\dagger}_{i\uparrow} c^{\dagger}_{i\downarrow}$$



Lattice Hamiltonian for Bose Metal?

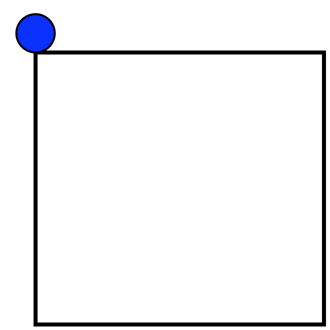
Hard core Bosons hopping on a lattice:

$$\mathcal{H}_J = -J \sum_{ij} (b_i^\dagger b_j + h.c.) \qquad b_i^\dagger = c_{i\uparrow}^\dagger c_{i\downarrow}^\dagger$$

Expect a superfluid phase

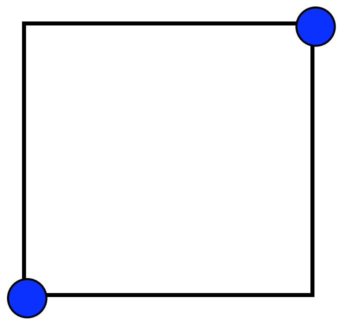
But hopping strength is "small"
due to anisotropy of Fermion hopping

$$J \sim \epsilon t^2 / U$$

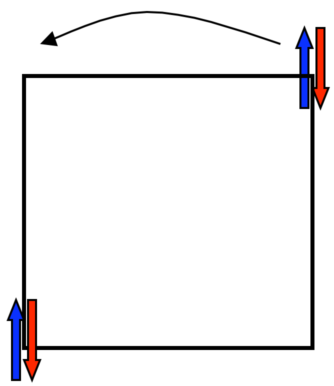


Consider higher order process which
takes advantage of anisotropic hopping:
Ring exchange interaction

Generate Boson ring exchange



Generate Boson ring exchange



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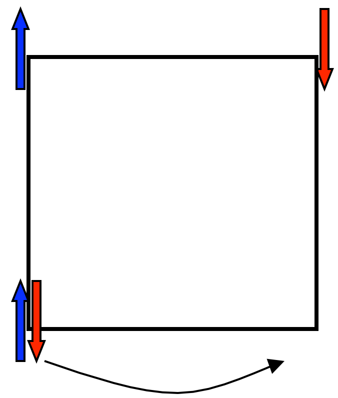

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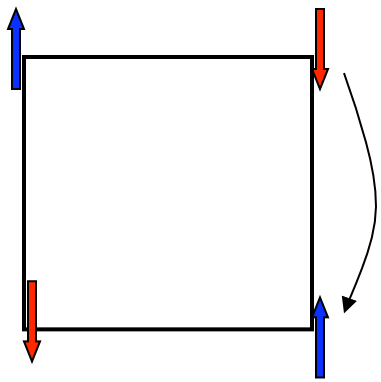

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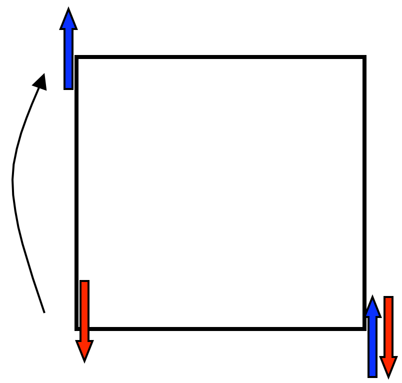

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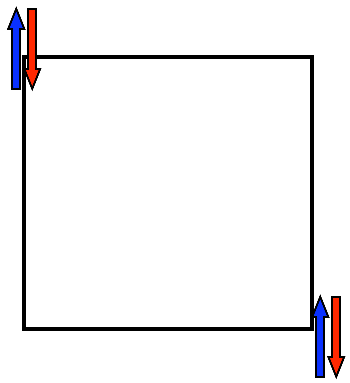

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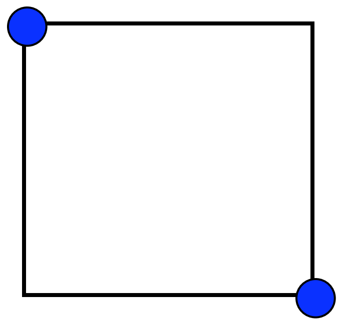
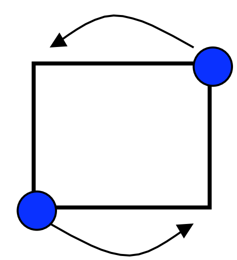


Boson Ring Exchange Model

$$\mathcal{H}_{JK} = -J\sum_{ij}(b_i^\dagger b_j + h.c.) + K \sum_{plackets} (b_1^\dagger b_2 b_3^\dagger b_4 + h.c.)$$

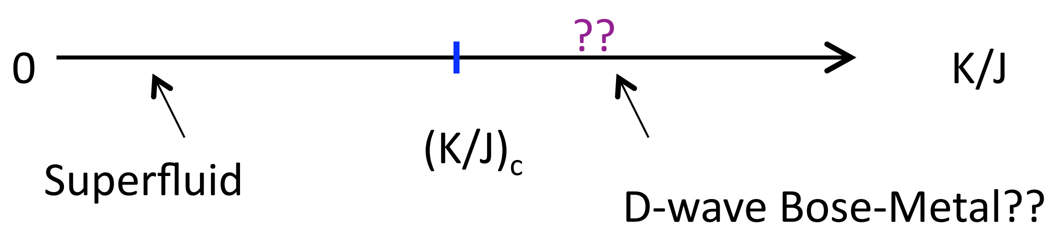
$$J \sim \epsilon t^2/U; \qquad K \sim t^4/U^3$$

K is positive and can be "large"
Frustrates the condensation of bosons



"Ring exchange"

Phase diagram: K/J and density of bosons

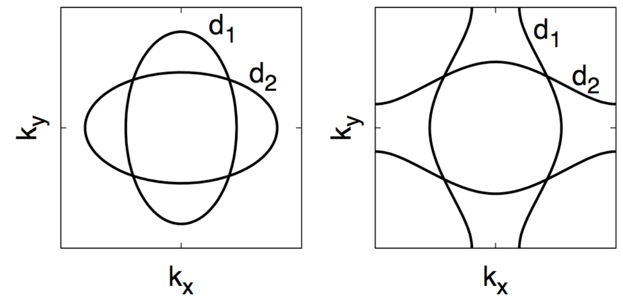


Boson Ring Exchange Hamiltonian has a ***sign problem*** (like Fermions), and cannot be attacked with Quantum Monte Carlo.   What to do??

Bose Surfaces in D-wave Bose-Metal

$$b = d_1 d_2$$

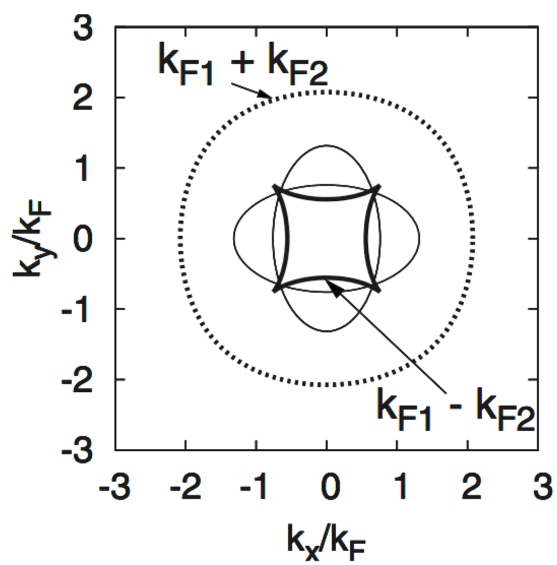
Mean Field Green's functions factorize:



$$G_b^{MF}(\mathbf{r},\tau) = G_{d_1}^{MF}(\mathbf{r},\tau)G_{d_2}^{MF}(\mathbf{r},\tau)/\bar{\rho}$$

Momentum distribution function:

$$n_b(\mathbf{k}) = \int G_b(\mathbf{r})e^{i\mathbf{k}\cdot\mathbf{r}}d\mathbf{r}$$

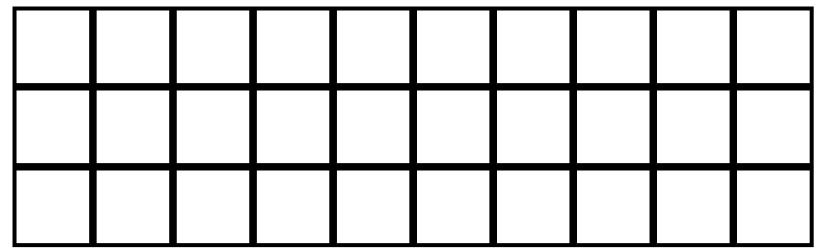


Two singular lines in
momentum space, Bose surfaces:

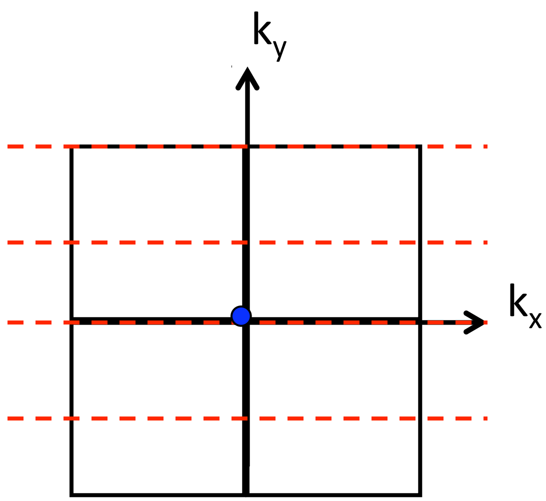
$$\mathbf{k}_{F_1}(\hat{\mathbf{r}}) \pm \mathbf{k}_{F_2}(\hat{\mathbf{r}})$$

Ladders to access Bose surfaces

Transverse y-components of momentum become quantized

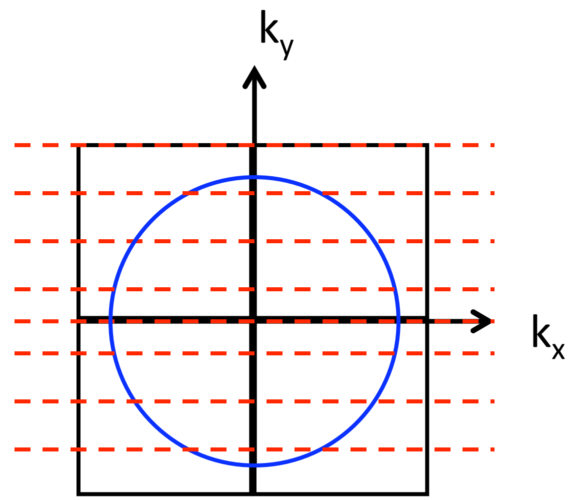


Put Bose superfluid on n-leg ladder



Single gapless 1d mode

Put D-wave Bose metal on n-leg ladder



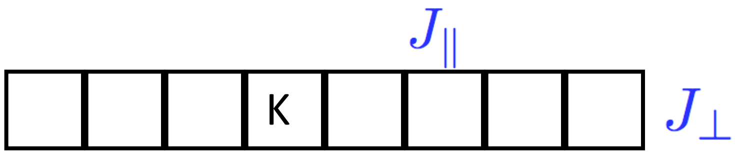
Many gapless 1d modes, one for each "Bose" point

Signature of 2d Bose surface present on ladders

Expectation: Signature of Bose surface in Bose-Metal on n-leg ladders

Boson ring model on the 2 and 4 Leg Ladder

- Exact Diag.
- Variational Monte Carlo
- DMRG
- Bosonization of quasi-1d gauge theory



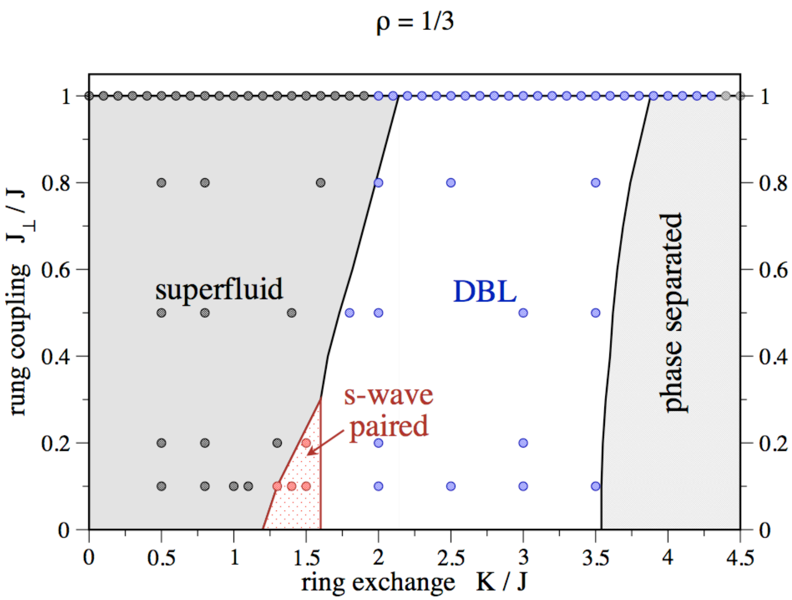
E. Gull et. Al. PRB 78, 54520 (2008)

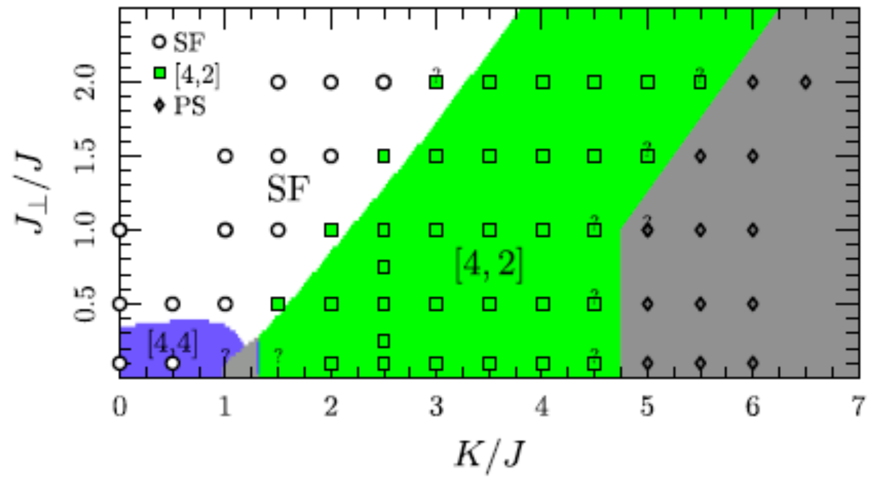
R. Mishmash et al. cond-mat 1110.4607

$$
\begin{aligned}
H &= H_J + H_4 \ , \\
H_J &= -J \sum_{\mathbf{r};\, \hat{\mu}=\hat{\mathbf{x}},\hat{\mathbf{y}}} (b_{\mathbf{r}}^{\dagger} b_{\mathbf{r}+\hat{\mu}} + h.c.) \ , \\
H_4 &= K_4 \sum_{\mathbf{r}} (b_{\mathbf{r}}^{\dagger} b_{\mathbf{r}+\hat{\mathbf{x}}} b_{\mathbf{r}+\hat{\mathbf{x}}+\hat{\mathbf{y}}}^{\dagger} b_{\mathbf{r}+\hat{\mathbf{y}}} + h.c.) \ ,
\end{aligned}
$$

Ladder descendant of 2D Bose-metal??

Phase Diagrams for 2 and 4-leg ladder



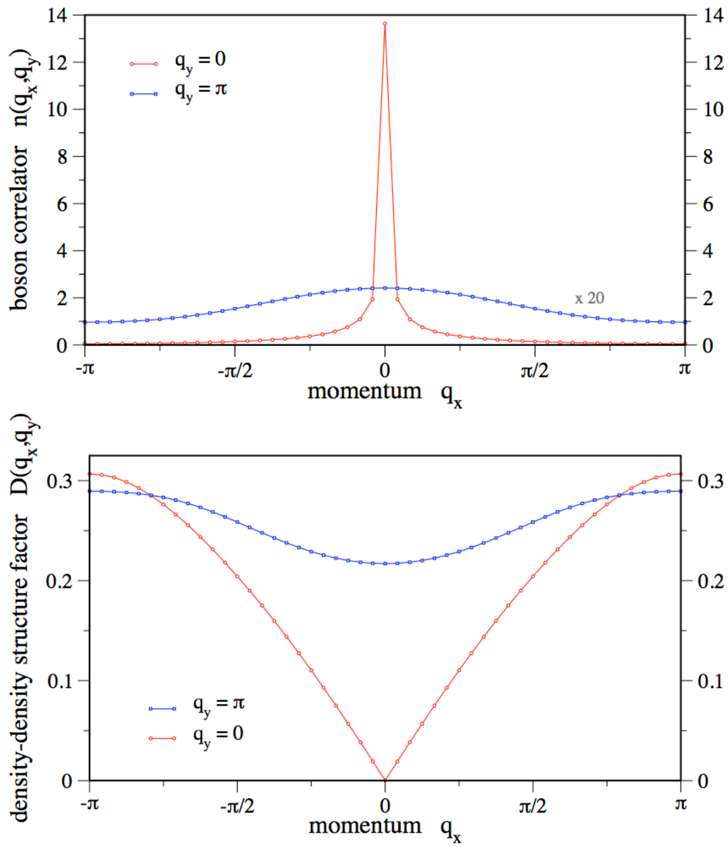


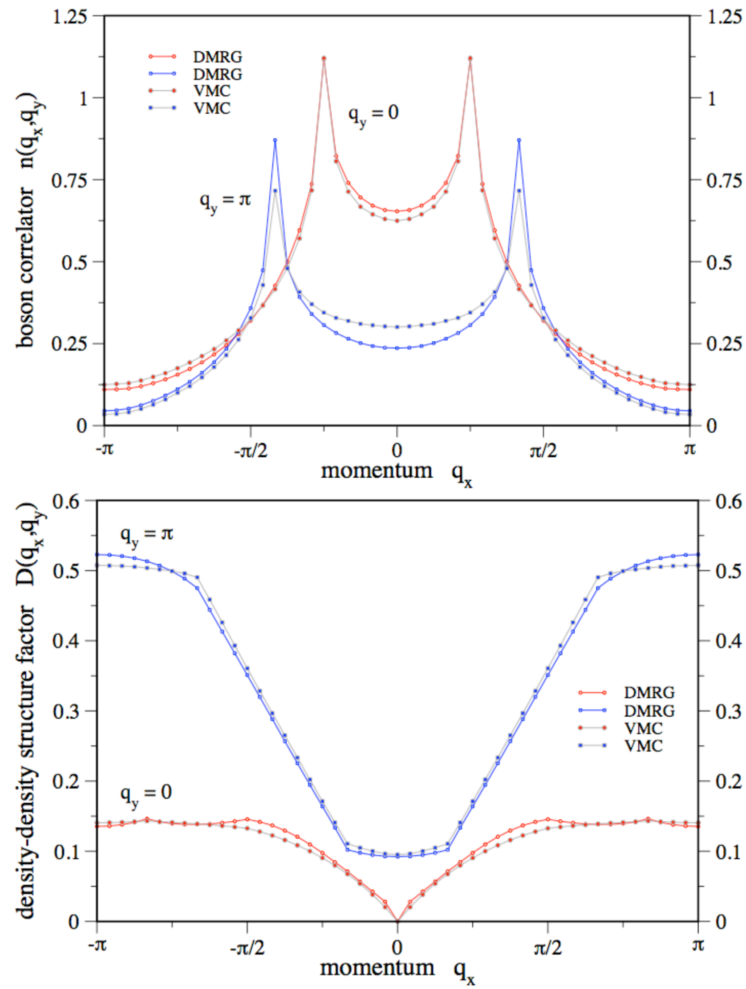
Phases:

1) Superfluid – “Bose condensate”
2) D-Wave Bose Metal - DBL
3) s-wave Pair-Boson “condensate”

D-wave Bose-Metal occupies large region of phase diagrams

Superfluid versus D-wave Bose-Metal (2-leg ladder)

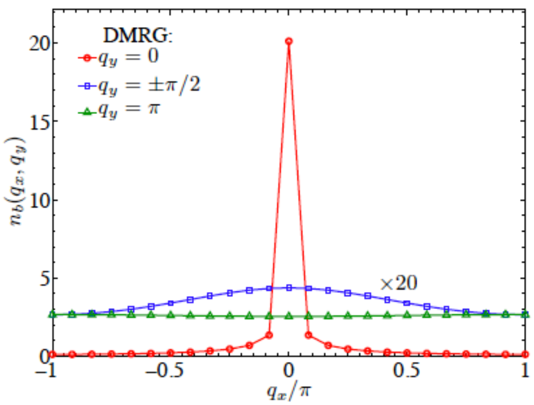


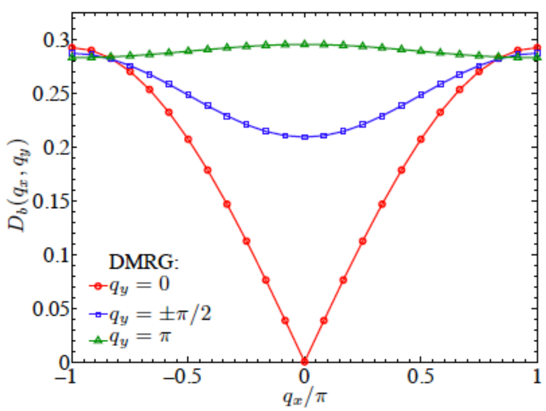


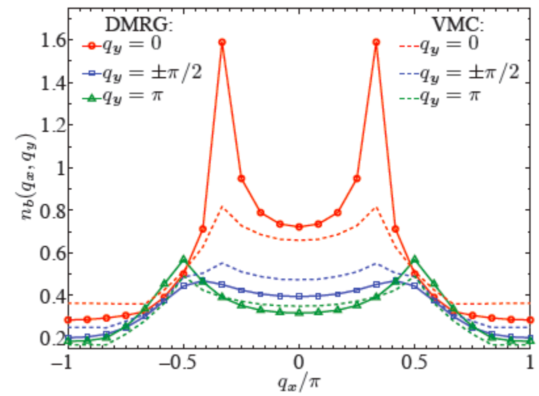
Superfluid - “condensed” at zero momentum

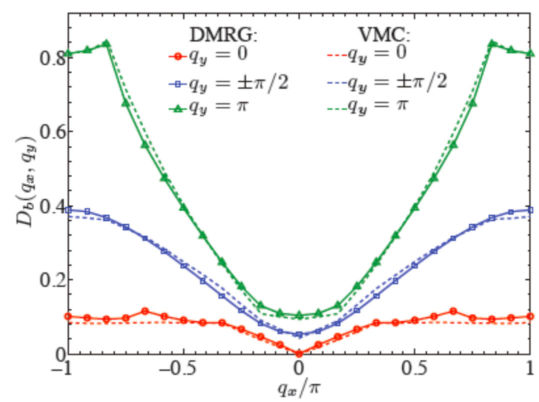
D-wave Bose-Metal; Singular “Bose points” at $q_y = 0, \pi$

Superfluid versus Bose-Metal (4-leg ladder)





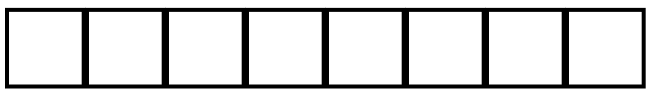


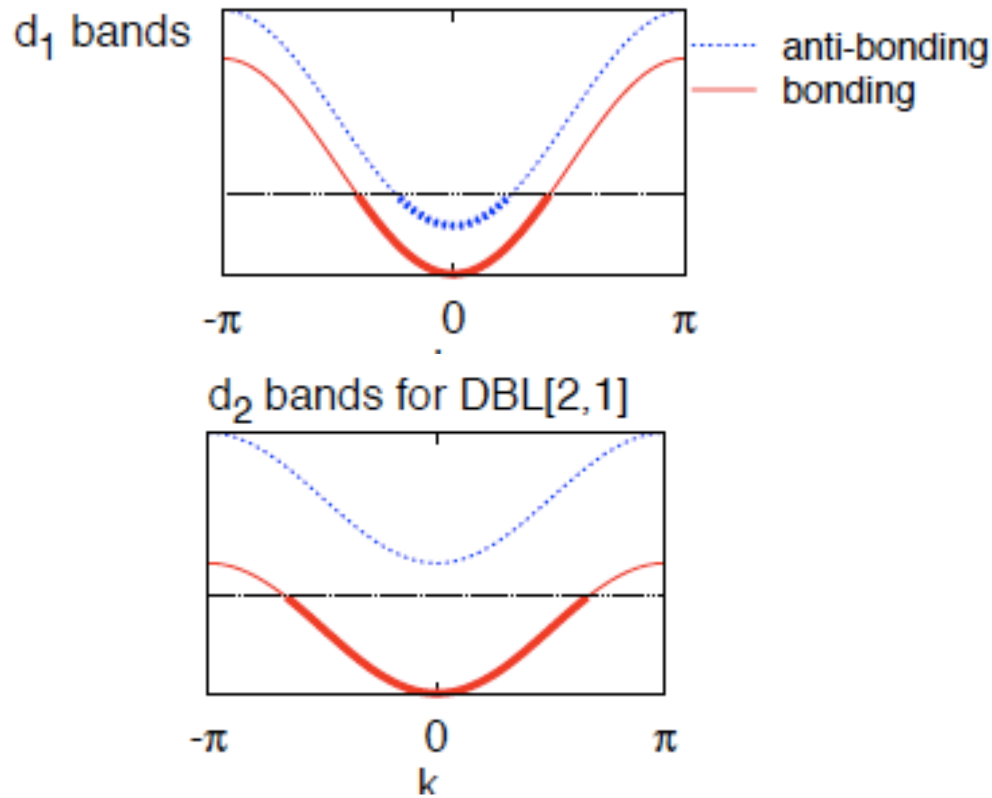


Superfluid: "Condensed at zero momentum

Bose Metal: Singular "Bose-surfaces" at $q_y = 0, \pm\pi/2, \pi$

Variational Wf for D-wave Bose-metal on 2-leg ladder





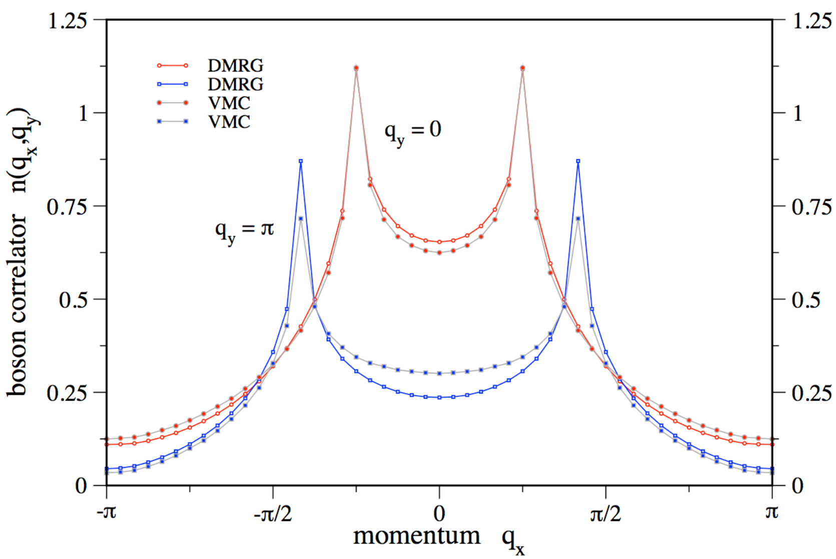
In DBM:

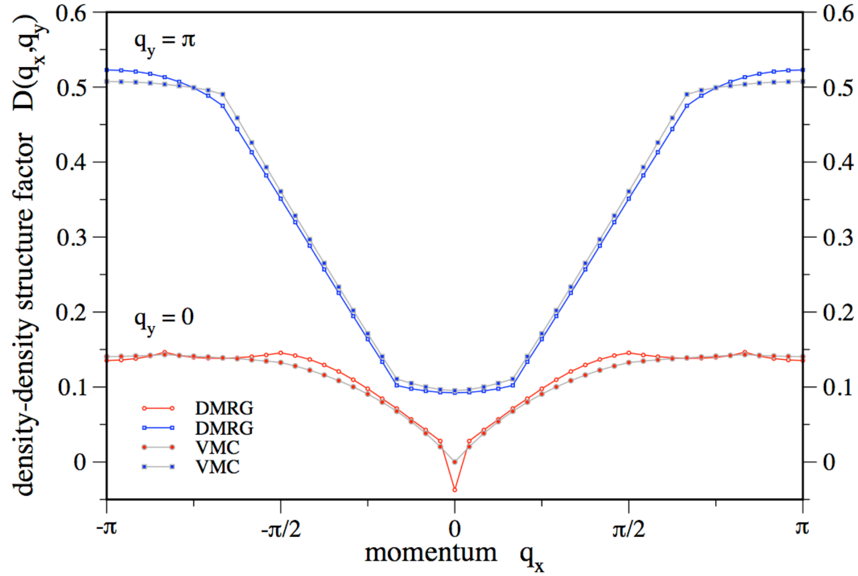
Bonding/Antibonding occupied for d_1 Fermion, just bonding occupied for d_2 Fermion

Variational parameter:
Fermi wavevectors in d_1 bands

$$\Psi_{\mathrm{bos}}(r_1, r_2, \ldots) = \Psi_{d_1}(r_1, r_2, \ldots) \cdot \Psi_{d_2}(r_1, r_2, \ldots).$$

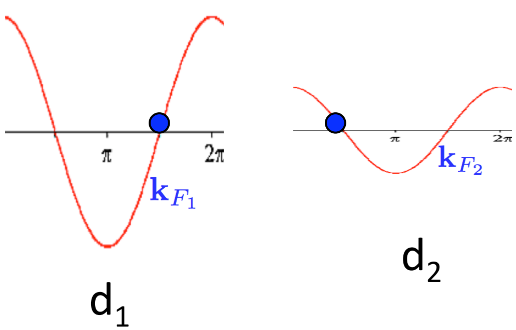
How good is variational wavefunction?





Gauge mean field theory predicts singularities in momentum distribution function at: $\mathbf{k}_{F_1} \pm \mathbf{k}_{F_2}$

Both DMRG and $\det_1$ x $\det_2$ Wavefunction show singular cusps *only* at $\mathbf{k}_{F_1} - \mathbf{k}_{F_2}$ (Ampere's law)



“D-Wave Metal”

NFL phase of 2d electrons?

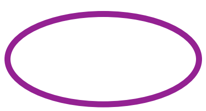
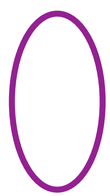
All fermionic Parton construction

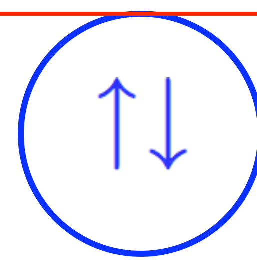
$$c^\dagger_\alpha(\mathbf{r}) = b^\dagger(\mathbf{r}) f^\dagger_\alpha(\mathbf{r}) = d^\dagger_x(\mathbf{r}) d^\dagger_y(\mathbf{r}) f^\dagger_\alpha(\mathbf{r})$$

Wavefunction; Product of determinants

$$\{\vec{R}_i\} = \{\vec{r}_{i\uparrow}, \vec{r}_{j\downarrow}\}$$

$$\Psi^{Metal}_{d_{xy}} = \det_x[e^{i\mathbf{K_i}\cdot\mathbf{R_j}}] \cdot \det_y[e^{i\mathbf{K_i}\cdot\mathbf{R_j}}] \times \det[e^{i\mathbf{k_i}\cdot\mathbf{r_{j\uparrow}}}] \cdot \det[e^{i\mathbf{k_i}\cdot\mathbf{r_{j\downarrow}}}]$$

x  y 



Filled Fermi sea

Can use Variational Monte Carlo to extract equal time correlation functions from wf

But what about energetics???

Hamiltonian for D-wave Metal?

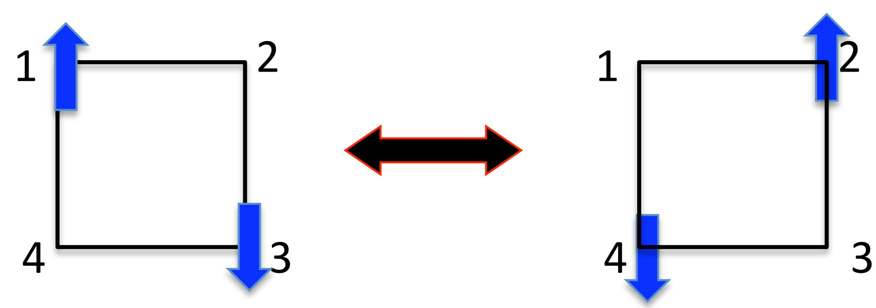
Strong coupling limit of parton gauge theory $c_\alpha = f_\alpha d_x d_y$

t-K "Ring" Hamiltonian (no double occupancy constraint)

$$\mathcal{H}_{tK} = -t \sum_{\langle ij \rangle} [c_{i\alpha}^\dagger c_{j\alpha} + h.c.] + K \sum_{\langle 1234 \rangle} [\mathcal{S}_{13}^\dagger \mathcal{S}_{24} + h.c.]$$

$$\mathcal{S}_{ij}^\dagger = \frac{1}{\sqrt{2}} [c_{i\uparrow}^\dagger c_{j\downarrow}^\dagger - c_{i\downarrow}^\dagger c_{j\uparrow}^\dagger]$$

Electron singlet pair
"rotation" term



Phase diagram of electron t-K Hamiltonian?

$$\mathcal{H}_{tK} = -t\sum_{\langle ij\rangle}[c^{\dagger}_{i\alpha}c_{j\alpha} + h.c.] + K\sum_{\langle 1234\rangle}[\mathcal{S}^{\dagger}_{13}\mathcal{S}_{24} + h.c.]$$

(Density and K/t)

Severe sign problem - intractable

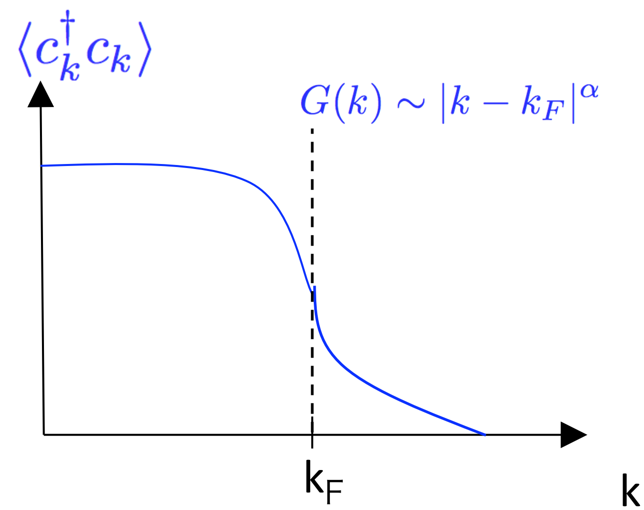
Once again: Analyze t-K electron ring Hamiltonian on ***2-leg ladder***

Possible to identify a NFL on a 2-leg ladder?

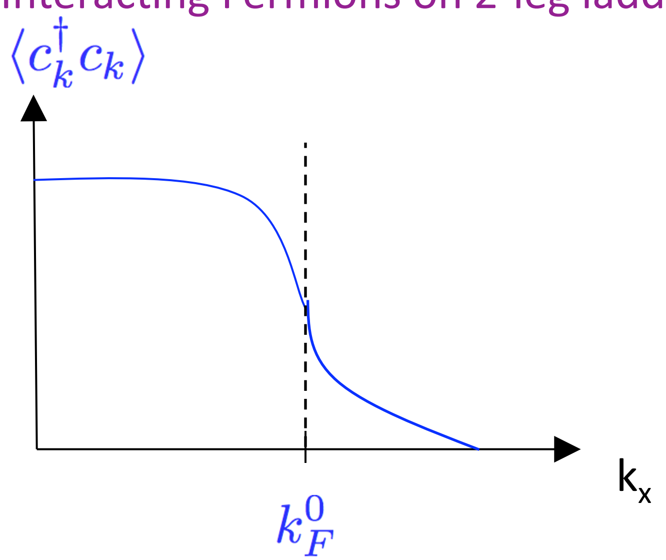
Interacting Fermions in 1d: A Luttinger liquid

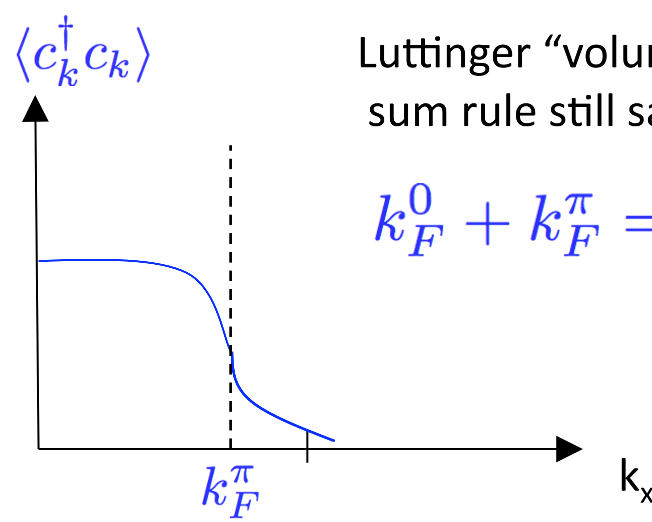
$G(x) \sim \sin(k_F x)/x^{1+\alpha}$ Luttinger liquid exponent: α

Momentum distribution function has (dominant) singularity at $k=k_F$ satisfying Luttinger sum rule



Interacting Fermions on 2-leg ladder: 2-bands





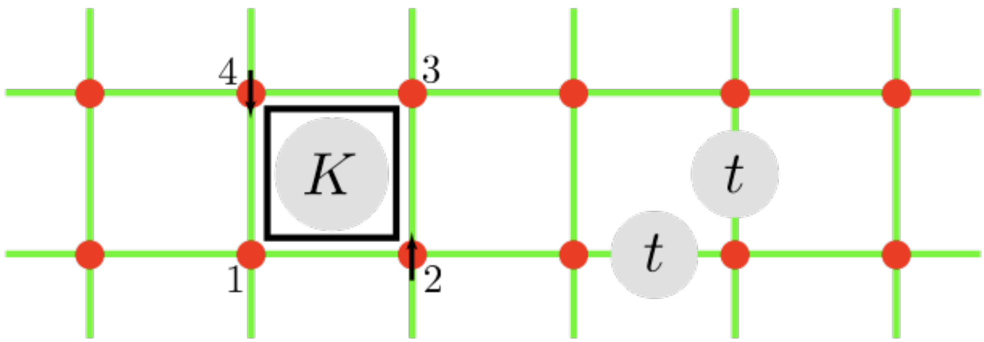
Luttinger "volume" sum rule still satisfied:

$$k_F^0 + k_F^\pi = \pi n_{1d}$$

Searching for a "non-Luttinger liquid" (ie. a Luttinger-liquid violating Luttinger's sum rule)

Electron t-K model on 2-leg ladder

$$\mathcal{H}_{tK} = -t\sum_{\langle ij \rangle}[c^{\dagger}_{i\alpha}c_{j\alpha} + h.c.] + K \sum_{\langle 1234 \rangle}[\mathcal{S}^{\dagger}_{13}\mathcal{S}_{24} + h.c.]$$

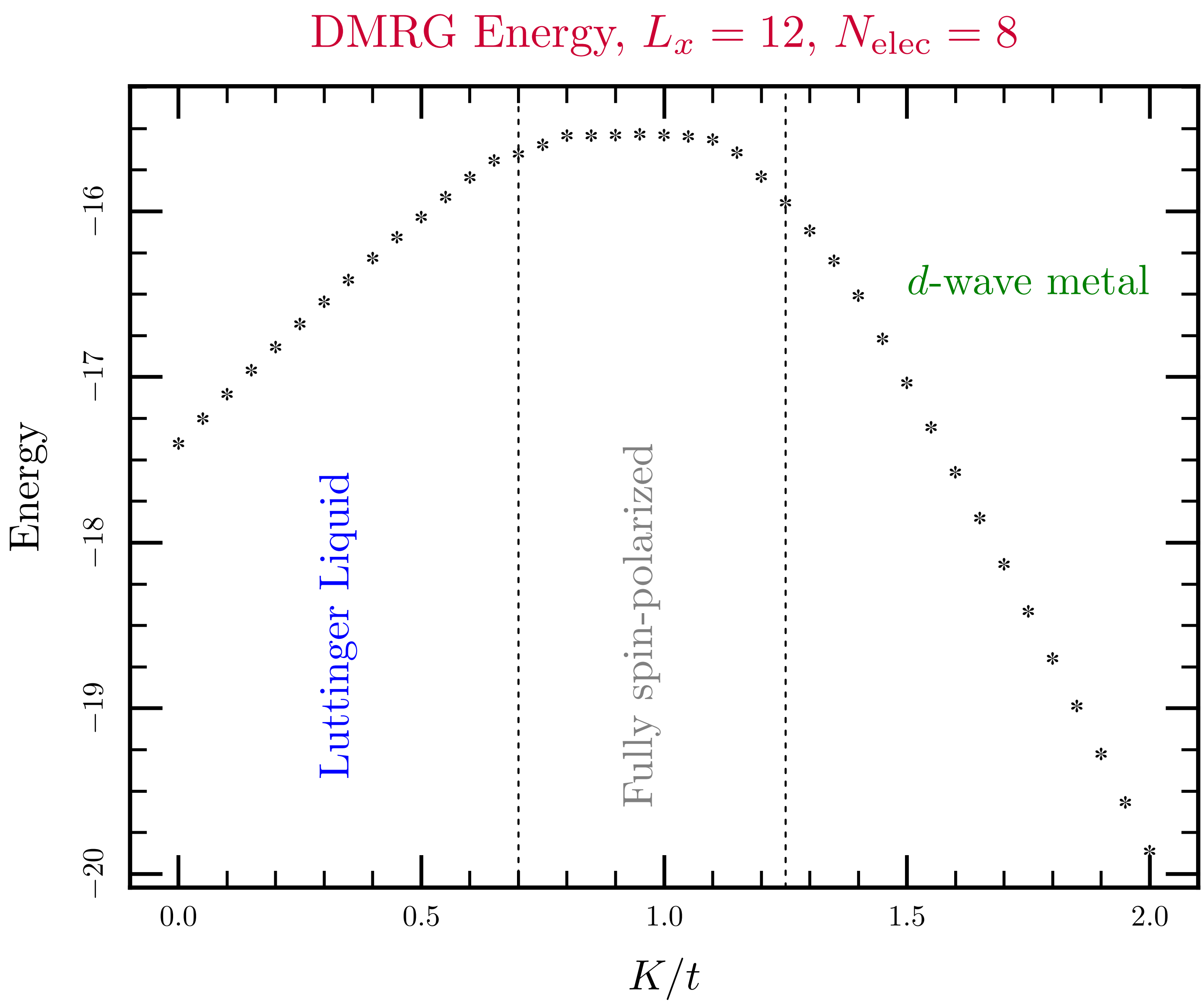


Two dimensionless parameters:
K/t and density n
(**n=1/3 henceforth**)
No double occupancy

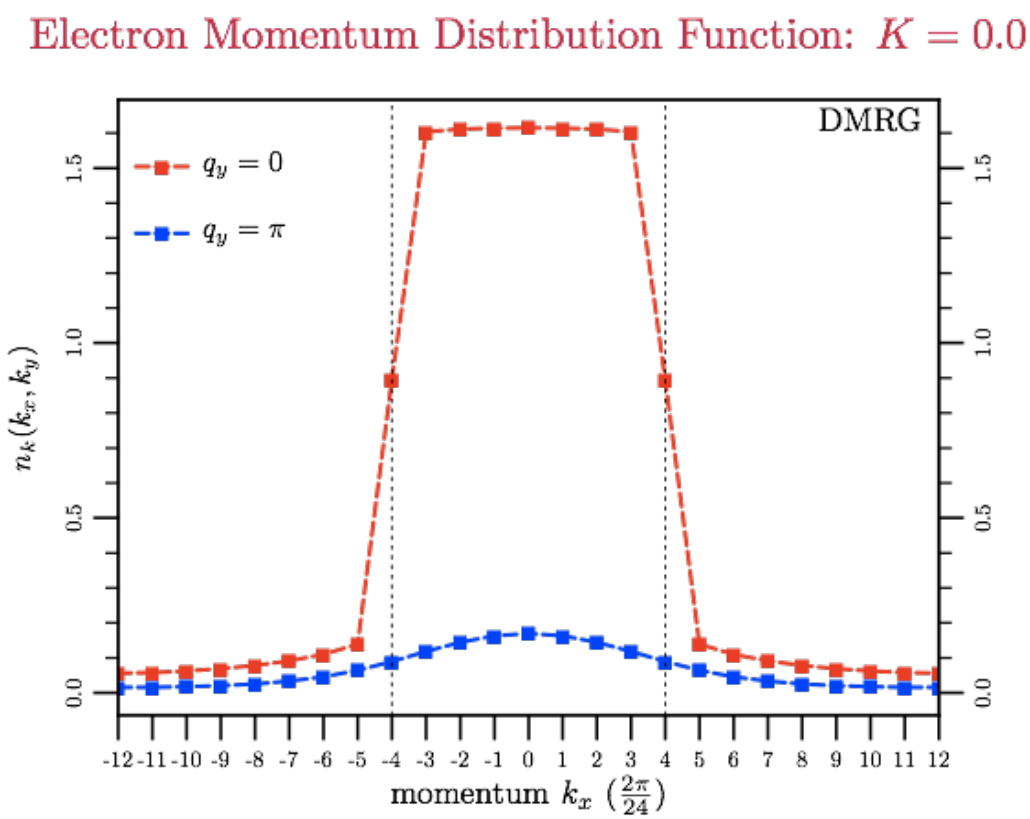
Hongchen Jiang, Matt Block, Ryan Mishmash, Donna Sheng, Lesik Motrunich and MPAF (in progress)

ED
DMRG
VMC
Bosonization of Quasi-1d U(1) gauge theory

Ground State energy: DMRG



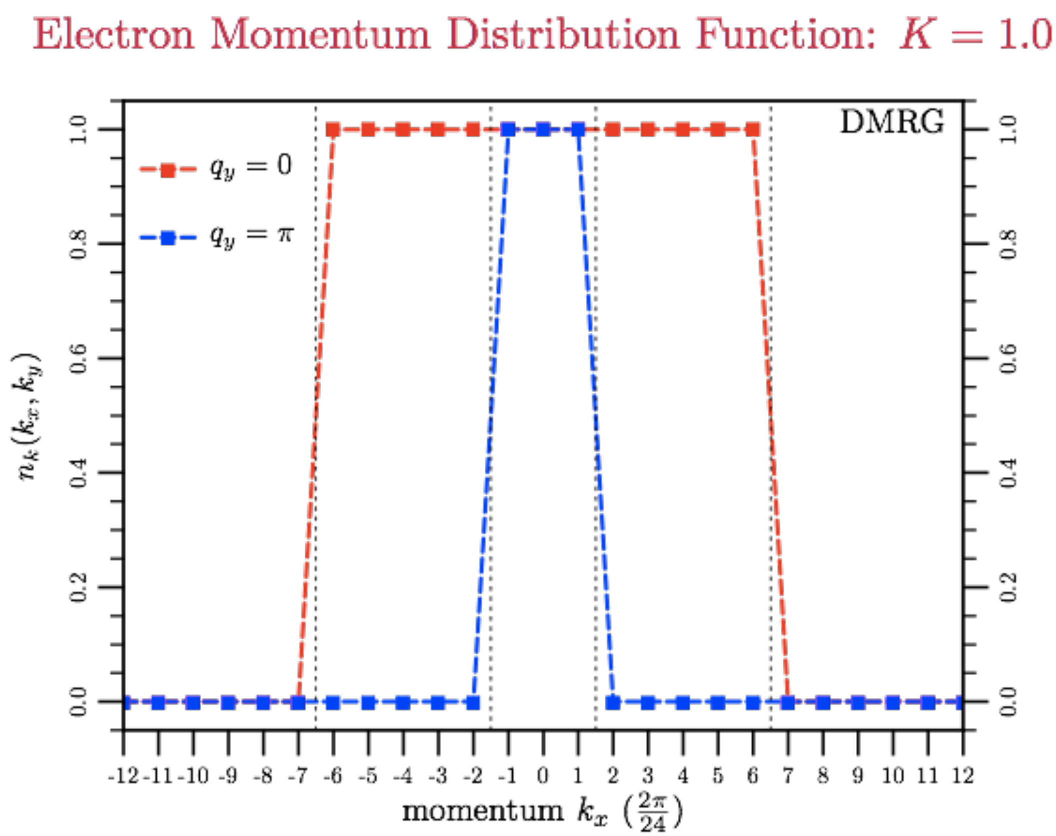
K/t <0.7 Luttinger Liquid



Satisfies Luttinger's Theorem: the volume enclosed by the "Fermi surface" yields the particle density. (16 particles, singlet, 8 up and 8 down)

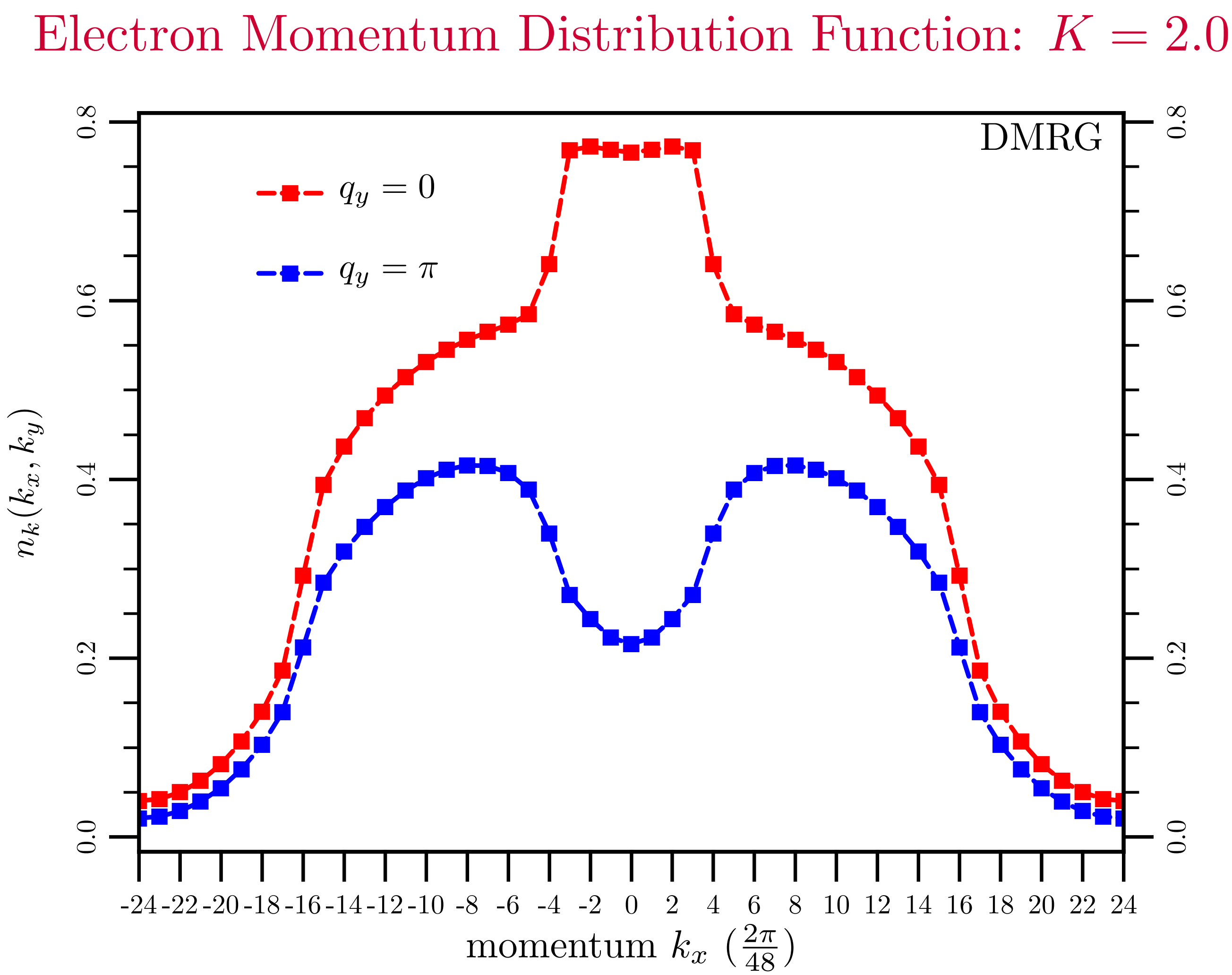
A canonical (single band) Luttinger liquid

0.7< K/t <1.25 : Spin Polarized



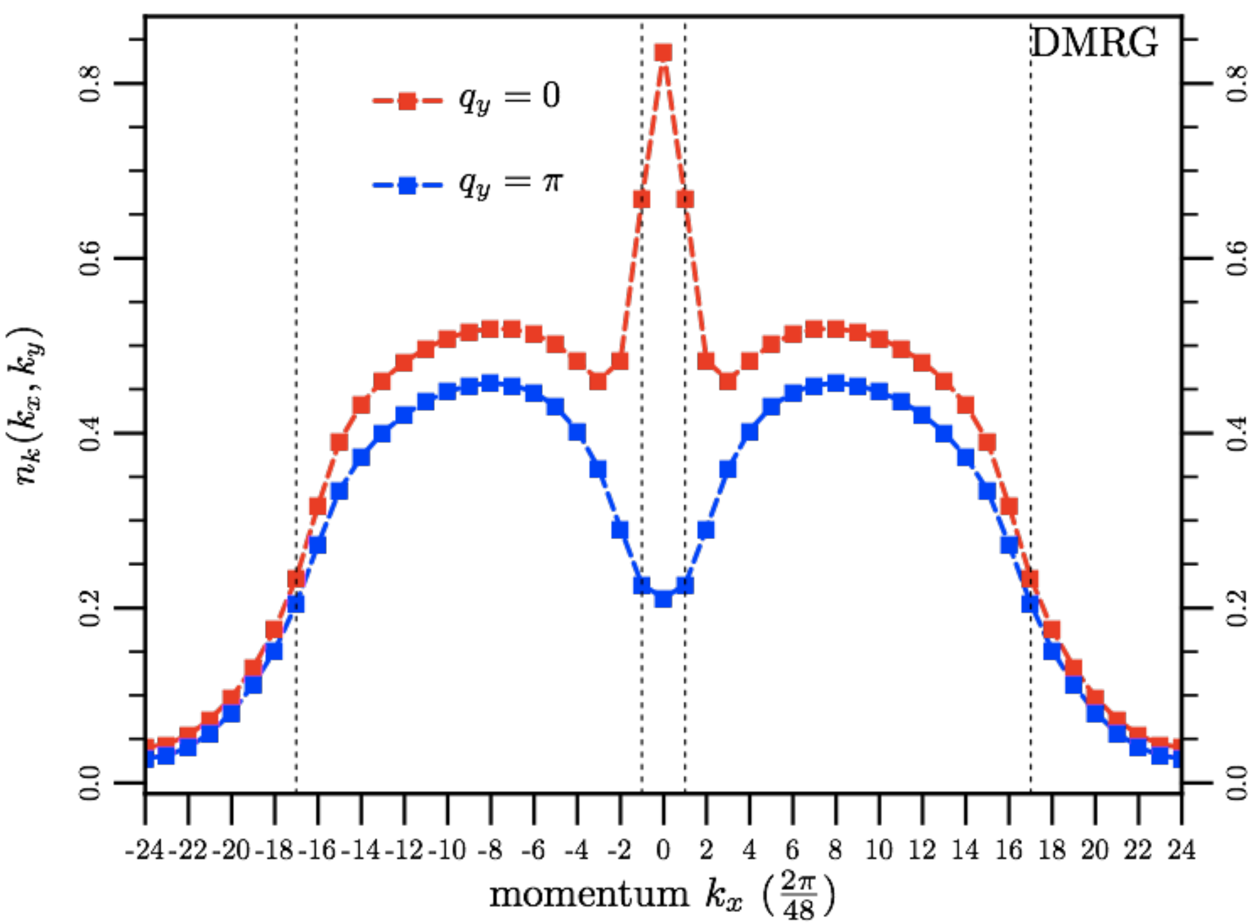
Non-interacting spin polarized Fermi sea is exact ground state here.
Luttinger theorem satisfied

K/t>1.25: Non-LL Phase



K/t > 1.25: Non-LL Phase

Electron Momentum Distribution Function: $K = 2.5$



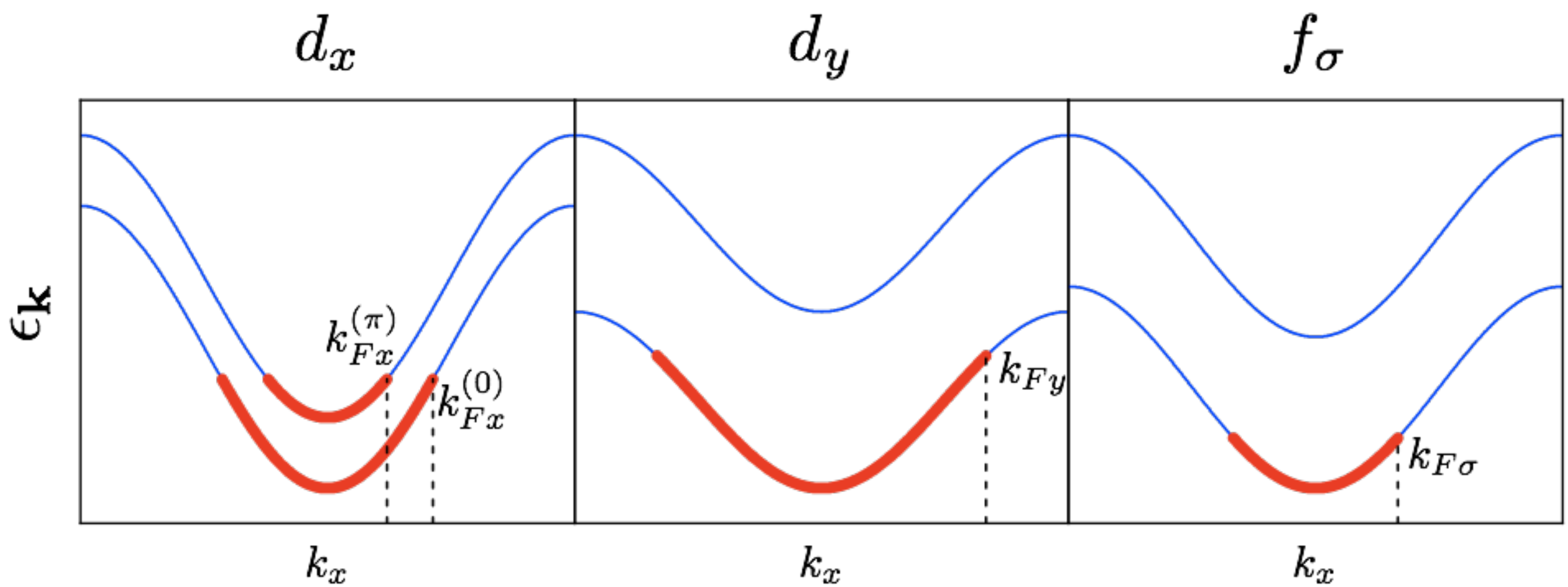
Non-monotonic momentum distribution function; No sign of Luttingers volume

Non-Luttinger-Liquid phase for K>1.25?

Electron momentum distribution function: Singular features, but at momenta which do not satisfy Luttinger's volume theorem

Can we understand in terms of D-wave Metal wavefunction??

Employ parton construction, gauge theory and VMC $c_\alpha = d_x d_y f_\alpha$



Electron momentum distribution function

Mean Field Theory: convolution of partons

$$n_c^{MFT}(k) = n_{d_x}(k) \otimes n_{d_y}(k) \otimes n_f(k) \qquad c_\sigma = d_x d_y f_\sigma$$

Gauge theory - certain wavevectors enhanced

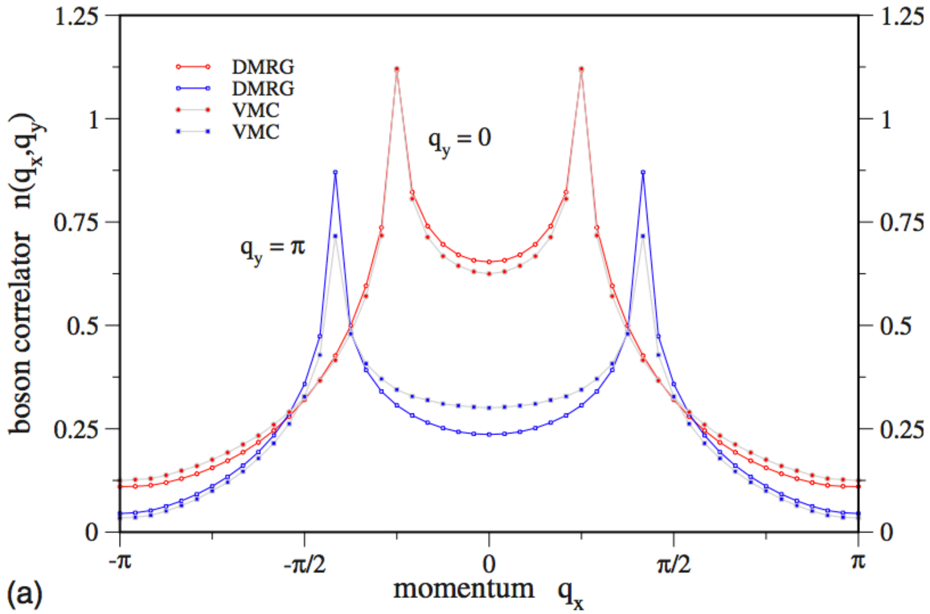
Illustrate with Boson ring model (MFT)

$$n_b^{MFT}(k) = n_{d_x}(k) \otimes n_{d_y}(k)$$

$$b = d_x d_y$$

Very sharp peaks in the **exact** boson momentum distribution function! (from DMRG)

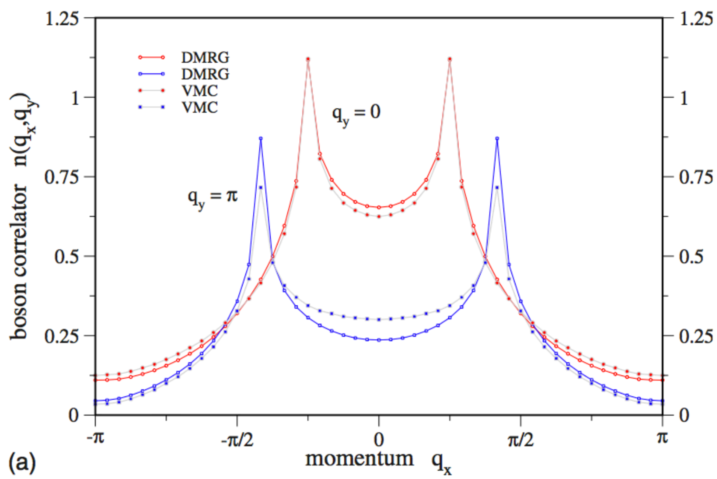
$$n_b(k)$$



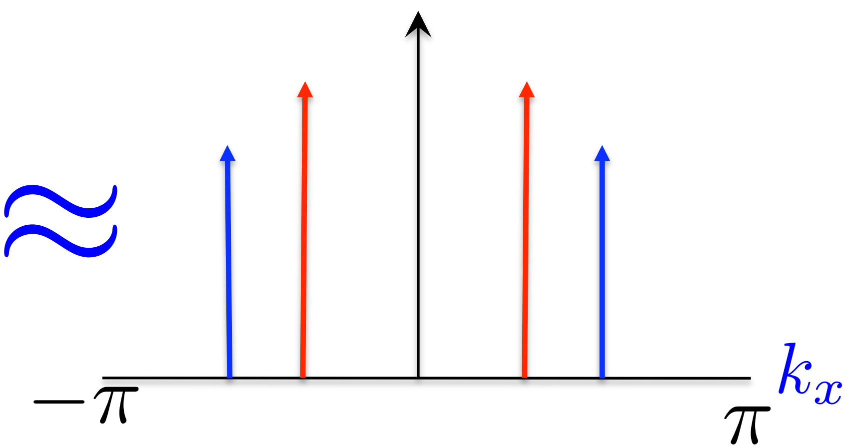
(a)

Momentum distribution function in the d-wave metal?

$$n_b(k) \qquad\qquad \tilde{n}_b(k)$$



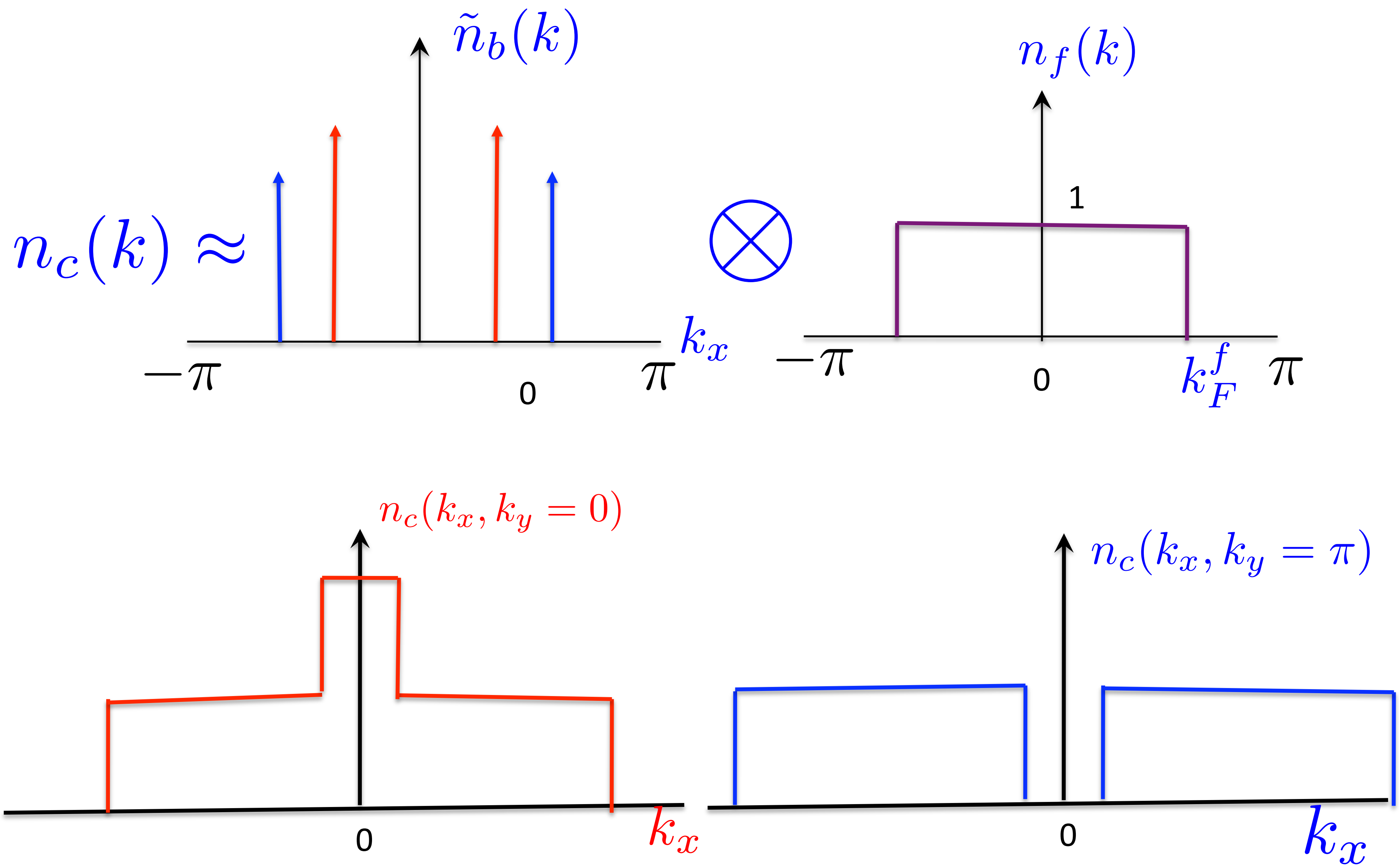
$$\approx$$



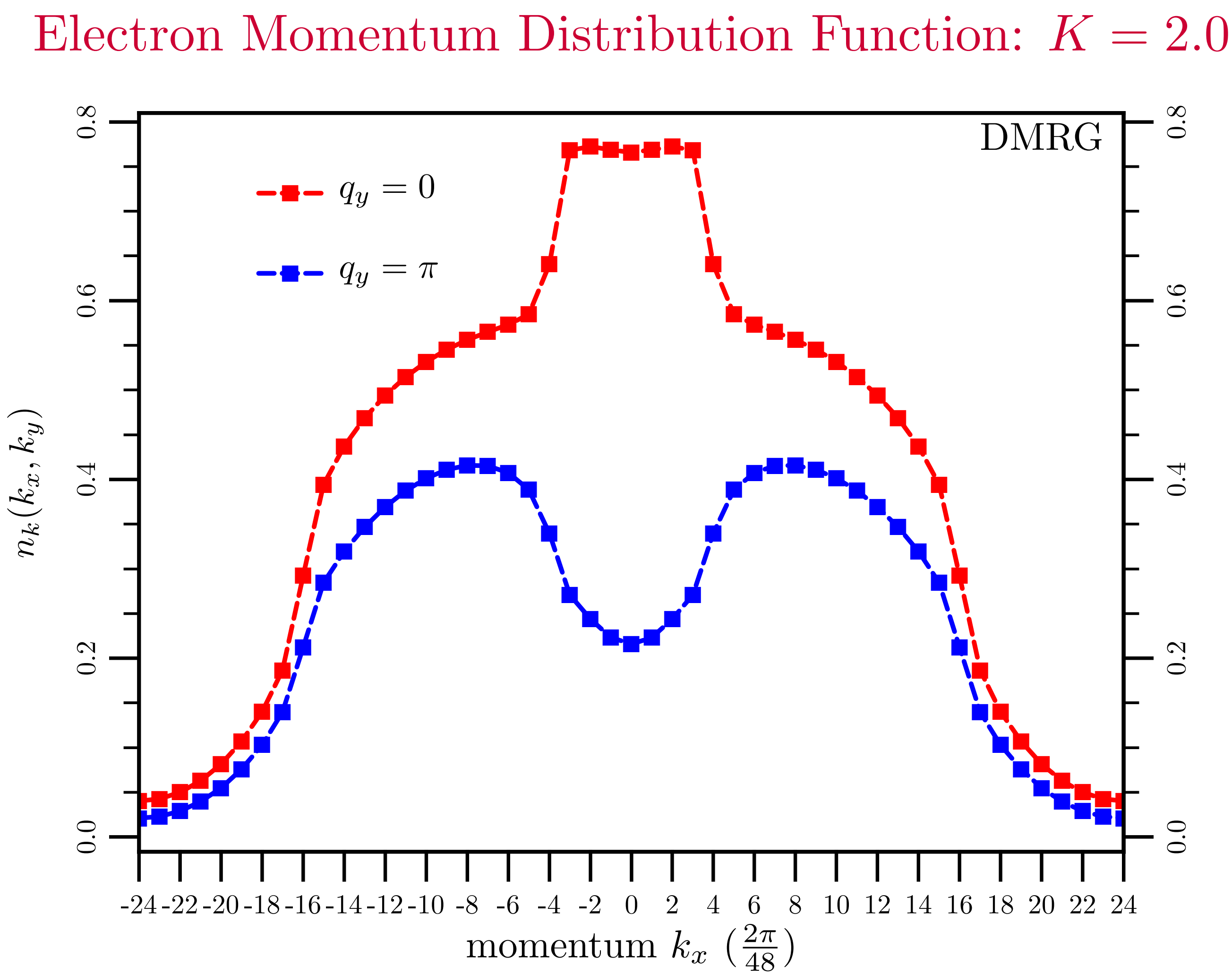
$$n_c(k) \overset{?}{\approx} \tilde{n}_b(k) \otimes n_f(k)$$

$n_f(k) = \Theta(K_F^f - |k|)$ (Free spinon sea)

Convolution: c = b f



K/t>1.25: Non-LL Phase



Variational Monte Carlo (VMC)

D-wave Metal: Product of Slater determinants

$$\Psi_{d_{xy}}^{Metal} = \det_x[e^{i\mathbf{K_i}\cdot\mathbf{R_j}}]\cdot\det_y[e^{i\mathbf{K_i}\cdot\mathbf{R_j}}]\times\det[e^{i\mathbf{k_i}\cdot\mathbf{r_{j\uparrow}}}]\cdot\det[e^{i\mathbf{k_i}\cdot\mathbf{r_{j\downarrow}}}]$$

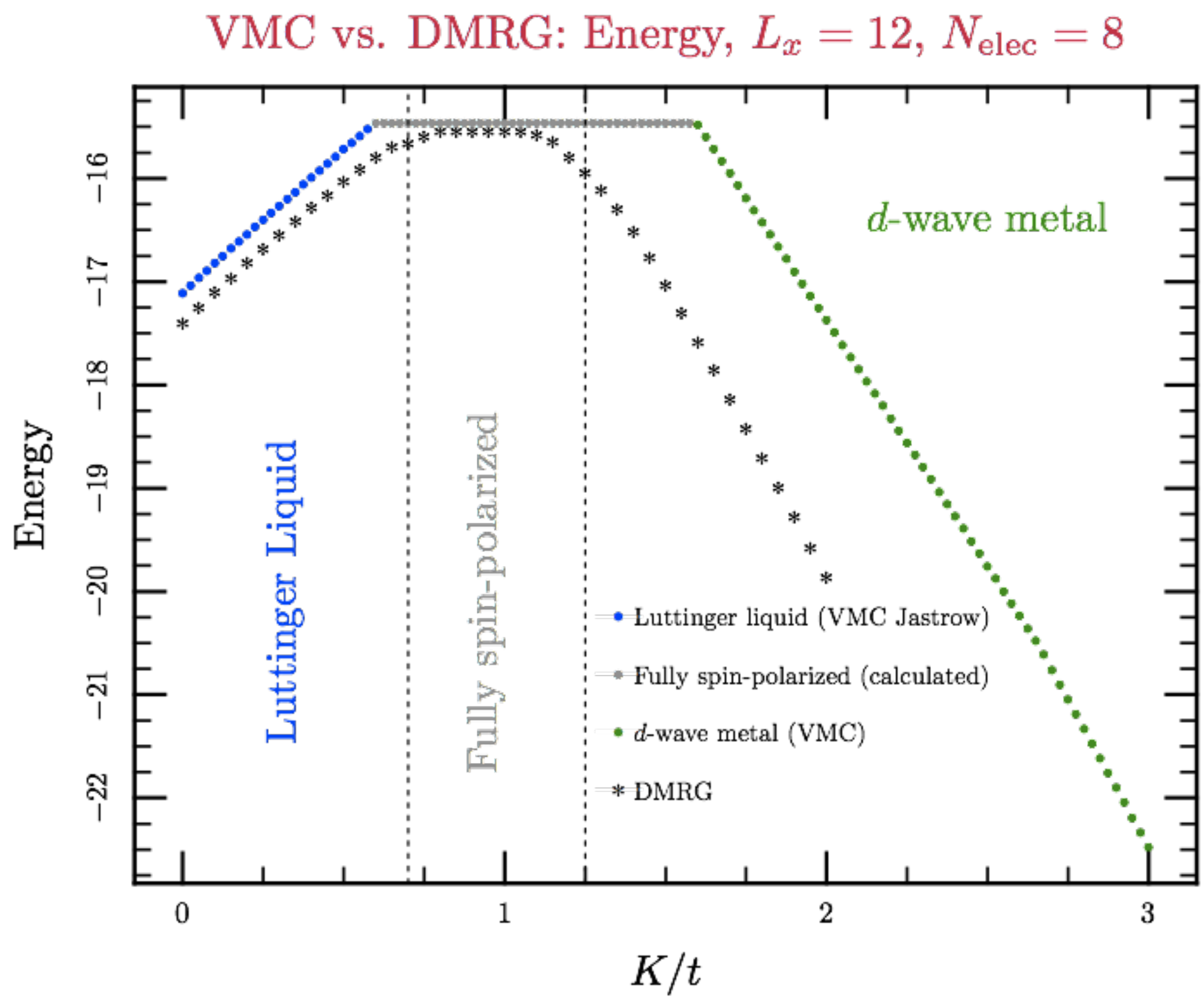
Variational Parameters:

Distribution of d_x partons between
bonding/anti-bonding bands (f-spinons and d_y partons
only in bonding band)

2 parameters to tune the Luttinger exponents

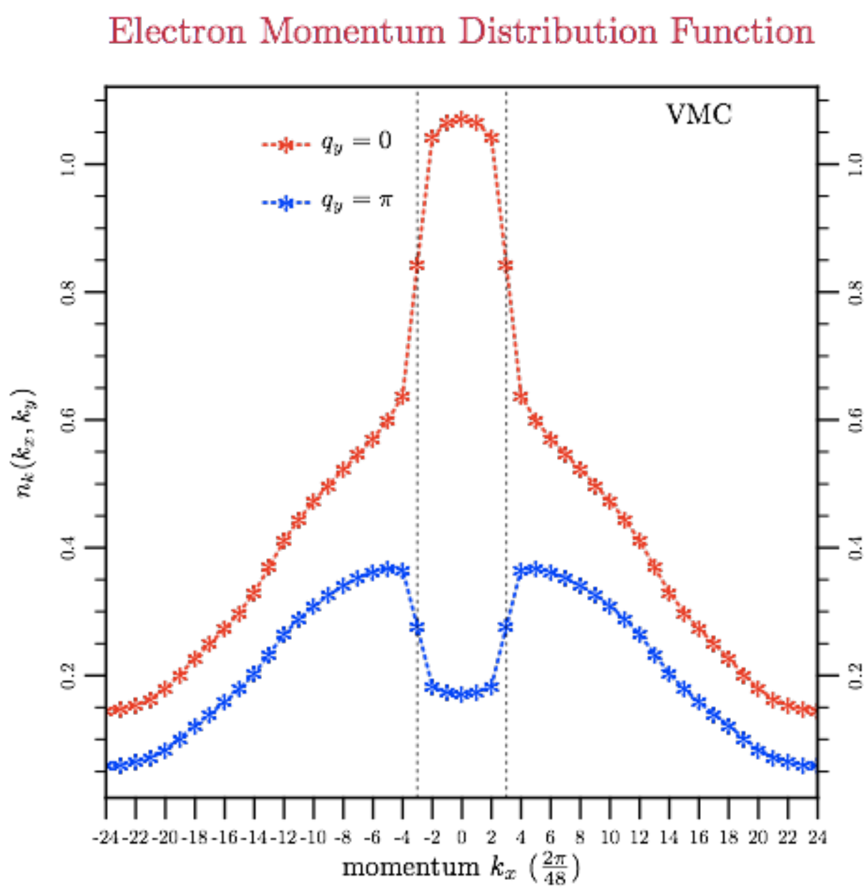
(Luttinger liquid phase: Jastrow factor multiplying filled Fermi sea)

Ground State energy: DMRG vs VMC



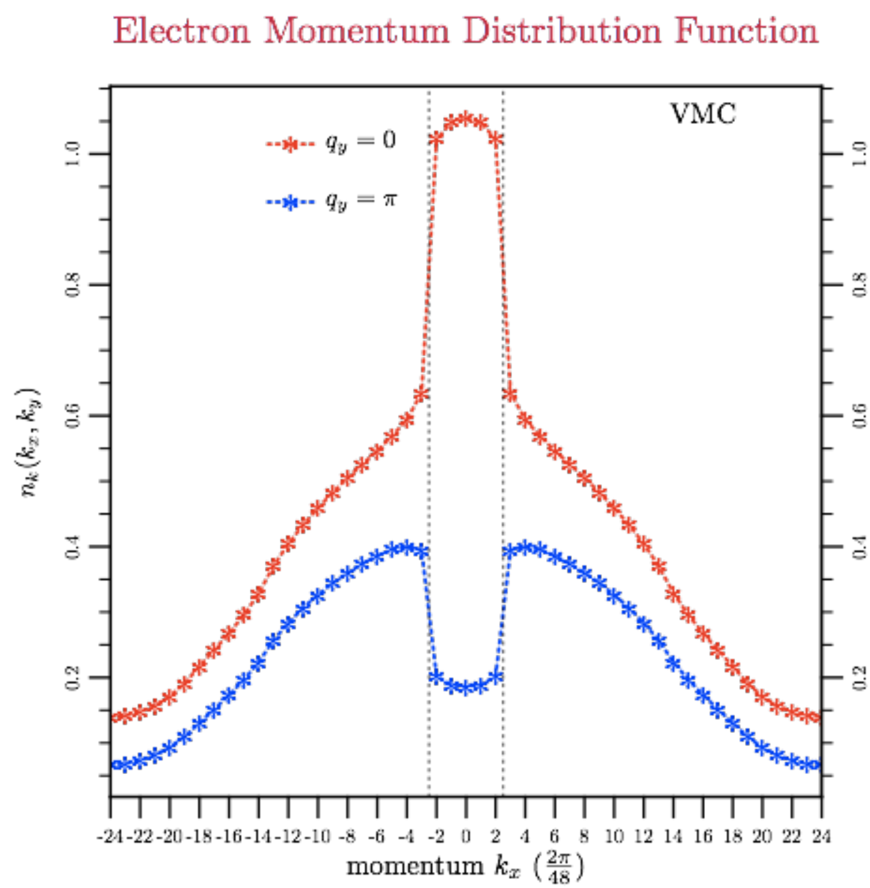
Evolution of VMC States

$$d_x : N_0 = 22, N_\pi = 10$$



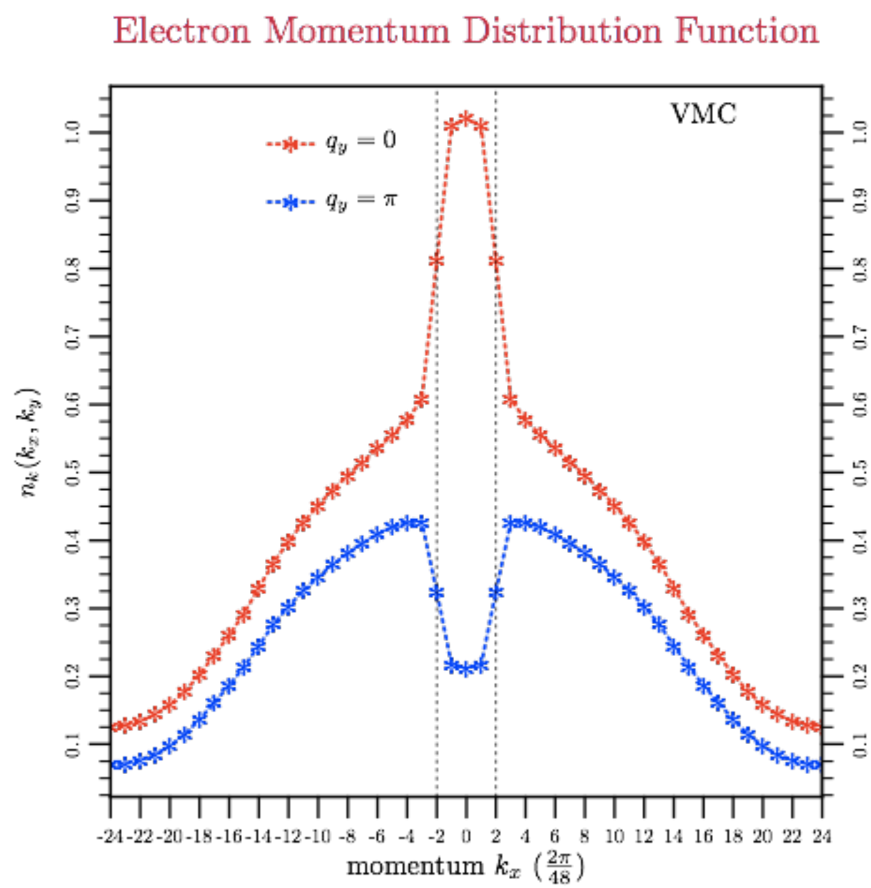
Evolution of VMC States

$$d_x : N_0 = 21, N_\pi = 11$$



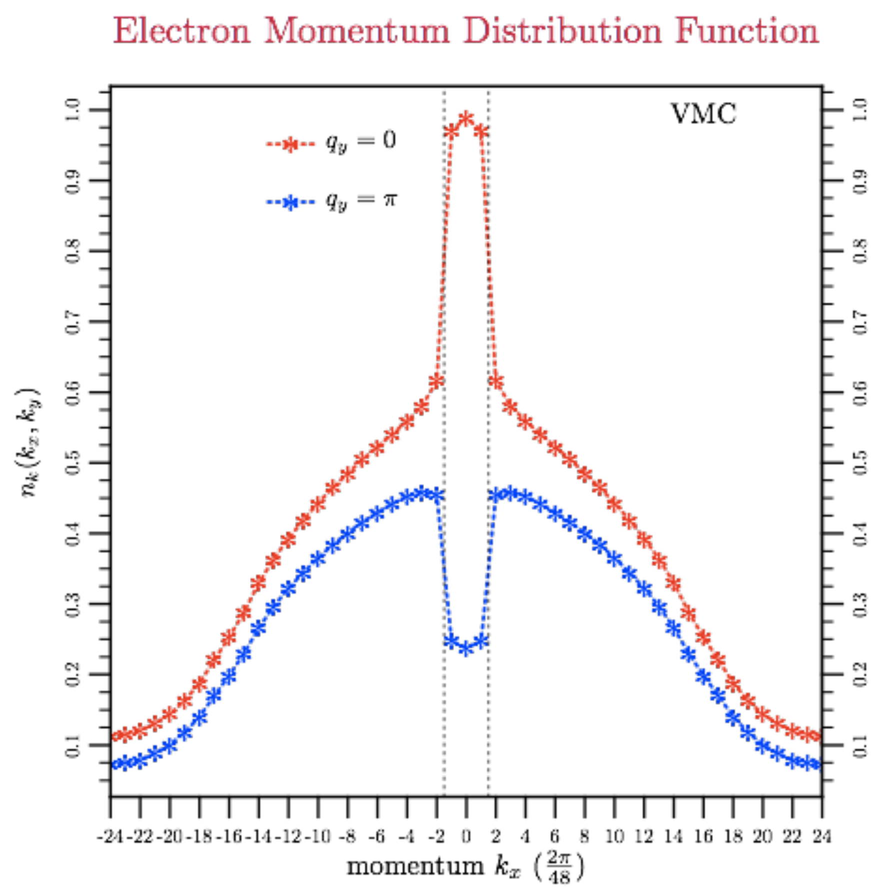
Evolution of VMC States

$$d_x : N_0 = 20, N_\pi = 12$$



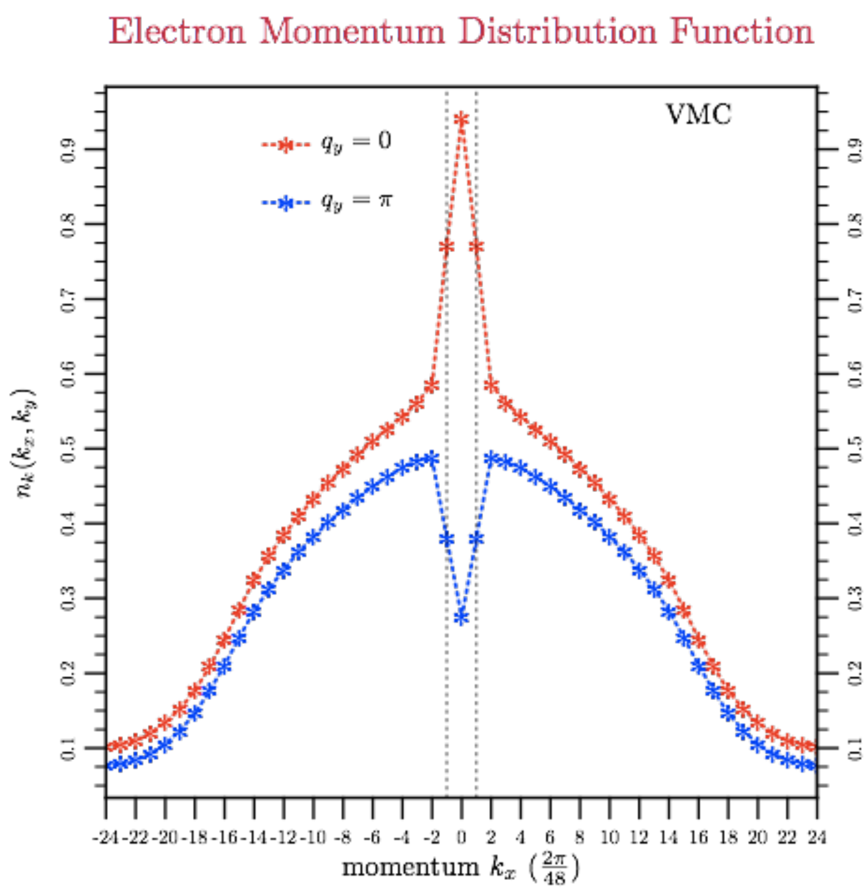
Evolution of VMC States

$$d_x : N_0 = 19, N_\pi = 13$$



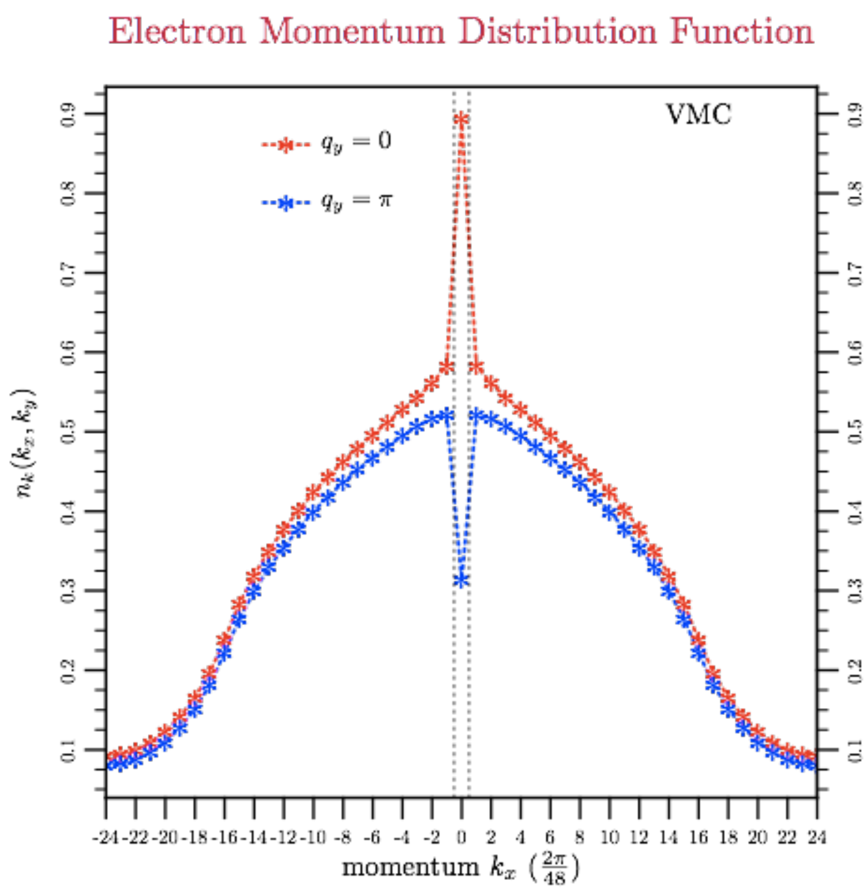
Evolution of VMC States

$$d_x : N_0 = 18, N_\pi = 14$$



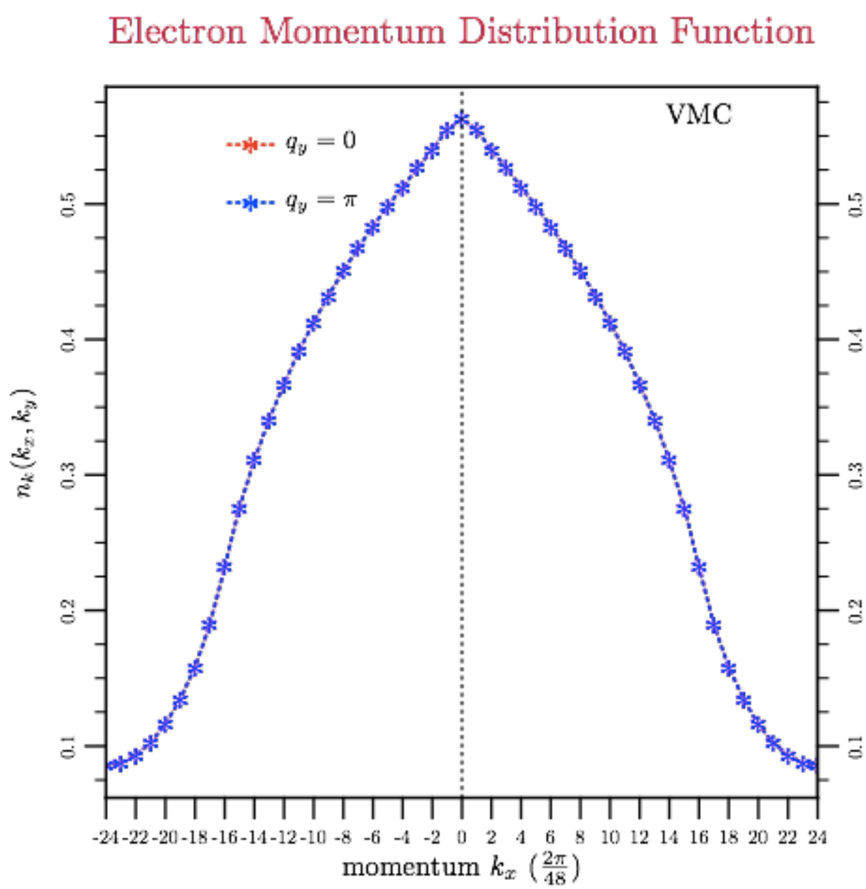
Evolution of VMC States

$$d_x : N_0 = 17, N_\pi = 15$$



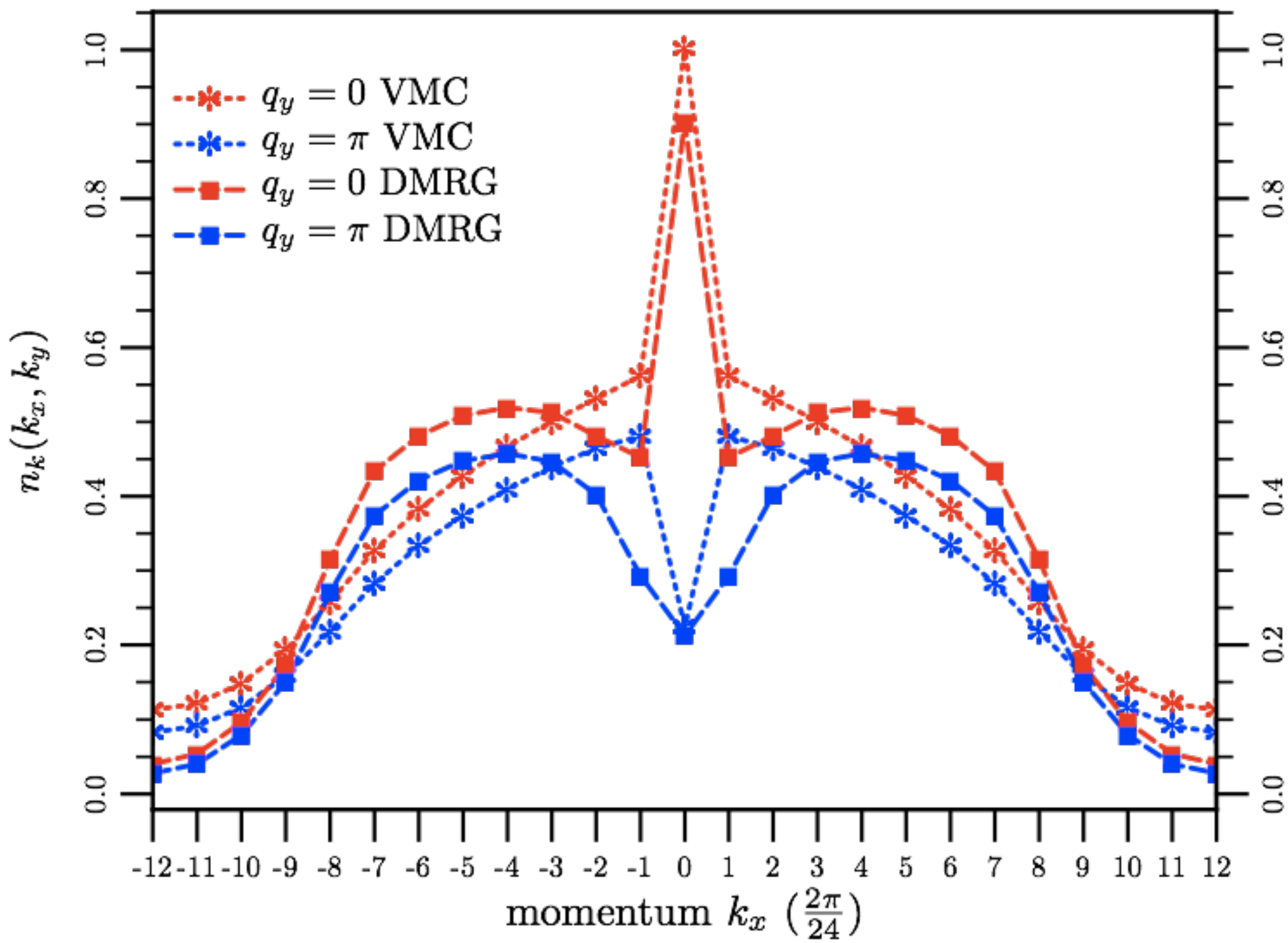
Evolution of VMC States

$$d_x : N_0 = 16, N_\pi = 16$$



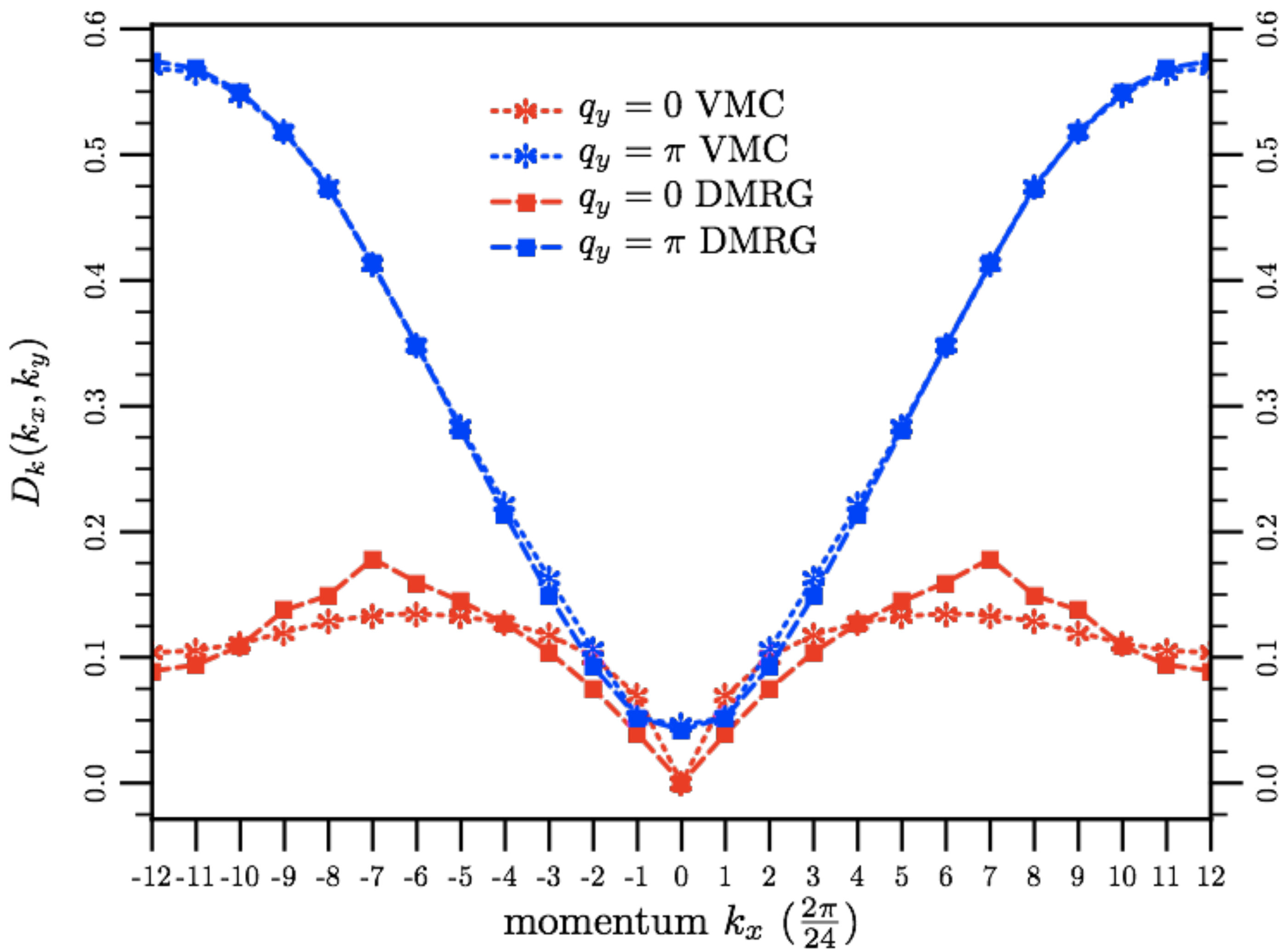
VMC vs. DMRG

Electron Momentum Distribution Function: $K = 2.5$



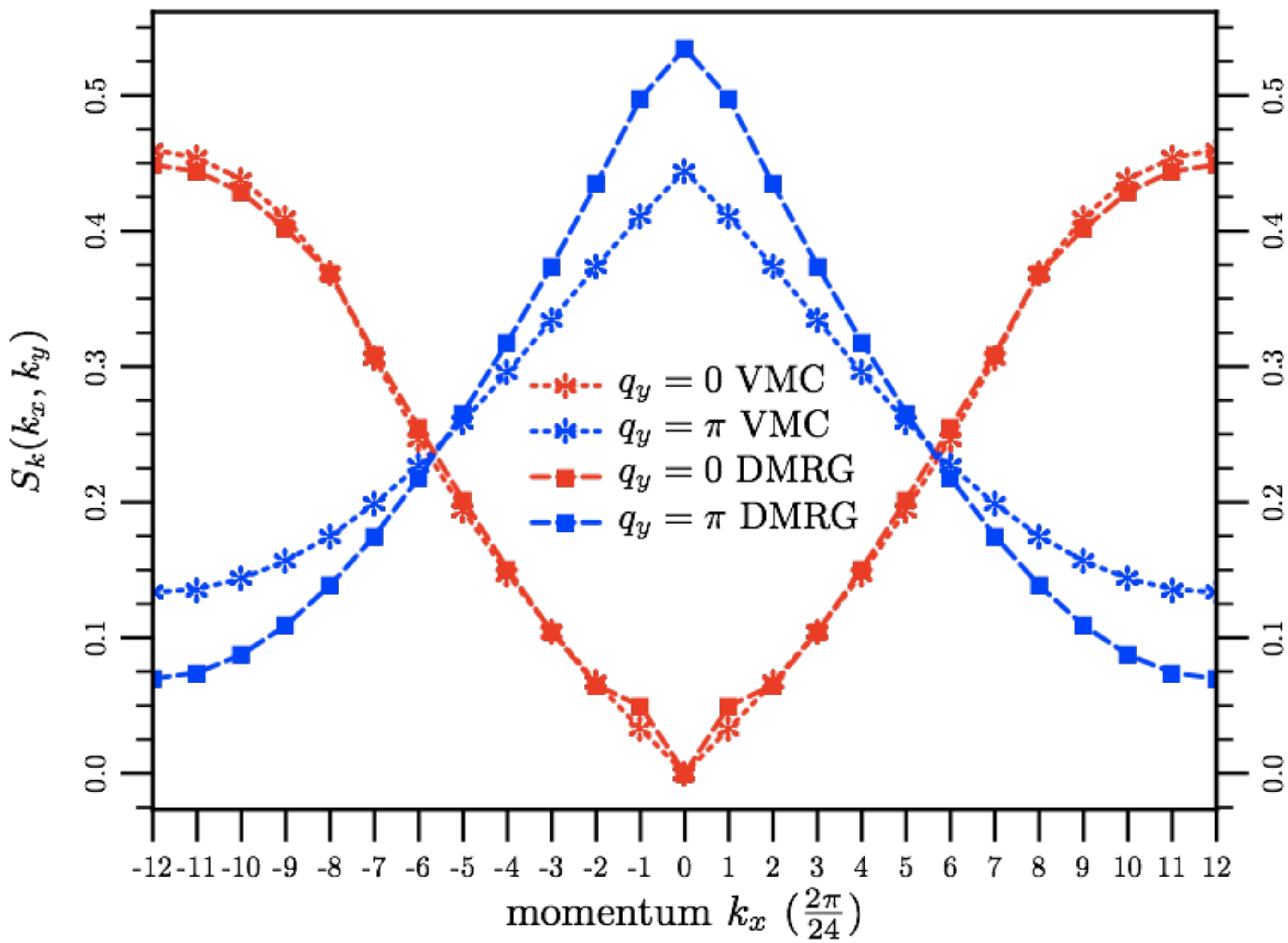
VMC vs. DMRG

Density-density Structure Factor: $K = 2.5$



VMC vs. DMRG

Spin-spin Structure Factor: $K = 2.5$



Conclusions

- ***Parton construction*** allows access of 2d ***Bose-Metal*** and ***Non-Fermi Liquid*** states
- Bose ring model on 2-leg and 4-leg has a ladder descendent of 2d Bose-Metal
- Electron Ring model on 2-leg ladder has "non-Luttinger liquid" phase, a descendent of 2d ***D-wave Metal*** phase

Open

- Bose and Electron NFL on Multi-leg ladders: entanglement entropy, dynamics…
- VMC energetics in 2d: Wavefunctions for FL, D-wave BCS, D-wave Metal,…
- Other wfs/Hamiltonians for 2d NFL phases??
- ***Other approaches besides partons??***: AdS/CFT, …

Correlators and Structure Factors

Electron Momentum Distribution Function:

$$n_{k\sigma}(\mathbf{k}) = \frac{1}{L_x L_y} \sum_{i,j} \langle c_{i\sigma}^{\dagger} c_{j\sigma} \rangle e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)} \qquad n_k = n_{k\uparrow} + n_{k\downarrow}$$

Density-density Structure Factor:

$$D_k(\mathbf{k}) = \frac{1}{L_x L_y} \sum_{i,j} \left[\langle \rho(\mathbf{r}_i)\, \rho(\mathbf{r}_j) \rangle - \langle \rho(\mathbf{r}_i) \rangle \langle \rho(\mathbf{r}_j) \rangle \right] e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)}$$

$$\rho(\mathbf{r}_i) = c_{i\uparrow}^{\dagger} c_{i\uparrow} + c_{i\downarrow}^{\dagger} c_{i\downarrow}$$

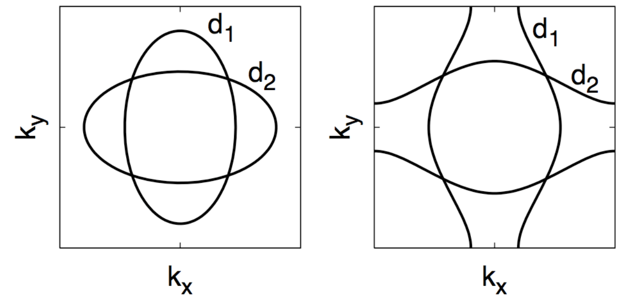
Spin-spin Structure Factor:

$$S_k(\mathbf{k}) = \frac{1}{L_x L_y} \sum_{i,j} \langle \mathbf{S}_i \cdot \mathbf{S}_j \rangle e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)}$$

Bose Surfaces in D-wave Bose-Metal

Mean Field Green's functions factorize:

$$G_b^{MF}(\mathbf{r},\tau) = G_{d_1}^{MF}(\mathbf{r},\tau) G_{d_2}^{MF}(\mathbf{r},\tau)/\bar{\rho}$$



$$G_{d_\alpha}^{MF}(\mathbf{r}) \approx \frac{1}{2^{1/2}\pi^{3/2}} \frac{\cos(\mathbf{k}_{F_\alpha}\cdot\mathbf{r} - 3\pi/4)}{c_\alpha^{1/2}|\mathbf{r}|^{3/2}}$$

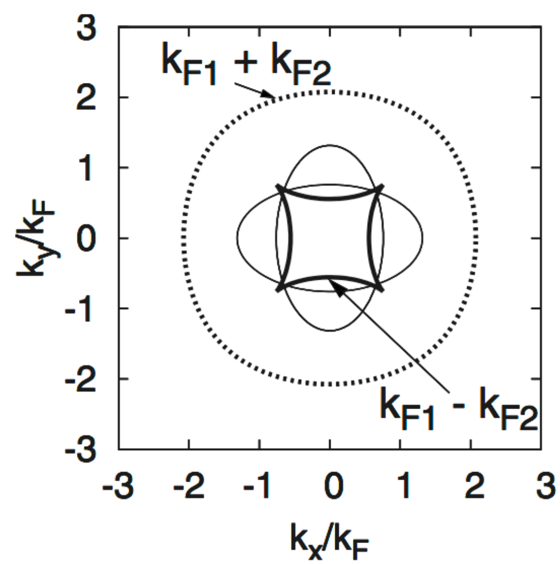
$$(\partial\epsilon_\alpha/\partial\mathbf{k})_{\mathbf{k}_{F\alpha}(\hat{\mathbf{r}})} = (const)\hat{\mathbf{r}}$$

Momentum distribution function:

$$n_b(\mathbf{k}) = \int G_b(\mathbf{r}) e^{i\mathbf{k}\cdot\mathbf{r}} d\mathbf{r}$$

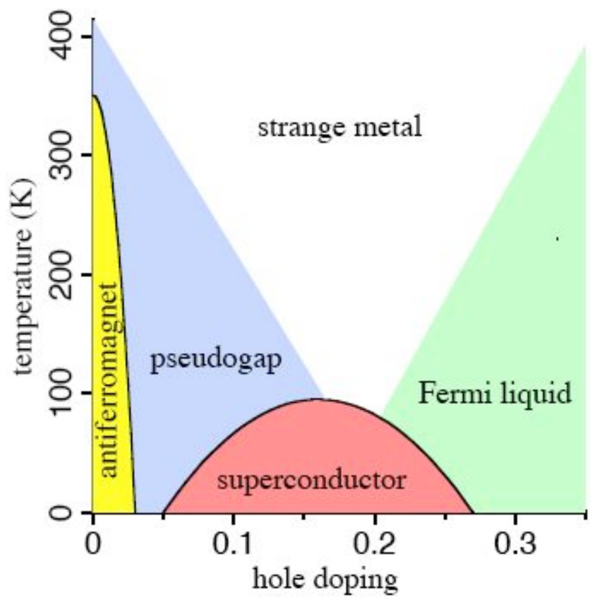
Two singular lines in momentum space, Bose surfaces:

$$\mathbf{k}_{F_1}(\hat{\mathbf{r}}) \pm \mathbf{k}_{F_2}(\hat{\mathbf{r}})$$



Motivation for Non-Fermi-Liquid Metal: “Abnormal” state of High T_c Superconductors

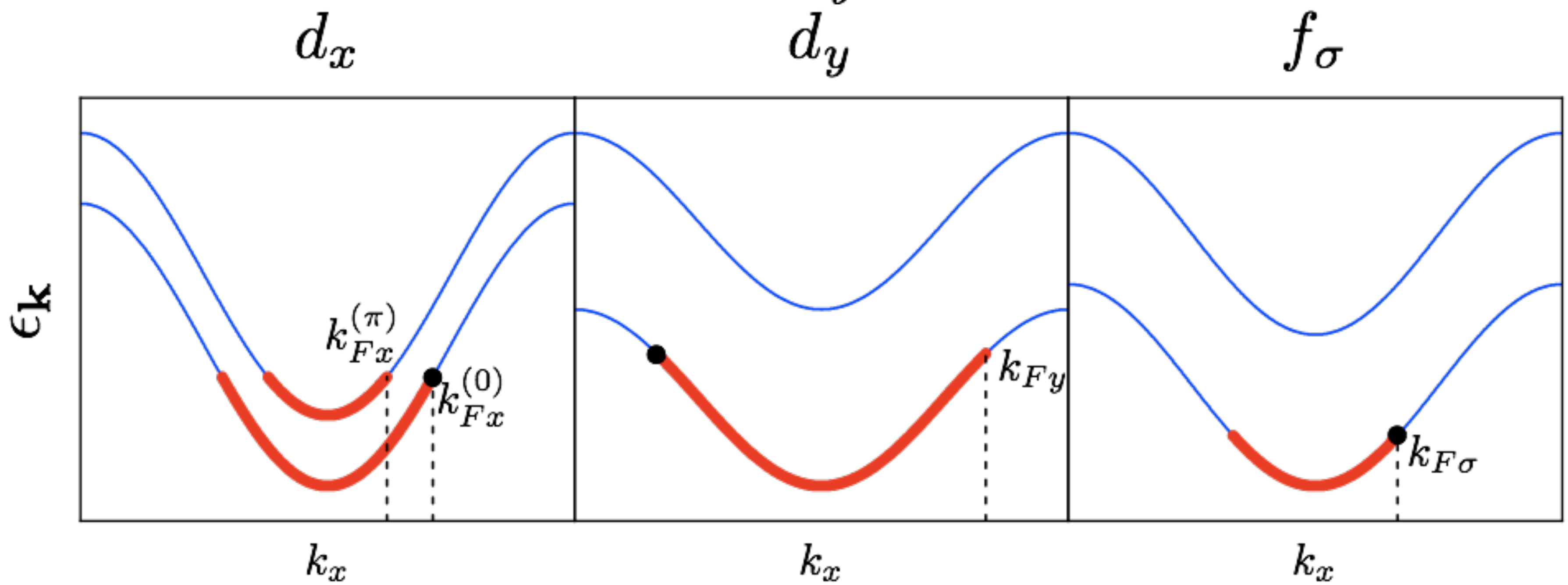
Phase Diagram



Strange metal: “Fermi surface” but quasiparticles are not “sharp”
Spectral function measured with ARPES suggests Z=0

The *d*-wave Metal on 2 Legs

$$c_\sigma^\dagger = d_x^\dagger d_y^\dagger f_\sigma^\dagger$$



In n_k, an enhanced singularity is predicted by the gauge theory at $k_{Fx}^{(k_y)} - k_{Fy} + k_{F\sigma}$

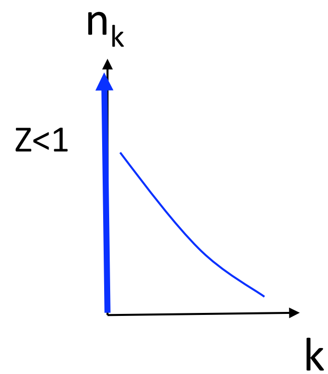
But what is a "Bose-Metal"?

First - A conventional interacting superfluid:

Boson Green's function $$G_b(\mathbf{r}) = \langle b^\dagger(\mathbf{r}) b(\mathbf{0}) \rangle$$

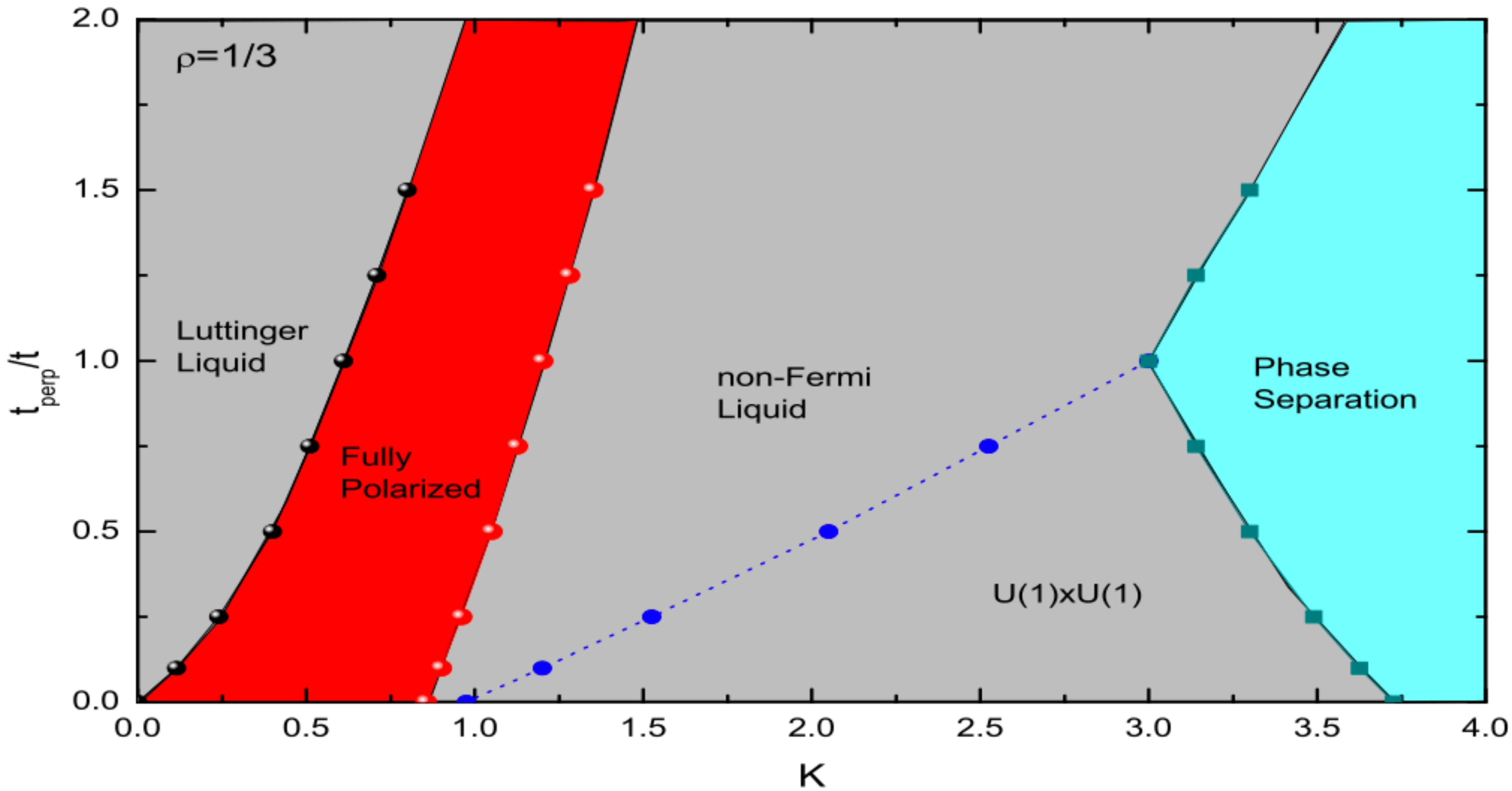
Off-diagonal long-ranged order $$G_b(\mathbf{r} \to \infty) = \rho_c = Z\rho; \qquad Z < 1$$

Momentum distribution function

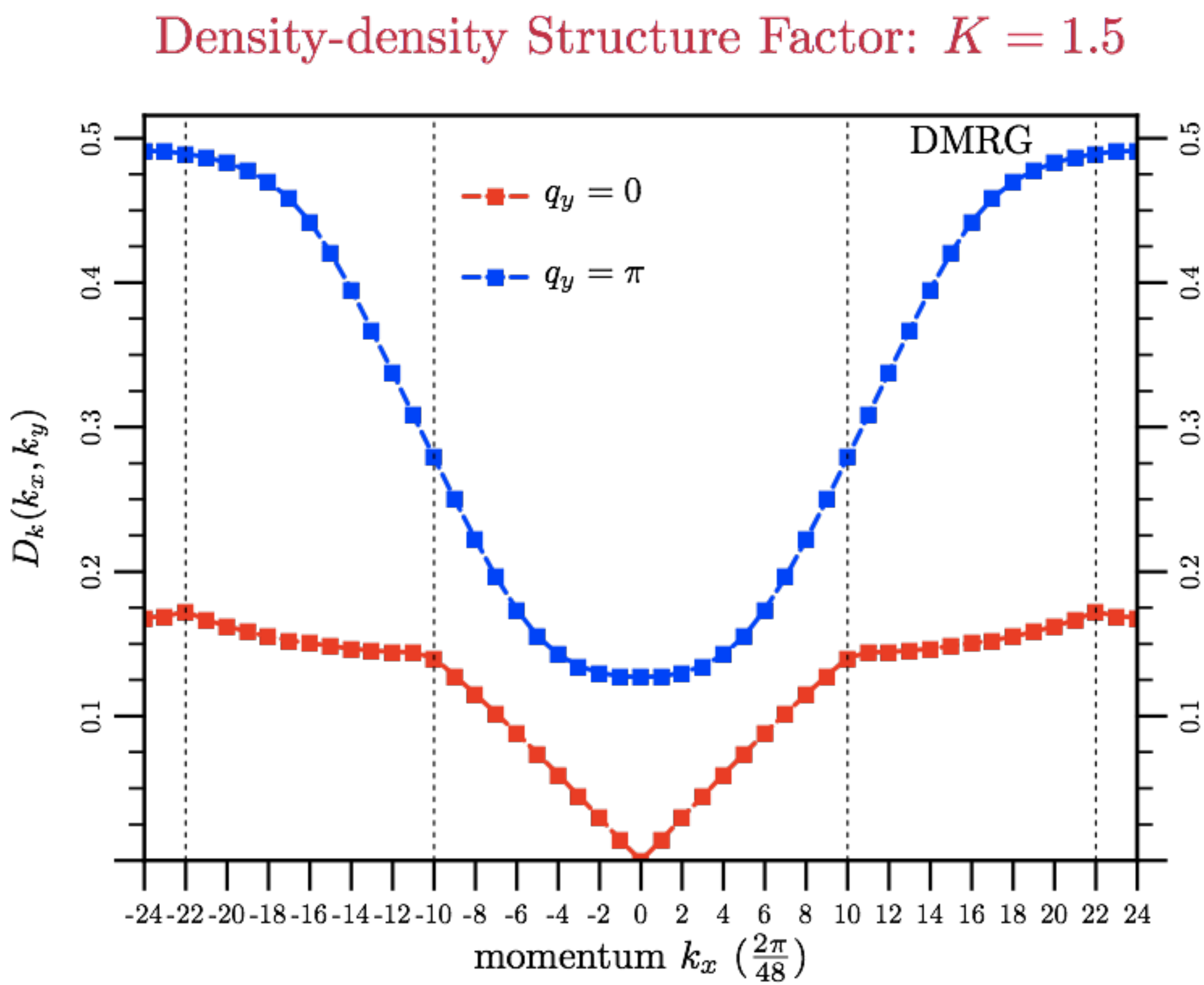


DMRG Phase diagram varying transverse electron hopping, t_{perp}

$$\mathcal{H}_{tK} = -t \sum_{\langle ij \rangle} [c^{\dagger}_{i\alpha} c_{j\alpha} + h.c.] + K \sum_{\langle 1234 \rangle} [\mathcal{S}^{\dagger}_{13} \mathcal{S}_{24} + h.c.]$$

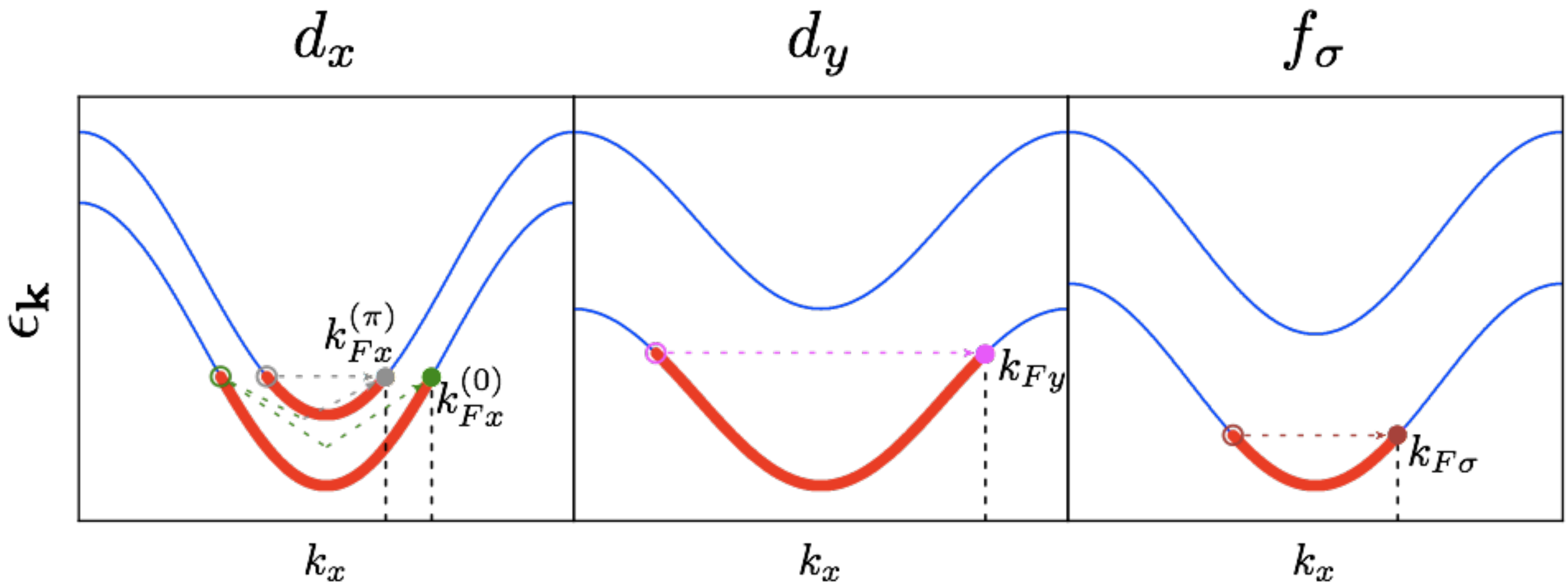


Density-density structure factor: DMRG



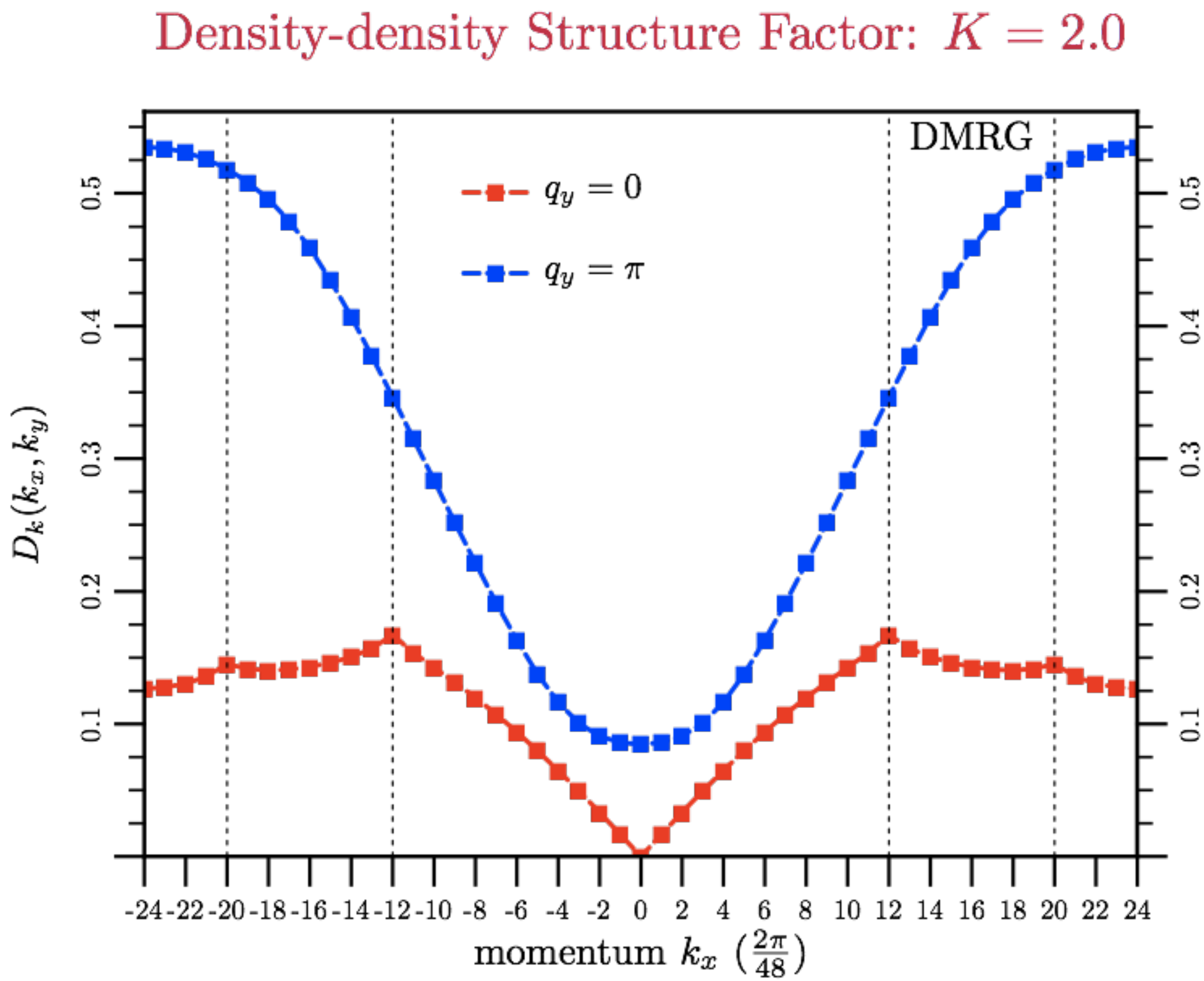
The *d*-wave Metal on 2 Legs

$$c_\sigma^\dagger = d_x^\dagger d_y^\dagger f_\sigma^\dagger$$



In D_k, enhanced singularities are predicted by the gauge theory at various "$2k_F$" wavevectors.

Evolution of Peak Locations



Evolution of Peak Locations