Gravity, Spin Models and Continuous Phase Transitions

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+ ongoing

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- Non-trivial critical exponents at T_c only computable by Monte-Carlo for $D > 2$.

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- Sound-speed of Goldstone mode: $\vec{m} \Leftrightarrow |\vec{m}|e^{i\psi}$ then
  $F_L \sim \int |\vec{M}|^2(\delta\psi)^2$
  In the MFA $c_\psi \sim |\vec{M}|^2 \sim |T - T_c|$

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- Gauge theories $\Rightarrow$ GR!
- A new approach to holographic super-fluids/super-conductors

Lattice gauge theory and Spin-models

Polyakov ’78; Susskind ’79

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- Any LGT with *arbitrary* gauge group G in d-dimensions with arbitrary *adjoint matter*
- Integrate out gauge invariant states $\Rightarrow$ generate effective theory for the Polyakov loop
- $Z_{LGT}(P;T) \sim Z_{SpM}(\vec{s};T^{-1})$
- Ferromagnetic spin model $\mathcal{H} = -J\sum_{\langle ij\rangle} \vec{s}_i \cdot \vec{s}_j + \cdots$
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- Inversion of temperature:
  Deconf. (high T) phase in LGT $\Leftrightarrow$ Ordered (low T) phase of SpM
  Conf. (low T) phase in LGT $\Leftrightarrow$ Disordered (high T) phase of SpM

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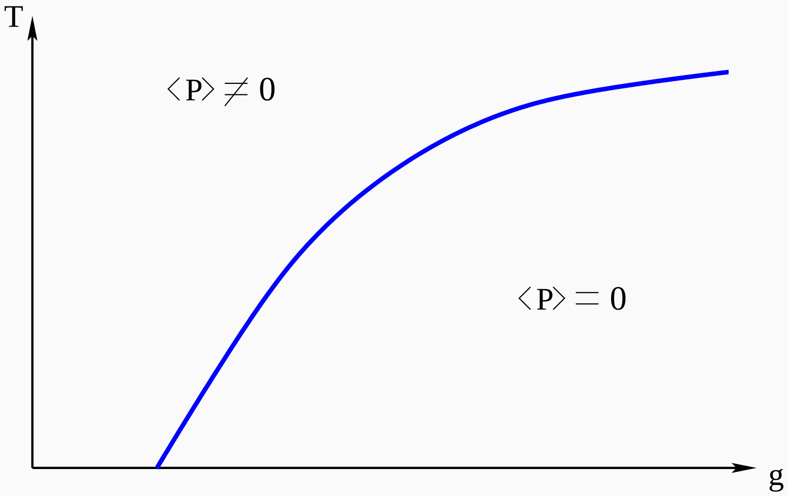

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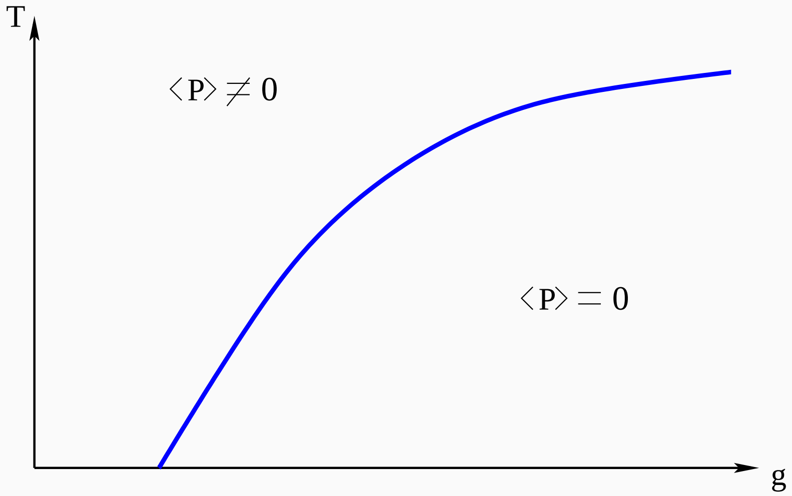

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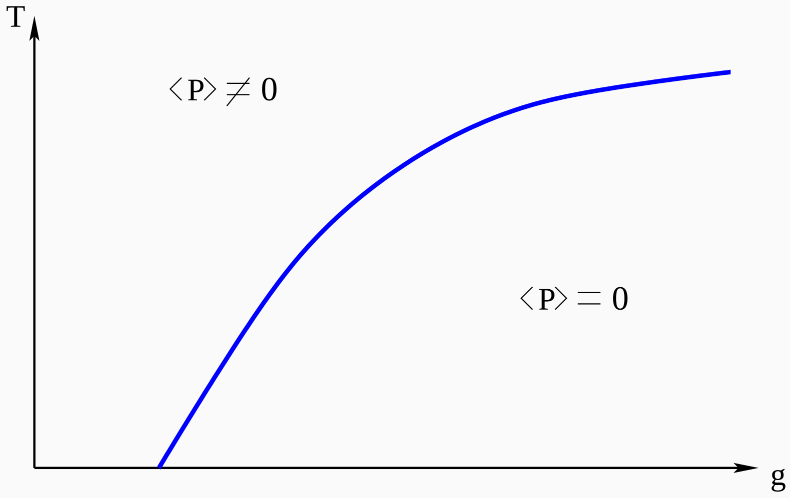

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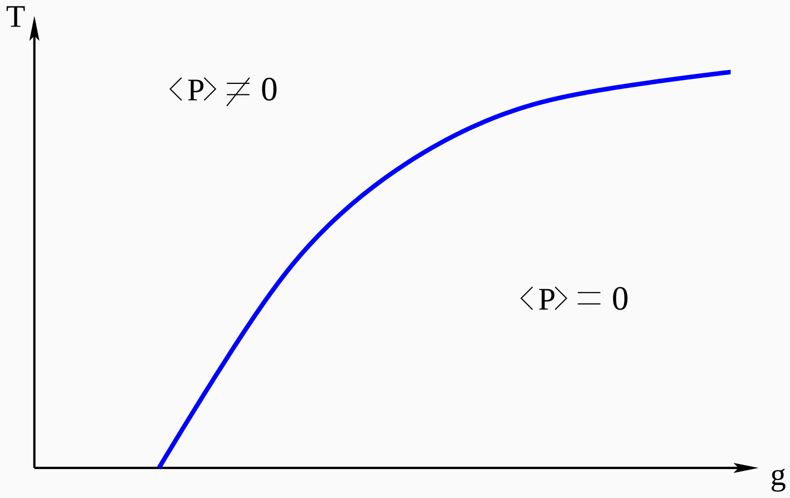

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- No long-range magnetic fluctuations ⇒ integrate out $U_{\vec{r},j}$
- The resulting theory $\mathcal{L}[P]$ describes long-range fluctuations at criticality
- Polyakov ’78; Susskind ’79: Can be mapped onto a spin-model with $P \Leftrightarrow \vec{s}$ (explicitly shown in the limit $g \gg 1$)

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- Some examples:
  1. Pure $SU(2)$ in $d = 4$ second order transition with Z_2 (Ising) critical exponents,
  2. $SU(N)$ with $N > 4$, $d > 3$
     Spin model with Z_N fixed point: $d = 4$ non-trivial $U(1)$ XY model exponents, $d > 4$ mean-field exponents.

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- Focus on
  $SU(N)$ with $N \to \infty$,
  Spin model with $Z_N \to U(1)$ fixed point.

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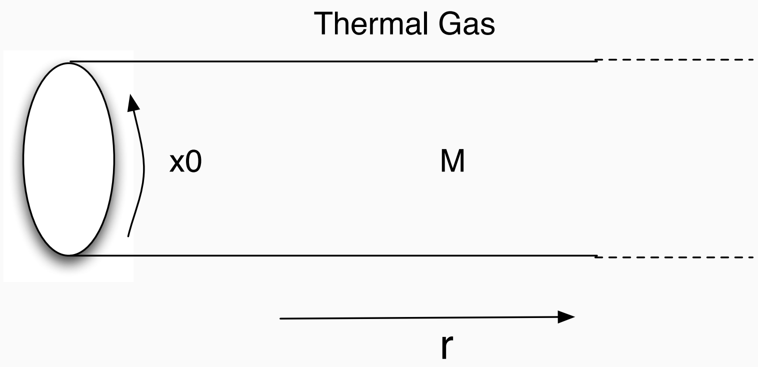

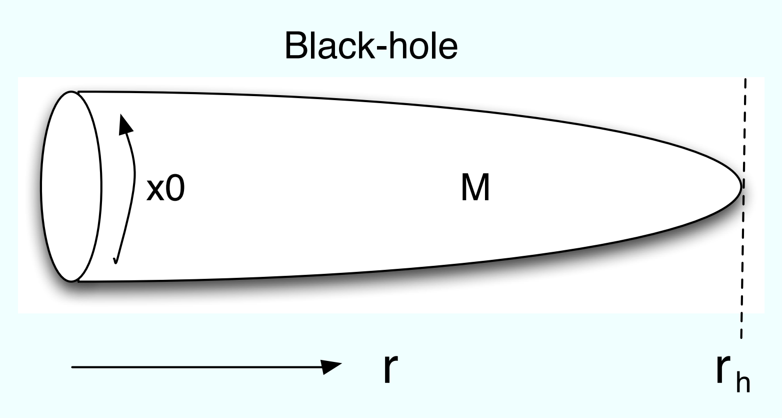

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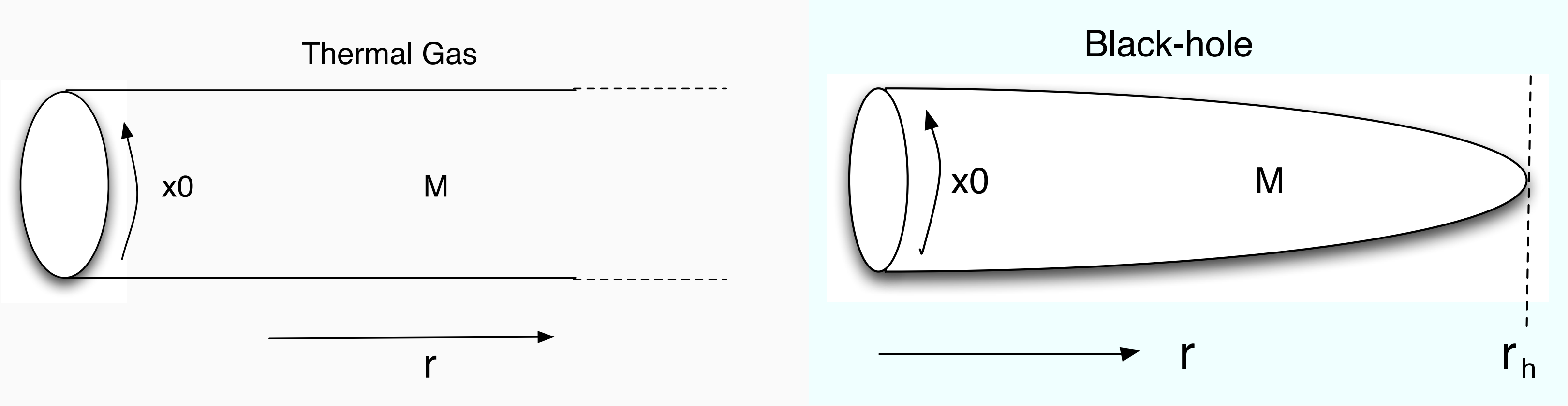

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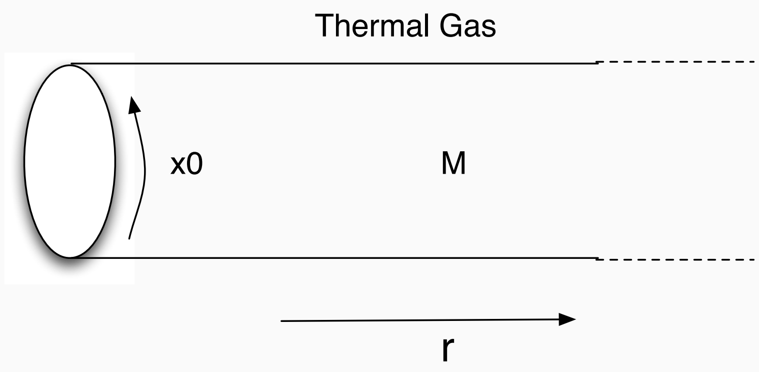

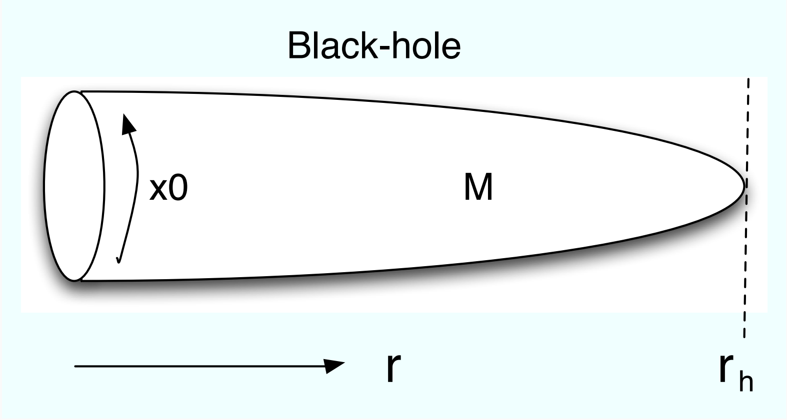

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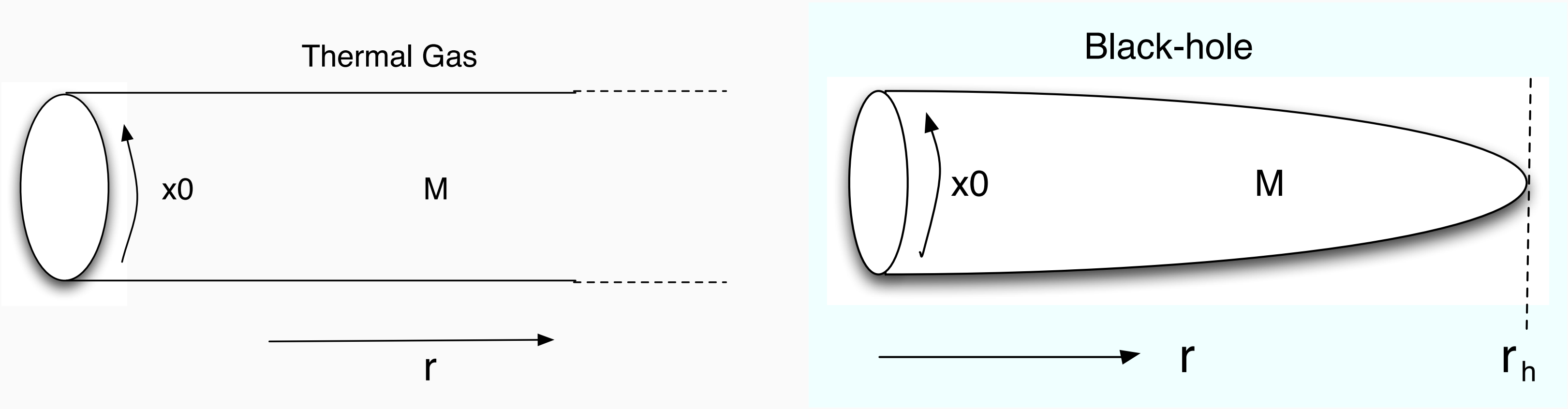

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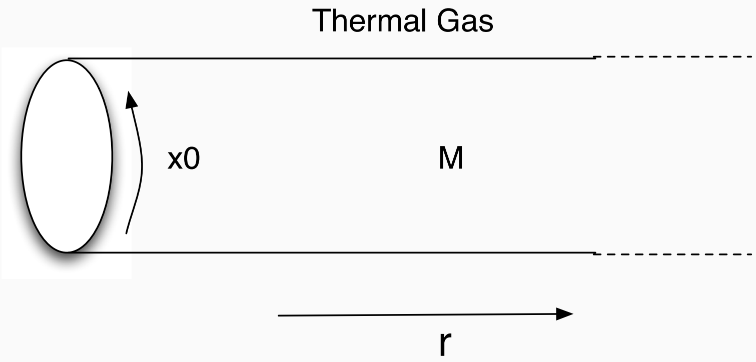

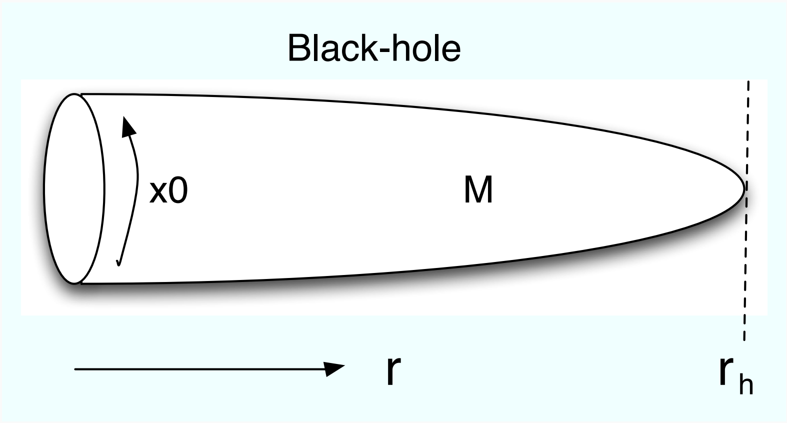

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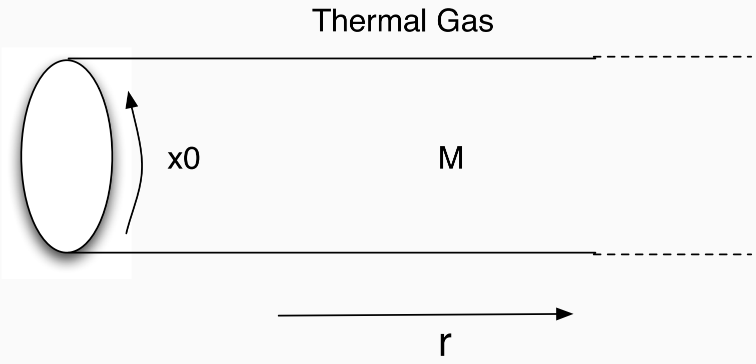

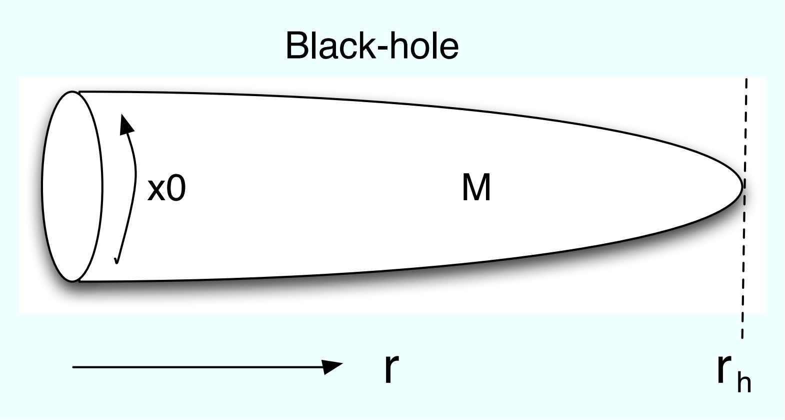

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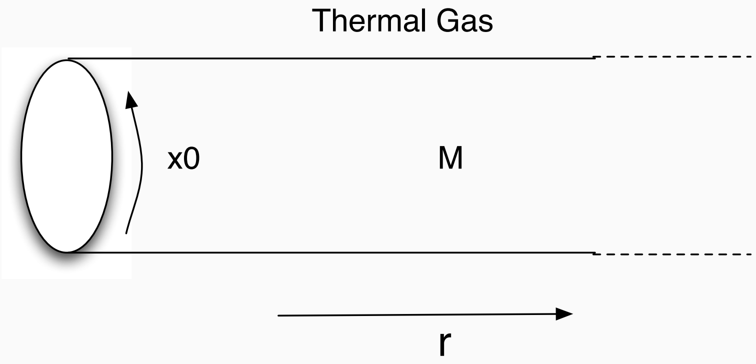

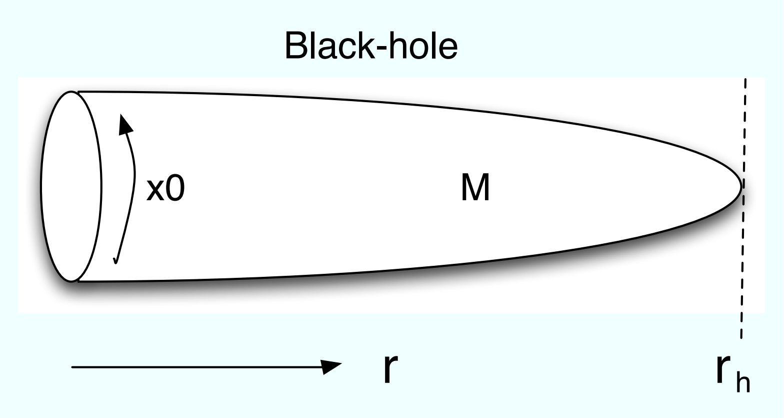

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- Fluctuations $\delta\Psi \Leftrightarrow$ Goldstone mode in the dual spin-model

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T

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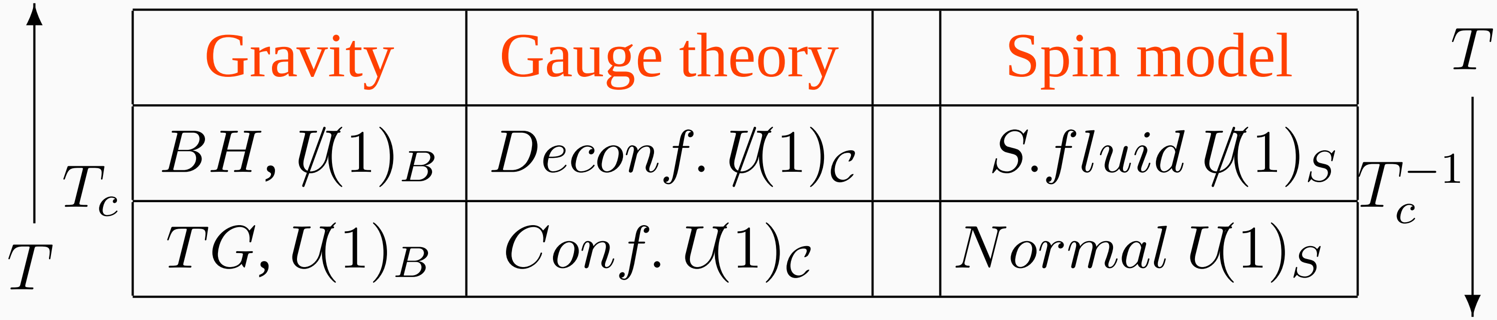

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Continuous Hawking-Page $\Leftrightarrow$ Normal-to-superfluid transition

GRAVITY/SPIN-MODEL CORRESPONDENCE

Continuous HP in dilaton-Einstein U.G. ’10

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Requirements for a second order Hawking-Page transition:

i.) There is a finite T_c at which:

ii.) $\Delta F(T_c) = 0$. TG(BH) dominates for $T < T_c$ $(T > T_c)$.

iii.) $\Delta S(T_c) = 0$

iv.) Make sure that this happens between the thermodynamically favored BH and TG branches.

Solution to the constraints

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All can be solved if as $T \rightarrow T_c$ horizon marginally traps the singularity!

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- Can be done because this regime is governed by a linear-dilaton CFT on the world-sheet!

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! 2nd order HP at T_c happens in a sub-dominant branch

! Background i the string frame is NOT linear-dilaton

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- One finds $c_\psi^2 \propto e^{-\sqrt{V_\infty}r_h} \sim (T - T_c)$.
- Second sound indeed vanishes at T_c with the mean-field exponent!

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Mean-field scaling in the magnetization!

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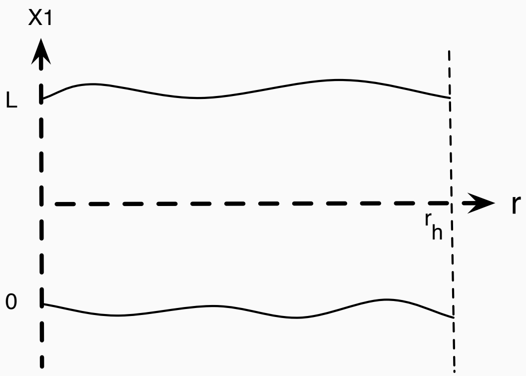
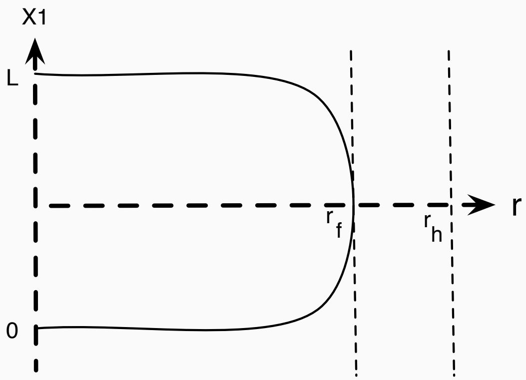
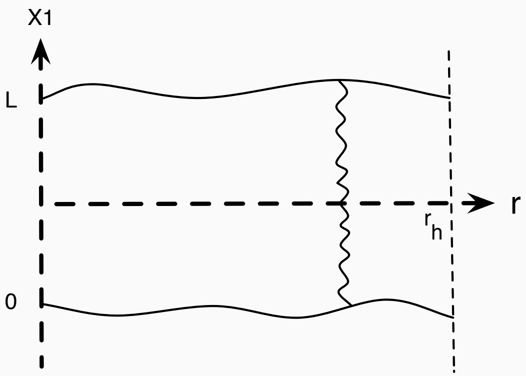


(a)



(b)



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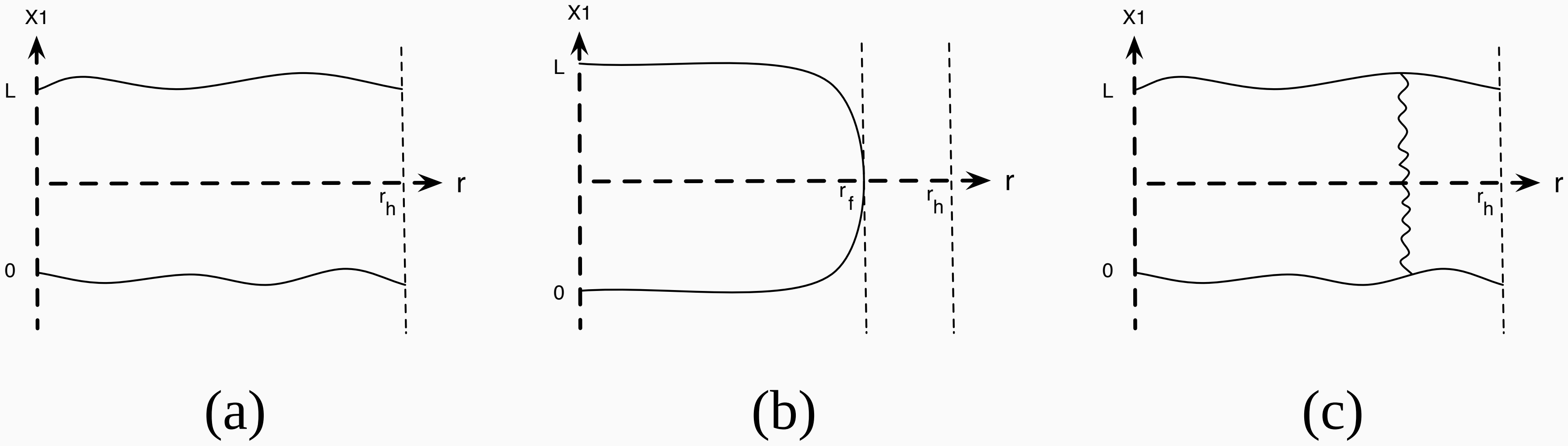

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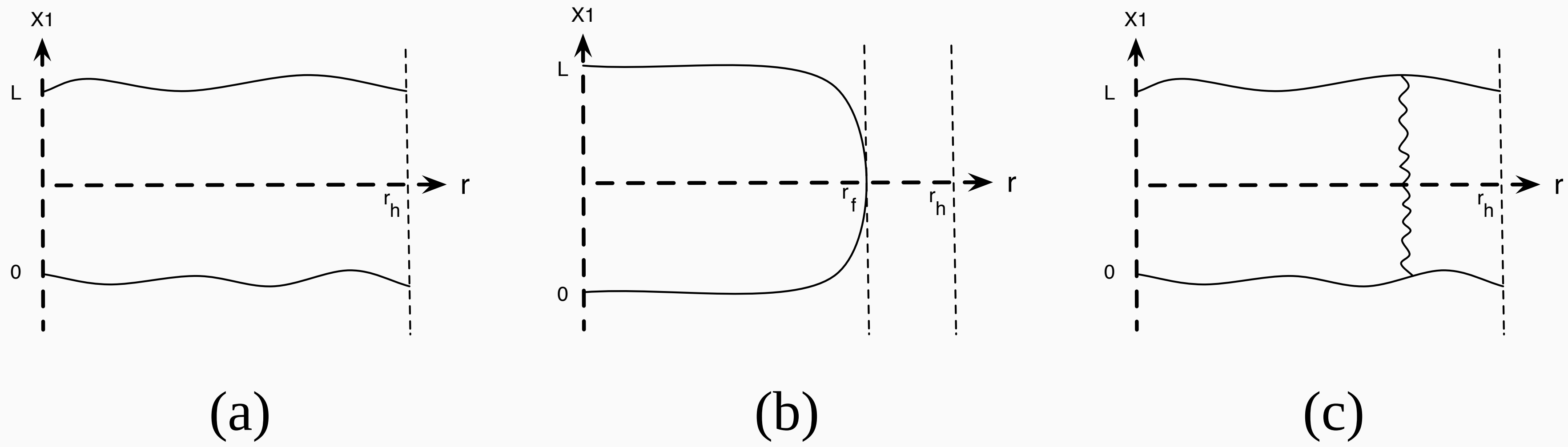

(a) $\langle \vec{m}(L) \cdot \vec{m}(0) \rangle_a = |\vec{M}|^2$. Finite in BH, 0 for TG.

(b) $S_{F1} \to m_T L + \cdots$

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Two-point function, cont'ed

(c) bulk exchange diagrams:

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CT^- include a zero-mode: $m_- = 0$ as $\psi = \int_M B$ is modulus: Goldstone mode!

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Precisely the expected behavior from the XY model,

with $\xi_{\|}^{-1} \to min(m_T, m_+)$ for $L \gg 1$.

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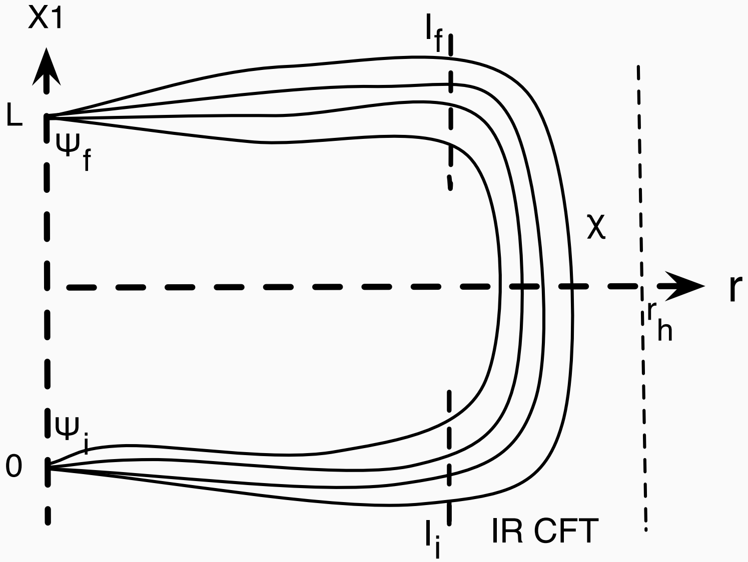

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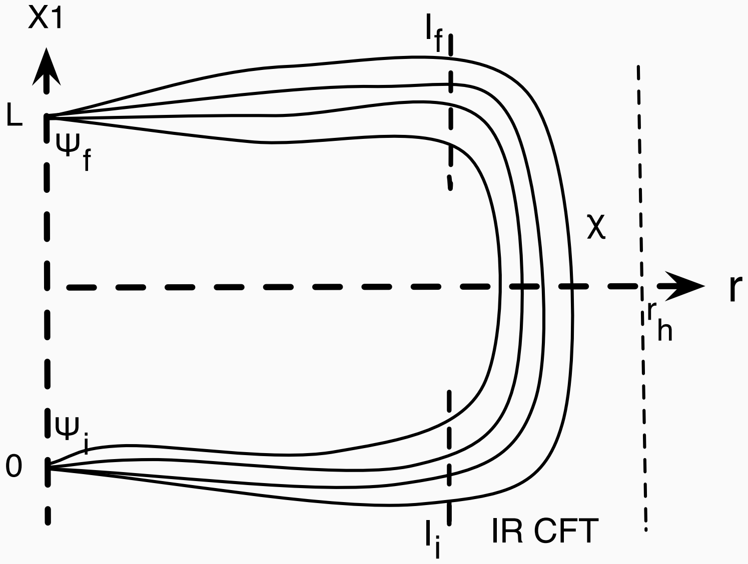

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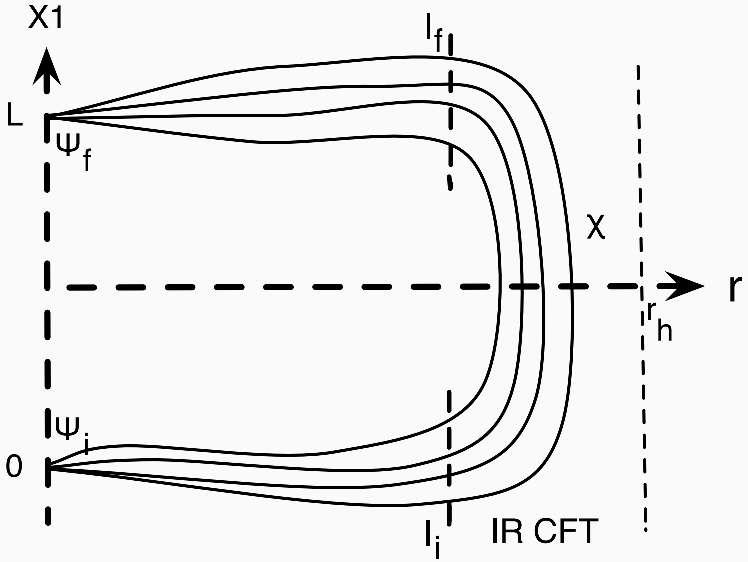

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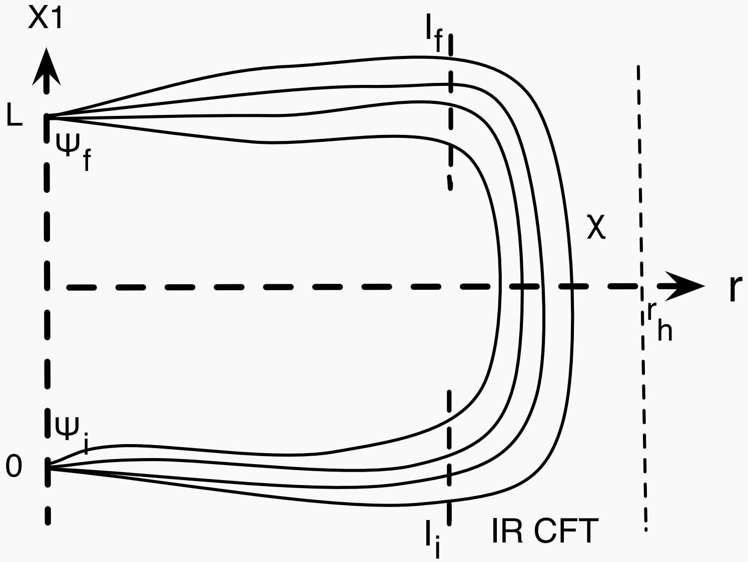

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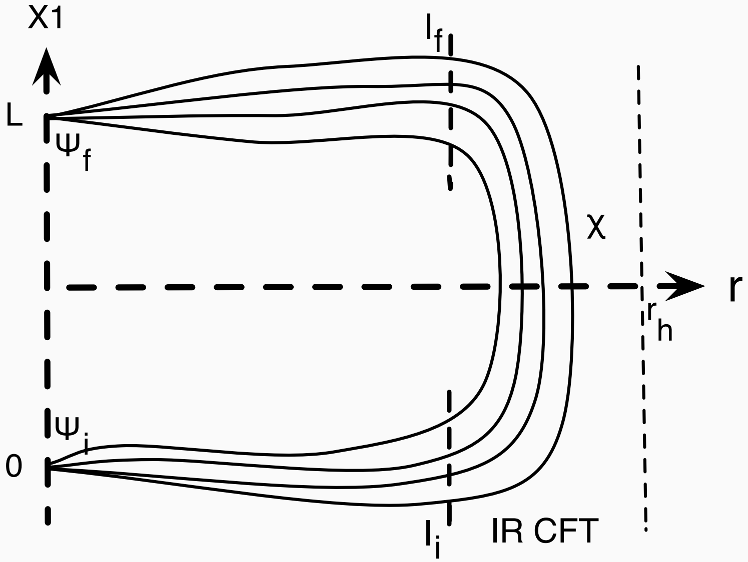
Indeed diverges if identify with Hagedorn a la Atick-Witten

$T_c = \frac{1}{4\pi\ell_s}$ and $\xi \to \frac{\ell_s}{2\sqrt{2}} \left(\frac{T-T_c}{T_c}\right)^{-\frac{1}{2}}$

Correlation length ξ

(b) Connected paths:



Propagator in the IR: $\Delta_{IR}(\chi) \sim \int dp_r d^{d-2}p_\perp e^{-ip_x^*(\chi)L}$

Dominant mode is the "winding tachyon":

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Mean-field scaling again!

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- Exponents of quantites controlled by bulk fluctuations, suppressed by $1/N \Rightarrow$ mean-field
- Exponents of stringy quantites $\Rightarrow$, controlled by $\alpha' \Rightarrow$ beyond mean-field possible

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THANK YOU !

Solving for the conditions

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- Then condition ii) is automatic.

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$V(\Phi) \to V_\infty \, e^{2\sqrt{\frac{\xi}{d-1}}\Phi} \left(1 + V_{sub}(\Phi)\right), \qquad \Phi \to \infty$