Semi-local Quantum Liquids

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Nabil Iqbal, HL, Mark Mezei, arxiv:1105.4621

Nabil Iqbal, HL, Mark Mezei, arxiv:1108.0425

Holographic non-Fermi liquids:

HL, John McGreevy, David Vegh, 0903.2477 (PRD)
Tom Faulkner, HL, JM, DV, 0907.2694 (PRD)
TF, Nabil Iqbal, HL, JM, DV, 1003.1728 (Science)

Holographic quantum phase transitions:

Nabil Iqbal, HL, Mark Mezei, Qimiao Si arxiv:1003.0010 (PRD)

Some related papers:

Sung-Sik Lee, 0809.3402, Cubrovic, Zaanen, Schalm, 0904.1933

Faulkner, Polchinski, 1001.5049

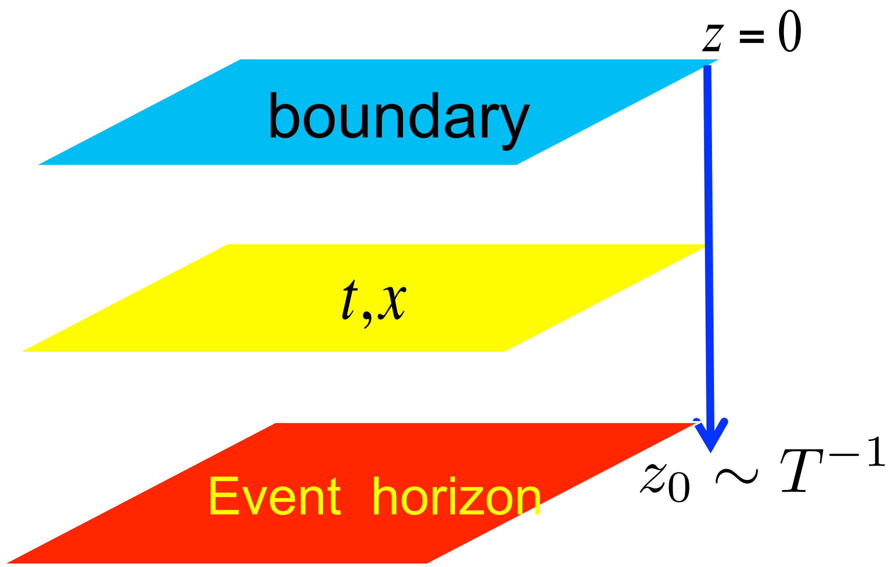
Hartnoll, Polchinski, Silverstein, Tong, 0912.1061

Faulkner, Horowitz, Roberts, arxiv:1008.1581

Jensen, Karch, Son, Thompson, arxiv: 1002.3159 Jensen: 1108.0421

Gauge/gravity duality

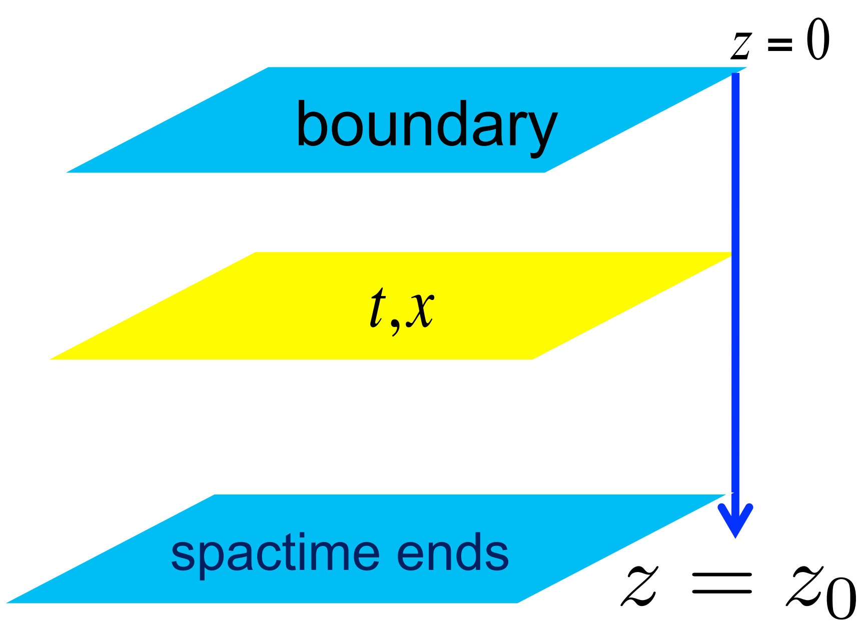
highly quantum mechanical, strong coupling phenomena ⟺ simple geometric picture or gravitational dynamics



finite T ⟺ Black hole

QGP: universality

$\frac{\eta}{s}$ ....



confining theory:
spacetime ends smoothly at a finite proper distance from any interior point.

$$\text{mass gap} \sim \frac{1}{z_0}$$

Can we extract some universal physics for a finite density system from gauge/gravity duality?

Charged black hole

Take a U(1) global symmetry. Put the system at a finite chemical potential for this U(1).

J_μ ⟷ A_M Minimal bulk description: Einstein-Maxwell system

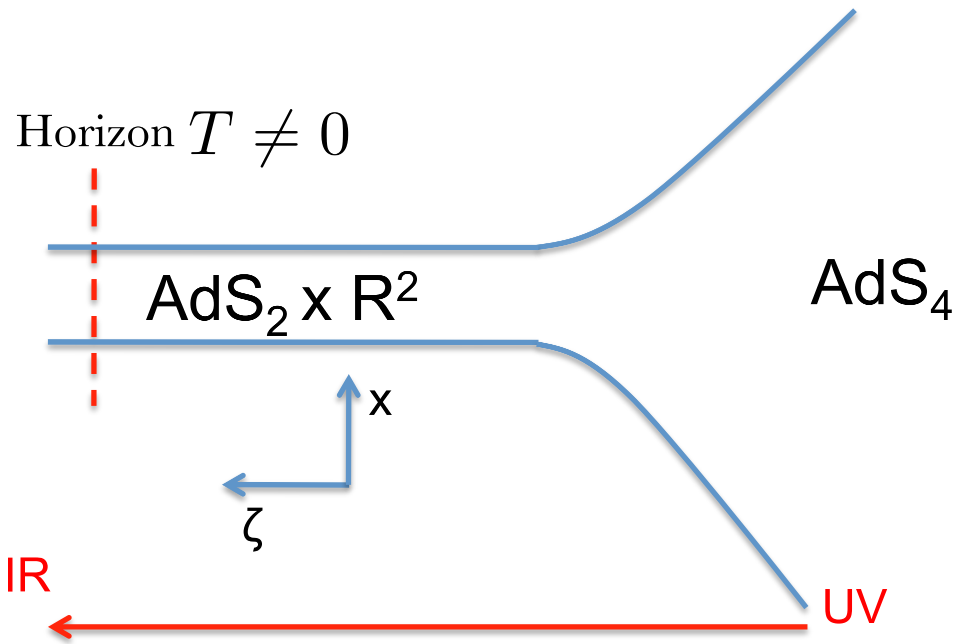
⟹ charged AdS black hole (T=0: extremal)

Since the Einstein-Maxwell sector is contained in any holographic description of a finite density system,

charged BH has a chance to be something "universal"

i.e. systems with different UV descriptions may share similar IR physics at a finite chemical potential.

Extremal charged BH



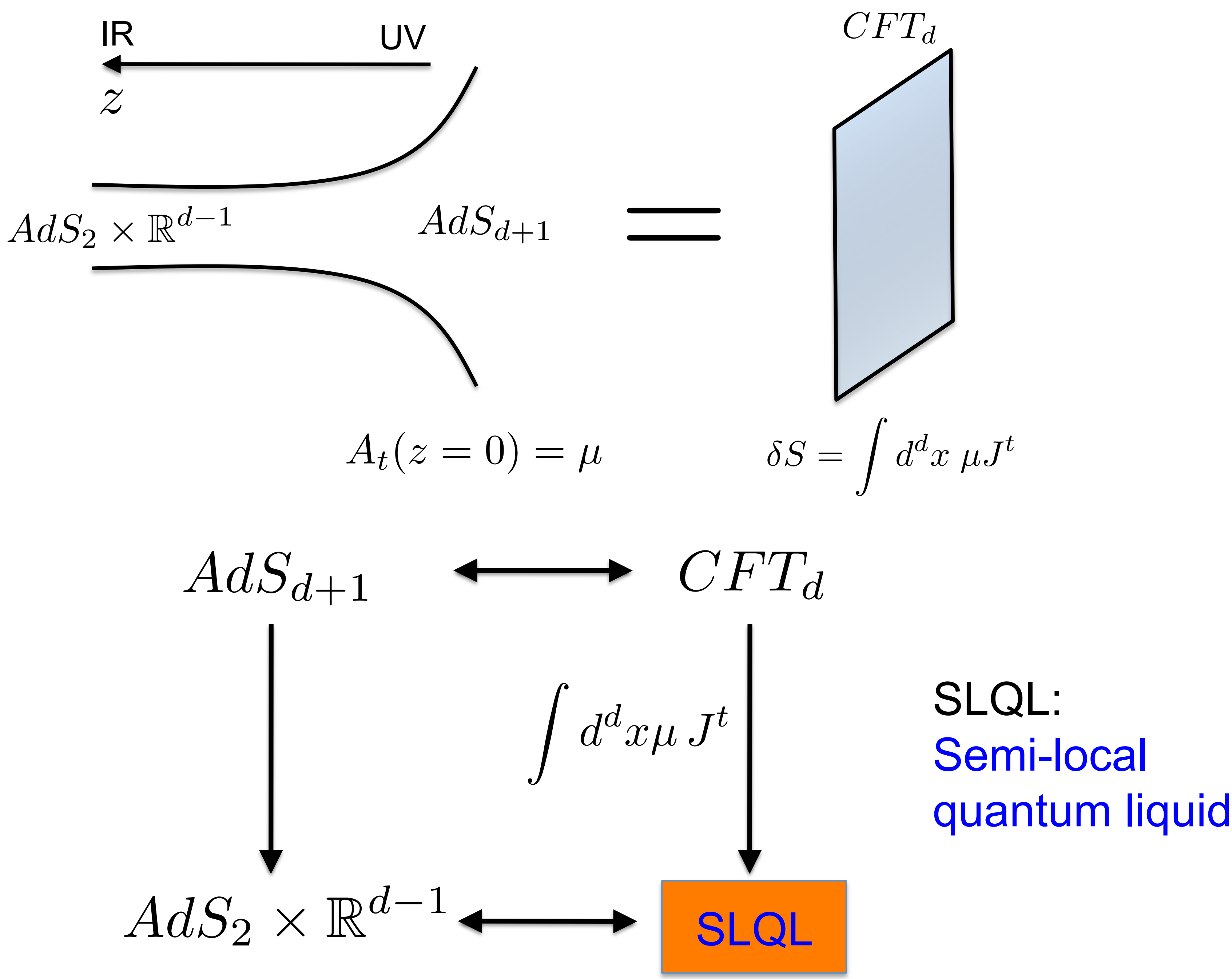
1. Degenerate horizon

Horizon infinite proper distance away

2. Transverse R^2 finite size

Finite entropy density at T=0

When T << μ, only AdS_2 region is heated up.


IR
UV
z
CFT_d
$AdS_2 \times \mathbb{R}^{d-1}$
AdS_{d+1}
$A_t(z=0) = \mu$
$\delta S = \int d^d x \, \mu J^t$
AdS_{d+1}
CFT_d
$\int d^d x \mu \, J^t$
SLQL:
Semi-local
quantum liquid
$AdS_2 \times \mathbb{R}^{d-1}$
SLQL

Semi-local quantum liquid

SLQL: scaling symmetry in time, SL(2,R), ω/T scaling

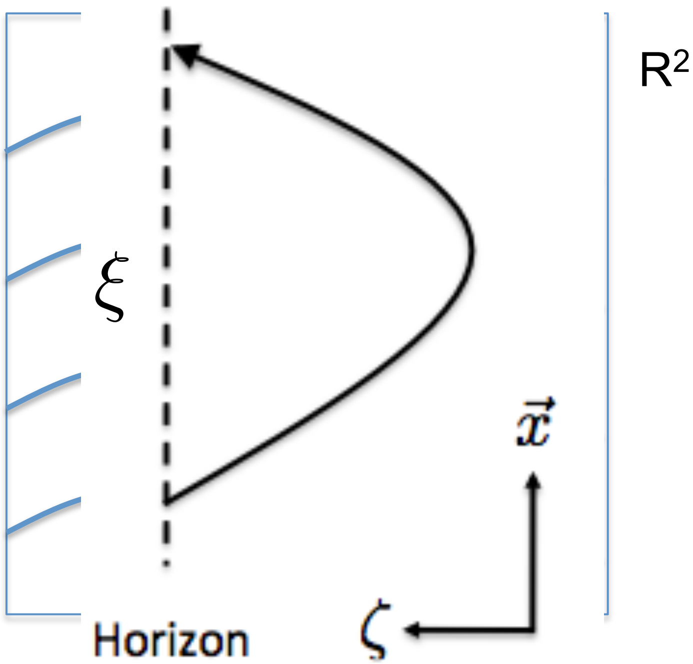
Consider $\phi \quad \Leftrightarrow \quad \Phi$

Scaling dimensions for $\Phi_{\vec{k}}$: $\quad \delta_k = \frac{1}{2} + \nu_k ,$

$$\nu_k = \frac{1}{\sqrt{d(d-1)}} \sqrt{m^2 R^2 - \frac{q^2}{2} + \frac{d(d-1)}{4} + \frac{k^2}{\mu_*^2}} \qquad \mathcal{G}_k(\omega) = c(\nu_k)(-i\omega)^{2\nu_k}$$

- Gapless excitations for all k
- the dimension increases with k
- the dimension decreases with q
- Both $\mathcal{G}_k$ and ν_k become independent of k when $k \ll \mu$
- infinite correlation time, but a finite correlation length

Semi-local behavior



$$\xi = \frac{1}{\mu_*}\frac{1}{\sqrt{m^2R^2 - \frac{q^2}{2} + \frac{d(d-1)}{4}}}$$

- For $x \equiv |\vec{x}| \ll \xi$

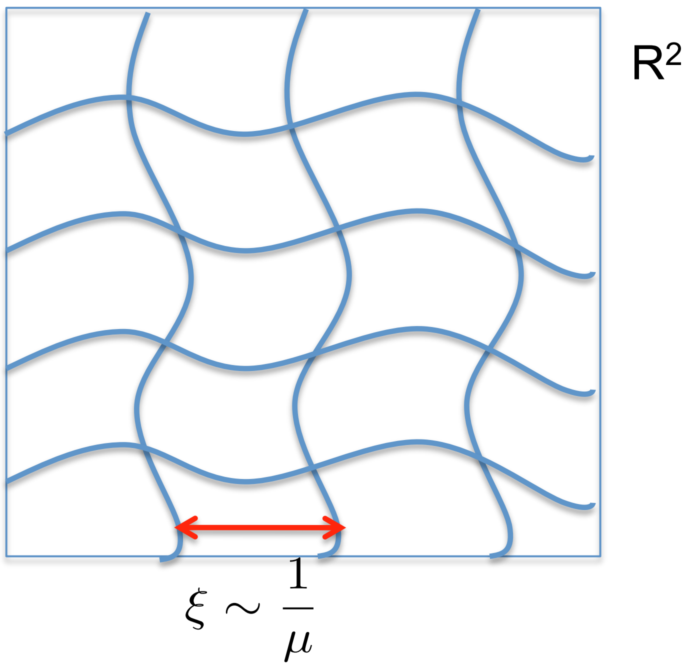
$$G_E(\tau, x) \sim \frac{1}{\tau^{2\delta_{k=0}}}$$

- For $x \gg \xi$

$$G_E(\tau, x) \sim e^{-\frac{x}{\xi}}$$

- The system separates into regions of size ξ, with no strong correlations between different regions.
- Within each region, the system behaves like a conformal quantum mechanical system with an infinite correlation time.

This is rather similar, but different from local quantum criticality discussed in the condensed matter literature:



- This describes a phase, not a critical point.

- dimensions and correlation functions depend on momentum, not like in impurity models

Correlation length diverges near critical point

→ Semi-local quantum liquid (gravity in $AdS_2 \times R^{d-1}$)

Also characterized by a finite entropy density

Prediction from gravity:

A class of QFTs

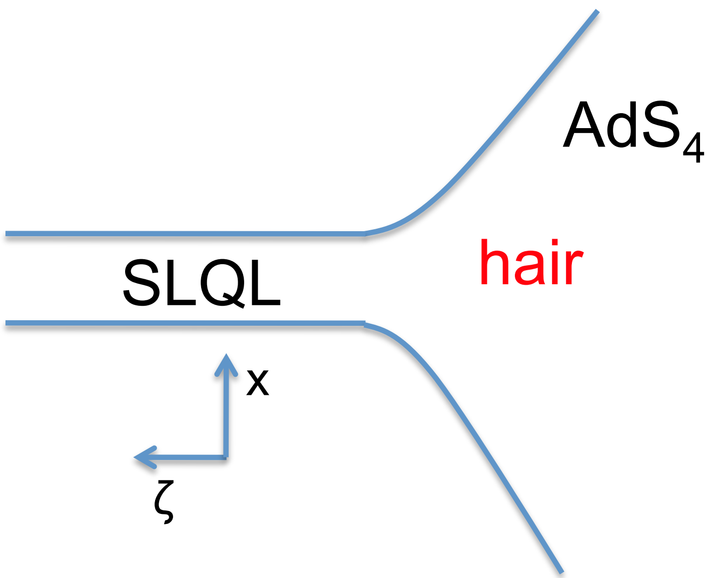
$$\Big\downarrow \int d^d x \mu \, J^t$$

SLQL

Physics of SLQL

Black hole hair

It turns out SLQL may not be the only low energy degrees of freedom.



New low energy degrees of freedom arise if the charged black hole admits hair.

Scalar hair: static, normalizable solution for a scalar field.

Gravity theory contains various matter fields (scalar, spinor, ….).

………

Fermionic hair and Fermi Surface

Consider a spinor field: $\psi \quad \Leftrightarrow \quad \Psi$ (boundary operator)

$$m, q \quad \Leftrightarrow \quad \Delta, q$$

$$\psi(z, \vec{x}) = \psi_k(z) e^{i\vec{k}\cdot\vec{x}}$$

There exists certain range of (m,q), $\psi_k(z)$ can have a normalizable solution at some $|\vec{k}| = k_F$

When this happens, there are gapless excitations of Ψ at the momentum shell

$$|\vec{k}| = k_F$$

➡ A Fermi surface in the boundary theory

Small excitations at the Fermi surface

$$G_R(\omega,k) = \frac{h_1}{\omega - v_F(k-k_F) + \Sigma(\omega)} \qquad \Sigma(\omega) = h\mathcal{G}_{k_F}(\omega) = c\omega^{2\nu}$$

$$\mathcal{G}_{k_F}(\omega): \text{ SLQL correlation function at } k = k_F$$

$$\nu: \text{ SLQL scaling dimension of } \Psi \text{ evaluated at } k_F$$

Quasi-particle decay rate:

$$\Gamma \propto \omega^{2\nu}$$

(decay by falling into the black hole)

$\nu > \frac{1}{2}$ long-lived quasi-particles,

$\nu \leq \frac{1}{2}$ No long-lived quasi-particles

$\nu = \frac{1}{2}$ $\Gamma \propto \omega$ as in high Tc cuprates !

Finite temperature: Replace $\omega^{2\nu}$ by a universal scaling function $T^{2\nu} g(\omega/T)$ (known analytically)

Marginal Fermi liquid

For $\nu_{k_F} = \frac{1}{2}$

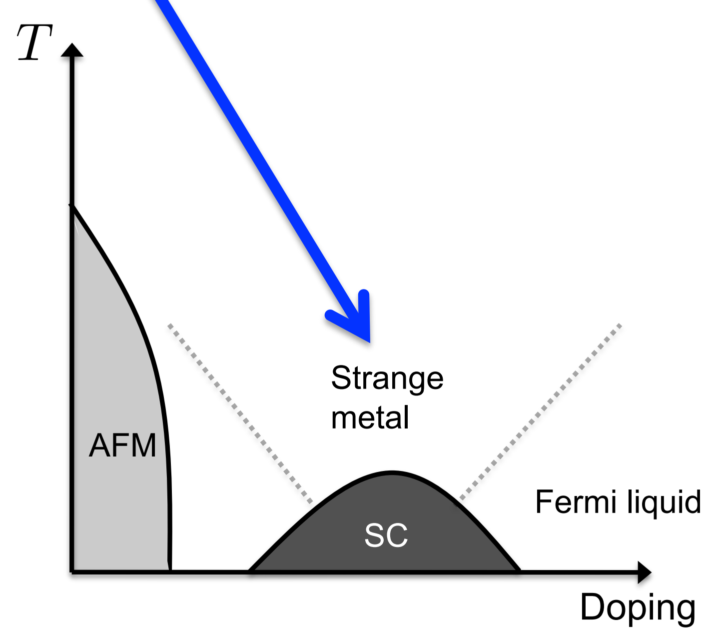
$$G_R \approx \frac{h_1}{k_\perp + \tilde{c}_1 \omega \log \omega + c_1 \omega}$$

$\tilde{c}_1$: real

c_1 : complex

Precisely that for
``Marginal Fermi liquid''
proposed on phenomenologic
ground for high Tc cuprates
near optimal doping.

Varma, Littlewood, Schmitt-Rink,
Abrahams, Ruckenstein (89)



Strange metals

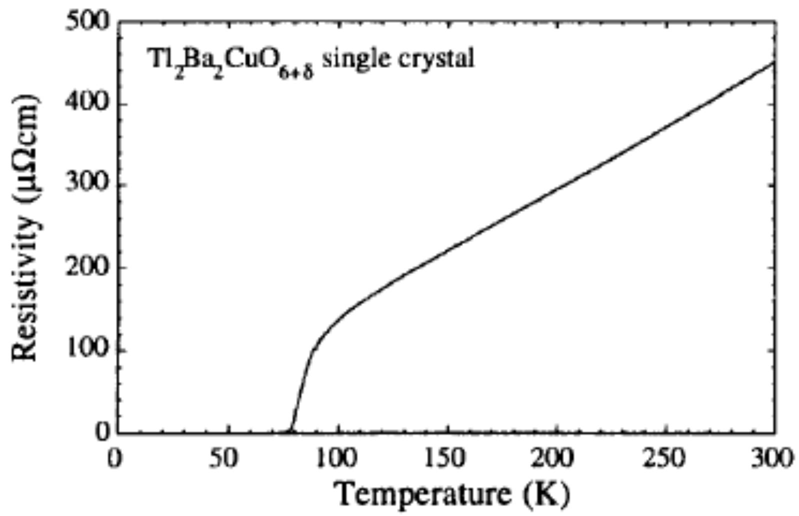
From photoemission experiments:

1. There is still a k_F 2. Width of the peak $\Gamma \propto \omega$ Quasi-particle decay rate

There is a Fermi surface, but no long-lived quasi-particles !

Resistivity linear in temperature:

Mackenzie 97

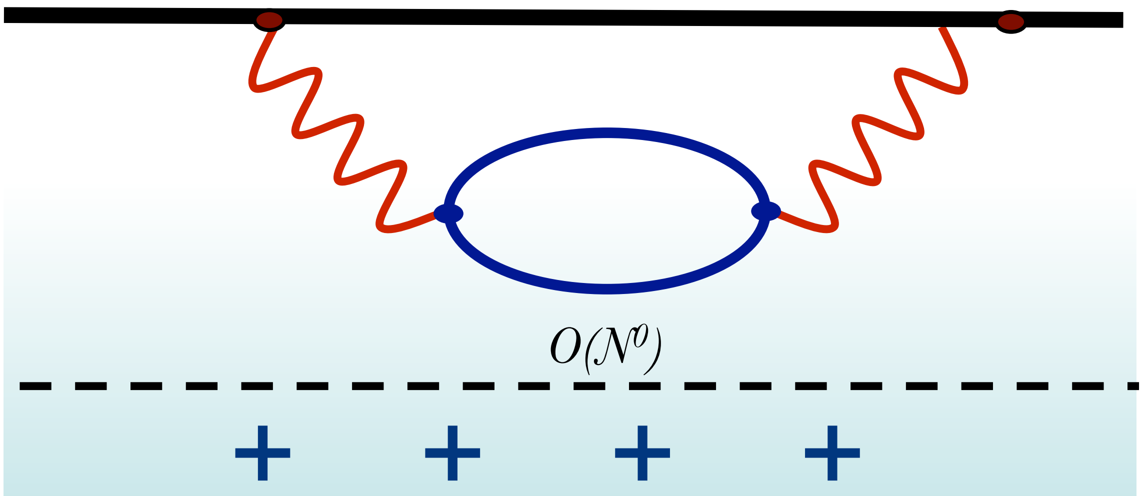


In sharp contrast with that of a Fermi Liquid:

$$\rho = \rho_0 + cT^2$$

Simple, robust, universal (for all cuprates and many heavy fermion materials), long standing puzzle ..

Conductivity from fermions



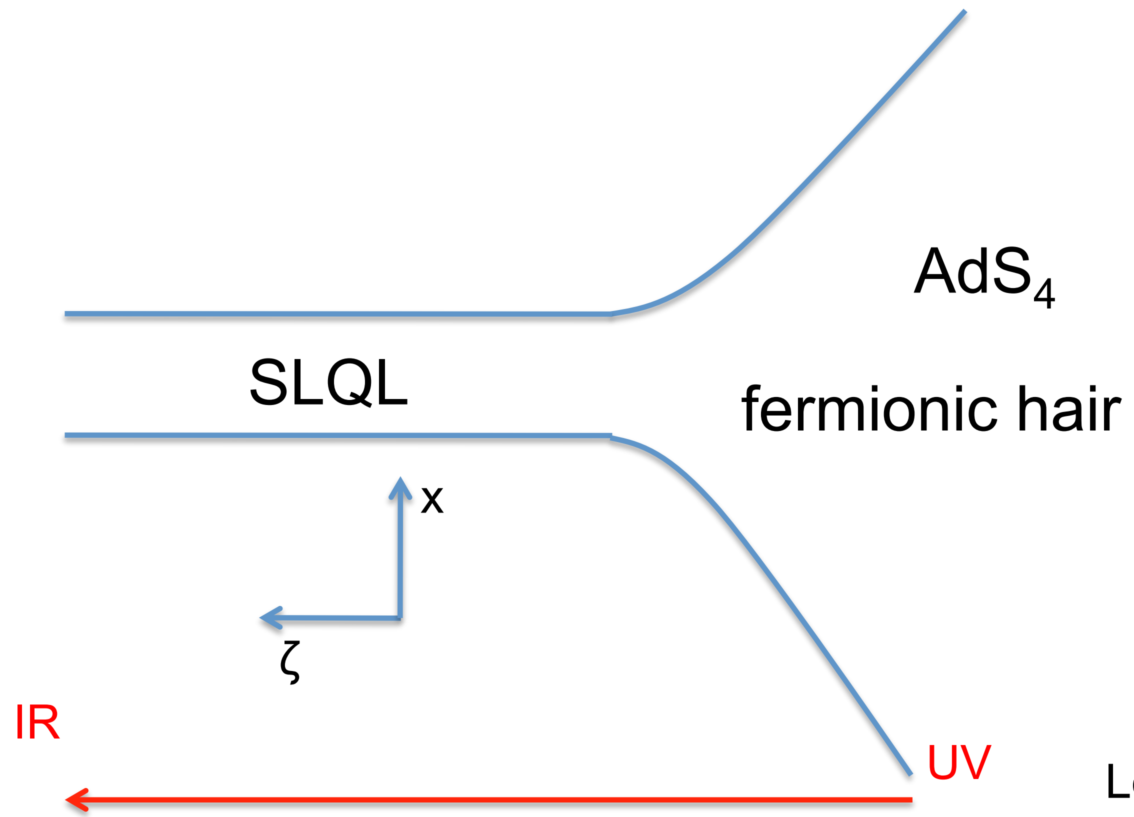
One-loop calculation in gravity:

many subtleties and potential pitfalls ……

$$\sigma_{FS} \propto T^{-\alpha} \quad \text{with} \quad \alpha = 2\nu$$

For marginal fermi liquid (relevant for cuprates) $\nu = \frac{1}{2}$

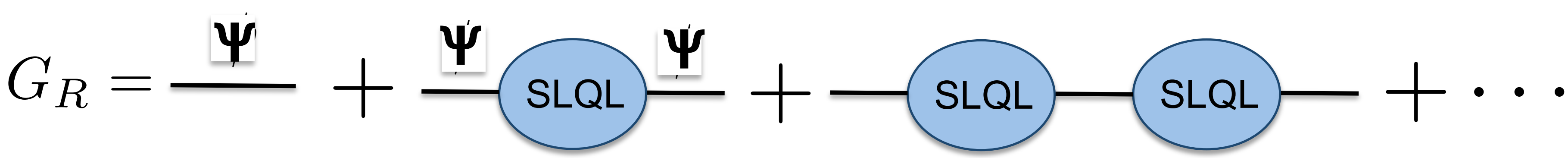
$$\sigma_{FS} \propto T^{-1}$$ leading to linear resistivity !



Faulkner, HL, McGreevy,Vegh
Faulkner, Polchinski
Faulkner, HL, Rangamani

Low energy effective theory:

$$S_{eff} = \tilde{S}_{\rm SLQL}[\Phi] + \int \lambda(k,\omega)\Phi_{\vec{k}}\Psi_{-\vec{k}} + S_{\rm free\ fermion}[\Psi]$$



Hybridization between free fermions and SLQL

Scalar hair: instability

When there is a scalar hair at some k_F, the dynamical susceptibility has a pole in the upper half frequency plane, which indicates instability to condensation of the corresponding scalar.

Faulkner, HL, McGreevy,Vegh

scalar hair → Superconductors (charged)

Ising-nematic, or AFM (neutral)

It is possible to tune certain external parameter g (double trace coupling) to a quantum critical point g_c, where the instability disappear.

Faulkner, Horowitz, Roberts

Hybridized quantum critical point

Faulkner, Horowitz, Roberts; Iqbal, HL, Mezei; Jensen

Near the critical point, the dynamical susceptibility is given by:

$$\chi(\omega, \vec{k}) = \frac{Z}{g - g_c + c(\vec{k} - \vec{Q})^2 + T^\alpha f(\frac{\omega}{T})}$$

This is of the same form as the magnetic suscpetibility for $CeCu_{6-x}Au_x$ near x_c=0.1 (Schroeder etal, 1998)

$CeCu_{6-x}Au_x$: x<0.1, Fermi liquid, x > 0.1 antiferromagnet

Low energy effective theory

$$S_{eff} = S_{\rm SLQL}[\Phi] + \lambda \int dt\, \Phi\psi + S_{LG}[\psi]$$

Landau-Ginsburg sector hybridized with a strongly coupled SLQL.

Hybridized quantum critical point

Summary

Black hole hair: gapless degrees of freedom (described by meant field) hybridized with degrees of freedom in SLQL:

Fermion: non-Fermi liquids, Fermi surface without quasiparticles, strange metal behavior as in cuprates.

Scalar: hybridized QCP, as in $CeCu_{6-x}Au_x$

- Spatial sector mean field;
- Self-energy nontrivial scaling in frequency (weakly dependent spatial momentum)

Naturally lead to local quantum critical behavior

Instabilities of SLQL

Instabilities of SLQL

SLQL develops instability when ν_k becomes imaginary

$$\nu_k = \frac{1}{\sqrt{d(d-1)}}\sqrt{m^2R^2 - \frac{q^2}{2} + \frac{d(d-1)}{4} + \frac{k^2}{\mu_*^2}}$$

(Scalar, similarly for spinor)

When $$u \equiv m^2R^2 - \frac{q^2}{2} + \frac{d(d-1)}{4} < 0$$

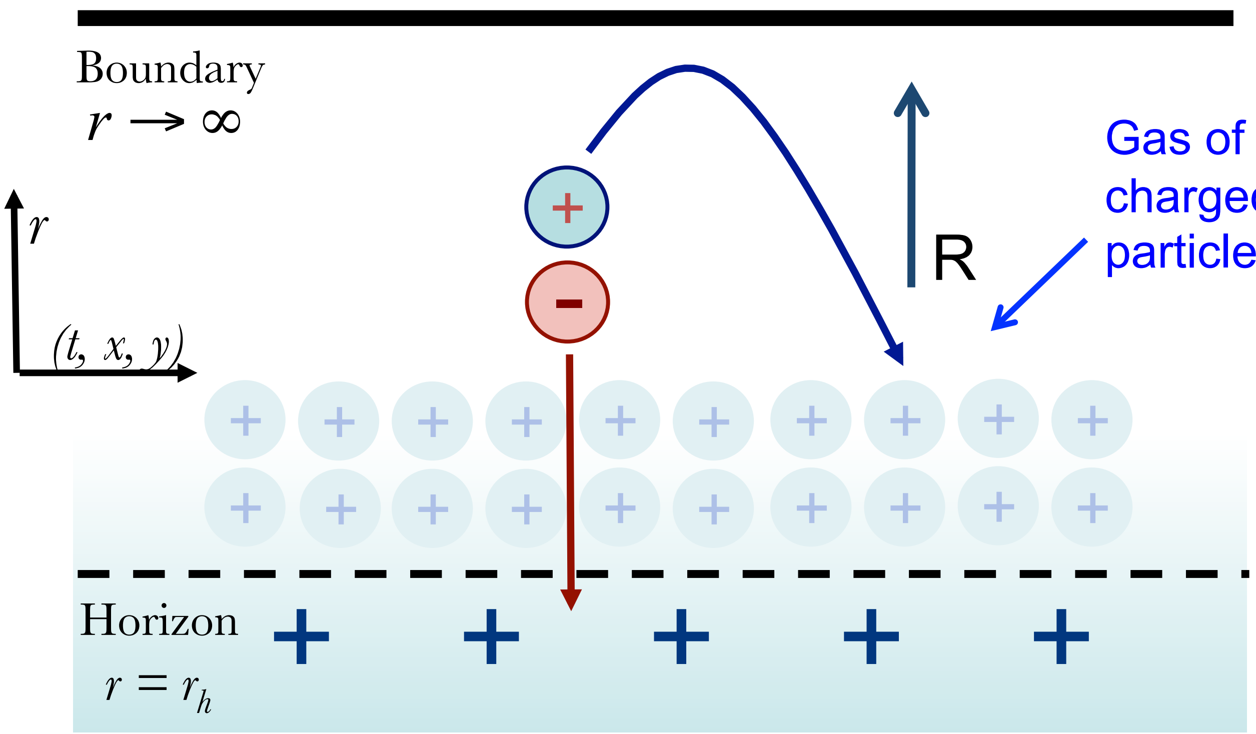
Scalar: below BF bound of AdS_2 Hartnoll, Herzog, Horowitz

Gravity: black hole can pair produce these particles

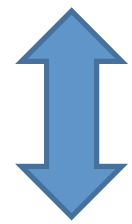
Density of states: $$\rho(E) = \text{const} + \frac{\lambda}{E} + \cdots$$

Fermion gas in $AdS_2 \times R^2$

Consider a charged field outside the black hole:

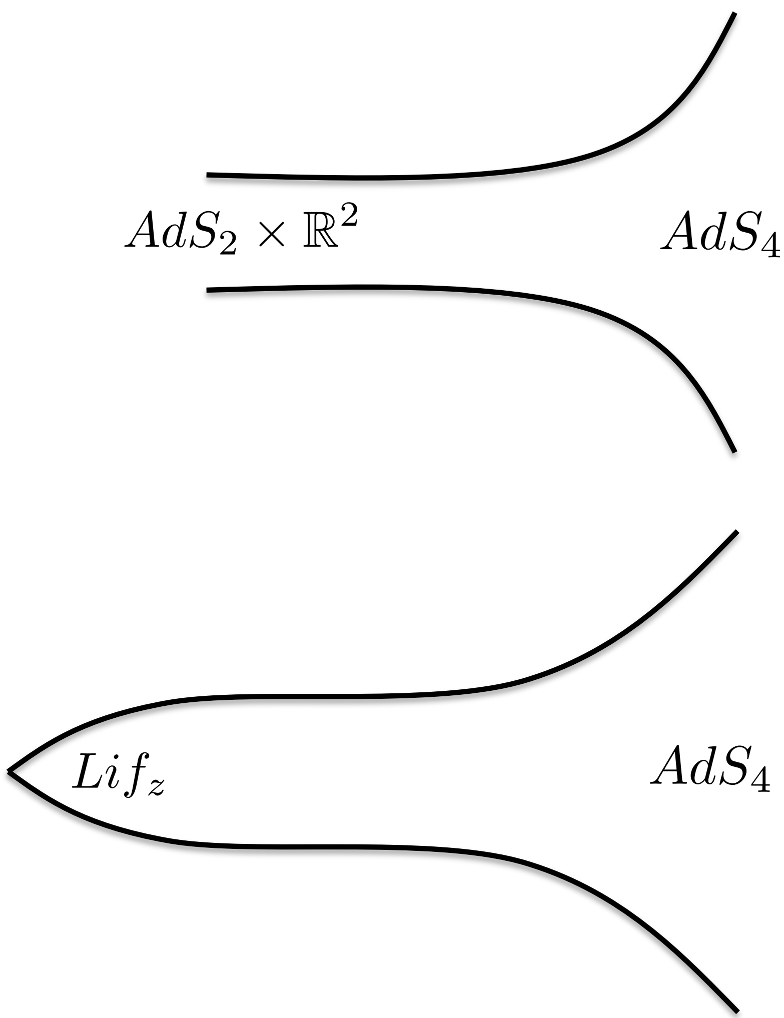


bulk charged fermion gas



a finite O(1) density of fermions in a strongly interacting boundary theory

Finite N and back reaction of fermionic gas



Hartnoll, Polchinski, Silverstein, Tong

The total density of the fermionic gas turns out to be infinite.

Backreaction of the fermionic gas: the spacetime becomes Lifshitz at $\Lambda \sim e^{-cN^2}$

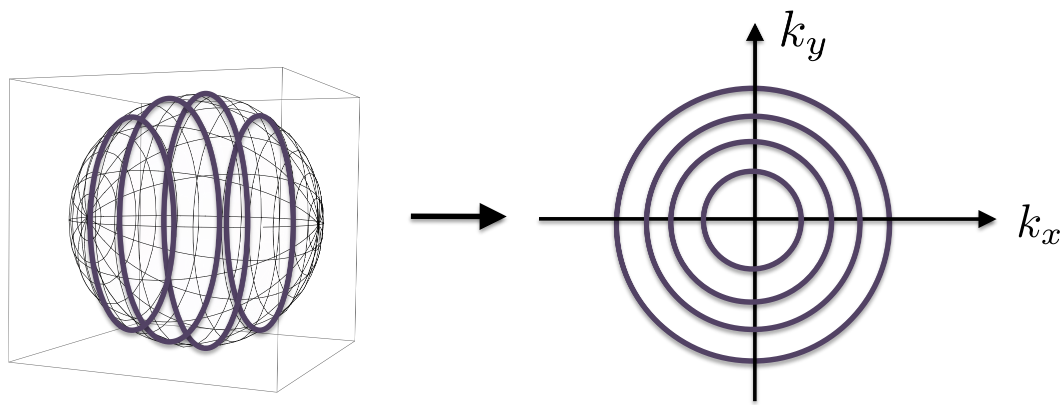
- Charged horizon disappears, all charges carried by bulk fermions
- The new geometry has zero entropy density.

Heavy Fermi liquid

Iqbal, HL, Mezei, Hartnoll, Hofman, Vegh

Cubrovic, Liu, Schalm, Sun, Zaanen

Fermions in Backreacted geometry → Multiple Fermi surfaces

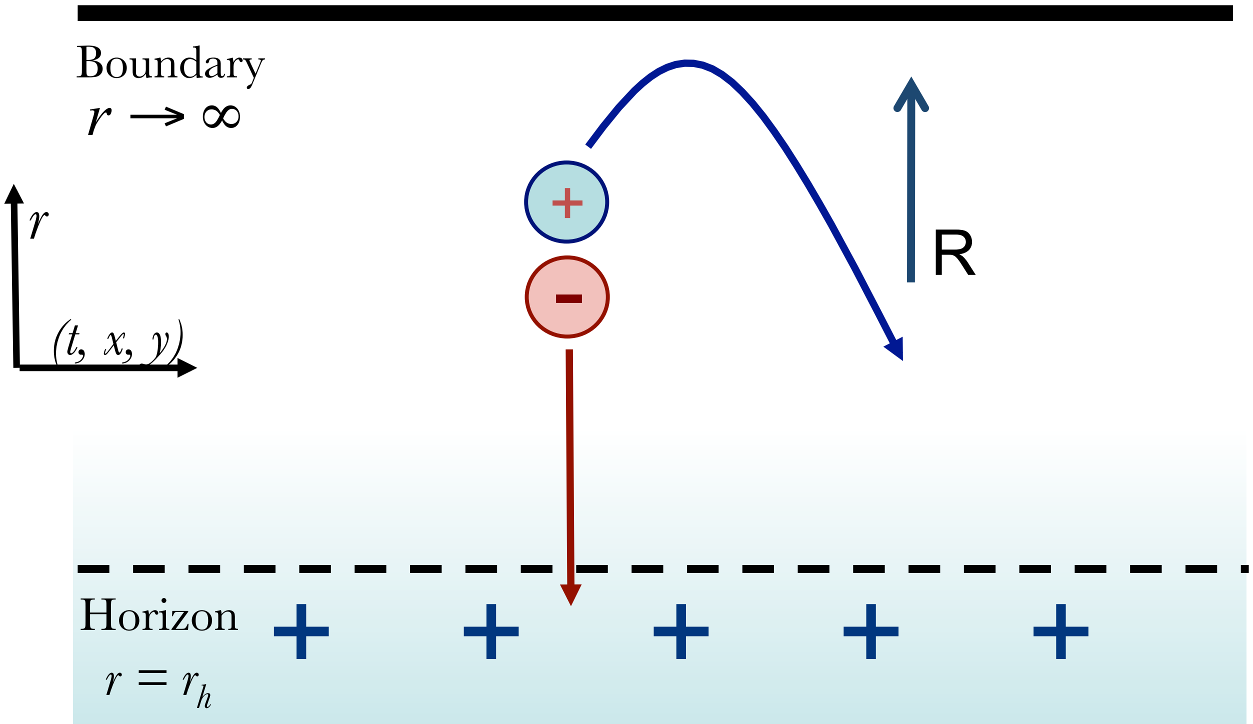


Each Fermi surface corresponds to a different fermionic species (different bound states).

Each Fermi surface is a Fermi liquid of heavy fermions.

Scalar instability

Consider a charged scalar field outside the black hole:



Fermion: reflection probability R < 1 , leading to an equilibrium.

Scalar: R > 1 (superradiance), will grow and condense.

$$u \equiv m^2 R^2 - \frac{q^2}{2} + \frac{d(d-1)}{4} < 0$$

Critical point: u_c=0

Bifurcating critical point (I)

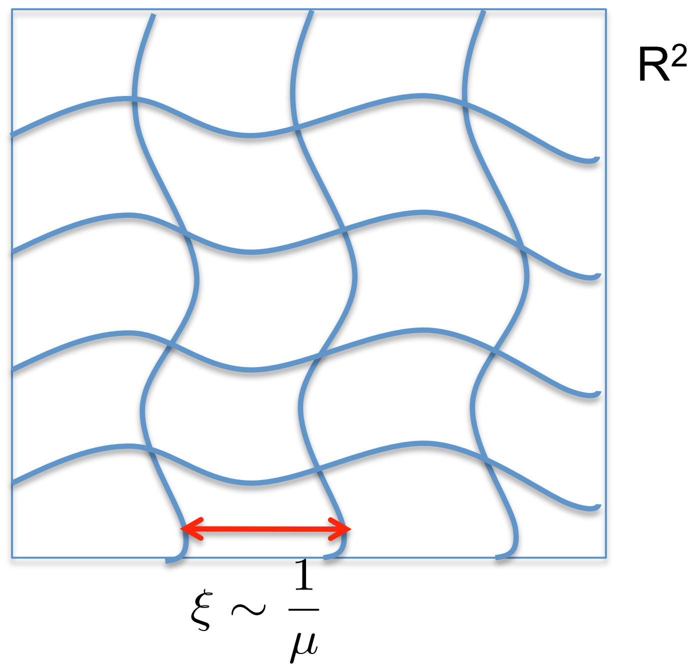
Near the critical point $u_c = 0$, the static, uniform susceptibility:

$$\chi = \mu_*^{2\nu_U} \frac{\beta + \sqrt{u}\tilde{\beta}}{\alpha + \sqrt{u}\tilde{\alpha}}$$

It remains finite at the critical point. Becomes complex when u < 0. (branch point at u=0)

Correlation length diverges as in mean field

$$\xi = \frac{1}{\sqrt{2\mu\sqrt{u}}}$$



Landau-Ginsburg-Wilson: critical point with relevant operators.

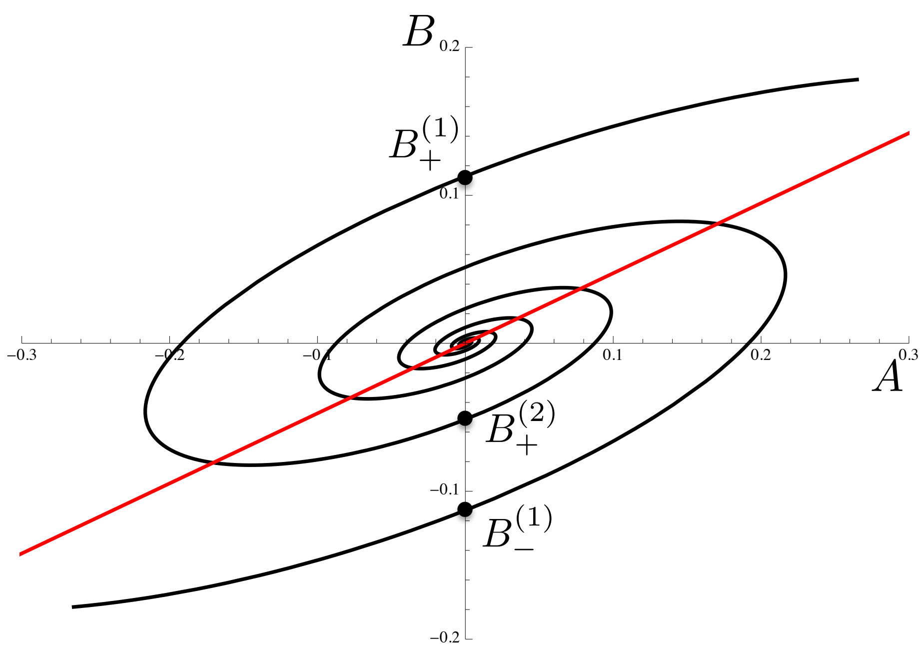
Bifurcating QCP: collision of two fixed points

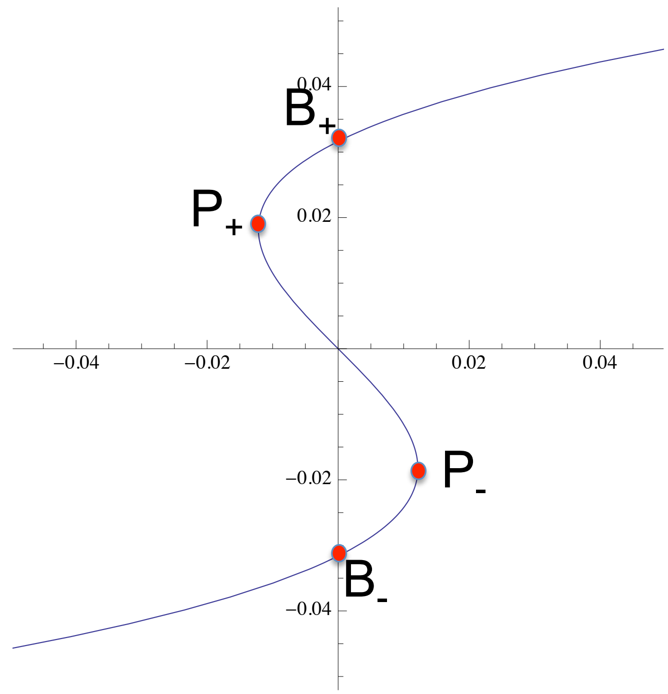
Bifurcating critical point (II)

There are in fact an infinite number of condensed states:

$$\langle O \rangle_n \sim \exp\left(-\frac{n\pi}{2\sqrt{-u}}\right), \qquad n = 1, 2, \cdots$$

The response of all these states can be characterized by a spiral:





Physical interpretation

Recall that we are consider a non-Abelian gauge theory: O is a gauge invariant operator.

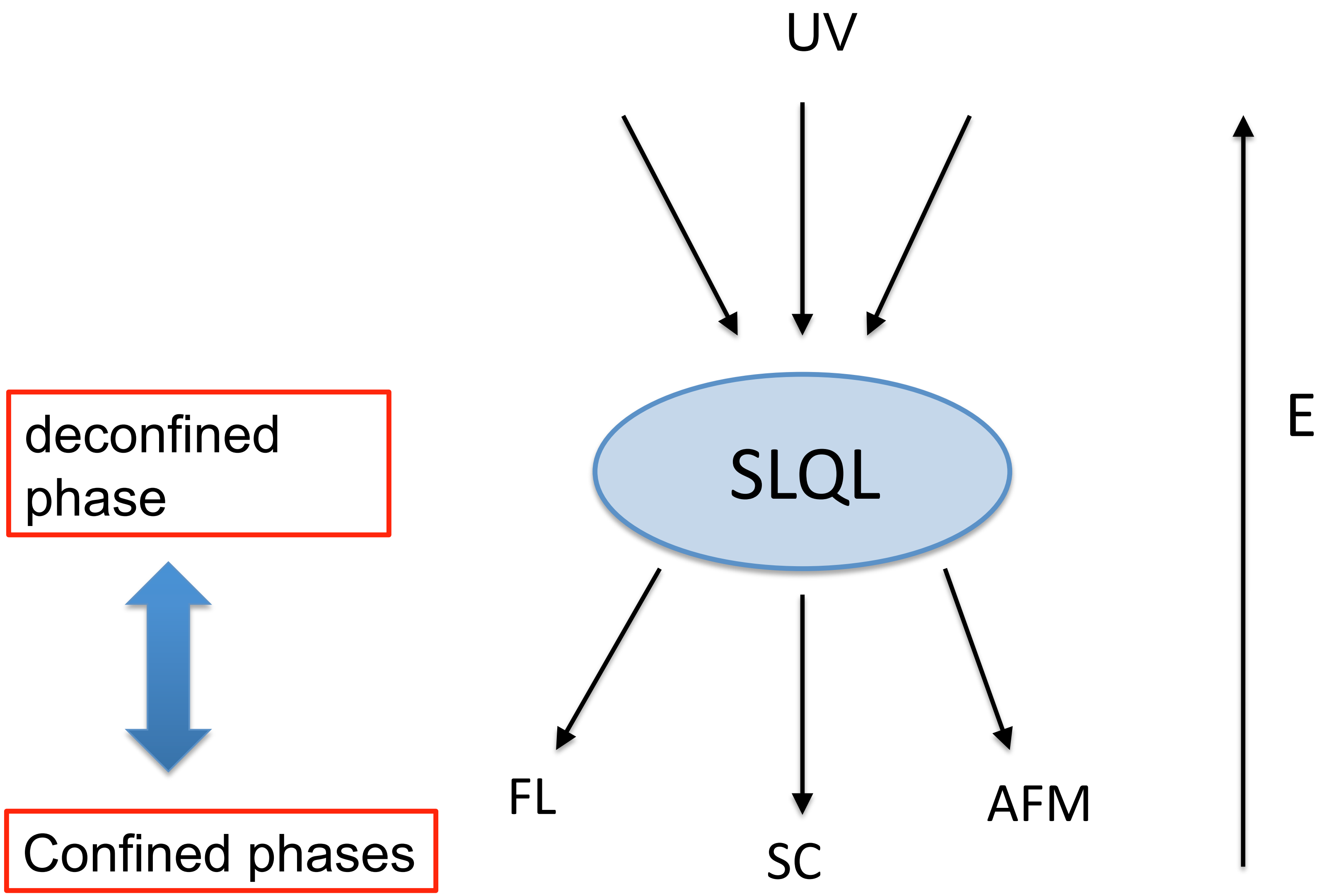
SLQL: finite entropy density $O(N^2)$, excitations of O gapless, no quasiparticles.

When O develops a complex scaling dimension in SLQL, it forms a tower of gapped bound states, each of which then Bose-Einstein condenses.

(infinite geometric series has to do with discrete scaling symmetry of a complex scaling dimension)

Similarly with a spinor operator: each bound state now forms a Fermi surface.

SLQL is a deconfined phase of these lower energy phases.


UV
deconfined phase
SLQL
E
Confined phases
FL
SC
AFM

Summary

A. Black hole has hair

→ Fermion: non-Fermi liquid

Scalar: instability, hybridized QCP

B. Dimension (i.e ν_k) of the operator in the SLQL becomes complex.

(black hole can pair produce particles)

→ Fermion: Heavy Fermi liquid

Scalar: Instability, bifurcating QCP

Interpretation of SLQL

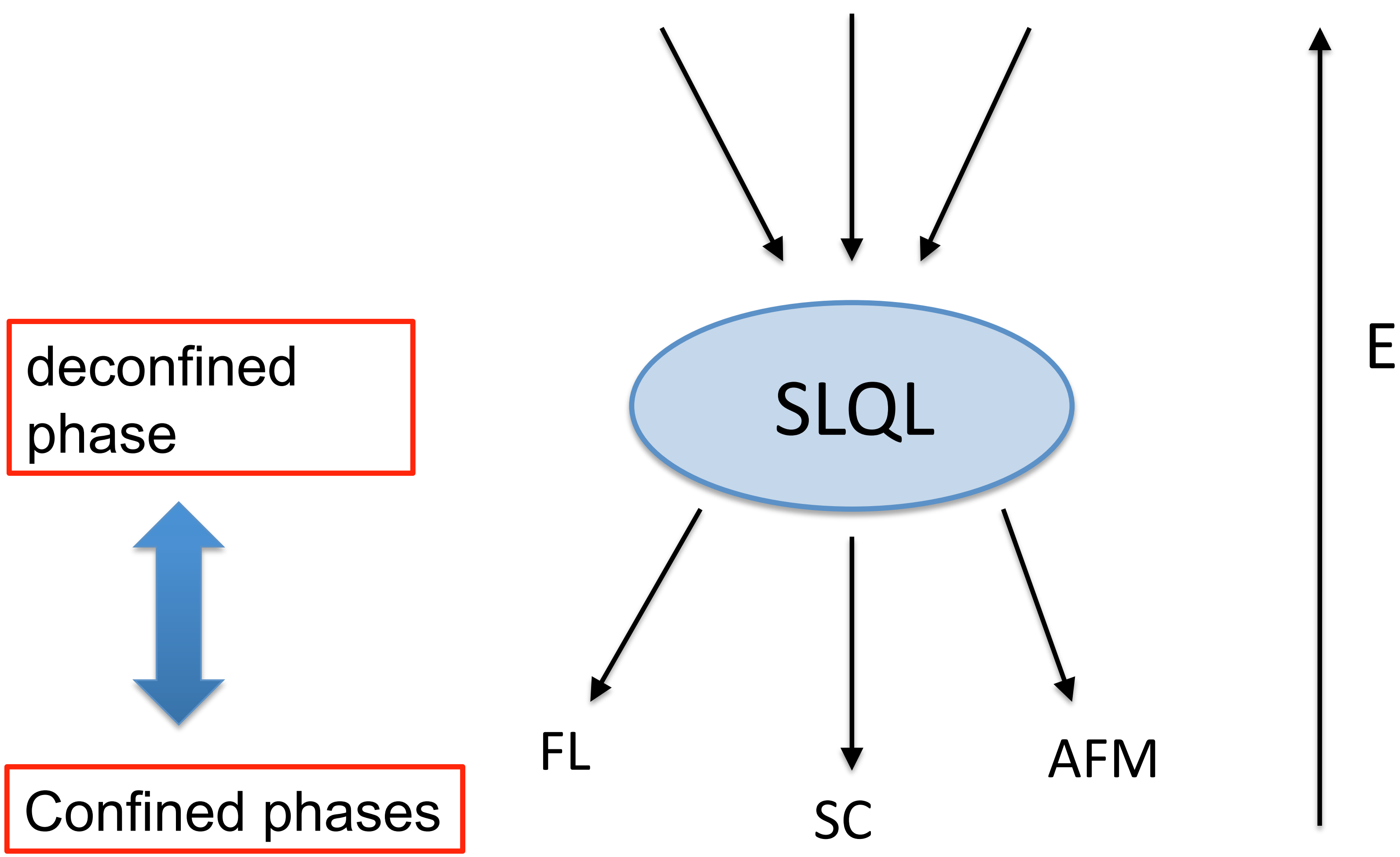
How do we interpret the entropy of SLQL?

Without supersymmetry, the entropy density should go away at finite N, i.e. the system will pick a unique ground state.

The precise physical nature of the ground state will depend on specific dynamics of an individual system.

The examples we have seen may be generic, i.e. SLQL may always be unstable to order into some lower energy phase.

Universal intermediate energy phase



Possible condensed matter applications

Could the SLQL phase underlie some known condensed matter systems?

Encountering some scaling behavior

vacuum or intermediate-energy effects?

Scaling has traditionally been associated with a QCP.

SLQL phase provides a nontrivial example scaling could arise from intermediate-energy effects.

- Systems which exhibit frustrated or competing interaction terms in their Hamiltonian.
- Systems which involve strong competition between tendencies towards itinerancy and localization.

Candidates of SLQL

In our context, strange metal behavior and hybridized quantum critical behavior are intermediate-energy effects,
NOT vacuum effects.

Could the strange metal phase of cuprates and certain critical behavior of heavy fermions be due to an intermediate-energy phase such as SLQL?

SLQL offers novel superconducting instabilities, could Superconductivity in cuprates or certain heavy fermion materials be of similar origin?

More generally, could an intermediate phase like SLQL be a generic phenomenon in condensed matter physics?

(note: similar features with DMFT)

Thank You !

Marginal critical point

One can also tune parameters so that a bifurcating and a hybridized critical point coincide. Here one finds postulated bosonic fluctuations underling marginal Fermi liquid:

$$\chi(k,\omega) = -\chi_* \log\left(-\frac{i\omega}{2}\right) + \cdots$$

At finite T:

$$\chi(k,\omega) = \frac{\pi\chi_*}{2}\tanh\left(\frac{\omega}{2\pi T}\right) + \cdots$$

$$\mathrm{Im}\chi(\omega,k) \sim \begin{cases} \frac{\omega}{T} & |\omega| \ll T \\ \mathrm{sgn}(\omega) & |\omega| \gg T \end{cases}$$

Varma, Littlewood, Schmitt-Rink, Abrahams, Ruckenstein (1989)

Sachdev, Ye (1992)

Bifurcating critical point (II)

$$\chi(\omega,\vec{k}) = \chi_0 \frac{\log\left(\frac{\omega}{\omega_b}\right) - i\frac{\pi}{2}}{\log\left(\frac{\omega}{\omega_a}\right) - i\frac{\pi}{2}}$$