Holographic zero sound at finite temperature

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In a system of interacting fermions at low (or zero) temperature there exists a collective excitation known as the zero sound (Landau, 1957; observed in liquid He-3 in 1960s)

Temperature dependence of the zero sound mode is described (at sufficiently low temperatures) by Landau Fermi-liquid theory

A similar mode (holographic zero sound) was shown to exist in holographic models at zero temperature

We would like to understand its temperature dependence

Outline

Zero sound in Landau Fermi-liquid theory

Zero sound in He-3

Holographic zero sound in the D3/D7 system

Zero sound in Landau Fermi-liquid theory

Original Landau papers

L. D. Landau, “The theory of a Fermi liquid,” Zh. Eksp. Teor. Fiz. 30, 1058 (1956) [Soviet Phys. JETP 3, 920 (1957)].

L. D. Landau, “Oscillations in a Fermi liquid,” Zh. Eksp. Teor. Fiz. 32, 59 (1957) [Soviet Phys. - JETP 5, 101 (1959)].

Relativistic version

G.Baym and S.A.Chin, “Landau theory of relativistic Fermi liquids”, Nucl Phys **A262** (1976), 527

Microscopic (effective field theory) version

R.Shankar, “Renormalization group approach to interacting fermions”,
Rev Mod Phys, **66** (1994), 129

J.Polchinski, “Effective Field Theory and the Fermi Surface”, hep-th/9210046

Zero sound in Landau Fermi-liquid theory

In LFL, the dominant low energy excitations are quasiparticles carrying the same quantum numbers as fundamental particles

Quasiparticle energy: $\varepsilon_{\vec{k}}$ Width: $\sim (\varepsilon_{\vec{k}} - \mu)^2$

Equilibrium distribution function:
$$n_{\vec{k}} = \left(\exp\left(\frac{\varepsilon_{\vec{k}} - \mu)}{T} \right) + 1 \right)^{-1}$$

At low temperature:
$$\varepsilon_{\vec{k}} \simeq \mu + v_F \left(|\vec{k}| - k_F \right)$$

$$N/V = k_F^3 / 3\pi^2 \hbar^3 \qquad m^* = k_F / v_F$$

Specific heat: $c_V = m^* k_F / 3\hbar^3 \, T = \left(\frac{\pi}{3}\right)^{2/3} (m^*/\hbar^2) \, (N/V)^{1/3} \, T$

Zero sound in Landau Fermi-liquid theory

$$\delta E = \sum_{\sigma} \int \varepsilon_{\vec{k},\sigma} \delta n_{\vec{k},\sigma} \frac{d^3 k}{(2\pi)^3}$$

$$\delta \varepsilon_{\vec{k},\sigma} = \sum_{\sigma'} \int f_{\vec{k}\sigma,\vec{k}'\sigma'} \delta n_{\vec{k}',\sigma'} \frac{d^3 k'}{(2\pi)^3}$$

Landau interaction function $f_{\vec{k}\sigma,\vec{k}'\sigma'}$ can be expanded in Legendre polynomials yielding Landau parameters

$$F_l \, , G_l$$

In a weakly-interacting theory, $f_{\vec{k}\sigma,\vec{k}'\sigma'}$ can be computed perturbatively

Zero sound in Landau Fermi-liquid theory

For small deviations from equilibrium we have

$$n(\vec{k},\vec{r},t) = n_0(\vec{k}) + \delta n(\vec{k},\vec{r},t)$$

where the function δn obeys the Landau-Silin transport equation

$$\frac{\partial \delta n}{\partial t} + \frac{\partial \varepsilon_0}{\partial \vec{k}}\frac{\partial \delta n}{\partial \vec{r}} - \frac{\partial \delta \varepsilon}{\partial \vec{r}}\frac{\partial n_0}{\partial \vec{p}} = I[n]$$

whose third term involves the interaction function $f_{\vec{k}\sigma,\vec{k}'\sigma'}$

Solutions in the zero-temperature limit (with vanishing r.h.s.) are known as *zero sound* (Landau, 1957)

Zero sound in Landau Fermi-liquid theory

Zero sound dispersion relation

$$\omega = \pm v_s q - i\Gamma(q,\mu)$$

appears as a pole in the density-density correlation function

$$\langle J^0(-\omega,-q) J^0(\omega,q)\rangle$$

and thus as the lowest quasinormal frequency in the spectrum of a dual gravity background

$$\omega = \pm \frac{1}{\sqrt{3}} q - \frac{iq^2}{6\mu} + O(q^3)$$

e.g. in the D3/D7 system (in the probe brane limit) holographically describing

$\mathcal{N}=4\ SU(N_c)\ \mathrm{SYM} + N_f\ \mathcal{N}=2$ hypermultiplet fields in the limit $N_c \to \infty\,,\ g_{YM}^2 N_c \to \infty\,,\ N_f/N_c \to 0$

Zero sound at finite temperature

Zero sound mode is affected by quasiparticle interactions

Let τ be the mean time between quasiparticle collisions

Three regimes can be distinguished for the collective mode with frequency ω :

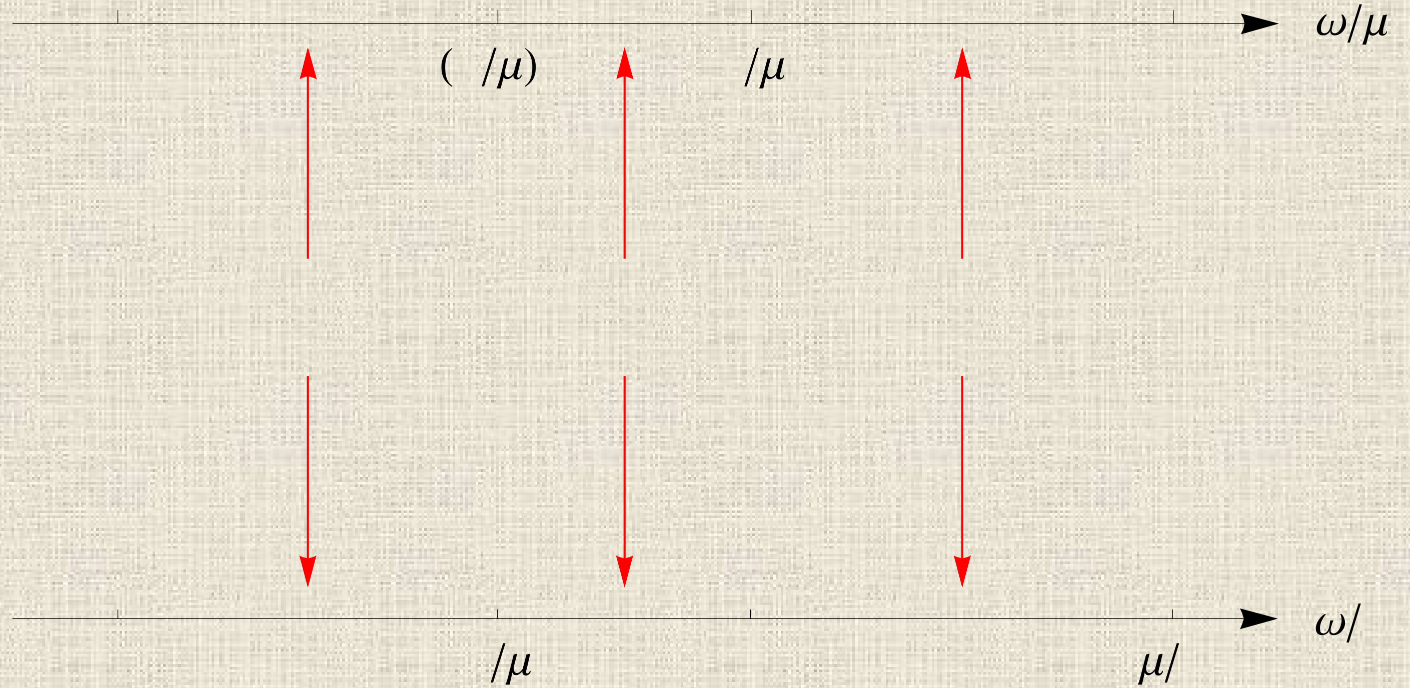
Collisionless quantum regime: $\omega\tau \gg 1\,, \quad \omega \gg T$

Collisionless thermal regime: $\omega\tau \gg 1\,, \quad \omega \ll T$

Hydrodynamic regime: $\omega\tau \ll 1\,, \quad \omega \ll T$

In LFL kinetic theory: $$1/\tau \sim \frac{\pi^2 T^2 + \omega^2}{\mu(1+e^{-\omega/T})}$$

Zero sound at finite temperature



LFL applicability conditions

$$T \ll \mu\,, \quad \omega \ll \mu$$

Zero sound at finite temperature

$$\omega(q) = v_s q - i\Gamma_\omega$$

$$q(\omega) = \omega/v_s + i\Gamma_q$$

$$\arg q(\omega) = \mathrm{Im}\, q/\mathrm{Re}\, q$$

TABLE II. Sound attenuation coefficients in a Landau Fermi-liquid

	Γ_ω	Γ_q	$\mathrm{Arg}\, q$
Hydrodynamic regime	$\left(\frac{\theta}{T}\right)^2 \frac{q^2}{\theta}$	$\frac{\theta\, \omega^2}{T^2}$	$\left(\frac{\theta}{T}\right)^2 \frac{\omega}{\theta}$
Collisionless thermal regime	$\frac{T^2}{\theta}$	$\frac{T^2}{\theta}$	$\left(\frac{T}{\theta}\right)^2 \frac{\theta}{\omega}$
Collisionless quantum regime	$\frac{q^2}{\theta}$	$\frac{\omega^2}{\theta}$	$\frac{\omega}{\theta}$

Zero sound at finite temperature

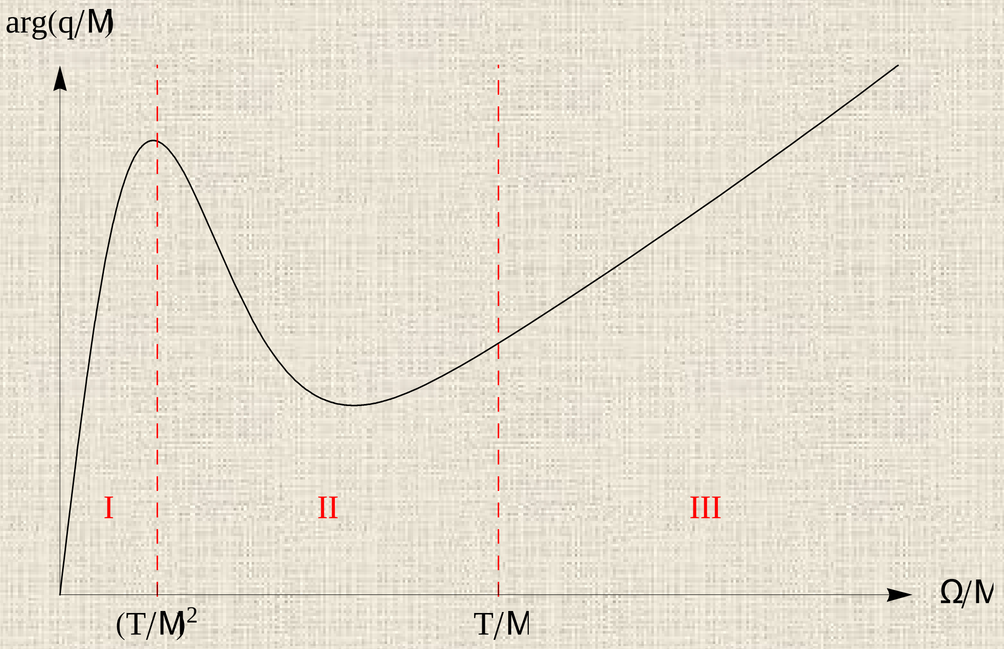
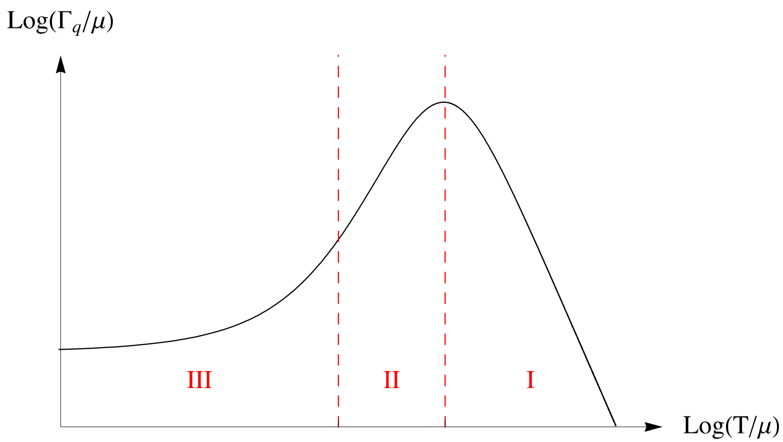
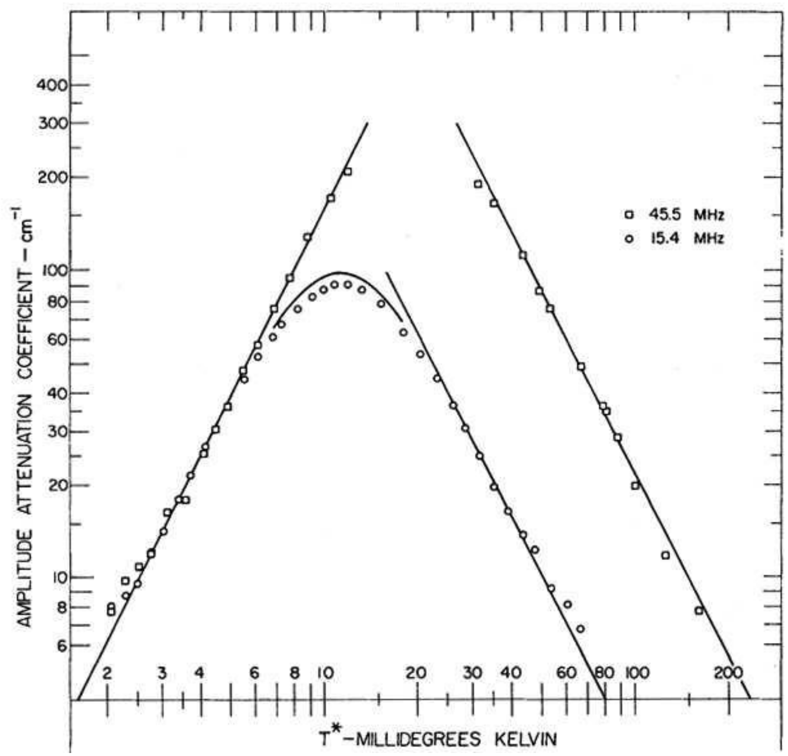


FIG. 2. A sketch of the dependence of the sound mode damping on frequency in the hydrodynamic (I), collisionless thermal (II) and collisionless quantum (III) regimes of a Landau Fermi-liquid. First sound propagates in region I while the zero sound mode exists in regions II and III.

Zero sound at finite temperature





D3/D7 zero sound at finite temperature

		x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
N_c	D3	$\times$	$\times$	$\times$	$\times$						
N_f	D7	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$		

$$S = S_{\text{adjoint}} + S_{\text{fundamental}}$$

$$= S_{\text{adjoint}} - N_f T_{D7} \int d^8\xi \sqrt{-\det\left(g_{ab} + F_{ab}\right)}$$

$$\mathcal{N} = 4 \;\; SU(N_c) \;\; \text{SYM} \quad + \quad N_f \;\; \mathcal{N} = 2 \;\; \text{fund. fermions and scalars}$$

$$\lambda = g_{YM}^2 N_c \to \infty\,, \quad N_c \to \infty\,, \quad N_f/N_c \to 0$$

D3/D7 zero sound at finite temperature

$$ds_{10}^2 = \frac{r^2}{R^2}\left[-\left(1-\frac{r_H^4}{r^4}\right)dt^2 + d\vec{x}^2\right] + \left(1-\frac{r_H^4}{r^4}\right)^{-1}\frac{R^2}{r^2}dr^2 + R^2 ds_{S^5}^2$$

$$T = r_H \triangleleft \pi R^2$$

$$S_{\text{fundamental}} = -\frac{N r_H^4}{2}\int_0^1 du\, d^4x \frac{\cos^3\theta}{u^3}\sqrt{1 + 4u^2 f \theta'^2 - 4\frac{u^3}{r_H^2}A_t'^2}$$

$$U(N_f) = U(1)_B \times SU(N_f)$$

Parameters:

T (temperature), μ ("baryonic" chemical potential), m (hypermultiplet mass)

D3/D7 zero sound at finite temperature

$$\langle J^0(-\omega,-q)J^0(\omega,q)\rangle^{ret}_{T,\mu}$$

Zero temperature, finite density, zero hypermultiplet mass

$$\tilde{\omega} = \pm\frac{\tilde{q}}{\sqrt{3}} - i\frac{\Gamma\left(\frac{1}{2}\right)}{\Gamma\left(\frac{1}{6}\right)\Gamma\left(\frac{1}{3}\right)}\tilde{q}^2 + O\left(\tilde{q}^3\right)$$

Zero temperature, finite density, finite hypermultiplet mass

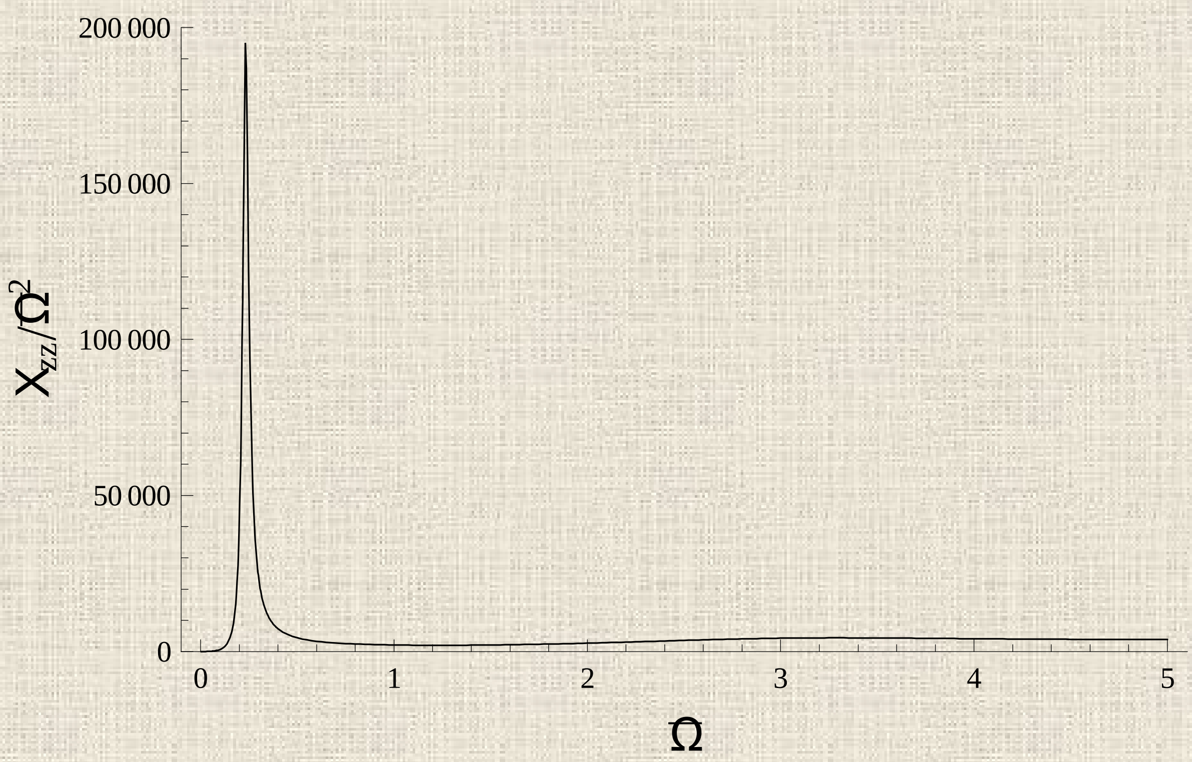
$$\tilde{\omega} = \pm\frac{1}{\sqrt{3}}\left(\frac{1-\tilde{m}^2}{1-\tilde{m}^2/3}\right)^{1/2}\tilde{q} - i\frac{\Gamma\left(\frac{1}{2}\right)}{\Gamma\left(\frac{1}{3}\right)\Gamma\left(\frac{1}{6}\right)}\frac{(1-\tilde{m}^2)^{4/3}}{(1-\tilde{m}^2/3)^2}\tilde{q}^2 + O\left(\tilde{q}^3\right)$$

High temperature, finite density, zero hypermultiplet mass

$$\tilde{\omega} = -iD\,\tilde{q}^2 + O(\tilde{q}^3)$$

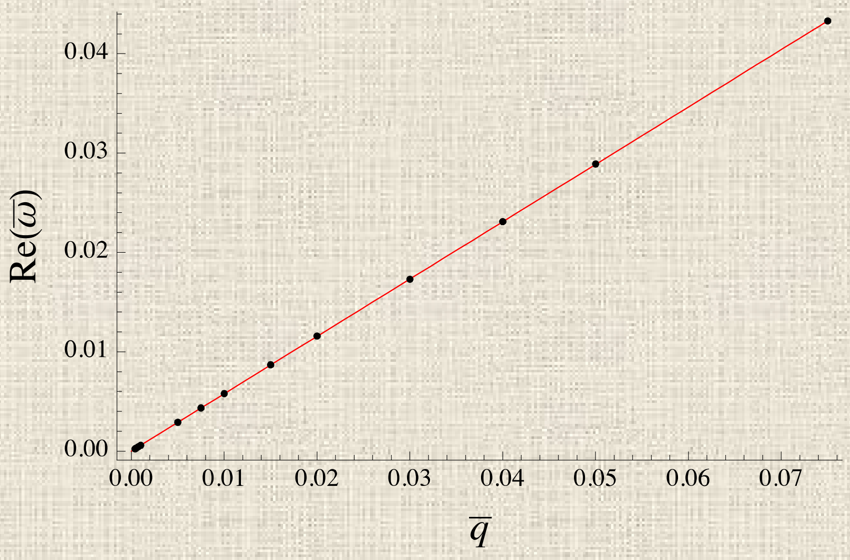
$$D(\tilde{d}) = \frac{\tilde{d}^{\frac{1}{3}}}{2}\sqrt{1+\tilde{d}^2}\;{}_2F_1\left[\frac{3}{2},\frac{1}{3};\frac{4}{3};-\tilde{d}^2\right]$$

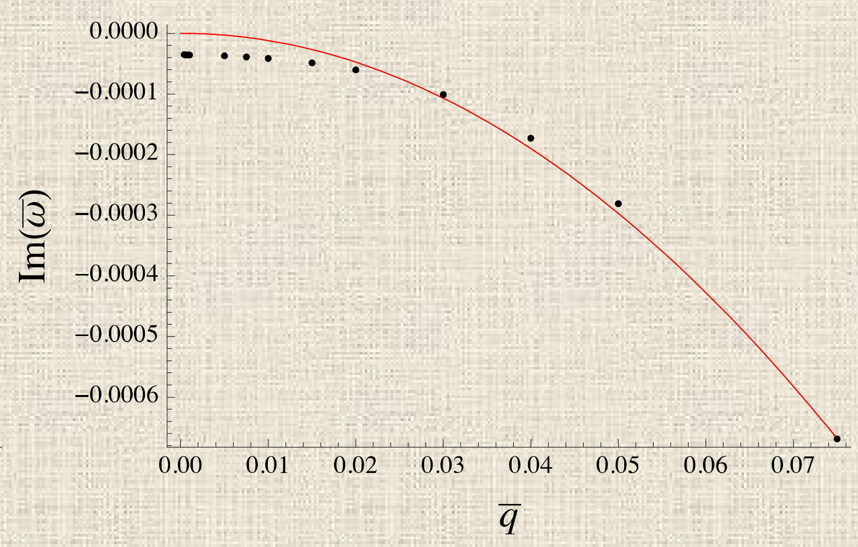
D3/D7 zero sound at finite temperature



Holographic zero sound peak in the collisionless quantum regime

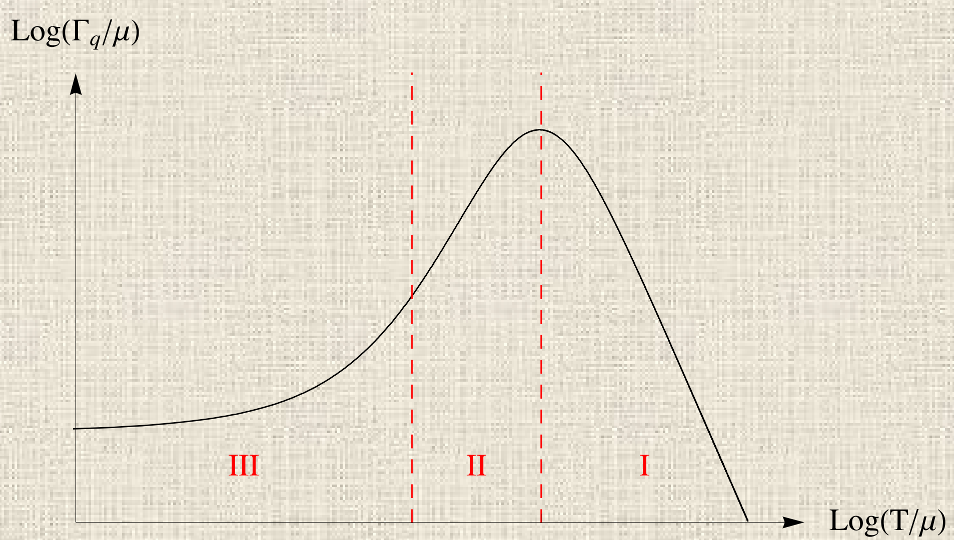
D3/D7 zero sound at finite temperature

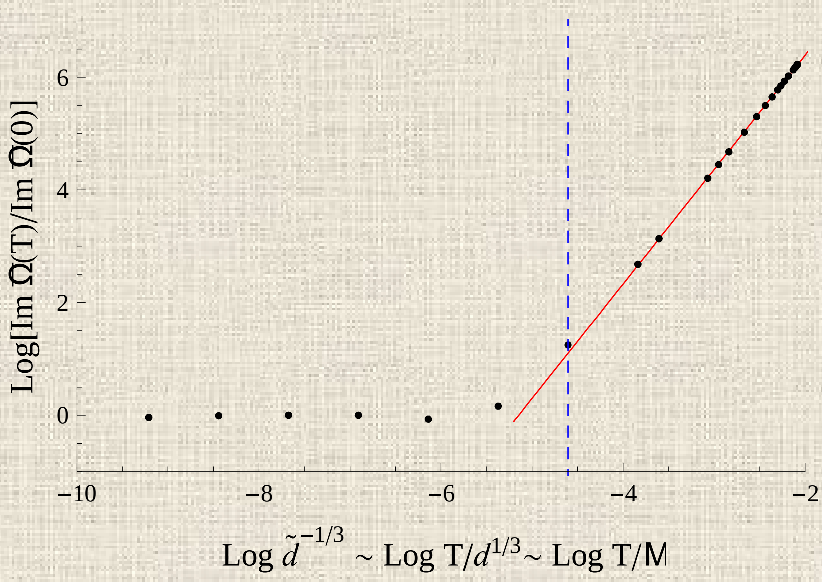




Zero sound dispersion relation in the collisionless quantum regime

D3/D7 zero sound at finite temperature





Collisionless quantum – collisionless thermal crossover

D3/D7 zero sound at finite temperature

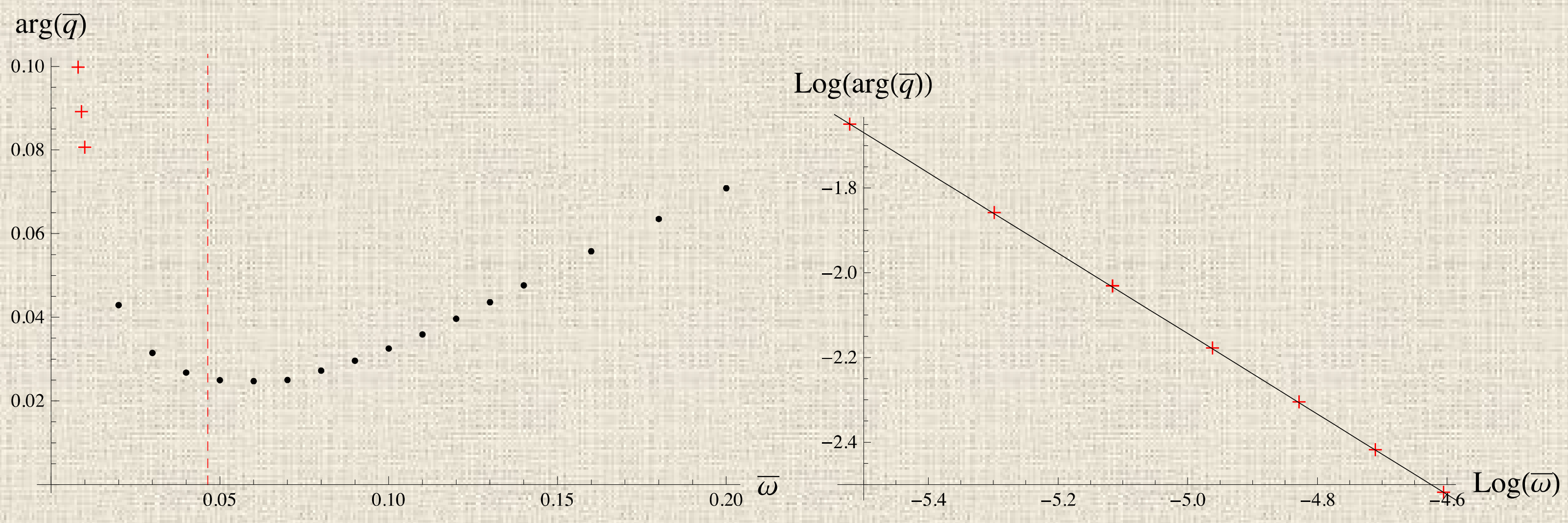
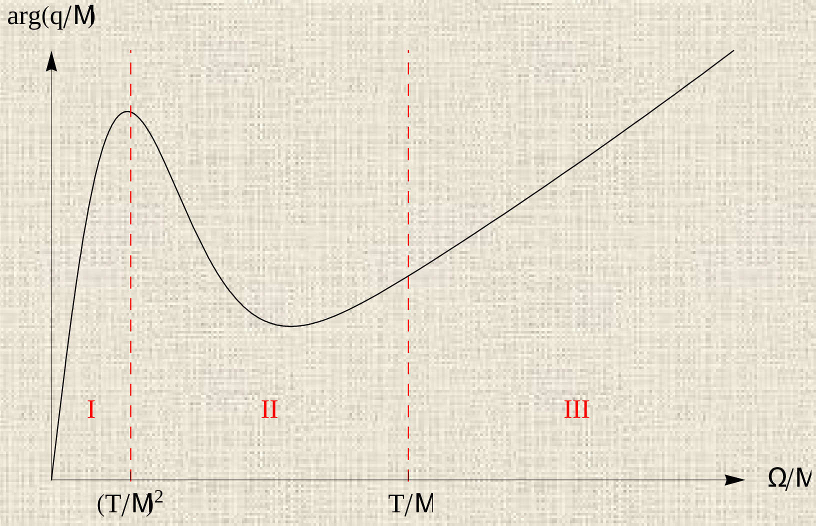
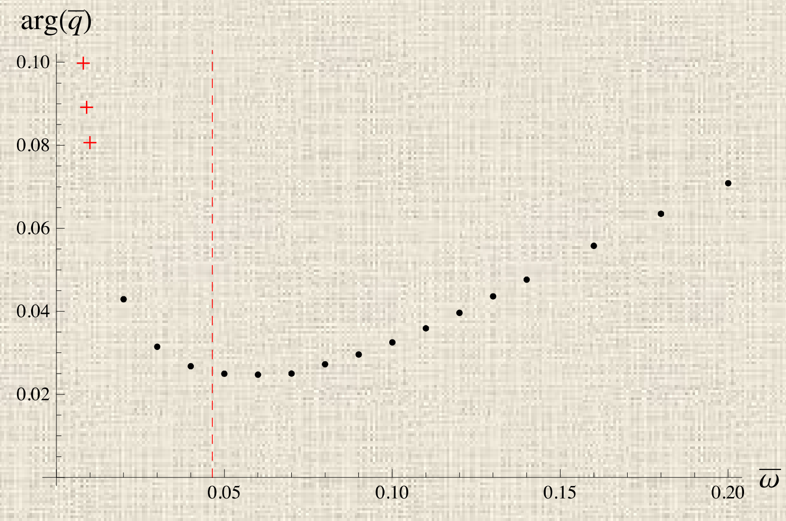


FIG. 9. The frequency dependence of the D3/D7 acoustic mode when $\bar{m} = 0$, $\tilde{d} = 10^4$ in the collisionless regime. The dots and crosses are our numerical results, the dashed line denotes $\bar{\omega} = \pi T/d^{1/3} \sim T/\mu$, and the solid line shows the best-fit straight line with gradient $\alpha \approx -0.95$. The points on the left of the left hand plot correspond to the rightmost points on the right hand plot

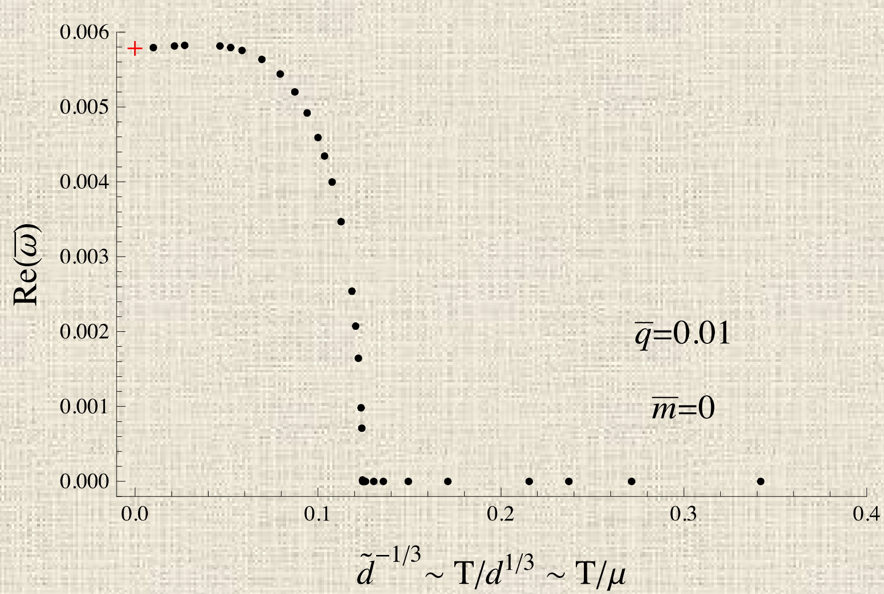
D3/D7 zero sound at finite temperature

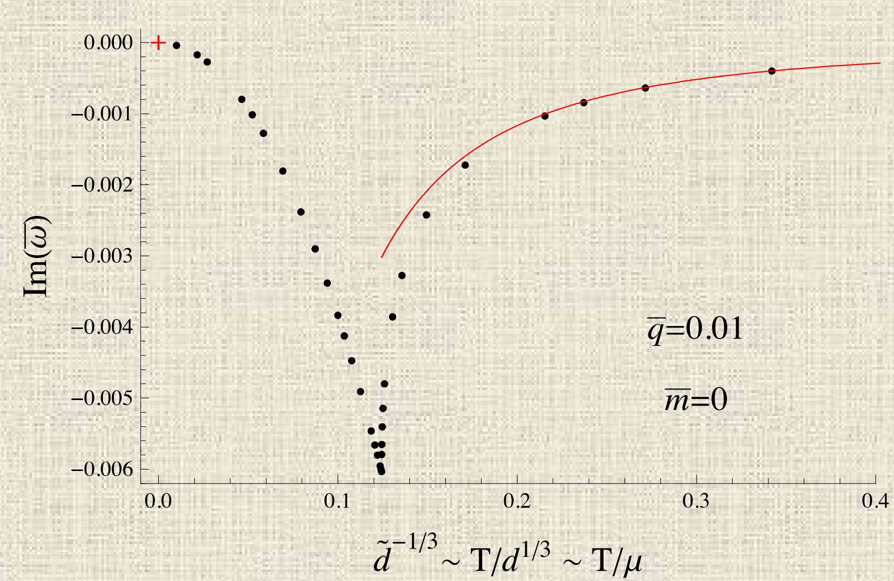




Collisionless quantum – collisionless thermal crossover

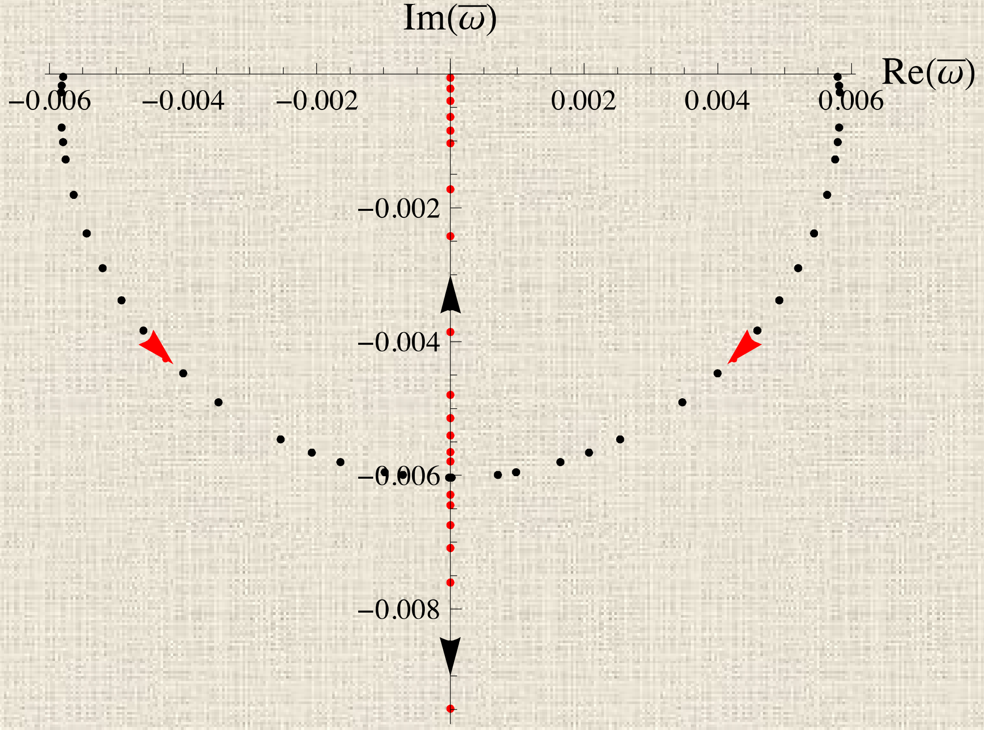
D3/D7 zero sound at finite temperature



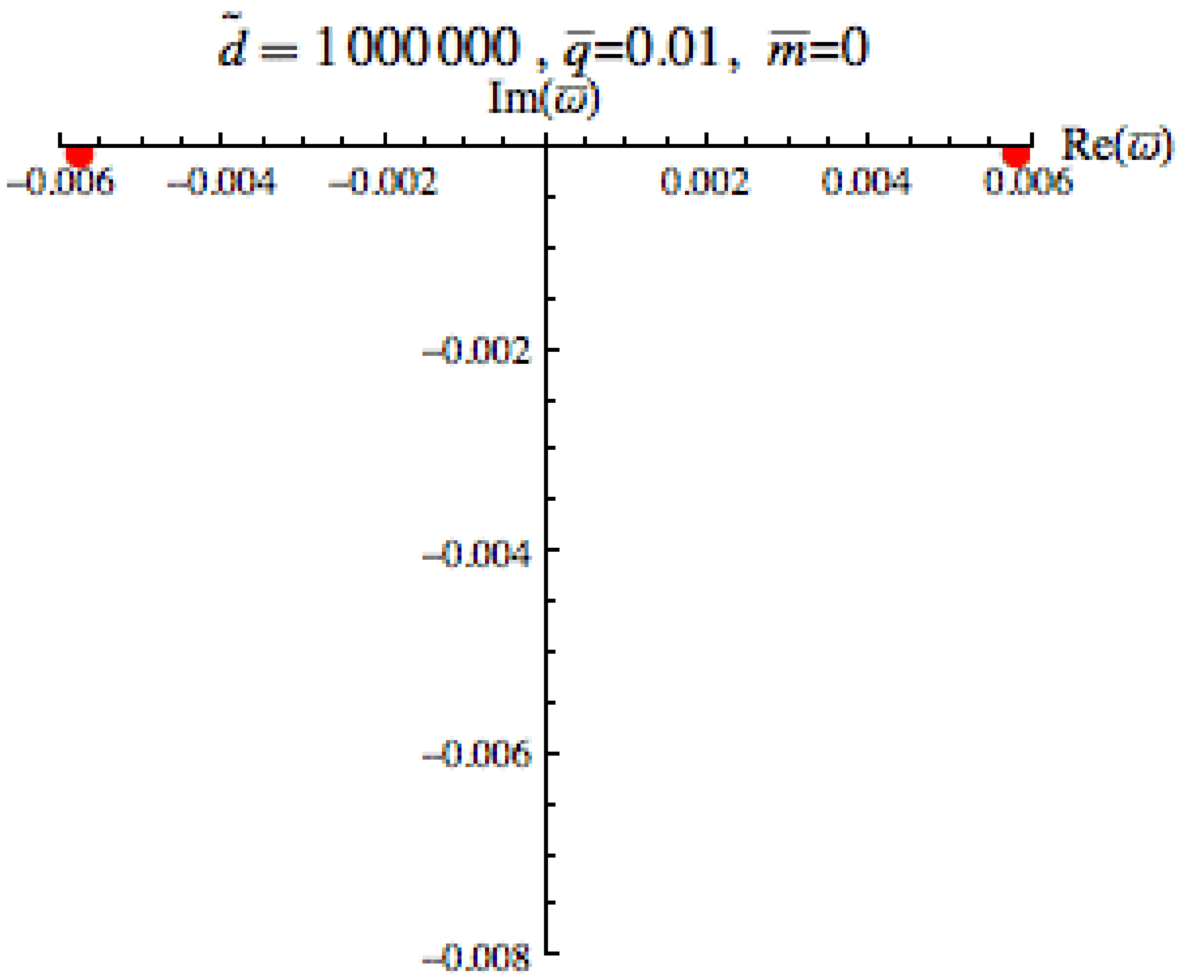


Collisionless thermal – hydrodynamic crossover

D3/D7 zero sound at finite temperature



Poles of the density-density correlator:
The collisionless thermal – hydrodynamic crossover



$\tilde{d} = 1\,000\,000$, $\bar{q}$=0.01 , $\bar{m}$=0
Im($\bar{\omega}$)
Re($\bar{\omega}$)
−0.006
−0.004
−0.002
0.002
0.004
0.006
−0.002
−0.004
−0.006
−0.008

D3/D7 zero sound at finite temperature

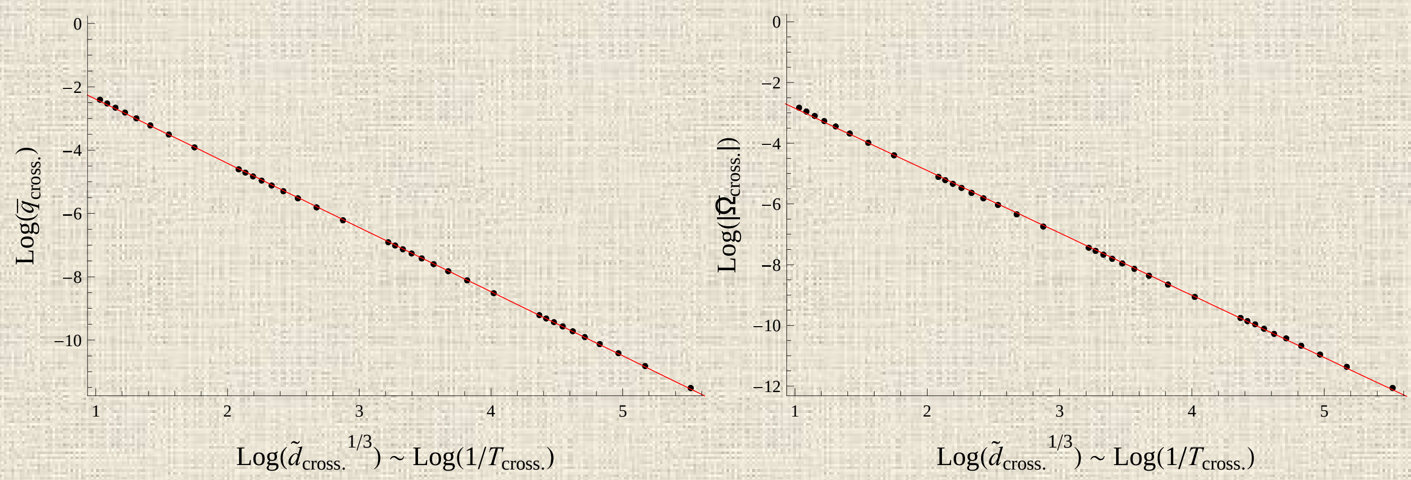


FIG. 13. The temperature dependence of the collisionless-hydrodynamic crossover value of frequency and momentum for $\tilde{m} = 0$. The points are our numerical results and the solid lines are the best-fit straight lines which both have gradient -2.0.

$$l_{\rm mfp} \sim \tau \propto d^{-1/3}\left(\frac{T}{d^{1/3}}\right)^{\alpha} \qquad\qquad l_{\rm mfp} \sim \tau \sim d^{1/3}\,T^{-2} \sim \theta\, T^{-2}$$

Unusual features of D3/D7 thermodynamics

$$c_v \sim N_c^2 T^3 + \cdots + \lambda N_f N_c T^6/\mu^3 + \cdots \,, \quad T \ll \mu$$

$$s \to s_0 \sim \mu^3 \neq 0 \quad \text{for} \quad T \to 0$$

Density-density correlator apparently shows no singularity at $q = 2q_F$

Conclusions

D3/D7 zero sound at finite temperature behaves exactly as LFL zero sound

D3/D7 thermodynamics (in the probe brane limit) appears to be incompatible with LFL

It would be interesting to:

a) Understand this discrepancy

b) Consider other holographic and field-theoretic systems

c) Understand this from the (holographic) RG perspective

THANK YOU!