Philip Phillips, UIUC

KITP AdScmt11, Nov 14, 2011

Holography and Mottness: a Discrete Marriage

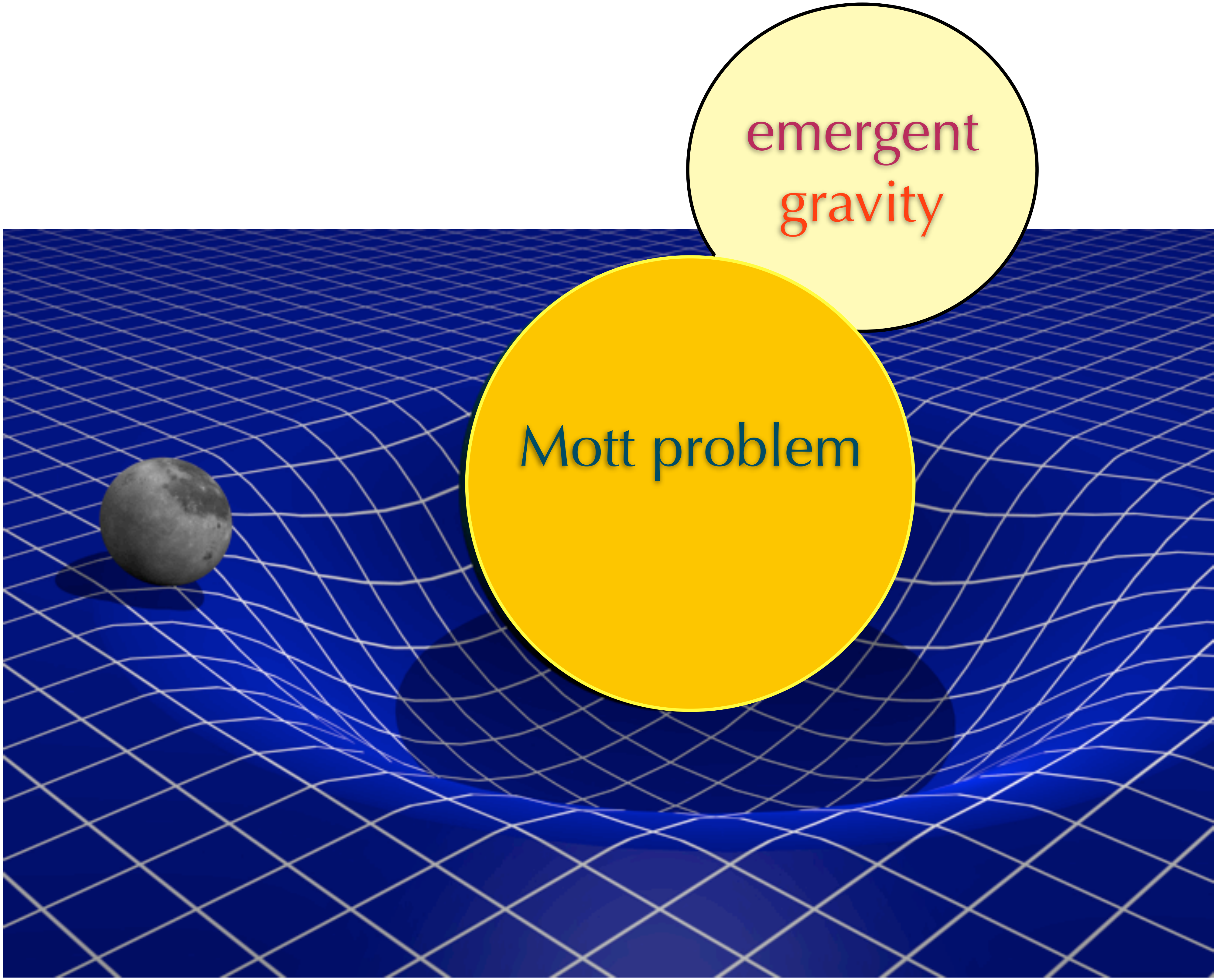
Thanks to: NSF, EFRC (DOE)

M. Edalati

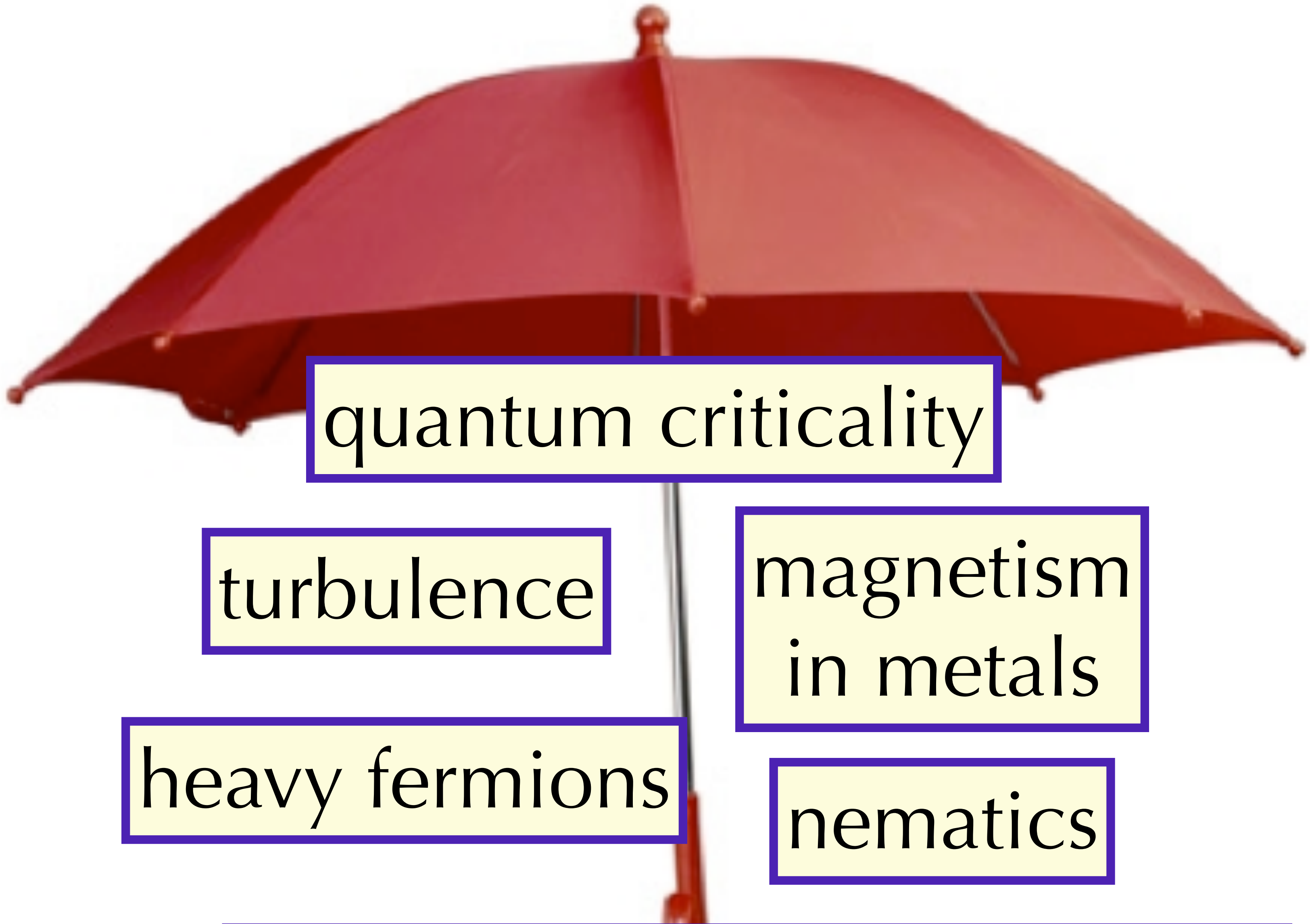
Ka Wai Lo

R. G. Leigh

Seungmin Hong


emergent
gravity
Mott problem

Unsolved condensed matter problems



quantum criticality

turbulence

magnetism in metals

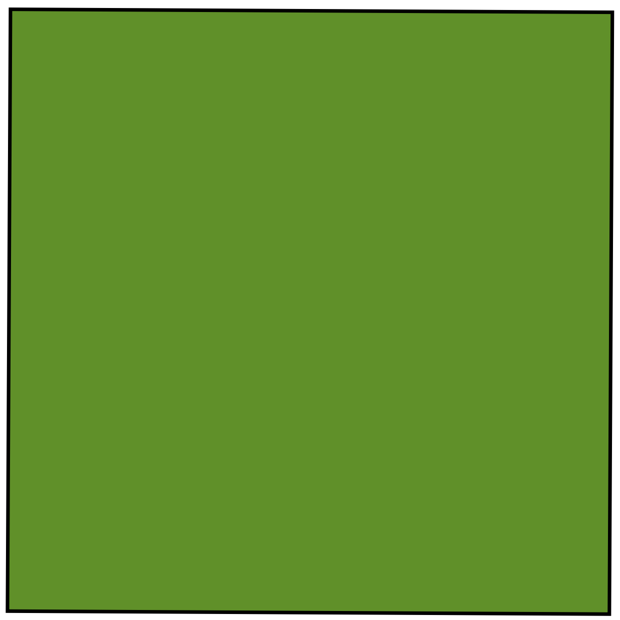
heavy fermions

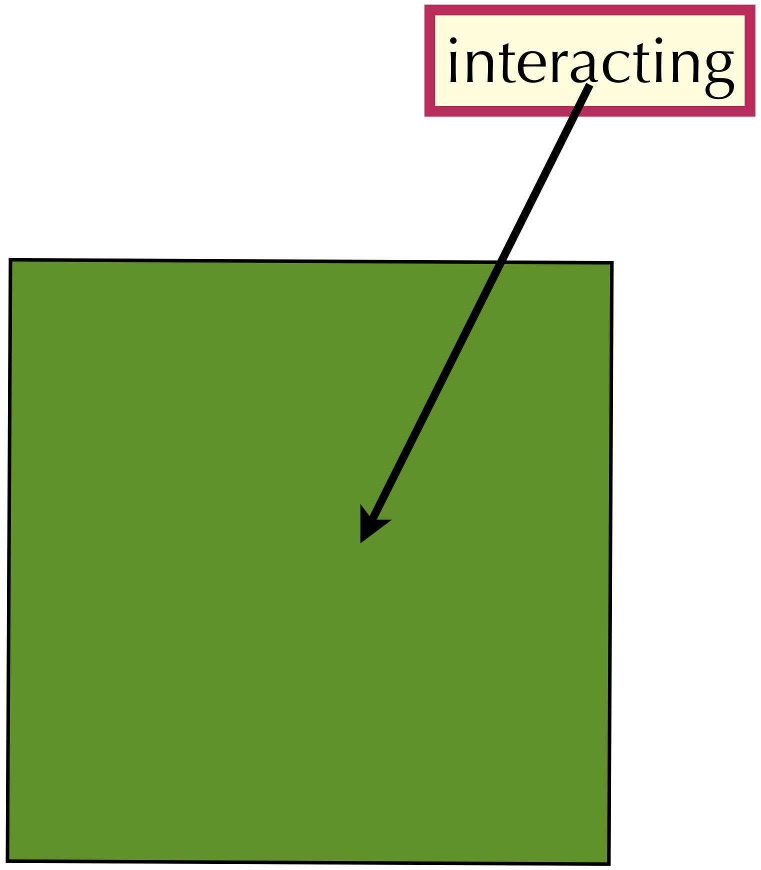
nematics

high T_c (cuprate, organics,...

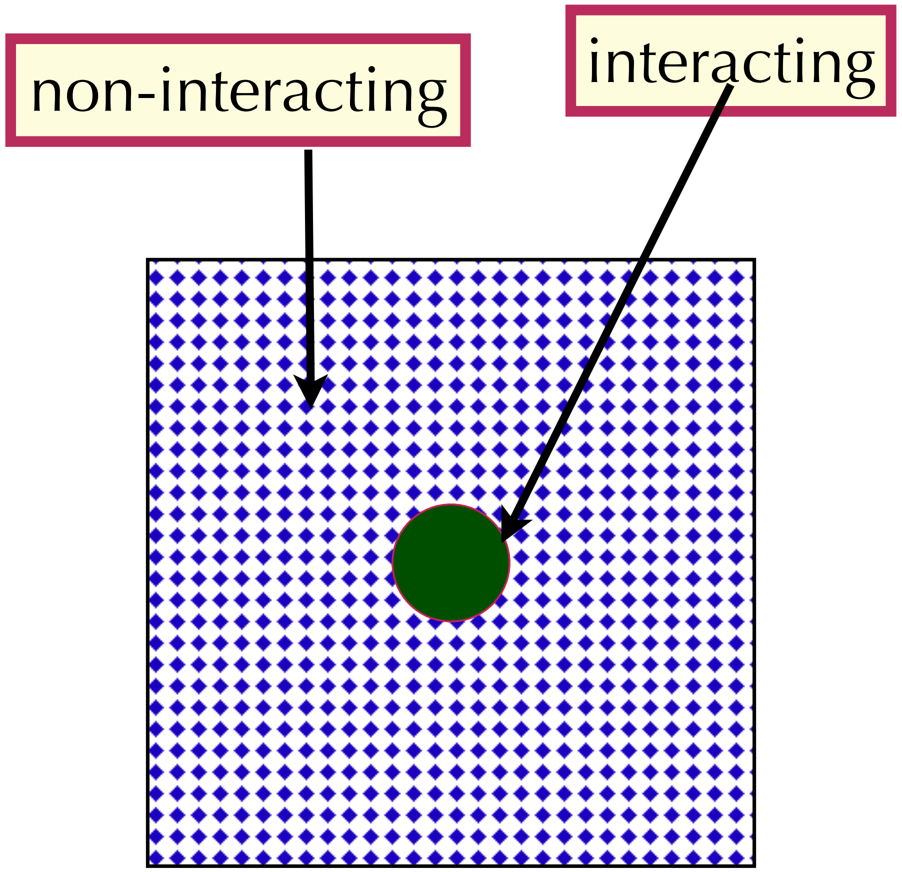
physics at strong coupling

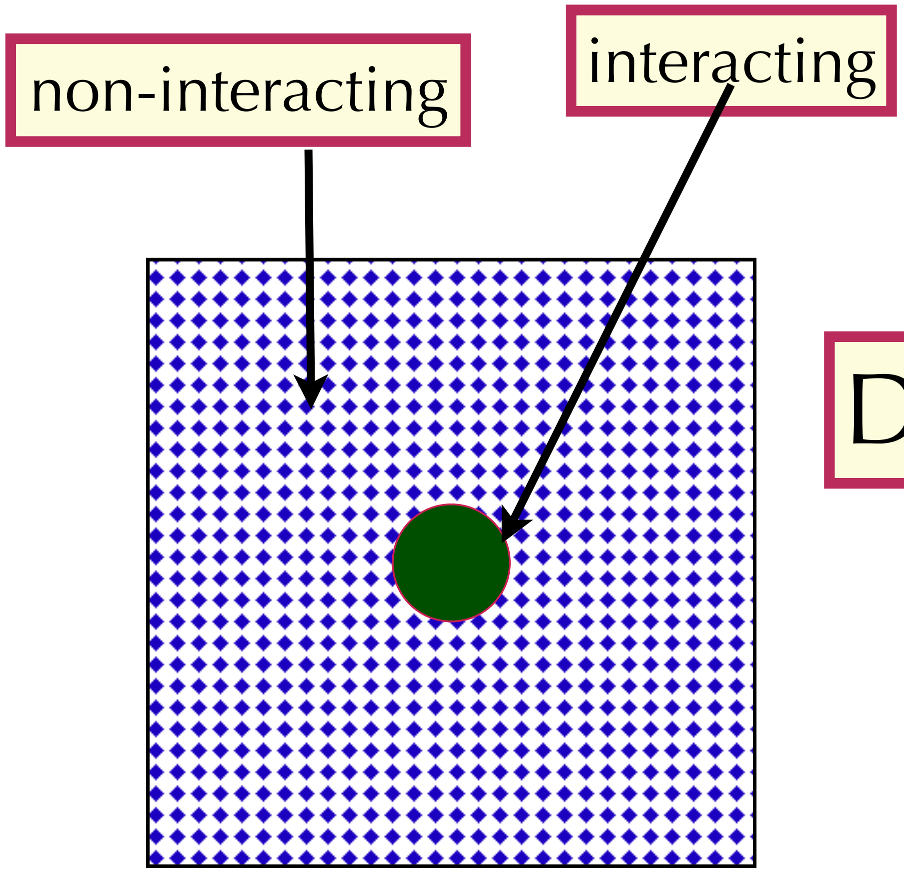
What computational tools do we have for strongly correlated electron systems?



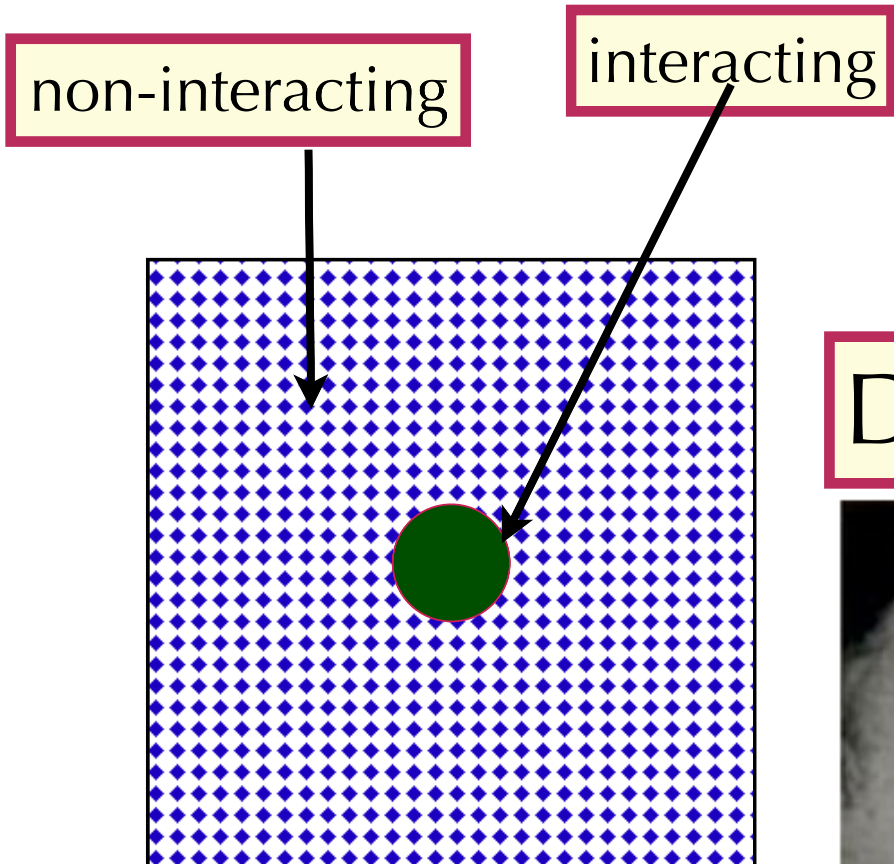

interacting

interacting


non-interacting
interacting

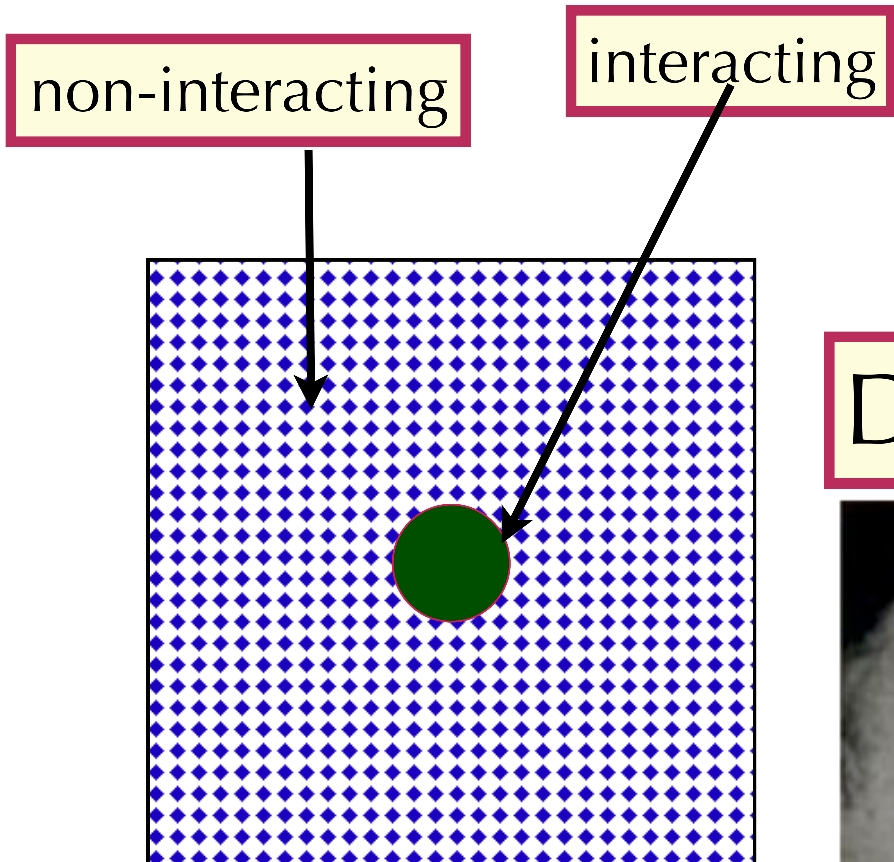


DMFT



the only difference
between this and
a theory is that this is not
a theory

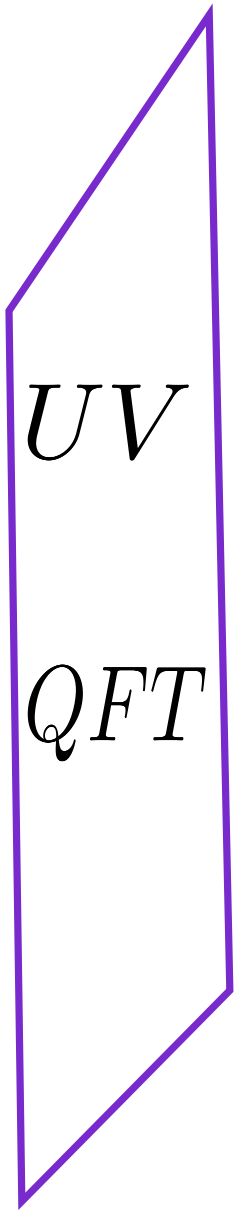
DMFT



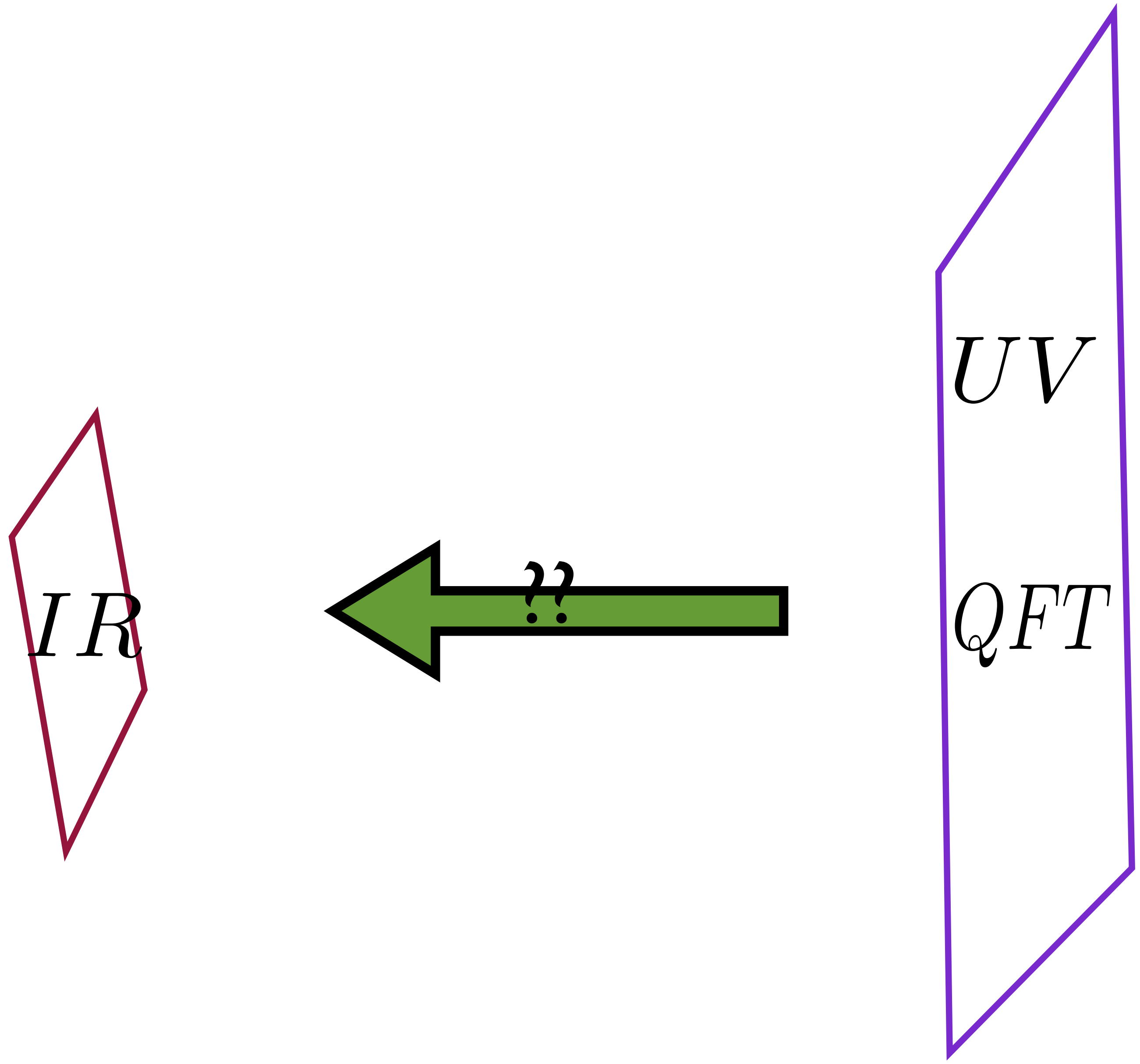
DMFT

the only difference between this and a theory is that this is not a theory

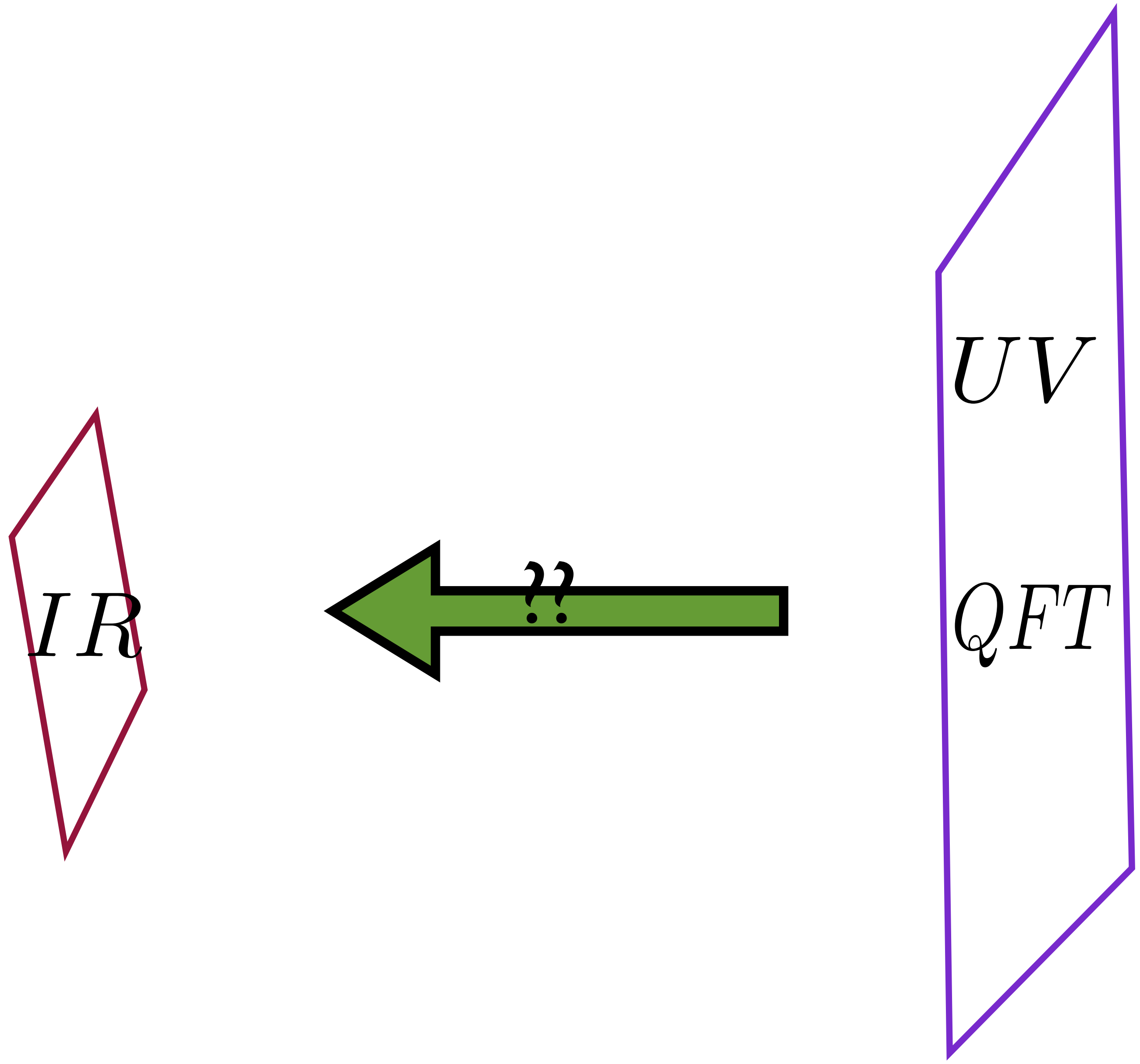
why does this work at long wavelengths?



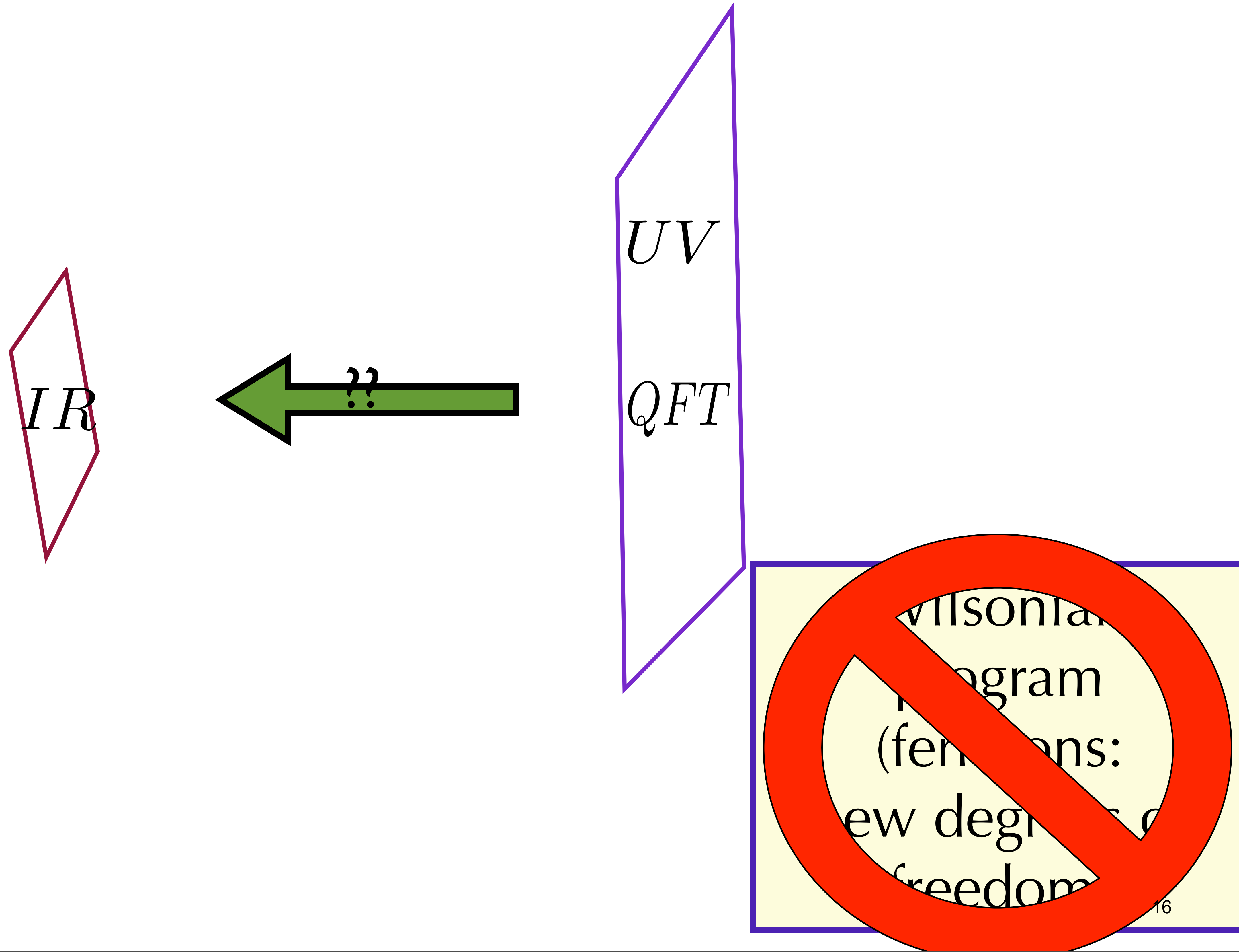
UV
QFT

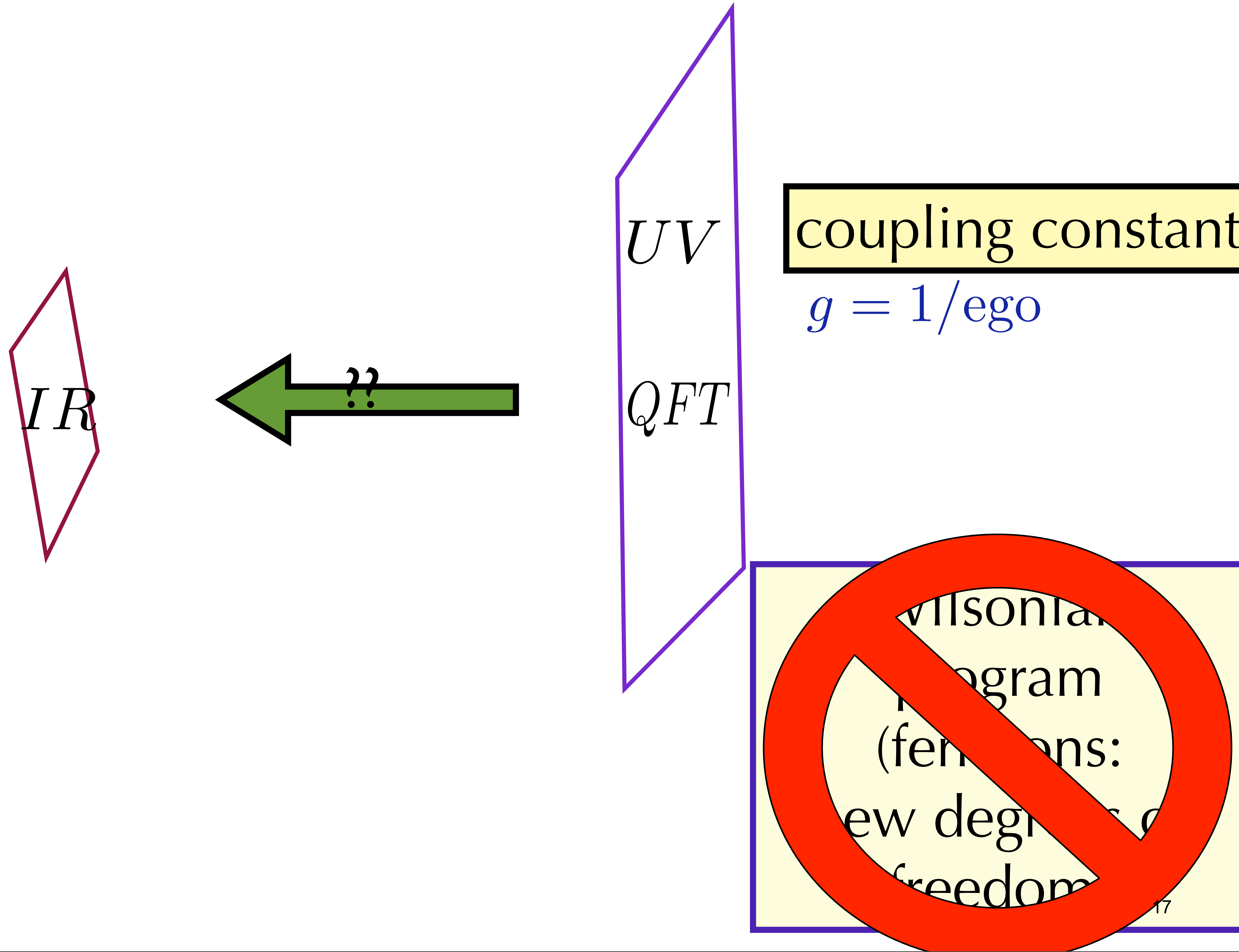


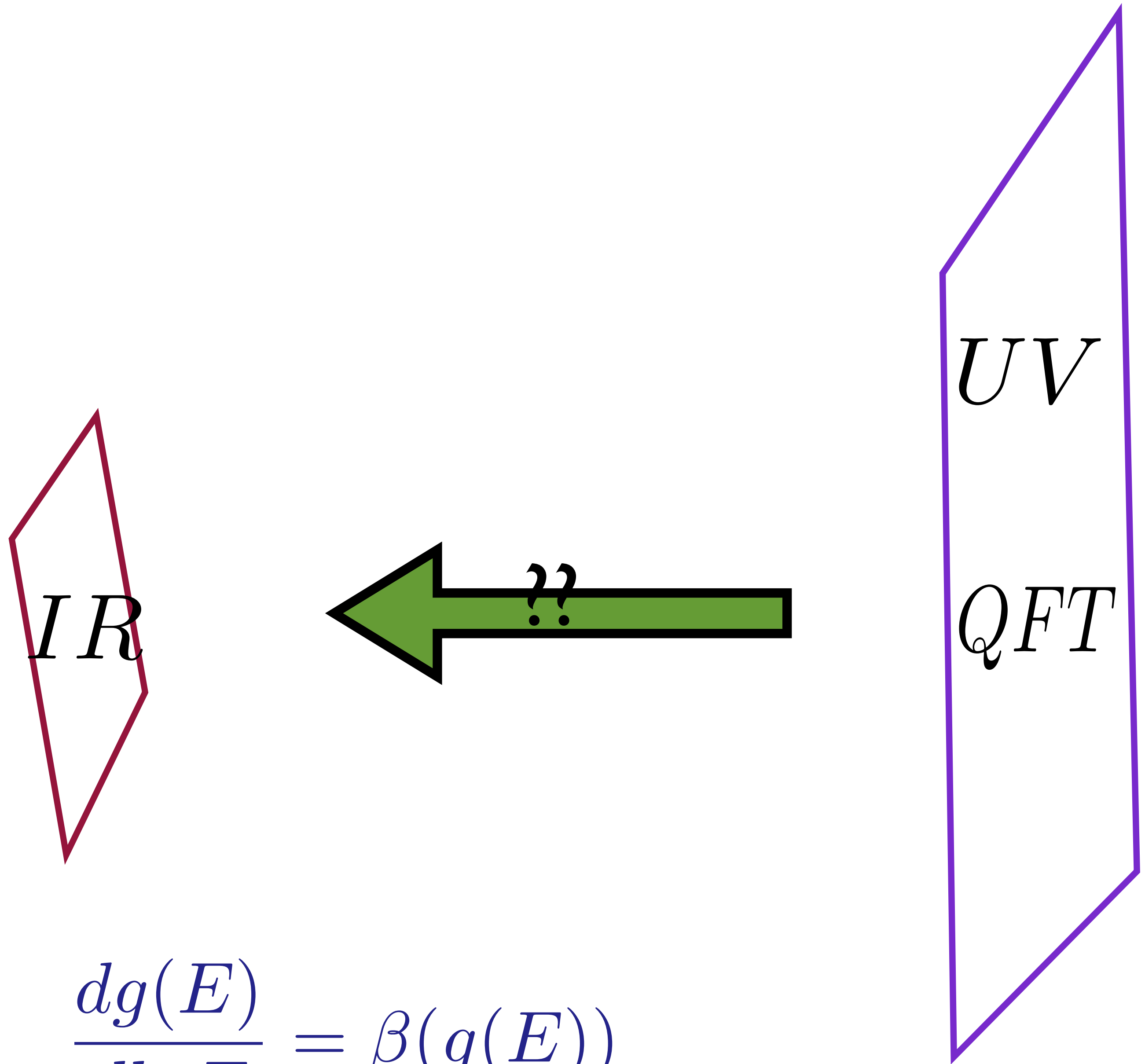
UV
QFT
??
IR



Wilsonian program (fermions: new degrees of freedom)


IR
??
UV
QFT


IR
??
UV
QFT
coupling constant
g = 1/ego



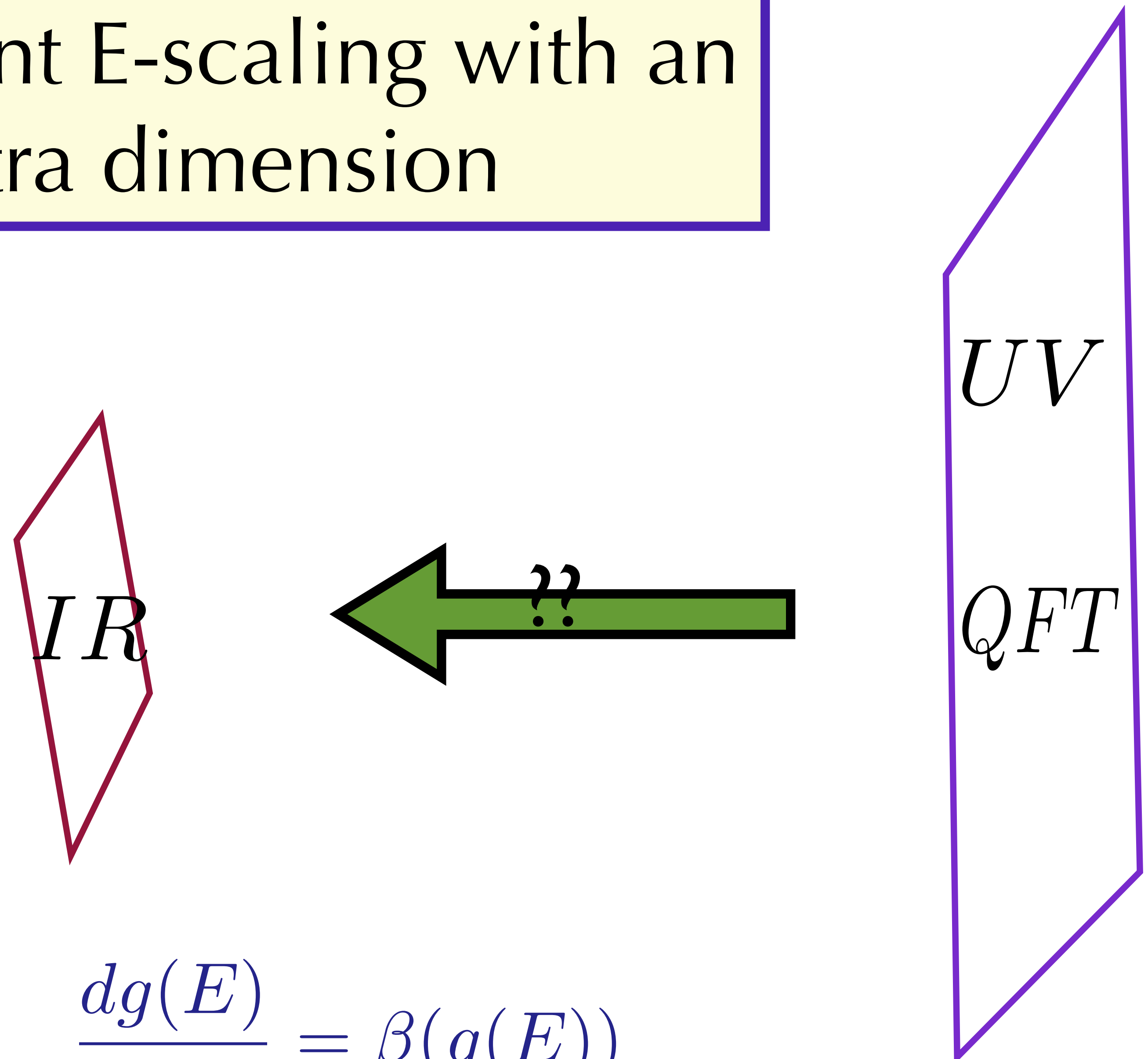
coupling constant

$g = 1/\mathrm{ego}$

$$\frac{dg(E)}{dlnE} = \beta(g(E))$$

locality in energy

implement E-scaling with an extra dimension



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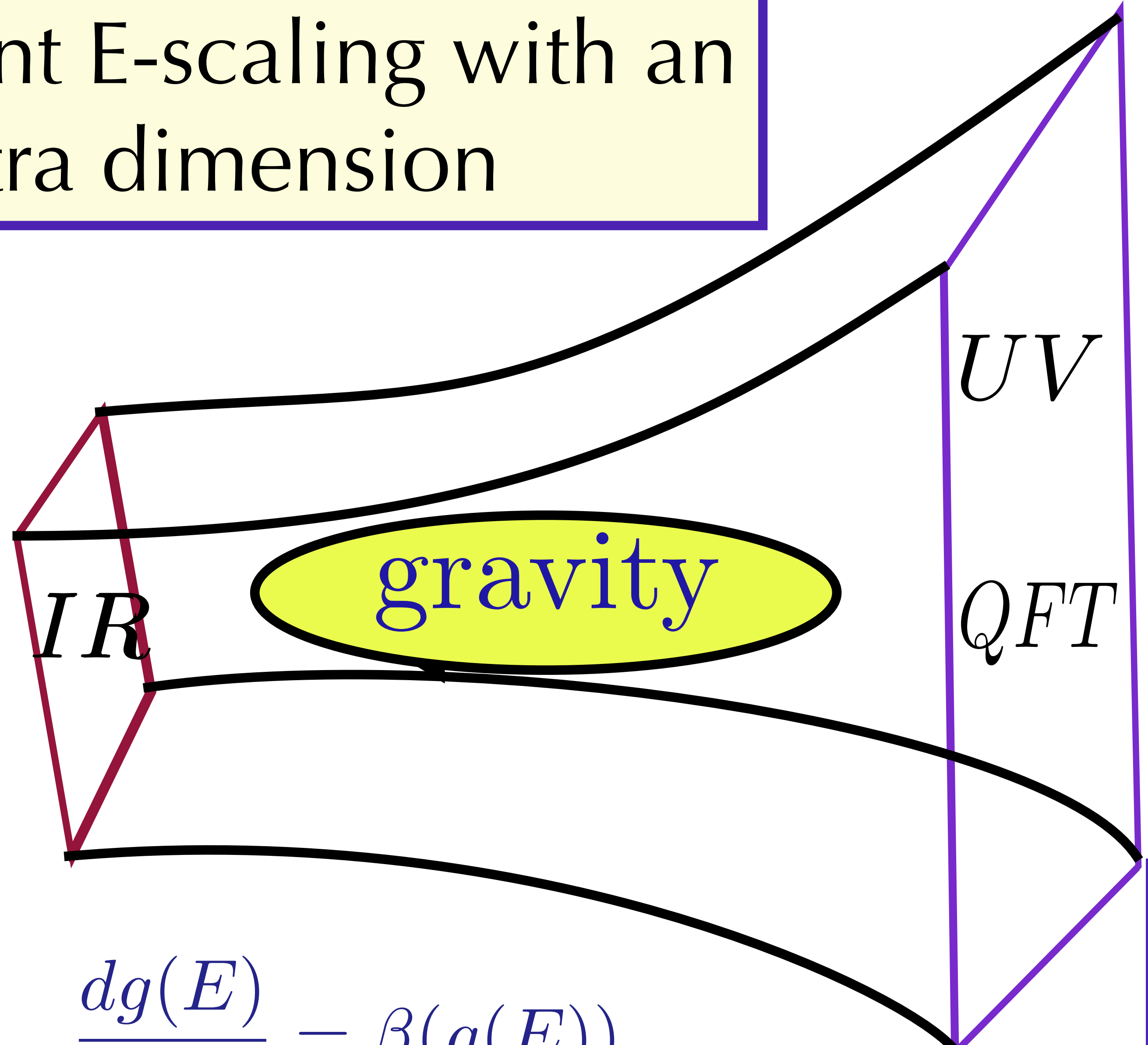
locality in energy

gauge-gravity duality
(Maldacena, 1997)

implement E-scaling with an extra dimension



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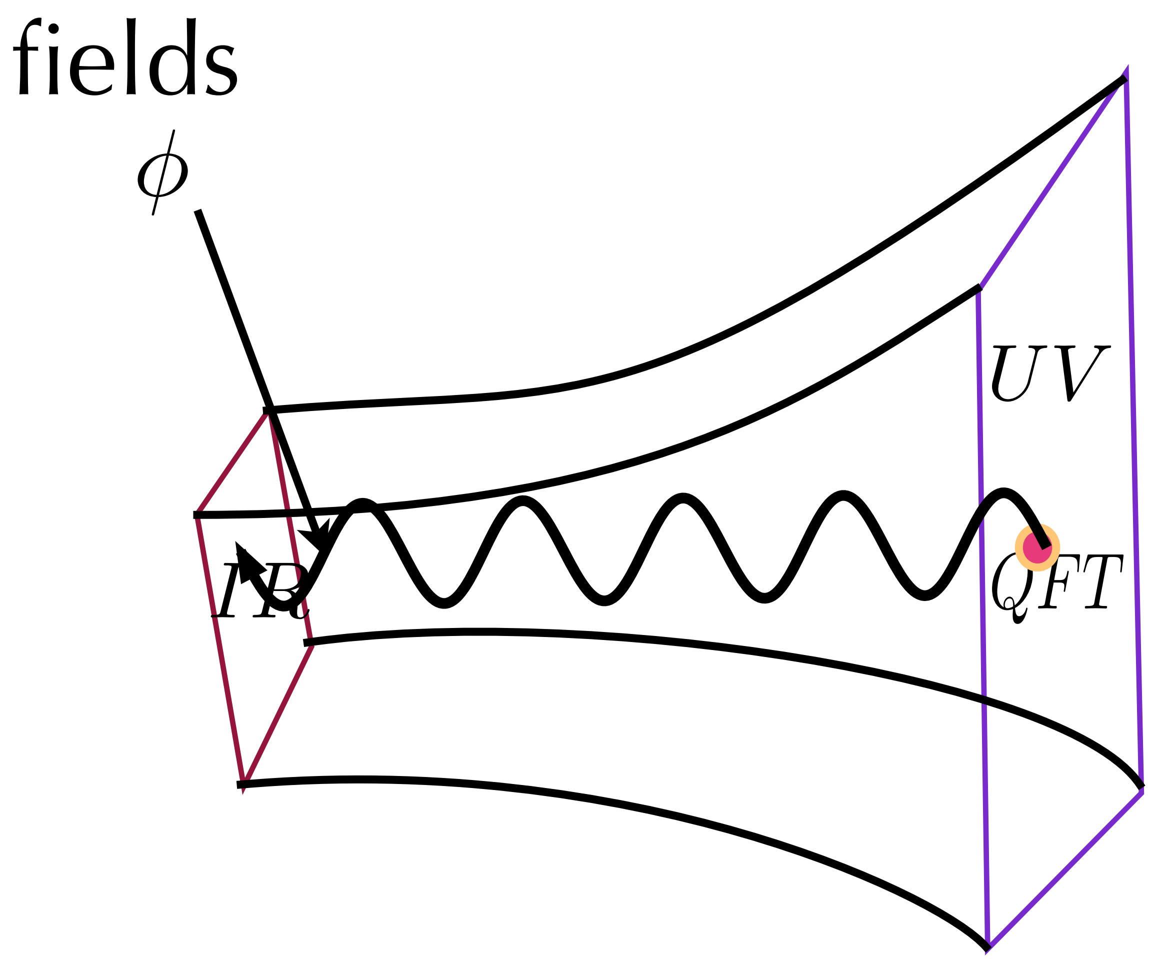
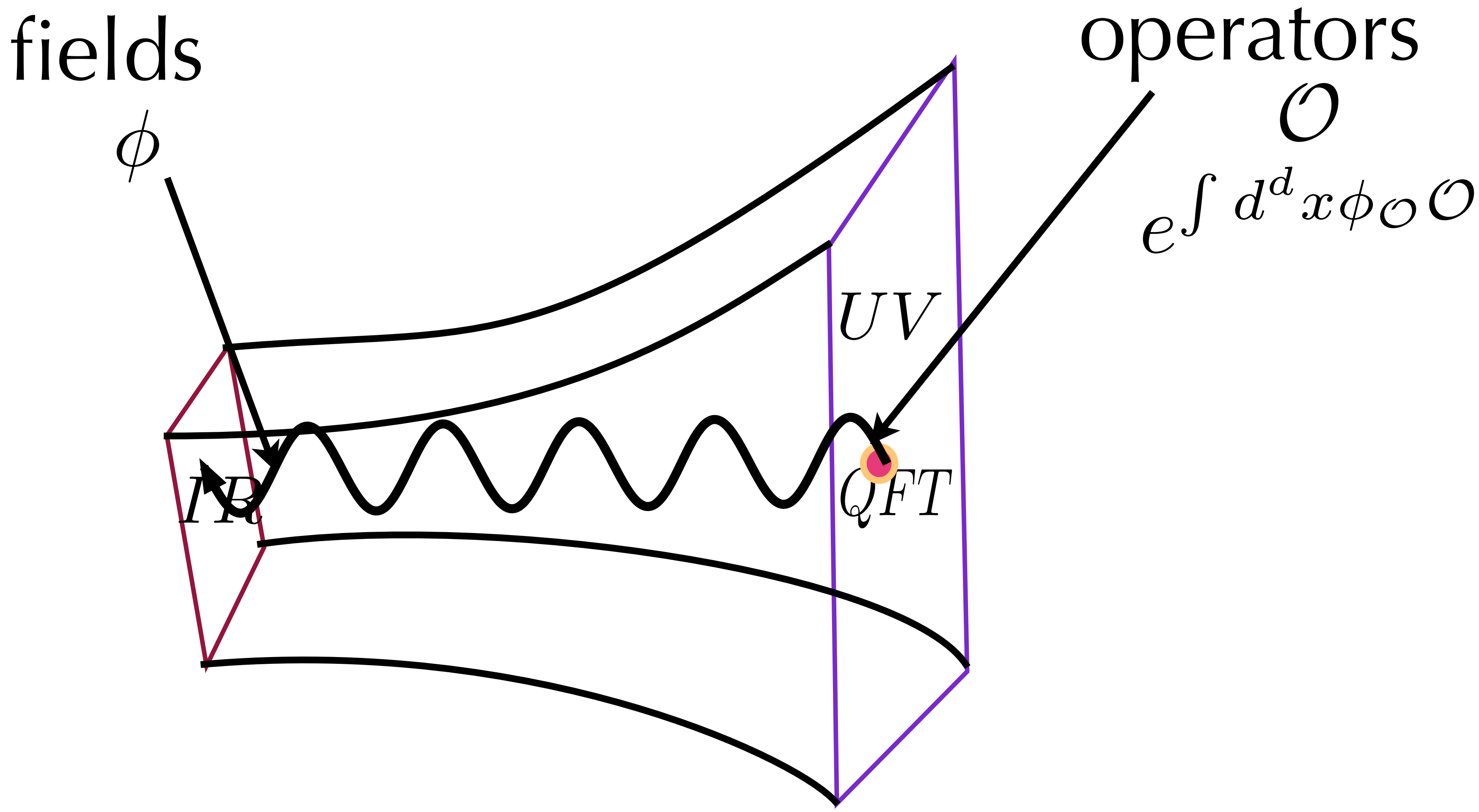


$$e^{\int d^d x \phi_{\mathcal{O}} \mathcal{O}}$$

Claim: $Z_{\mathrm{QFT}} = e^{-S^{\mathrm{on-shell}}_{\mathrm{ADS}}}(\phi(\phi_{\partial \mathrm{ADS}=\mathrm{J}_{\mathcal{O}}}))$



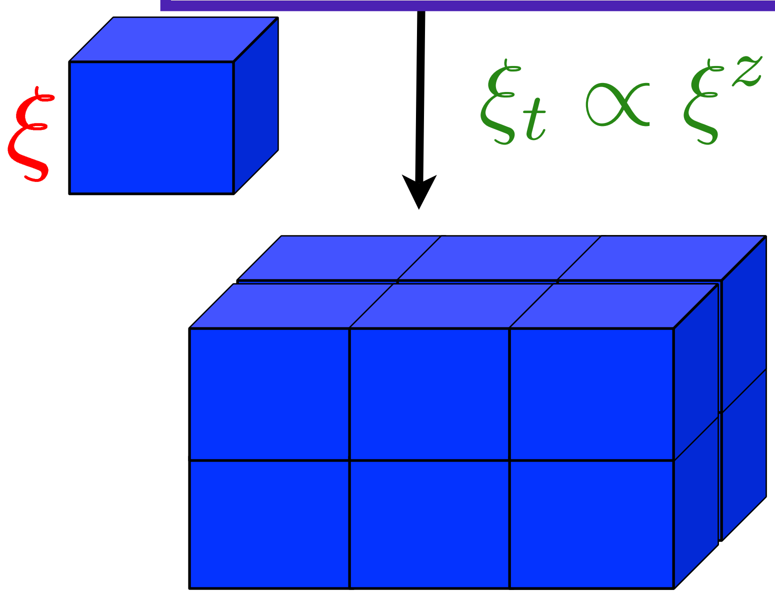
Claim: $Z_{\rm QFT} = e^{-S^{\rm on-shell}_{\rm ADS}(\phi(\phi_{\partial {\rm ADS}={\rm J}_{\mathcal{O}}}))}$

What holography does for you?

What holography does for you?

Landau-Wilson

Hamiltonian



long-wavelengths

RG equations

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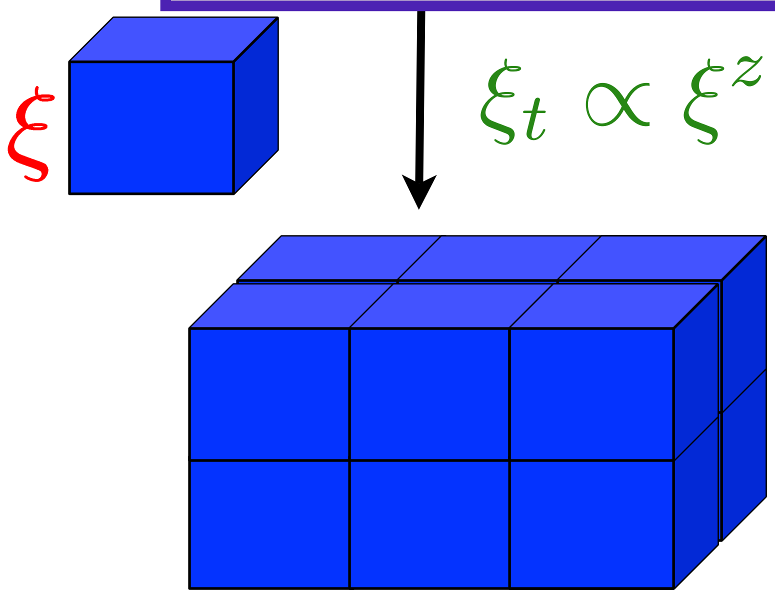

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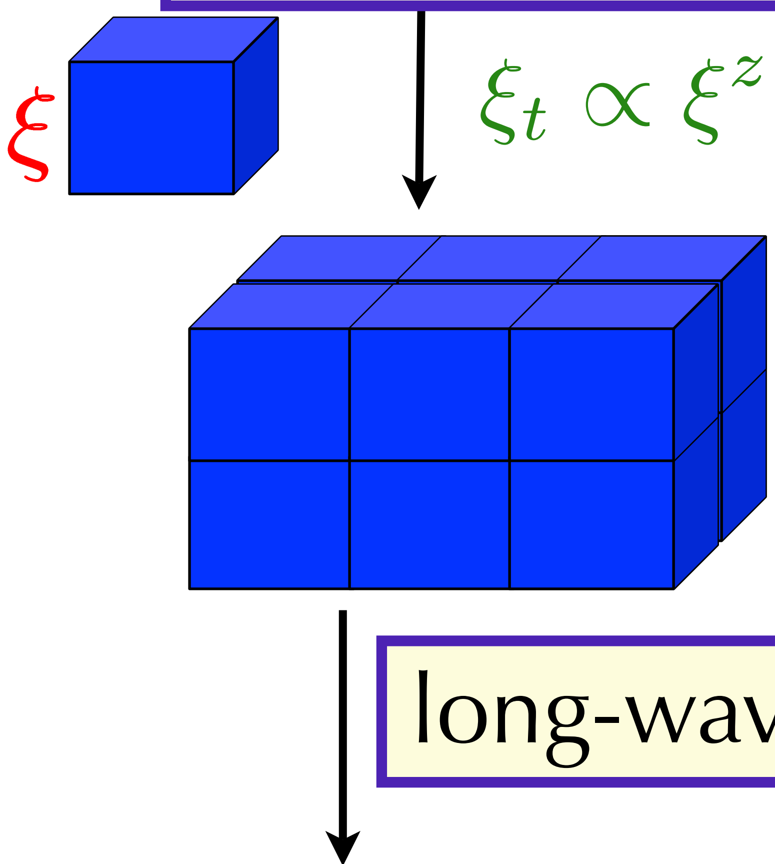
RG=GR

Hamiltonian



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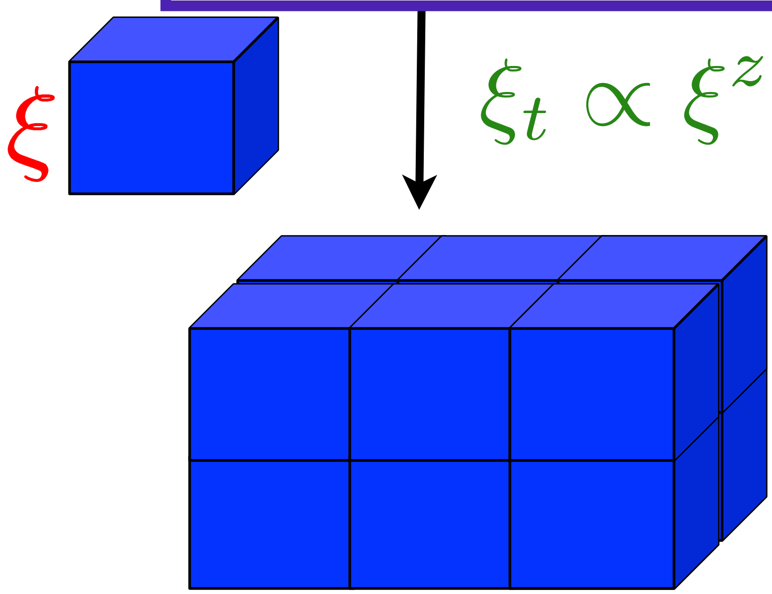
$\xi_t \propto \xi^z$



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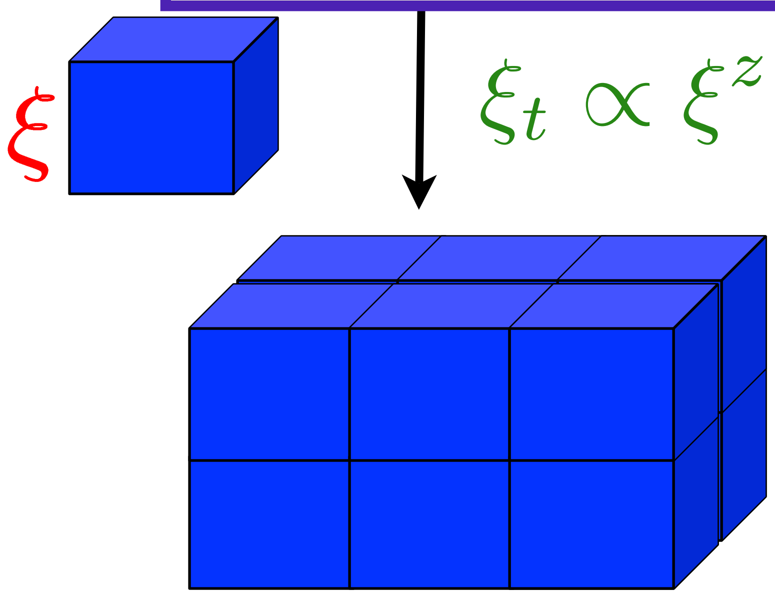

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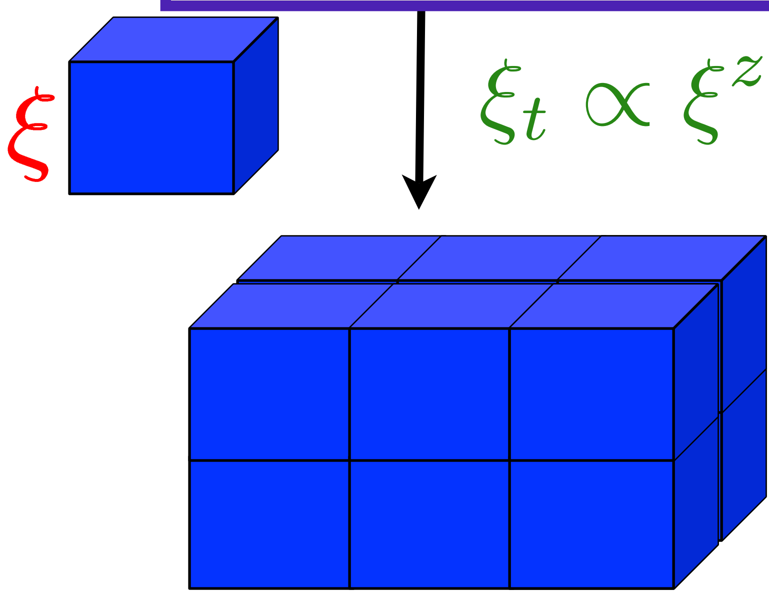

long-wavelengths

RG equations

holography

RG=GR

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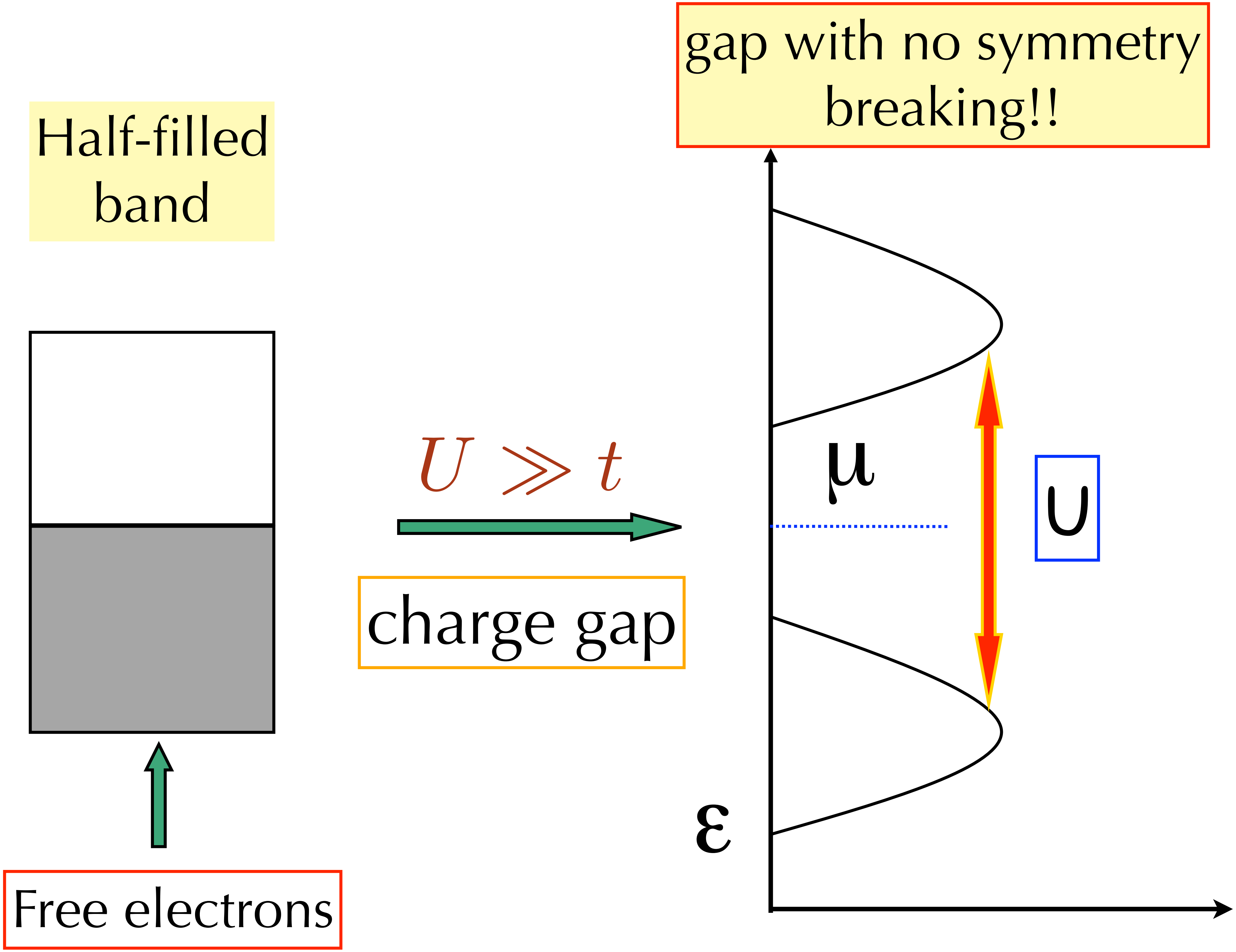
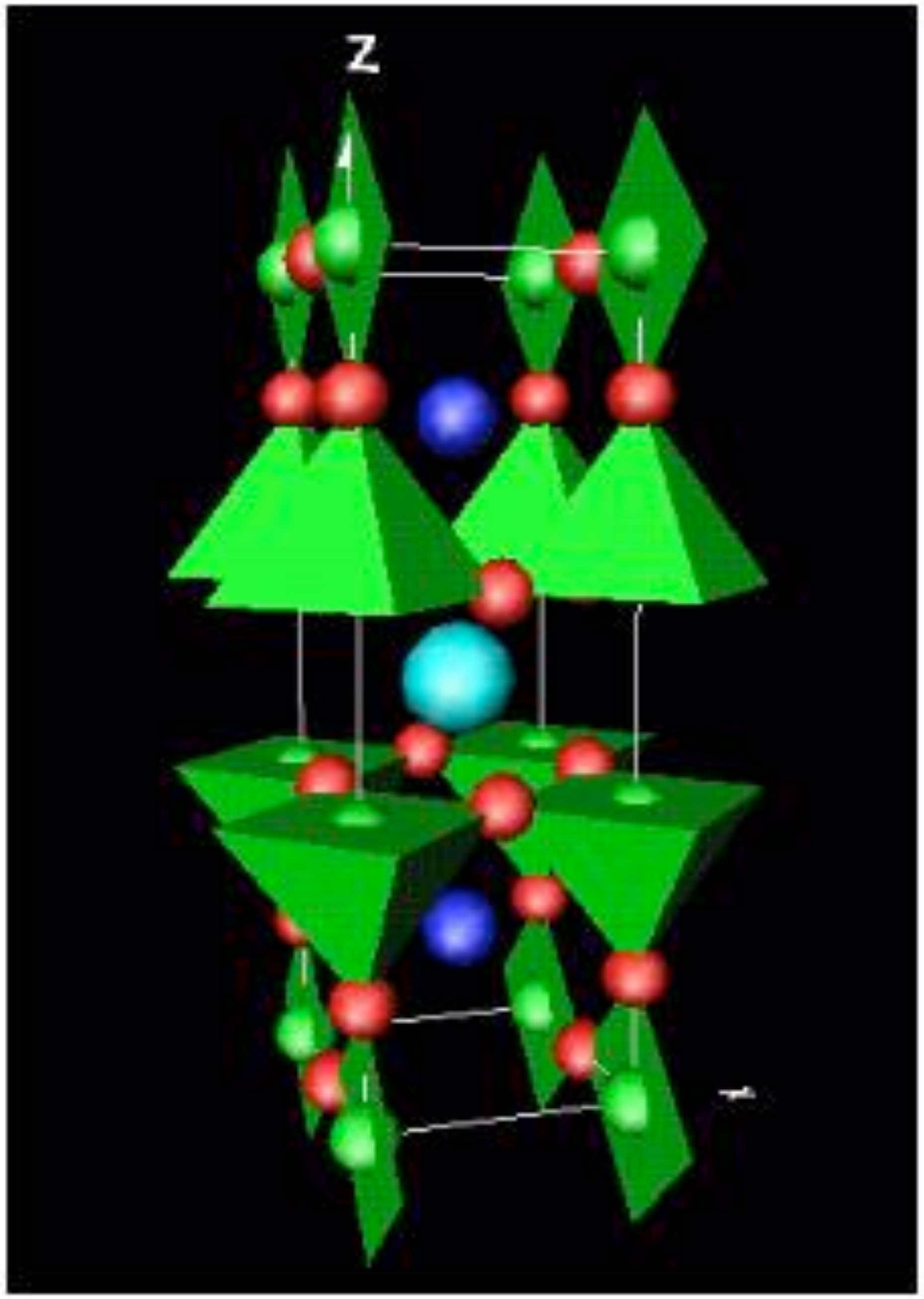
microscopic UV model not easy (need M-theory)

so what (currents, symmetries)

Can holography solve the Mott problem?


gap with no symmetry breaking!!
Half-filled band
$U \gg t$
charge gap
μ
U
ε
Free electrons

Why is the Mott problem important?



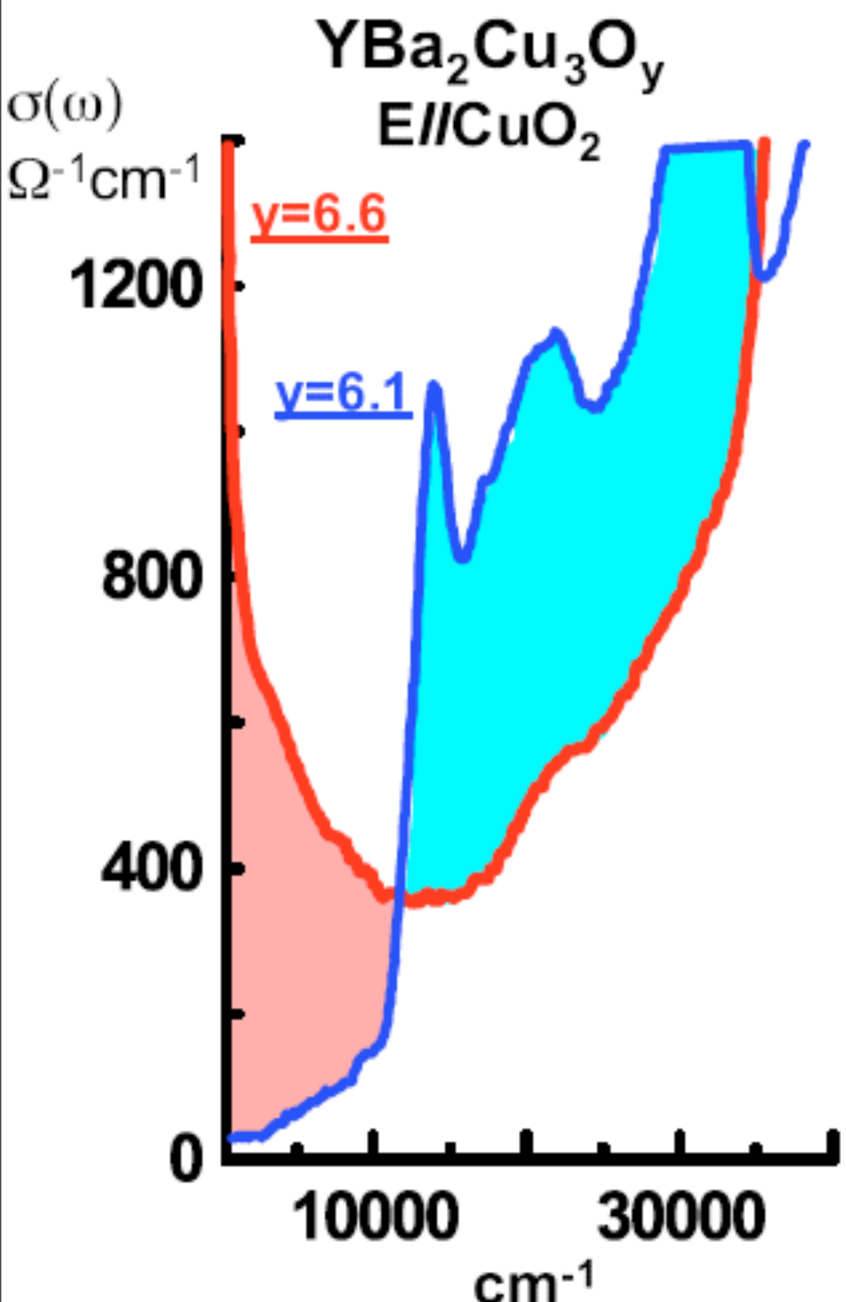
$Y\,Ba_2Cu_3O_7$
Cuprate Superconductors

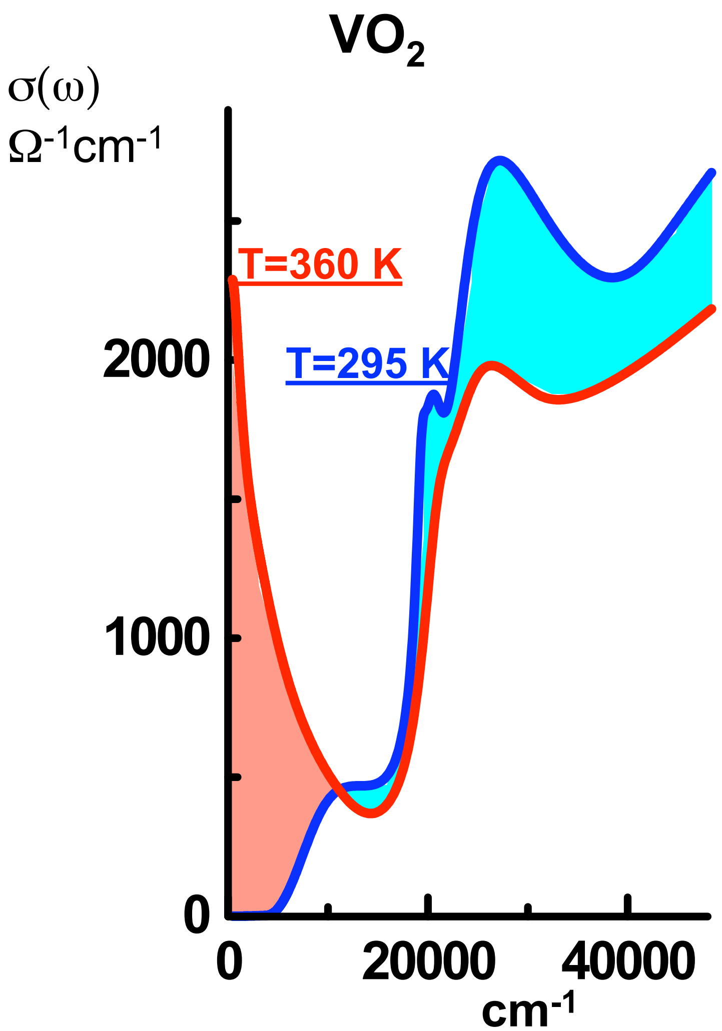
$$U/t = 10 \gg 1$$

interactions dominate:
Strong Coupling Physics

Experimental facts: Mottness

$$\Delta = 0.6eV > \Delta_{\text{dimerization}}$$ (Mott, 1976) $$\frac{\Delta}{T_{\text{crit}}} \approx 20$$





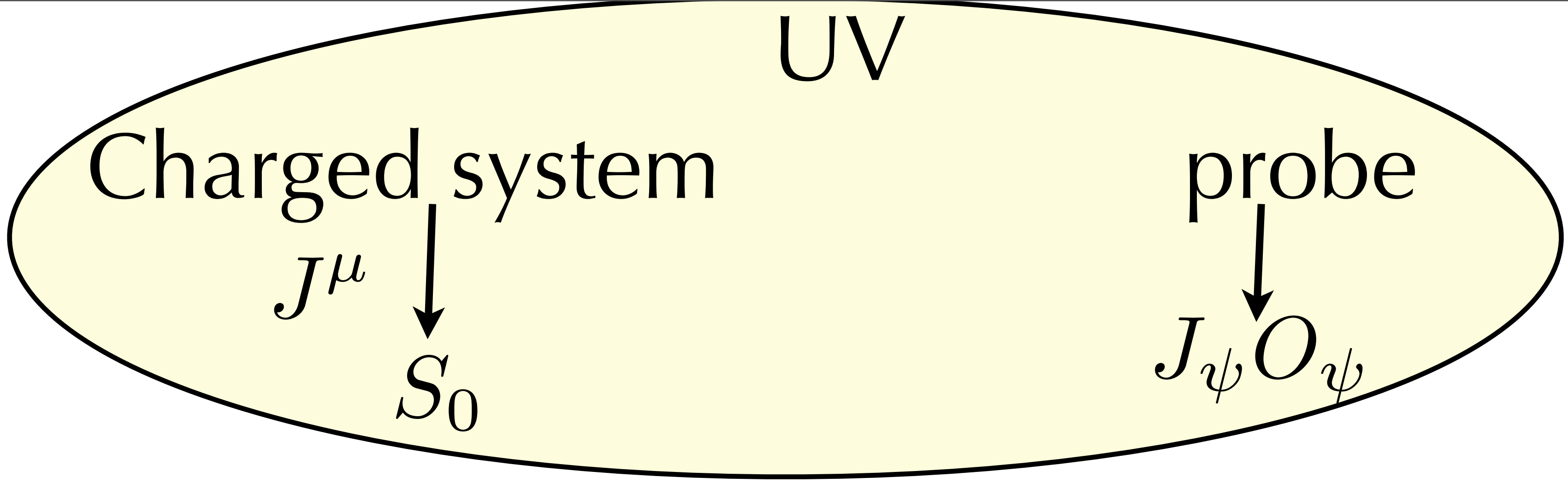
M. M. Qazilbash, K. S. Burch, D. Whisler, D. Shrekenhamer, B. G. Chae, H. T. Kim, and D. N. Basov PRB 74, 205118 (2006)

transfer of spectral weight to high energies beyond any ordering scale

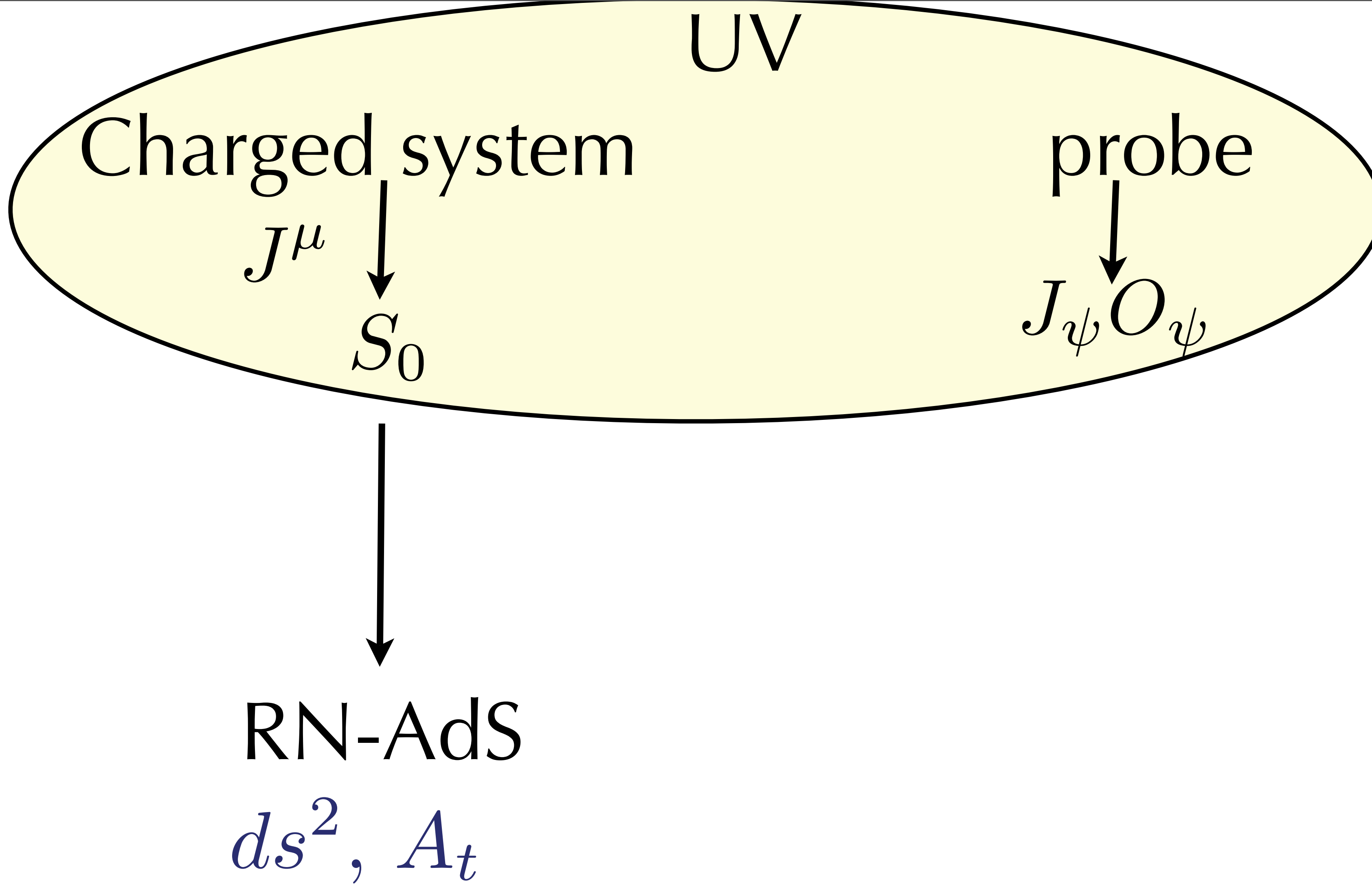
Recall, $eV = 10^4 K$

What bulk gravitational theory gives rise to a gap in ImG without
`spontaneous' symmetry breaking?

dynamically generated gap: Mott gap
(for probe fermions)



what has been done?
MIT, Leiden, McMaster,...



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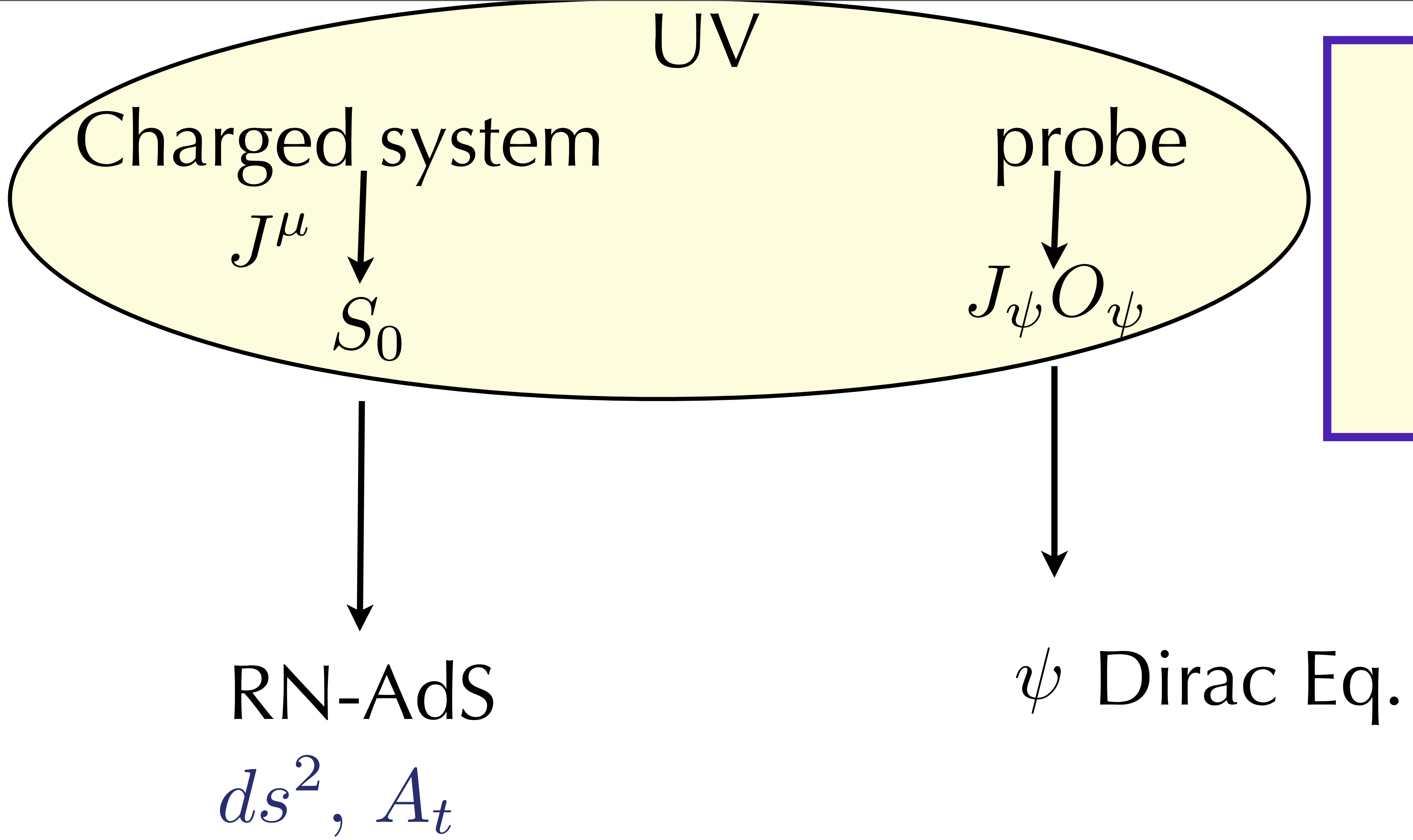

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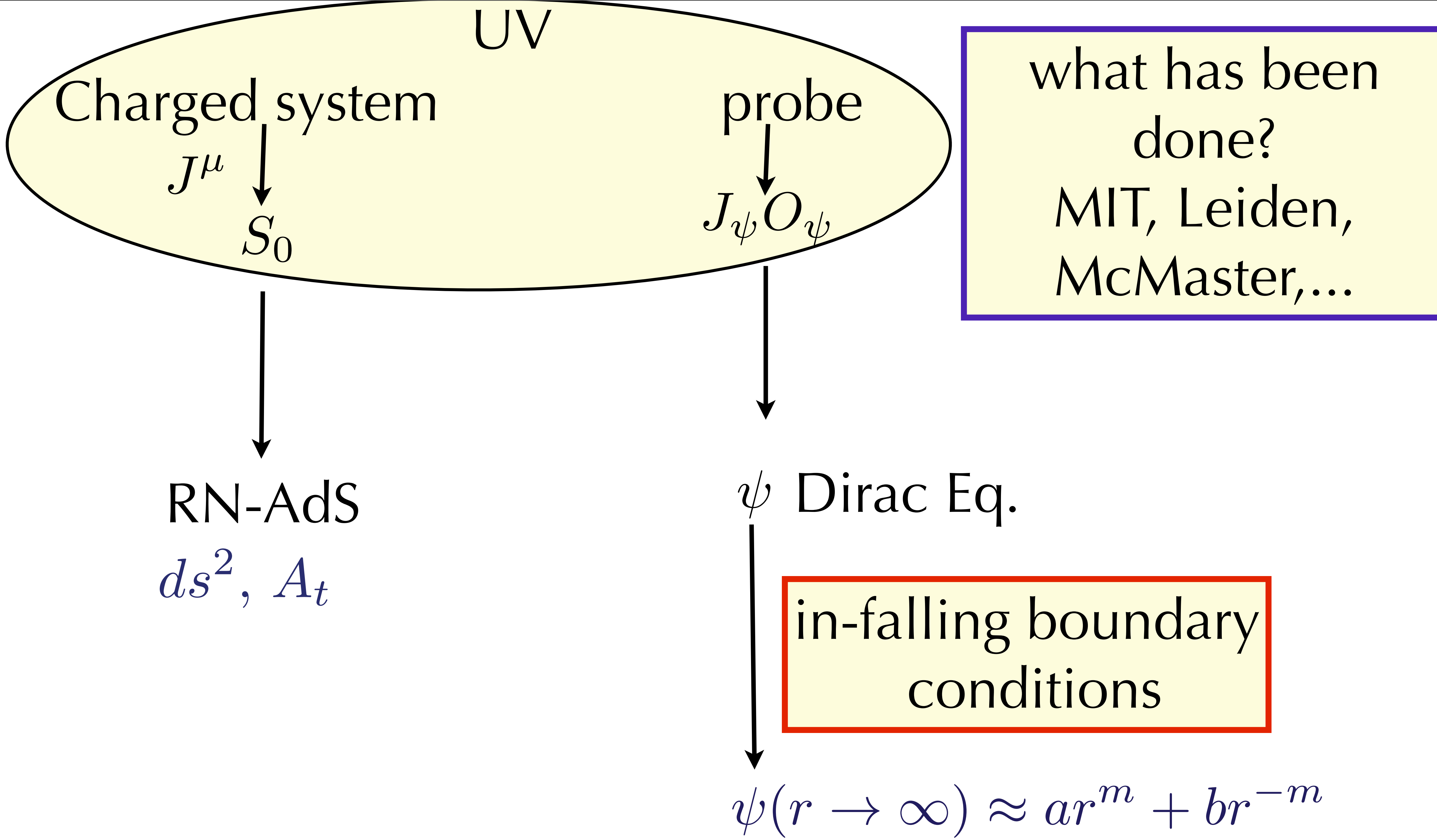


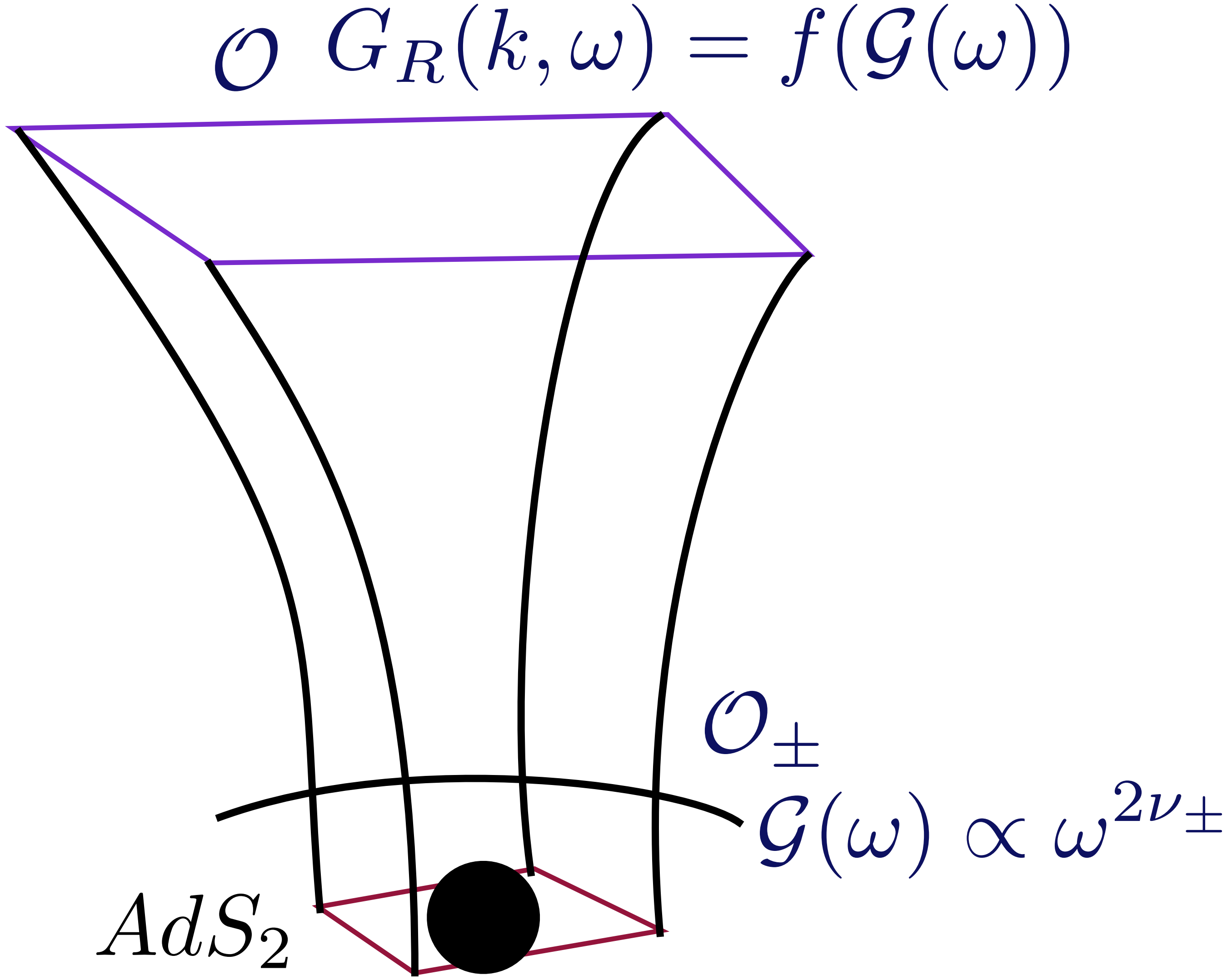
Retarded Green function: $G = \dfrac{b}{a}$ =f(UV (k_F), IR (q,m))

a=0 defines FS

$$ds^2 = \frac{L_2^2}{\zeta^2}\left(-d\tau^2 + d\zeta^2\right) + \frac{r_0^2}{R^2} d\vec{x}^2$$

$$\zeta = \omega L_2^2 / (r - r_0)$$

boundary physics:
$AdS_2 \times R^2$



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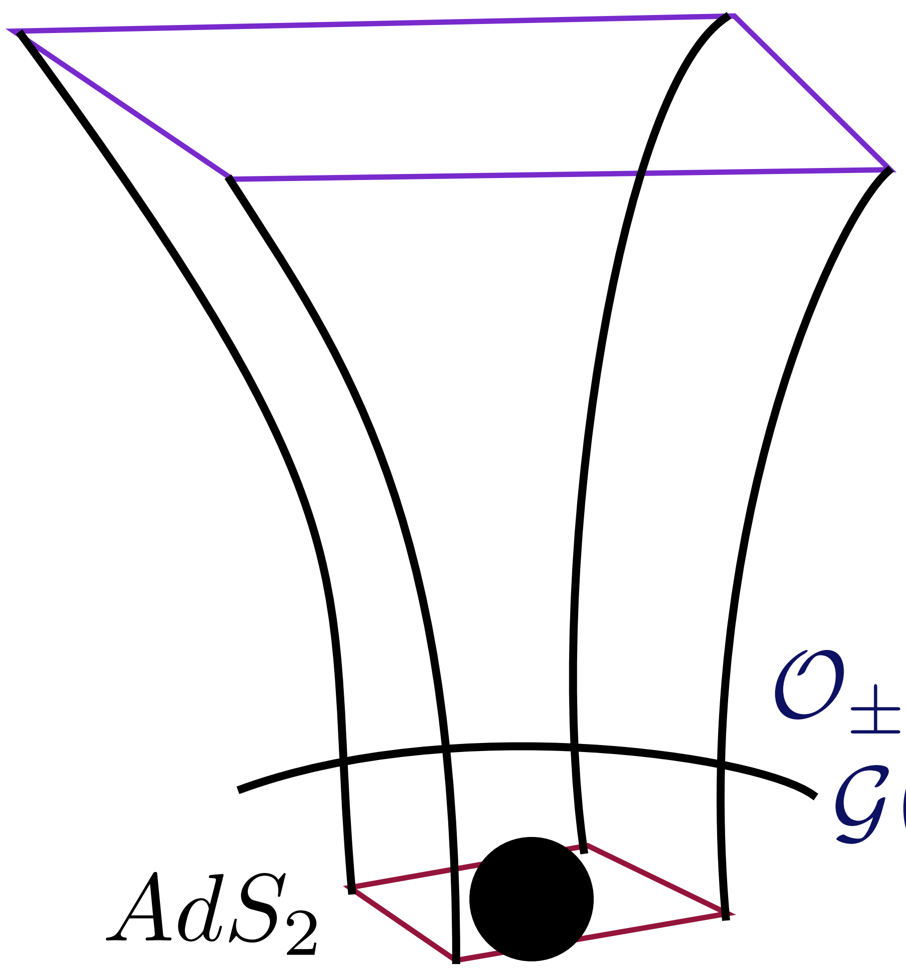
boundary physics:
$AdS_2 \times R^2$

$$\{\vec{x}, \tau, \zeta\} \to \{\vec{x}, \lambda\tau, \lambda\zeta\}$$

$\mathcal{O}$ $G_R(k,\omega) = f(\mathcal{G}(\omega))$



$\mathcal{O}_\pm$

$\mathcal{G}(\omega) \propto \omega^{2\nu_\pm}$

AdS_2

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boundary physics:
$AdS_2 \times R^2$

finite T=0 entropy

$$S \propto T^{2/z} \neq 0 \quad T \to 0$$

$$\vec{x} \to \lambda^{(1/z=\infty)} \vec{x} = \vec{x}$$

$$\{\vec{x}, \tau, \zeta\} \to \{\vec{x}, \lambda\tau, \lambda\zeta\}$$

$$ds^2 = \frac{L_2^2}{\zeta^2}\left(-d\tau^2 + d\zeta^2\right) + \frac{r_0^2}{R^2} d\vec{x}^2$$

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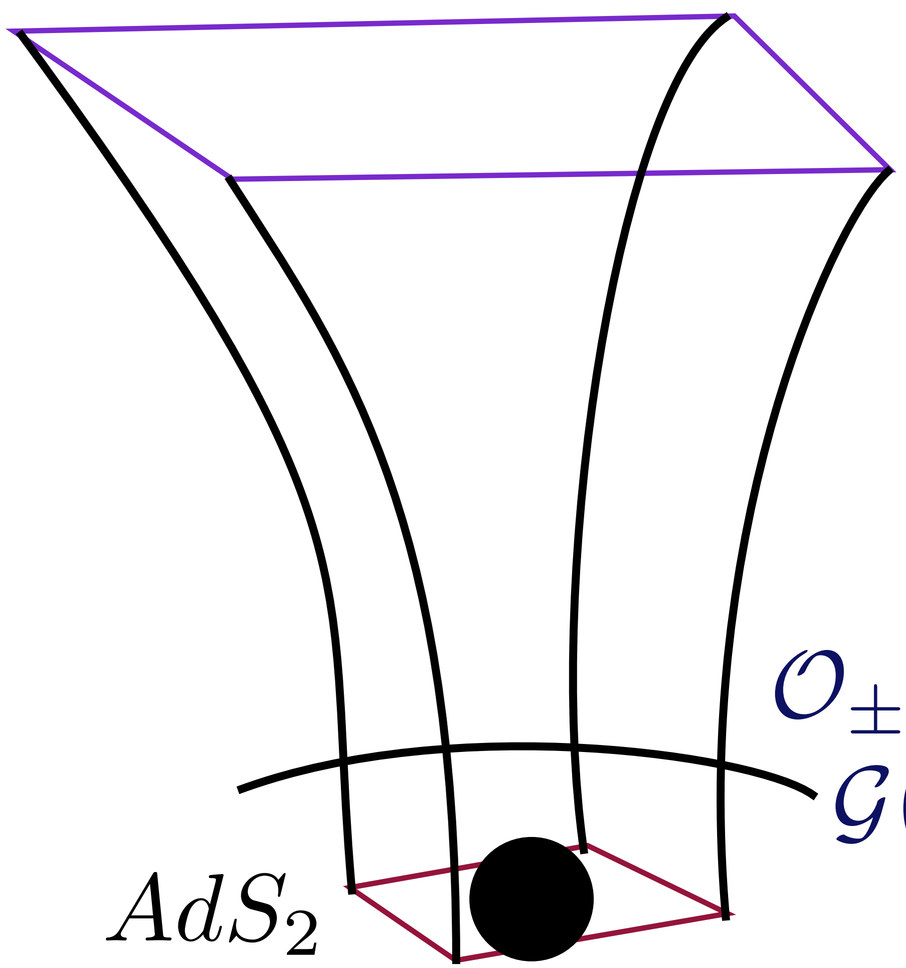

boundary physics:
$AdS_2 \times R^2$

$$\sqrt{-g}i\bar{\psi}(D-m)\psi$$

AdS-RN
MIT, Leiden group

→

$$G(\omega,k)=\frac{Z}{v_F(k-k_F)-\omega-h\omega\ln\omega}$$

marginal Fermi liquid

??

→

Mott gap

How to Destroy the Fermi surface?

decoherence

$$\sqrt{-g} i\bar{\psi}(D - m)\psi$$

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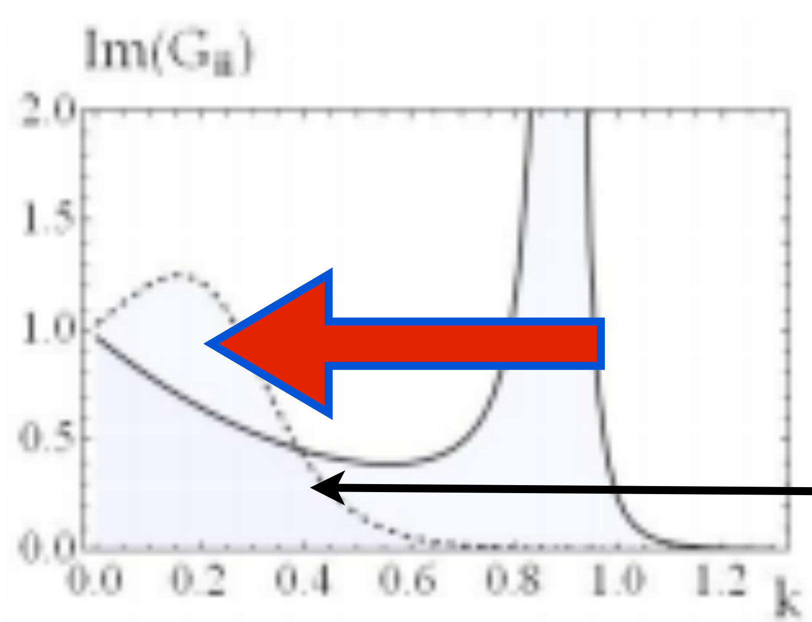

log-oscillatory regime

$$S_{\mathrm{probe}}(\psi, \bar{\psi}) = \int d^d x \sqrt{-g}\, i \bar{\psi} (\Gamma^M D_M - m + \cdots) \psi$$

$$S_{\text{probe}}(\psi, \bar{\psi}) = \int d^{\text{d}}x\sqrt{-g}\,i\bar{\psi}(\Gamma^M D_M - m + \cdots)\psi$$

what is hidden here?

$$S_{\rm probe}(\psi, \bar{\psi}) = \int d^d x \sqrt{-g} i \bar{\psi} (\Gamma^M D_M - m + \cdots) \psi$$

what is hidden here?

$$F_{\mu\nu} \Gamma^{\mu\nu}$$

consider $\sqrt{-g} i \bar{\psi} (\not{D} - m - i p \not{F}) \psi$

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consider $\sqrt{-g}\, i\bar{\psi}(\not{D} - m - ip\not{F})\psi$

QED anomalous magnetic moment of an electron
(Schwinger 1949)

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$$F_{\mu\nu}\Gamma^{\mu\nu}$$

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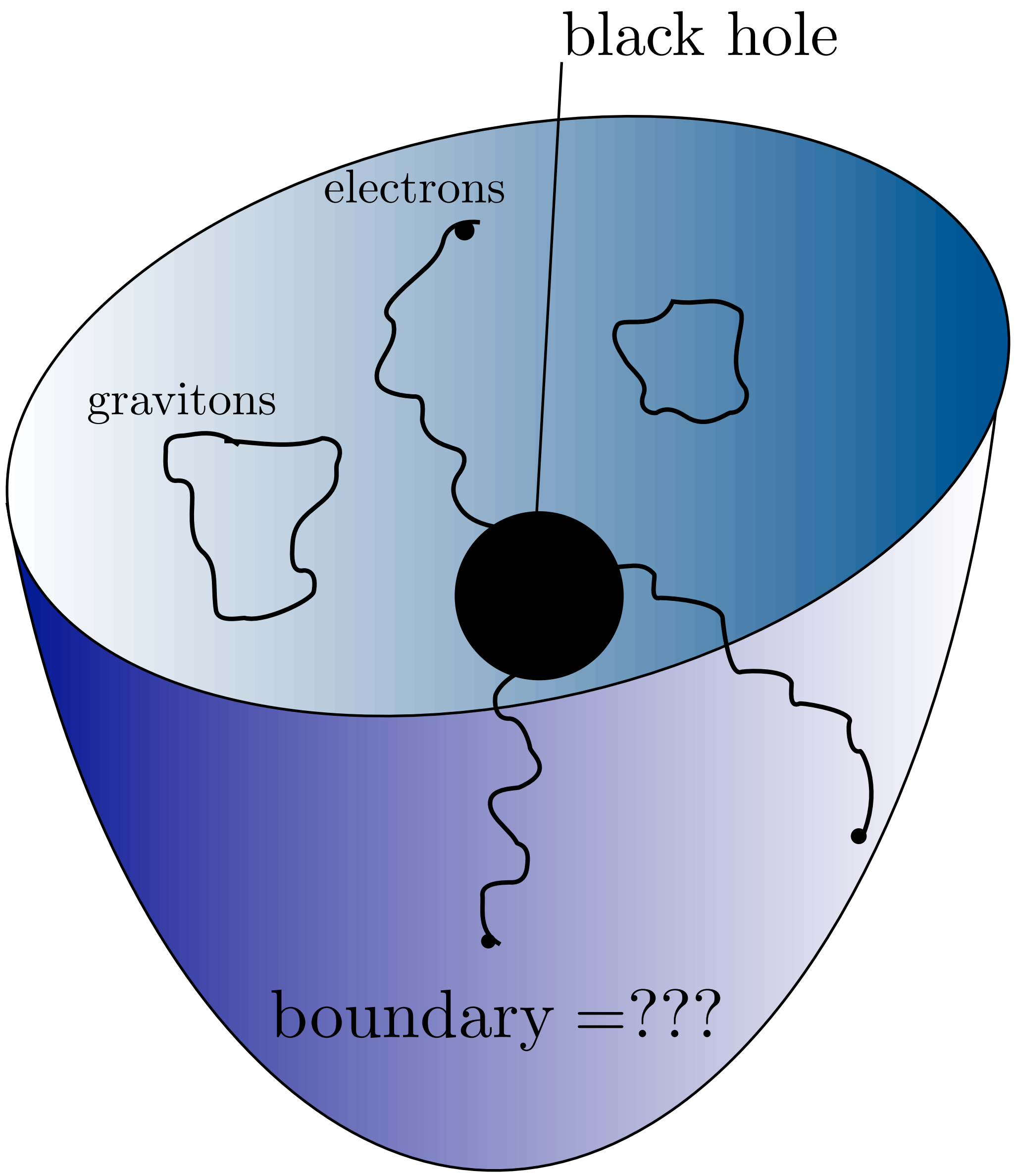
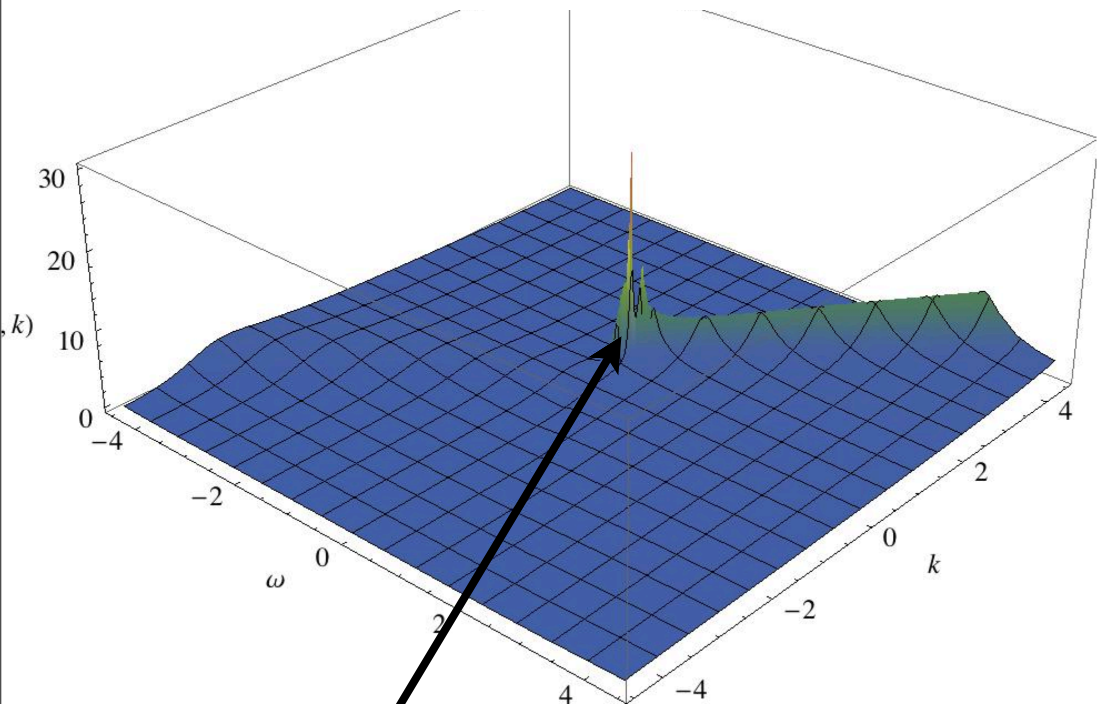
QED anomalous magnetic moment of an electron
(Schwinger 1949)

fermions in RN Ads_{d+1} coupled to a gauge field
through a dipole interaction


black hole
electrons
gravitons
boundary =???

How is the spectrum modified?

P=0

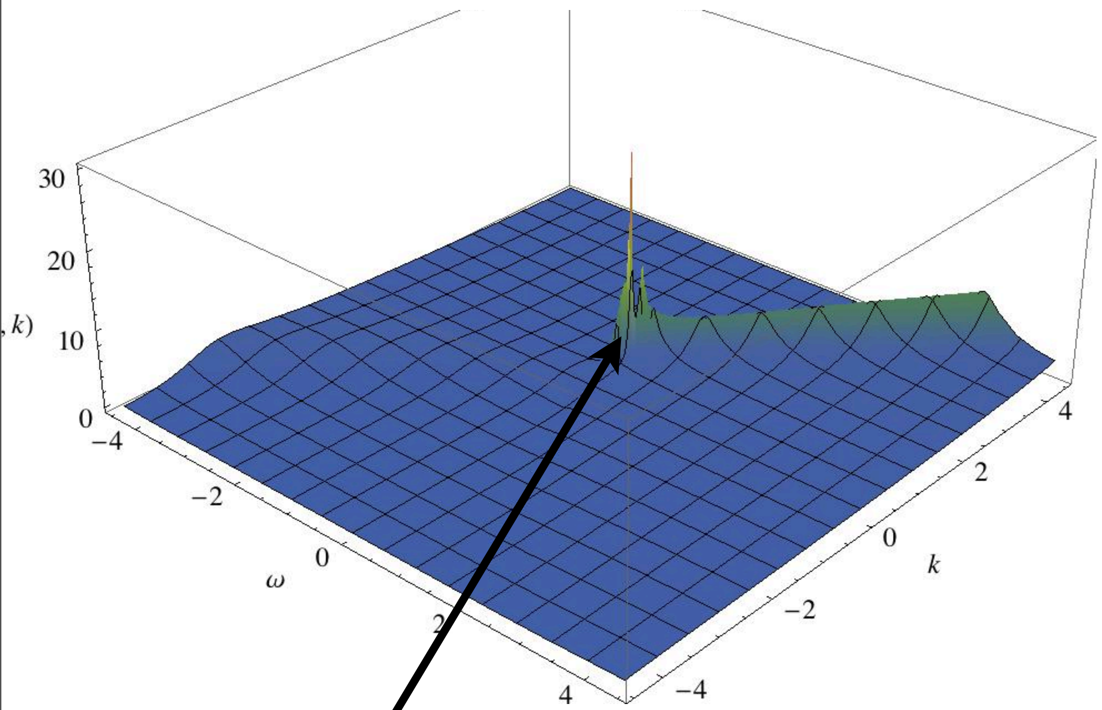


Fermi
surface
peak

How is the spectrum modified?

P

P=0



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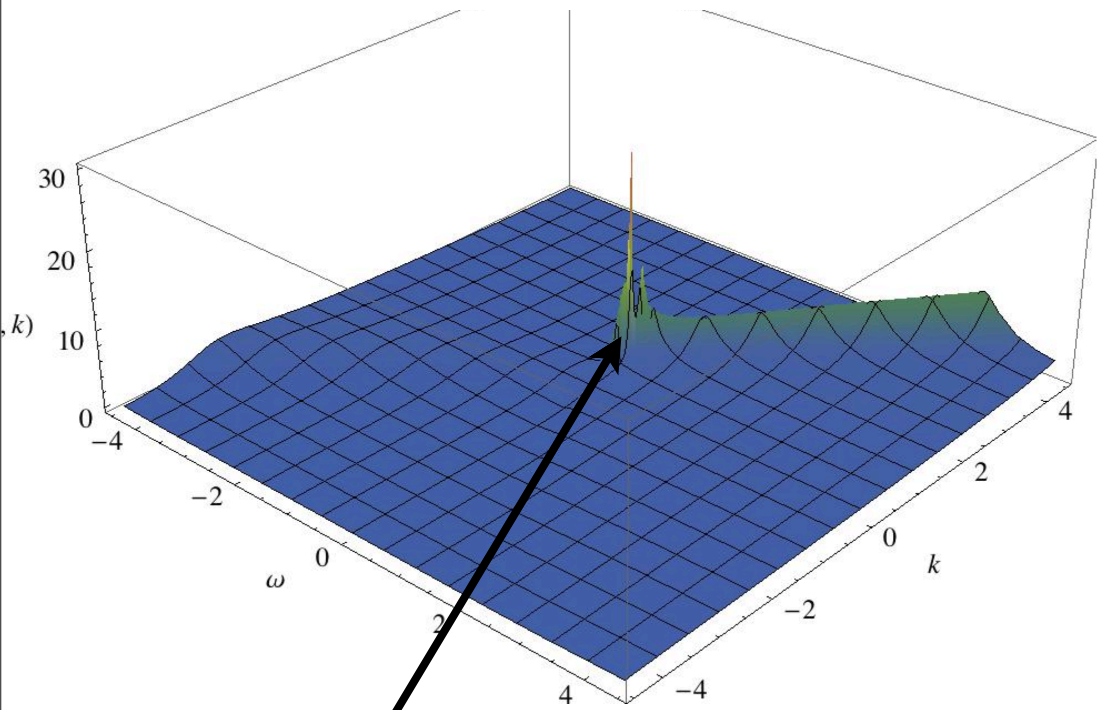

$$-1.54 < p < -0.53$$

$$1 > \nu_{k_F} > 1/2$$

$$\Re\omega \propto k - k_F$$

$$\Im\omega \propto (k - k_F)^{2\nu_{k_F}}$$

`Fermi Liquid'

Fermi surface peak

How is the spectrum modified?

P

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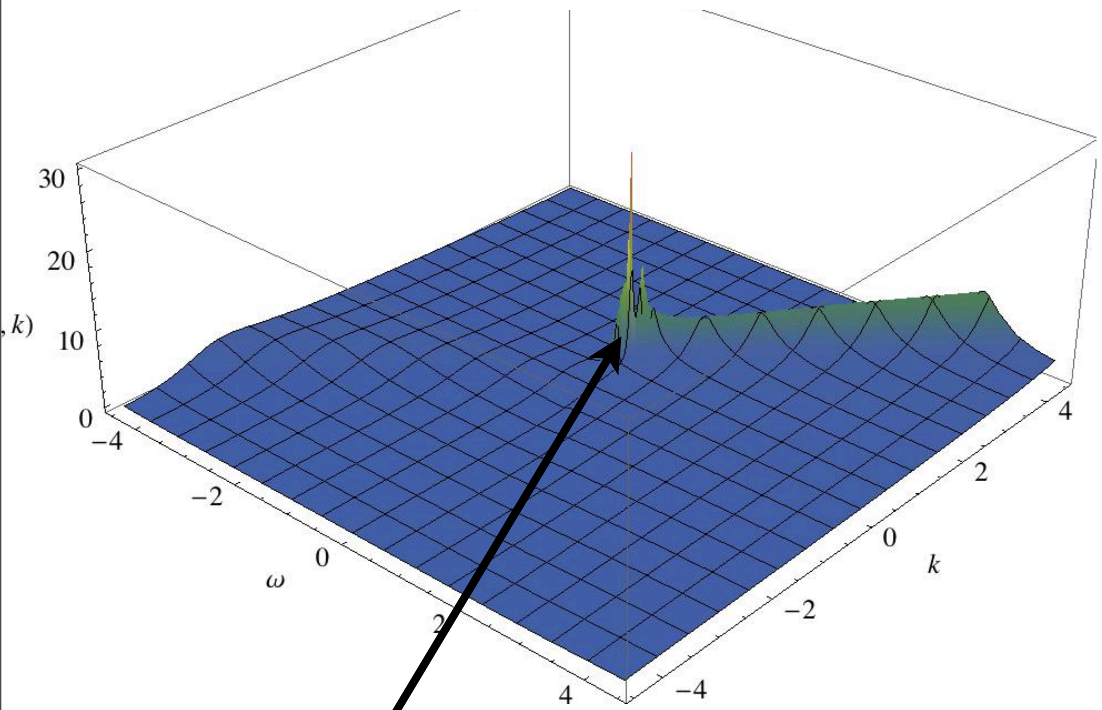

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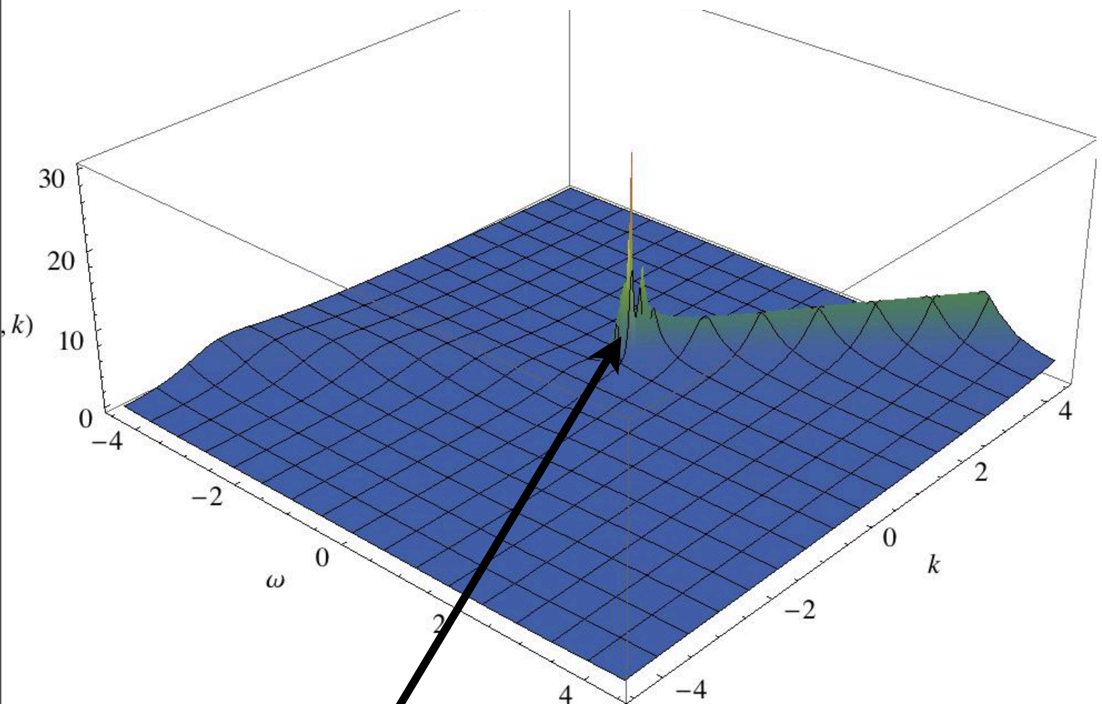


Fermi surface peak

$p = -0.53$

$\nu_{k_F} = 1/2$

MFL

$-0.53 < p < 1/\sqrt{6}$

$1/2 > \nu_{k_F} > 0$

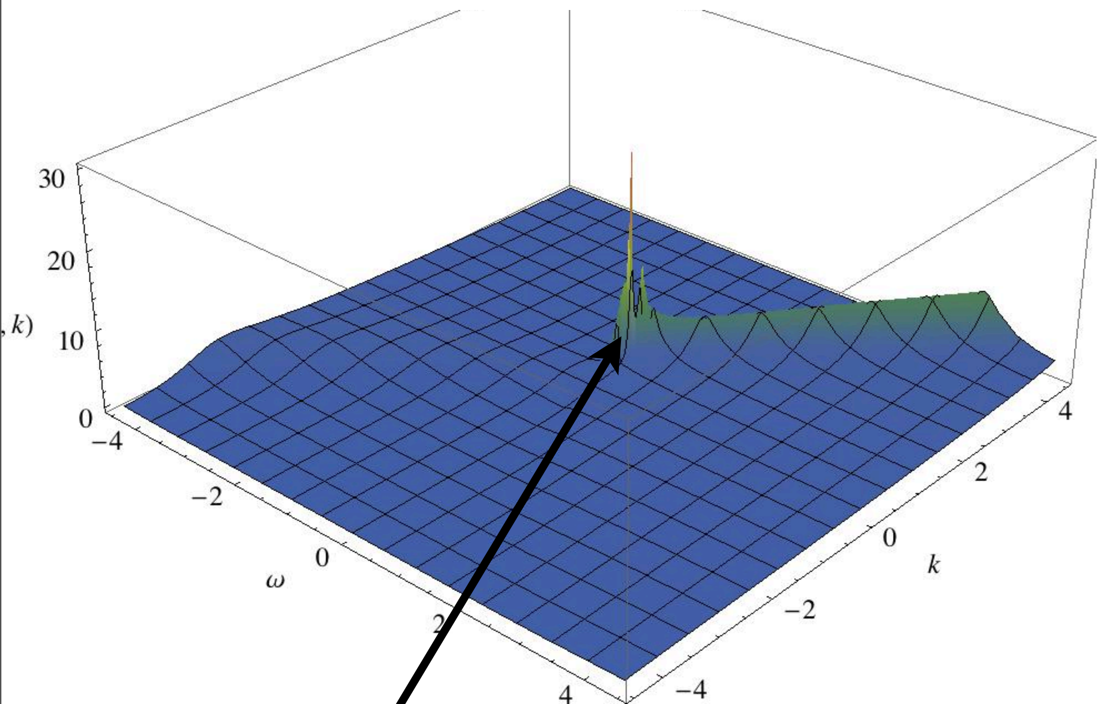
$\Re\omega = \Im\omega \propto (k - k_F)^{1/(2\nu_{k_F})}$

NFL

How is the spectrum modified?

P

P=0



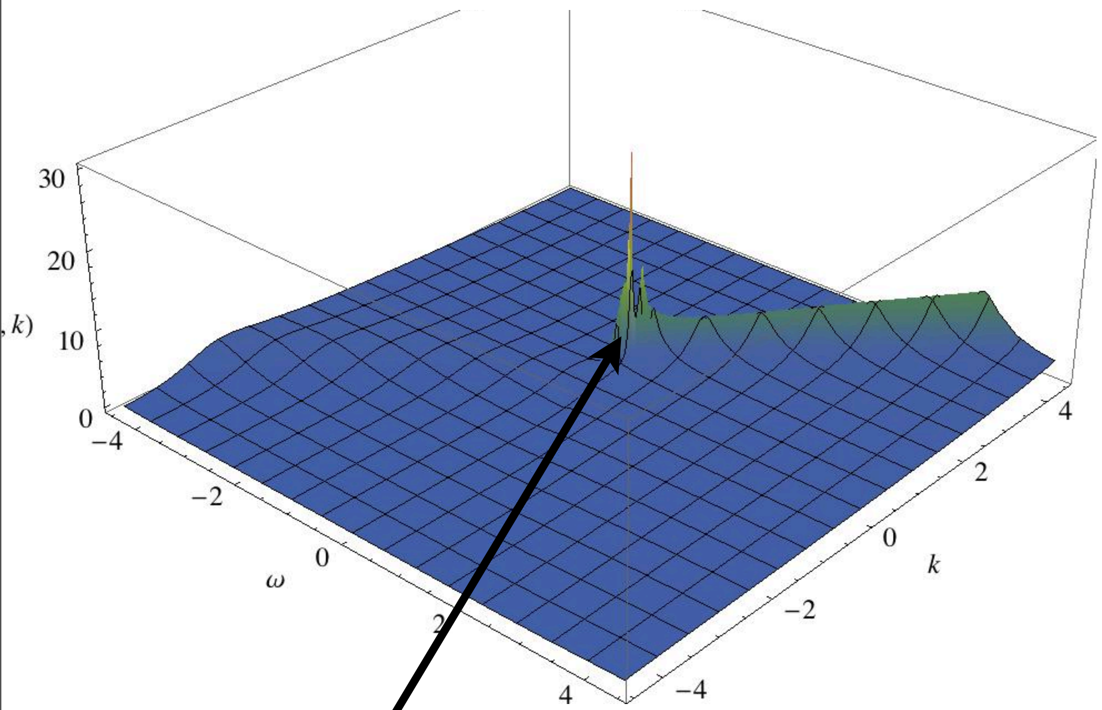
Fermi surface peak

How is the spectrum modified?

$P > 4.2$

P

P=0



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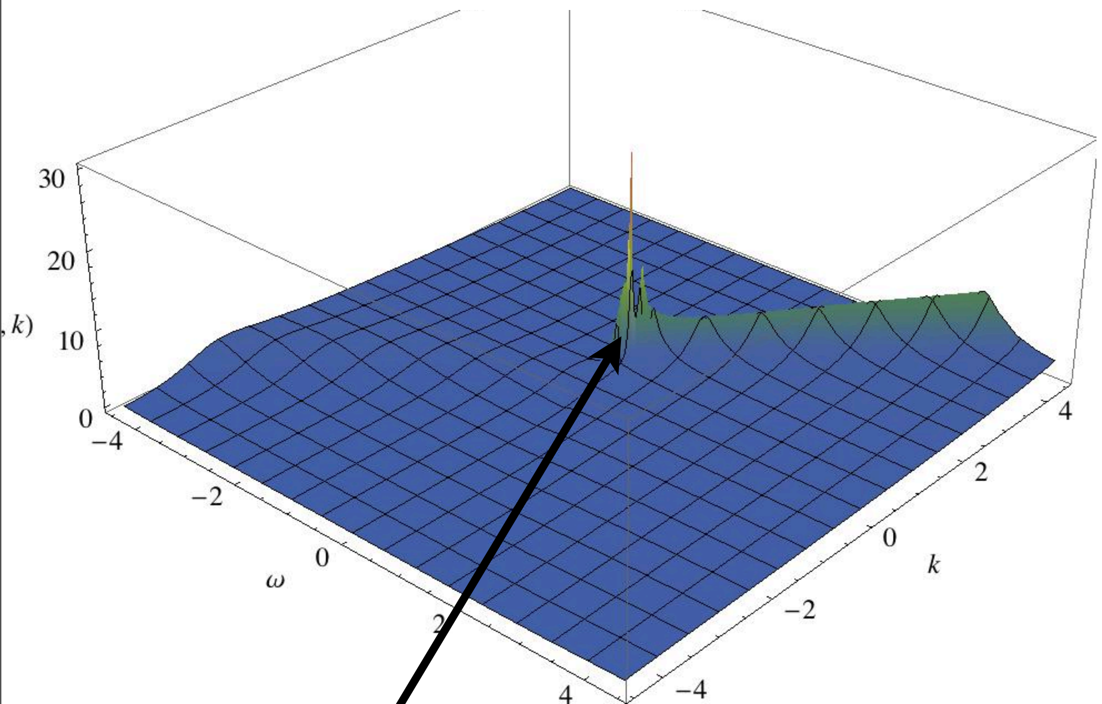

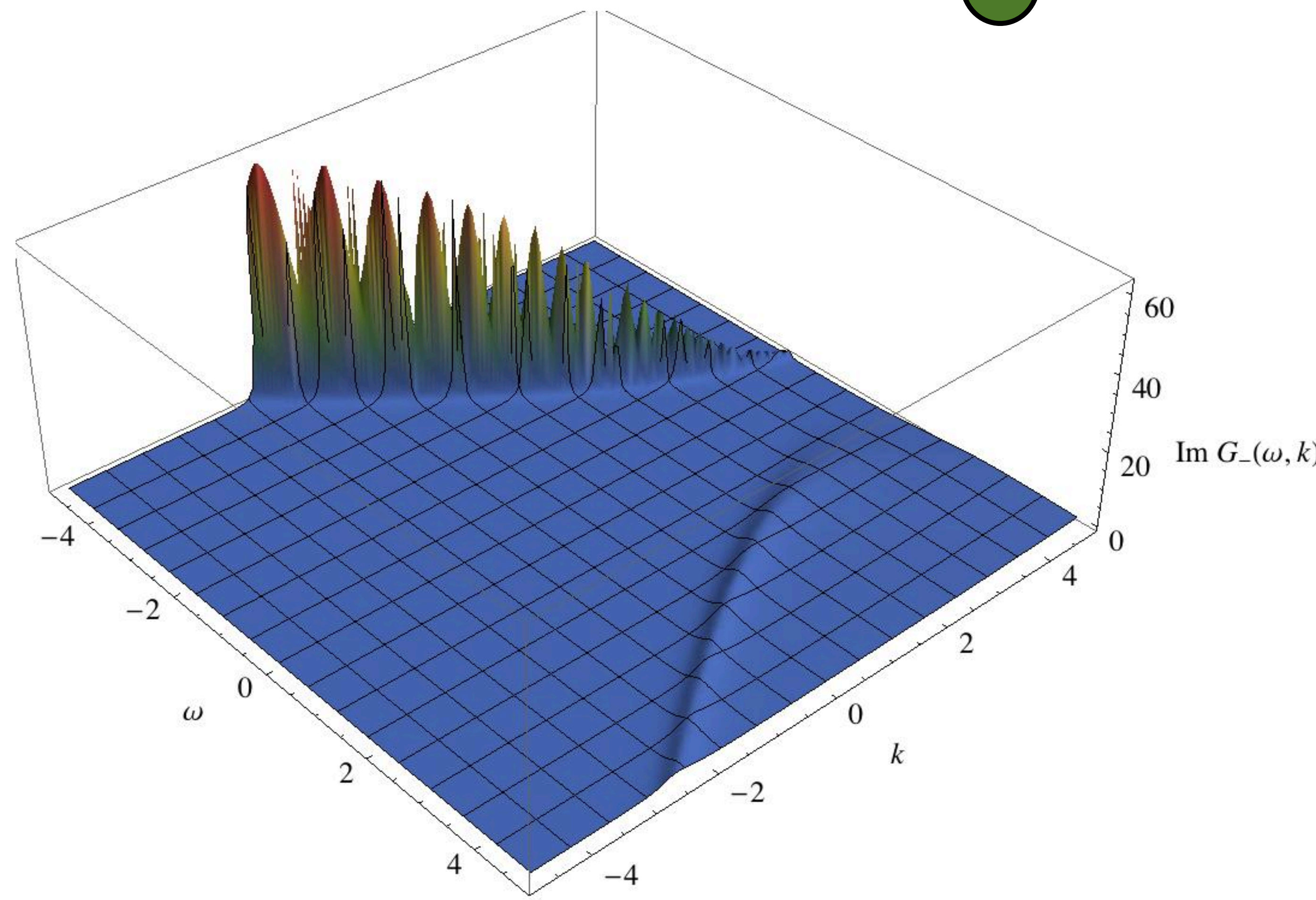

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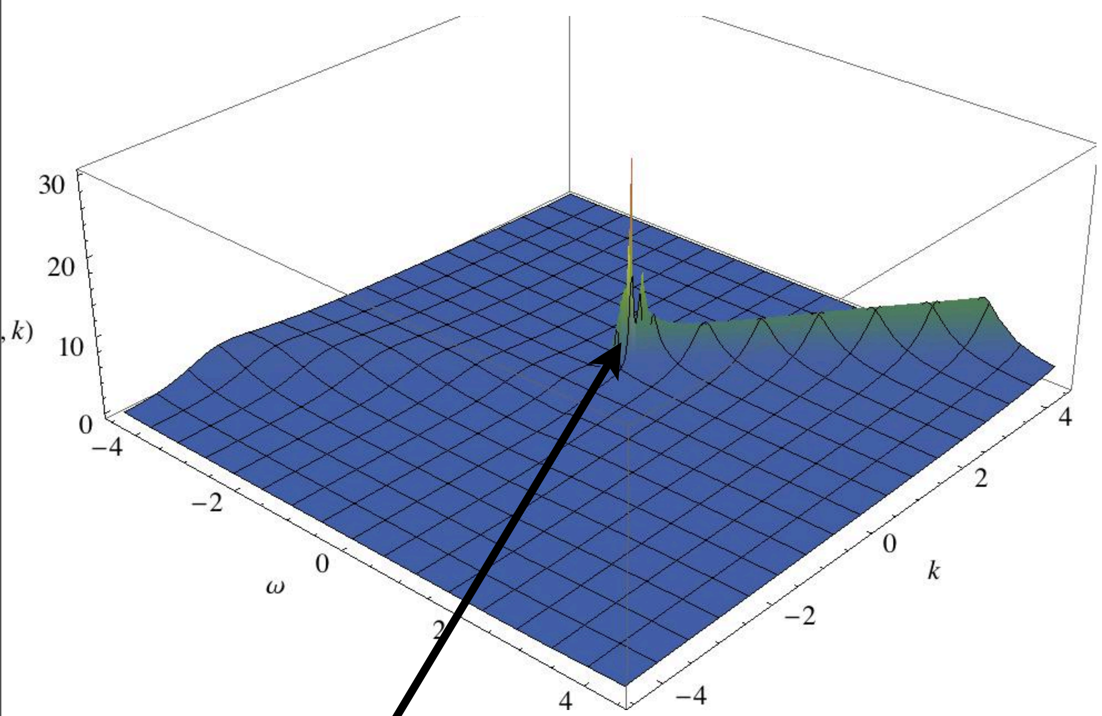

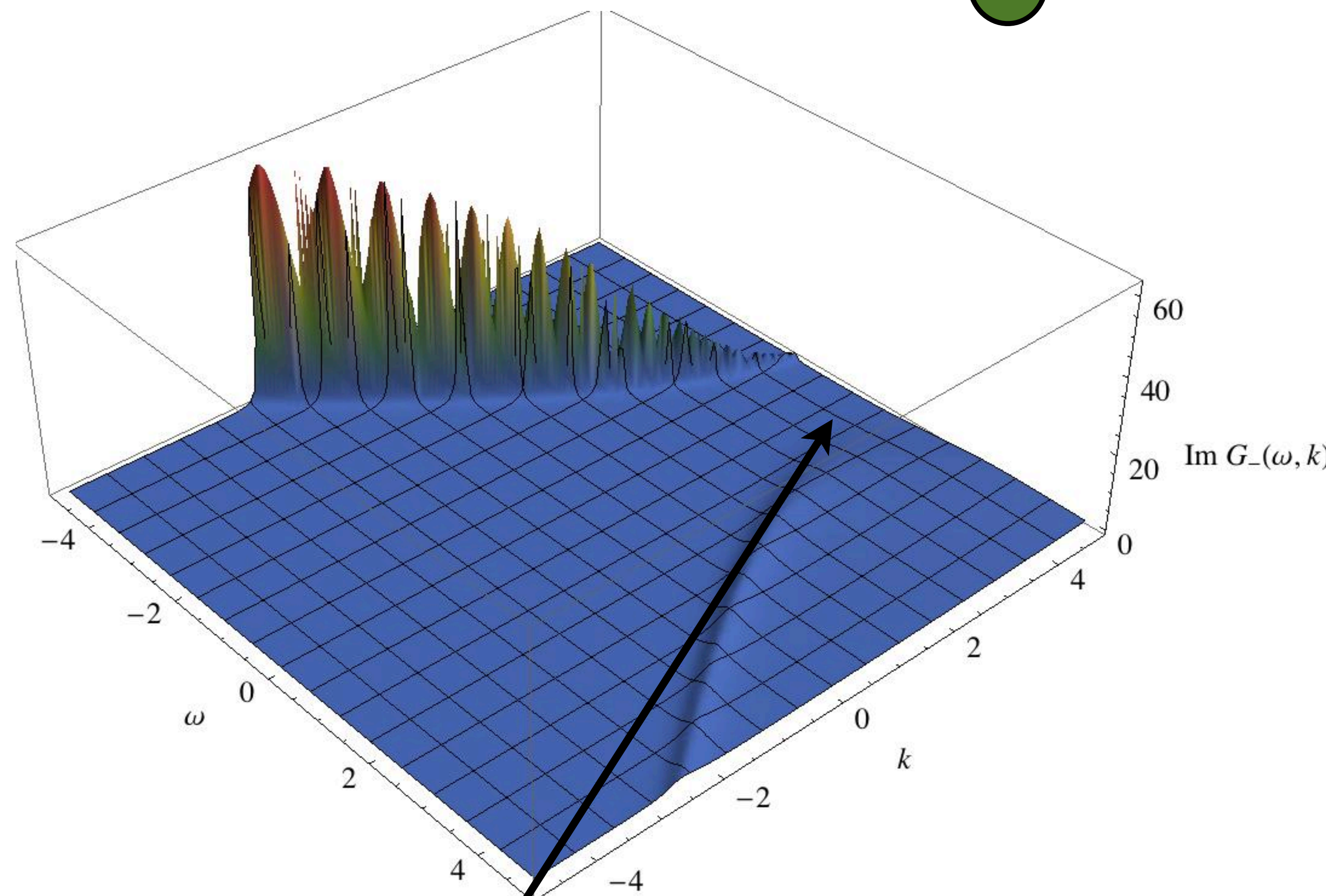

Fermi surface peak

dynamically generated gap:

How is the spectrum modified?

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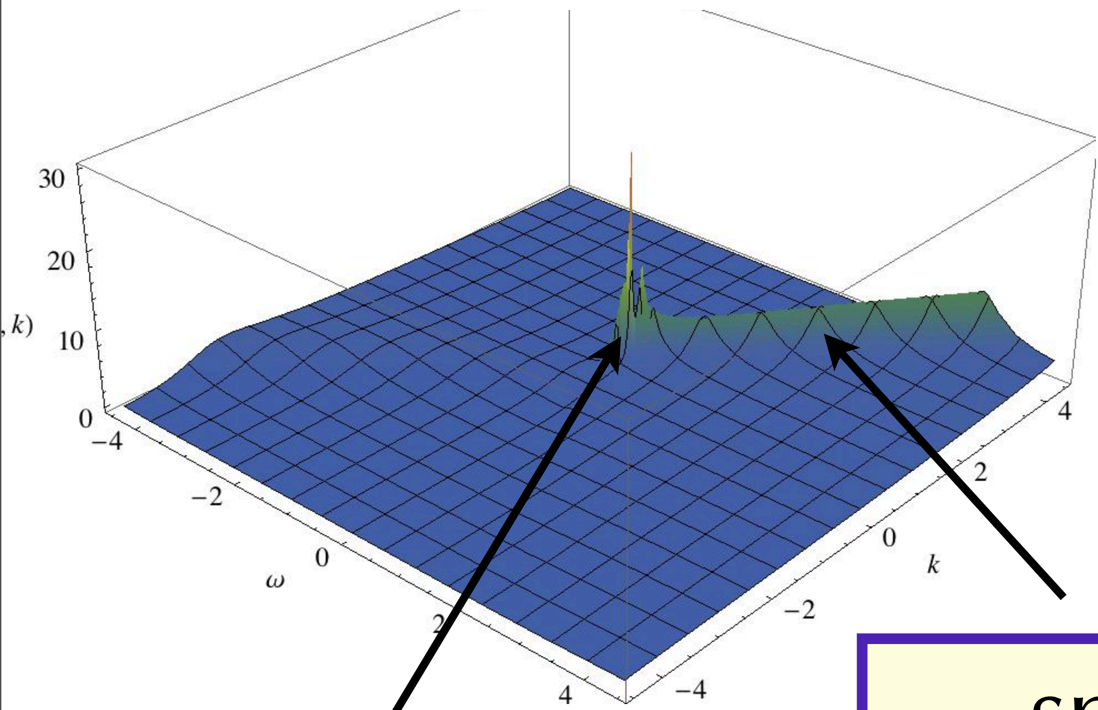
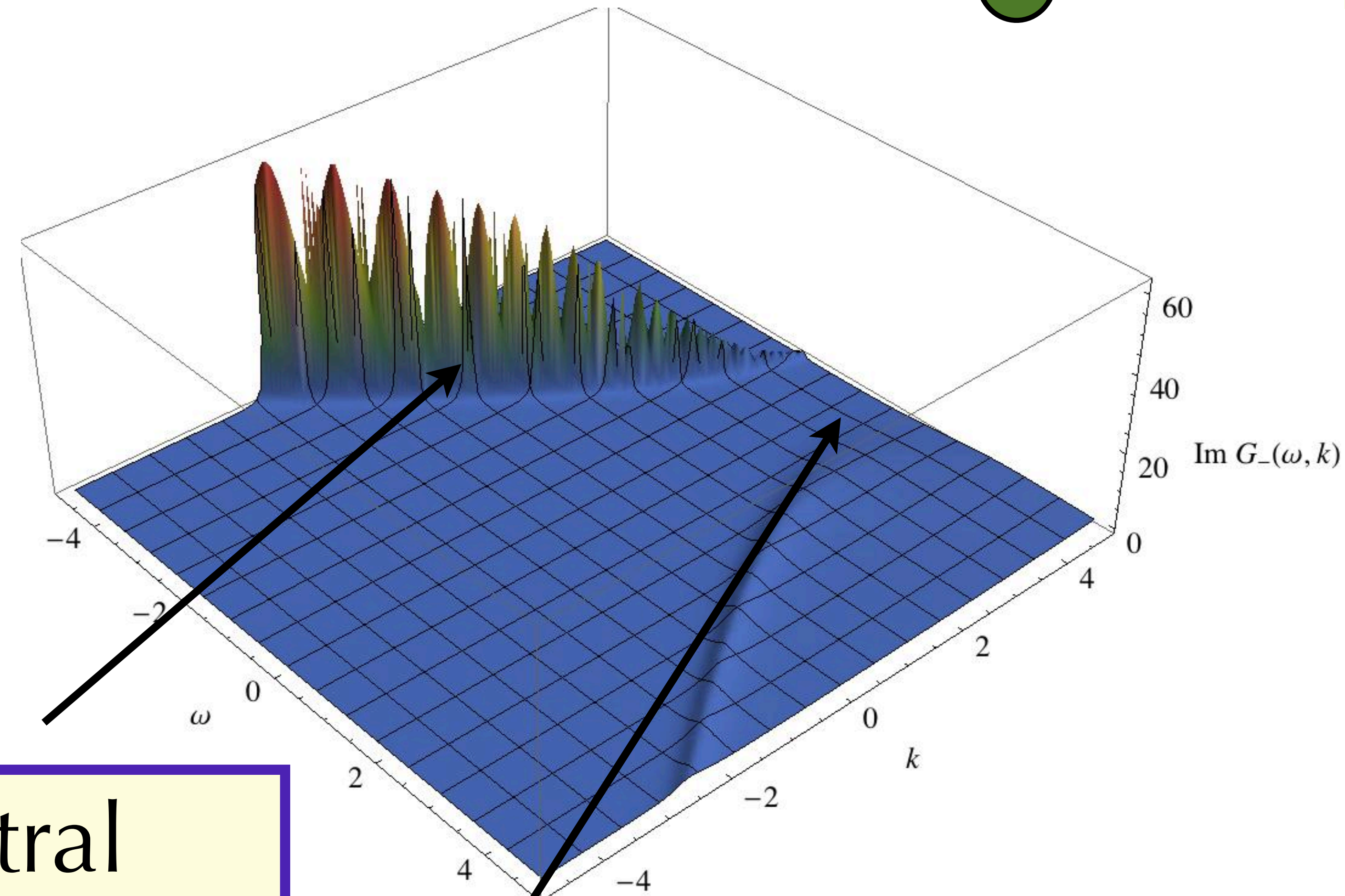
P

P=0



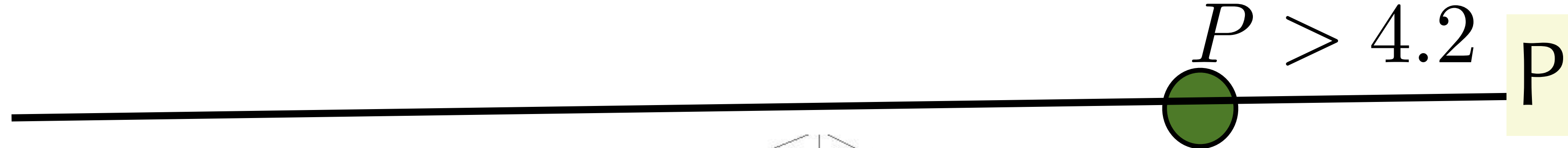


Fermi surface peak

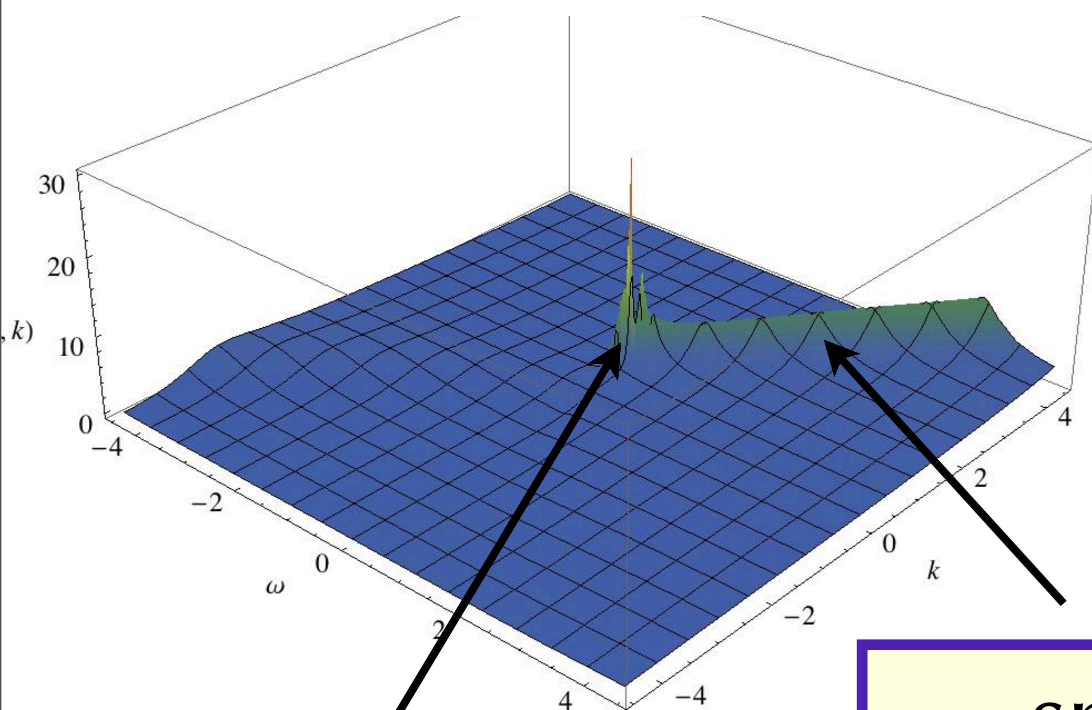
spectral weight transfer

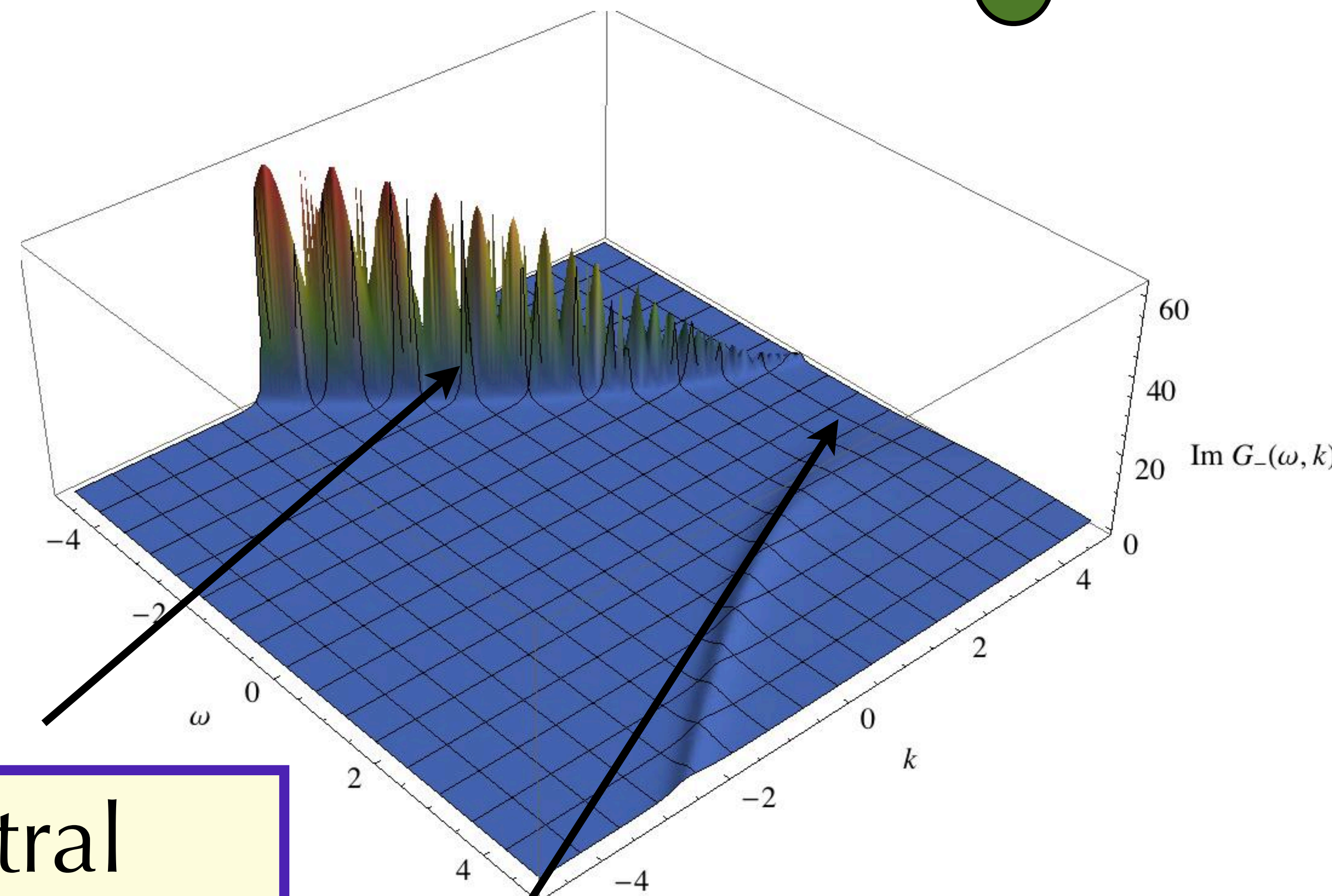
dynamically generated gap:

How is the spectrum modified?



P=0



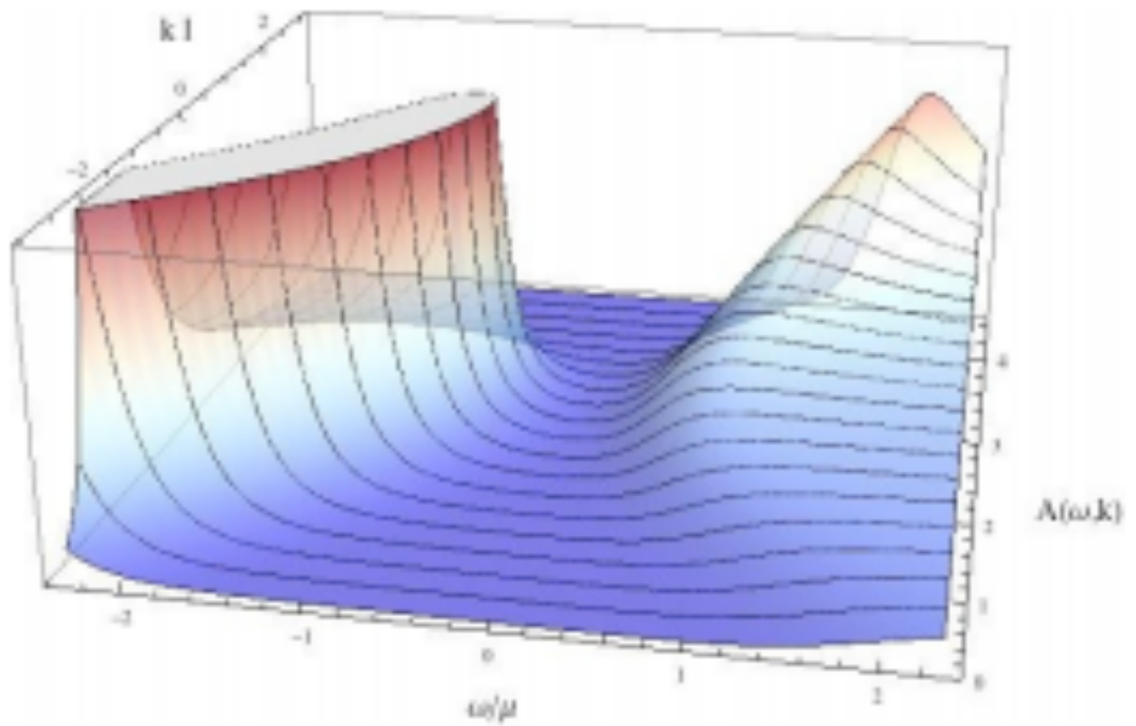


spectral weight transfer

Fermi surface peak

dynamically generated gap:

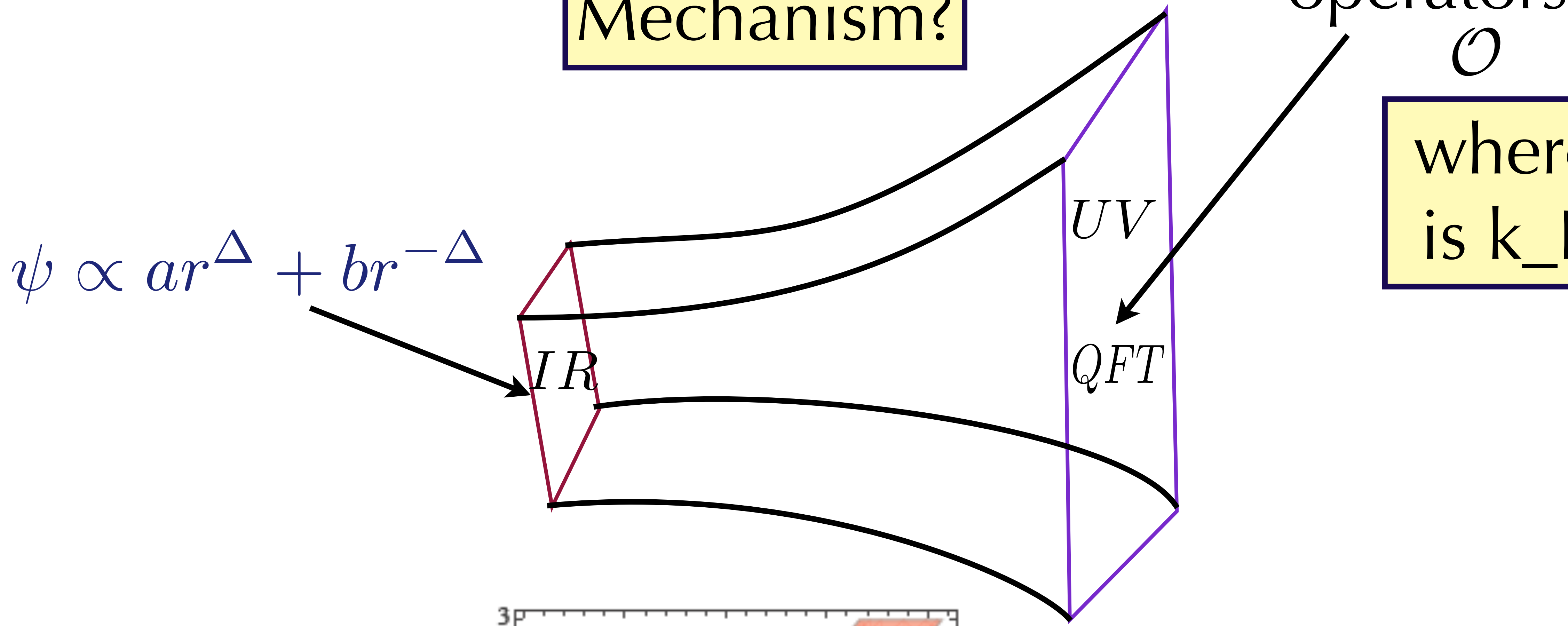
Gubser, Gauntlett, 2011 similar results



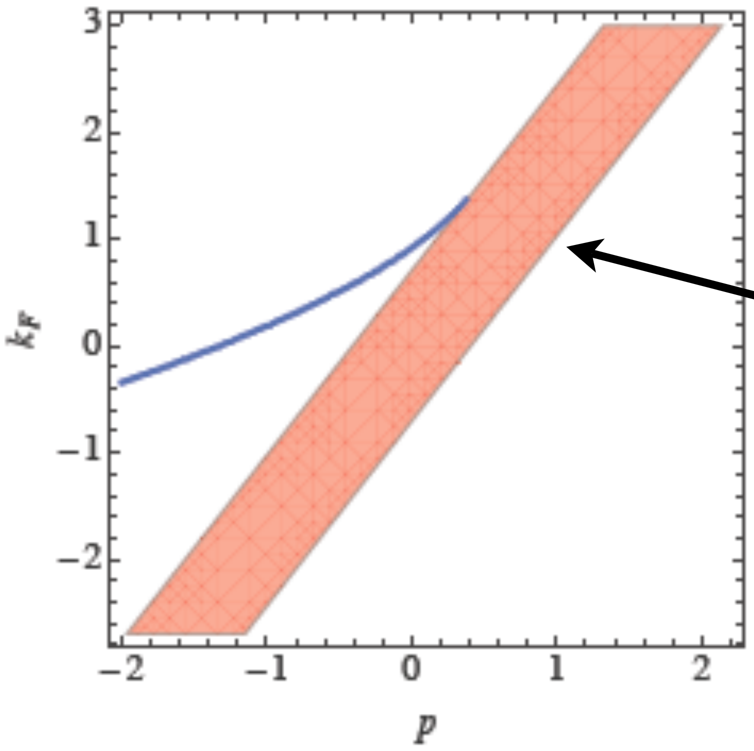
a)

Sonner, 2011 (top-down gravitino model)

Mechanism?



where is k_F



log-oscillatory

k_F moves into log-oscillatory region: IR $\mathcal{O}_{\pm}$ acquires a complex dimension near horizon

radial Dirac Equation

$$\psi_{I\pm}(\zeta) \quad = \psi^{(0)}_{I\pm}(\zeta) + \omega\,\psi^{(1)}_{I\pm}(\zeta) + \omega^2 \psi^{(2)}_{I\pm}(\zeta) + \cdots$$

$$-\psi^{(0)\prime\prime}_{I\pm}(\zeta) = i\sigma_2 \left(1 + \frac{qe_d}{\zeta}\right) - \frac{L_2}{\zeta}\left[m\sigma_3 + \left(pe_d \pm \frac{kL}{r_0}\right)\sigma_1\right]\psi^{(0)}_{I\pm}(\zeta),$$

$$e_d = 1/\sqrt{2d(d-1)}$$

$$m_k^2 = m^2 + \left(pe_d \pm \frac{kL}{r_0}\right)^2$$

p: time-reversal breaking mass term (in bulk)

$$\nu_k^{\pm} = \sqrt{m_{k\pm}^2 L_2^2 - q^2 e_d^2 - i\epsilon}, \delta_{\pm} = \nu_k^{\pm} + 1/2$$

near horizon

radial Dirac Equation

$$\psi_{I\pm}(\zeta) \quad = \psi^{(0)}_{I\pm}(\zeta) + \omega\,\psi^{(1)}_{I\pm}(\zeta) + \omega^2\psi^{(2)}_{I\pm}(\zeta) + \cdots$$

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$$\nu_k^{\pm} = \sqrt{m^2_{k\pm}L_2^2 - q^2e_d^2 - i\epsilon}, \delta_{\pm} = \nu_k^{\pm} + 1/2$$

scaling dimension is complex!

What does a complex scaling dimension mean?

continuous scale invariance

$$\mathcal{O} = \mu(\lambda)\mathcal{O}(\lambda r)$$

$$1 = \mu(\lambda)\lambda^{\Delta}$$

$$\Delta = -\frac{\ln\mu}{\ln\lambda}$$

Δ is real, independent of scale

what about complex Δ

$$1 = \mu\lambda^{\Delta}$$

$$e^{2\pi i n} = 1 = \mu\lambda^{\Delta}$$

Discrete scale invariance (DSI)

$$e^{2\pi i n} = 1 = \mu\lambda^{\Delta}$$

n=0:CSI

$$\Delta = -\frac{\ln \mu}{\ln \lambda} + \frac{2\pi i n}{\ln \lambda}$$

Discrete scale invariance (DSI)

$$e^{2\pi i n} = 1 = \mu\lambda^{\Delta}$$

n=0:CSI

$$\Delta = -\frac{\ln\mu}{\ln\lambda} + \frac{2\pi i n}{\ln\lambda}$$

scaling dimension depends on scale

$$\lambda_n = \lambda^n$$

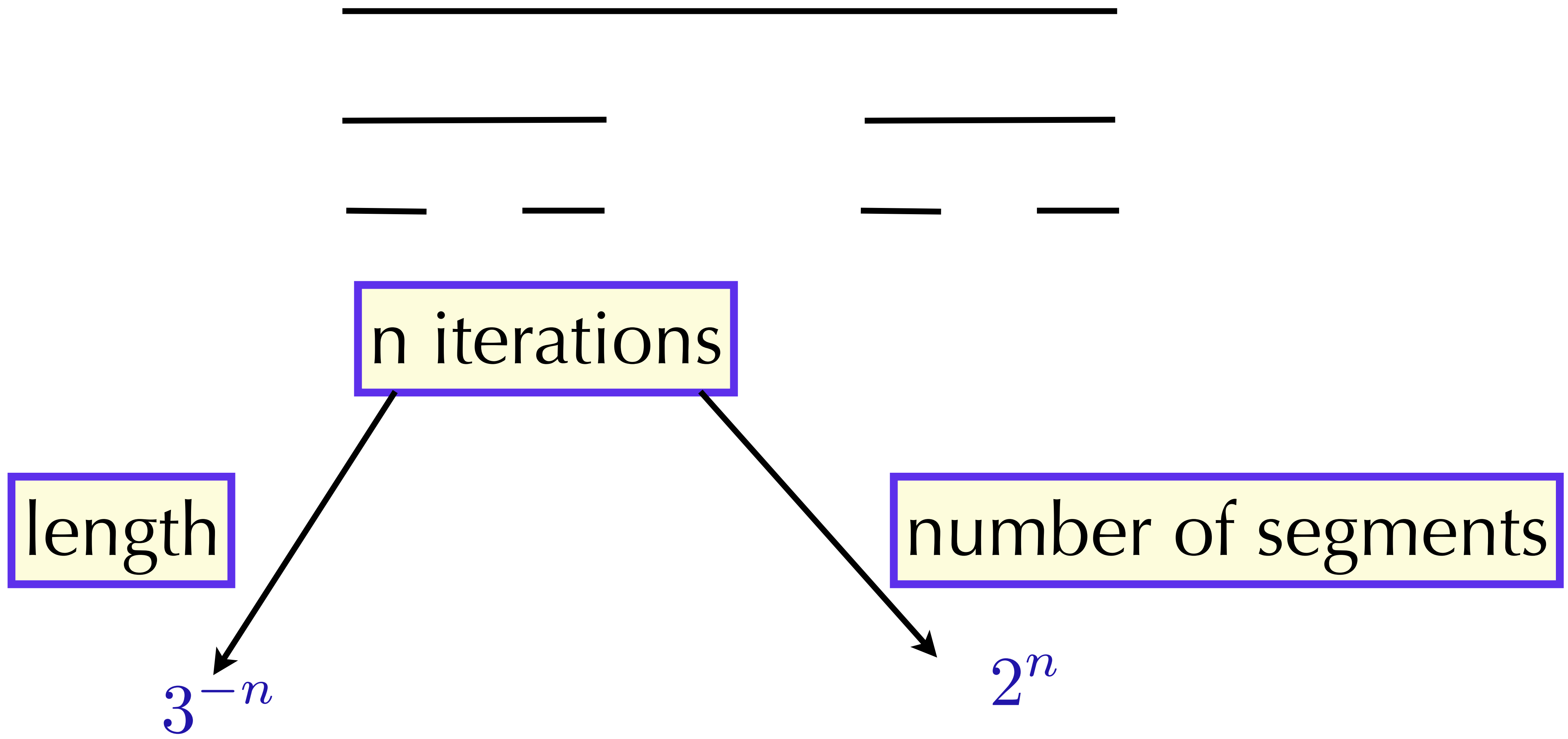
magnification

example

example

example

example



example

n iterations

length

3^{-n}

number of segments

2^n

scale invariance only for $\lambda_p = 3^p$

example

n iterations

length

3^{-n}

number of segments

2^{n}

scale invariance only for $\lambda_p = 3^p$

discrete scale invariance: $D = -\frac{\ln 2}{\ln 3} + \frac{2\pi i n}{\ln 3}$

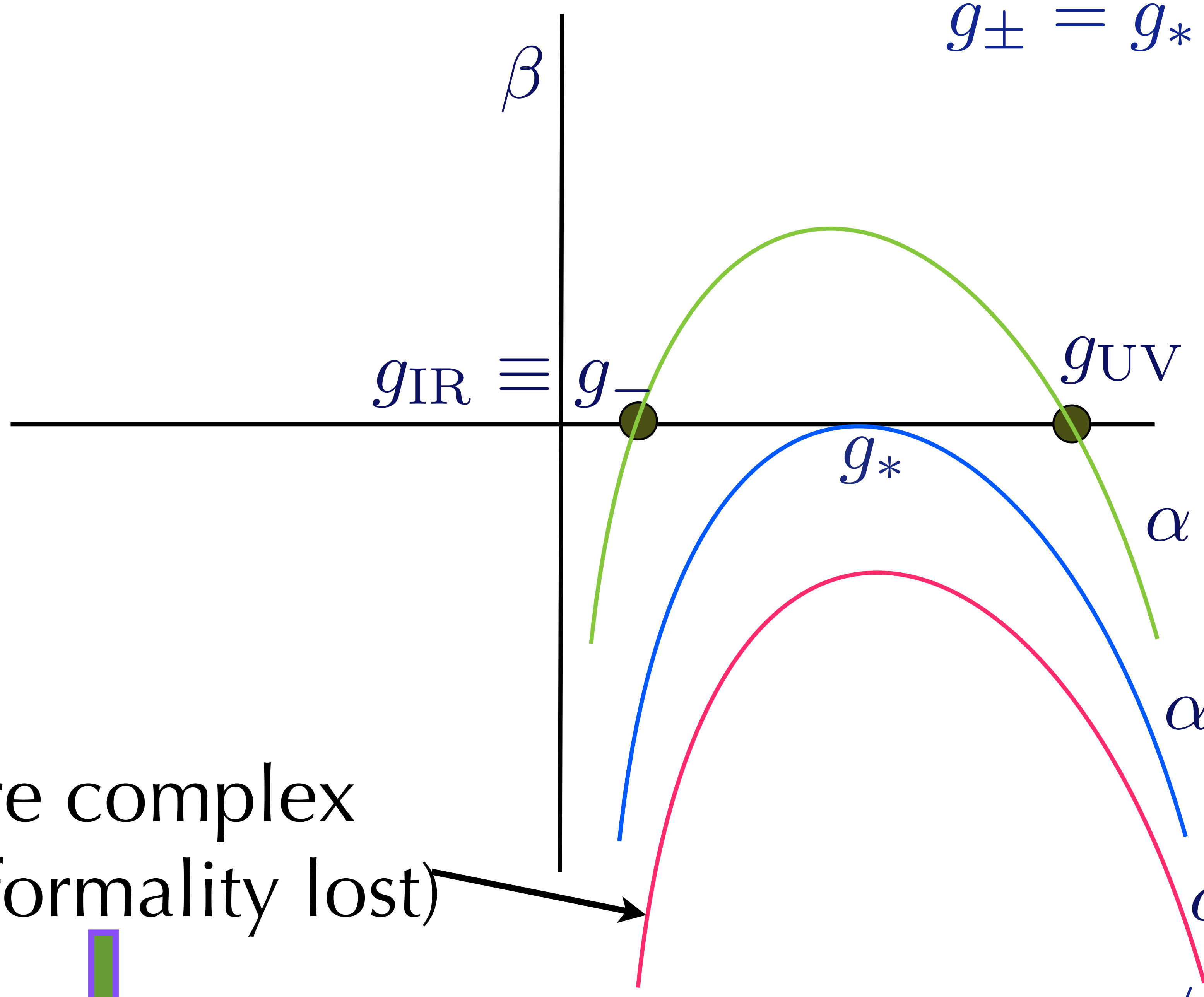
discrete scale invariance

hidden scale
(length, energy,...)

toy model: merging of UV and IR fixed points

$$\beta = (\alpha - \alpha_*) - (g - g_*)^2$$

$$g_\pm = g_* \pm \sqrt{\alpha - \alpha_*}$$



$g_\pm$ are complex (conformality lost)

$$\Lambda_{\rm IR} = \Lambda_{\rm UV} e^{-\pi/\sqrt{\alpha_* - \alpha}}$$

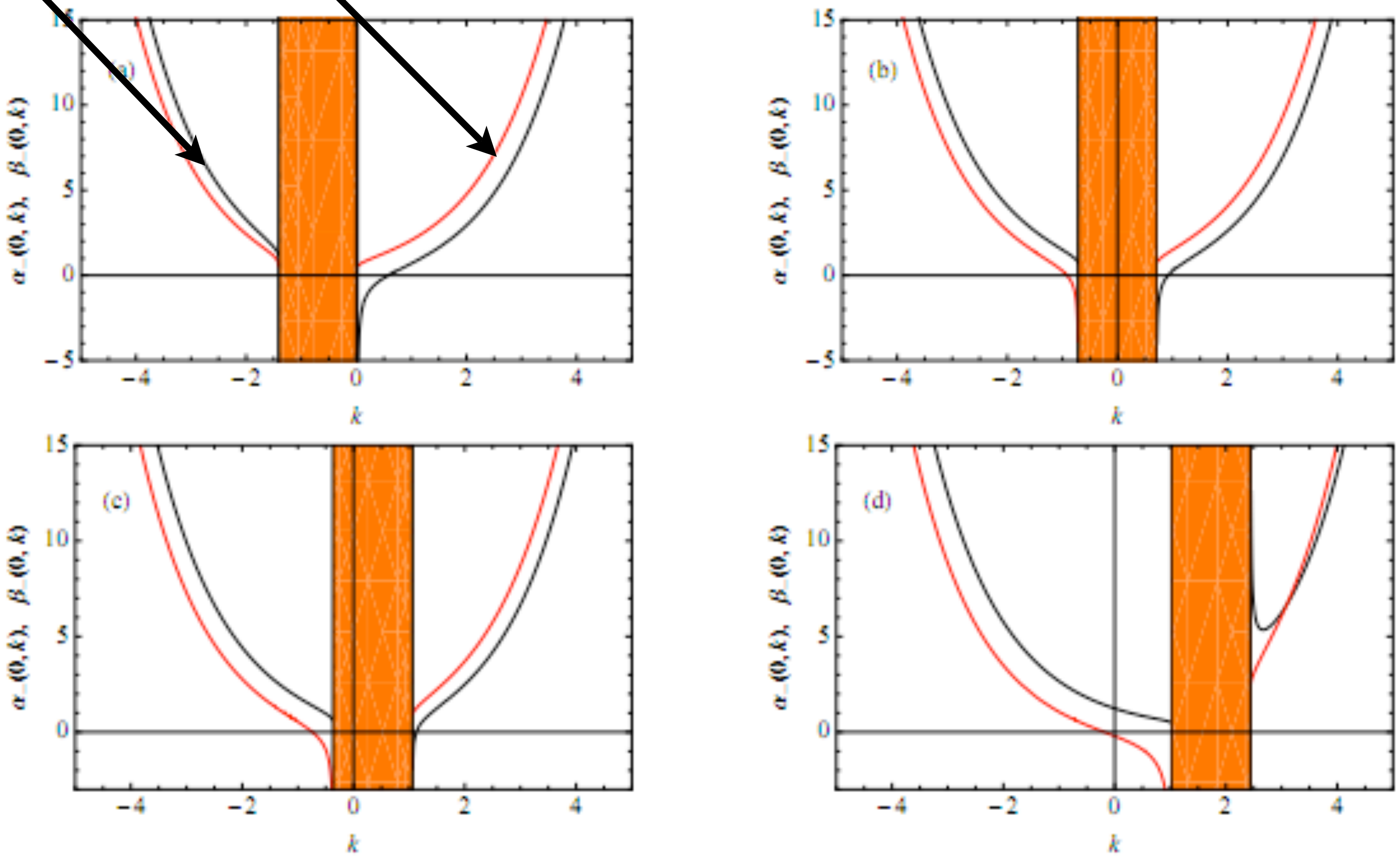
DSI

BKT transition

Kaplan, arxiv:0905.475

$$G = \frac{\beta_-(0,k)}{\alpha_-(0,k)}$$

p=-0.4 p=0.0

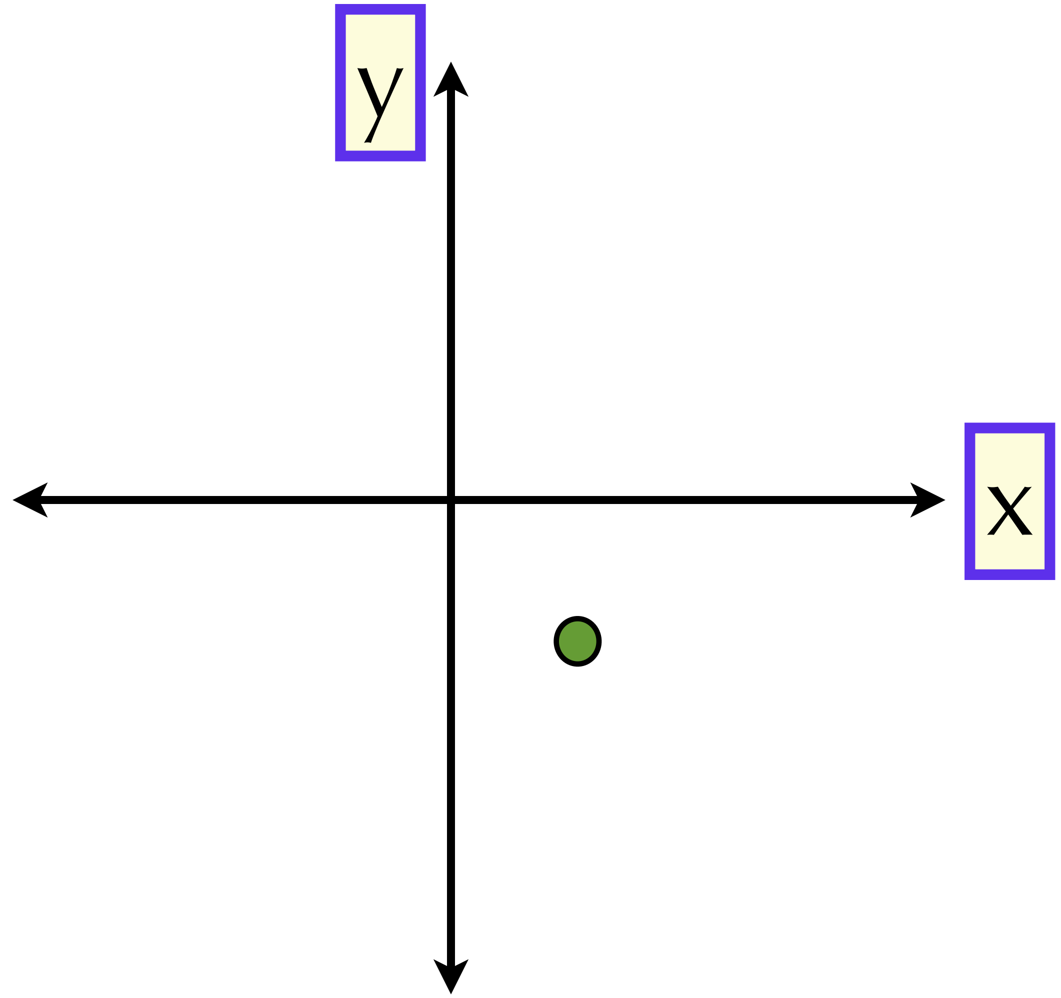


p=0.2 p=1.0

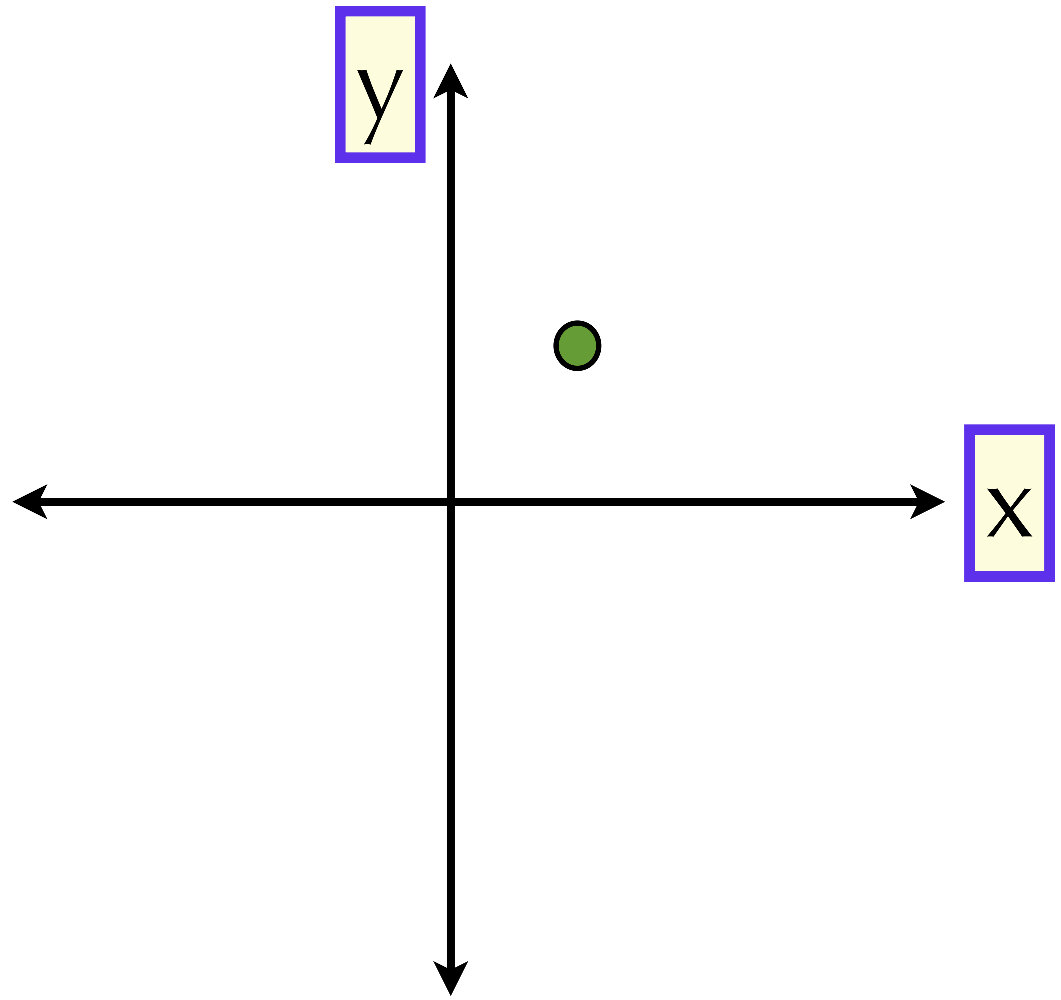
no poles outside log-oscillatory region for $p > 1/\sqrt{6}$

Mott physics and DSI are linked!

is there an instability? (violation of BF bound)

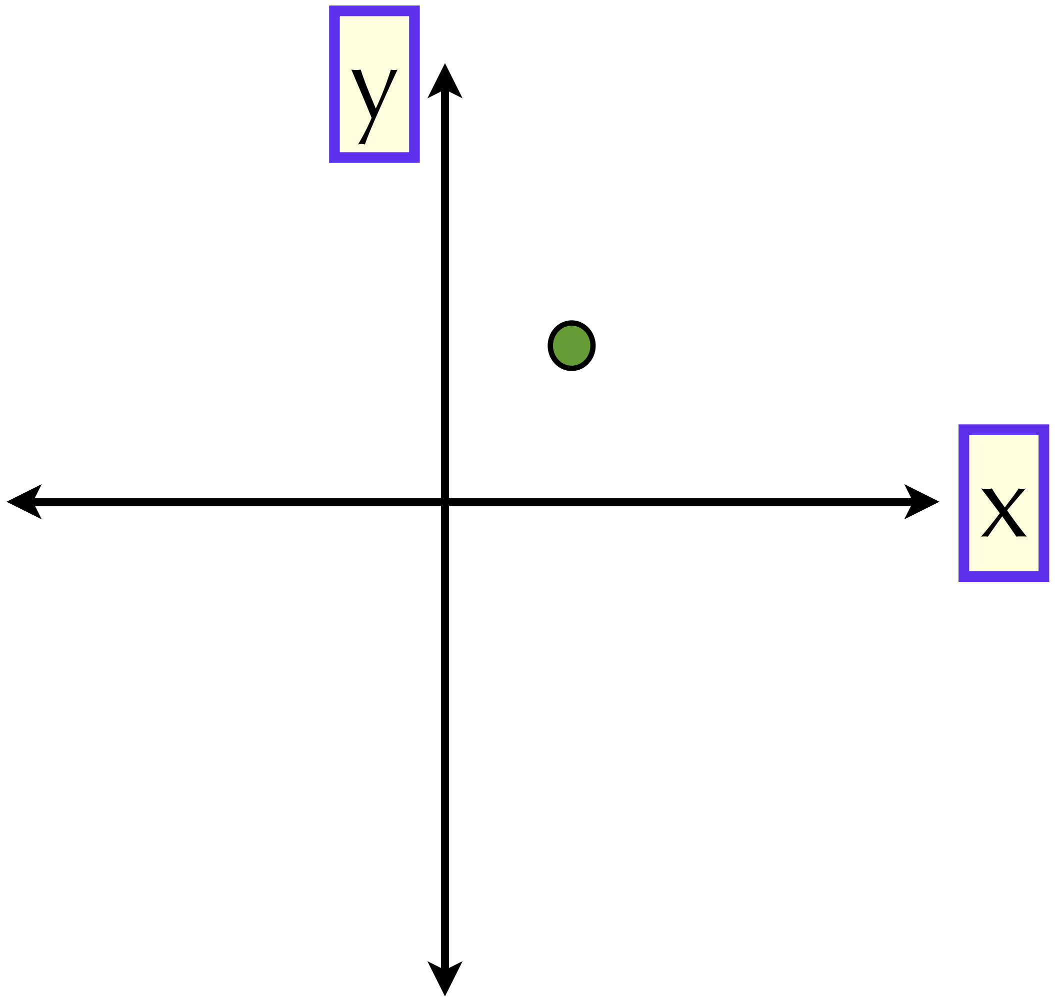


is there an instability? (violation of BF bound)



is there an instability? (violation of BF bound)

$$E \to E + i\gamma \quad \gamma > 0$$



is there an instability? (violation of BF bound)

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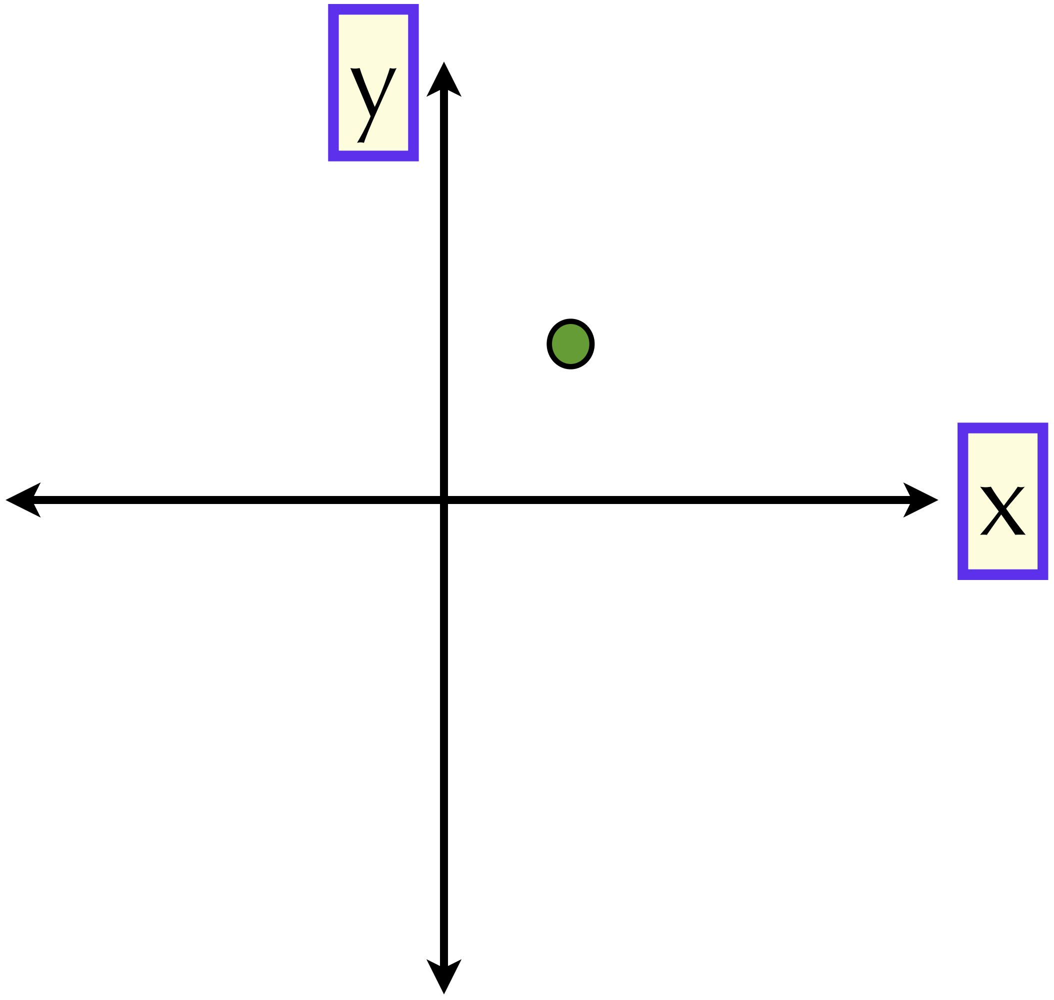
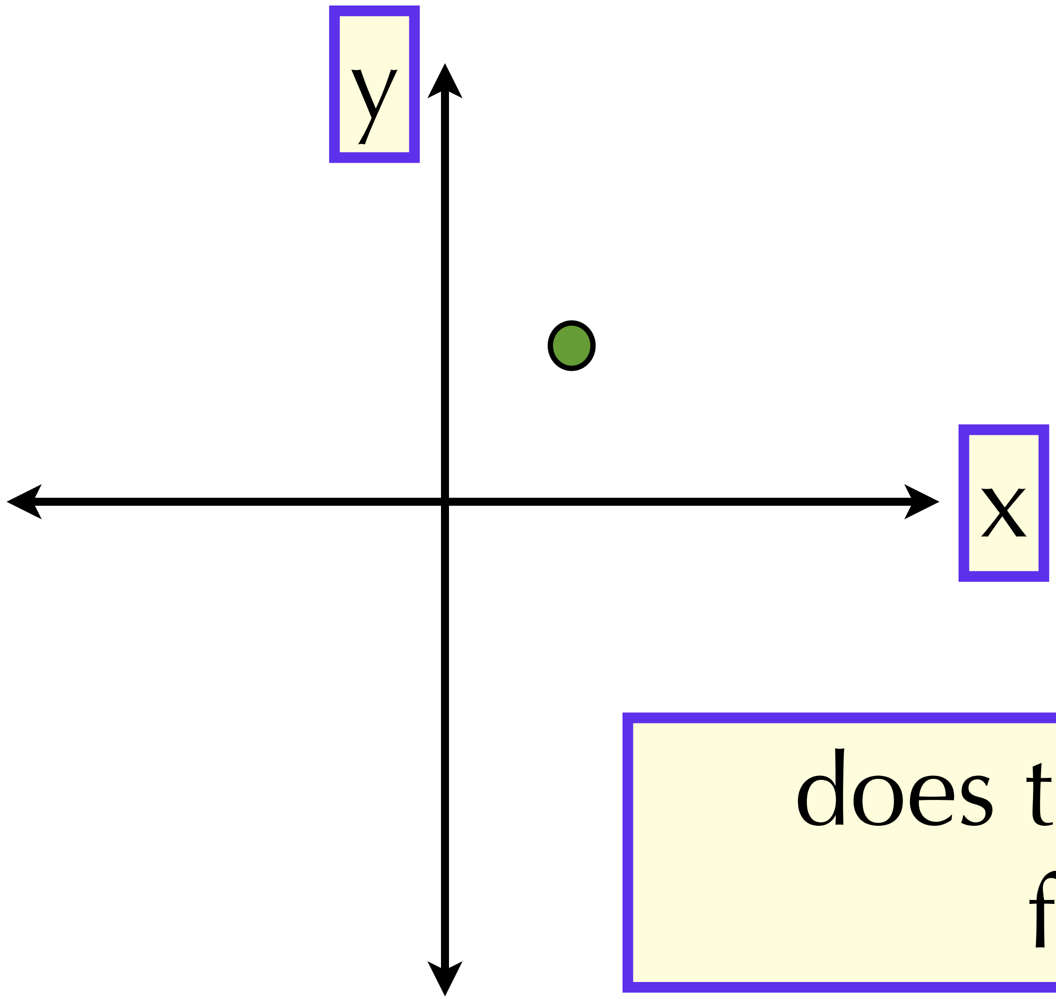


$$\langle \phi \rangle \neq 0$$

condensate (bosonic)

is there an instability? (violation of BF bound)

$$E \to E + i\gamma \quad \gamma > 0$$

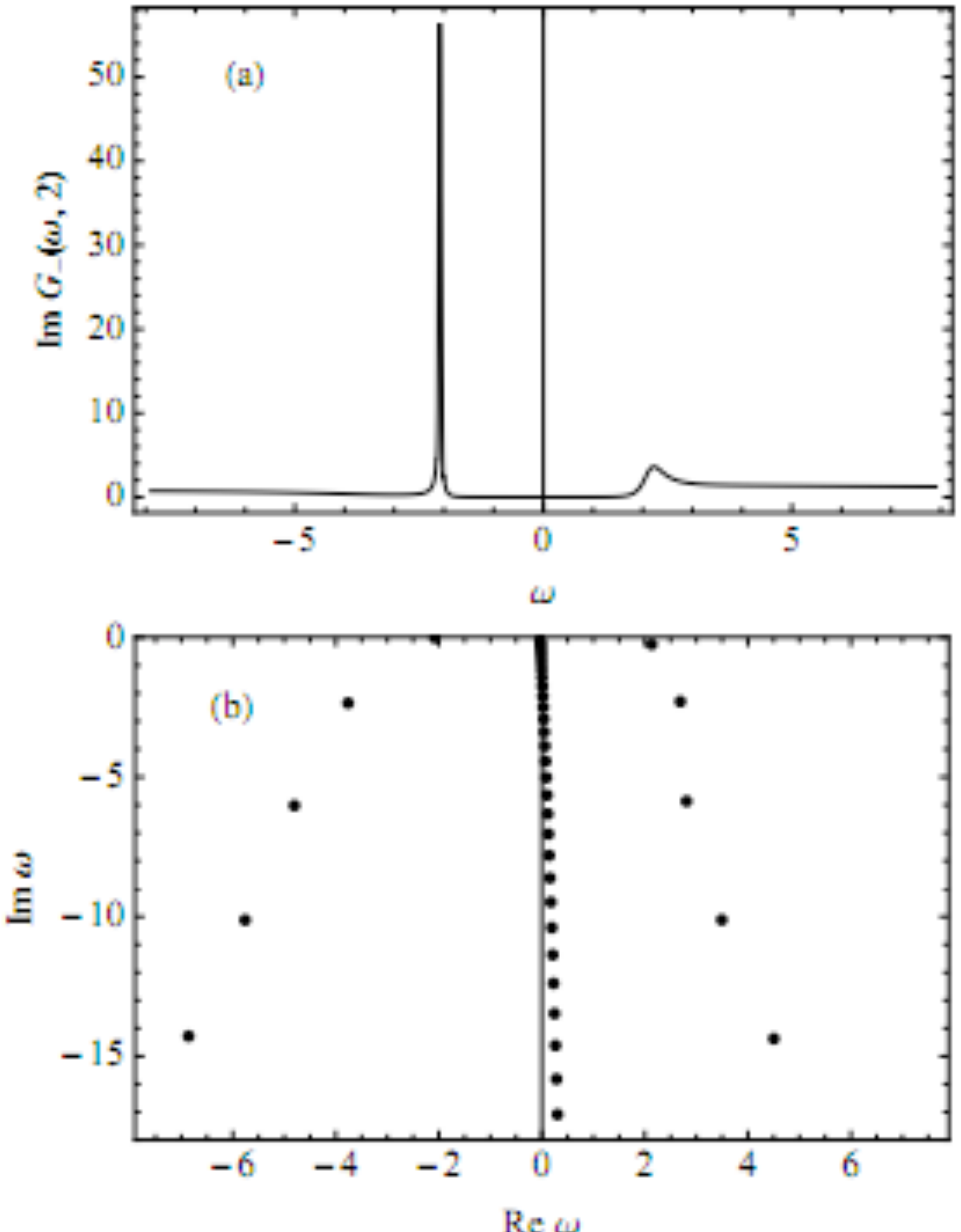


$$\langle \phi \rangle \neq 0$$

condensate (bosonic)

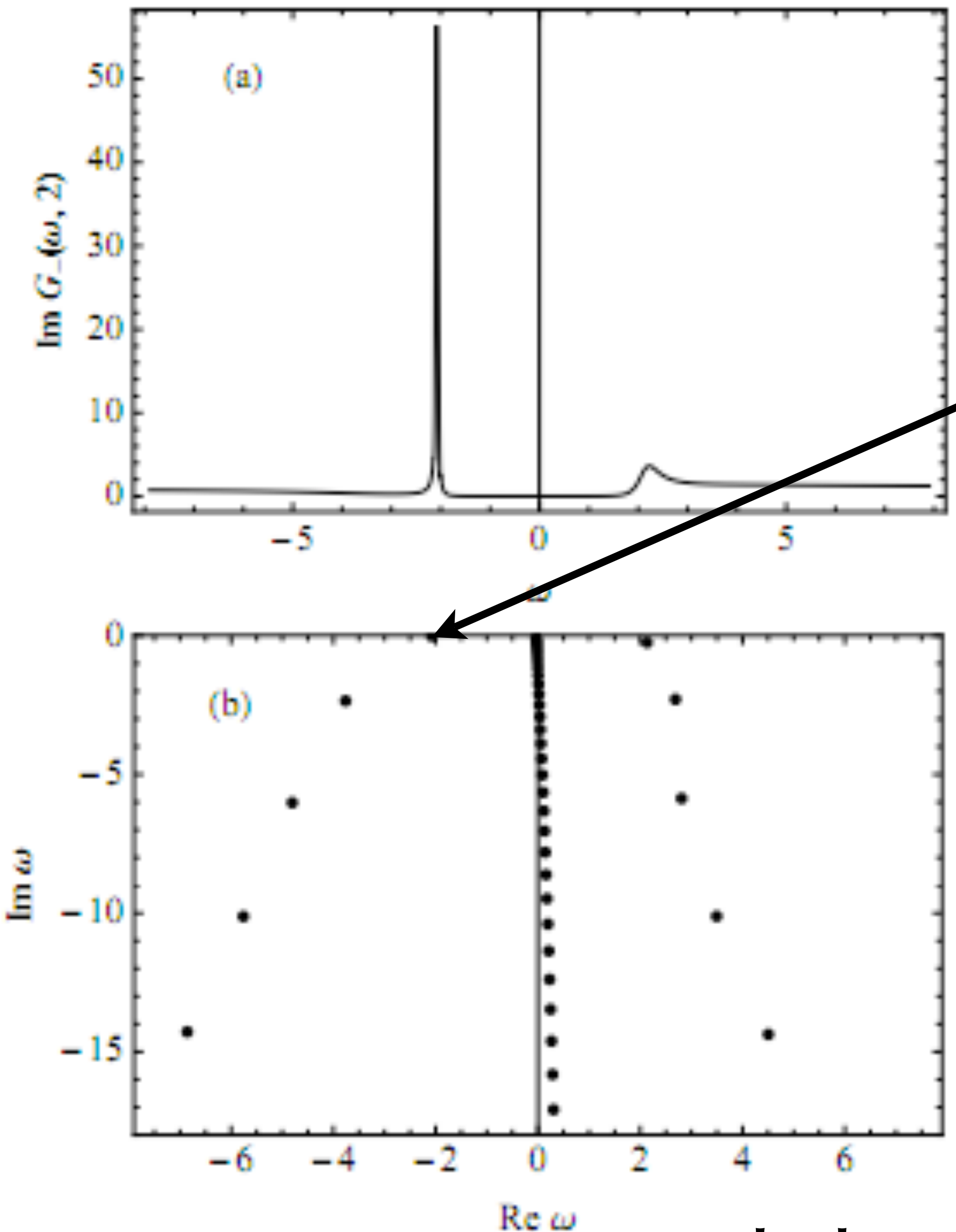
does this happen for fermions?

quasi-normal modes

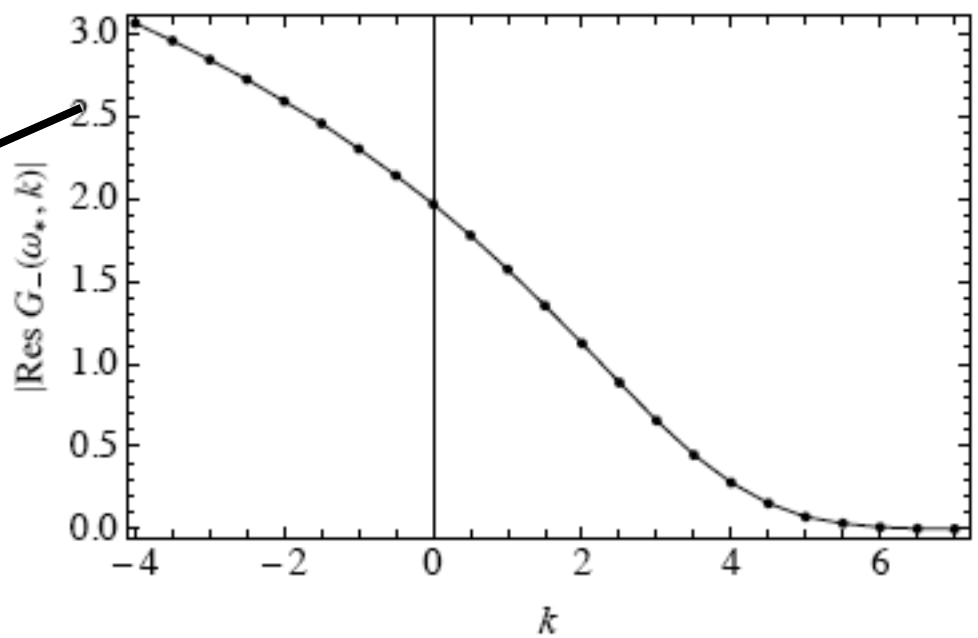


$Im\omega < 0$ no instability

quasi-normal modes



quasi-particle residue

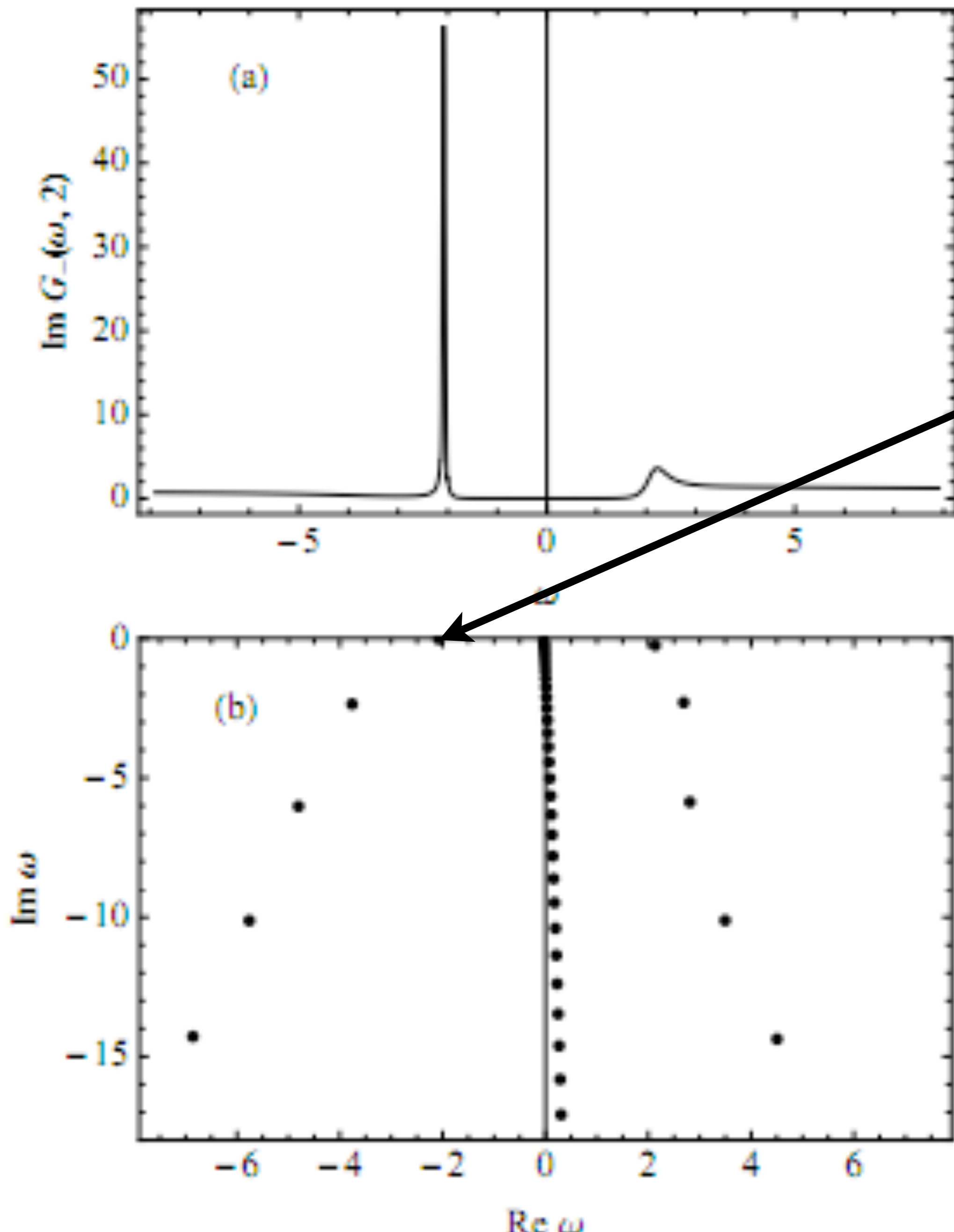


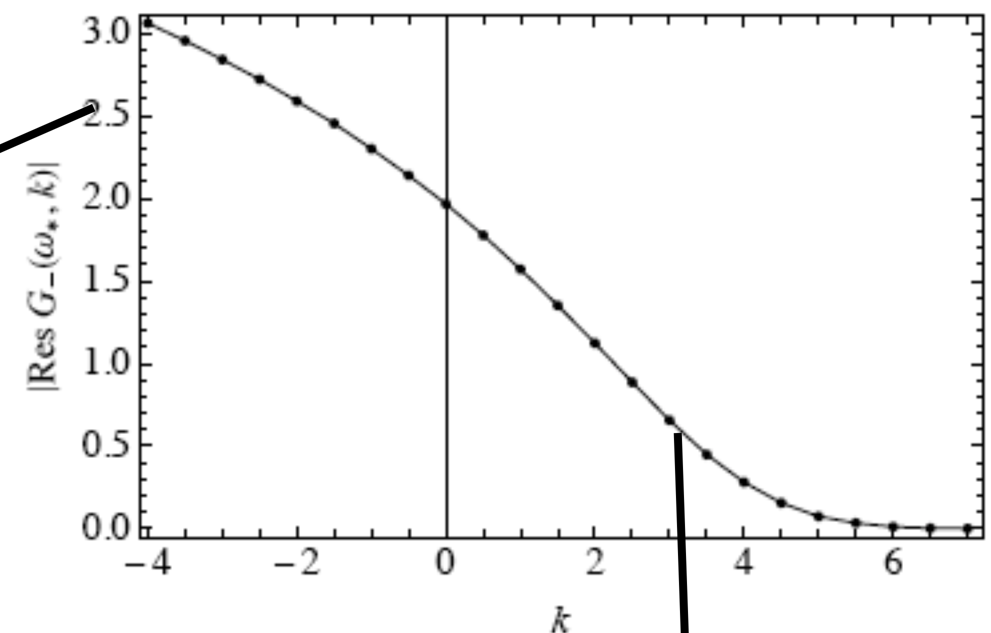
$Im\omega < 0$ no instability

quasi-normal modes

but the residue drops to zero: opening of a gap

quasi-particle residue



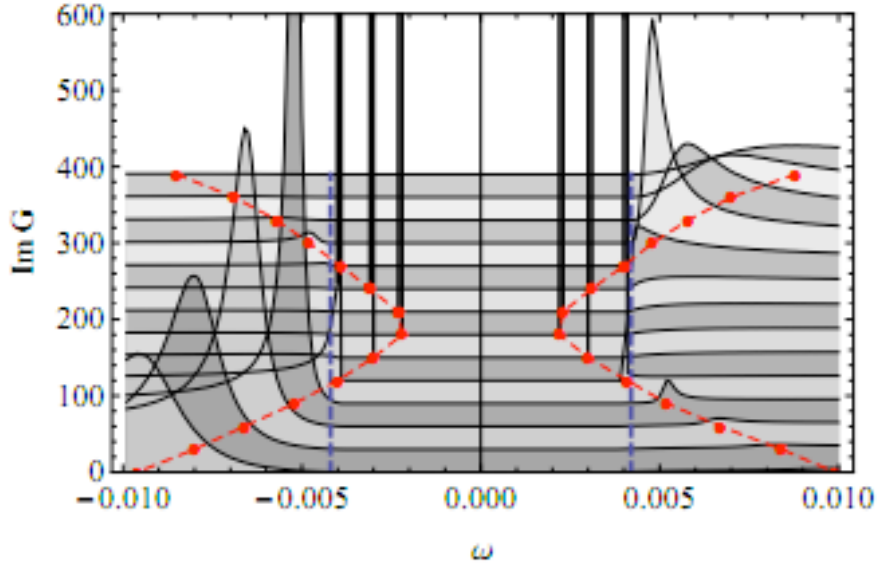


$$G_R(\omega, \mathbf{k}) = \frac{\mathbf{b}^{(0)} + \omega \mathbf{b}^{1} \mathcal{G}_{\mathbf{k}}(\omega)}{\mathbf{a}^{(0)} + \omega \mathbf{a}^{(1)} \mathcal{G}_{\mathbf{k}}(\omega)}$$

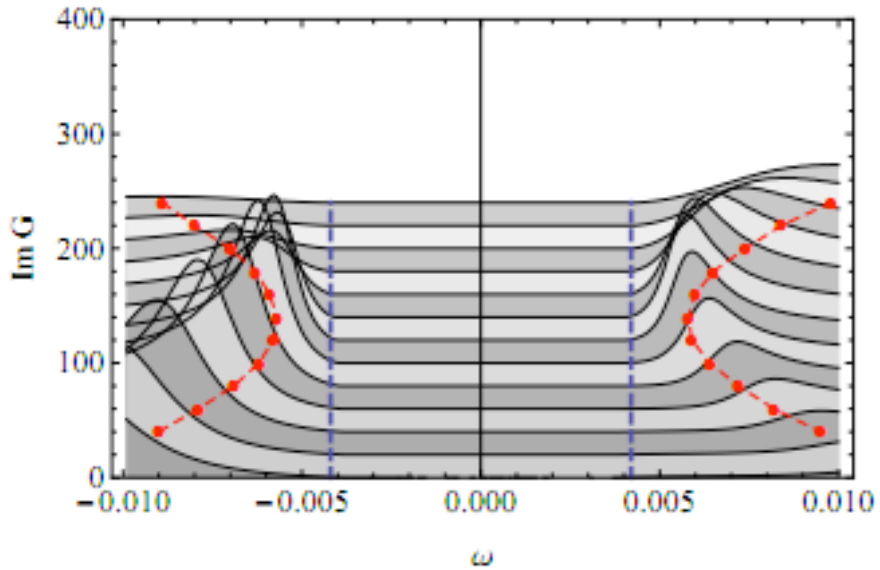
$Im\omega < 0$ no instability

$$S(\varphi) + S_D(\zeta) + \eta_5 \int d^{d+1}x\sqrt{-g}\varphi\bar{\zeta}C\Gamma^5\bar{\zeta}^T + cc$$

$\eta_5 = 0.025$

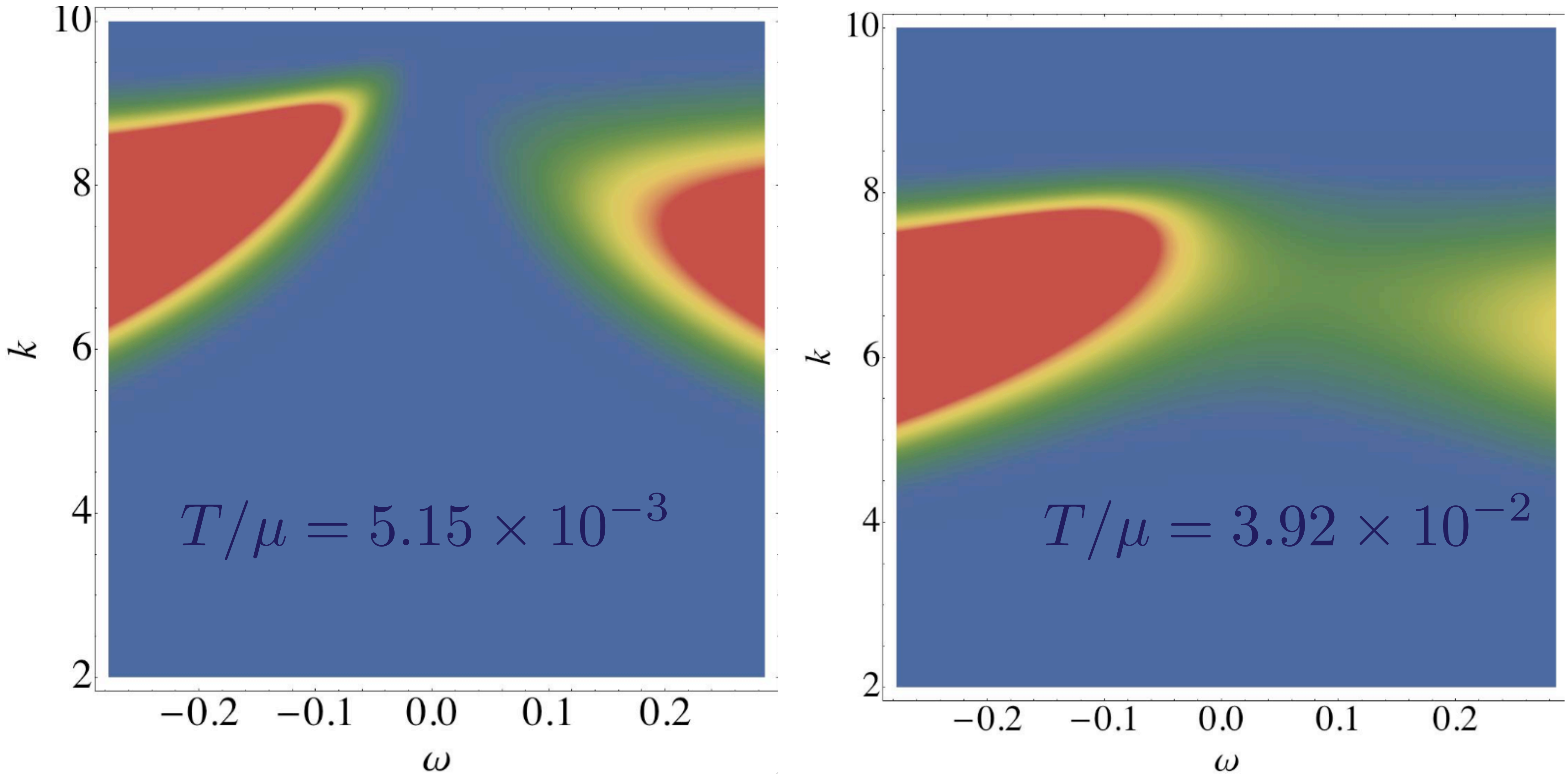


$\eta_5 = 0.075$



majorana coupling kills qp peaks
through coupling to scaling dimension of spinor field

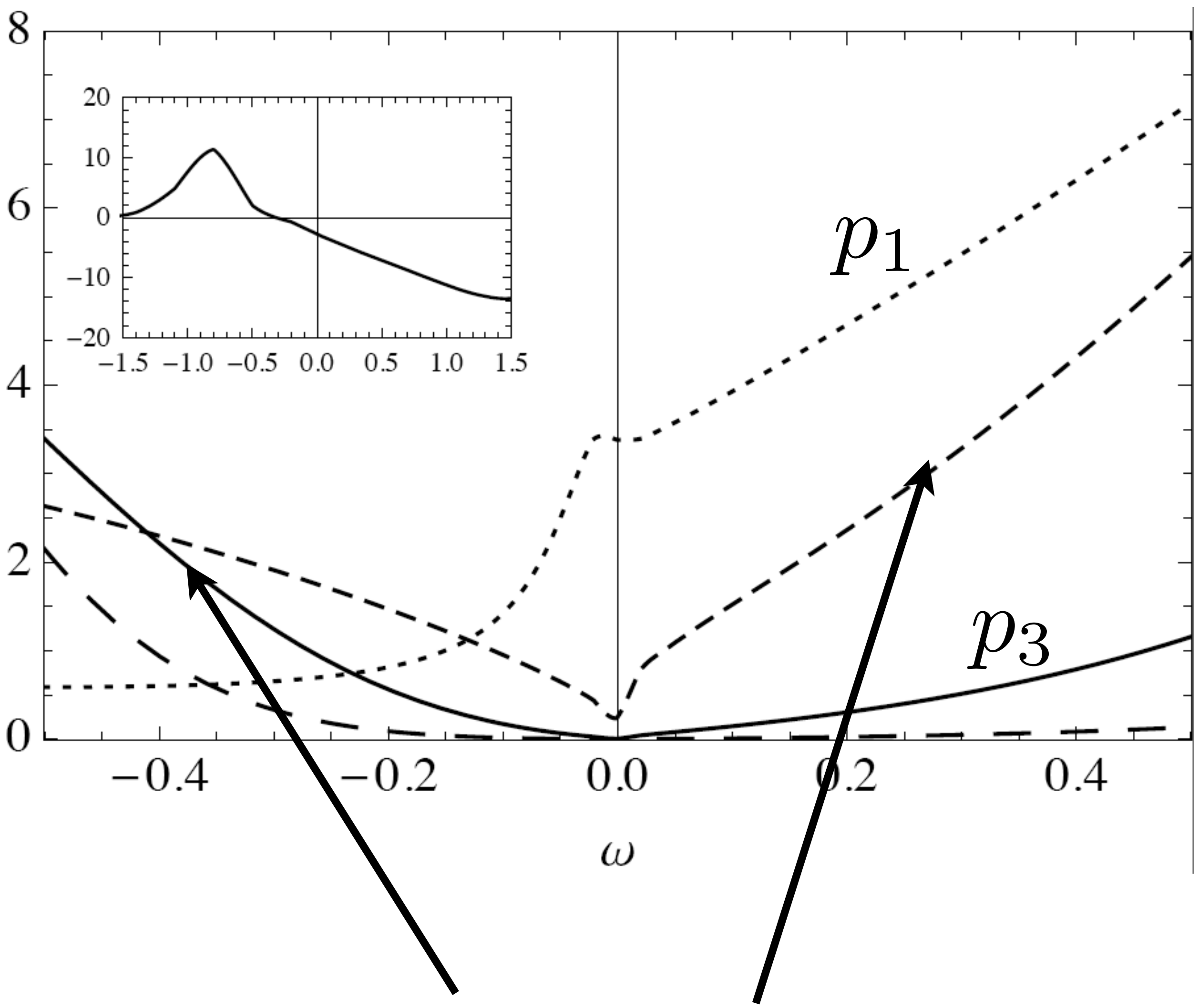
Finite Temperature Mott transition from Holography

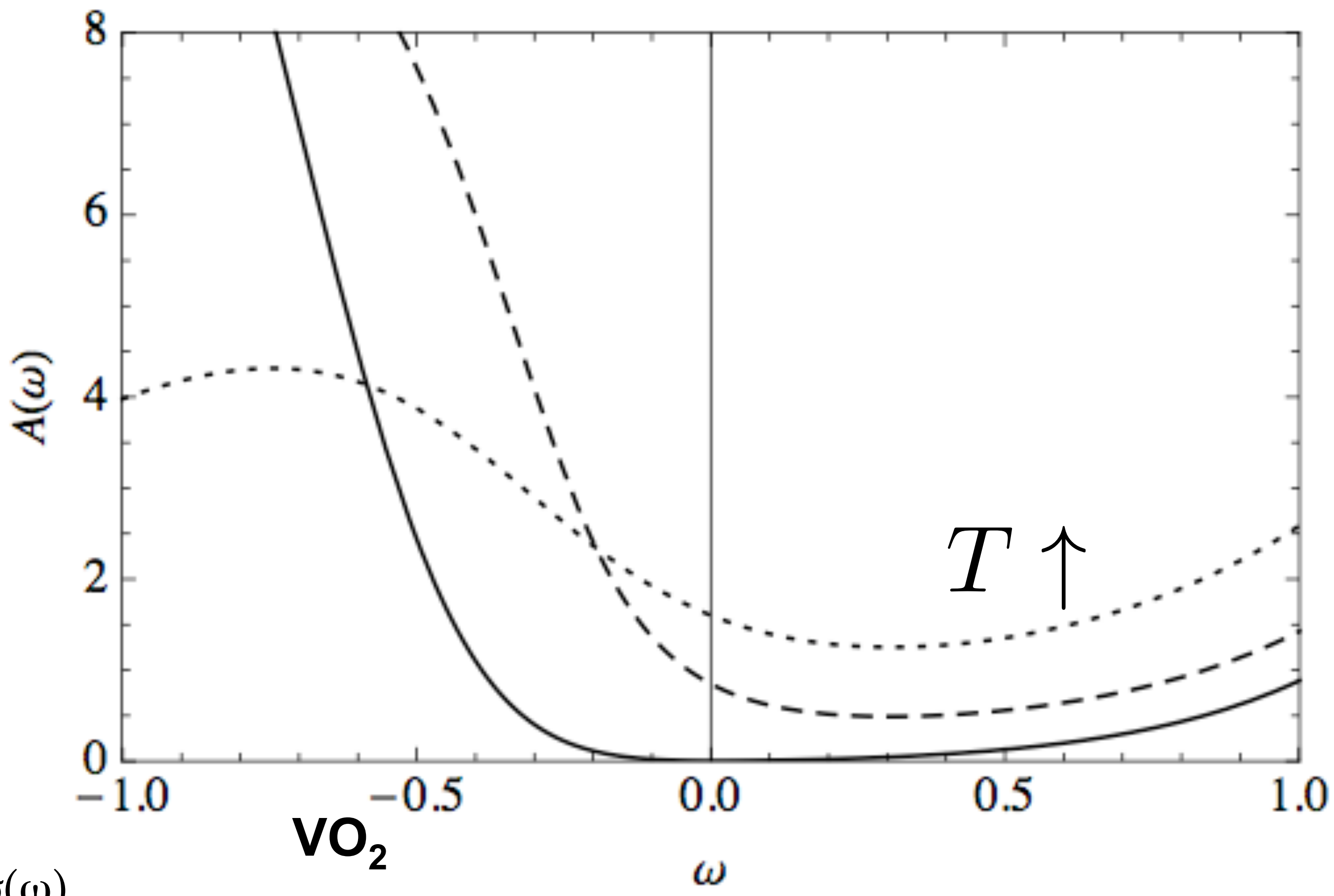


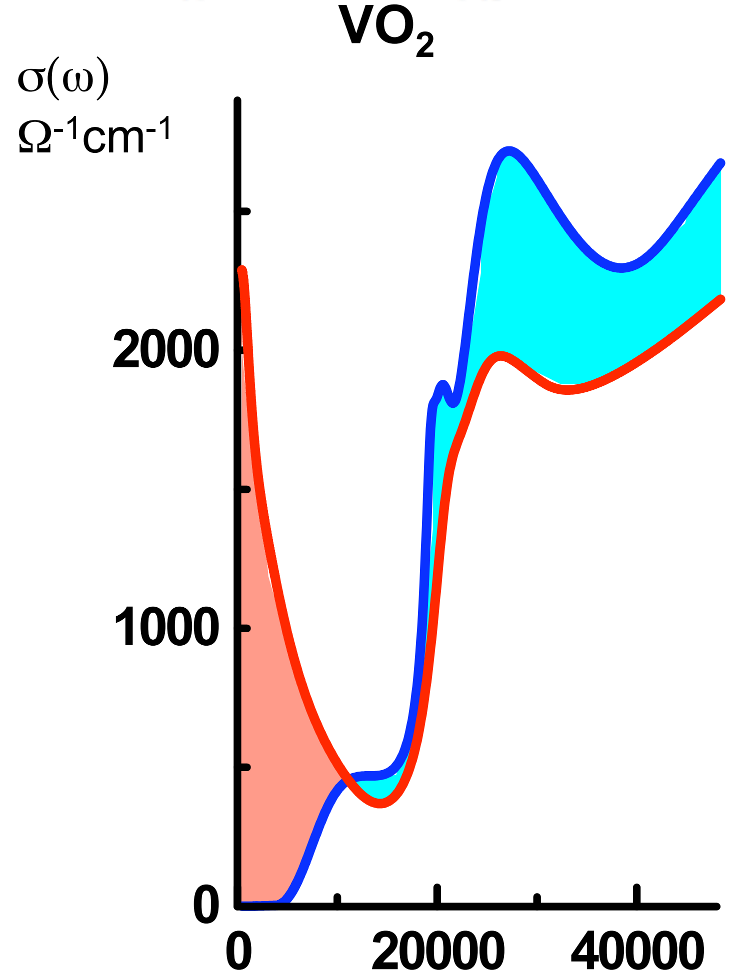
$$\frac{\Delta}{T_{\text{crit}}} \approx 10$$

$$\frac{\Delta}{T_{\text{crit}}} \approx 20$$

vanadium oxide







spectral weight
transfer
UV-IR mixing

$$G(\omega, k) = G(\omega \lambda^n, k)$$

↓

discrete scale invariance in energy

↓

emergent IR scale

↓

energy gap: Mott gap (Mottness)

$$G(\omega, k) = G(\omega\lambda^n, k)$$

↓

discrete scale invariance in energy

↓

emergent IR scale

↓ no condensate

energy gap: Mott gap (Mottness)

continuous scale invariance

↓

discrete scale invariance
in energy

is this the symmetry that is ultimately
broken in the Mott
problem?

a.) yes

b.) no

holography

a.) yes	√
b.) no	

	holography	VO_2, cuprates,...
a.) yes	√	
b.) no		

	holography	VO_2, cuprates,...
a.) yes	√	
b.) no		

Does scaling in VO_2 obey: $\Lambda_{\mathrm{IR}} = \Lambda_{\mathrm{UV}} e^{-\pi/\sqrt{U_c - U}}$

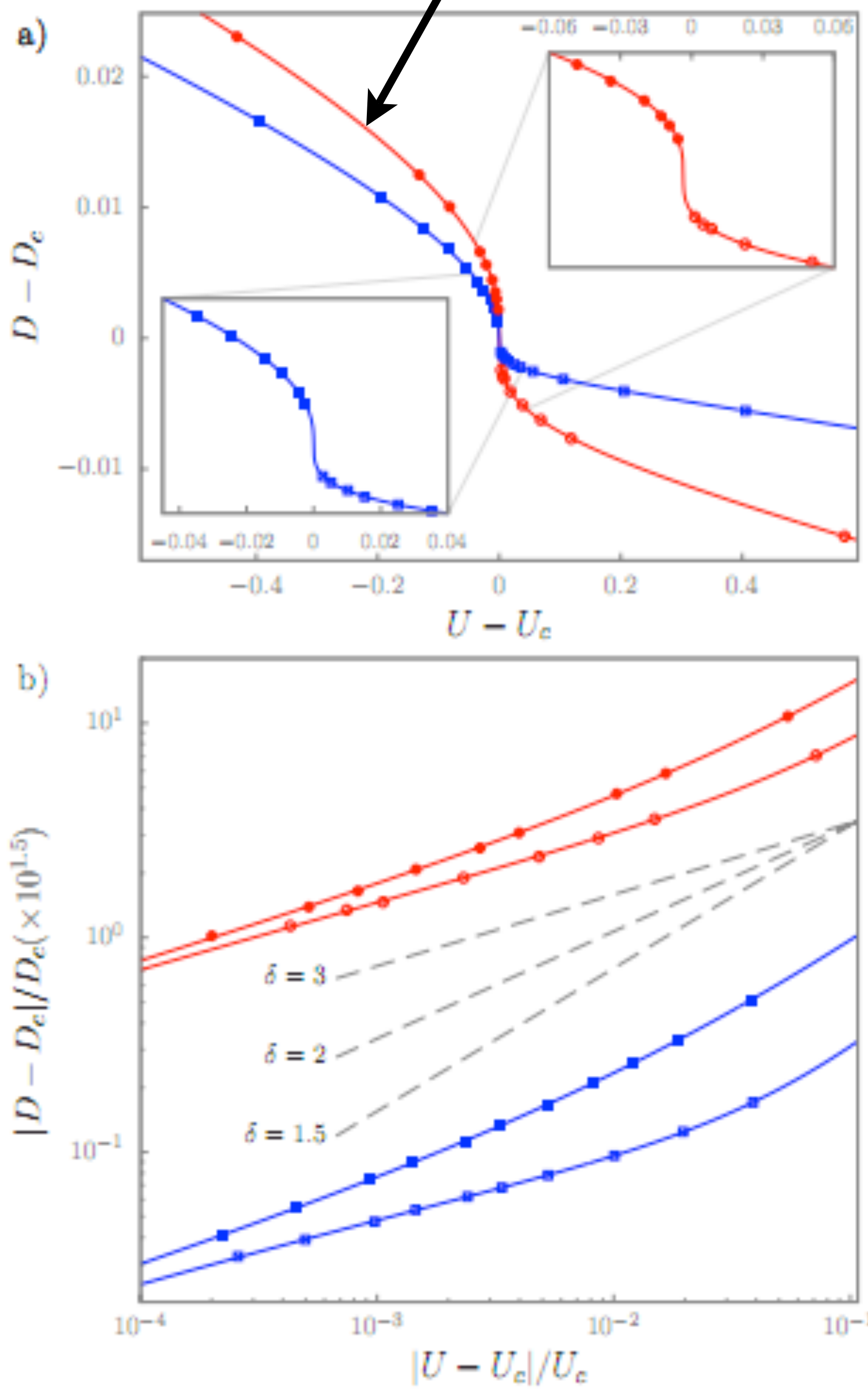
	holography	VO_2, cuprates,...
a.) yes	√	
b.) no		

Does scaling in VO_2 obey: $\Lambda_{\rm IR} = \Lambda_{\rm UV} e^{-\pi/\sqrt{U_c - U}}$

if yes: holography has solved the Mott problem

$$f(U) = c_1 \mathrm{sgn}(\delta U)|\delta U|^{1/\delta} + c_2|\delta|^{2/\delta} + c_3\delta U + D_c$$

4 parameters



Semon, Tremblay, arxiv:1110.6195

$$f(U) = c_1 \mathrm{sgn}(\delta U)|\delta U|^{1/\delta} + c_2|\delta|^{2/\delta} + c_3\delta U + D_c$$

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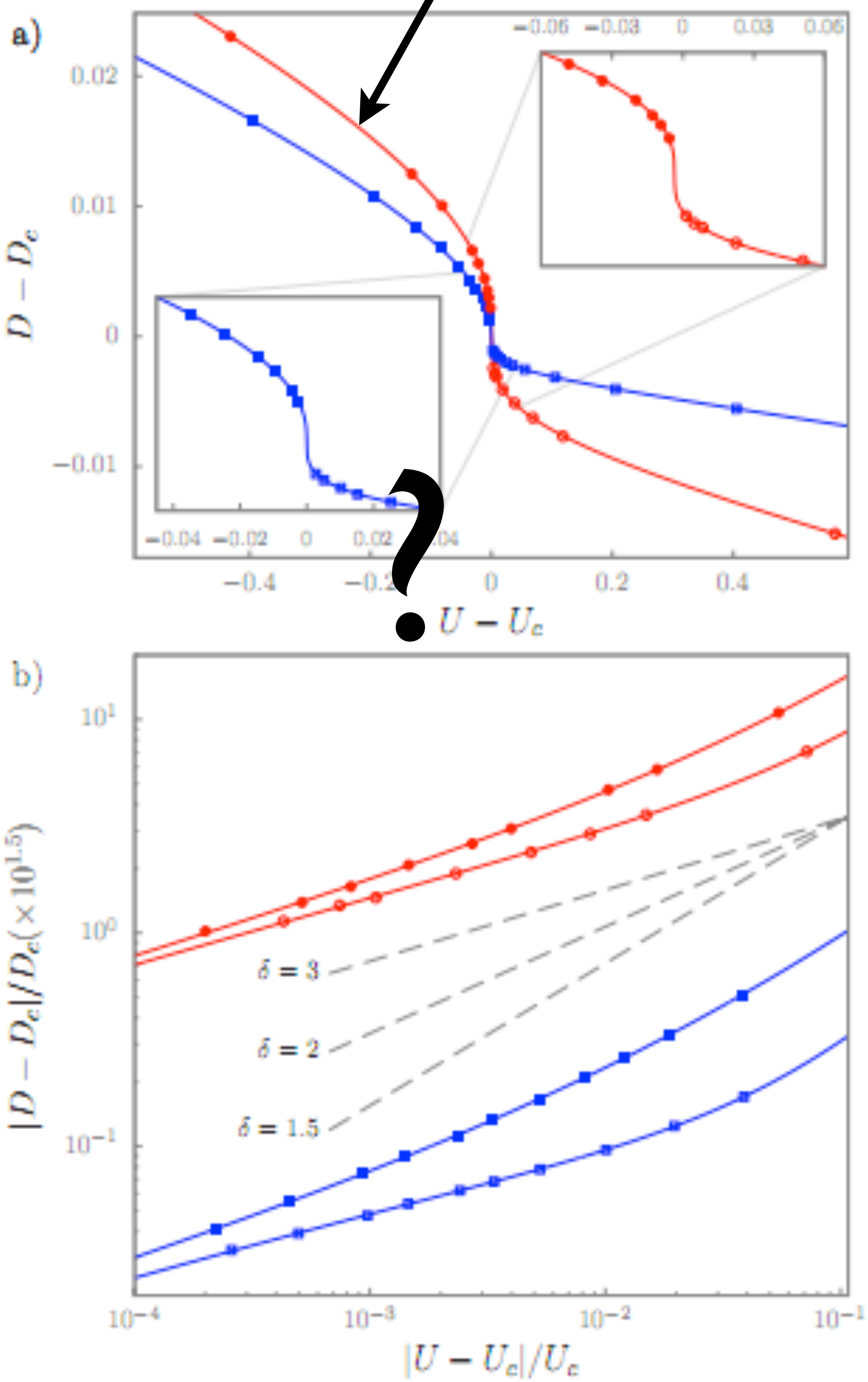

Semon, Tremblay, arxiv:1110.6195

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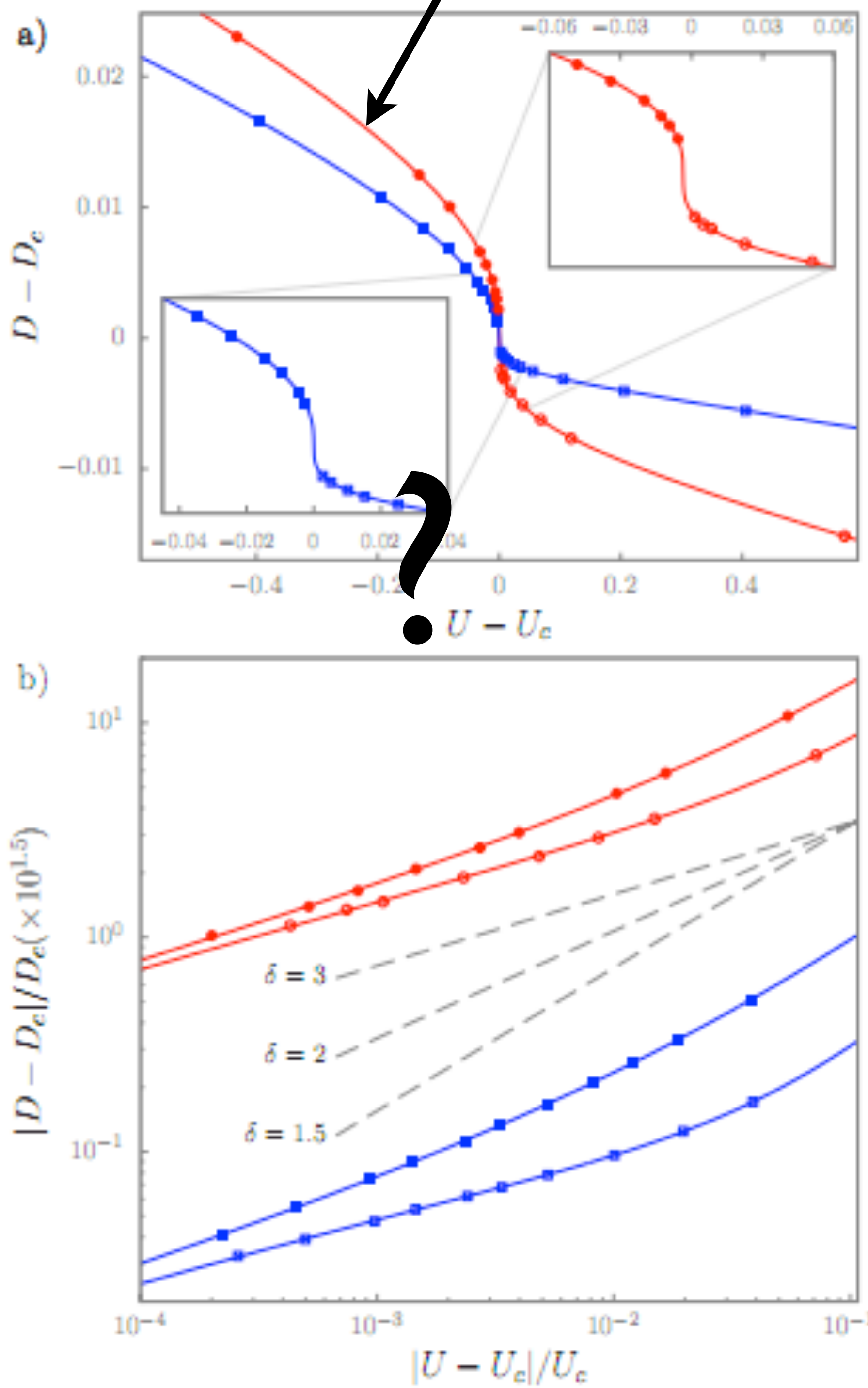

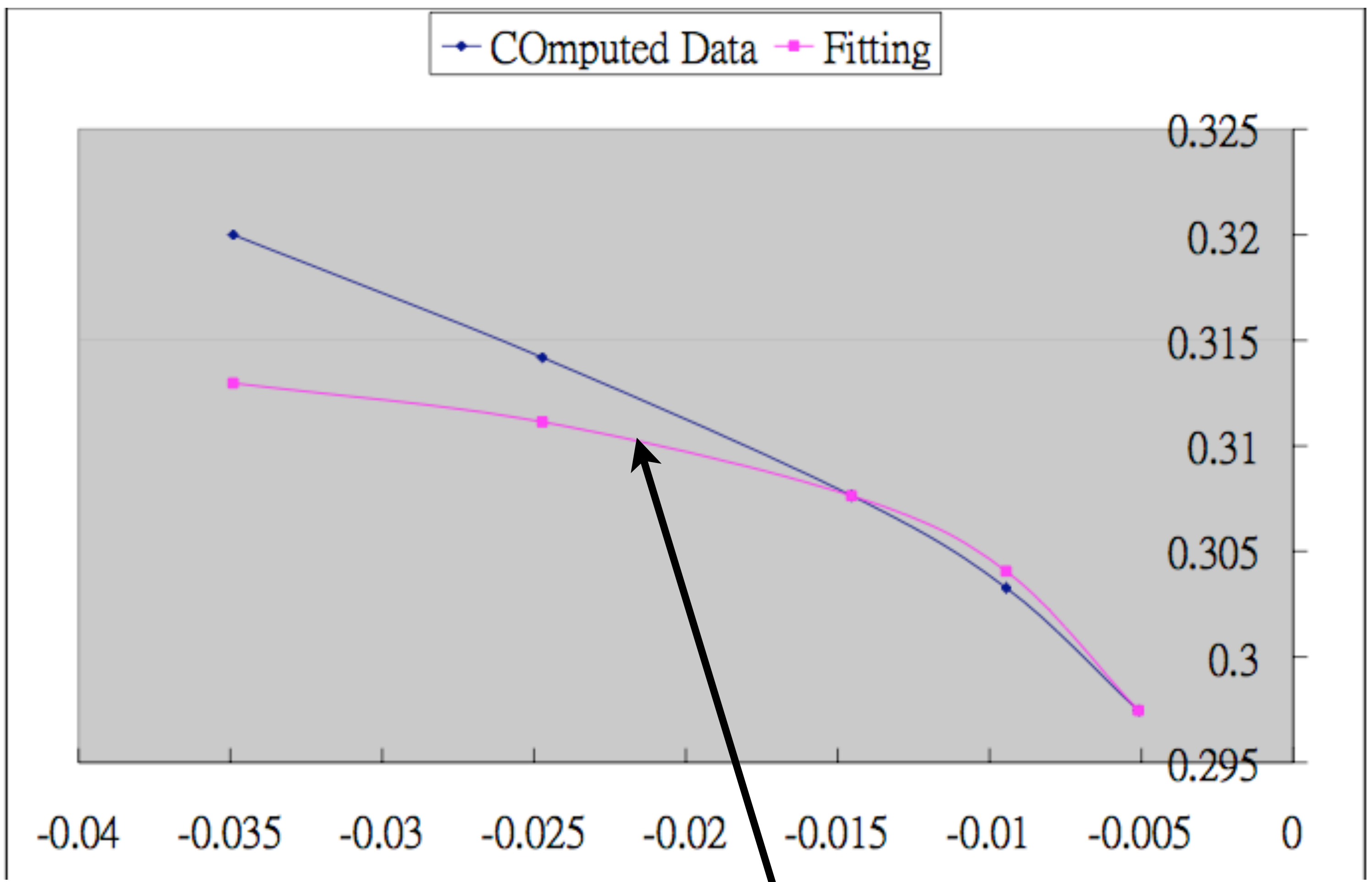

$$\Lambda_{\mathrm{IR}} = \Lambda_{\mathrm{UV}} e^{-\gamma/\sqrt{U_c - U}}$$

Semon, Tremblay, arxiv:1110.6195

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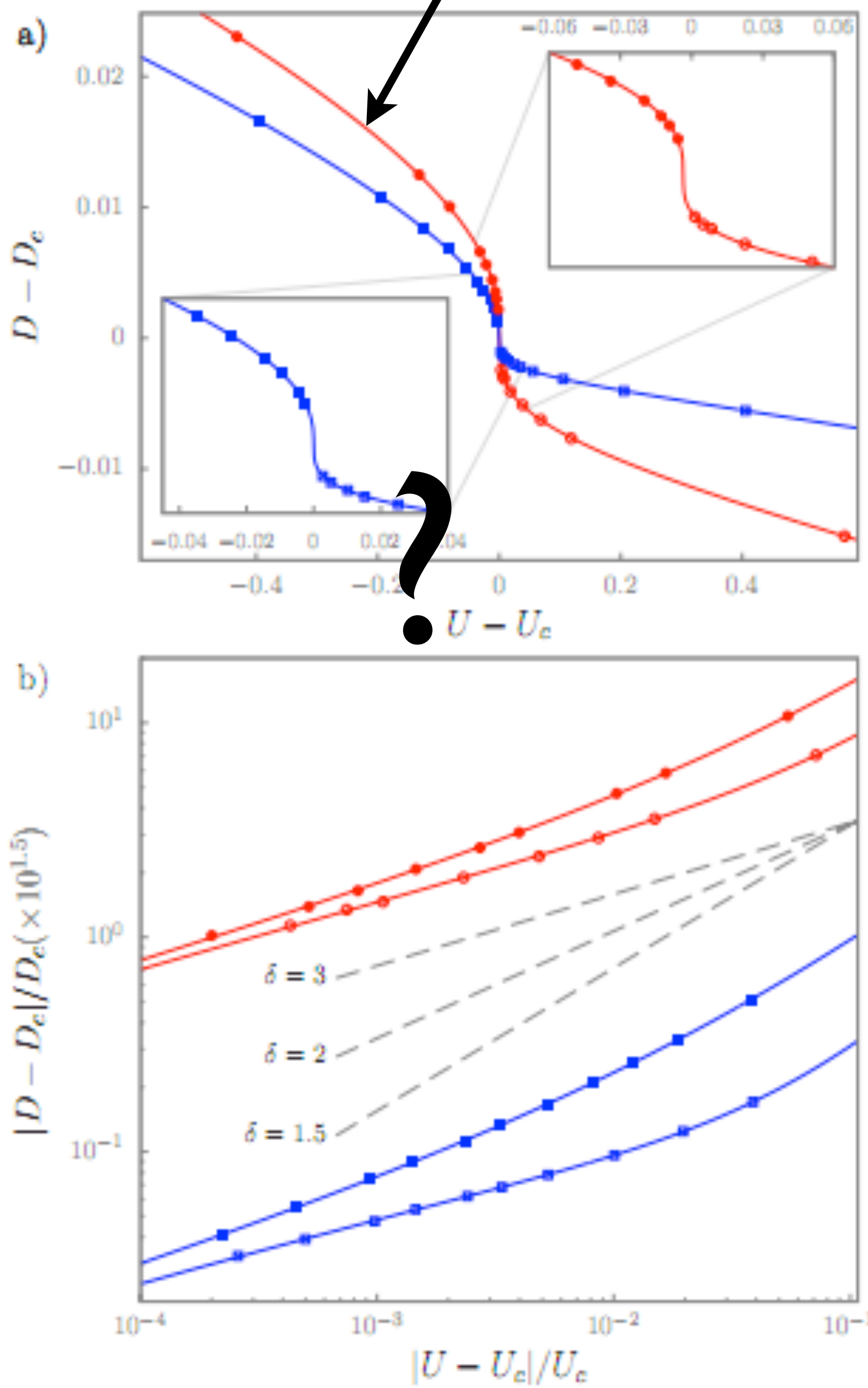
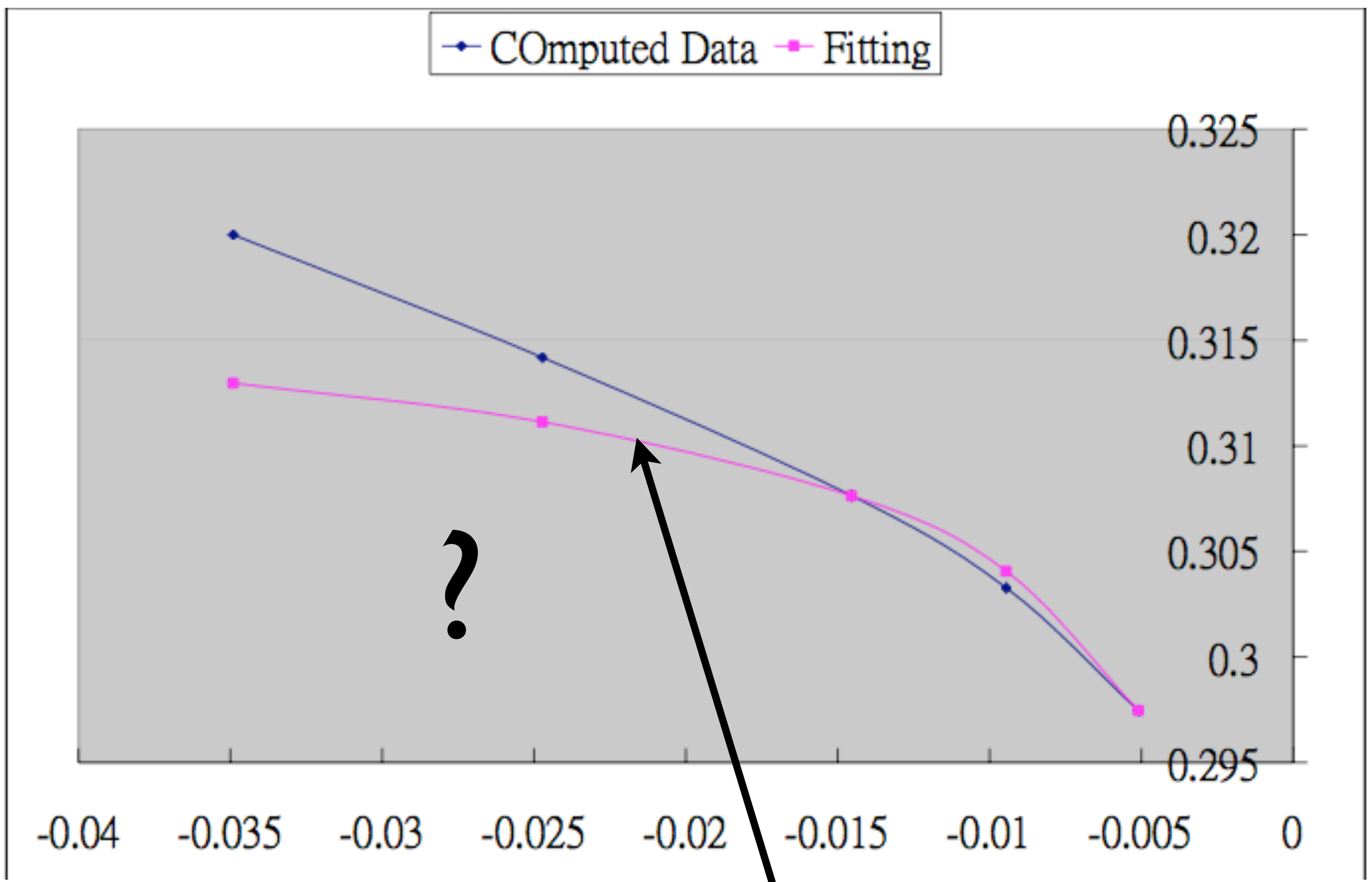
4 parameters





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