Holography of compressible quantum phases

KITP, September 29, 2011

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Conformal quantum matter

Compressible quantum matter

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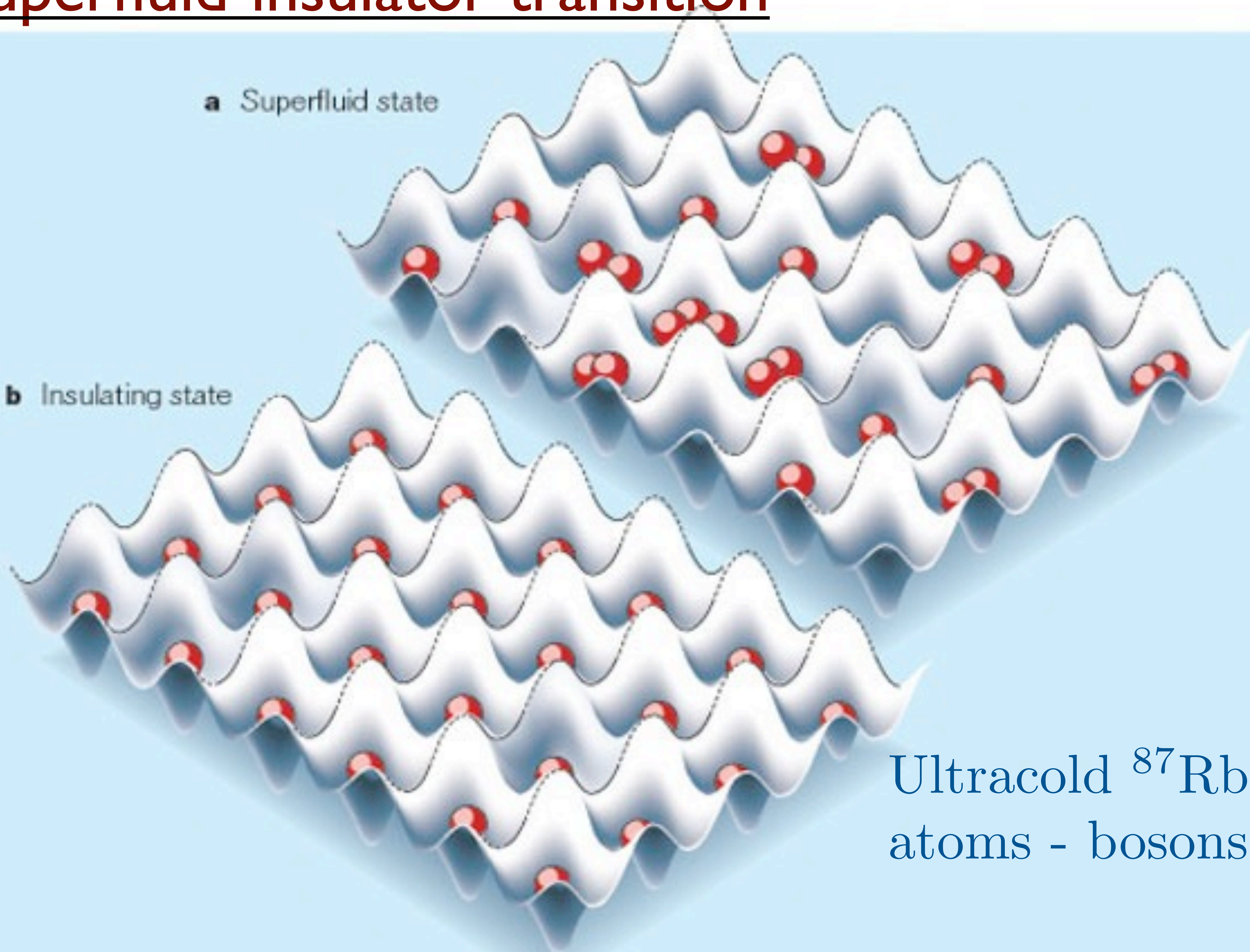
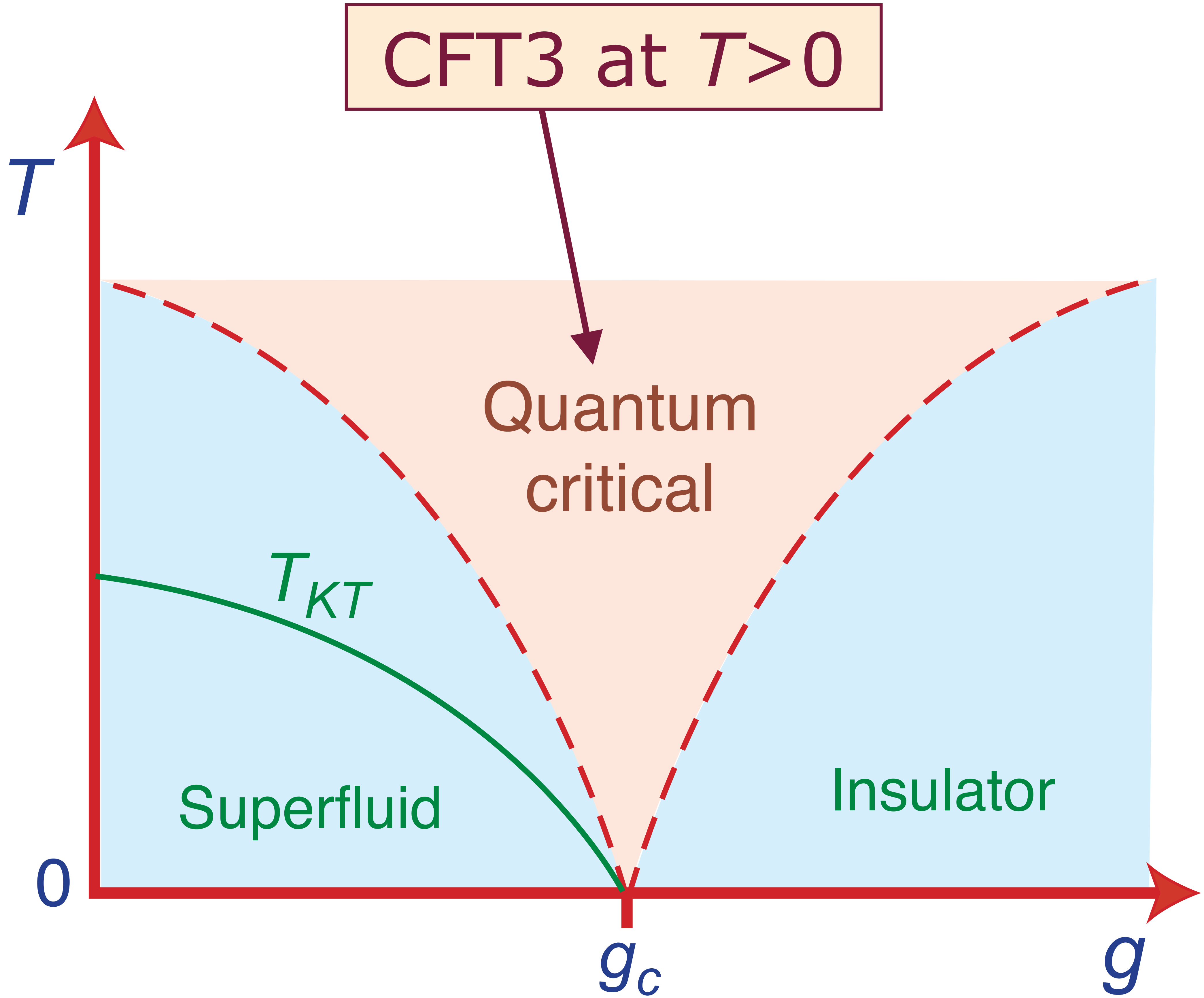
Compressible quantum matter

Superfluid-insulator transition



Ultracold ^{87}Rb
atoms - bosons

M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).


CFT3 at $T>0$
T
Quantum
critical
T_{KT}
Superfluid
Insulator
0
g_c
g

Quantum critical transport

Quantum *"nearly perfect fluid"*
with shortest possible
equilibration time, $\tau_{\rm eq}$

$$\tau_{\rm eq} = \mathcal{C}\,\frac{\hbar}{k_B T}$$

where $\mathcal{C}$ is a *universal* constant

S. Sachdev, *Quantum Phase Transitions,* Cambridge (1999).

AdS_4 theory of "nearly perfect fluids"

To leading order in a gradient expansion, charge transport in an infinite set of strongly-interacting CFT3s can be described by Einstein-Maxwell gravity/electrodynamics on AdS_4-Schwarzschild

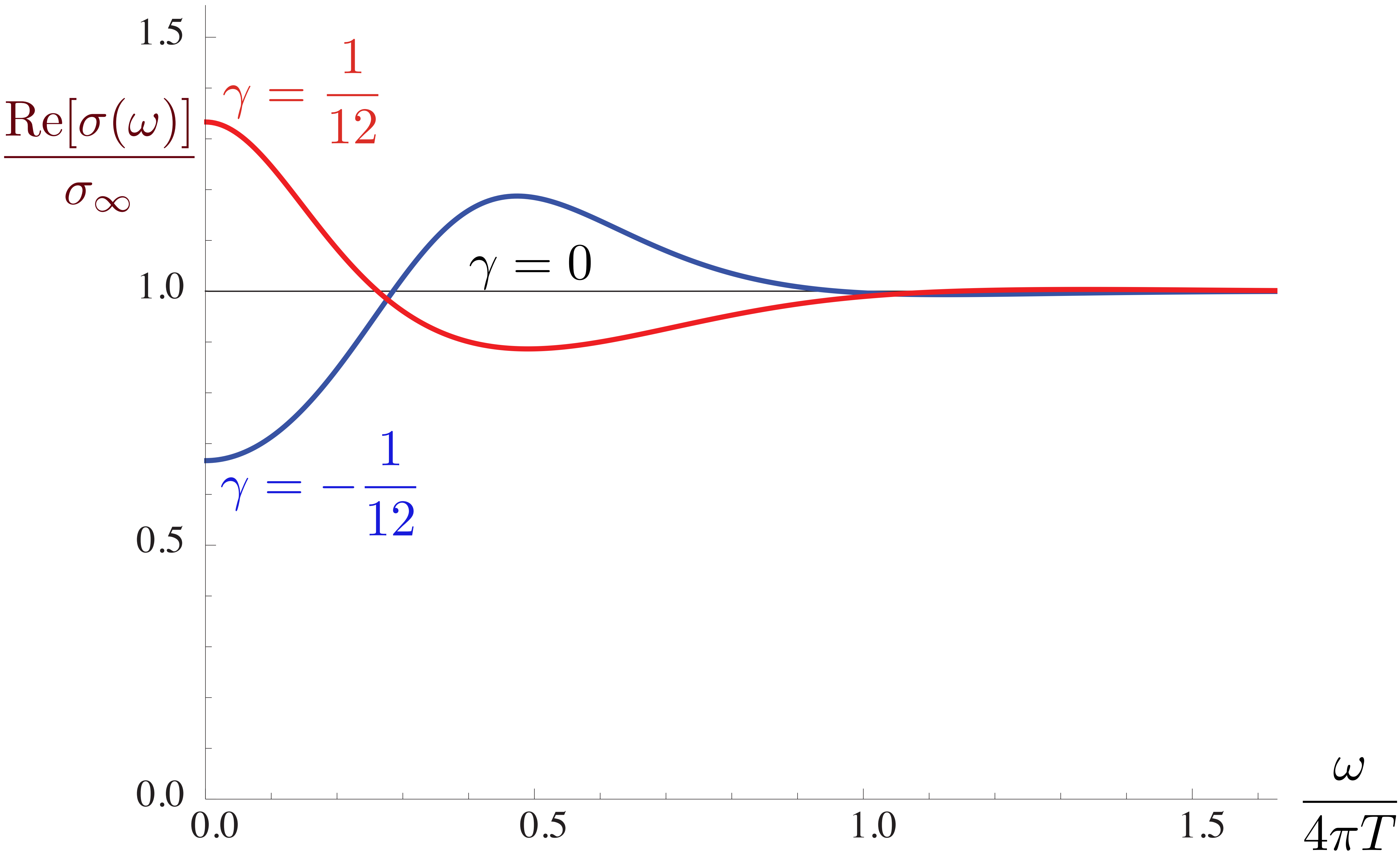
We include all possible 4-derivative terms: after suitable field redefinitions, the required theory has only *one* dimensionless constant γ (L is the radius of AdS_4):

$$\mathcal{S}_{EM} = \int d^4x\sqrt{-g}\left[-\frac{1}{4e^2}F_{ab}F^{ab} + \frac{\gamma L^2}{e^2}C_{abcd}F^{ab}F^{cd}\right],$$

where C_{abcd} is the Weyl curvature tensor.
Stability and causality constraints restrict $|\gamma| < 1/12$.

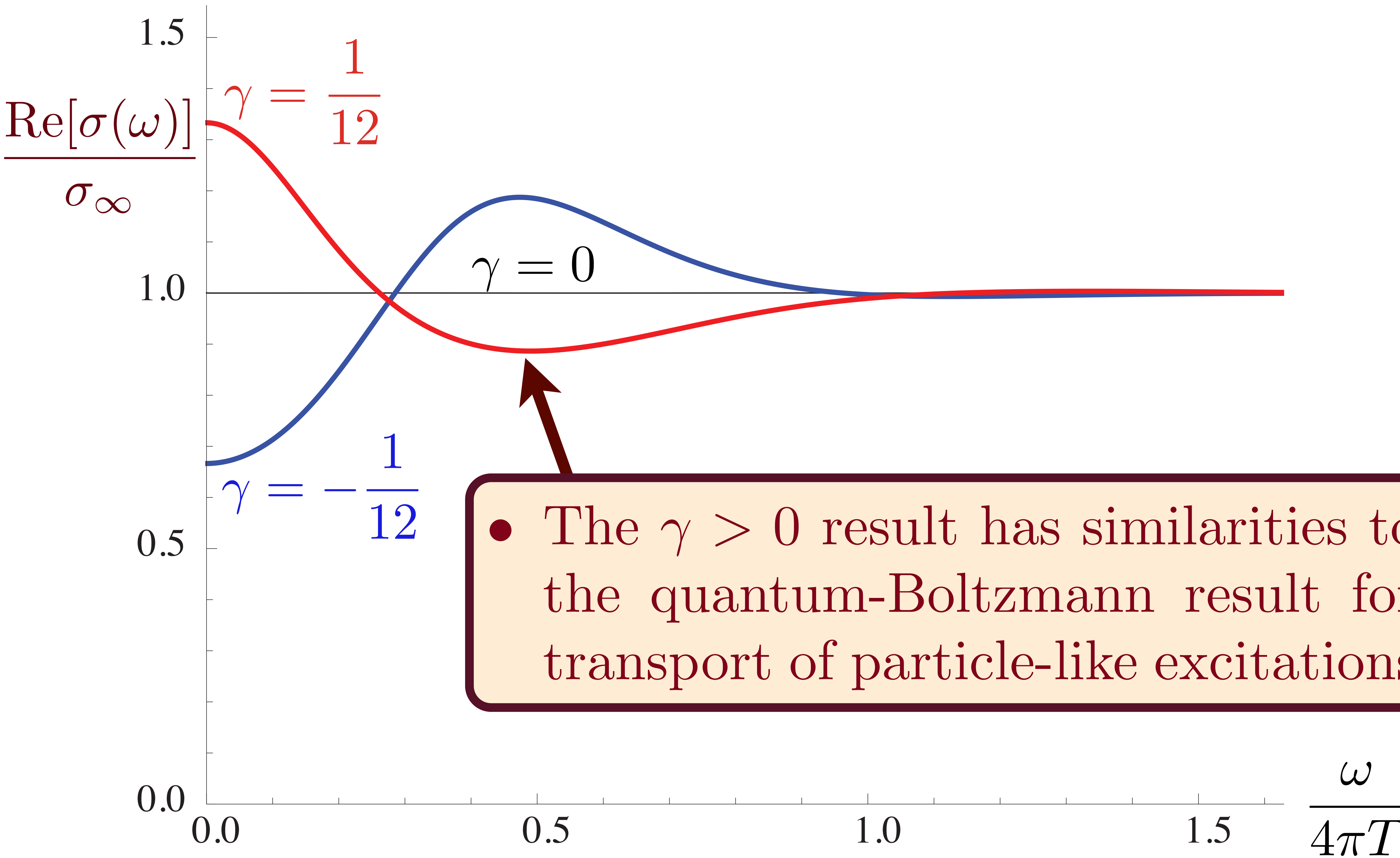
R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

AdS_4 theory of strongly interacting "perfect fluids"



R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

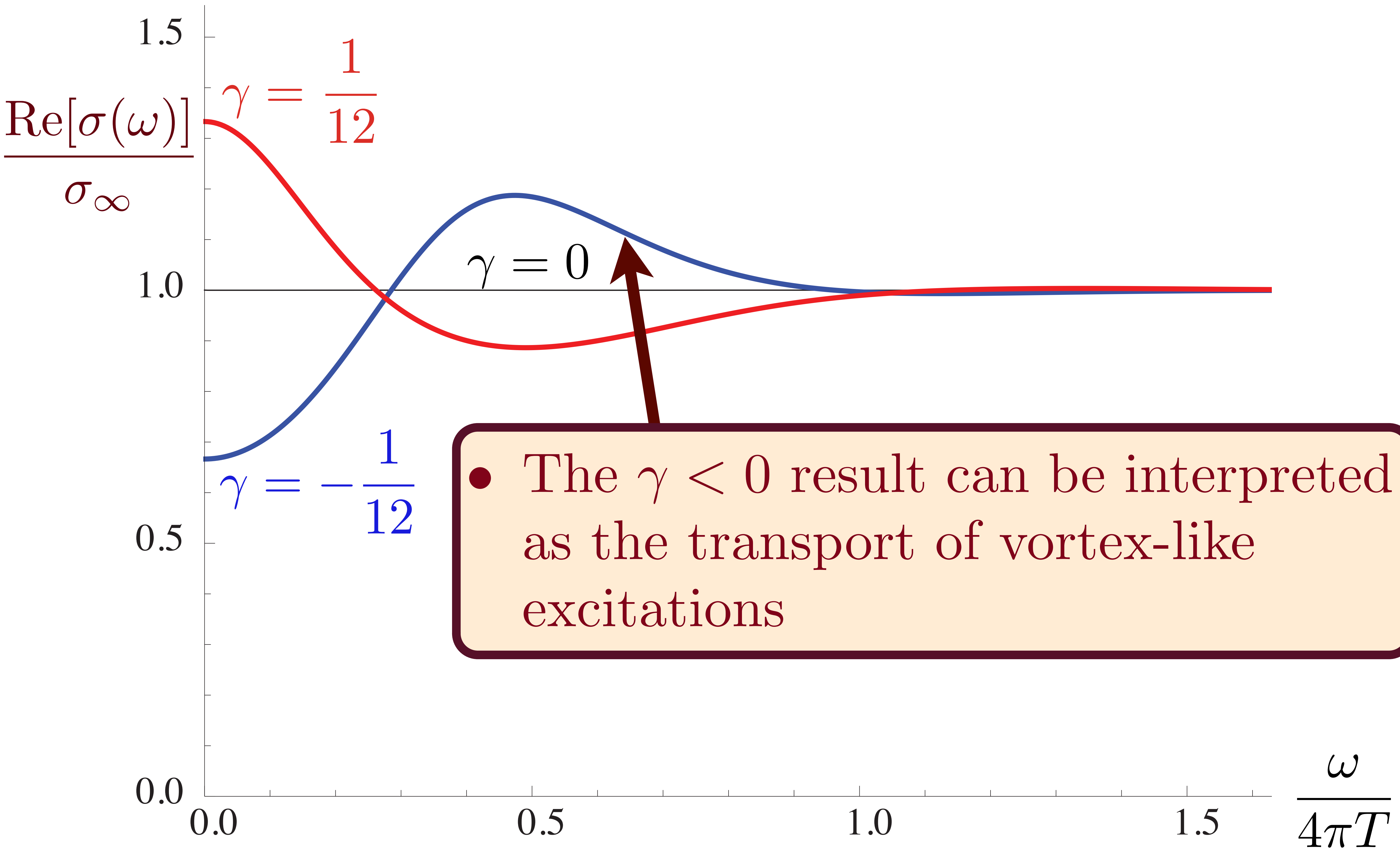
AdS$_4$ theory of strongly interacting "perfect fluids"



- The $\gamma > 0$ result has similarities to the quantum-Boltzmann result for transport of particle-like excitations

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

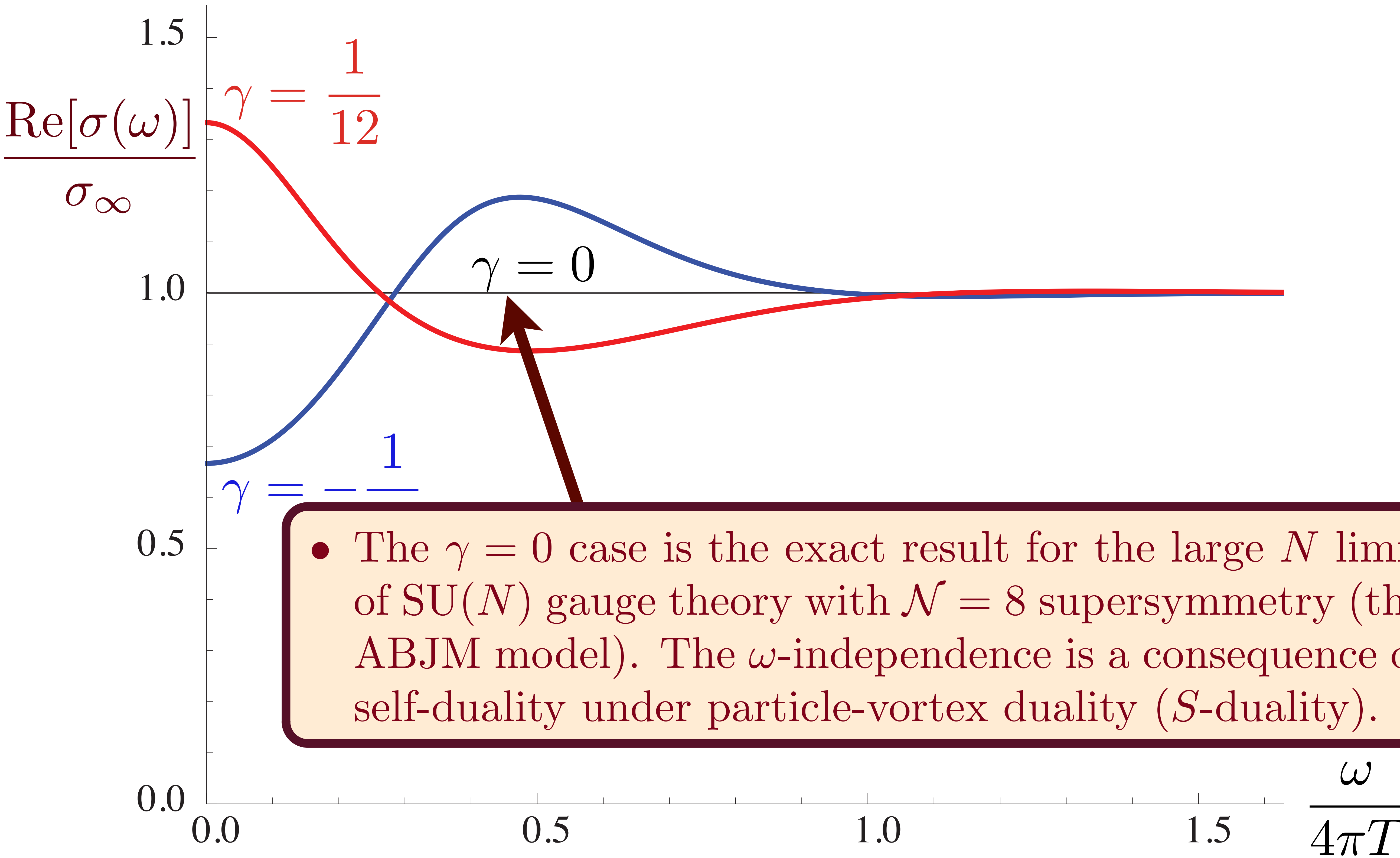
AdS_4 theory of strongly interacting "perfect fluids"



- The $\gamma < 0$ result can be interpreted as the transport of vortex-like excitations

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

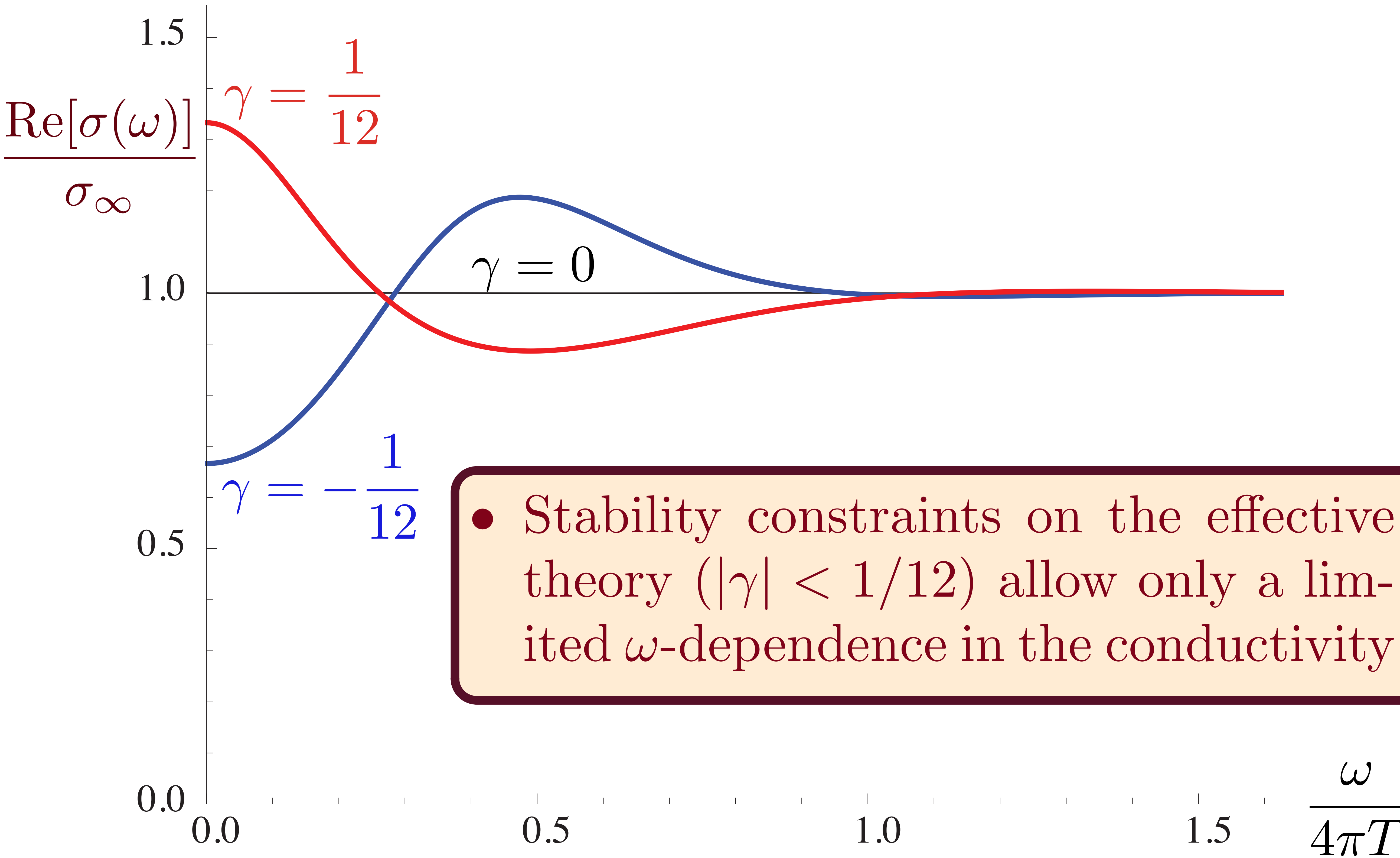
AdS$_4$ theory of strongly interacting "perfect fluids"



- The $\gamma = 0$ case is the exact result for the large N limit of SU(N) gauge theory with $\mathcal{N} = 8$ supersymmetry (the ABJM model). The ω-independence is a consequence of self-duality under particle-vortex duality (S-duality).

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

AdS_4 theory of strongly interacting "perfect fluids"



- Stability constraints on the effective theory ($|\gamma| < 1/12$) allow only a limited ω-dependence in the conductivity

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

Compressible quantum matter

Conventional phases

1. Holographic theory of the Fermi liquid (FL)

Exotic phases

1. Continuum models with gauge theories: the fractionalized Fermi liquid (FL*)

2. Holographic approach

3. Connections to models and experiments on the heavy fermion compounds and the cuprate superconductors

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Compressible quantum matter

- Consider an infinite, continuum, translationally-invariant quantum system with a globally conserved U(1) charge $\mathcal{Q}$ (the "electron density") in spatial dimension $d > 1$.

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- Compressible systems must be gapless.

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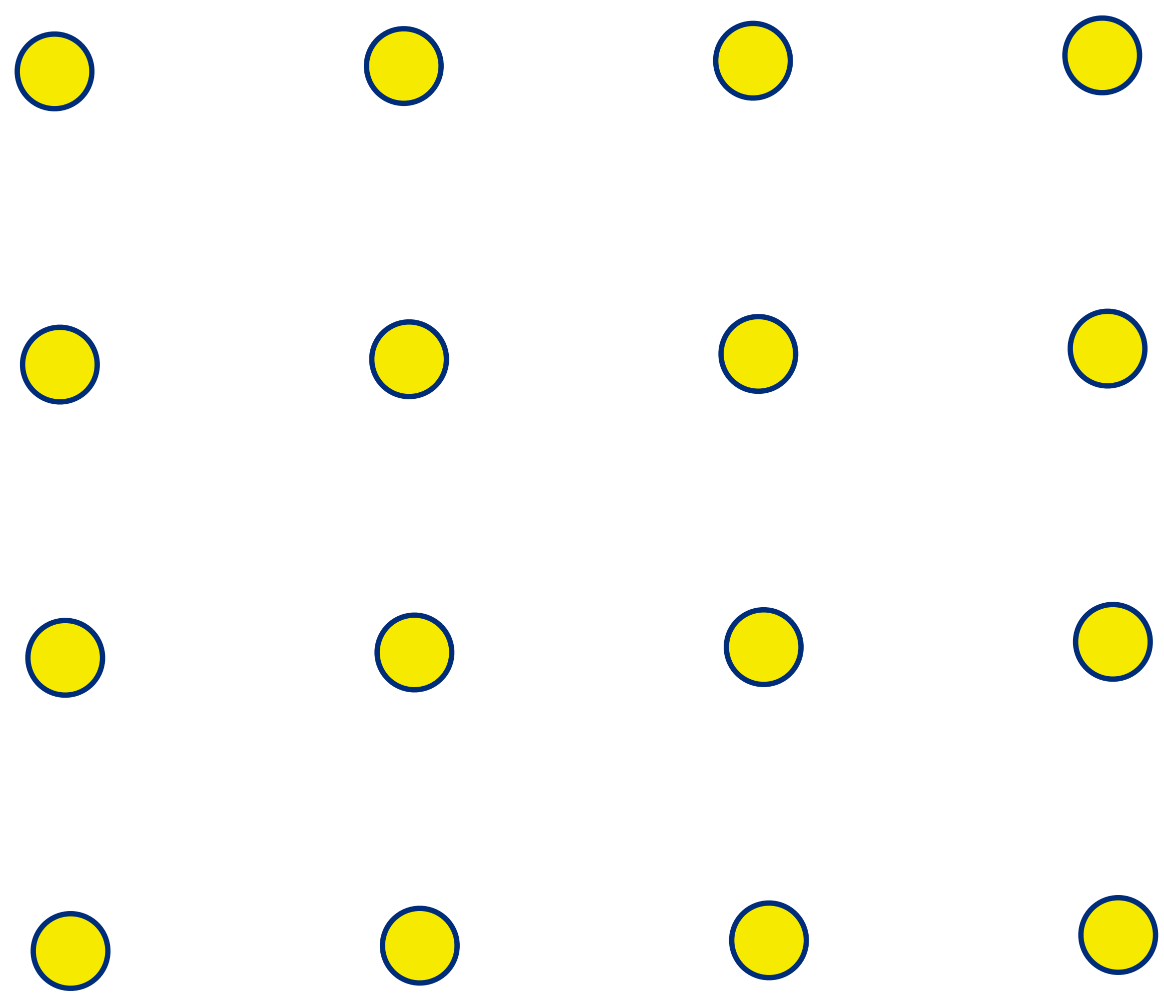
- Compressible systems must be gapless.

- Conformal systems are compressible in $d = 1$, but not for $d > 1$.

Compressible quantum matter

One compressible state is the **solid** (or "Wigner crystal" or "stripe").
This state breaks translational symmetry.

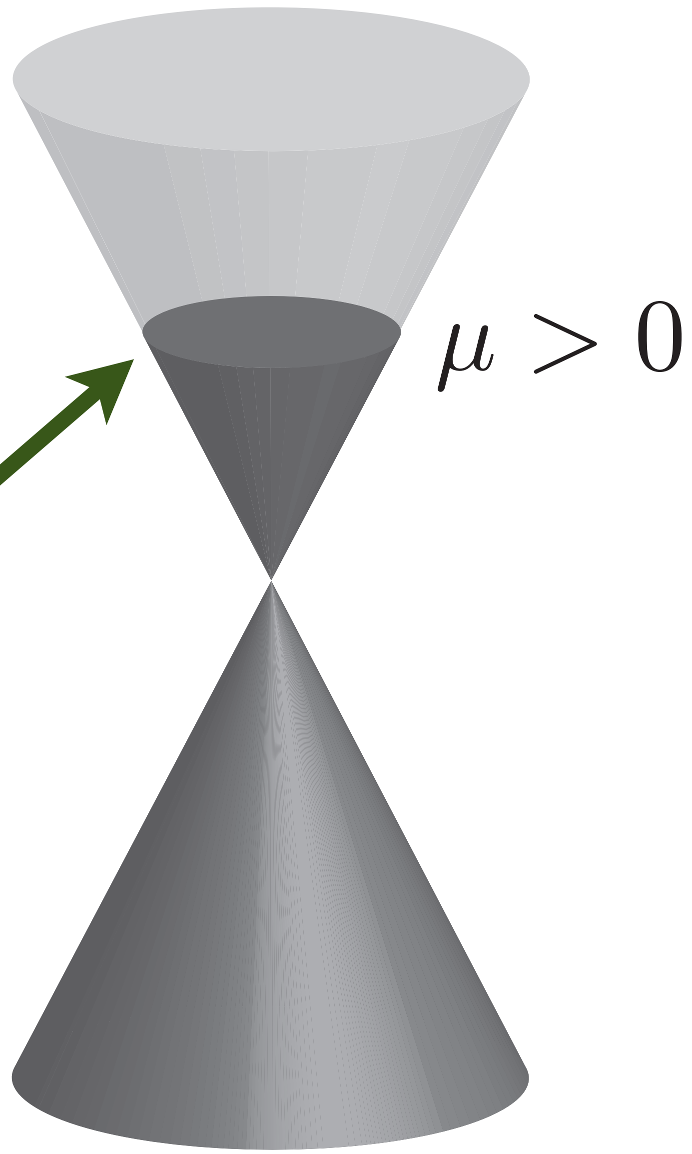


Compressible quantum matter

Another familiar compressible state is the **superfluid**.
This state breaks the global U(1) symmetry associated with $\mathcal{Q}$

Condensate of fermion pairs

The only other familiar compressible phase is a **<u>Fermi Liquid</u>** with a **<u>Fermi surface</u>**

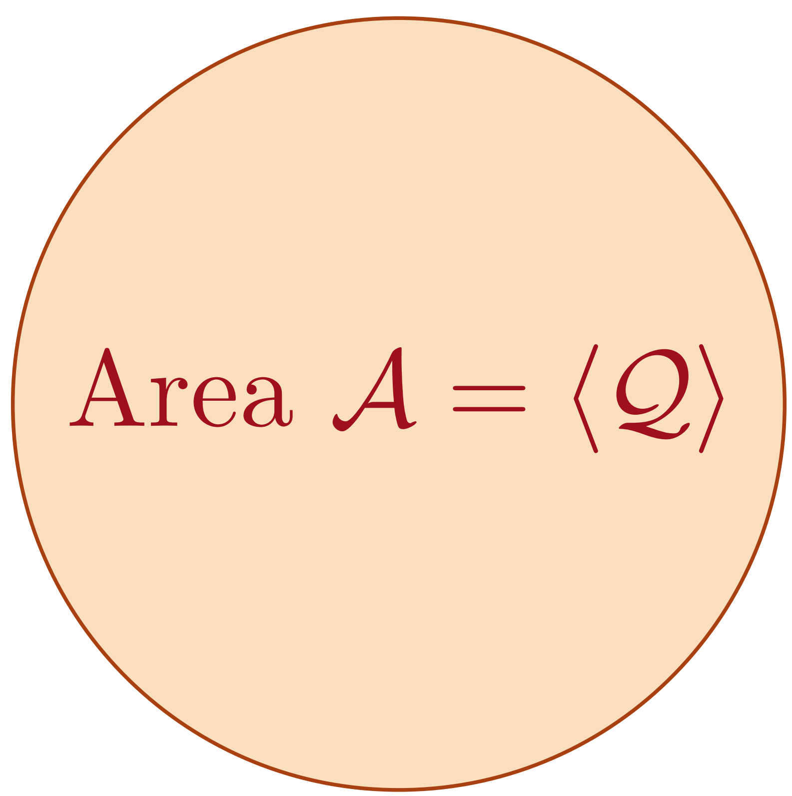


Graphene

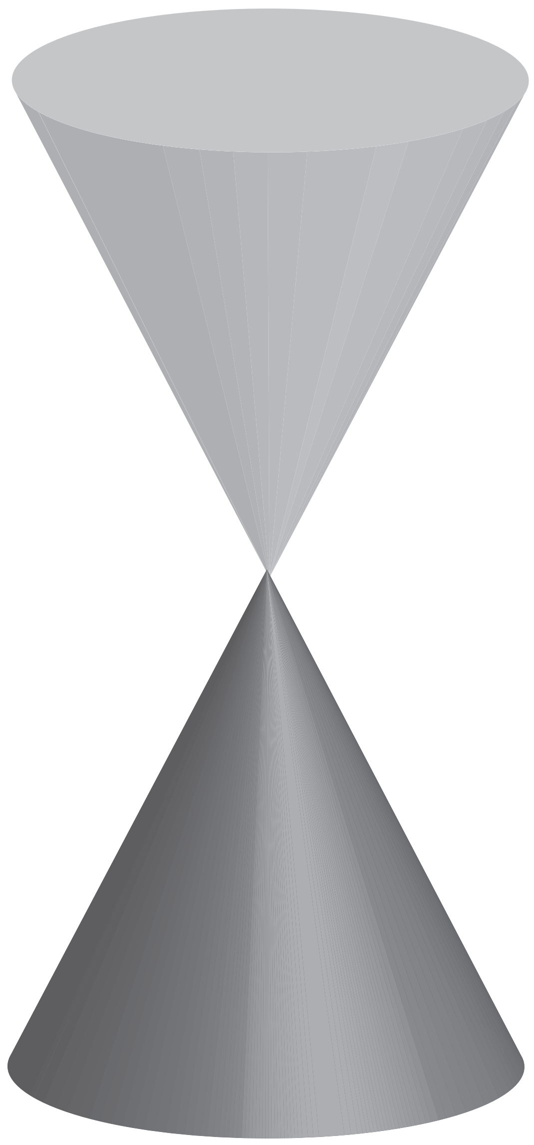
The Landau Fermi liquid

- The *only* low energy excitations are long-lived quasiparticles near the Fermi surface.

- **Luttinger relation:** The total "volume (area)" $\mathcal{A}$ enclosed by the Fermi surface is equal to $\langle \mathcal{Q} \rangle$. This is a *key* constraint which allows extrapolation from weak to strong coupling, and also holds for "non-Fermi liquid" compressible phases to be discussed later.

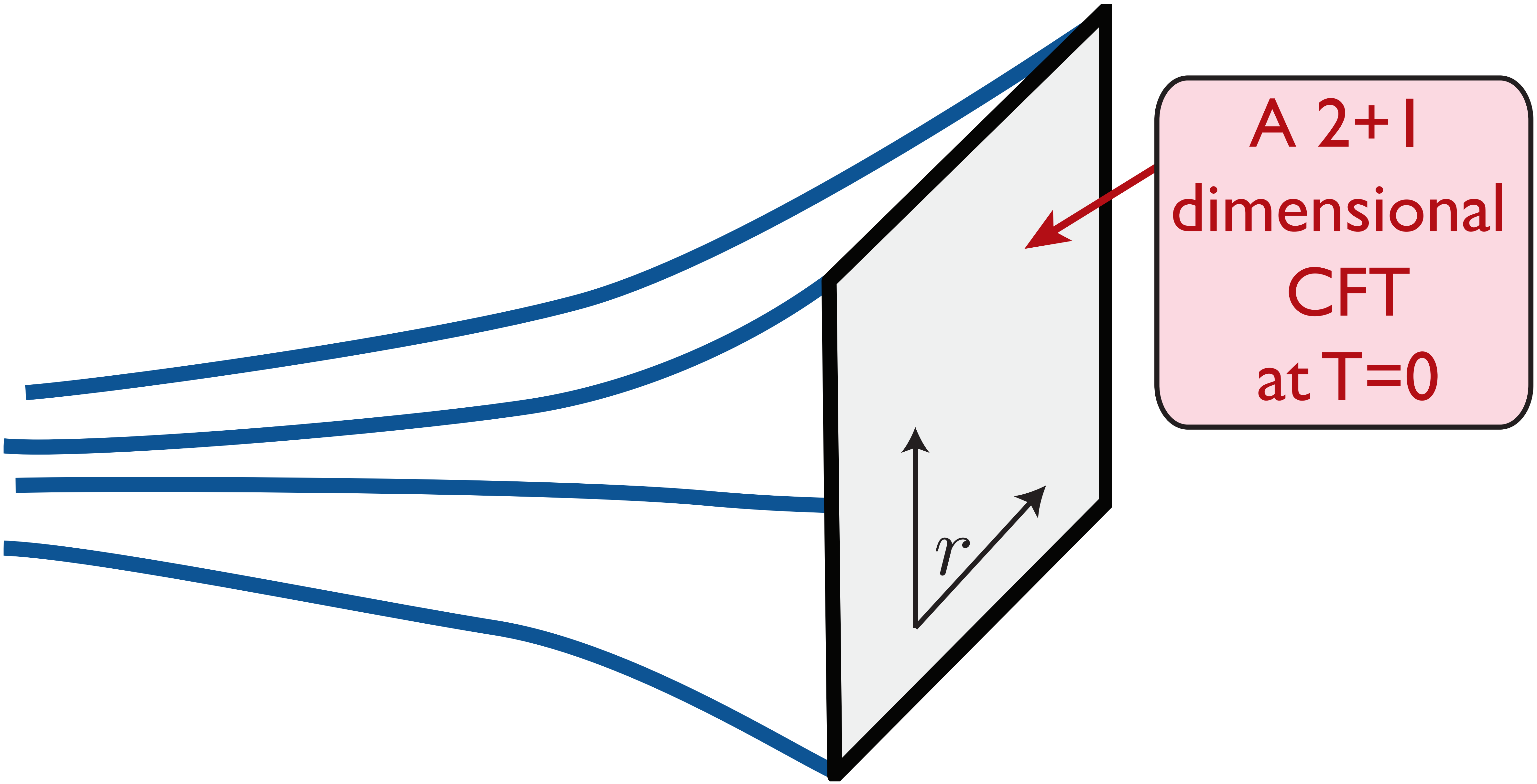


Begin with a CFT



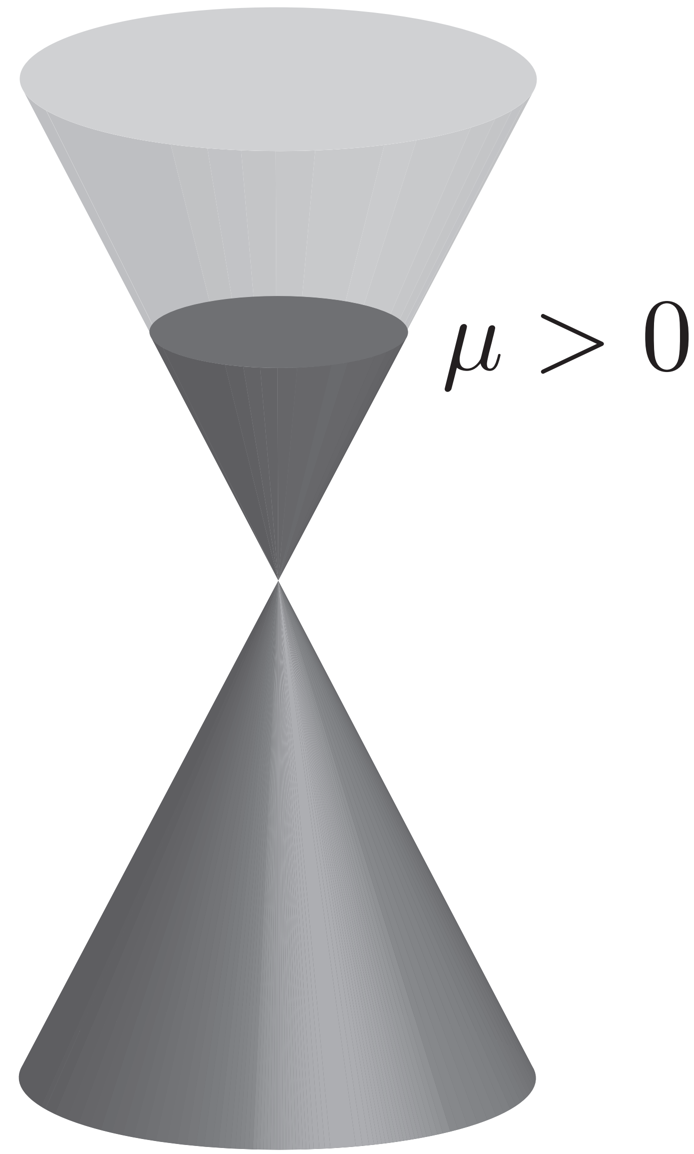
Dirac fermions + gauge field + ...

Holographic representation: AdS4

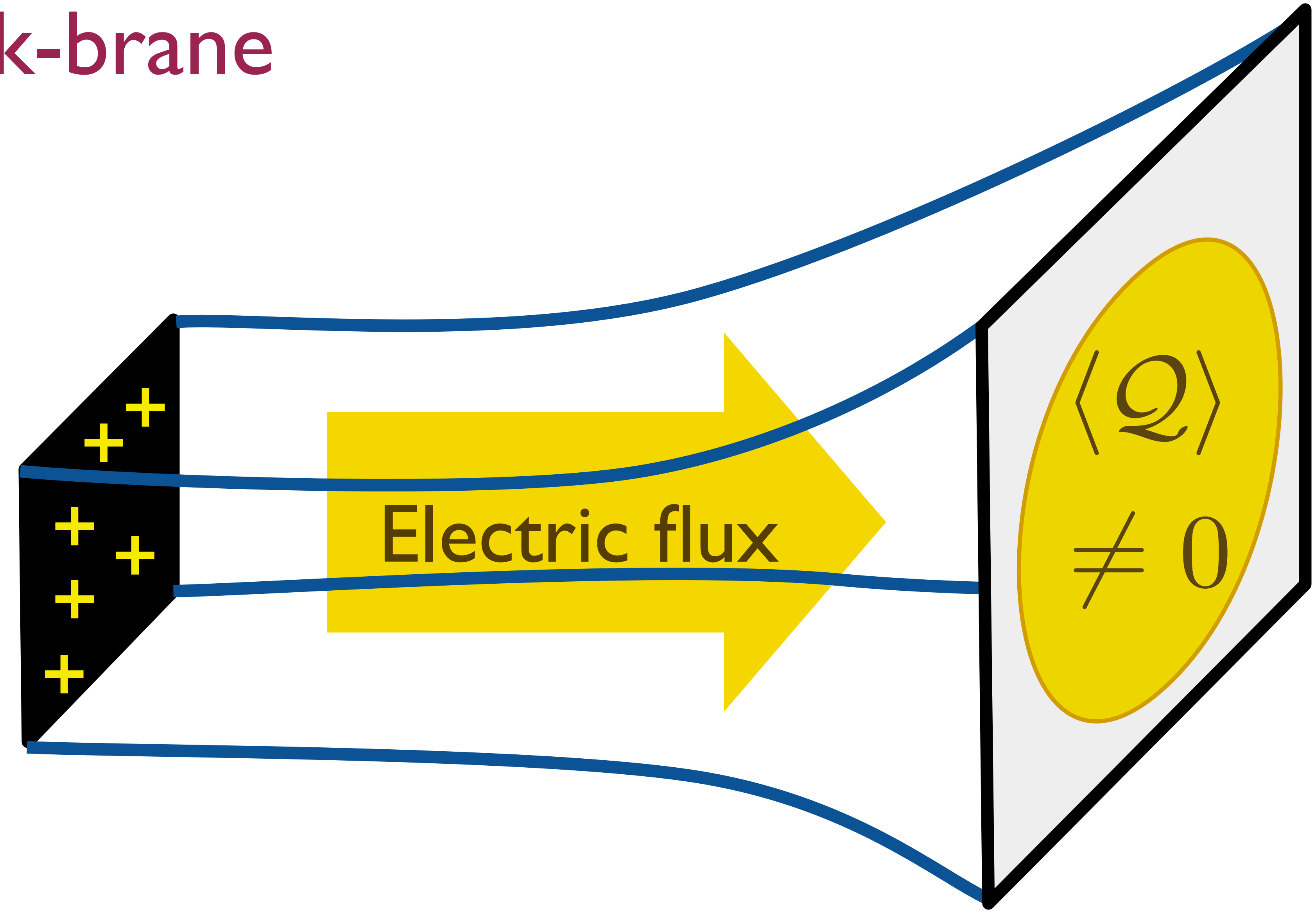


$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) \right]$$

Will describe a Landau Fermi liquid obtained by applying a chemical potential to the "deconfined" CFT



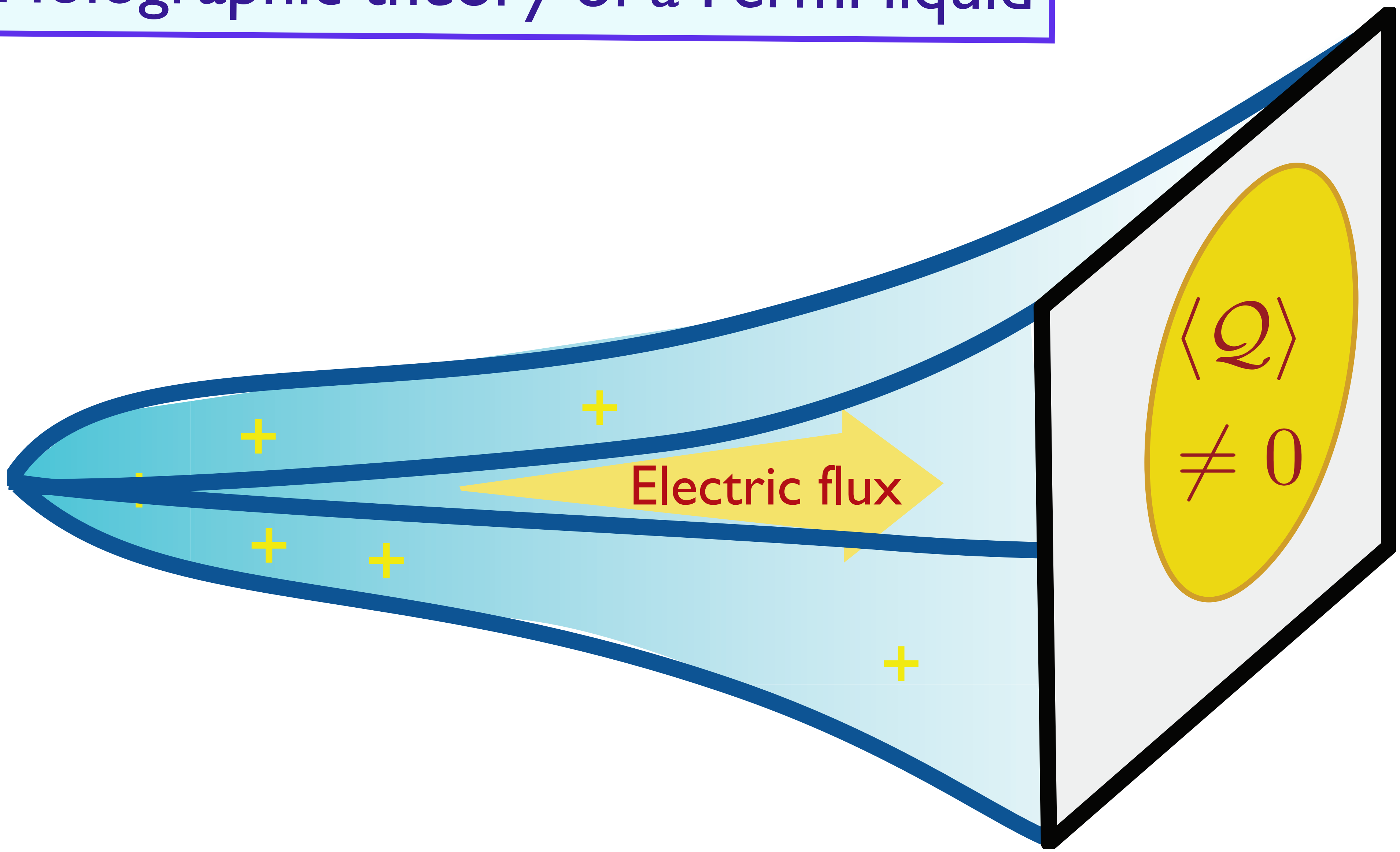
The Maxwell-Einstein theory of the applied chemical potential yields a AdS_4-Reissner-Nordtröm black-brane



$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) - \frac{1}{4e^2} F_{ab} F^{ab} \right]$$

Holographic theory of a Fermi liquid

S. Sachdev
arXiv:1107.5321



$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) - \frac{Z(\phi)}{4e^2} F_{ab} F^{ab} + \mathcal{L}[\text{matter}, \phi] \right]$$

In a confining phase, the horizon disappears,
there is charge density delocalized in the bulk spacetime,
and a Fermi liquid phase is obtained on the boundary

Holographic theory of a Fermi liquid

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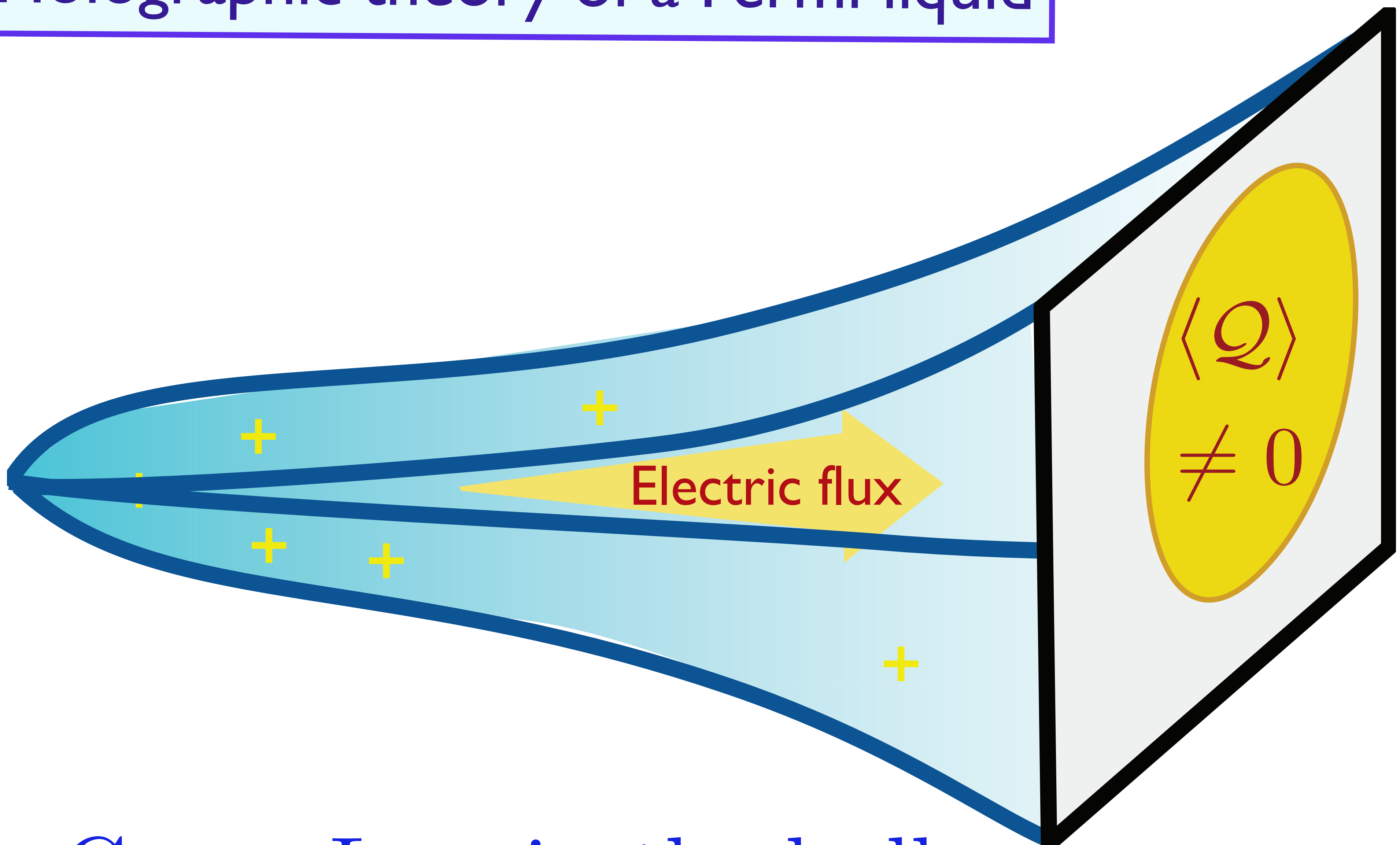

Gauss Law in the bulk
$\Leftrightarrow$ Luttinger theorem on the boundary

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Consider QED_4, with *full* quantum fluctuations,

$$S = \int d^4x \sqrt{g} \left[\frac{1}{4e^2} F_{ab} F^{ab} + i \left(\overline{\psi} \Gamma^M D_M \psi + m \overline{\psi} \psi \right) \right].$$

in a metric which is AdS_4 in the UV, and confining in the IR. A simple model

$$ds^2 = \frac{1}{z^2}(dz^2 - dt^2 + dx^2 + dy^2) \quad , \quad z < z_m$$

with z_m determined by the confining scale.

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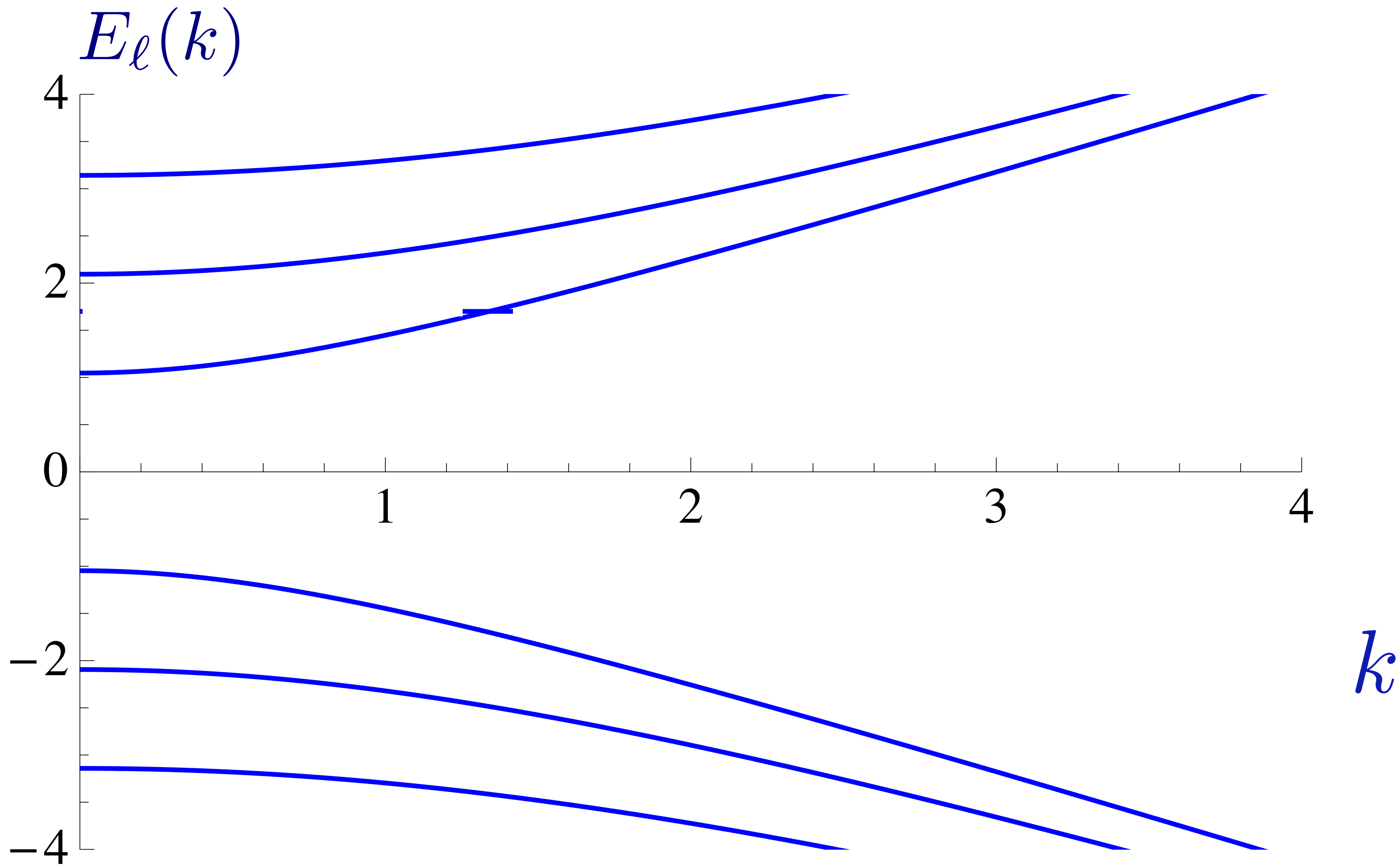

Massive Dirac fermions at zero chemical potential

Dispersion $E_\ell(k) = \sqrt{k^2 + M_\ell^2}$

Masses $M_\ell \sim 1/z_m$

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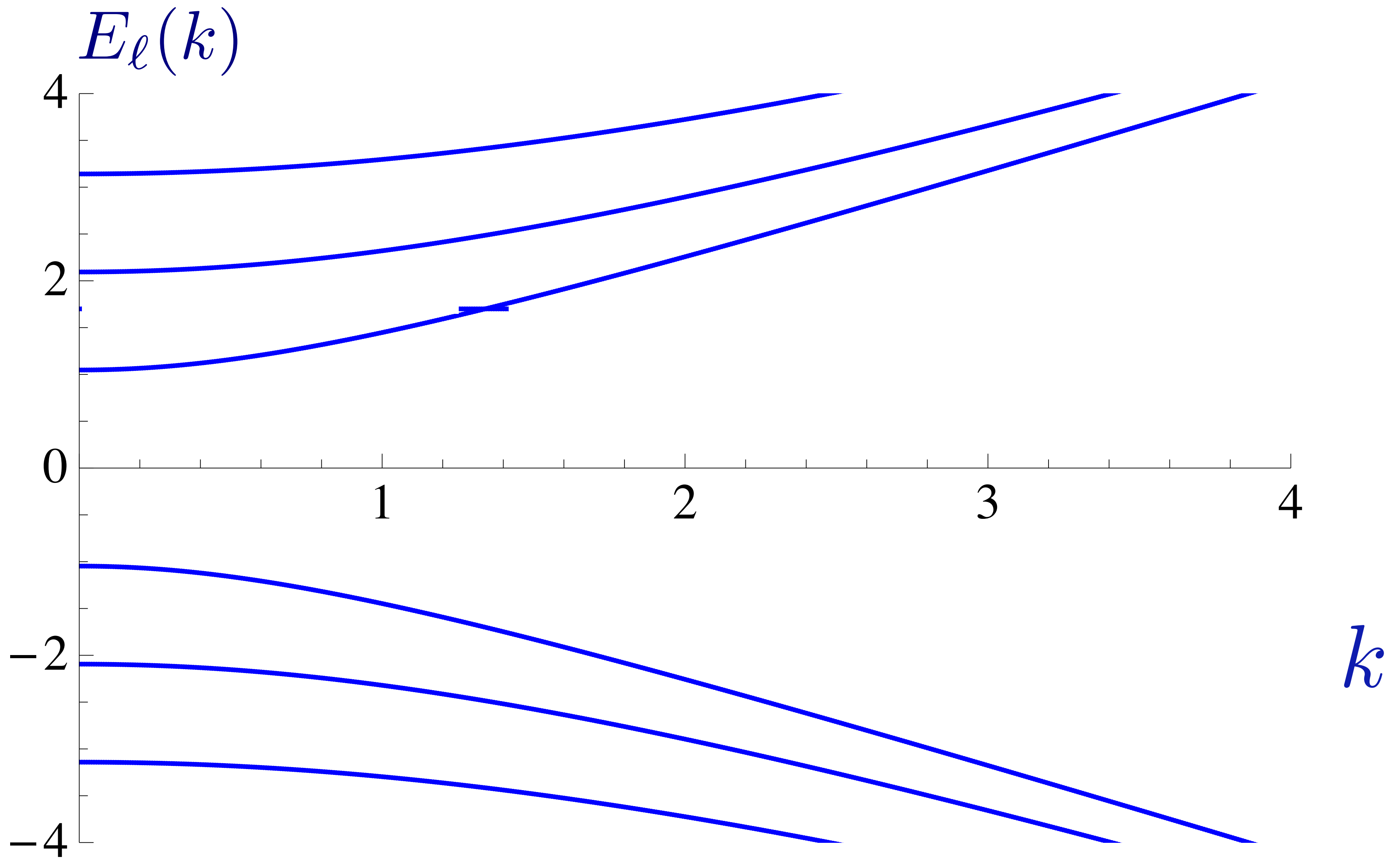

Massive Dirac fermions at zero chemical potential
Dispersion $E_\ell(k) = \sqrt{k^2 + M_\ell^2}$
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Almost all previous holographic theories have considered the situation where the spacing between the $E_\ell(k)$ vanishes, and an infinite number of $E_\ell(k)$ are relevant.

Holographic theory of a Fermi liquid

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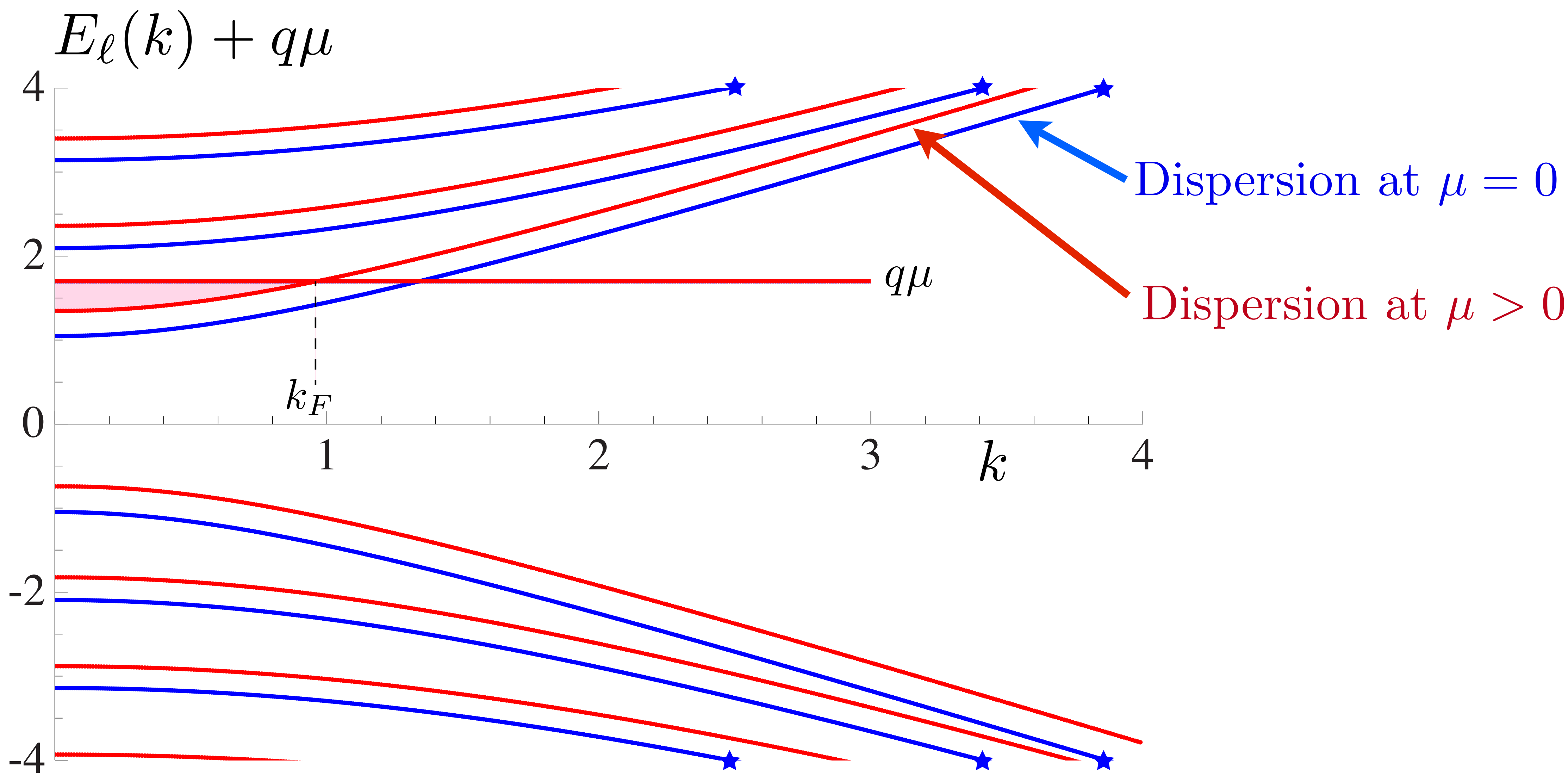

The spectrum at non-zero chemical potential is determined by self-consistently solving the Dirac equation and Gauss's law:

$$\left(\vec{\Gamma} \cdot \vec{D} + m\right) \Psi_\ell = E_\ell \Psi_\ell \;\; ; \;\; \nabla_z \mathcal{E}_z = \sum_\ell \int \frac{d^2k}{4\pi^2} \Psi_\ell^\dagger(k,z) \Psi_\ell(k,z) f(E_\ell(k))$$

where $\mathcal{E}$ is the electric field, and $f(E)$ is the Fermi function

Holographic theory of a Fermi liquid

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- The confining geometry implies that all gauge and graviton modes are gapped.

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Holographic theory of a Fermi liquid

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- The confining geometry implies that all gauge and graviton modes are gapped.
- We can apply standard many body theory results, treating this multi-band system in 2 dimensions, like a 2DEG at a semiconductor surface.
- Integrating Gauss's Law, we obtain

$$\mathcal{E}_z(\text{boundary}) - \mathcal{E}_z(\text{IR}) = \mathcal{A}$$

But $\mathcal{E}_z(\text{boundary}) = \langle \mathcal{Q} \rangle$, but the rules of AdS/CFT. So we obtain the usual Luttinger theorem of a Landau Fermi liquid,

$$\mathcal{A} = \langle \mathcal{Q} \rangle$$

provided $\mathcal{E}_z(\text{IR}) = 0$.

Holographic theory of a Fermi liquid

S. Sachdev
arXiv:1107.5321

Technical notes:

- No source term is included at the boundary for the fermions
- The boundary fermion Green's function is computed by taking a suitable limit of the bulk Green's function (Klebanov, Witten):

$$G(r, r') = \lim_{z, z' \to 0} (zz')^{\alpha} G_B(r, z; r', z')$$

Holographic theory of a Fermi liquid

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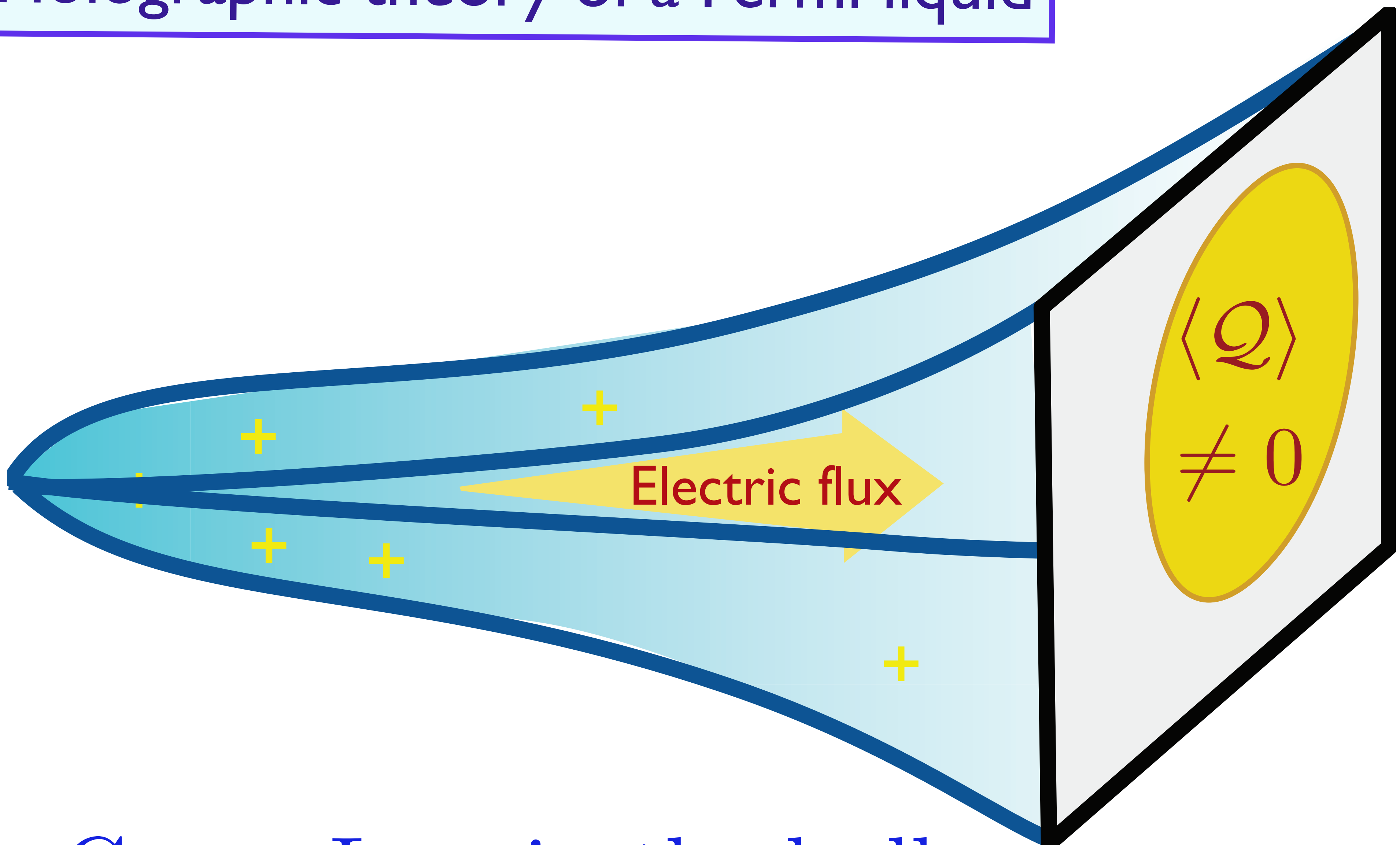

$$\text{Gauss Law in the bulk}$$
$$\Leftrightarrow \text{Luttinger theorem on the boundary}$$

In a confining FL phase, the metric terminates, the bulk charge equals the boundary charge, and the electric flux vanishes in the IR.

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The Fermi surface

This is the locus of zero energy singularities in momentum space in the two-point correlator of fermions carrying charge $\mathcal{Q}$.

$$G^{-1}_{\text{fermion}}(k = k_F, \omega = 0) = 0.$$

Excitations with $k < k_F$ are 'hole'-like (negative energy), and those with $k > k_F$ are 'particle'-like (positive energy), or vice-versa.

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Luttinger relation: Applies as long as the global U(1) symmetry associated with $\mathcal{Q}$ is unbroken. The total "volume (area)" $\mathcal{A}$ enclosed by the Fermi surface is equal to $\langle \mathcal{Q} \rangle$. Here $\langle \mathcal{Q} \rangle$ includes the charge carried by the bosons. This is a *key* constraint which allows extrapolation from weak to strong coupling.

Consider mixture of fermions f and bosons b.

$$\begin{aligned}\mathcal{L} &= f^\dagger \left(\partial_\tau - \frac{\nabla^2}{2m} - \mu\right) f \\ &+ b^\dagger \left(\partial_\tau - \frac{\nabla^2}{2m_b} - \mu_b\right) b + s|b|^2 - g\, b^\dagger f^\dagger f b + \ldots\end{aligned}$$

Consider mixture of fermions f and bosons b.
There is a $\mathrm{U}(1)\times\mathrm{U}_b(1)$ symmetry
and 2 conserved charges:

$$\mathcal{Q} = f^\dagger f$$
$$\mathcal{Q}_b = b^\dagger b$$

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The 2 symmetries imply 2 Luttinger constraints. However, bosons at non-zero density invariably Bose condense at $T = 0$, and so $\mathrm{U}_b(1)$ is broken. So there is only the single constraint on the f Fermi surface. This describes mixtures of ^{3}He and ^{4}He.

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Consider mixture of fermions f and bosons b.
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and 2 conserved charges:

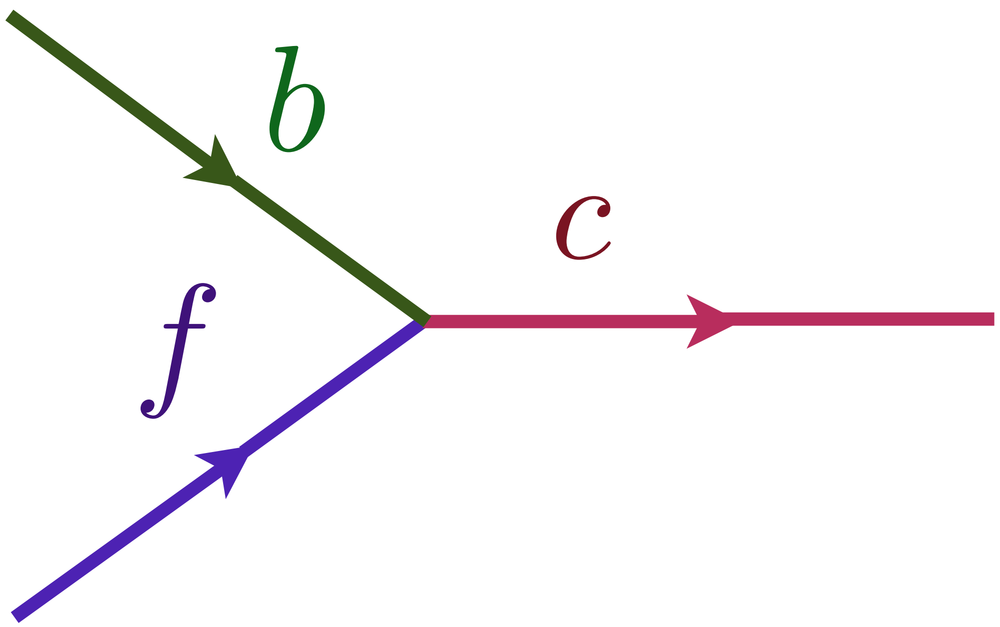
$$\mathcal{A} = \langle \mathcal{Q} \rangle$$

$$\mathcal{Q} = f^\dagger f$$
$$\mathcal{Q}_b = b^\dagger b$$

Superfluid: $\langle b \rangle \neq 0$
U$_b$(1) broken; U(1) unbroken

$$\begin{aligned} \mathcal{L} &= f^\dagger \left(\partial_\tau - \frac{\nabla^2}{2m} - \mu \right) f \\ &+ b^\dagger \left(\partial_\tau - \frac{\nabla^2}{2m_b} - \mu_b \right) b + s|b|^2 - g\, b^\dagger f^\dagger f b + \ldots \end{aligned}$$

Increase the coupling g until the boson, b, and fermion, f, can bind into a 'molecule', the fermion c.



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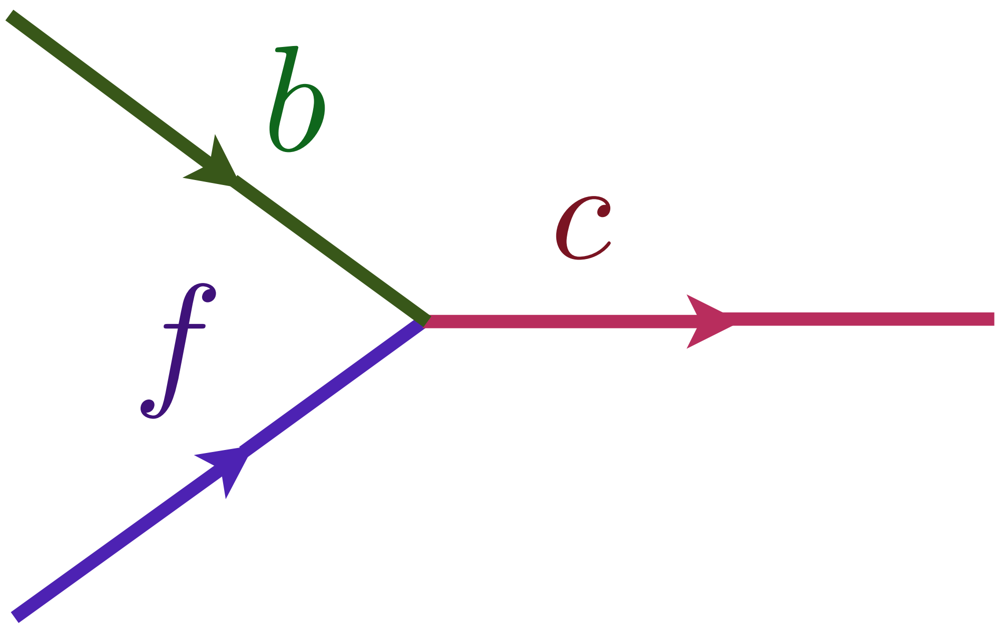
S. Powell, S. Sachdev, and H. P. Büchler, *Physical Review* B **72**, 024534 (2005)

Increase the coupling g until the boson, b, and fermion, f, can bind into a 'molecule', the fermion c.

Decouple the interaction between b and f by a fermion c



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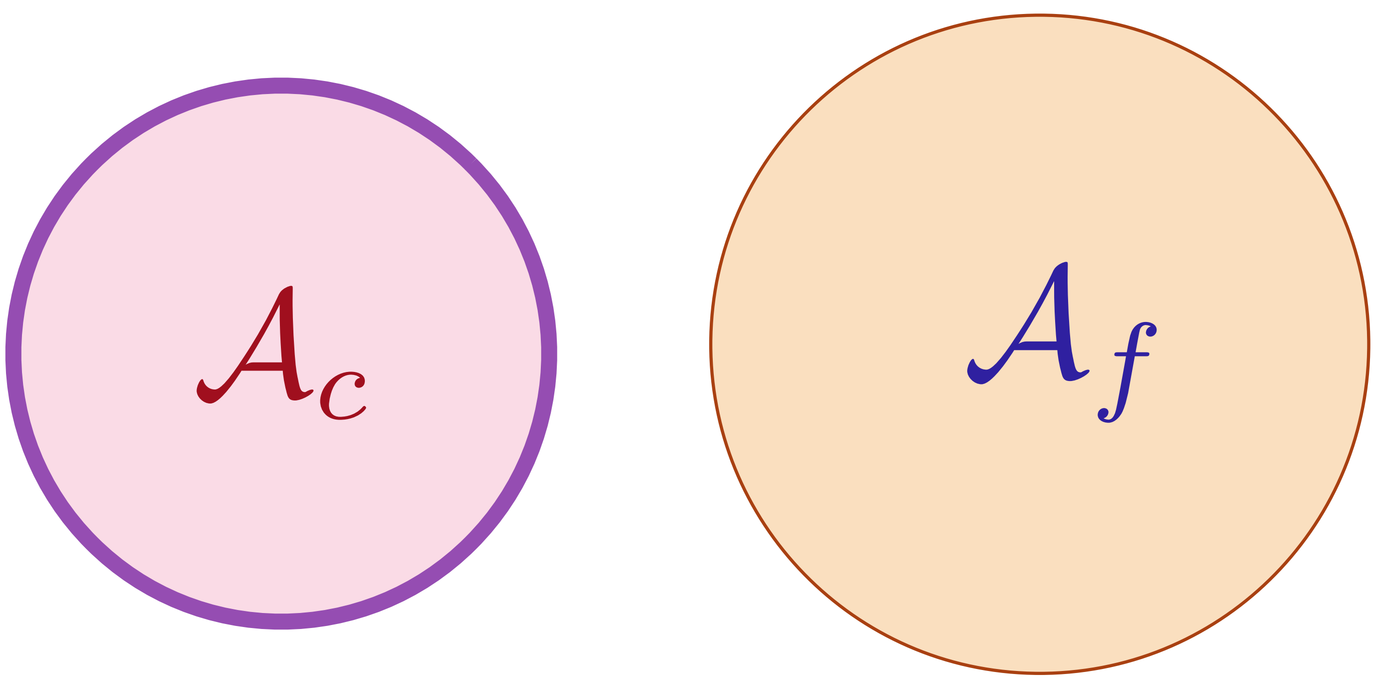
In a phase with $\mathrm{U}_b(1)$ unbroken, there is a Luttinger relation for each conserved U(1) charge. However, the boson, b cannot have a Fermi surface in its Green's function, and so there is no area associated with it, although the boson density is *included* in the Luttinger relation

$$\begin{aligned} \mathcal{A}_c + \mathcal{A}_f &= \left\langle f^\dagger f \right\rangle = \langle \mathcal{Q} \rangle \\ \mathcal{A}_c &= \left\langle b^\dagger b \right\rangle = \langle \mathcal{Q}_b \rangle \end{aligned}$$



The b bosons have bound with f fermions to form c "molecules"

S. Powell, S. Sachdev, and H. P. Büchler, *Physical Review* B **72**, 024534 (2005)
P. Coleman, I. Paul, and J. Rech, *Physical Review* B **72**, 094430 (2005)

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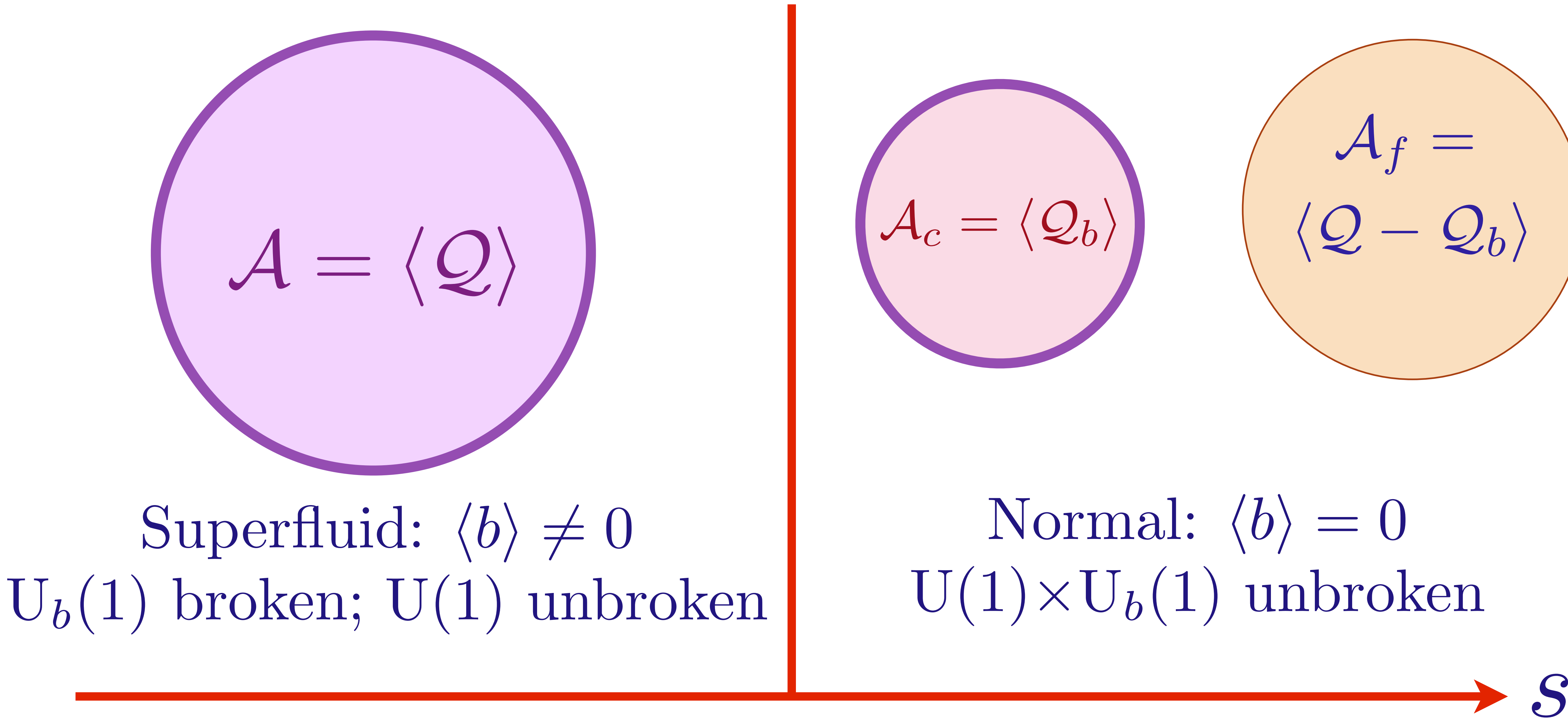
Phase diagram of boson-fermion mixture



$$
\begin{aligned}
\mathcal{L} &= f^\dagger \left(\partial_\tau - \frac{\nabla^2}{2m} - \mu \right) f \\
&+ b^\dagger \left(\partial_\tau - \frac{\nabla^2}{2m_b} - \mu_b \right) b + s|b|^2 - g\, b^\dagger f^\dagger f b + \ldots
\end{aligned}
$$

- Now gauge $\mathcal{Q} - \mathcal{Q}_b$ by a dynamic gauge field A_a. This leaves fermion c gauge-invariant

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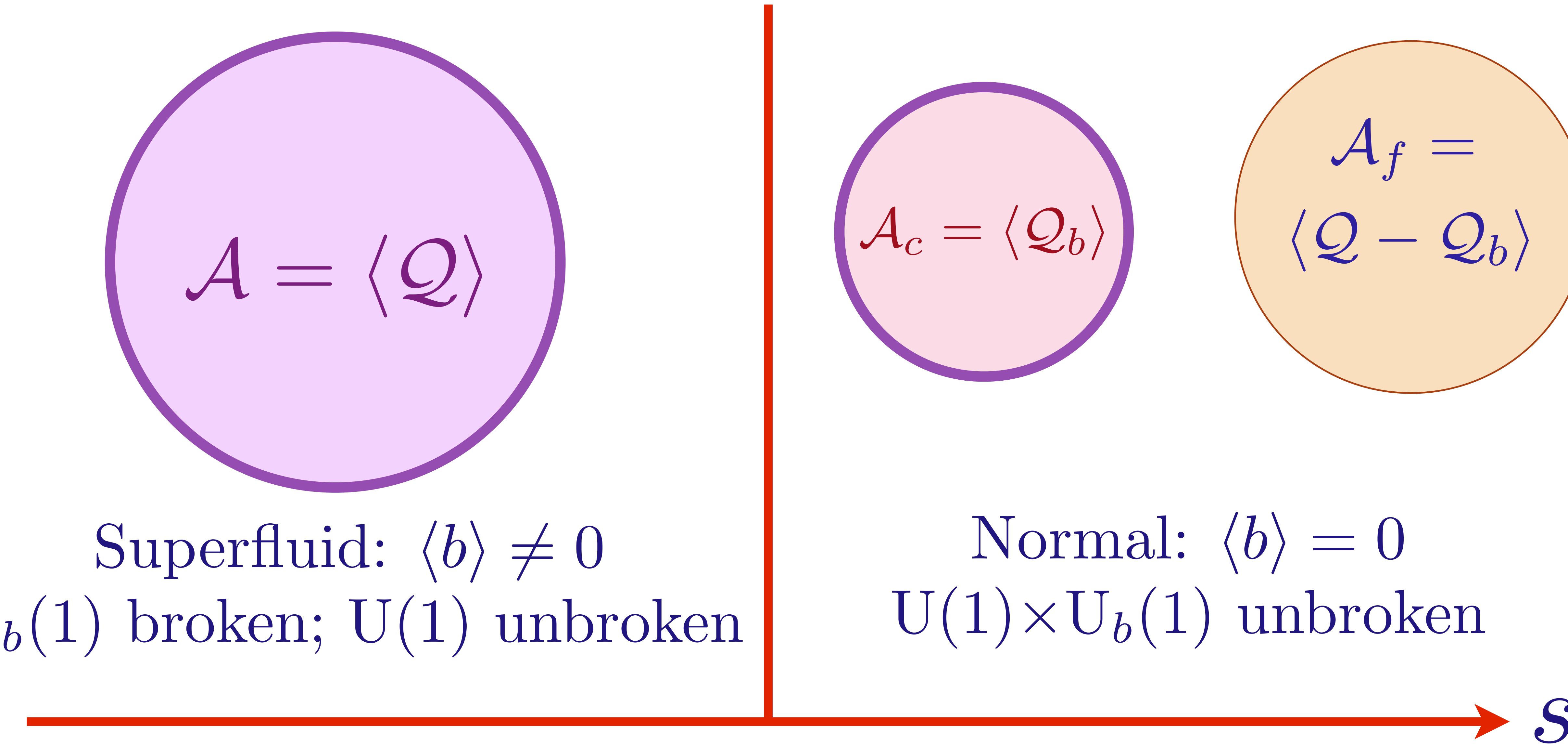
(Need a background neutralizing charge)

$$\begin{aligned}\mathcal{L} &= f^\dagger \left(\partial_\tau - i A_\tau - \frac{(\nabla - i\mathbf{A})^2}{2m} - \mu \right) f \\ &+ b^\dagger \left(\partial_\tau + i A_\tau - \frac{(\nabla + i\mathbf{A})^2}{2m_b} - \mu_b \right) b + s|b|^2 - g\, b^\dagger f^\dagger f b + \ldots\end{aligned}$$

S. Powell, S. Sachdev, and H. P. Büchler, *Physical Review* B **72**, 024534 (2005)
P. Coleman, I. Paul, and J. Rech, *Physical Review* B **72**, 094430 (2005)

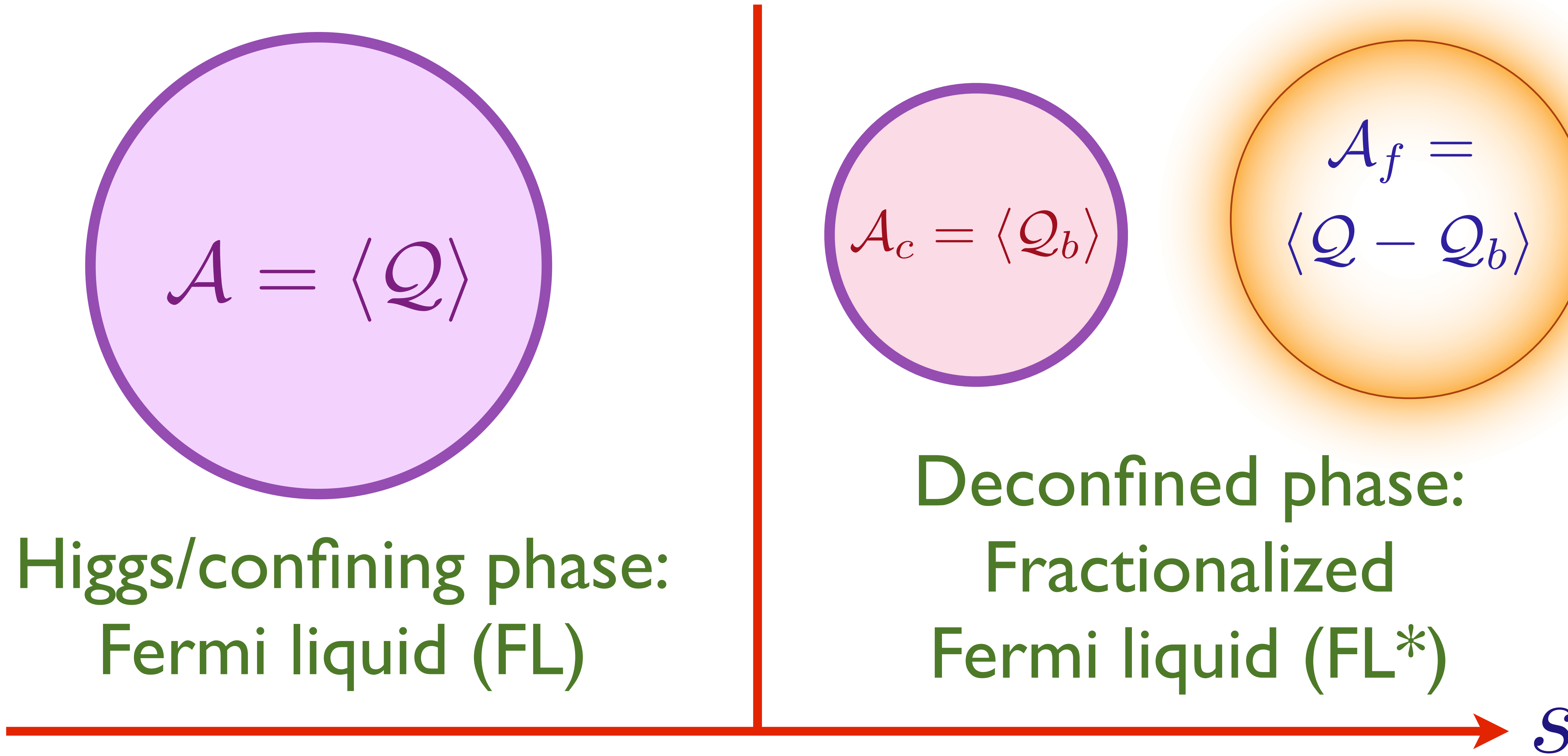
Phase diagram of boson-fermion mixture



$$
\begin{aligned}
\mathcal{L} &= f^\dagger \left(\partial_\tau - \frac{\nabla^2}{2m} - \mu \right) f \\
&+ b^\dagger \left(\partial_\tau - \frac{\nabla^2}{2m_b} - \mu_b \right) b + s|b|^2 - g\, b^\dagger f^\dagger f b + \ldots
\end{aligned}
$$

T. Senthil, M. Vojta, and S. Sachdev, *Physical Review* B **69**, 035111 (2004)
P. Coleman, I. Paul, and J. Rech, *Physical Review* B **72**, 094430 (2005)

Phase diagram of U(1) gauge theory



$$\begin{aligned} \mathcal{L} &= f^\dagger \left(\partial_\tau - iA_\tau - \frac{(\nabla - i\mathbf{A})^2}{2m} - \mu \right) f \\ &+ b^\dagger \left(\partial_\tau + iA_\tau - \frac{(\nabla + i\mathbf{A})^2}{2m_b} - \mu_b \right) b + s|b|^2 - g\, b^\dagger f^\dagger f b + \ldots \end{aligned}$$

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P. Coleman, I. Paul, and J. Rech, *Physical Review* B **72**, 094430 (2005)

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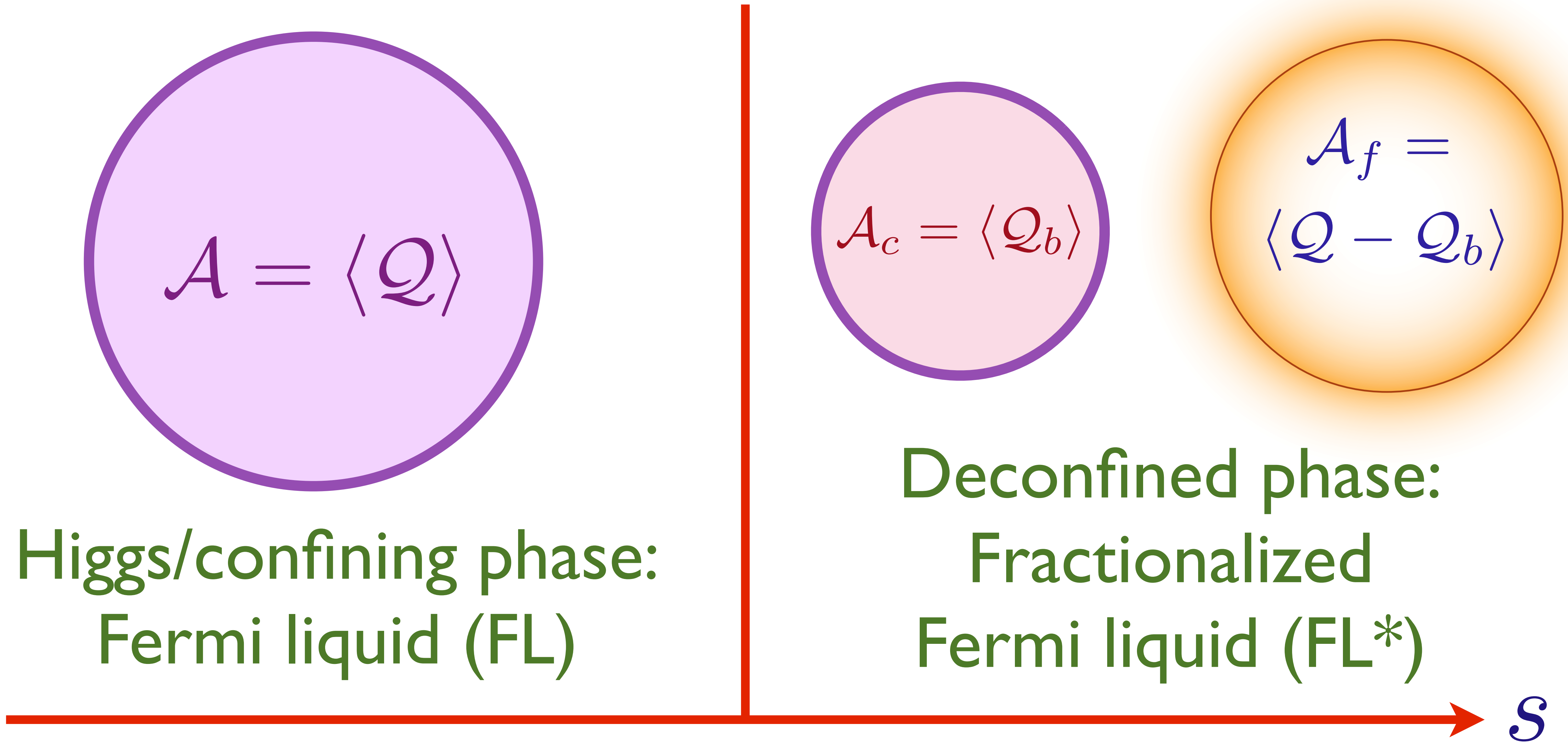

$$\mathcal{L} = f^\dagger \left(\partial_\tau - i A_\tau - \frac{(\nabla - i\mathbf{A})^2}{2m} - \mu \right) f$$
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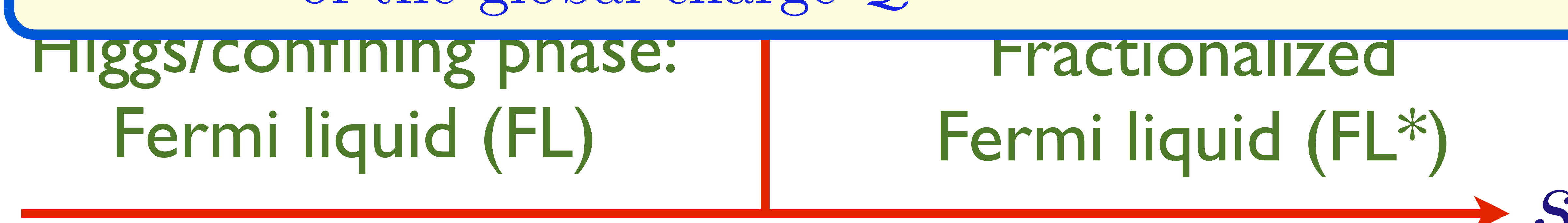
T. Senthil, M. Vojta, and S. Sachdev, *Physical Review* B **69**, 035111 (2004)
P. Coleman, I. Paul, and J. Rech, *Physical Review* B **72**, 094430 (2005)

Phase diagram of U(1) gauge theory

- FL phase: Fermi surface of gauge-neutral fermions encloses total global charge Q
- FL* phase: Fermi surface of gauge neutral fermions encloses only part of the global charge Q



$$\mathcal{L} = f^\dagger \left(\partial_\tau - iA_\tau - \frac{(\nabla - i\mathbf{A})^2}{2m} - \mu \right) f$$
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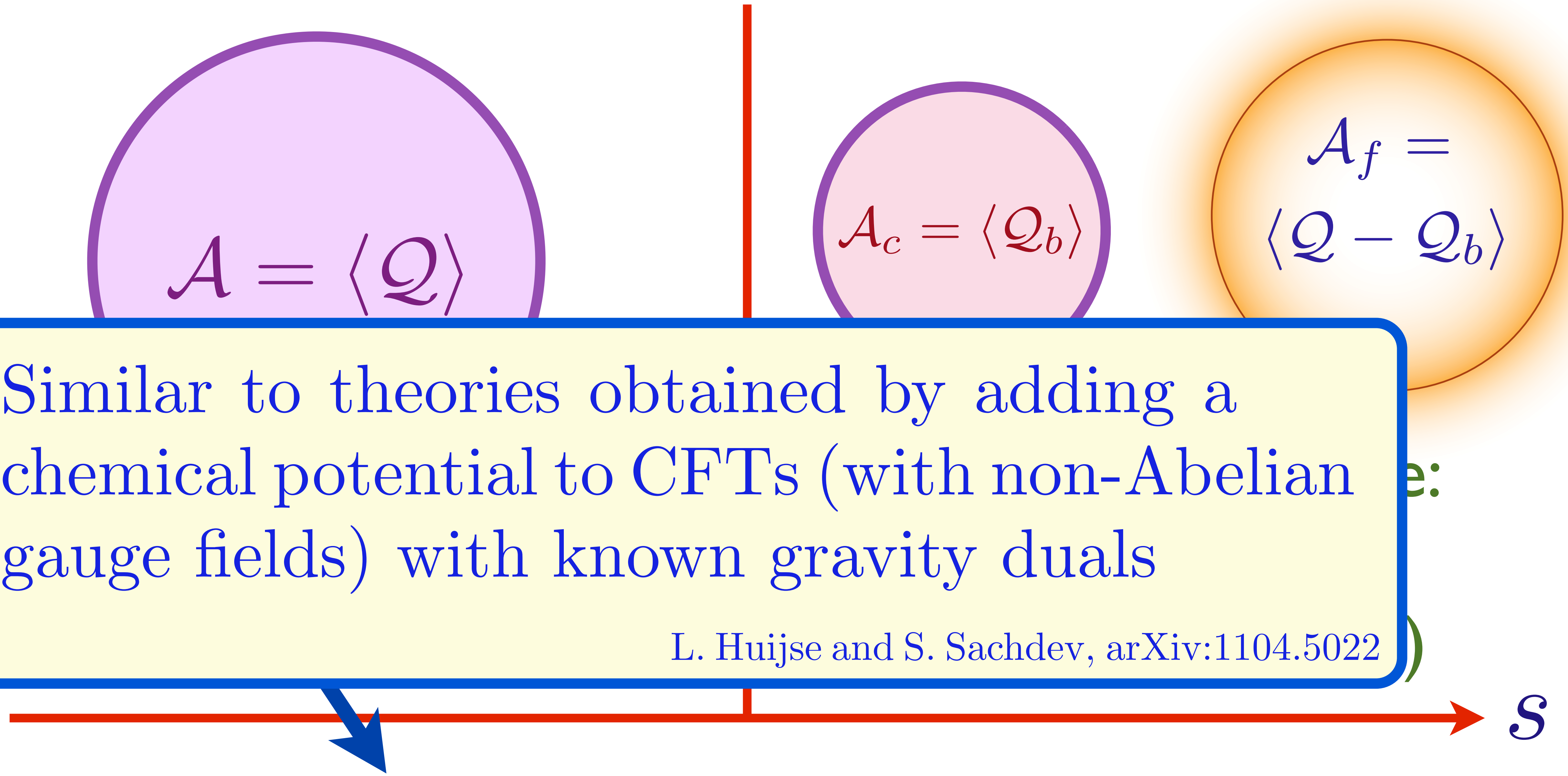

Similar to theories obtained by adding a chemical potential to CFTs (with non-Abelian gauge fields) with known gravity duals

L. Huijse and S. Sachdev, arXiv:1104.5022

$$\mathcal{L} = f^\dagger \left(\partial_\tau - i A_\tau - \frac{(\nabla - i\mathbf{A})^2}{2m} - \mu \right) f$$
$$+ \; b^\dagger \left(\partial_\tau + i A_\tau - \frac{(\nabla + i\mathbf{A})^2}{2m_b} - \mu_b \right) b + s|b|^2 - g\, b^\dagger f^\dagger f b + \ldots$$

Compressible quantum matter

Conjecture: All compressible states which preserve translational and global U(1) symmetries must have FERMI SURFACES, *but they are not necessarily Fermi liquids.*

- Such states obey the Luttinger relation

$$\sum_{\ell} q_{\ell} \mathcal{A}_{\ell} = \langle \mathcal{Q} \rangle,$$

where the ℓ'th Fermi surface has fermionic quasiparticles with global U(1) charge q_ℓ and encloses area $\mathcal{A}_\ell$.

- Non-Fermi liquids have quasiparticles coupled to deconfined gauge fields (or gapless bosonic modes at quantum critical points).

Compressible quantum matter

Conventional phases

1. Holographic theory of the Fermi liquid (FL)

Exotic phases

1. Continuum models with gauge theories: the fractionalized Fermi liquid (FL*)

2. Holographic approach

3. Connections to models and experiments on the heavy fermion compounds and the cuprate superconductors

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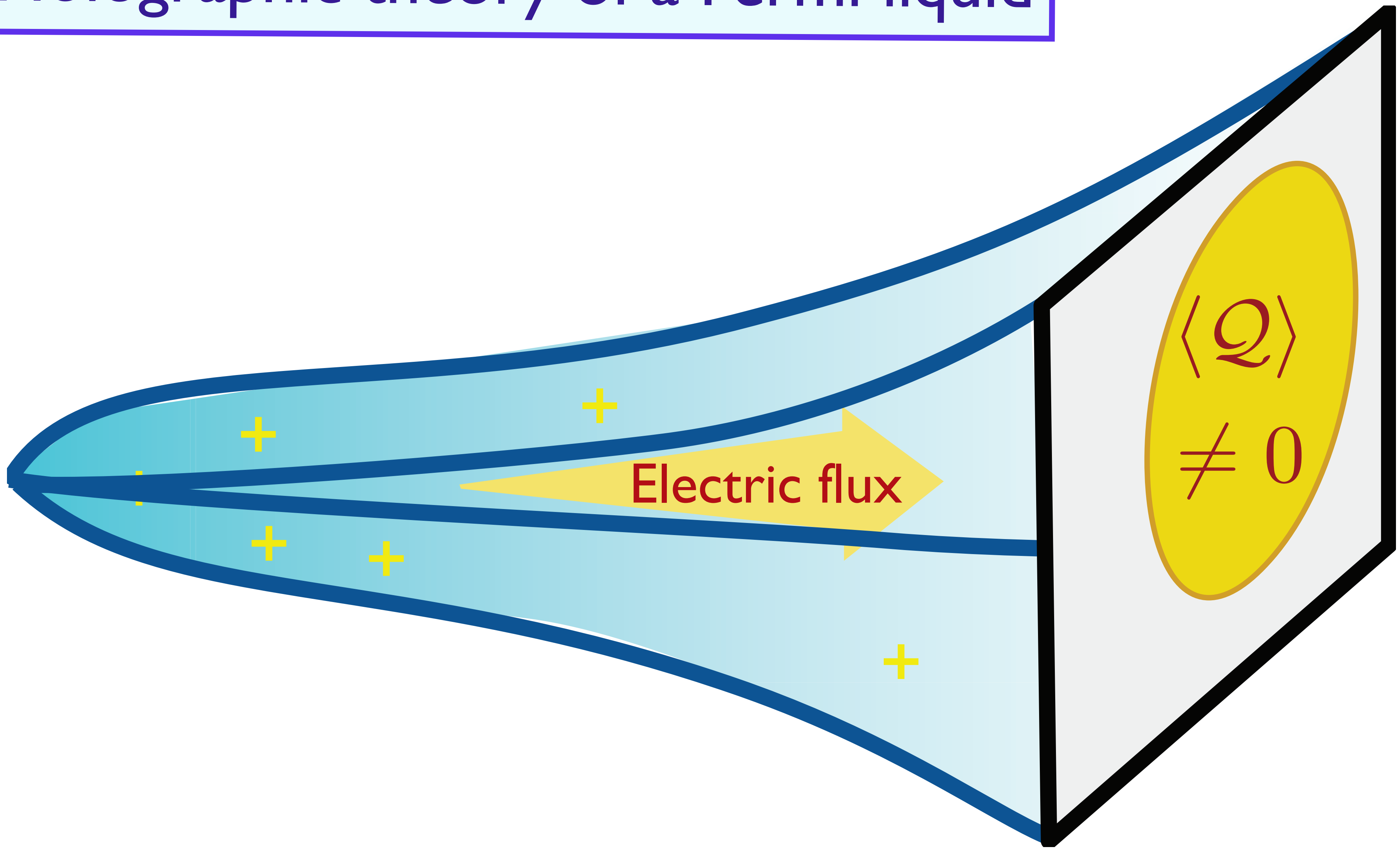
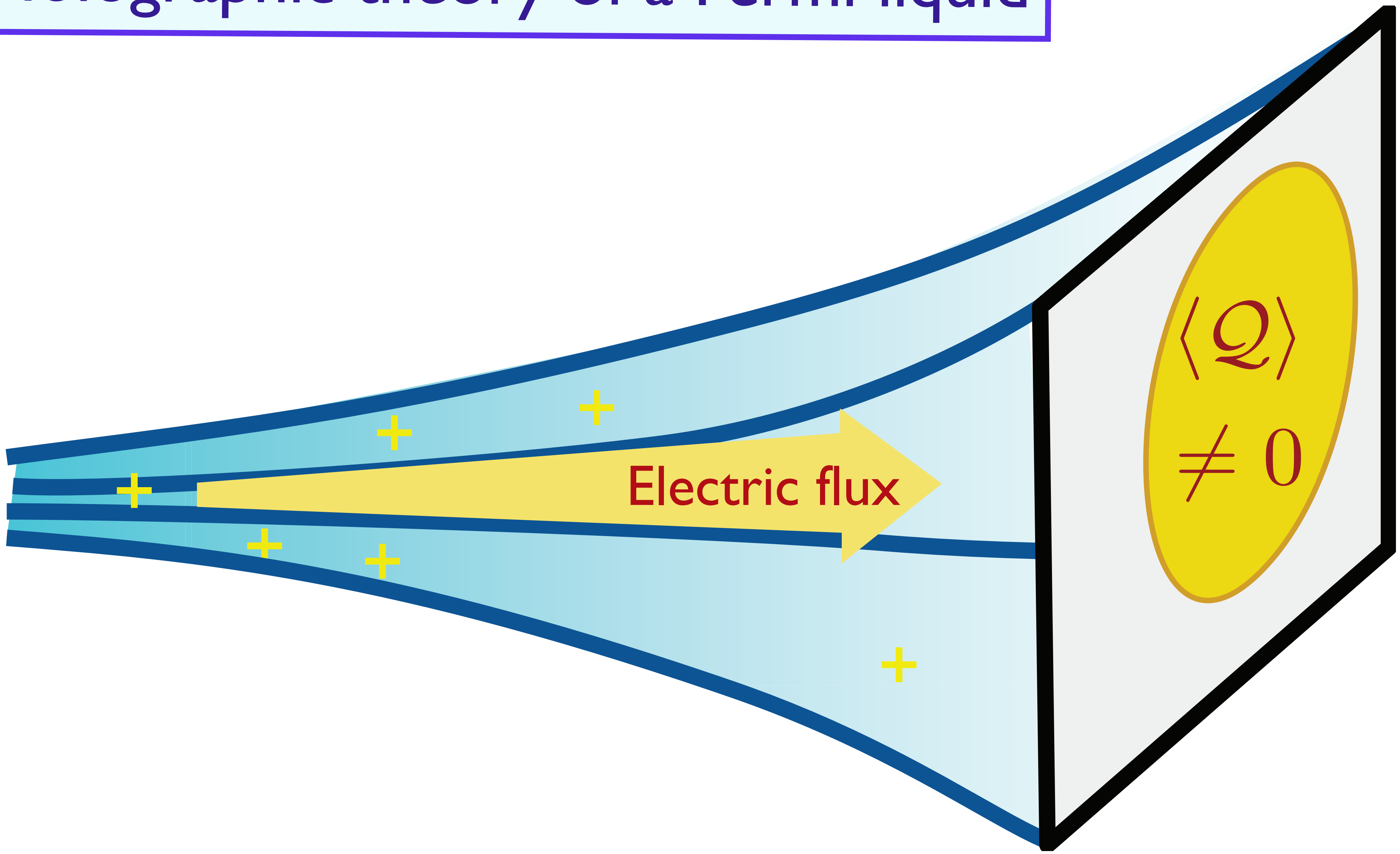
Holographic theory of a Fermi liquid

S. Sachdev
arXiv:1107.5321



In a confining FL phase, the metric terminates, the bulk charge equals the boundary charge, and the electric flux vanishes in the IR.

Holographic theory of a Fermi liquid



In a deconfined FL* phase, the metric extends to infinity (representing critical IR modes), and part of the electric flux "leaks out".

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Compressible quantum matter

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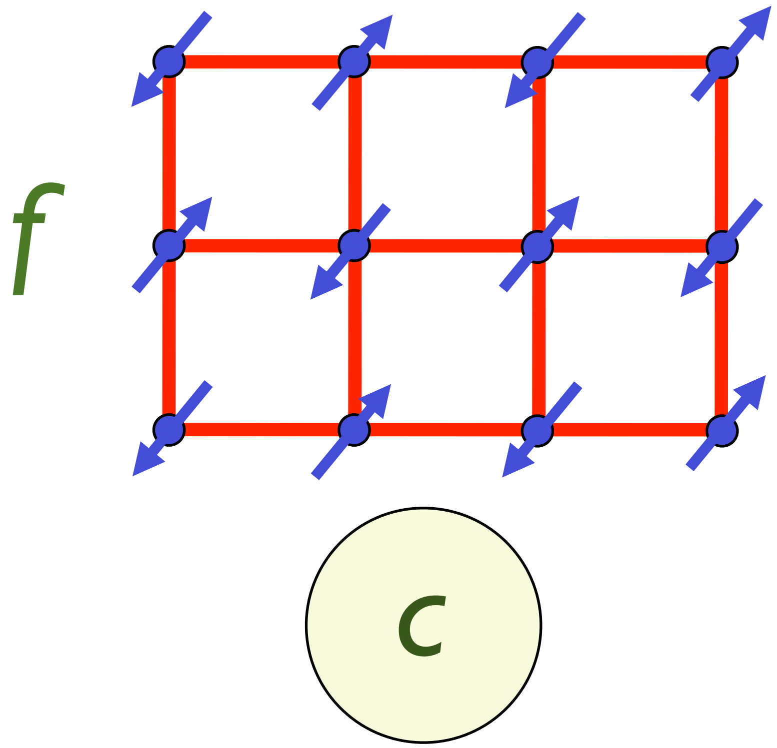
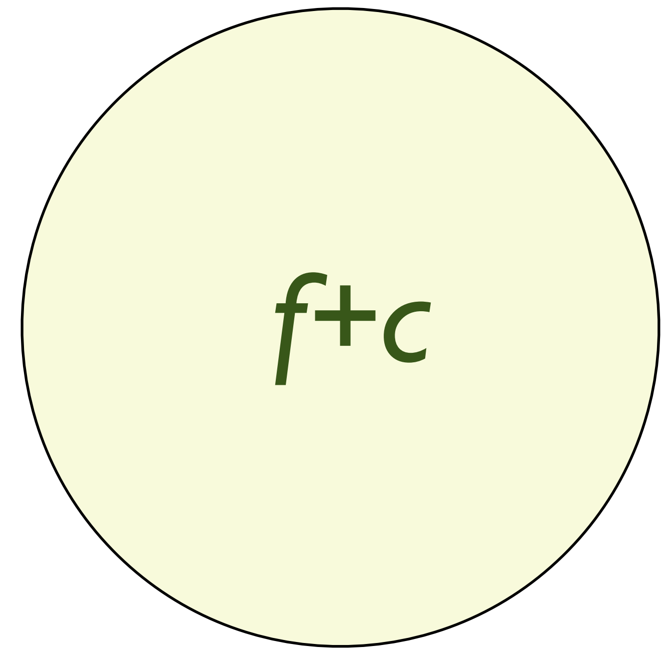
Magnetic order and the heavy Fermi liquid in the Kondo lattice



$$\langle \vec{\varphi} \rangle \neq 0$$

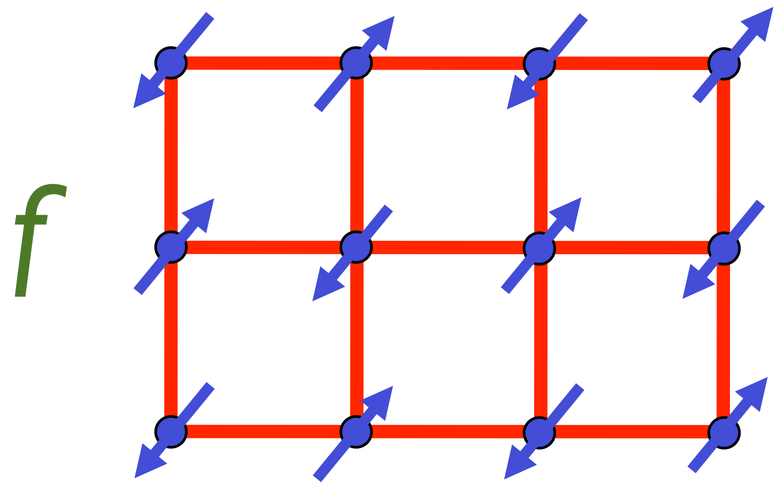
Magnetic Metal:
f-electron moments
and
c-conduction electron
Fermi surface

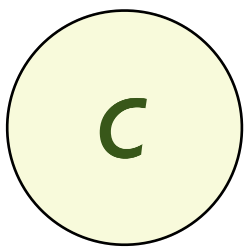


$$\langle \vec{\varphi} \rangle = 0$$

Heavy Fermi liquid
with “large” Fermi
surface of
hydridized *f* and
c-conduction
electrons

Separating onset of SDW order and the heavy Fermi liquid in the Kondo lattice





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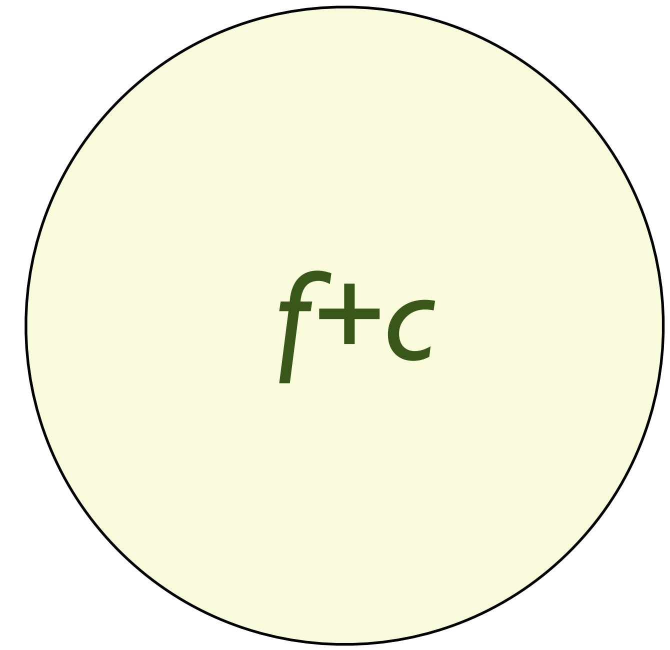

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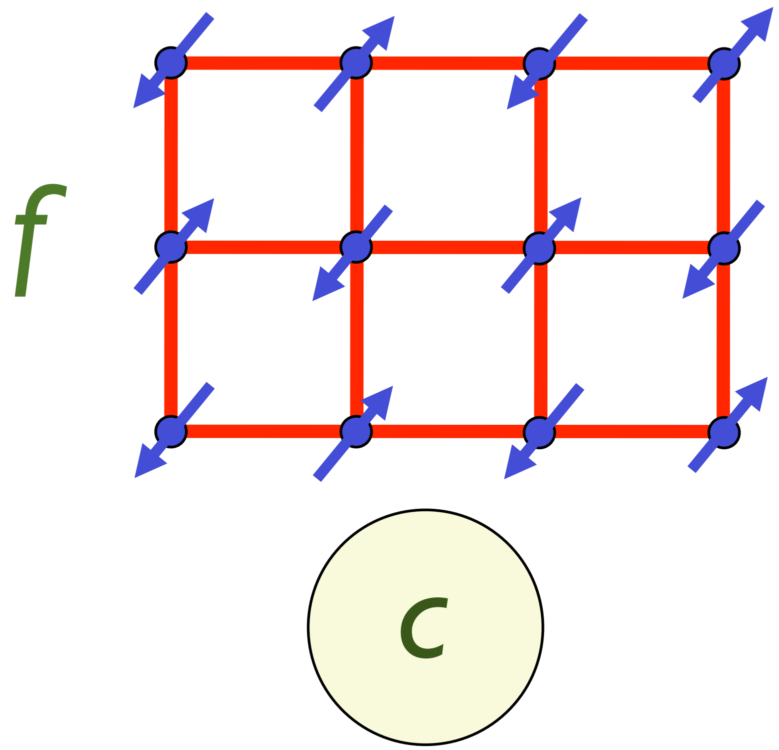

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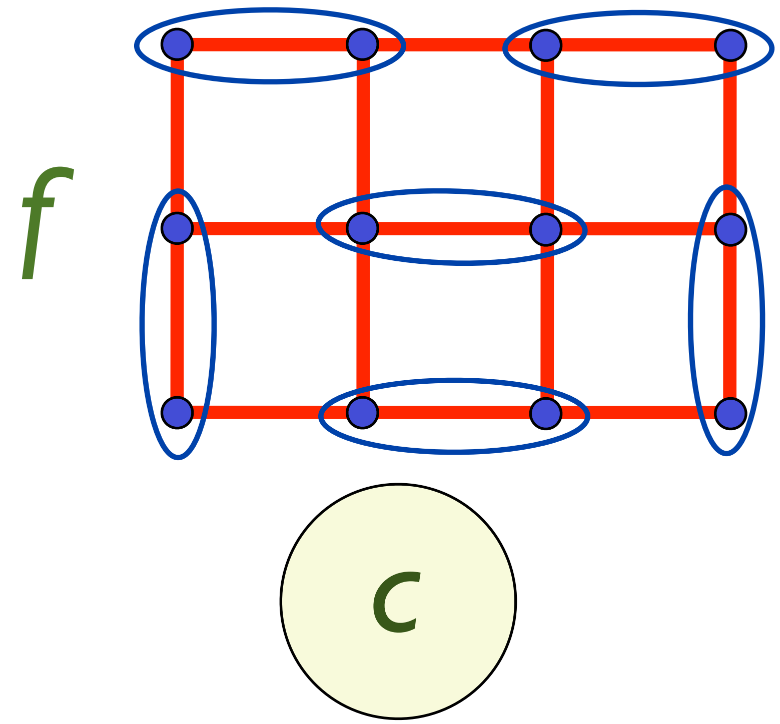
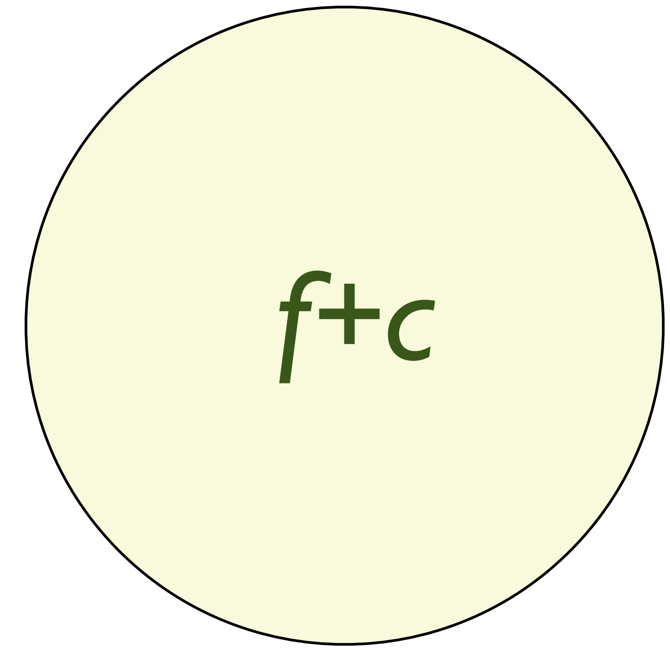


$$\langle \vec{\varphi} \rangle = 0$$

Conduction electron
Fermi surface
and
spin-liquid of
f-electrons



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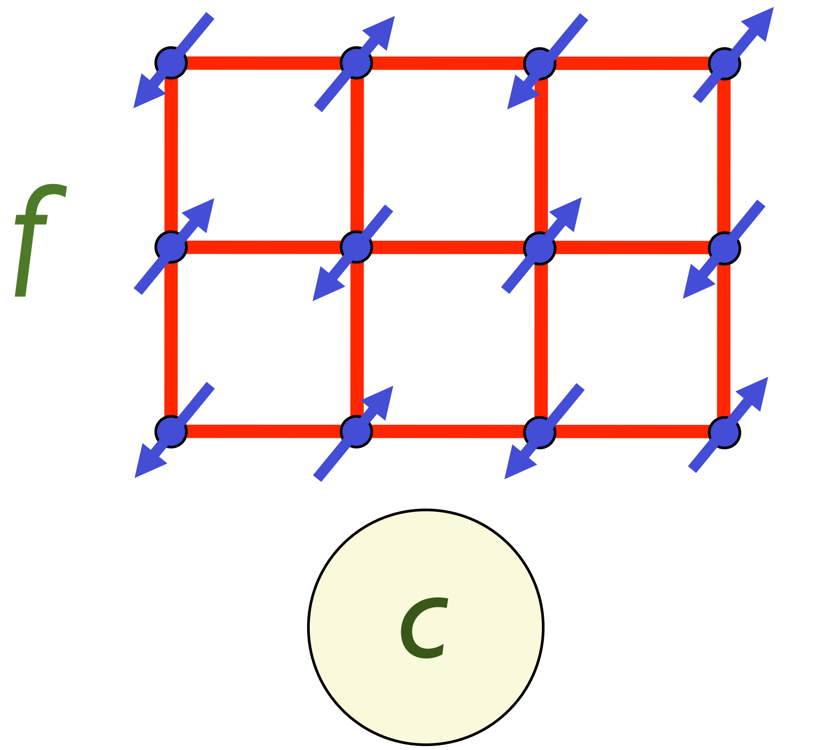

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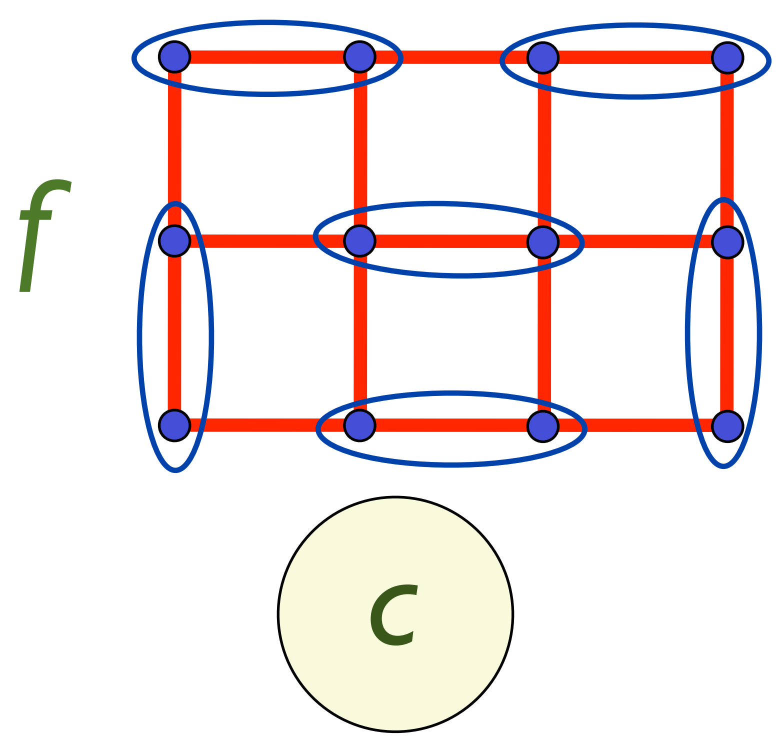
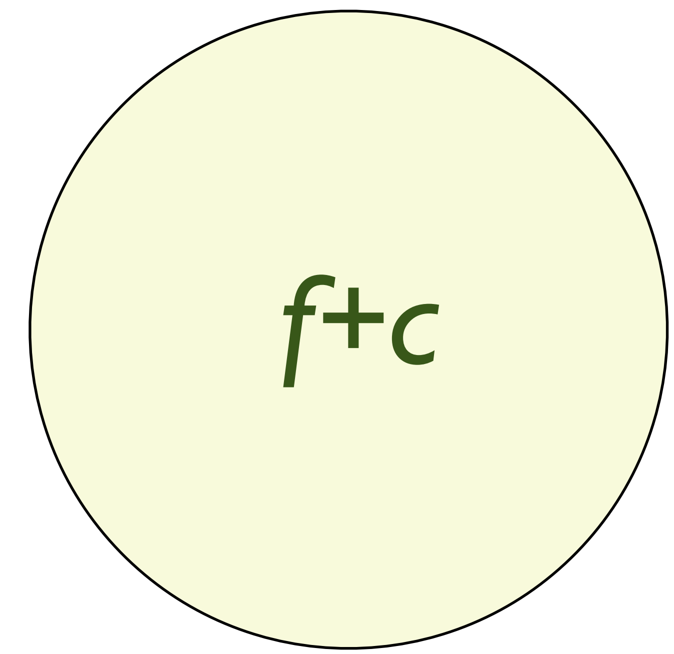


$\langle \vec{\varphi} \rangle = 0$

Fractionalized Fermi liquid (FL*) phase with no symmetry breaking and "small" Fermi surface

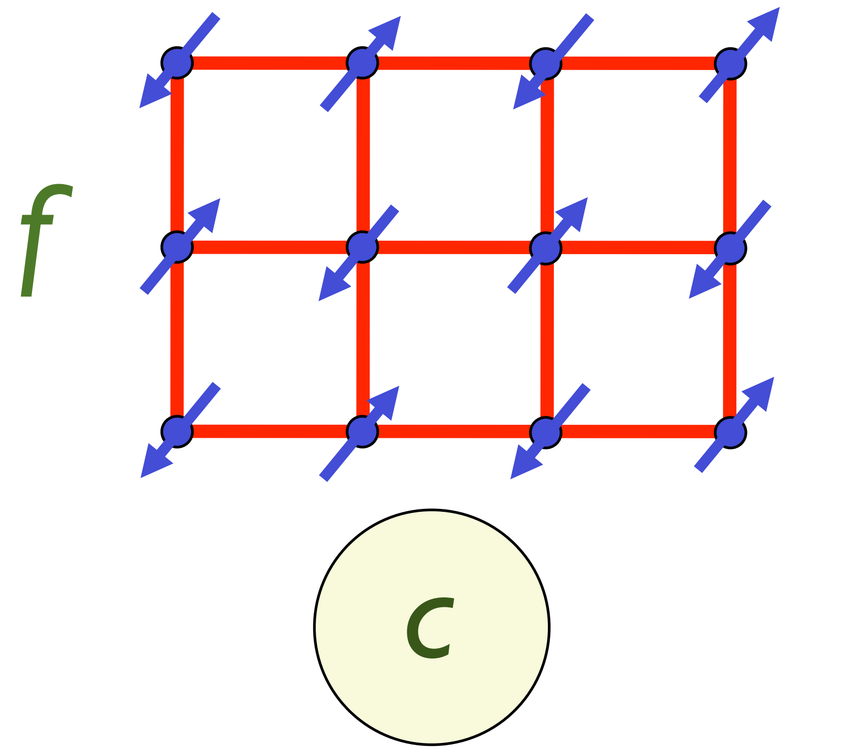


$\langle \vec{\varphi} \rangle = 0$

Heavy Fermi liquid with "large" Fermi surface of hydridized *f* and *c*-conduction electrons

T. Senthil, S. Sachdev, and M. Vojta, *Phys. Rev. Lett.* **90**, 216403 (2003)

Separating onset of SDW order and the heavy Fermi liquid in the Kondo lattice



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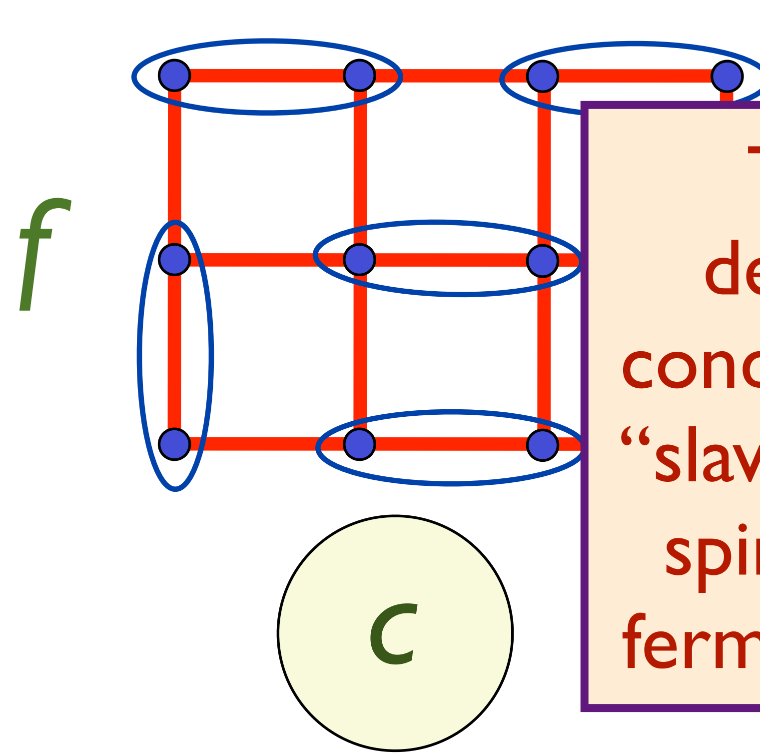

Transition described by condensation of a "slave boson" in a spin liquid with fermionic spinons

$\langle \vec{\varphi} \rangle = 0$

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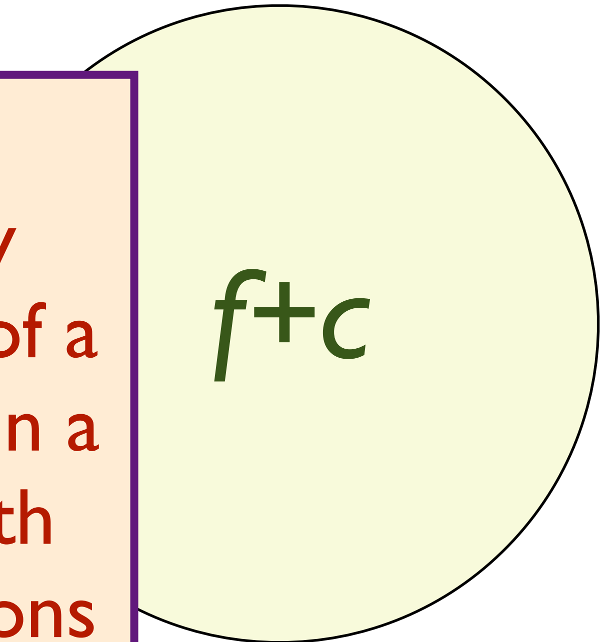

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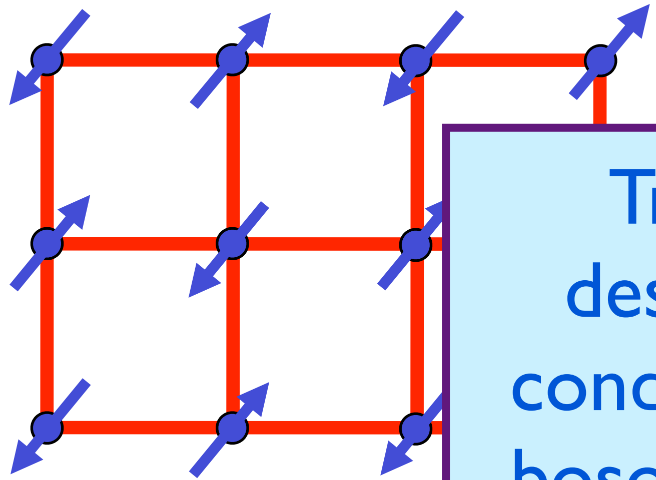
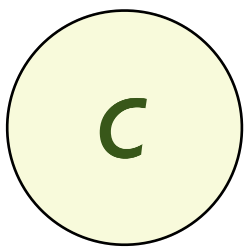
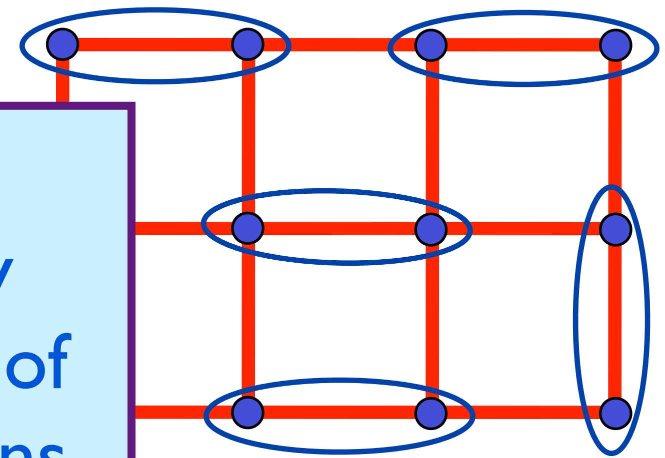
f

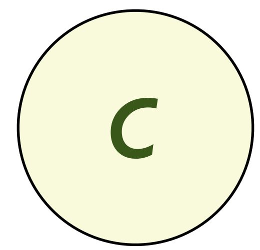


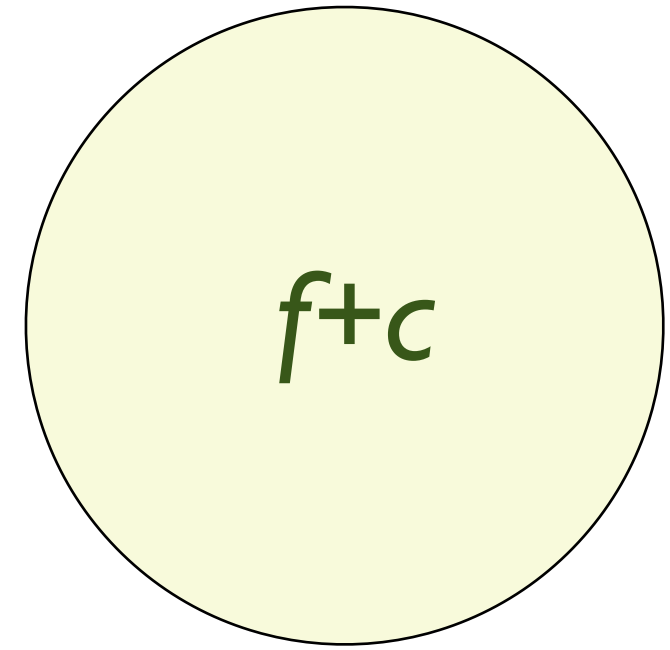


Transition described by condensation of bosonic spinons

T. Grover and T. Senthil, *Phys. Rev. B* **81**, 205102 (2010)







$\langle \vec{\varphi} \rangle \neq 0$

Magnetic Metal: *f*-electron moments and *c*-conduction electron Fermi surface

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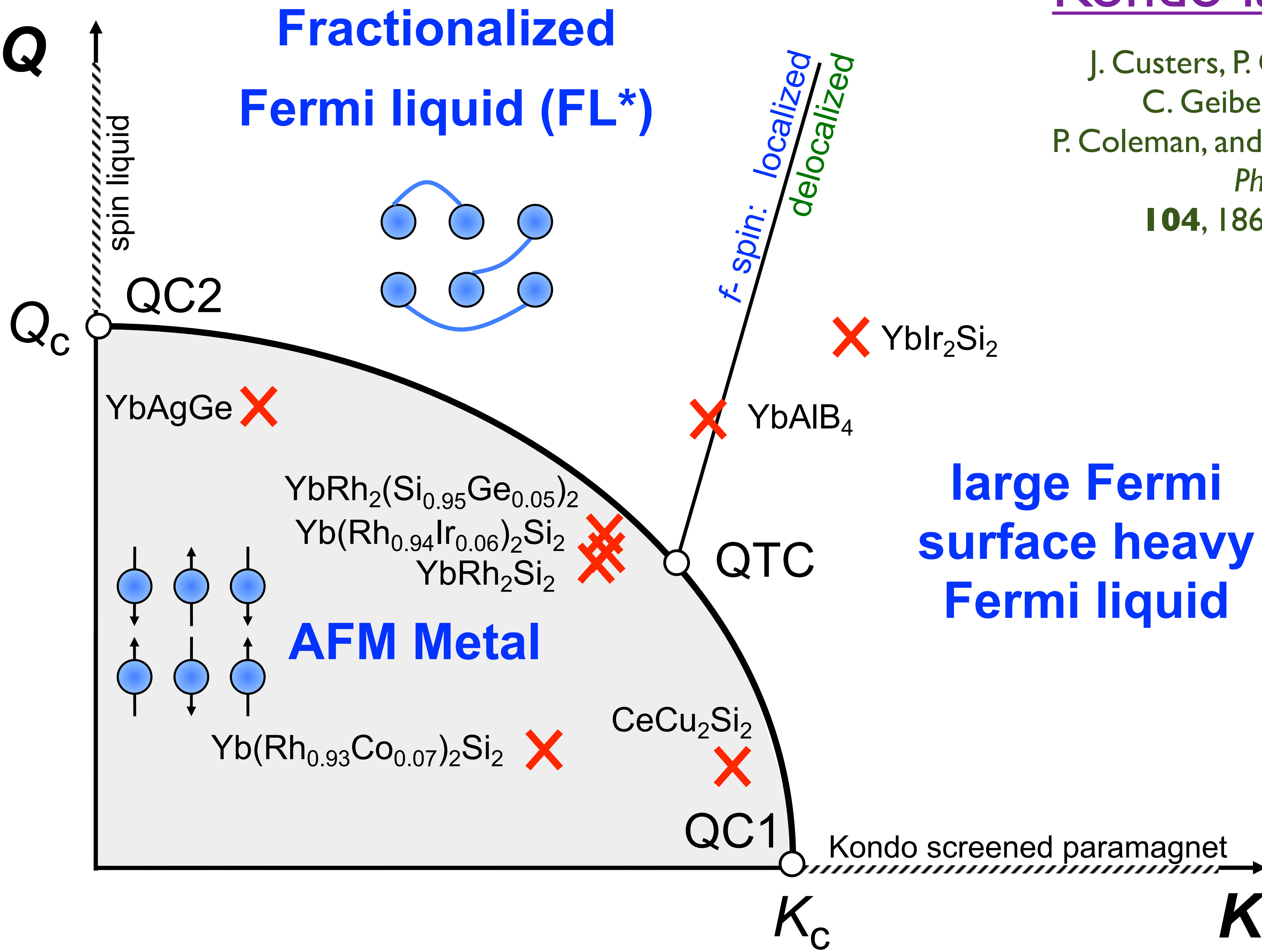
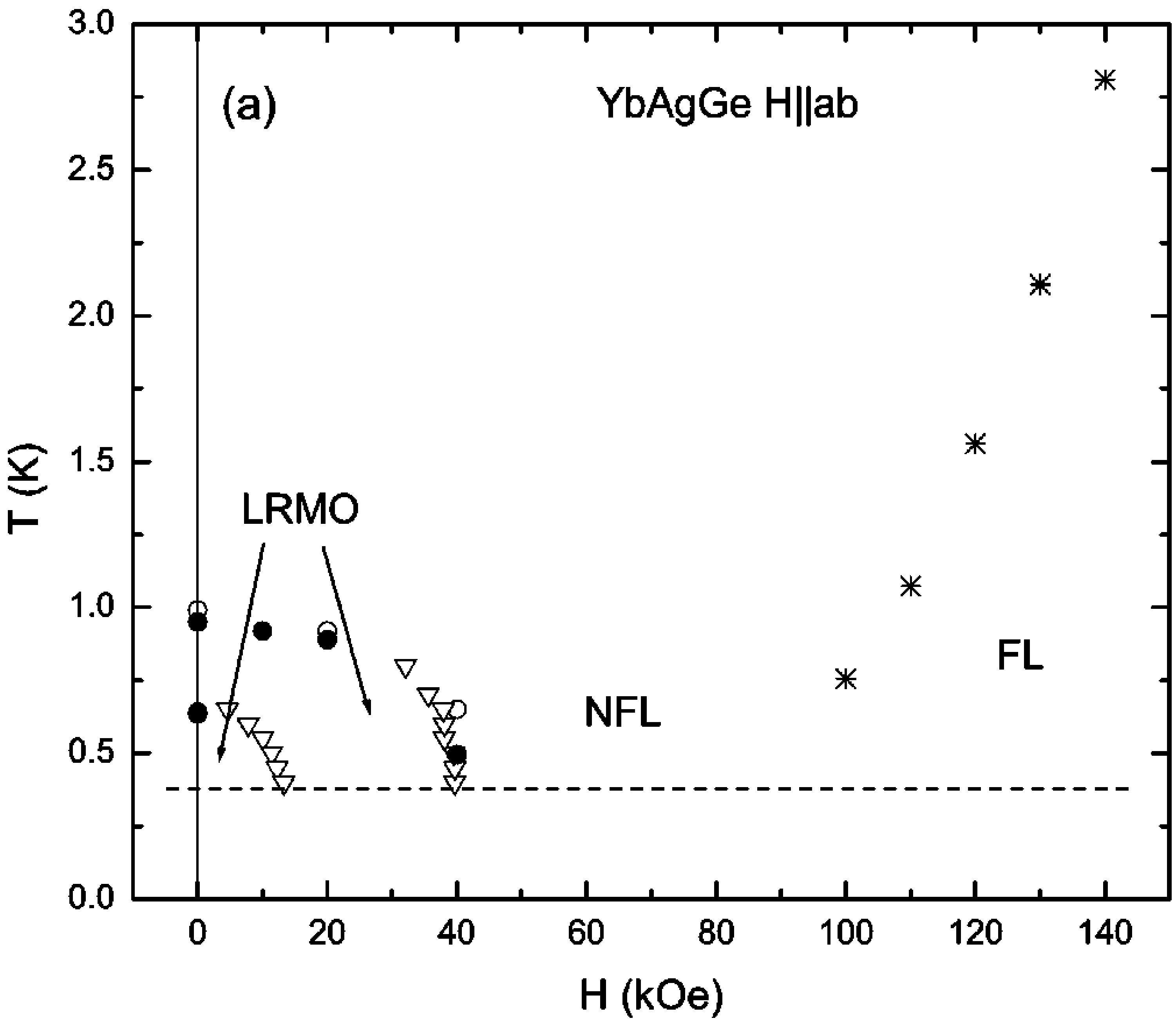
Experimental perpective on same phase diagrams of Kondo lattice

J. Custers, P. Gegenwart, C. Geibel, F. Steglich, P. Coleman, and S. Paschen, *Phys. Rev. Lett.* **104**, 186402 (2010)

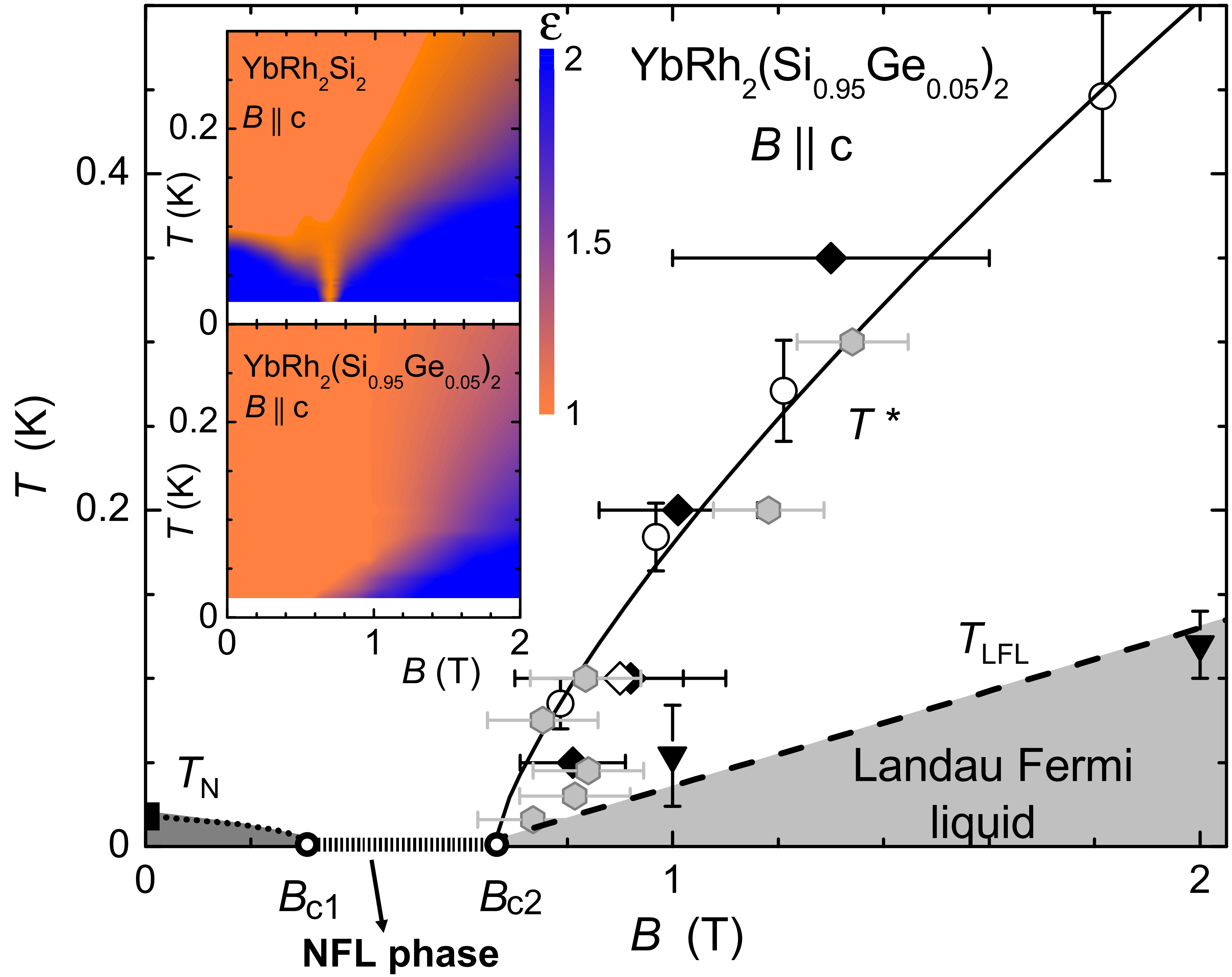




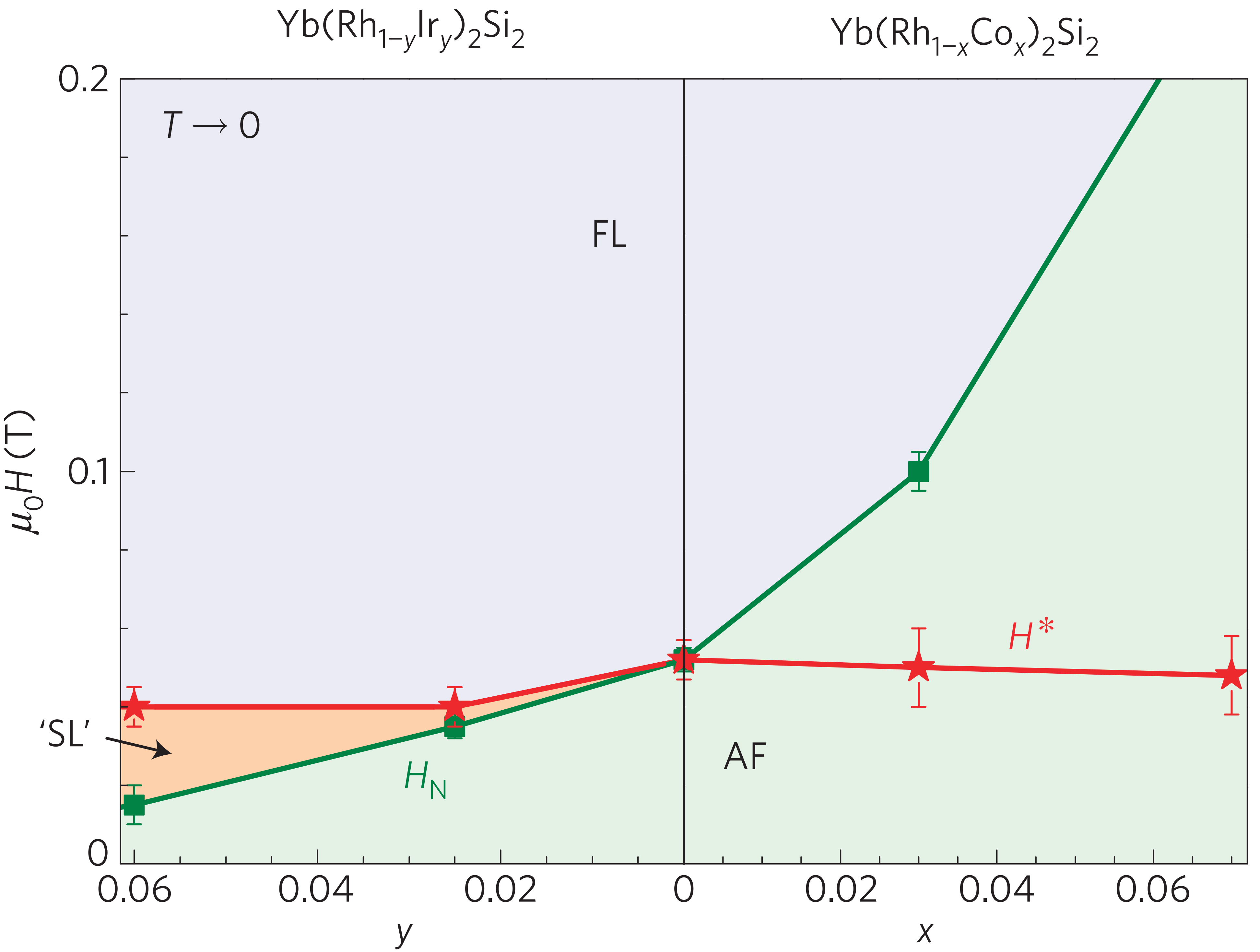
PHYSICAL REVIEW B **69**, 014415 (2004)

Magnetic field induced non-Fermi-liquid behavior in YbAgGe single crystals

S. L. Bud'ko,[1] E. Morosan,[1,2] and P. C. Canfield[1,2]



J. Custers, P. Gegenwart, C. Geibel, F. Steglich, P. Coleman, and S. Paschen, *Phys. Rev. Lett.* **104**, 186402 (2010)

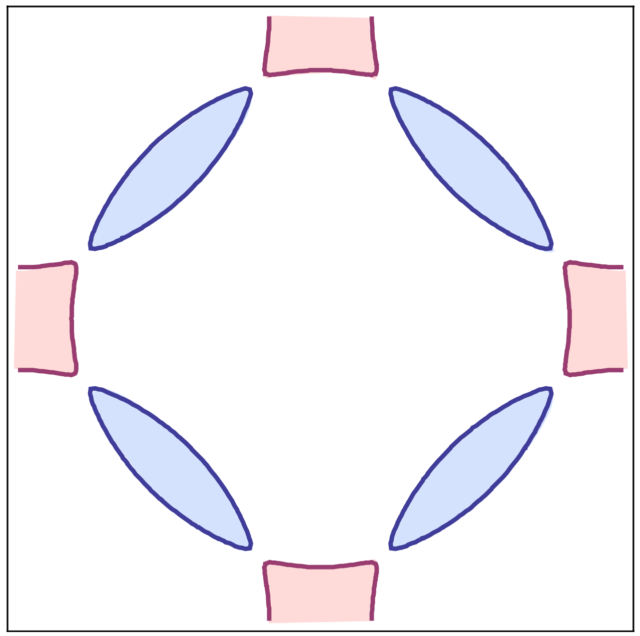


Detaching the antiferromagnetic quantum critical point from the Fermi-surface reconstruction in $YbRh_2Si_2$

Nature Physics 5, 465 (2009)

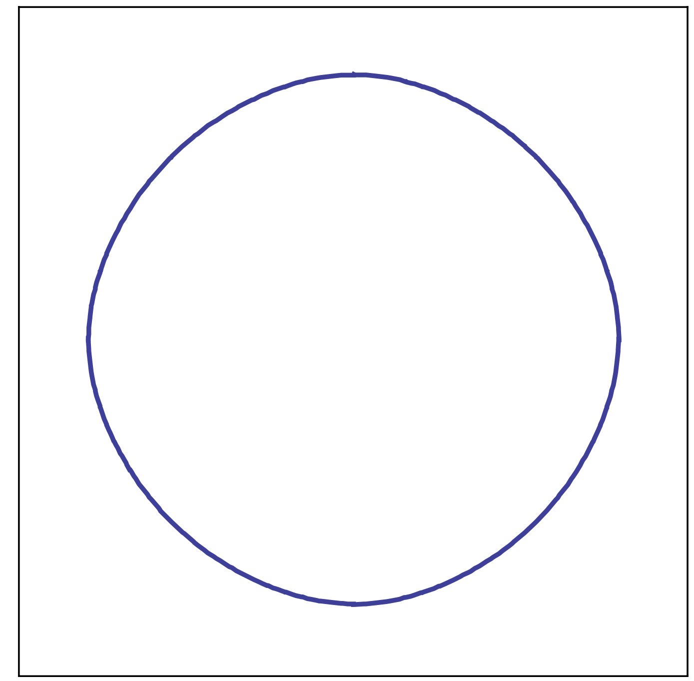
S. Friedemann[1]*, T. Westerkamp[1], M. Brando[1], N. Oeschler[1], S. Wirth[1], P. Gegenwart[1,2], C. Krellner[1], C. Geibel[1] and F. Steglich[1]*

Fermi surface reconstruction in a single band model



$\langle \vec{\varphi} \rangle \neq 0$

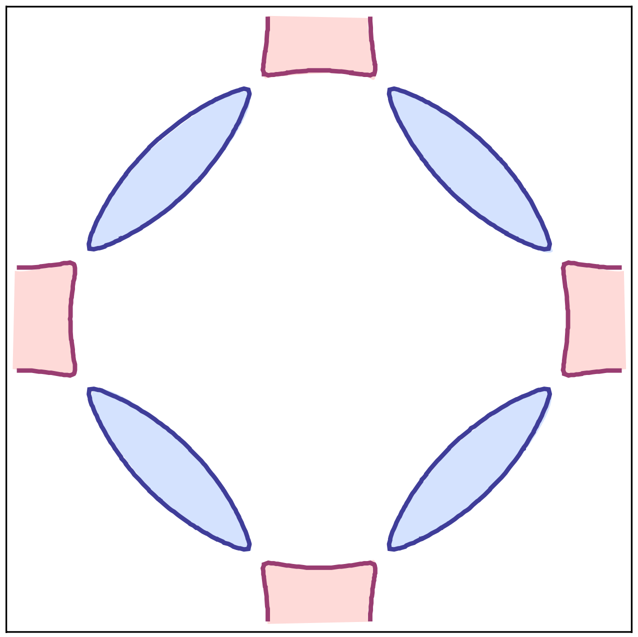
Metal with electron and hole pockets



$\langle \vec{\varphi} \rangle = 0$

Metal with “large” Fermi surface

Separating onset of SDW order and Fermi surface reconstruction



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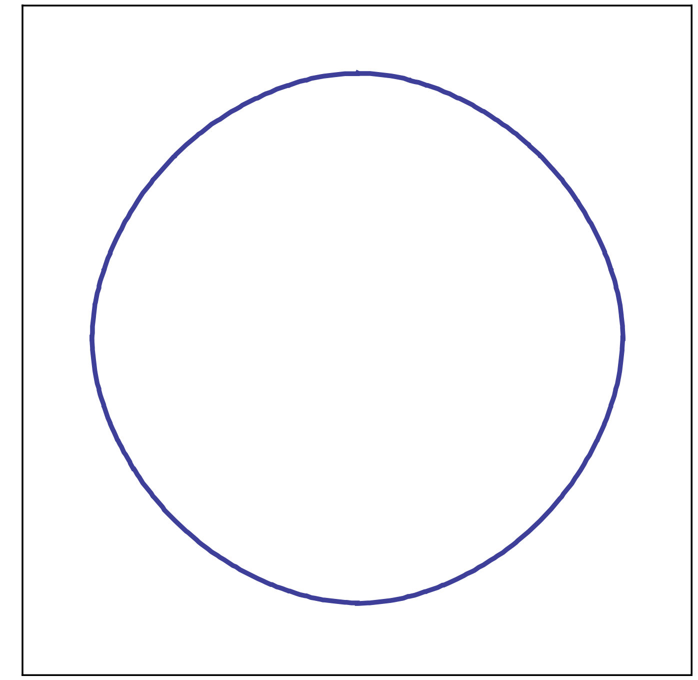

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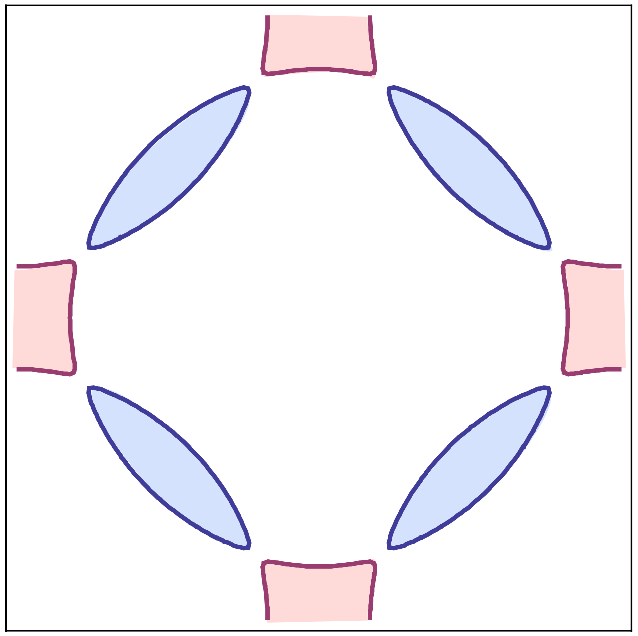
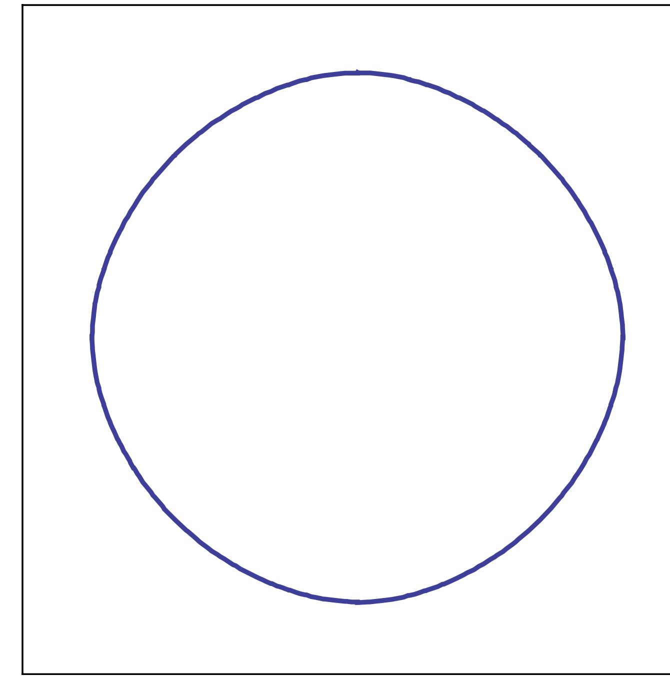


Electron and/or hole Fermi pockets form in "local" SDW order, but quantum fluctuations destroy long-range SDW order



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$\langle \vec{\varphi} \rangle = 0$

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Fractionalized Fermi liquid (FL*) phase with no symmetry breaking and "small" Fermi surface

Metal with "large" Fermi surface

Y. Qi and S. Sachdev, *Physical Review B* **81**, 115129 (2010); M. Punk and S. Sachdev, to appear;
see also T. C. Ribeiro and X.-G. Wen, *Physical Review B* **74**, 155113 (2006)

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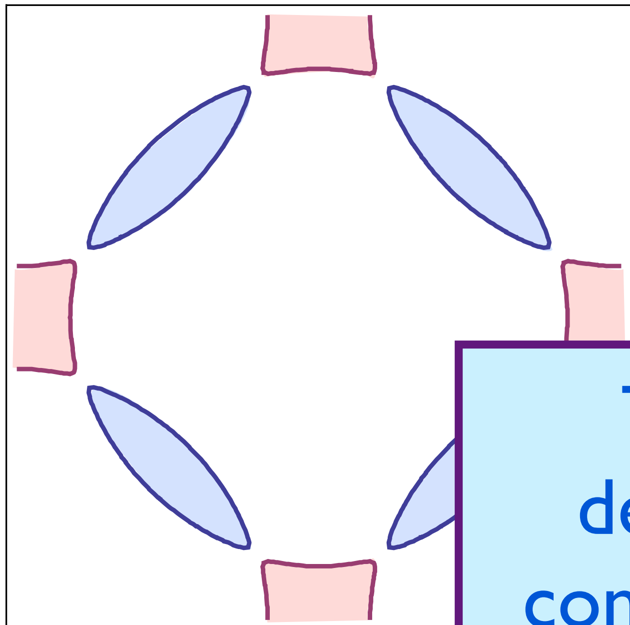

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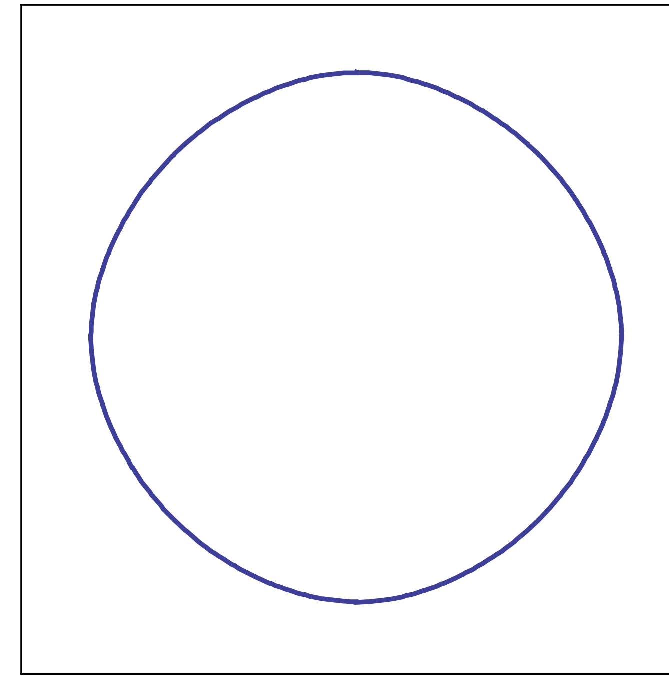

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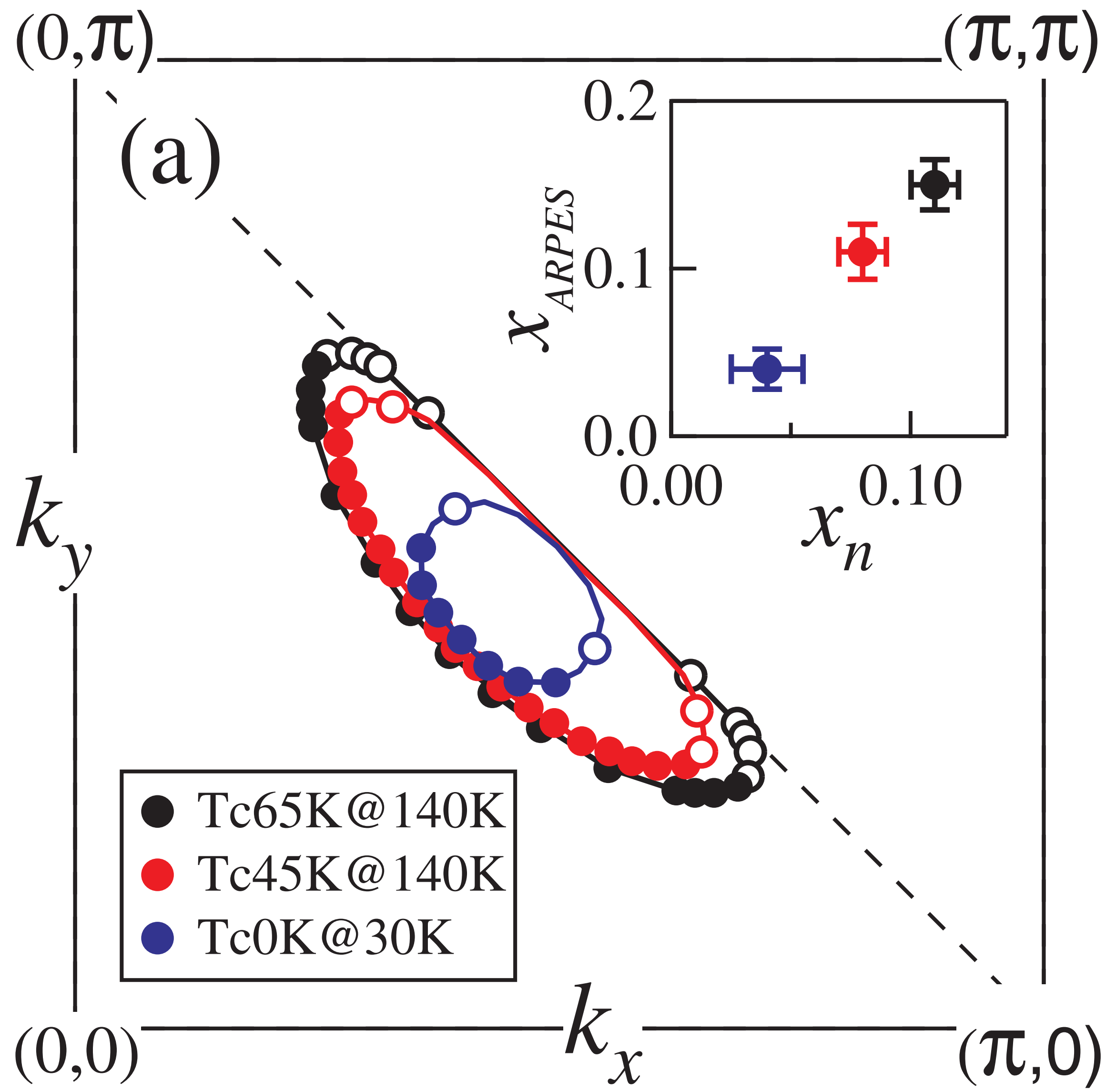

PRL **107,** 047003 (2011) PHYSICAL REVIEW LETTERS week ending 22 JULY 2011

Reconstructed Fermi Surface of Underdoped $Bi_2Sr_2CaCu_2O_{8+\delta}$ Cuprate Superconductors

H.-B. Yang,[1] J. D. Rameau,[1] Z.-H. Pan,[1] G. D. Gu,[1] P. D. Johnson,[1] H. Claus,[2] D. G. Hinks,[2] and T. E. Kidd[3]

Thursday, September 29, 2011

Conclusions

Quantum criticality and conformal field theories

- New insights and solvable models for diffusion and transport of strongly interacting systems near quantum critical points

- The description is far removed from, and complementary to, that of the quantum Boltzmann equation which builds on the quasiparticle/vortex picture.

- Prospects non-linear, and non-equilibrium transport

Conclusions

Compressible quantum matter

- Presented a holographic model of a Fermi liquid

- Fractionalized Fermi liquid (FL*), appears in deconfined gauge theories, holographic models, and lattice theories of the heavy-fermion compounds and cuprates superconductors.

- Numerous plausible sightings of the FL* phase in recent experiments