Maximally Supersymmetric "Dirt"

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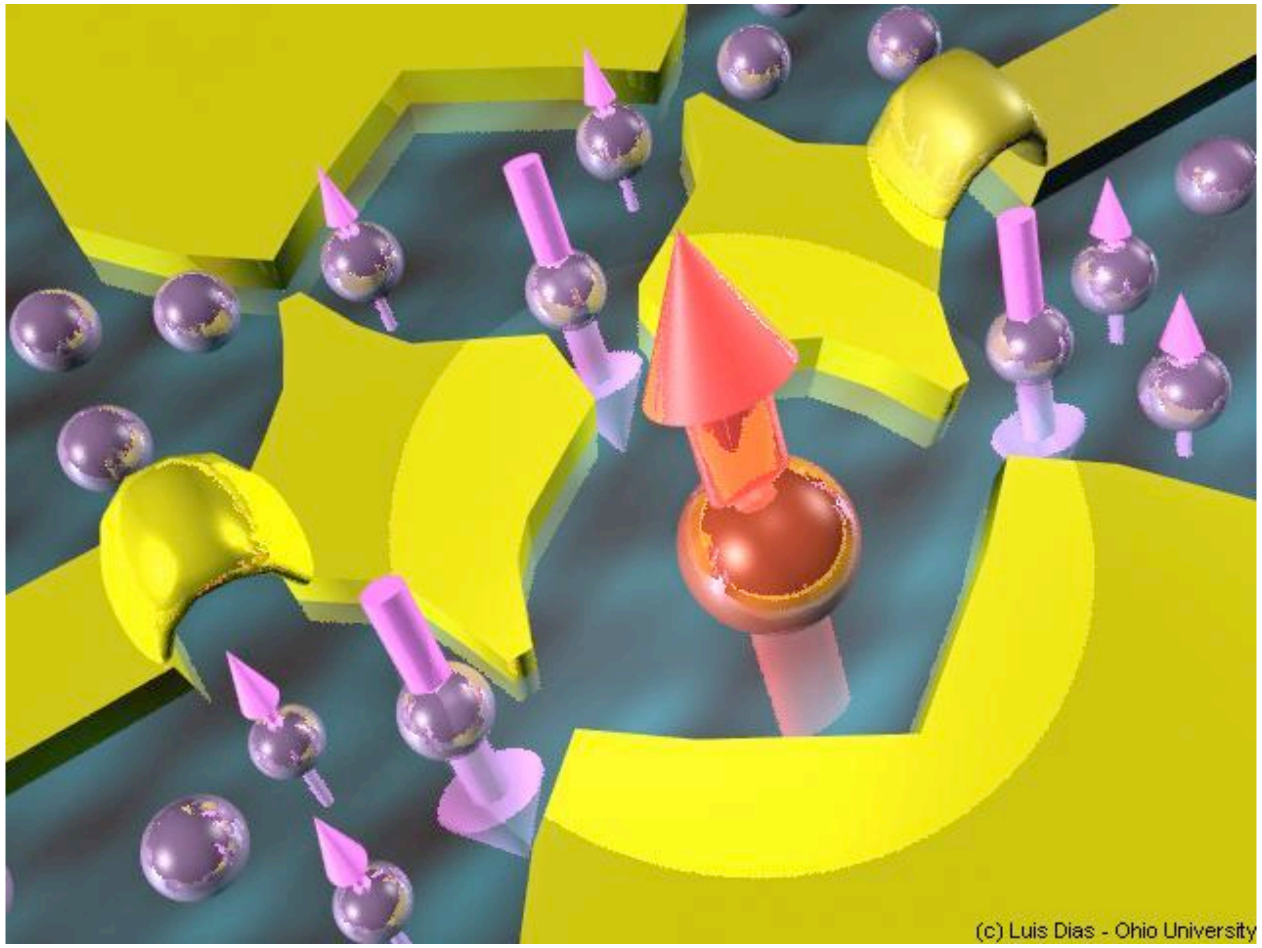


Based on work with:
S. Harrison, G. Torroba arXiv:1110.5325
K. Jensen, A. Karch, J. Polchinski, E. Silverstein arXiv:1105.1772
+ older papers with A. Karch, S. Yaida

I. Introduction and motivation

The Kondo effect was in a sense the first example of a system exhibiting asymptotically free running of a coupling constant:

$$H = \sum_{\vec{k}\alpha} \psi^{\dagger\alpha}_{\vec{k}} \psi_{\vec{k}\alpha} \epsilon(k) + J\vec{S} \cdot \sum_{\vec{k}\vec{k}'} \psi^{\dagger}_{\vec{k}} \frac{\vec{\sigma}}{2} \psi_{\vec{k}'}$$

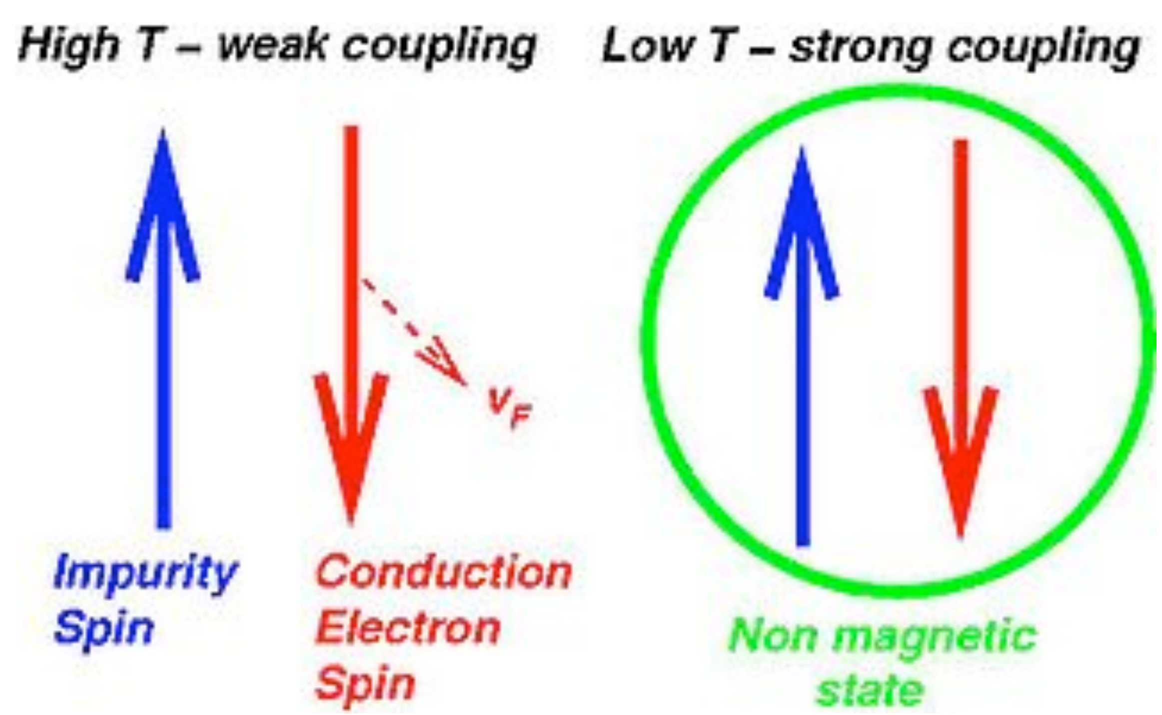
The effective coupling of the impurity spin to the itinerant electrons grows logarithmically at low energies

$$\lambda \equiv J\nu, \qquad \lambda(T) \approx \lambda + \lambda^2 \ln \frac{D}{T} + \dots$$

leading to interesting phenomena at the Kondo temperature:

$$T_K \approx D \exp[-1/\lambda],$$

below which one electron "sacrifices itself" to neutralize the spin:



Variants of this model exhibit other interesting behaviours. One natural generalisation is the multi-channel model:

$$H = \sum_{\vec{p}, i, \alpha} \epsilon(\vec{p}) \psi^{\dagger}_{\vec{p} i \alpha} \psi_{\vec{p} i \alpha} + J \sum_{\vec{p} \vec{p'} \, i \alpha \beta} S_{\alpha\beta} \, \psi^{\dagger}_{\vec{p} i \alpha} \psi_{\vec{p'} \, i \beta}$$

with i=1,...,K labelling channel, and alpha the index for the global SU(2) spin symmetry.

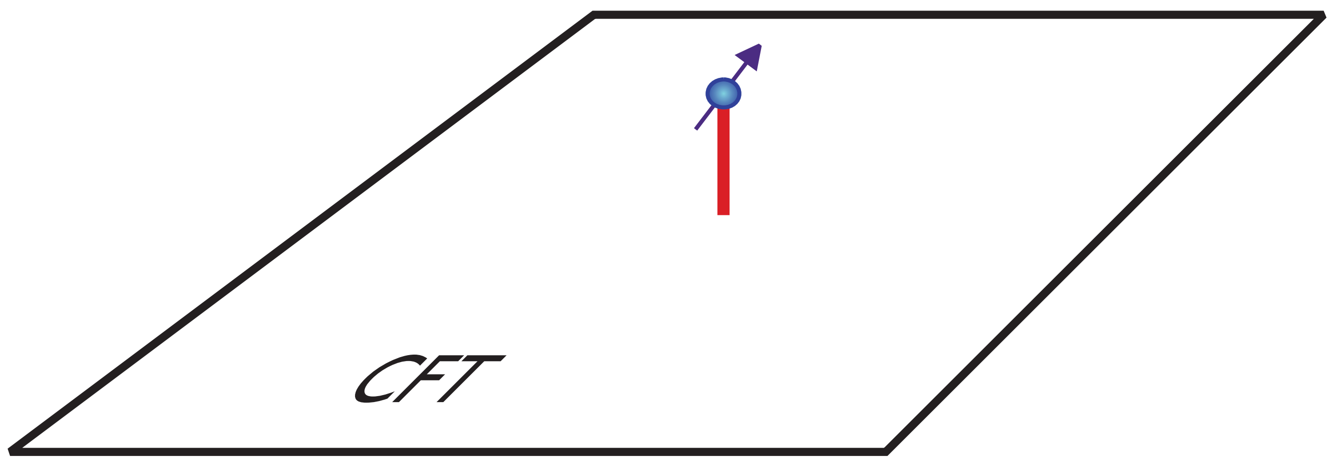
If the defect has spin s, then the IR fate depends on the # of channels compared to s:

$K > 2s$ "Overscreened," non-Fermi liquid behavior

$K < 2s$ "Underscreened," free partially screened spin in IR

c.f. exact solution by Affleck, Ludwig

Another interesting generalisation arises when instead of considering the impurity interacting with a free Fermi liquid, one considers a non-trivial bulk CFT (as would happen if one tunes such a system through a quantum critical point):



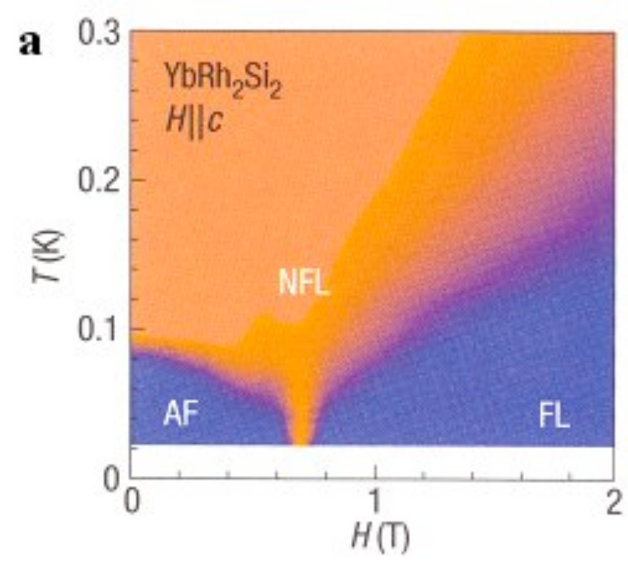
c.f. Sachdev, Buragohain, Vojta

We will be considering such models in the context of gauge/gravity duality, momentarily.

A last and even more interesting generalisation is to consider the Kondo lattice model:

$$H = H_J + \sum_k \epsilon_k c^\dagger_{k\alpha} c^\alpha_k + \frac{J_K}{2} \sum_i \hat{S}^a_i c^\dagger_{i\alpha} (\sigma^a)^\alpha_\beta c^\beta_i .$$

Now competition between the Kondo interaction and RKKY spin-spin interactions, is thought to potentially explain the existence of phase diagrams like those of the heavy fermion metals:



We will be studying highly idealized models of this general sort in the talk today. The bulk will be a highly supersymmetric CFT, coupled supersymmetrically to the defect spin. There are many drawbacks to the supersymmetry, but it has the virtue of allowing us to reliably solve for some features of the physics, in some limits.

Plan:

II. SUSY Kondo model: probe approximation
III. SUSY Kondo lattice model: probe approximation
IV. SUSY Kondo model: including backreaction

II. The maximally supersymmetric Kondo model

We will be studying the system realised by the following configuration of D3 and D5 branes in type IIB superstring theory:

	0	1	2	3	4	5	6	7	8	9
N $D3$	$\times$	$\times$	$\times$	$\times$						
M $D5$	$\times$				$\times$	$\times$	$\times$	$\times$	$\times$	
k $F1$	$\times$									$\times$

$$S = S_{D3} + S_{D5} + S_{\text{defect}}$$

$$S_{\text{defect}} = \int dt\ \left[i\bar{\chi}_i^I \partial_t \chi_I^i + \bar{\chi}_i^I \left(A_0(t,0)_j^i + n_a \phi^a (t,0)_j^i \right) \chi_I^j + \bar{\chi}_i^I (\tilde{A}_0)_I^J \chi_J^i - k(\tilde{A}_0)_I^I \right]$$

In the standard supergravity limit, this system is dual to N=4 SYM coupled to a defect fermion with:

$$S_{\text{field theory}} = S_{\mathcal{N}=4} + \int dt \left[i\chi_b^\dagger \partial_t \chi^b + \chi_b^\dagger \left\{ (A_0(t, \overrightarrow{0}))^b_c + v^I (\phi_I(t, \overrightarrow{0}))^b_c \right\} \chi^c \right],$$

$$\sum_{\alpha=1}^{N} \chi_\alpha^\dagger \chi_\alpha = k$$

The bosonic symmetries preserved by the defect are:

$$SL(2,\mathbb{R}) \times SO(3) \times SO(5)$$

It is useful to write the $AdS_5 \times S^5$ metric in a way that makes these symmetries manifest:

$$ds^2 = R^2 \left(du^2 + \cosh^2 u\, ds^2_{AdS_2} + \sinh^2 u\, d\Omega_2^2 + d\theta^2 + \sin^2\theta\, d\Omega_4^2 \right)$$

In the probe approximation M << N, the D5 worldvolume is an $AdS_2 \times S^4$, given by the embedding conditions:

$$u = 0 \ , \ \ \theta = \theta_k \ .$$

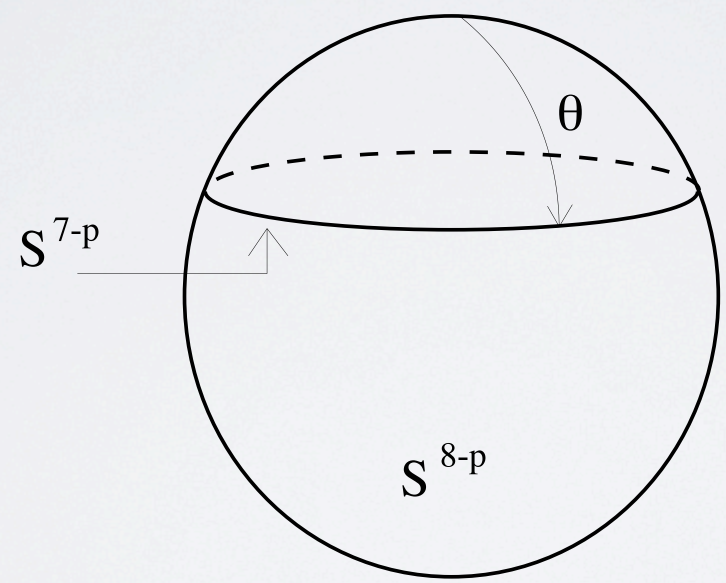


Figure 1: The points of the S^{8-p} sphere with the same polar angle θ define a S^{7-p} sphere. The angle θ represents the latitude on S^{8-p}, measured from one of its poles.

The allowed angles are:

$$k = \frac{N}{\pi}\left(\theta_k - \frac{1}{2}\sin 2\theta_k\right) \ .$$

The defect free energy and entropy can be computed by evaluating the DBI action immersed in the AdS black brane:

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + \frac{r^2}{R^2}\left(\sum_{i=1}^{3} dx_i^2\right) + R^2(d\theta^2 + \sin^2\theta\, d\Omega_4^2) \ ,$$

$$f(r) = \frac{r^2}{R^2}\left(1 - \frac{r_+^4}{r^4}\right) \ .$$

(The field theory temperature is given by $T = r_+/\pi R^2.$)

Regularizing by subtracting the Euclidean action of the analogous D5 in pure AdS space-time, one finds for a single brane:

$$F_{\text{defect}} = -\sqrt{\lambda}\,\frac{\sin^3\theta_k}{3\pi}NT$$

The impurity entropy or "g-function" is defined by:

$$\log g = \mathcal{S}_{\rm imp} \equiv \lim_{T\to 0} \lim_{V\to\infty} \left[\mathcal{S}(T) - \mathcal{S}_{\rm ambient}(T)\right]$$

In this case, now restoring M, we find:

$$\log g = \mathcal{S}_{\rm imp} = \sqrt{\lambda}\,\frac{\sin^3\theta_k}{3\pi} M N$$

By way of comparison, the multi-channel Kondo model with K channels and SU(N) spin symmetry (@ large N), with a defect in the kth antisymmetric representation, has:

$$\mathcal{S}_{\rm imp} = \frac{2}{\pi} M N \left[f\left(\frac{\pi}{1+K/N}\right) - f\left(\frac{\pi}{1+K/N}(1-k/N)\right) - f\left(\frac{\pi}{1+K/N} k/N\right)\right]$$

$$f(x) = \int_0^x du \, \log \sin u$$

Parcollet, Georges, Kotliar, Sengupta

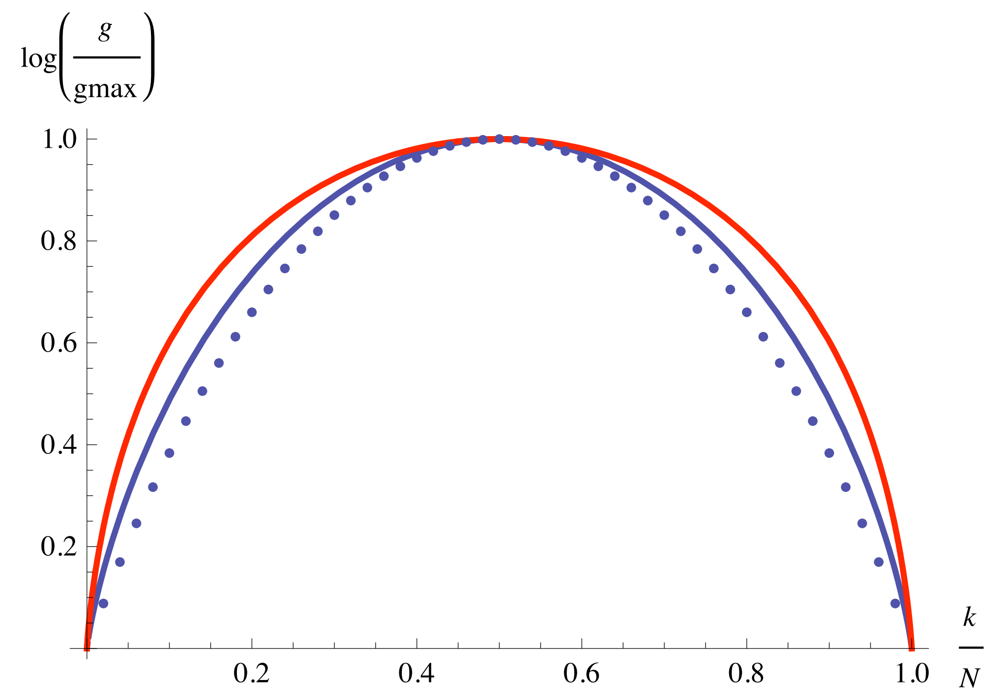


Figure 1: Impurity entropy as a function of k/N for the supersymmetric model (dotted curve) and nonsupersymmetric multichannel model with number of channels $K/N = 1$ (blue) and $K/N = 0.1$ (red).

* The plot is symmetric about k/N = 1/2 due to particle/hole symmetry

* We see the results for the SUSY model are closest to those for the standard multi-channel model with # of channels equal to N

* From the exact result, or its small k/N expansion

$$\log g = \frac{1}{2} k M \sqrt{\lambda} \left[1 - \frac{3}{10} \left(\frac{3\pi k}{2N} \right)^{2/3} - \frac{3}{280} \left(\frac{3\pi k}{2N} \right)^{4/3} + \ldots \right]$$

we see that the answer is far from being that of a free spin with integer number of possible spin states. This is also true of overscreened (but not underscreened) Kondo models.

Defect specific heat and susceptibility

In the "real" model, these vanish at the fixed point, and are governed by the leading irrelevant operator that would be present in the flow.

In our model too,

$$C_{\text{defect}} = -T \frac{\partial^2 F_{\text{defect}}}{\partial T^2}$$

will clearly vanish at the fixed point (even after backreaction), and so will be governed by the leading irrelevant operator.

We define susceptibility with respect to the "magnetic field" that couples to the SO(5) R-current:

$$S \supset \int d^4x \, \mathcal{A}_\alpha J_R^\alpha \qquad \chi_{\text{total}} \equiv \left. \frac{\partial^2 F}{\partial B^2} \right|_{B=0}$$

The defect susceptibility will vanish trivially in the probe approximation before including the leading irrelevant operator. Backreaction will change this.

* We should classify defect operator spectrum, find lowest dimension SO(3) x SO(5) singlet.

  * The system is highly symmetric, enjoying the $OSp(4^*|4)$ supergroup of symmetries. The lowest weight representations of this supergroup are classified.

Gunaydin, Scalise

The even subgroup of the supergroup is

$$SL(2,\mathbb{R}) \times SO(3) \times SO(5)$$

and the states are classified by the quantum numbers h,j,m1,m2 (the SL(2,R) dimension, the SO(3) spin, and the SO(5) Dynkin labels, respectively).

* Intuitively, in the limit we're working, we expect the lowest dimension operators to be the short multiplets of this algebra, of schematic form:

$$(\mathcal{O}^{(n)})_I^{\ J} = \sum_{i,j} \bar{\chi}_j^J \, (\phi^{a_1} \ldots \phi^{a_n})_i^{\ j} \, \chi_I^i$$

To see which sets of quantum numbers are present in the spectrum, we need to do a KK reduction of the D5 fluctuations. The anomalous dimensions are then determined via:

$$h^{scalar} = \frac{d}{2} + \frac{1}{2}\sqrt{d^2 + 4m^2}$$

(and its analogues for vectors and spinors).

The explicit spectrum is determined by linearising the D5 brane action

$$S_5 = -T_5 \int d^6\xi \, \sqrt{-\det(G+F)} + T_5 \int F \wedge C_4$$

around the embedding. The worldvolume metric and gauge field are:

$$ds^2_{D5} = R^2(ds^2_{AdS_2} + \sin^2\theta_k \, d\Omega_4^2)$$

$$F = \cos\theta_k \, e^0 \wedge e^1$$

and we consider fluctuations of the 6d gauge field, as well as $\delta u, \delta\theta$.

The calculations are unpalatable. The results (which are exact for chiral primary operators, even away from the probe limit, due to SUSY) are:

D5 field	defect operator	$SL(2,\mathbb{R})\times SO(3)\times SO(5)$
$(\delta\theta, f_{rt})^{(1)}_{l=1}$	$\mathcal{O}\equiv\bar{\chi}\phi_\perp\chi$	$(1,0;0,1)$
$\delta u_{l=0}$	$Q^2\mathcal{O}\sim\bar{\chi}(n^a D_\alpha\phi_a)\chi$	$(2,1;0,0)$
$(\delta\theta, f_{rt})^{(1)}_{l}$	$\mathcal{O}^{(l)}\equiv\bar{\chi}(\phi_\perp^{(a_1}\dots\phi_\perp^{a_l)})\chi$	$(l,0;0,l)$
δu_{l-1}	$Q^2\mathcal{O}^{(l)}\sim\bar{\chi}(n^a D_\alpha\phi_a\phi_\perp^{(a_1}\dots\phi_\perp^{a_{l-1})})\chi$	$(l+1,1;0,l-1)$
$(a_i)_l$	$Q^2\mathcal{O}^{(l)}\sim\bar{\chi}(\Gamma_i n^a\phi_a\,\phi_\perp^{[a_1}\phi_\perp^{(a_2]}\dots\phi_\perp^{a_l)})\chi$	$(l+1,0;2,l-2)$
$(\delta\theta, f_{rt})^{(2)}_{l}$	$Q^4\mathcal{O}^{(l)}\sim\bar{\chi}((n^a D_\alpha\phi_a)^2\phi_\perp^{(a_1}\dots\phi_\perp^{a_{l-2})})\chi$	$(l+2,0;0,l-2)$

There is one marginal operator, that transforms as an SO(5) vector; geometrically, it correponds to the fluctuation of D5 scalar fields that rotates the embedding of $SO(5)\subset SO(6)$.

In general, when the leading irrelevant operator $\mathcal{O}_0$ has dimension h_0 and we consider

$$S_{\text{defect}} \to S_{\text{defect}} + \int dt\,(\lambda_0 \mathcal{O}_0 + \text{h.c.})$$

(for defect operators with vanishing one-point function), we'll find:

$$C_{\text{defect}} \sim \left(\frac{T}{T_K}\right)^{2(h_0-1)}, \quad \chi_{\text{defect}} \sim \left(\frac{T}{T_K}\right)^{2(h_0-1)} \frac{1}{T}.$$

The overscreened Kondo model with N=K has $h_0 = 3/2$.

Here, instead the leading global singlet perturbation is:

$$(\chi^\dagger \phi_\perp \chi) \cdot (\chi^\dagger \phi_\perp \chi)\,,$$

of dimension two.

III. SUSY Kondo lattice models

* It would be very nice to also get a handle on the lattice models. They can be tied to non-Fermi liquids; in these gravity models, this is readily visible in the probe approximation.

* The probes naturally live on AdS_2 geometries.

* Such geometries are dual to "locally critical" sectors, that is sectors which enjoy dynamical scaling

$$x \to \lambda x, \ t \to \lambda^z t$$

with $z = \infty$.

* Fermions coupled to such locally critical sectors can naturally be deformed into non-Fermi liquids.

S.S. Lee;
Cubrovic, Schalm, Zaanen;
Liu, McGreevy, Vegh

* A good intuitive way to understand this was emphasized by Faulkner and Polchinski.

Consider a quantum field theory whose action takes the schematic form:

$$S = \quad S_{\text{strong}} + \sum_{J,J'} \int dt \ \left[c_J^\dagger (i\delta_{J,J'}\partial_t + \mu\delta_{J,J'} + t_{J,J'}) c_{J'} \right]$$
$$+g \sum_J \int dt \left[c_J^\dagger \mathcal{O}_J^F + (\text{Hermitian conjugate}) \right].$$

* There is a strongly coupled sector which we'll assume is a large N theory that we can describe using gravity.

* There is a free (lattice) fermion with a Fermi surface.

* Deform these field theories by coupling them together with coupling constant “g”.

* In perturbation theory in g, there is a simple set of graphs that correct the free fermion propagator:

* In the large N limit, this geometric series gives the exact result for the corrected c propagator.

Then, the resulting "dressed" c propagator can be written purely in terms of the two-point function of $\mathcal{O}$ in the strongly coupled sector:

$$G_g(\mathbf{k},\omega) \sim \frac{1}{\omega - v|\mathbf{k}-\mathbf{k}_F(\mathbf{k})| - g^2\mathcal{G}(\mathbf{k},\omega)} \ .$$

$$\mathcal{G}(\omega) = \int dt\, e^{i\omega t} \langle \mathcal{O}_J^F(t) \mathcal{O}_J^{F\dagger}(0)\rangle \ .$$

If we make the strong dynamical assumption that the strongly coupled sector exhibits local quantum criticality, then the two-point function is constrained:

$$\mathcal{G}(\omega) \ = \ c_\Delta \omega^{2\Delta-1}$$

* For any $\Delta < 1$ one obtains a non-Fermi liquid.

$$\Delta = 1 \rightarrow \mathcal{G} \sim \omega \log(\omega)$$

Varma et al, 1989

"Marginal Fermi liquid."

* In defect models, the lowest dimension operator coupled to "c" is often a defect-localised operator.

* Local criticality then automatic in probe approx.!

* Unfortunately, the D3/D5 system does not lead to an interesting non-Fermi liquid.

However, in a close relative (a natural analogue constructed using M2 branes and probe M2' branes, instead of D3 and D5 branes):

	0	1	2	3	4	5	6	7	8	9	10
M2	x	x	x								
$M2'$	x	::	::	x	x						

one can naturally obtain precisely the scalings required for the marginal Fermi liquid of Varma et al.

These theories involve defects (both fermionic and bosonic) coupled to the N=6 supersymmetric doubled Chern-Simons theories studied by ABJM.

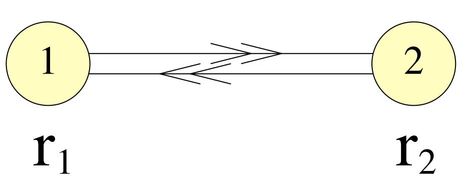
A rather general 3d N=2 supersymmetric Chern-Simons theory has a Lagrangian of the form:

$$S = \int d^3x \, \frac{k}{4\pi} \mathrm{Tr}(A \wedge dA + \frac{2}{3} A^3) + D_\mu \bar{\phi}_i D^\mu \phi_i + i \bar{\psi}_i \gamma^\mu D_\mu \psi_i$$
$$- \frac{16\pi^2}{k^2} (\bar{\phi}_i T^a_{R_i} \phi_i)(\bar{\phi}_j T^b_{Rj} \phi_j)(\bar{\phi}_k T^a_{R_k} T^b_{R_k} \phi_k)$$
$$- \frac{4\pi}{k} (\bar{\phi}_i T^a_{R_i} \phi_i)(\bar{\psi}_j T^a_{R_j} \psi_j) - \frac{8\pi}{k} (\bar{\psi}_i T^a_{R_i} \phi_i)(\bar{\phi}_j T^a_{R_j} \psi_j) \, .$$

Gaiotto, Yin; many earlier

* There can also be a superpotential.

* Our theory has groups and superpotential summarised by the quiver below:



$$W = \frac{2\pi}{k} \epsilon^{ab} \epsilon^{\dot{a}\dot{b}} Tr(A_a B_{\dot{a}} A_b B_{\dot{b}}) \, .$$

At each lattice point, the defect fields are "hypermultiplets" with quantum numbers:

-

$$Q_1\ (N,1),\ \ Q_2(1,N)$$

$$\tilde{Q}_1\ (\bar{N},1),\ \ \tilde{Q}_2(1,\bar{N})$$

They couple to the bulk ABJM fields with couplings of the schematic form:

$$\Delta S = \int dt \sum_i |(A_1B_1 - A_2B_2)Q_i|^2 + |(A_1B_2 - A_2B_1)Q_i|^2 + |(A_1B_2 + A_2B_1)Q_i|^2 \quad (6)$$

This class of theories can produce marginal Fermi liquid for the following simple reason. The most obvious defect-localised fermionic operator is of the form:

$$\tilde{\chi}_1 \psi_A \chi_2$$

* At weak coupling, this has h=1.

* Gravity analysis shows that this remains true at strong coupling; this is the "right value" to yield a marginal Fermi liquid in our previous discussion.

This theory is of course very unrealistic. One leading worry: backreaction.

Three comments on backreaction in lattice models:

1. Intuitively, we expect the backreaction to become important when we go deep enough into the IR to "see" many defects enclosed in our characteristic length scale.

We can estimate the temperature/energy scale at which this becomes important using the free energy (here in the D=3+1 case):

$$\mathcal{F} = N^2 T^4 + N \frac{T}{a^3}$$

By the time

$$T \leq N^{-1/3} \times \frac{1}{a}$$

the backreaction will surely be important; the defects now dominate the free energy of the dual field theory.

2. Backreaction almost surely eliminates the locally critical behavior evident in the probe approximation in these systems.

* This can be seen from the following (crude) energetics argument.

E.g. in the M2 case, we might look for a stable solution of the form:

$$AdS_2 \times T^2 \times X$$

Call the three radii of the factors in the geometry A, T and S. The 1+1 dimensional effective action governing the radions takes the schematic form:

$$\mathcal{S} = \int d^2x \left(-T^2 S^7 + A^2 T^2 S^5 - N_2' A^2 S - \frac{N_2^2 A^2 T^2}{S^7} \right) . \quad (17)$$

The four terms come from the AdS and internal curvatures; the M2' brane tensions; and the 7-form flux from the M2 branes. We have smeared the M2' branes, averaging their energy over the internal directions.

In this approximation one finds an AdS2 vacuum with:

$$A \sim S \sim N_2^{1/6}\,, \quad T \sim N_2'^{1/2}/N_2^{1/3}\,.$$

However, in the real system the branes are unsmeared and there is another preferred circle. Including its radion, we find no AdS2 vacua in the SUGRA regime.

3. There is a generic field theory argument that suggests that local criticality can never persist down to zero energy. The general form of the density of states, for a locally critical theory, should take the form:

$$\rho(E) = A\delta(E) + B/E$$

* B should be non-zero in a non-trivial theory

* But then $\int \rho(E)\ dE$ has an IR divergence.

* This should be cut off in a realistic system. But it is a logarithmic divergence, so locally critical behavior could conceivably persist down to exponentially low energies.

IV. SUSY Kondo model: including backreaction

Let us return to the single-site model, with M D5 branes and $g^2_{YM} M \gg 1.$

Can we find a smooth backreacted solution with no "probes"?

In the Kondo model itself, in the simplest cases, the fermionic defect "disappears" in the IR, just leaving a disturbance on a region of order the confinement scale to the behaviour of the bulk electrons.

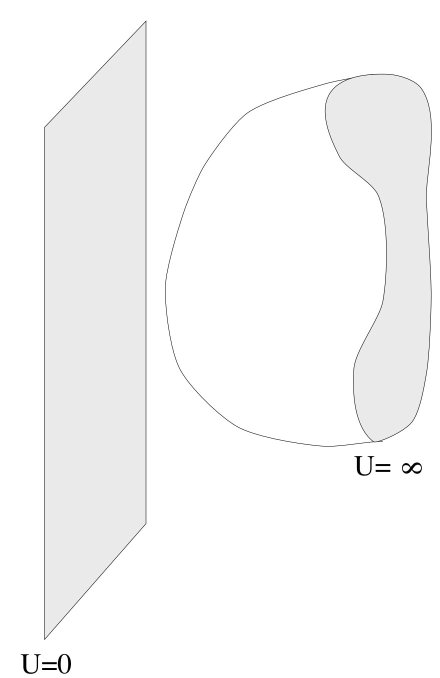
Could the D5 defects similarly "disappear" in our problem?

They would have to leave behind a signature of their D5 charge. This can happen; if a non-trivial three-sphere is created, M units of three-form flux could replace the D5s.

We'll see that this does happen. The D5 branes squash the 5-sphere so much that is splits into two, and replace themselves with three-form flux in a new smooth geometry.

In fact, the relevant supergravity solutions have already been found, by D'Hoker, Estes and Gutperle.

They were not studying impurity models. Their interest was BPS Wilson loops in maximally supersymmetric Yang-Mills theory.



But, the two problems turn out to be equivalent.

Yamaguchi; Hartnoll, Kumar; Gomis, Passerini

Lets sketch this equivalence in the simplest case, for the case of the k-fold antisymmetric representation of SU(N).

This is the case M=1, with k fermions present at the D3/D5 intersection.

* Recall that the action of our full gauge theory is:

$$S_{\text{field theory}} = S_{\mathcal{N}=4} + \int dt \left[i\chi_b^\dagger \partial_t \chi^b + \chi_b^\dagger \left\{ (A_0(t, \vec{0}))^b_c + v^I (\phi_I(t, \vec{0}))^b_c \right\} \chi^c \right],$$

* Choose a gauge where the combination $A_0 + v^I \phi_I$ has constant eigenvalues $(m_1, \ldots, m_N)$.

The equation of motion for the defect fermions is then:

$$(i\partial_t + m_i)\chi_i = 0 \;\;,\;\; i = 1, \ldots, N\,.$$

We wish to write a defect partition function summing only over the states with k fermions present. This is given by:

$$Z_{\rm defect} = \sum_{i_1<i_2<\ldots<i_k} e^{i\int dt\, m_{i_1}} \ldots e^{i\int dt\, m_{i_k}}\,,$$

But we can recognise this as the trace of the Wilson line in the kth antisymmetric representation of SU(N):

$$\sum_{i_1<i_2<\ldots<i_k} e^{i\int dt\, m_{i_1}} \ldots e^{i\int dt\, m_{i_k}} = {\rm Tr}_{A_k}\, P\, \exp\left(i\int dt(A_0 + n^a\phi_a)\right)\,.$$

I.e. integrating out the defect fermions produces a supersymmetric Wilson-loop insertion.

The representations are a bit more complicated for M > 1, but the same basic idea holds.

Most basic properties of DEG solutions

A natural ansatz for the metric building in the symmetries we are guaranteed to have, is to take:

$$\frac{ds^2}{R^2} = f_1^2 ds^2_{AdS_2} + f_2^2 d\Omega_2^2 + f_4^2 d\Omega^4 + d\Sigma^2$$

Here Σ is a Riemann surface with boundary, and the functions f vary over the surface.

For instance in the case of $AdS_5 \times S^5$

$$ds^2 = R^2 \left(du^2 + \cosh^2 u \, ds^2_{AdS_2} + \sinh^2 u \, d\Omega_2^2 + d\theta^2 + \sin^2 \theta \, d\Omega_4^2\right)$$

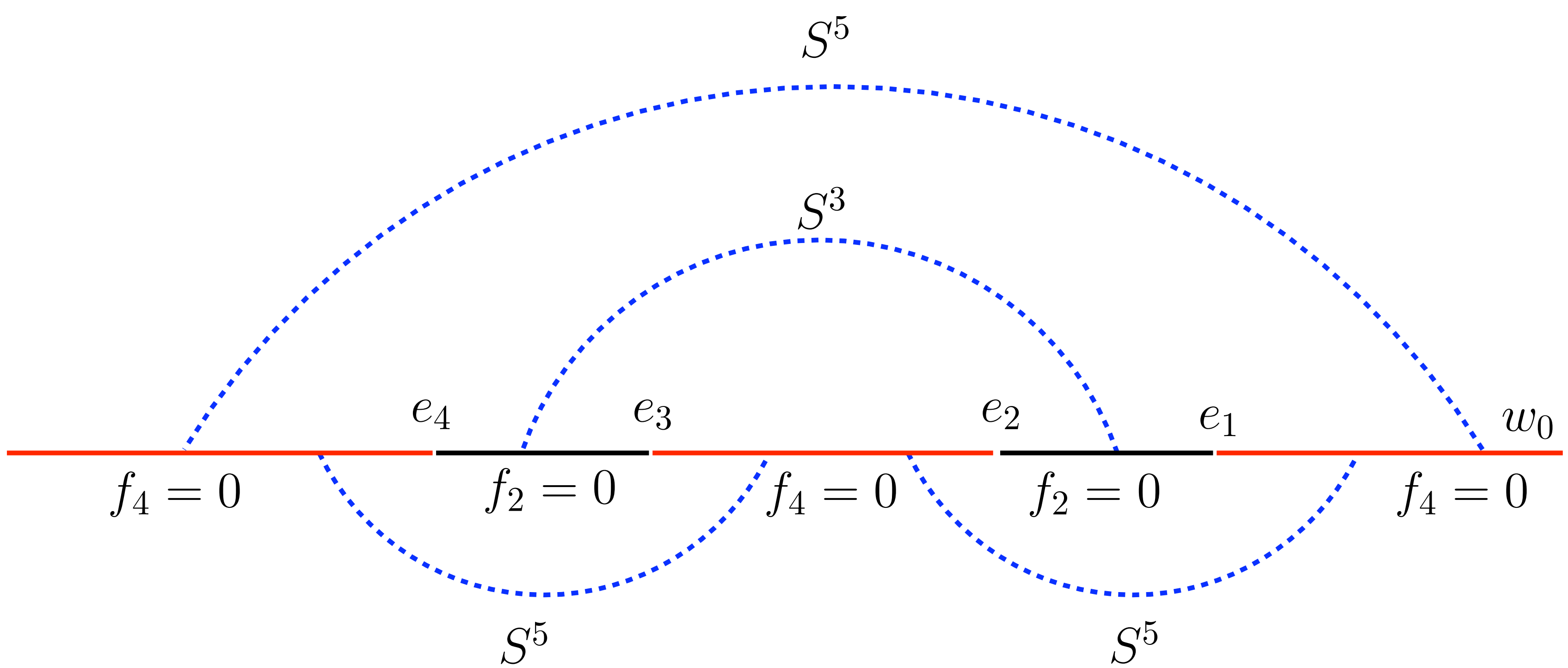
the Riemann surface is coordinatized by $u, \theta.$

The general solution is determined in terms of two real harmonic functions h_1, h_2 on Σ :

$$h_1^2 = \frac{1}{4} e^{-\phi} f_1^2 f_4^2 \; , \; h_2^2 = \frac{1}{4} e^{\phi} f_2^2 f_4^2$$

At each point on $\partial\Sigma$, one of the spheres shrinks.

We can therefore visualize the boundary as being divided into red and black segments, on which the four-sphere / two-sphere vanishes.



The non-trivial three-sphere is constructed by fibering two-spheres over a one-cycle connecting different black regions. The non-trivial five-spheres arise by fibering four-spheres over cycles connecting different red regions.

The full set of allowable solutions involves rather complicated "topology and regularity conditions" on the harmonic functions.

We will not discuss these conditions here.

The basic intuition should be clear: the boundary conditions on the harmonic functions are given by where they vanish at the boundary together with the nature of their pole at the AdS5 asymptotic, and they are then uniquely fixed. We give the explicit form of h for the one-stack transition, in our paper.

One can be painfully explicit about the solutions in terms of h. Introducing conformally flat coordinates on the Riemann surface

$$d\Sigma^2 = 4\rho^2 dv d\bar{v}$$

and defining the combinations

$$\begin{aligned} W &= \partial h_1 \bar{\partial} h_2 + \bar{\partial} h_1 \partial h_2 \\ V &= \partial h_1 \bar{\partial} h_2 - \bar{\partial} h_1 \partial h_2 \\ N_1 &= 2 h_1 h_2 |\partial h_1|^2 - h_1^2 W \\ N_2 &= 2 h_1 h_2 |\partial h_2|^2 - h_2^2 W \end{aligned}$$

one finds that the IIB supergravity fields are:

$$f_1 = \left(-4\sqrt{\frac{-N_2}{N_1}}h_1^4\frac{W}{N_1}\right)^{1/4}, \; f_2 = \left(-4\sqrt{\frac{-N_1}{N_2}}h_2^4\frac{W}{N_2}\right)^{1/4}$$

$$f_4 = \left(-4\sqrt{\frac{-N_2}{N_1}}\frac{N_2}{W}\right)^{1/4}, \; \rho = \left(-\frac{W^2N_1N_2}{h_1^4h_2^4}\right)^{1/8} .$$

$$e^{2\phi} = -\frac{N_2}{N_1} > 0 .$$

And writing $h_1 = \mathcal{A} + \overline{\mathcal{A}}, \;\; h_2 = \mathcal{B} + \overline{\mathcal{B}}\,,$

the fluxes are given by (for the case we drew, with I=1,2):

$$\int_{S^3} F_3 = 4\pi^2\alpha' M = 8\pi\int_{e_2}^{e_3} (i\partial\mathcal{A} + c.c.)$$

$$\int_{S_I^5} F_5 = 4\pi^4(\alpha')^2 N_I = 8\pi^2\int_{e_{2I}}^{e_{2I-1}} (\mathcal{A}\partial\mathcal{B} - \mathcal{B}\partial\mathcal{A} + c.c.) .$$

What's next

* We would like to "enjoy" the backreacted solutions (or bottom-up versions of similar solutions) by computing corrections to transport, e.g. looking for analogues of the famous "resistivity minimum" that initiated interest in the Kondo problem.

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* We would like to solve lattice models at the same level of explicitness. This is probably hard.

* The boundary CFT methods of Affleck and Ludwig, applied to s-wave reduction, might allow a direct solution of the maximally supersymmetric Kondo model. It would be fun to obtain this and compare to supergravity.

* There is a matrix model which captures the correlation functions of certain bulk operators in the presence of a Wilson loop; its eigenvalue distribution mimics beautifully aspects of the supergravity solution we sketched. It would be interesting to try and develop the matrix model to answer questions about correlators of boundary operators.

Gomis, Matsuura, Okuda, Trancanelli; Yamaguchi; Pestun; many earlier works