# Maximally Supersymmetric "Dirt"

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Based on work with: S. Harrison, G. Torroba arXiv:1110.5325 K. Jensen, A. Karch, J. Polchinski, E. Silverstein arXiv:1105.1772 + older papers with A. Karch, S. Yaida I. Introduction and motivation

The Kondo effect was in a sense the first example of a system exhibiting asymptotically free running of a coupling constant:

$$H = \sum_{\vec{k}\alpha} \psi_{\vec{k}}^{\dagger\alpha} \psi_{\vec{k}\alpha} \epsilon(k) + J\vec{S} \cdot \sum_{\vec{k}\vec{k'}} \psi_{\vec{k}}^{\dagger} \frac{\vec{\sigma}}{2} \psi_{\vec{k'}}$$

The effective coupling of the impurity spin to the itinerant electrons grows logarithmically at low energies

$$\lambda \equiv J\nu, \qquad \qquad \lambda(T) \approx \lambda + \lambda^2 \ln \frac{D}{T} + \dots$$

# leading to interesting phenomena at the Kondo temperature:

 $T_K \approx D \exp[-1/\lambda],$ 

# below which one electron "sacrifices itself" to neutralize the spin:



Variants of this model exhibit other interesting behaviours. One natural generalisation is the multi-channel model:

$$H = \sum_{\vec{p},i,\alpha} \epsilon(\vec{p}) \psi^{\dagger}_{\vec{p}\,i\alpha} \psi_{\vec{p}\,i\alpha} + J \sum_{\vec{p}\,\vec{p'}\,i\alpha\beta} S_{\alpha\beta} \,\psi^{\dagger}_{\vec{p}\,i\alpha} \psi_{\vec{p'}\,i\beta}$$

with i=1,...,K labelling channel, and alpha the index for the global SU(2) spin symmetry.

If the defect has spin s, then the IR fate depends on the # of channels compared to s:

> > c.f. exact solution by Affleck, Ludwig

Another interesting generalisation arises when instead of considering the impurity interacting with a free Fermi liquid, one considers a non-trivial bulk CFT (as would happen if one tunes such a system through a quantum critical point):



We will be considering such models in the context of gauge/gravity duality, momentarily.

A last and even more interesting generalisation is to consider the Kondo lattice model:

$$H = H_J + \sum_k \epsilon_k c_{k\alpha}^{\dagger} c_k^{\alpha} + \frac{J_K}{2} \sum_i \hat{S}_i^a c_{i\alpha}^{\dagger} (\sigma^a)_{\beta}^{\alpha} c_i^{\beta}.$$

Now competition between the Kondo interaction and RKKY spin-spin interactions, is thought to potentially explain the existence of phase diagrams like those of the heavy fermion metals:



We will be studying highly idealized models of this general sort in the talk today. The bulk will be a highly supersymmetric CFT, coupled supersymmetrically to the defect spin. There are many drawbacks to the supersymmetry, but it has the virtue of allowing us to reliably solve for some features of the physics, in some limits.

# Plan:

II. SUSY Kondo model: probe approximationIII. SUSY Kondo lattice model: probe approximationIV. SUSY Kondo model: including backreaction

II. The maximally supersymmetric Kondo model

We will be studying the system realised by the following configuration of D3 and D5 branes in type IIB superstring theory:

	0	1	2	3	4	5	6	7	8	9
N D3	×	$\times$	$\times$	$\times$						
M D5	×				×	×	×	×	×	
k F1	$  \times$									Х

 $S = S_{D3} + S_{D5} + S_{defect}$ 

$$S_{\text{defect}} = \int dt \, \left[ i \bar{\chi}_i^I \partial_t \chi_I^i + \bar{\chi}_i^I \left( A_0(t,0)_j^i + n_a \phi^a(t,0)_j^i \right) \chi_I^j + \bar{\chi}_i^I (\tilde{A}_0)_I^J \chi_J^i - k(\tilde{A}_0)_I^I \right]$$

# In the standard supergravity limit, this system is dual to N=4 SYM coupled to a defect fermion with:

$$S_{\text{field theory}} = S_{\mathcal{N}=4} + \int dt \left[ i\chi_b^{\dagger} \partial_t \chi^b + \chi_b^{\dagger} \left\{ (A_0(t, \overrightarrow{0}))_c^b + v^I(\phi_I(t, \overrightarrow{0}))_c^b \right\} \chi^c \right],$$
$$\sum_{\alpha=1}^N \chi_\alpha^{\dagger} \chi_\alpha = k.$$

The bosonic symmetries preserved by the defect are:

$$SL(2,\mathbb{R}) \times SO(3) \times SO(5)$$

It is useful to write the  $AdS_5 \times S^5$  metric in a way that makes these symmetries manifest:

$$ds^{2} = R^{2} \left( du^{2} + \cosh^{2} u \, ds^{2}_{AdS_{2}} + \sinh^{2} u \, d\Omega^{2}_{2} + d\theta^{2} + \sin^{2} \theta \, d\Omega^{2}_{4} \right)$$

# In the probe approximation $M \leq N$ , the D5 worldvolume is an given by the embedding conditions: $(\tau, \rho, \theta_1, \theta_2, \theta_3, \phi_4) \mapsto (\tau, \rho, 0, \theta_{\nu}, \theta_1, \theta_2, \theta_3, \phi_4)$ with $\rho \in (0, \infty)$ and $\tau \in (-\infty, \infty)$ ,



Figure 1: The points of the  $S^{8-p}$  sphere with the same polar angle  $\theta$  define a  $S^{7-p}$  sphere. The angle  $\theta$  represents the latitude on  $S^{8-p}$ , measured from one of its poles.

Wednesday, November 201 allowed angles are:

$$k = \frac{N}{\pi} \left( \theta_k - \frac{1}{2} \sin 2\theta_k \right)$$

The defect free energy and entropy can be computed by evaluating the DBI action immersed in the AdS black brane:

$$ds^{2} = -f(r)dt^{2} + \frac{dr^{2}}{f(r)} + \frac{r^{2}}{R^{2}} \left(\sum_{i=1}^{3} dx_{i}^{2}\right) + R^{2}(d\theta^{2} + \sin^{2}\theta \, d\Omega_{4}^{2}) ,$$

$$f(r) = \frac{r^2}{R^2} \left( 1 - \frac{r_+^4}{r^4} \right) \,.$$

(The field theory temperature is given by  $T = r_+/\pi R^2$ .)

Regularizing by subtracting the Euclidean action of the analogous D5 in pure AdS space-time, one finds for a single brane:

$$F_{\text{defect}} = -\sqrt{\lambda} \, \frac{\sin^3 \theta_k}{3\pi} NT$$

The impurity entropy or "g-function" is defined by:

$$\log g = \mathcal{S}_{\rm imp} \equiv \lim_{T \to 0} \lim_{V \to \infty} \left[ \mathcal{S}(T) - \mathcal{S}_{\rm ambient}(T) \right]$$

In this case, now restoring M, we find:

$$\log g = \mathcal{S}_{\rm imp} = \sqrt{\lambda} \, \frac{\sin^3 \theta_k}{3\pi} MN$$

By way of comparison, the multi-channel Kondo model with K channels and SU(N) spin symmetry (@ large N), with a defect in the kth antisymmetric representation, has:

$$\mathcal{S}_{imp} = \frac{2}{\pi} MN \left[ f\left(\frac{\pi}{1+K/N}\right) - f\left(\frac{\pi}{1+K/N}(1-k/N)\right) - f\left(\frac{\pi}{1+K/N}k/N\right) \right]$$

$$f(x) = \int_0^x du \, \log \sin u$$

Parcollet, Georges, Kotliar, Sengupta



Figure 1: Impurity entropy as a function of k/N for the supersymmetric model (dotted curve) and nonsupersymmetric multichannel model with number of channels K/N = 1 (blue) and K/N = 0.1 (red).

# \*The plot is symmetric about k/N = 1/2 due to particle/ hole symmetry

# \*We see the results for the SUSY model are closest to those for the standard multi-channel model with # of channels equal to N

\* From the exact result, or its small k/N expansion

$$\log g = \frac{1}{2} k M \sqrt{\lambda} \left[ 1 - \frac{3}{10} \left( \frac{3\pi k}{2N} \right)^{2/3} - \frac{3}{280} \left( \frac{3\pi k}{2N} \right)^{4/3} + \dots \right]$$

we see that the answer is far from being that of a free spin with integer number of possible spin states. This is also true of overscreened (but not underscreened) Kondo models.

#### Defect specific heat and susceptibility

In the "real" model, these vanish at the fixed point, and are governed by the leading irrelevant operator that would be present in the flow. In our model too,

$$C_{\rm defect} = -T \frac{\partial^2 F_{\rm defect}}{\partial T^2}$$

will clearly vanish at the fixed point (even after backreaction), and so will be governed by the leading irrelevant operator.

We define susceptibility with respect to the "magnetic field" that couples to the SO(5) R-current:

$$S \supset \int d^4x \,\mathcal{A}_{\alpha} J^{\alpha}_R \qquad \qquad \chi_{\text{total}} \equiv \left. \frac{\partial^2 F}{\partial B^2} \right|_{B=0}$$

The defect susceptibility will vanish trivially in the probe approximation before including the leading irrelevant operator. Backreaction will change this.

### perator of lowest dimension arises from an ultrashort supermultiplet of ull supermultiplet is \*We should classify defect operator spectrum, find lowest dimension SO(3) $\times$ SO(5) $\oplus$ (21:0,0).

rst state has dimension 1, is an SO(3) singlet and transforms as a vect next\*sTate system is in glidyn symmetric and joying the  $OSp(4^{*}e)$  references in SO(5) and SO(5) efferolate of a symmetric and SO(5) efferolate of a symmetric and SO(5) efferolate of a symmetric being by effective and SO(5) in purpresental tions not obtain the superpresentation of the supergroup is (f, 0; 0, f) + (f + 1/2, 1/2; 1, f - 1) + (f + 1, 1; 0, f - 1) + (f + 1, 0; 2, f + SD(2) + SO(3))

and the states are classified by the quantum numbers Db, junct, m2: (the Su(RR), dimension, the SO(3) spin, and the SO(5) Dynkin labels, respectively).

Monday, November 7, 2011 d the multiplet structure of the superconformal algebra, the

representations of this supergroup were analyzed by Gunaydin and Scalise some time ago.

\* Intuitively, in the limit we're working, we expect the lowest dimension operators to be the short multiplets of this algebra, of schematic form:

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To see which sets of quantum numbers are present in the spectrum, we need to do a KK reduction of the D5 fluctuations. The anomalous dimensions are then determined via:

$$h^{scalar} = \frac{d}{2} + \frac{1}{2}\sqrt{d^2 + 4m^2}$$

(and its analogues for vectors and spinors).

The explicit spectrum is determined by linearising the D5 brane action

$$S_5 = -T_5 \int d^6\xi \sqrt{-\det(G+F)} + T_5 \int F \wedge C_4$$

around the embedding. The worldvolume metric and gauge field are:

$$ds_{D5}^2 = R^2 (ds_{AdS_2}^2 + \sin^2 \theta_k \, d\Omega_4^2)$$

 $F = \cos \theta_k \, e^0 \wedge e^1$ 

and we consider fluctuations of the 6d gauge field, as well as  $\delta u, \delta \theta$ .

# The calculations are unpalatable. The results (which are exact for chiral primary operators, even away from the probe limit, due to SUSY) are:

D5 field	defect operator	$SL(2,\mathbb{R}) \times SO(3) \times SO(5)$
$(\delta\theta, f_{rt})_{l=1}^{(1)}$	${\cal O}\equiv ar\chi\phi_{\perp}\chi$	(1, 0; 0, 1)
$\delta u_{l=0}$	$Q^2 \mathcal{O} \sim \bar{\chi} (n^a D_\alpha \phi_a) \chi$	(2, 1; 0, 0)
$(\delta\theta, f_{rt})_l^{(1)}$	$\mathcal{O}^{(l)} \equiv \bar{\chi}(\phi_{\perp}^{(a_1}\dots\phi_{\perp}^{a_l})\chi$	(l,0;0,l)
$\delta u_{l-1}$	$Q^2 \mathcal{O}^{(l)} \sim \bar{\chi} (n^a D_\alpha \phi_a \phi_\perp^{(a_1} \dots \phi_\perp^{a_{l-1})}) \chi$	(l+1, 1; 0, l-1)
$(a_i)_l$	$Q^2 \mathcal{O}^{(l)} \sim \bar{\chi} (\Gamma_i n^a \phi_a  \phi_\perp^{[a_1} \phi_\perp^{(a_2]} \dots \phi_\perp^{a_l)}) \chi$	(l+1, 0; 2, l-2)
$(\delta  heta, f_{rt})_l^{(2)}$	$Q^4 \mathcal{O}^{(l)} \sim \bar{\chi}((n^a D_\alpha \phi_a)^2 \phi_\perp^{(a_1} \dots \phi_\perp^{a_{l-2})})\chi$	(l+2, 0; 0, l-2)

There is one marginal operator, that transforms as an SO(5) vector; geometrically, it correponds to the fluctuation of D5 scalar fields that rotates the embedding of  $SO(5) \subset SO(6)$ .

In general, when the leading irrelevant operator  $O_0$  has dimension  $h_0$  and we consider

$$S_{\text{defect}} \to S_{\text{defect}} + \int dt \left(\lambda_0 \mathcal{O}_0 + \text{h.c.}\right)$$

(for defect operators with vanishing one-point function), we'll find:

$$C_{\text{defect}} \sim \left(\frac{T}{T_K}\right)^{2(h_0-1)} , \ \chi_{\text{defect}} \sim \left(\frac{T}{T_K}\right)^{2(h_0-1)} \frac{1}{T}$$

The overscreened Kondo model with N=K has  $h_0 = 3/2$ . Here, instead the leading global singlet perturbation is:  $(\chi^{\dagger}\phi_{\perp}\chi) \cdot (\chi^{\dagger}\phi_{\perp}\chi),$ 

#### of dimension two.

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### III. SUSY Kondo lattice models

\* It would be very nice to also get a handle on the lattice models. They can be tied to non-Fermi liquids; in these gravity models, this is readily visible in the probe approximation.

\*The probes naturally live on  $AdS_2$  geometries.

\* Such geometries are dual to "locally critical" sectors, that is sectors which enjoy dynamical scaling

$$x \to \lambda x, \ t \to \lambda^z t$$

with  $z = \infty$ .

\* Fermions coupled to such locally critical sectors can naturally be deformed into non-Fermi liquids. S.S.Lee: Cubrovic, Schalm, Zaanen; Liu, McGreevy, Vegh

\* A good intuitive way to understand this was emphasized by Faulkner and Polchinski.

Consider a quantum field theory whose action takes the schematic form:

$$S = S_{\text{strong}} + \sum_{J,J'} \int dt \left[ c_J^{\dagger} (i\delta_{J,J'}\partial_t + \mu\delta_{J,J'} + t_{J,J'}) c_{J'} \right] + g \sum_J \int dt \left[ c_J^{\dagger} \mathcal{O}_J^F + (\text{Hermitian conjugate}) \right].$$

\* There is a strongly coupled sector which we'll assume is a large N theory that we can describe using gravity.

\* There is a free (lattice) fermion with a Fermi surface.

\* Deform these field theories by coupling them together with coupling constant "g".

\* In perturbation theory in g, there is a simple set of graphs that correct the free fermion propagator:

\_\_\_\_\_ + \_\_\_\_\_\_ + \_\_\_\_\_\_ + ...

\* In the large N limit, this geometric series gives the exact result for the corrected c propagator.

function for the the manipulation of the transfer of the transfer the transfer of the transfer clude that we can w Using large N factorization, it is then easy to show that the a marginal Fermi li exhibiting the strange metallic behavior discussed in Refs. [6, 8]. The calculation has second Ganio Red perator in Written iz puper ty ind terms of dimension  $\mathcal{O}_{in}$  the point of the property of the prope on each other. Thus tree-level mixing diagrams. single impurity inter  $G_g(\mathbf{k},\omega) \sim \frac{1}{\omega - v|\mathbf{k} - \mathbf{k}_F(\mathbf{k})| - g^2 \mathcal{G}(\mathbf{k},\omega)},$  $G_g(\mathbf{k},\omega) \approx \frac{1}{\omega - v|\mathbf{k} - \mathbf{k}_F(\mathbf{k})| - g^2 \mathcal{G}(\mathbf{k},\omega)},$ (14)gravity side exhibits Wedne branes each wrap an where In particular, for  $\mathcal{G}(\mathbf{k}, \omega) = c\omega^{2\Delta-1}$  with  $\Delta \leq 1$ , one finds a dominant low-frequence length of the critical interval of  $\mathcal{G}(\mathbf{k}, \omega) = c\omega^{2\Delta-1}$  with  $\Delta \leq 1$ , one finds a dominant low-frequence of the comparison o characteristic of a non-Ferm (ic) is which has value of the side of the second contrivial field theory is the second contribution of the second contributio behaviour]. For  $\Delta > 1$ , the residue does not vanish, but the theory is still novel in that th we make the strags dynamical as yon the that the strongly compled fector conicites society, white a lity, zero-tempthen Cthe's two-point function is constrained. true (fiai)e-Inuthe-tphaseture)Taheroperator lives on an slice Now, we are in a position for the bulking expression the two point point point advocated in [8]: in holographic tions are hoodstpained to the have tas: and Sec. II, the phase transition also drives an interesting transition if the structure of the |Monday, November 7, 2011 Ce. The main point is that the low frequency behavior of the Green's function

\* For any  $\Delta < 1$  one obtains a non-Fermi liquid.

 $\Delta = 1 \rightarrow \mathcal{G} \sim \omega \log(\omega)$ 

Varma et al, 1989

"Marginal Fermi liquid."

\* In defect models, the lowest dimension operator coupled to "c" is often a defect-localised operator.

\* Local criticality then automatic in probe approx.!

\* Unfortunately, the D3/D5 system does not lead to an interesting non-Fermi liquid.

(Z I)Acknowledgments We would like to thank S. Kivelson, However, Min Multionse Srelative (na hattarnal fan aloguesconstructed using M2 branes and proper M2, s.k., A.K. and E.S. acknowledge the hospi-D3 and proper M2, branes instead of D3 and (28)progress. S.K. also a cobrages he hospitality of the organisers of the 5th Asian Winter School at Jeju Island, and ween the  $\eta$ thanks the participants for asking many interesting questions criticality. about related subjects. This research was supported in part by ut  $A_0$  and the National Science Foundation under grants PHY-02-44728, not tied to PHY05-51164<sup>M2/</sup> and  $^{2}PHY07-57035$ , and by the DOE under oling term contracts DE-AC03-76SF00515 and DE-FG02-96ER40956. lets. [31] haturally obtain precisely the scalings required for it**one**thean g, where a the main give Ferrit miquid of Varma et al. To begin let us consider a variant of the construction of [11], is imporen the 3d who studied the brane configuration overns the set theories involve defects (both fermionic and bosonic) coupled  $\overline{159}$  the  $\sqrt{3}$  supersymmetric 3 oubled y to com-Chern-Simons theories studied by ABJM. pling the ns theory As before, an x indicates a direction in which the given branes mple tov-Monday, November 7, 2011 Are extended, and a :: indicates a direction in which they are in

s of the opervolves t lattice of d bersymmetrielen gennennts on the stranger with the second of the second cting D3 and the selection of the state of the selection sis for t  $\times S^4$  regionsken by the orbifolding and the presence of the N12' probes; added to D3-branceson SC(2) KdSC(2) to School 3 to (1) by the to the school and the school of the school and the school of is that t del is som nionic defected town to SU Gaiotto, Tin, Ty e  $(\psi_{1}(4))^{2} \times \mathcal{Z}_{4}$  for  $k \geq 4$ . The  $\mathcal{Z}_{4}$  factor many tearlier mee on a difference represents the same of the better. The here is the such 4), it follows from the analysis in [17] that njoys a matrix  $k_{k}^{3} \gg 1$ es of our dis- $-\langle\langle\psi_i T P_{B_i}^a, \psi_i\rangle\rangle\langle\psi_j T P_{B_i}^a, \psi_j\rangle$ eous bosonic (4)Wednesday, May 18, 2011 Xamples Vay 18, 2011  $\mathcal{F}_{i}$  in  $\mathcal{F}_{i}$  is  $\mathcal{F}_{i}$  and  $\mathcal{F}_{i}$  is  $\mathcal{F}_{i}$  and  $\mathcal{F}_{i}$  are superpartners figuration we in a chiral multiplet. These terms arise from integrating out nes: \* Our the or the scalars and fermions of the massive vector multiplet and flowing to the deep infrared limit of the theory. The field theory theory will a set of this (1)theory, with gauge groups  $U(N) \times U(N)$  appearing at levels  $\pm k$ . The 't Hooft coupling of this theory is N/k and so is large in the holographic limits. The matter fields  $\phi_i$  are e given brane four bi-fundamental fields  $A_{1,2}$  and  $B_{1,2}$ , in the  $(N, \overline{N})$  and en branes are  $(\bar{N}, N)$  representations  $\underline{\mathcal{H}}_{Aab}$ imensions in supersymmetric action written above for these fields, we add d but form a an  $\mathcal{N} = 3$  superpotential

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(anu is equivalent to the one in (1). We treat even and odd k symfrare At eachnatively posint, the defedd fields tardentif perintultiplets? different, mirror branes (rather than taking the  $g_k^{k/2}$  element volv to identify points on the same brane in the case k even). ing t sis fo The global symmetry of the M2-brane theory is partially broken by the orbifolding and the presence of the M2' probes; adde from  $SO(8) \times SO(2)$  to  $SO(6) \times U(1) \times Z_4$  for k = 1, is the and down to  $SU(2) \underset{O_1}{\times} U(1)^2 \times \underset{O_2}{\times} \underset{O_2}{\times} \underset{V_1}{\times} \underset{N}{\times} k > 1$ . The  $Z_4$  factor theor here represents the symmetry of the lattice. At large k (such tum that  $k^5 \gg N \gg 1$ ), it follows from the analysis in [17] that trans They couple to the bulk ABJM fields with couplings of the schematic form: Wednesday, May 18, 2011

 $\Delta S = \int dt \sum_{i} |(A_1 B_1 - A_2 B_2) Q_i|^2 + |(A_1 B_2 - A_2 B_1) Q_i|^2 + |(A_1 B_2 - A_2 B_1) Q_i|^2 + |(A_1 B_2 + A_2 B_1) Q_i|^2$ (6)

This class of theories can produce marginal Fermi liquid for the following simple reason. The most obvious defectlocalised fermionic operator is of the form:

 $\tilde{\chi}_1 \psi_A \chi_2$ 

### \*At weak coupling, this has h=1.

\* Gravity analysis shows that this remains true at strong coupling; this is the "right value" to yield a marginal Fermi liquid in our previous discussion.

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This theory is of course very unrealistic. One leading dimensions  $\Delta$  of scalar operators localized at the lattice sites as spinors in our construction, to the backsee of step KK modes arising in the M2' brane world-volume action, via the formula Three comments on backreaction in lattice models:

I. Intuitively, we expect the backreaction to become important when we go deep enough into the IR to "see" many defects enclosed in our characteristic length scale.

We can estimate the temperature/energy scale at which this becomes important using the free energy (here in the D=3+1 case):

$$\mathcal{F} = N^2 T^4 + N \frac{T}{a^3}$$

By the time

$$T \le N^{-1/3} \times \frac{1}{a}$$

the backreaction will surely be important; the defects now dominate the free energy of the dual field theory.

2. Backreaction almost surely eliminates the locally critical behavior evident in the probe approximation in these systems.

\*This can be seen from the following (crude) energetics argument.

relate to mose we have found at higher energy? cientiy uni Staying in the limit of strong 't Hooft coupling, good appro E.g. in the M2/geased we might have the formeary stable solution rak su the problem of finding the supergravity solution with backreat least with action. This can still be a challenging problem, but one can circle). It get insight from a simple energetics argument. We start with ration, for the M theory brane configuration (1). We are looking for an uration, an IR geometry  $AdS_2 \times R^2 \times X$ , which we will for convenience far enough compactify to  $AdS_2 \times T^2 \times X$ . We study this with the Ansatz typical tran Call the three radiie of inthe factors inty the logo metry A, the ande S. The intersective radii of the three factors  $AdS_2 \times T^2 \times S^2$ . The effect was disconstitutes at the schematic of the second states at the schematic of the second states at the second unstable sa the form is no a pri-Wednesday, May 18, 2011

$$\mathcal{S} = \int d^{2}x \left( -\frac{T^{2}S^{7} + A^{2}T^{2}S^{5} - N'_{2}A^{2}S}{T^{2}S^{7} + A^{2}T^{2}S^{5} - N'_{2}A^{2}S} - \frac{N^{2}A^{2}A^{2}T^{2}}{S^{7}} \right).$$

We work in units where the M theory scale is one, and ignore order one coefficients. The respective terms come from the curvatures of  $AdS_2$  and  $S^7$ , the 2'-brane tensions, and the 7form flux from the 2-branes. In other situations it would be natural to Weyl transform to an effective potential, but this is not possible for  $AdS_2$ ; instead we directly extremize with that might l from this contrivors contriside. As an ast also be ant ing for soluthe 3-4 dire

In Refs. [22

out having

<sup>2011</sup> respect to A in addition to T and S.

We work in units where the M theory scale is one, and ignore vors contribute to the vork in units where the M theory scale is one and ignore vors contribute to the vork in units where the M theory scale is one and ignore vors contribute to order one coefficients. The *J*-portion of the *J*-portio (1/) from this construction vors contribution  $A \stackrel{A}{\sim} \stackrel{S}{\sim} \stackrel{S}{\sim} \stackrel{N_{2}^{1/6}}{N_{2}^{1/6}} \stackrel{T}{\sim} \stackrel{T}{\sim} \stackrel{N_{2}^{\prime 1/2}}{N_{2}^{\prime 1/2}} \stackrel{N_{2}^{\prime 1/2}}{N_{2}^{1/3}} \stackrel{(18)}{(18)}$ out on their model of the second of the provided of the second However that is happening is that the fattice defects provide a force such impurity states of the second provide a force such impurity states acting against the contraction of the two spatial dimensions, Orbifolding by the the second provide a force of the second provide provide a force of the second provide provid Orbifolding by the bulk modes are locally critical. In the probe approxima- theory units tion, the innerant fields retained their relativistic scaling, and  $\Lambda$  A A Sednesday, May 18, 20 Gach independent impurity was invariant under a scale trans- $T \mathcal{L} \mathbb{N}$ formation leaving its position fixed effecte reais a common locardly critical analing of the whole geometry. andandsinstringt This statult in encouragingut but we should prope the AAS Monday, November 7, 2011 neat my We have averaged the action of the 12<sup>t</sup> branes over

3. There is a generic field theory argument that suggests that local criticality can never persist down to zero energy. The general form of the density of states, for a locally critical theory, should take the form:

$$\rho(E) = A\delta(E) + B/E$$

\* B should be non-zero in a non-trivial theory Sunday, July 3, 2011 \* But then  $\int \rho(E) dE$  has an IR divergence.

\*This should be cut off in a realistic system. But it is a logarithmic divergence, so locally critical behavior could conceivably persist down to exponentially low energies.

IV. SUSY Kondo model: including backreaction

Let us return to the single-site model, with M D5 branes and  $g_{YM}^2 M \gg 1$ .

Can we find a smooth backreacted solution with no "probes"?

In the Kondo model itself, in the simplest cases, the fermionic defect "disappears" in the IR, just leaving a disturbance on a region of order the confinement scale to the behaviour of the bulk electrons.

# Could the D5 defects similarly "disappear" in our problem?

They would have to leave behind a signature of their D5 charge. This can happen; if a non-trivial three-sphere is created, M units of three-form flux could replace the D5s.

We'll see that this does happen. The D5 branes squash the 5-sphere so much that is splits into two, and replace themselves with three-form flux in a new smooth geometry. In fact, the relevant supergravity solutions have already been found, by D'Hoker, Estes and Gutperle.

They were not studying impurity models. Their interest was BPS Wilson loops in maximally supersymmetric Yang-Mills theory.



The equation of motion for the defect fermions is then:

$$(i\partial_t + m_i)\chi_i = 0$$
,  $i = 1, \ldots, N$ .

We wish to write a defect partition function summing only over the states with k fermions present. This is given by:

$$Z_{\text{defect}} = \sum_{i_1 < i_2 < \dots < i_k} e^{i \int dt \, m_{i_1}} \dots e^{i \int dt \, m_{i_k}} \,,$$

But we can recognise this as the trace of the Wilson line in the kth antisymmetric representation of SU(N):

$$\sum_{i_1 < i_2 < \dots < i_k} e^{i \int dt \, m_{i_1}} \dots e^{i \int dt \, m_{i_k}} = \operatorname{Tr}_{A_k} P \, \exp\left(i \int dt (A_0 + n^a \phi_a)\right)$$

I.e. integrating out the defect fermions produces a supersymmetric Wilson-loop insertion.

The representations are a bit more complicated for M > I, but the same basic idea holds.

Most basic properties of DEG solutions

A natural ansatz for the metric building in the symmetries we are guaranteed to have, is to take:

$$\frac{ds^2}{R^2} = f_1^2 ds_{AdS_2}^2 + f_2^2 d\Omega_2^2 + f_4^2 d\Omega^4 + d\Sigma^2$$

Here  $\Sigma$  is a Riemann surface with boundary, and the functions f vary over the surface.

#### For instance in the case of $AdS_5 \times S^5$

 $ds^{2} = R^{2} \left( du^{2} + \cosh^{2} u \, ds^{2}_{AdS_{2}} + \sinh^{2} u \, d\Omega^{2}_{2} + d\theta^{2} + \sin^{2} \theta \, d\Omega^{2}_{4} \right)$ 

the Riemann surface is coordinatized by  $u, \theta$ .

The general solution is determined in terms of two real harmonic functions  $h_1, h_2$  on  $\Sigma$ :

$$h_1^2 = \frac{1}{4} e^{-\phi} f_1^2 f_4^2 \ , \ h_2^2 = \frac{1}{4} e^{\phi} f_2^2 f_4^2$$

At each point on  $\partial \Sigma$ , one of the spheres shrinks.

We can therefore visualize the boundary as being divided into red and black segments, on which the four-sphere / two-sphere vanishes.



The non-trivial three-sphere is constructed by fibering two-spheres over a one-cycle connecting different black regions. The non-trivial five-spheres arise by fibering fourspheres over cycles connecting different red regions. The full set of allowable solutions involves rather complicated "topology and regularity conditions" on the harmonic functions.

We will not discuss these conditions here.

The basic intuition should be clear: the boundary conditions on the harmonic functions are given by where they vanish at the boundary together with the nature of their pole at the AdS5 asymptotic, and they are then uniquely fixed. We give the explicit form of h for the one-stack transition, in our paper. One can be painfully explicit about the solutions in terms of h. Introducing conformally flat coordinates on the Riemann surface

$$d\Sigma^2 = 4\rho^2 dv d\bar{v}$$

### and defining the combinations

$$W = \partial h_1 \bar{\partial} h_2 + \bar{\partial} h_1 \partial h_2$$
$$V = \partial h_1 \bar{\partial} h_2 - \bar{\partial} h_1 \partial h_2$$
$$N_1 = 2h_1 h_2 |\partial h_1|^2 - h_1^2 W$$
$$N_2 = 2h_1 h_2 |\partial h_2|^2 - h_2^2 W$$

#### one finds that the IIB supergravity fields are:

$$f_{1} = \left(-4\sqrt{\frac{-N_{2}}{N_{1}}}h_{1}^{4}\frac{W}{N_{1}}\right)^{1/4}, f_{2} = \left(-4\sqrt{\frac{-N_{1}}{N_{2}}}h_{2}^{4}\frac{W}{N_{2}}\right)^{1/4}$$
$$f_{4} = \left(-4\sqrt{\frac{-N_{2}}{N_{1}}}\frac{N_{2}}{W}\right)^{1/4}, \rho = \left(-\frac{W^{2}N_{1}N_{2}}{h_{1}^{4}h_{2}^{4}}\right)^{1/8}.$$

$$e^{2\phi} = -\frac{N_2}{N_1} > 0 \; .$$

And writing  $h_1 = \mathcal{A} + \overline{\mathcal{A}}, h_2 = \mathcal{B} + \overline{\mathcal{B}},$ 

the fluxes are given by (for the case we drew, with I=I,2):

$$\int_{S^3} F_3 = 4\pi^2 \alpha' M = 8\pi \int_{e_2}^{e_3} (i\partial \mathcal{A} + c.c.)$$

$$\int_{S_I^5} F_5 = 4\pi^4 (\alpha')^2 N_I = 8\pi^2 \int_{e_{2I}}^{e_{2I-1}} (\mathcal{A}\partial\mathcal{B} - \mathcal{B}\partial\mathcal{A} + c.c.) \; .$$

### What's next

\*We would like to "enjoy" the backreacted solutions (or bottom-up versions of similar solutions) by computing corrections to transport, e.g. looking for analogues of the famous "resistivity minimum" that initiated interest in the Kondo problem.

\*We would like to solve lattice models at the same level of explicitness. This is probably hard.

\* The boundary CFT methods of Affleck and Ludwig, applied to s-wave reduction, might allow a direct solution of the maximally supersymmetric Kondo model. It would be fun to obtain this and compare to supergravity. \* There is a matrix model which captures the correlation functions of certain bulk operators in the presence of a Wilson loop; its eigenvalue distribution mimics beautifully aspects of the supergravity solution we sketched. It would be interesting to try and develop the matrix model to answer questions about correlators of boundary operators.

> Gomis, Matsuura, Okuda, Trancanelli; Yamaguchi; Pestun; many earlier works