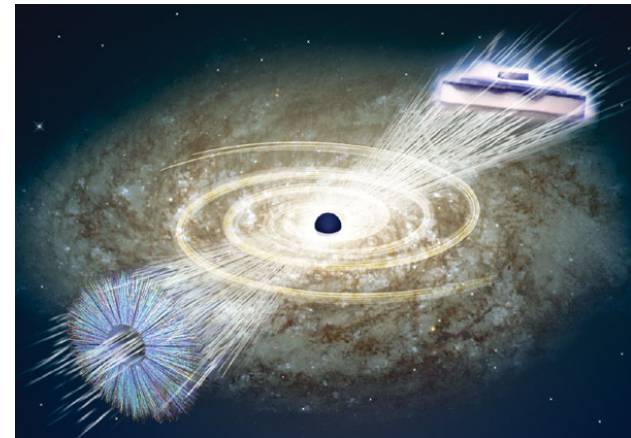
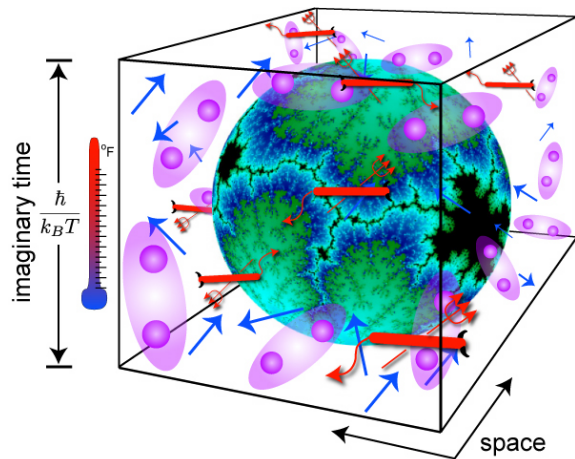


# Observing the origin of superconductivity in quantum critical metals.

Jan Zaanen

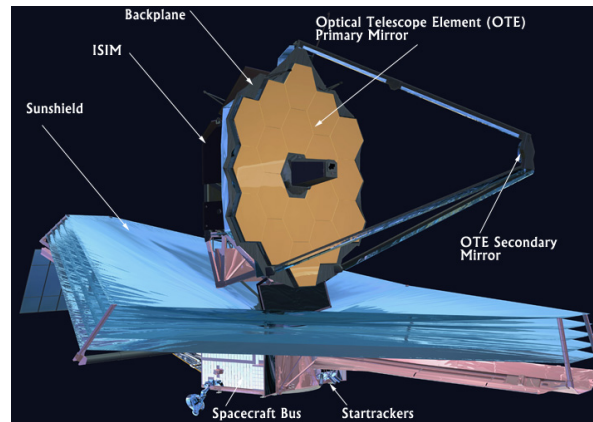


# Plan of the talk

---

**A proposal to build the condensed matter incarnation of**

## **The James Webb Space Telescope**

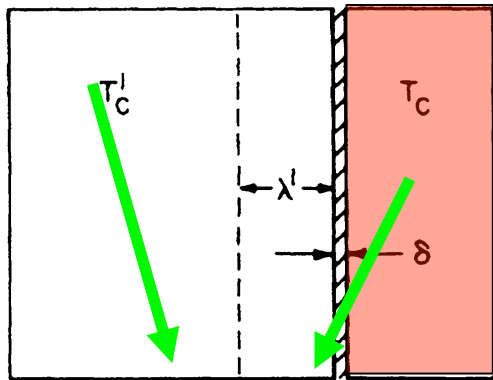


**To either: - Embarrass string theorists.**

**Or: - Win Nobel Prize(s) together with string theorists  
for high  $T_c$  superconductivity, quantum gravity, ...**

# Observing the origin of the pairing mechanism

SUPERCONDUCTOR 2    SUPERCONDUCTOR 1



$$T'_c > T > T_c$$

**2<sup>nd</sup> order Josephson effect**

$$\Delta c^\dagger c^\dagger$$



Ferrell    Scalapino

1969

1970

$$I_s(\mathbf{H}, V) \sim \frac{1}{R_N^2} \text{Im} \chi_{\text{pair}}(\mathbf{k}, \omega)$$

$$\omega = 2eV$$

# Promiscuous Leiden physics ...

J.-H. She et al., arXiv:1105.5377

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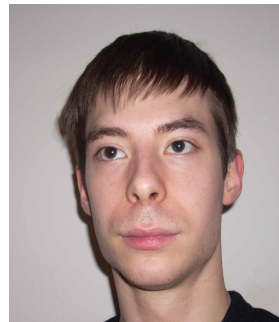
## Stringy folks



**Schalm**



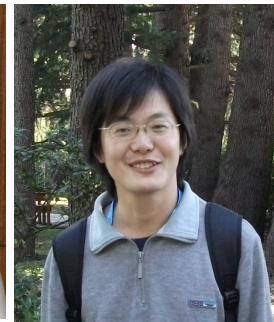
Parnachev



Cubrovic

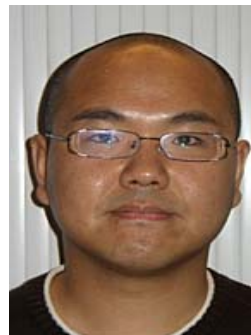


**Sun**



**Liu**

## Condensed matter people



**She**



**Overbosch**



**Mydosh**

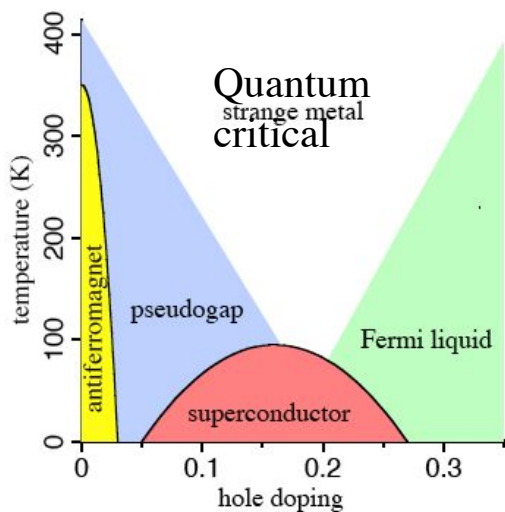


Hilgenkamp

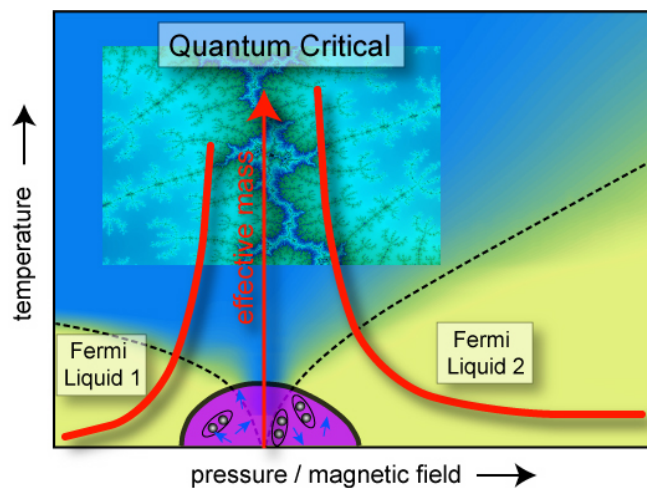


# A universal phase diagram

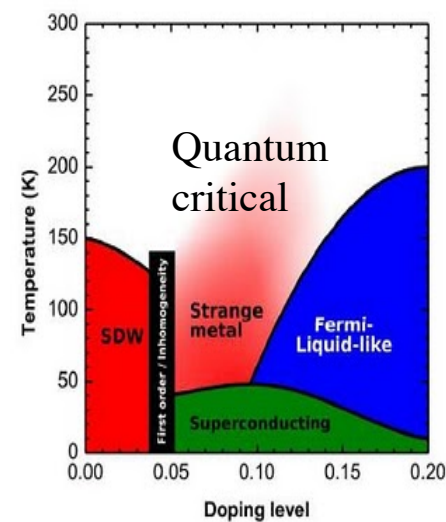
High  $T_c$   
superconductors



Heavy fermions



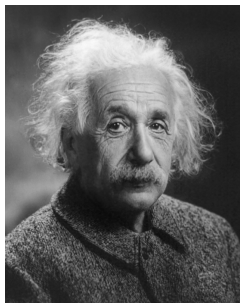
Iron  
superconductors (?)



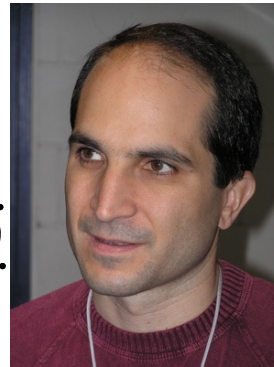
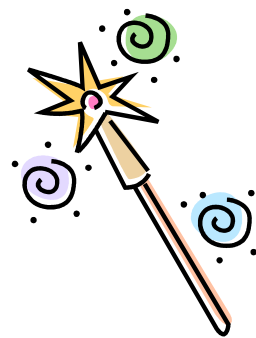
# General relativity “=” quantum field theory

---

## Gravity



In Anti-de-Sitter space



**Maldacena 1997**

**=**

**AdS/CFT  
correspondence**

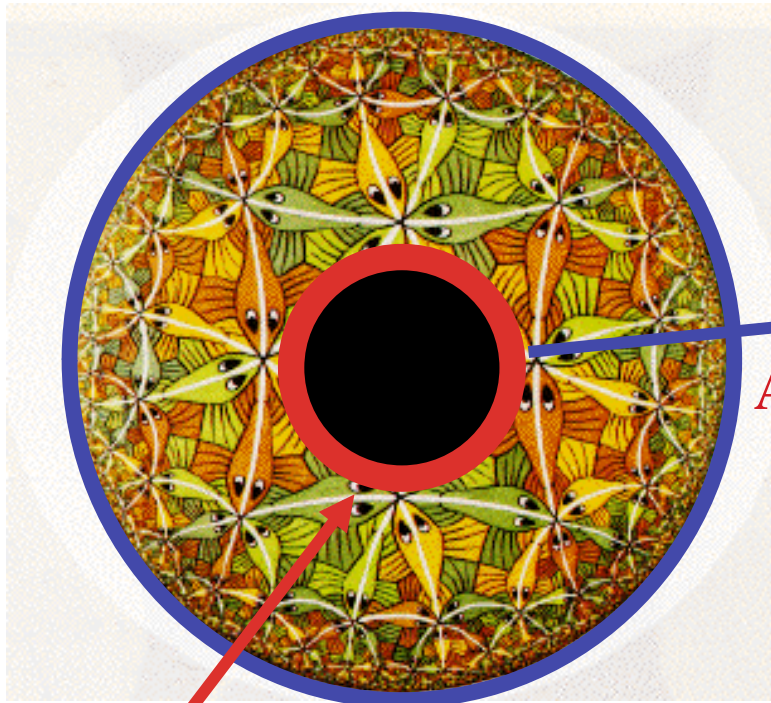
## Quantum fields



When they are conformal =  
**quantum critical**

# The holographic superconductor

Gubser; Hartnoll, Herzog, Horowitz



(Scalar) matter 'atmosphere'

Condensate (superconductor, ... ) on the boundary!



AdS-CFT

'Super radiance': in the presence of matter the extremal BH is unstable => zero T entropy always avoided by low T order!!!

# Plan

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1. On susceptibility, quantum criticality and instability.
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# Observing the pairing mechanism ...

---

**Claim: the maximal knowledge on the pairing mechanism is encoded in the temperature evolution of the normal state dynamical pair susceptibility,**

$$\chi_p(q, \omega) = -i \int_0^{\infty} dt e^{i\omega t - 0^+ t} \left\langle \left[ b^+(q, 0), b(q, t) \right] \right\rangle$$

$$b^+(q, t) = \sum_k c_{k+q/2, \uparrow}^+(t) c_{-k+q/2, \downarrow}^+(t)$$



# Observing the origin of magnetism ...

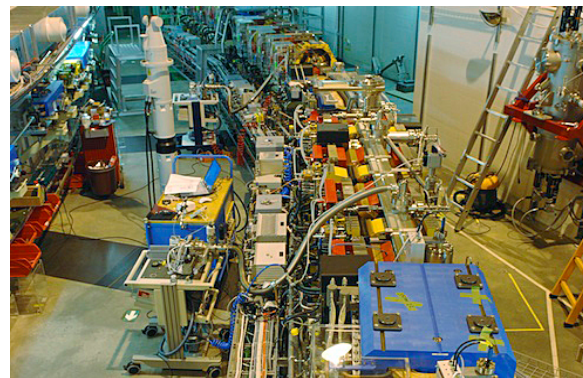
---

**The origin of magnetism: dynamical magnetic susceptibility.**

$$\chi_M(q, \omega) = -i \int_0^{\infty} dt e^{i\omega t - 0^+ t} \left\langle \left[ \vec{S}(q, 0), \vec{S}(q, t) \right] \right\rangle$$

$$\vec{S}(q, t) = \sum_{k\alpha\beta} c_{k+q,\alpha}^+(t) \vec{\sigma}_{\alpha\beta} c_{k,\beta}(t)$$

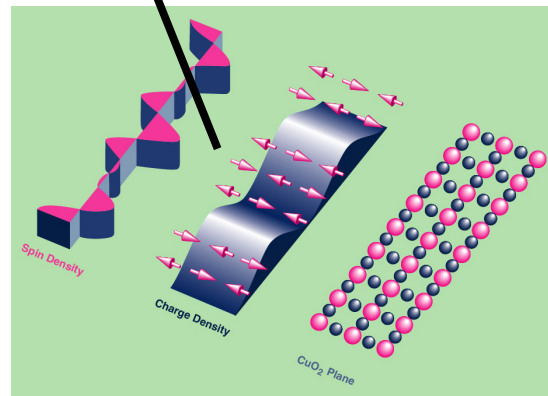
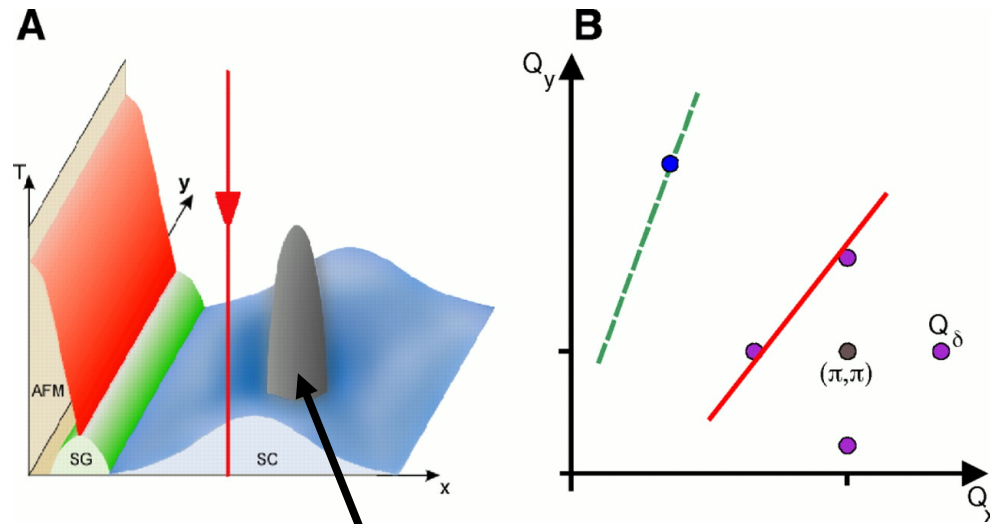
**Measured by inelastic neutron scattering:**



# Quantum critical spin fluctuations in underdoped cuprates



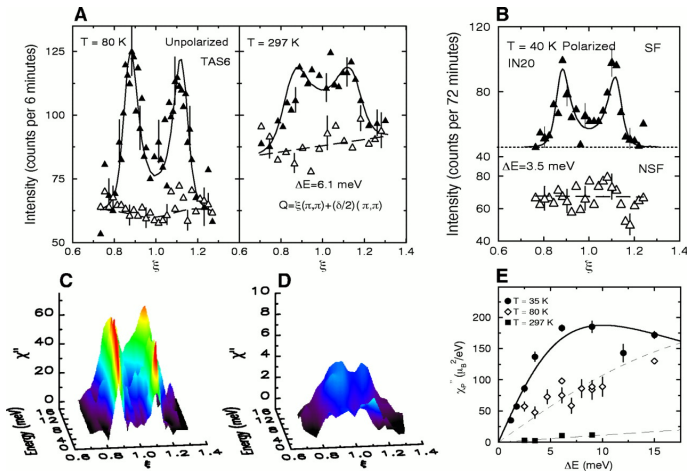
Aeppli et al.  
Science 1997.



# Quantum critical spin fluctuations in underdoped cuprates



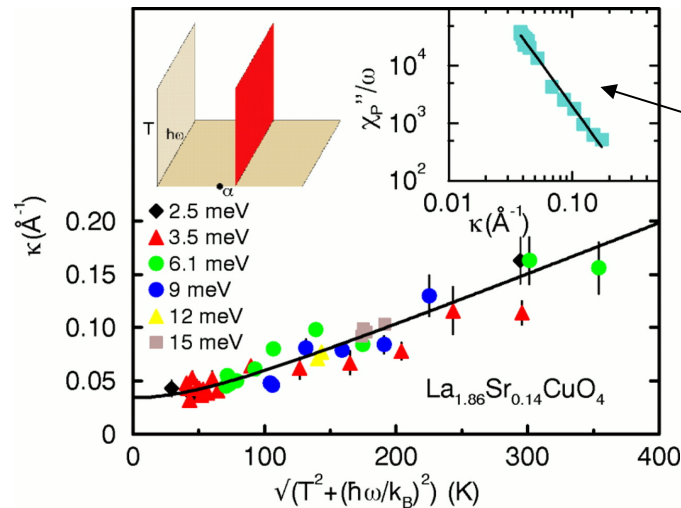
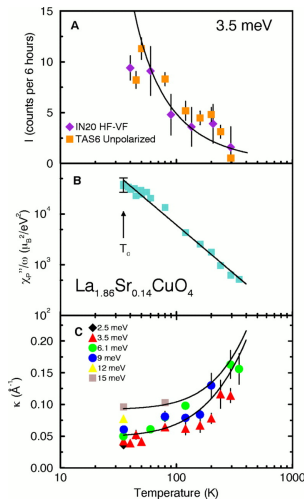
Aeppli et al.  
Science 1997.



**Peak widths reveal z=1 energy-temperature (conformal) scaling:**

$$\kappa^2 = \kappa_0^2 + a_0^{-2} \left( \frac{(k_B T)^2 + (\hbar\omega)^2}{(E_T)^2} \right)^{1/z} \quad z=1$$

**Real data**



Amplitude suggests

$$\eta \approx 0$$

2+1D Heisenberg

# Plan

---

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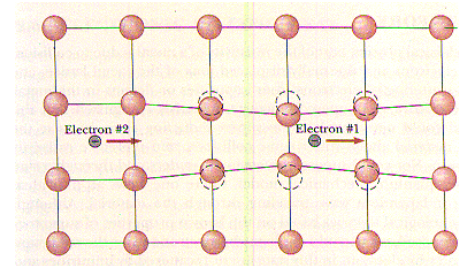


# BCS theory: fermions turning into bosons



Bardeen Cooper Schrieffer

Fermi-liquid + attractive interaction

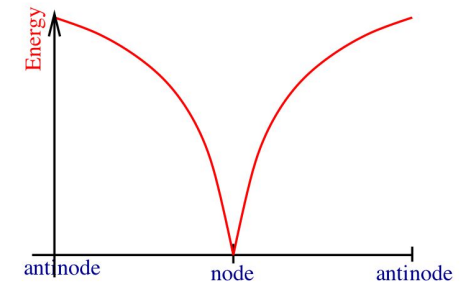
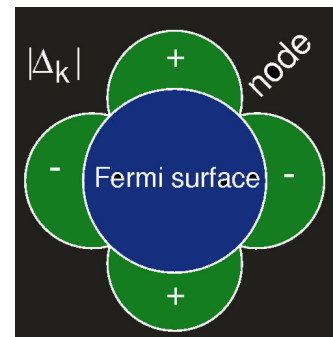


## Quasiparticles pair and Bose condense:

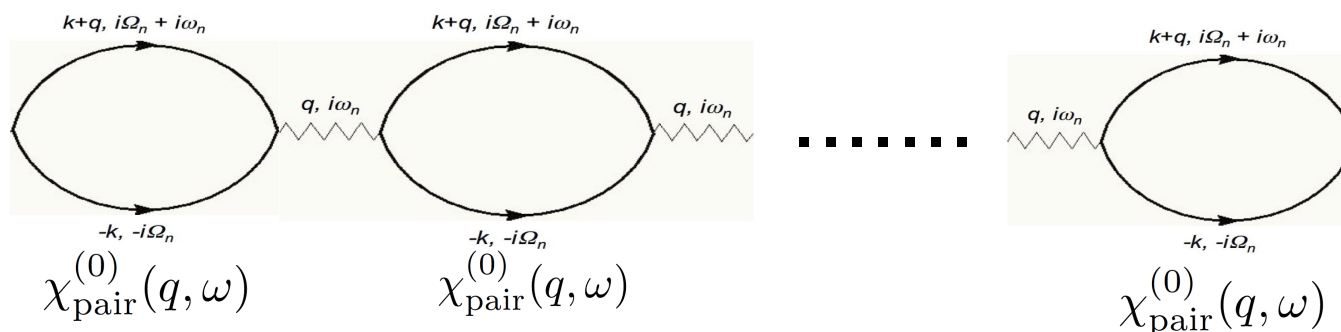
Ground state

$$\Psi_{BCS} = \prod_k (u_k + v_k c_{k\uparrow}^+ c_{-k\downarrow}^+) |vac.\rangle$$

D-wave SC: Dirac spectrum



# BCS and the pair susceptibility



$$\chi_{\text{pair}}(\omega) = \frac{\chi_{\text{pair}}^{(0)}(\omega)}{1 - g\chi_{\text{pair}}^{(0)}(\omega)} \quad \rightarrow \quad 1 - g\text{Re}\chi_{\text{pair}}^{(0)}(\omega = 0) = 0$$

$$\text{Im}\chi_{\text{pair}}^{(0)}(\omega) = \frac{1}{2E_F} \tanh\left(\frac{\hbar\omega}{4k_B T}\right)$$

$$\text{Re}\chi(\omega = 0) = 2 \int_0^{\omega_c} d\omega' \frac{\text{Im}\chi(\omega')}{\omega'}$$

$$\Delta = 2\omega_B e^{-1/\lambda}$$

# Computing the pair susceptibility: full Eliashberg



$$\chi(k, k'; q) = \chi_0(k, k'; q) + u^2 \sum_{k_1, k_2} \chi_0(k, k_1; q) D(k_2 - k_1) \chi(k_2, k'; q)$$

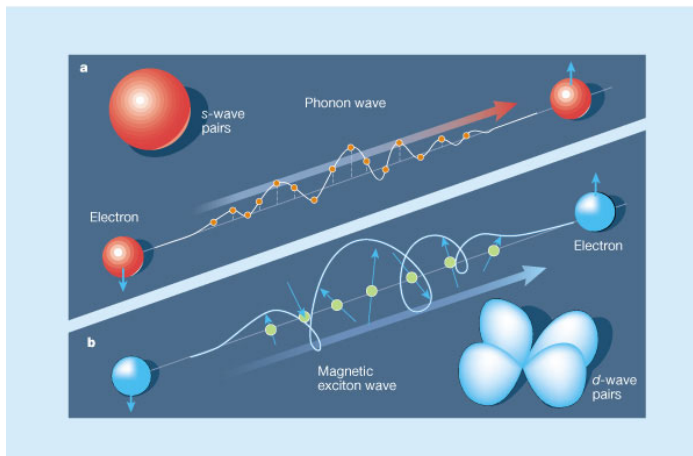
$$\Gamma(k; q) = \sum_{k'} \chi(k, k'; q)$$

$$\Gamma(i\nu; i\Omega) = \Gamma_0(i\nu; i\Omega) + \mathcal{A} \Gamma_0(i\nu; i\Omega) \sum_{\nu'} \lambda(i\nu' - i\nu) \Gamma(i\nu'; i\Omega)$$

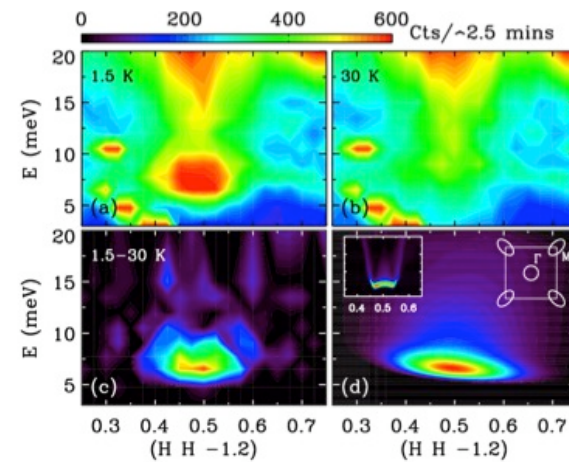
$$\chi_{\text{pair}}(i\Omega, \mathbf{q} = 0) = \sum_{\nu} \Gamma(i\nu; i\Omega) \quad i\Omega \rightarrow \omega + i\delta$$

$$\chi_{\text{pair}}(\omega, \mathbf{q} = 0)$$

# Superglue !



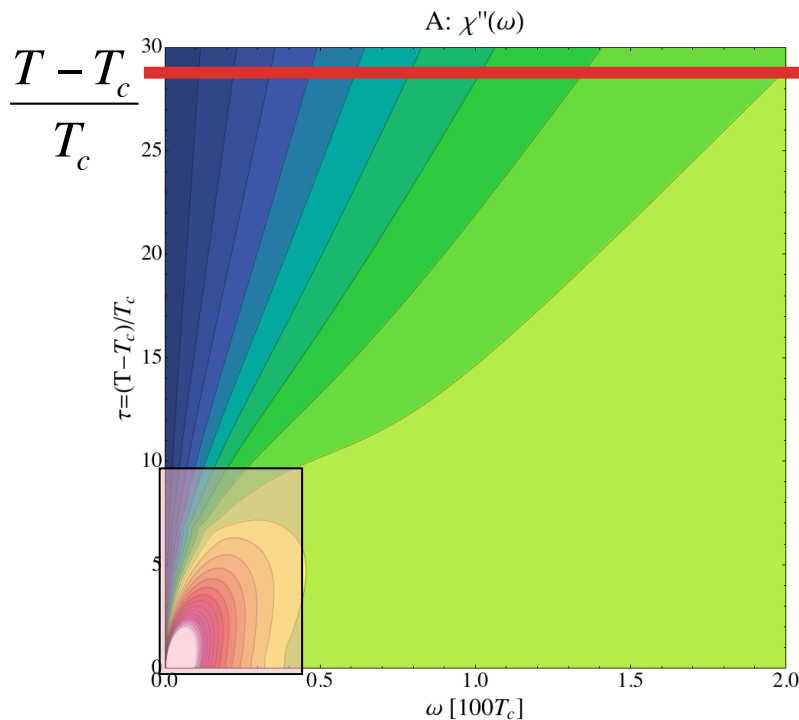
Magnetic resonance ...



$$\lambda(i\Omega) = \frac{g}{A} \frac{\omega_b^2}{\omega_b^2 + \Omega^2}$$

# Imaginary part of the “regular” BCS pair susceptibility

$$\lambda(i\Omega) = \frac{g}{A} \frac{\omega_b^2}{\omega_b^2 + \Omega^2}$$



**High temperature: the Fermi gas**

$$\text{Im} \chi_{pair}^{(0)}(\omega) = \frac{1}{2E_F} \tanh\left(\frac{\hbar\omega}{4k_B T}\right)$$

**Close to  $T_c$ : “relaxational peak”**

$$L = \frac{1}{\tau_r} \Psi \partial_t \Psi + |\nabla \Psi|^2 + \alpha_0 (T - T_c) |\Psi|^2 + w |\Psi|^4 + \dots$$

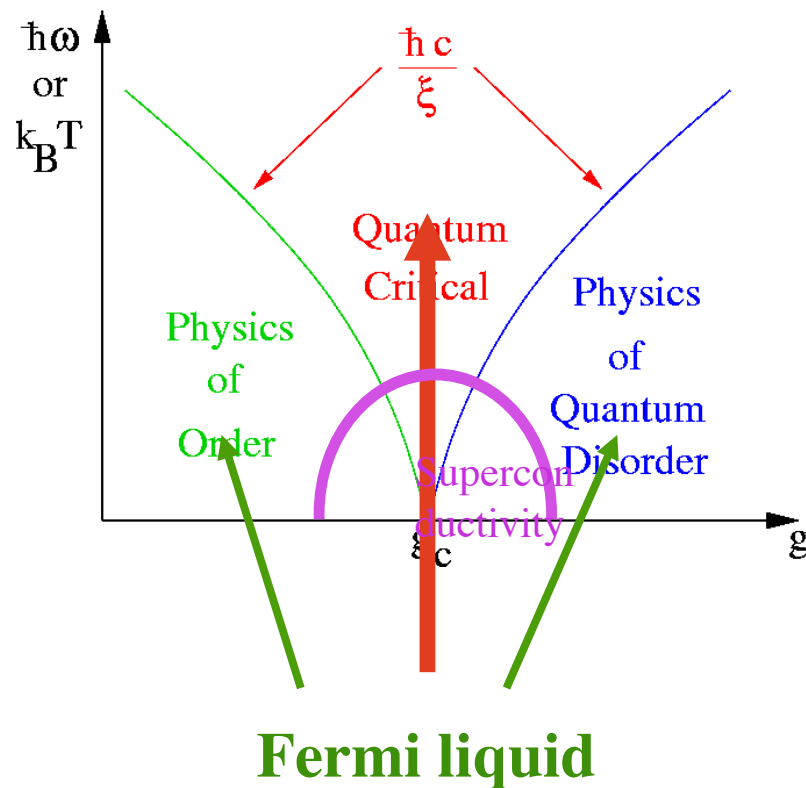
**Assume mean-field thermal transition  
(true in all cases)**

$$\chi_{pair}''(\omega) = \frac{\dot{\chi}_{pair}'(\omega = 0, T)}{1 - i\omega\tau_r}$$

$$\dot{\chi}_{pair}'(\omega = 0, T) = 1 / [\alpha_0 (T - T_c)] \quad \tau_r = \frac{8}{\pi} \frac{\hbar}{k_B (T - T_c)}$$



# Hertz-Millis and Chubukov's “critical glue” (Metlitski talk)



**Bosonic (magnetic, etc.) order  
parameter drives the phase transition**

**Electrons: fermion gas = heat bath  
damping bosonic critical fluctuations**

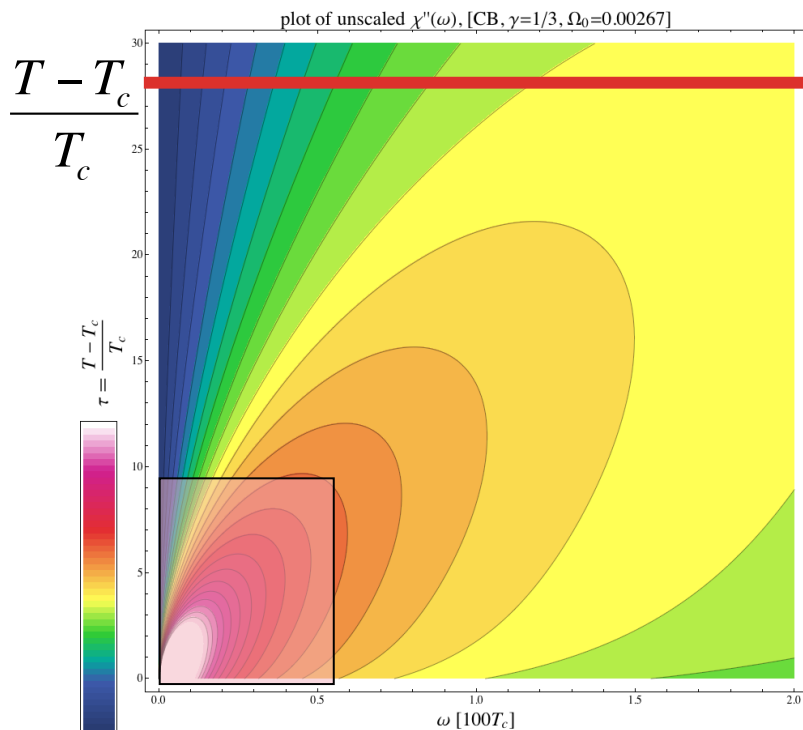
**Bosonic critical fluctuations ‘back  
react’ as pairing glue on the electrons**

$$\lambda(i\Omega) = \left( \frac{\Omega_0}{|\Omega|} \right)^\gamma$$

E.g.: Moon, Chubukov, J. Low Temp. Phys. 161,  
263 (2010)

# The Hertz-Millis-Chubukov “critical glue” pair susceptibility

$$\lambda(i\Omega) = \left( \frac{\Omega_0}{|\Omega|} \right)^\gamma$$



**High temperature: effectively strongly coupled, self energies produce a peak (yellow).**

**Close to  $T_c$ : “relaxational peak”**

$$\chi_{pair}''(\omega) = \frac{\chi_{pair}'(\omega = 0, T)}{1 - i\omega\tau_r}$$

# Plan

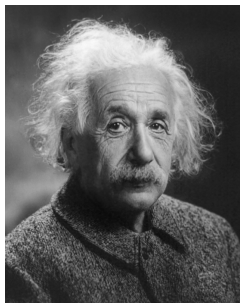
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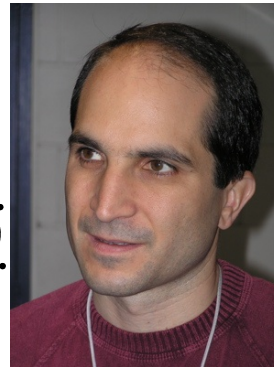
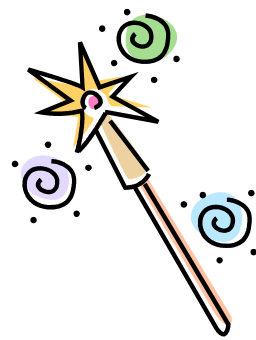
# General relativity “=” quantum field theory

---

## Gravity



In Anti-de-Sitter space



**Maldacena 1997**

**=**

**AdS/CFT  
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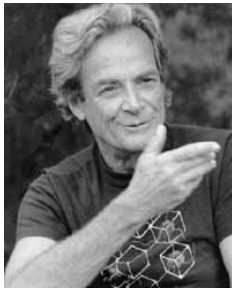
## Quantum fields



When they are conformal =  
**quantum critical**

# Fermion sign problem

Imaginary time path-integral formulation

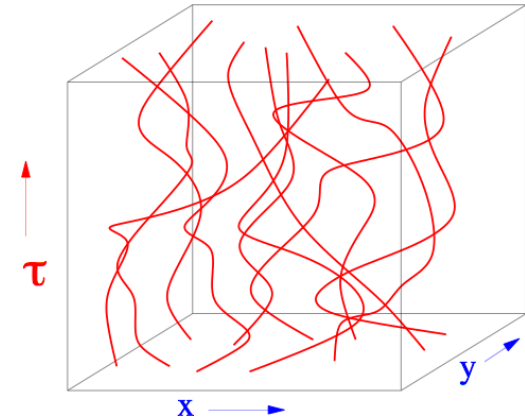


$$\begin{aligned}\mathcal{Z} &= \text{Tr} \exp(-\beta \hat{\mathcal{H}}) \\ &= \int d\mathbf{R} \rho(\mathbf{R}, \mathbf{R}; \beta)\end{aligned}$$

$$\mathbf{R} = (\mathbf{r}_1, \dots, \mathbf{r}_N) \in \mathbb{R}^{Nd}$$

$$\rho_{B/F}(\mathbf{R}, \mathbf{R}; \beta) = \frac{1}{N!} \sum_{\mathcal{P}} (\pm 1)^{\mathcal{P}} \rho_D(\mathbf{R}, \mathcal{P}\mathbf{R}; \beta)$$

$$= \frac{1}{N!} \sum_{\mathcal{P}} (\pm 1)^{\mathcal{P}} \int_{\mathbf{R} \rightarrow \mathcal{P}\mathbf{R}} \mathcal{D}\mathbf{R}(\tau) \exp \left\{ -\frac{1}{\hbar} \int_0^{\hbar/T} d\tau \left( \frac{m}{2} \dot{\mathbf{R}}^2(\tau) + V(\mathbf{R}(\tau)) \right) \right\}$$



Boltzmannons or Bosons:

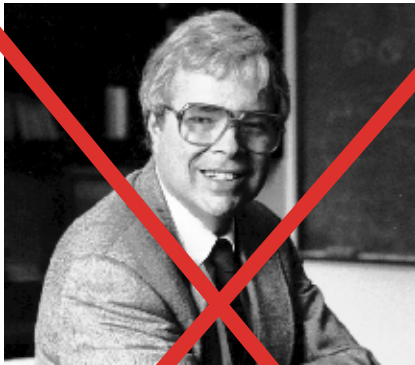
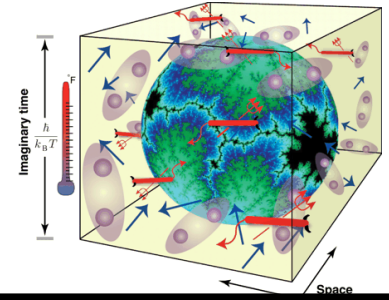
- integrand non-negative
- probability of equivalent classical system: (crosslinked) ringpolymers

Fermions:

- negative Boltzmann weights
- non probabilistic: NP-hard problem (Troyer, Wiese)!!!

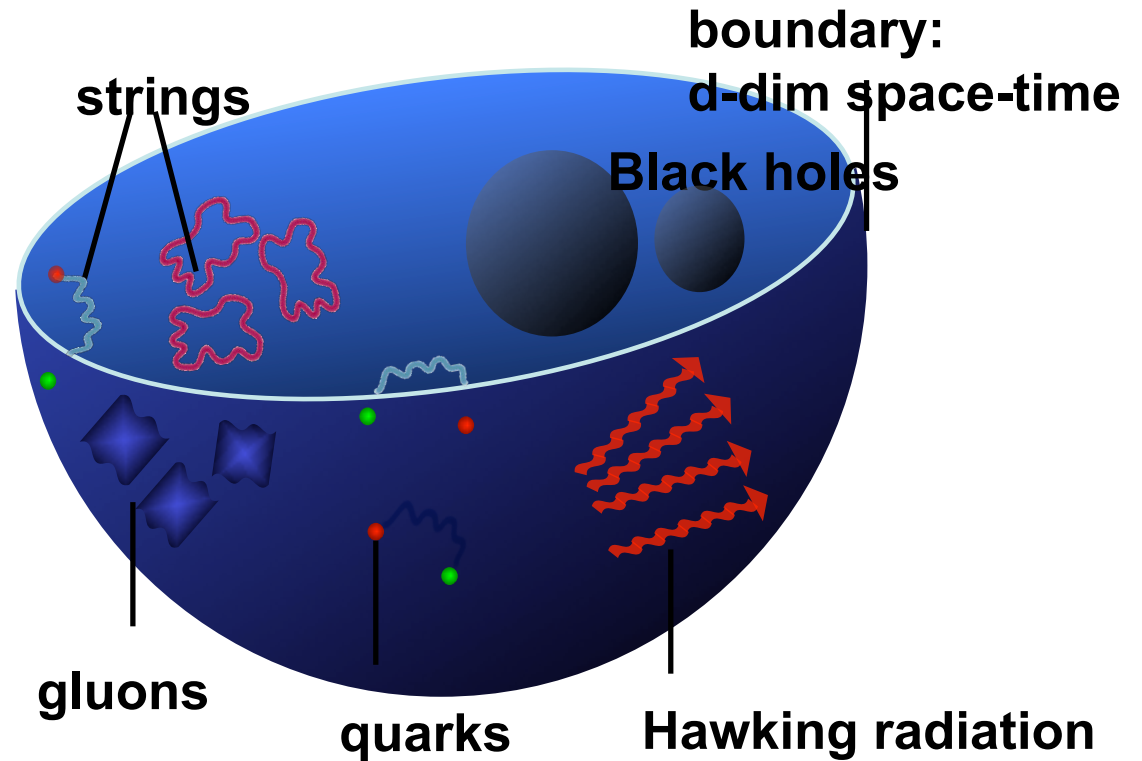


# Fermionic renormalization group



~~Wilson-Fisher RG:  
based on Boltzmannian  
statistical physics~~

## The Magic of AdS/CFT!



# Planckian dissipation

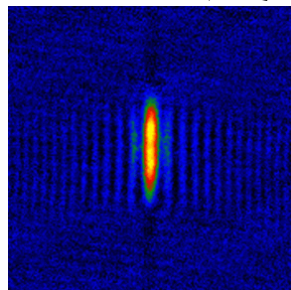
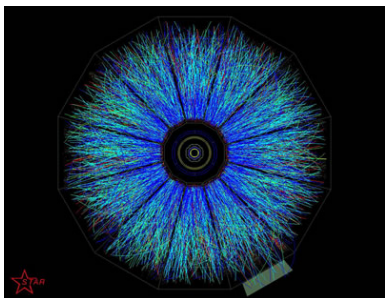


**Schwarzschild Black Hole: encodes for the finite temperature dissipative quantum critical fluid.**

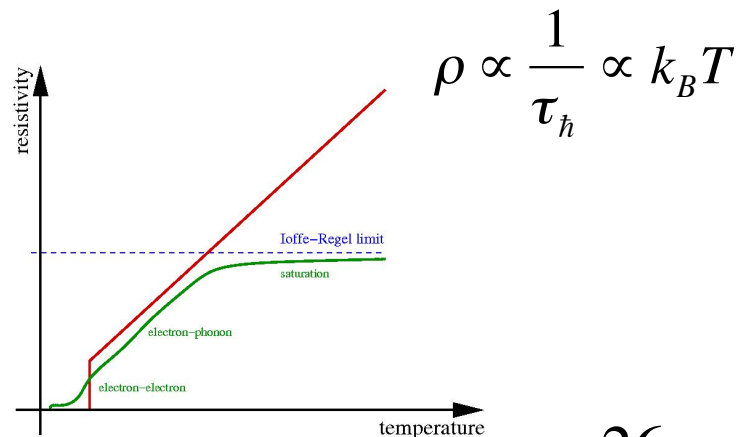
**Universal entropy production time:**

$$\tau = \tau_{\hbar} \approx \frac{\hbar}{k_B T}$$

Minimal viscosity: quark gluon plasma,  
unitary cold atom fermion gas  $\frac{\eta}{\mu} = \frac{\hbar}{4\pi k_B T}$



**Linear resistivity high Tc metals:**



# Quantum critical hydrodynamics: Planckian dissipation & viscosity

## Planckian dissipation:



Sachdev,  
1992

In a finite temperature quantum critical state the time it takes to convert work in heat (relaxation time) has to be

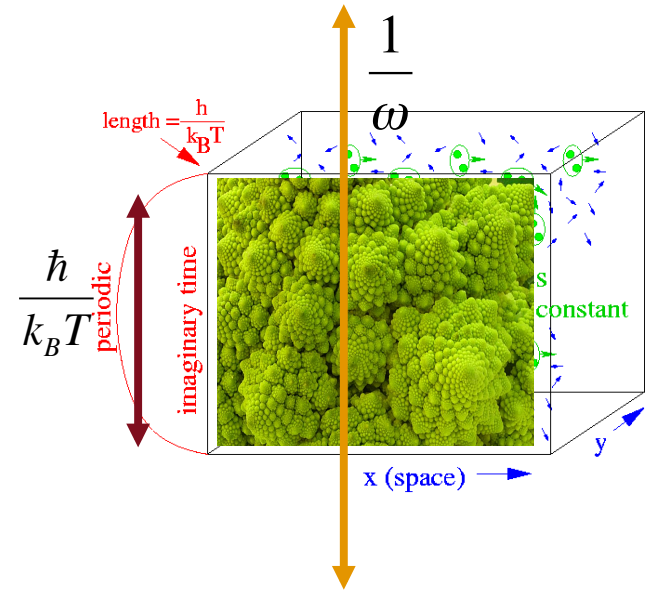
$$\tau = \tau_{\hbar} \approx \frac{\hbar}{k_B T}$$

Viscosity, entropy density:

$$\eta = (\varepsilon + p)\tau, s = \frac{\varepsilon + p}{T} \Rightarrow \frac{\eta}{s} = T\tau$$

Planckian viscosity:

$$\frac{\eta}{s} \approx \frac{\hbar}{k_B}$$

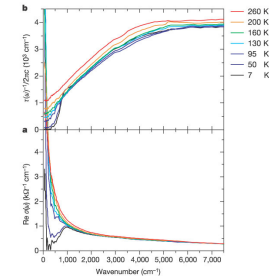


# Critical Cuprates are Planckian Dissipators



van der Marel, JZ, ... Nature 2003:

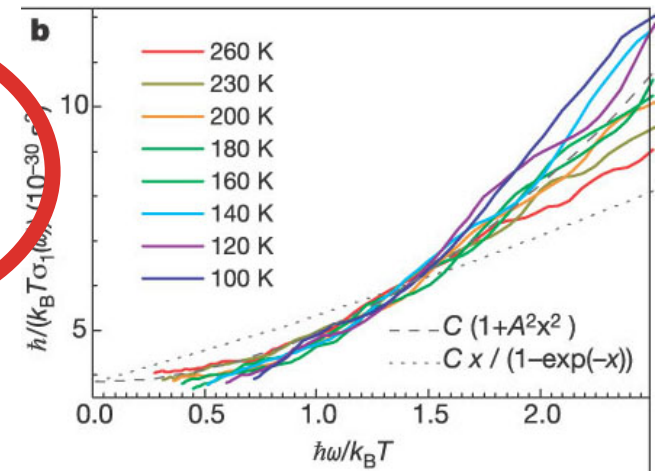
Optical conductivity QC cuprates



Frequency less than temperature:

$$\sigma_1(\omega, T) = \frac{1}{4\pi} \frac{\omega_{pr}^2 \tau_r}{1 + \omega^2 \tau_r^2}, \quad \tau_r = A \frac{\hbar}{k_B T}$$

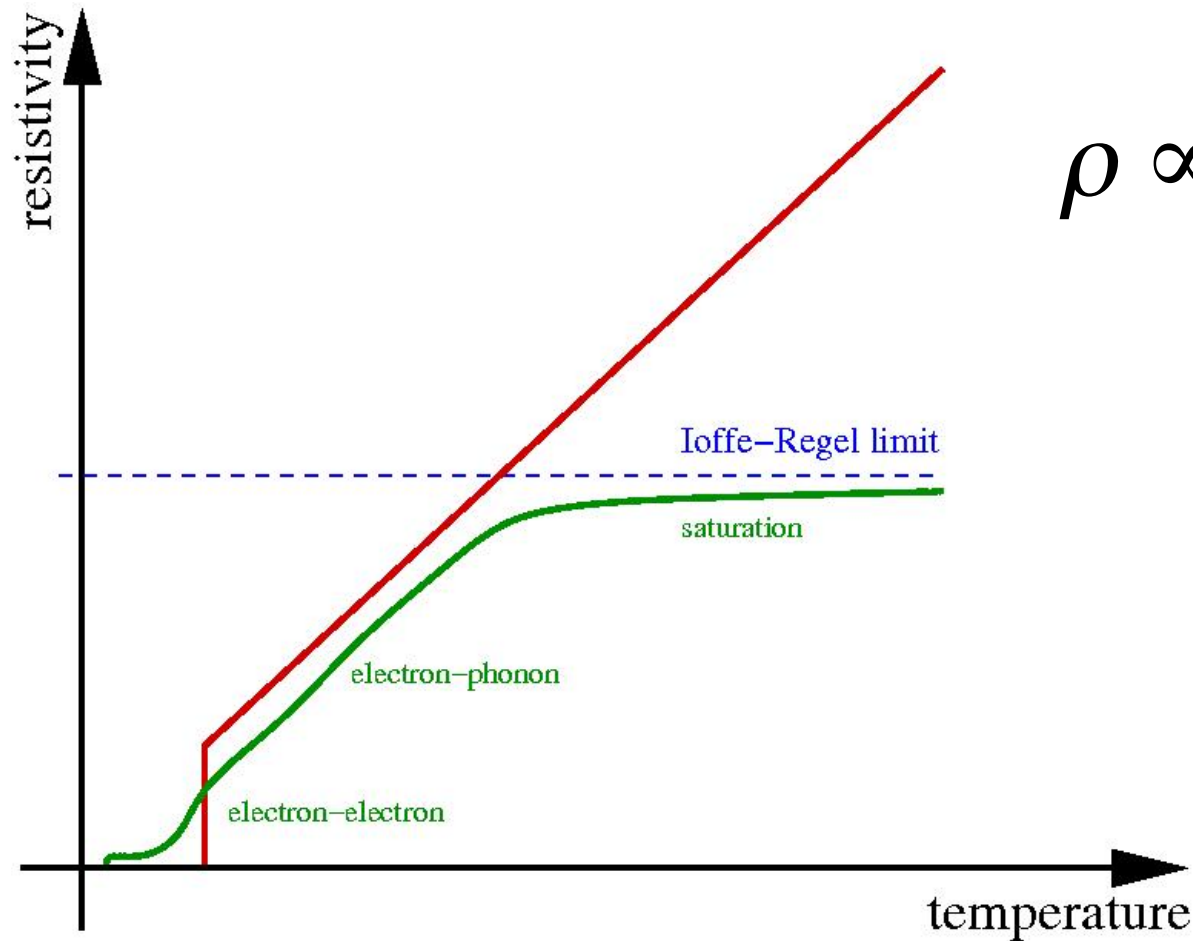
$$\Rightarrow \left[ \frac{\hbar}{k_B T \sigma_1} \right] = \text{const.} \left( 1 + A^2 \left[ \frac{\hbar \omega}{k_B T} \right]^2 \right)$$



**A=0.7: the normal state of optimally doped cuprates is a Planckian dissipator!**

# Divine resistivity = Planckian Dissipation!

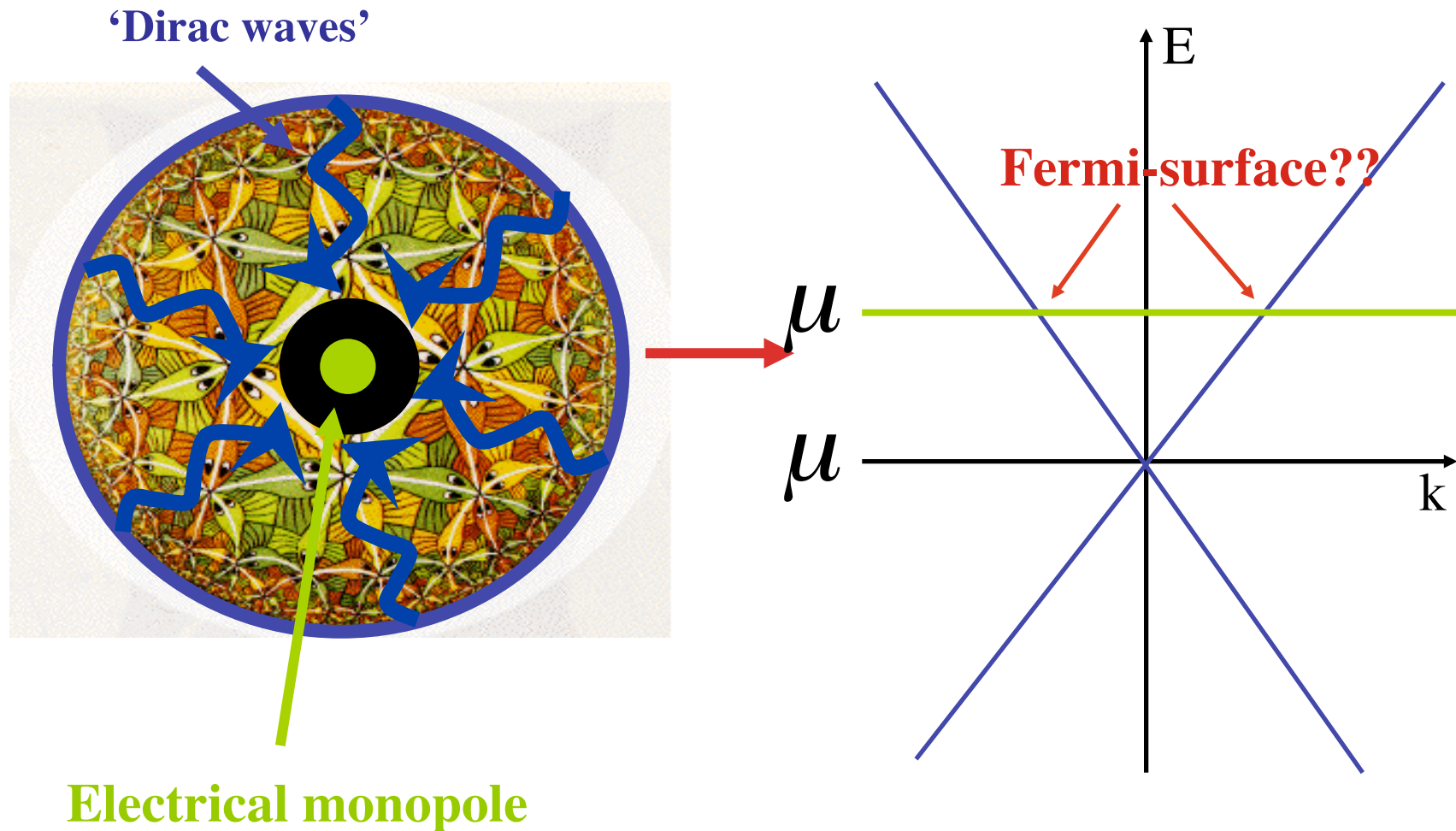
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$$\rho \propto \frac{1}{\tau_{\hbar}} \propto k_B T$$

# Breaking fermionic criticality with a chemical potential

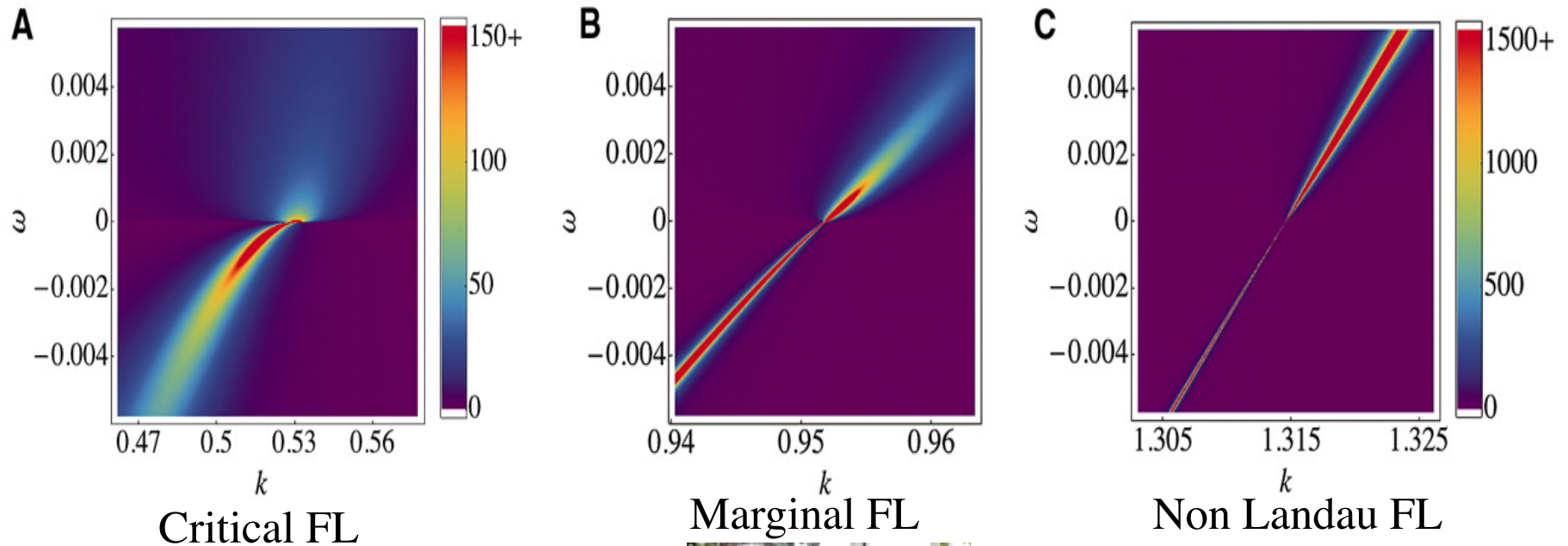
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# AdS/ARPES for the Reissner-Nordstrom non-Fermi liquids

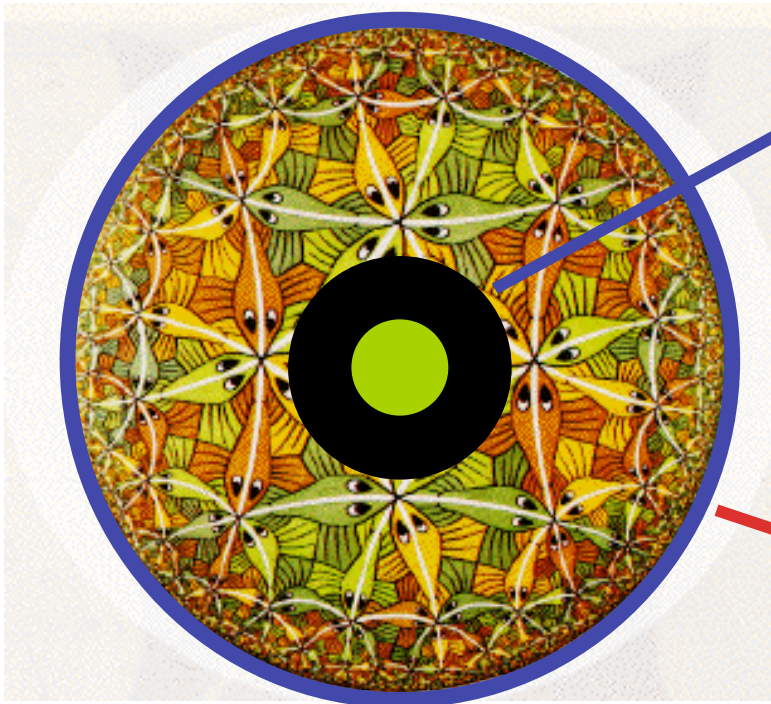
**Fermi surfaces but no quasiparticles!**





# The zero temperature extensive entropy ‘disaster’

---



The ‘extremal’ charged black hole with  $\text{AdS}^2$  horizon geometry has zero Hawking temperature but a finite horizon area.

**AdS-CFT**

The ‘seriously entangled’ quantum critical matter at zero temperature should have an extensive ground state entropy (?\*##!!)

# Why is $T_c$ high?

---

“Because there is superglue binding the electrons in pairs”

**Wrong!**

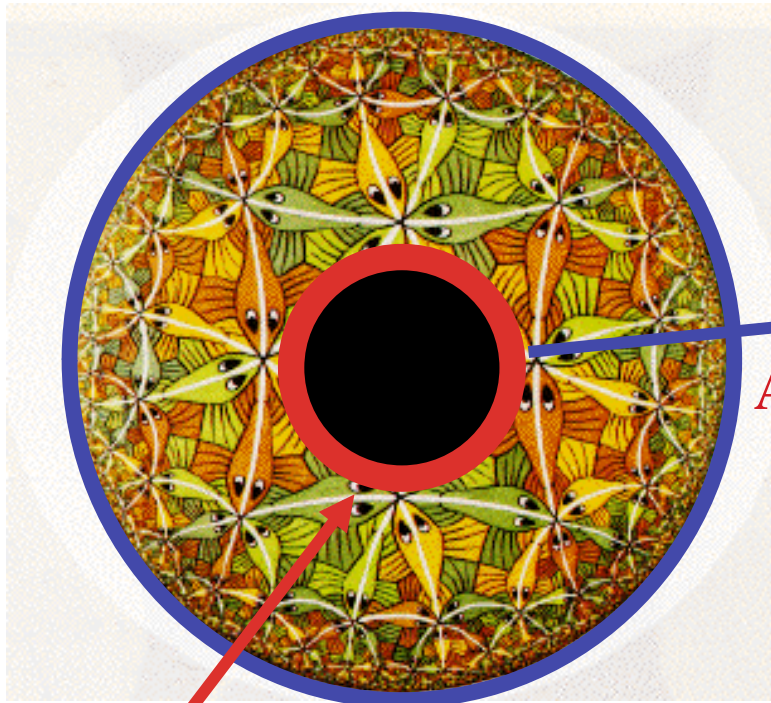
The superfluid density is tiny, it is very easy to ‘bend and twist’ a high  $T_c$  superconductor. **Its cohesive energy sucks.**

$T_c$ 's are set by the competition between the two sides ...

**The theory of the mechanism should explain why the free energy of the metal is seriously BAD.**

# The holographic superconductor

Gubser; Hartnoll, Herzog, Horowitz



(Scalar) matter ‘atmosphere’

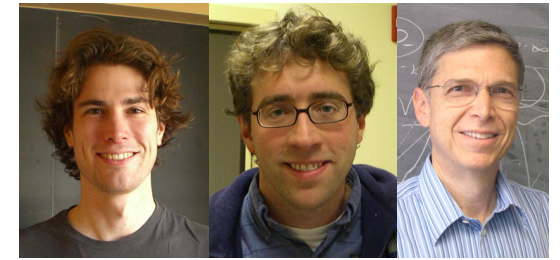
Condensate (superconductor, ... ) on the boundary!



AdS-CFT

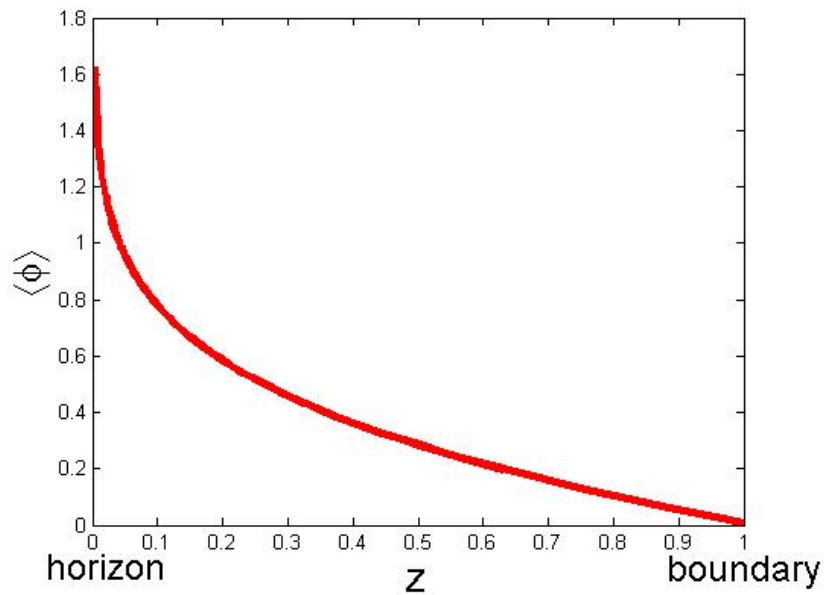
‘Super radiance’: in the presence of matter the extremal BH is unstable => zero T entropy always avoided by low T order!!!

# The Bose-Einstein Black hole hair

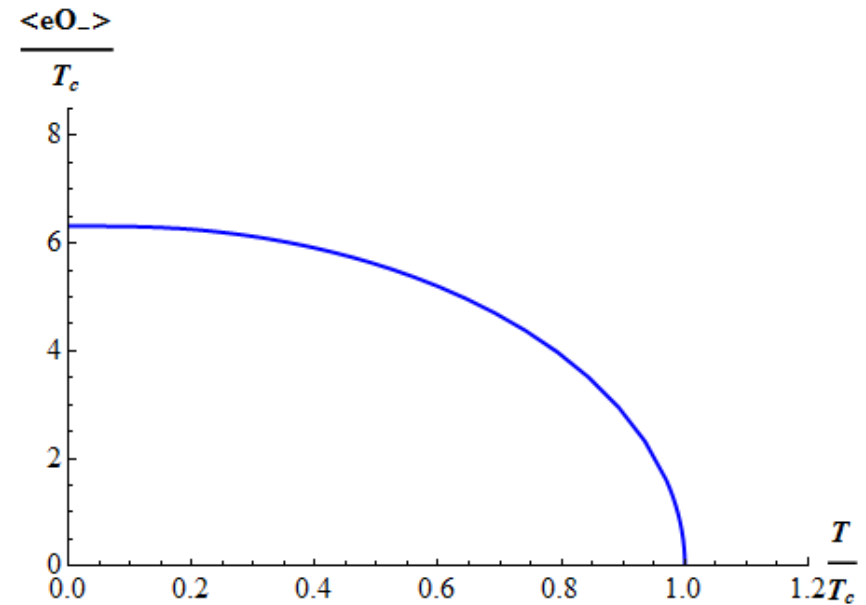


Hartnoll Herzog Horowitz

Scalar hair accumulates at the horizon

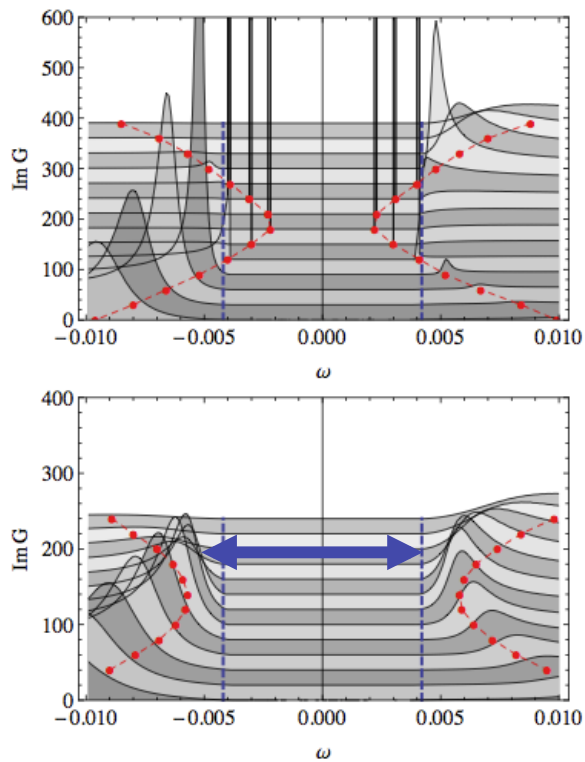


Mean field thermal transition.

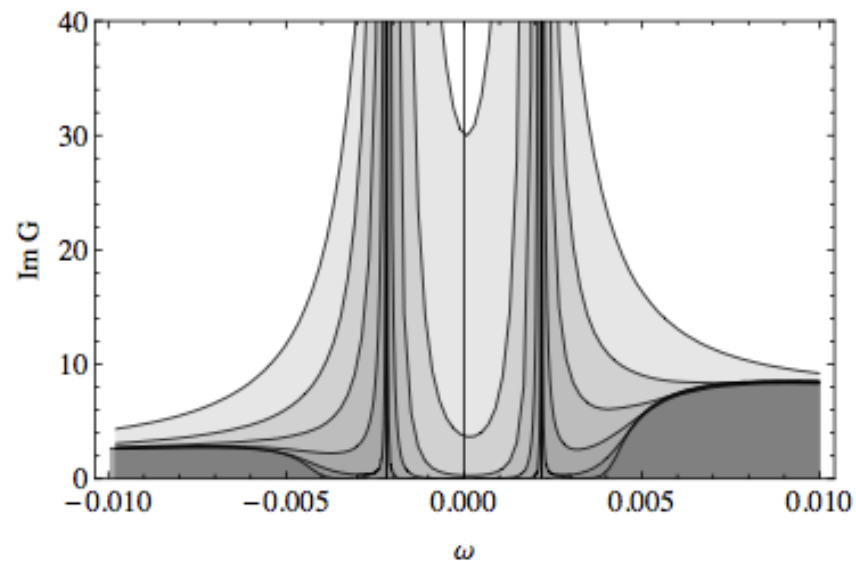


# Holographic superconductivity: stabilizing the fermions.

Fermion spectrum for scalar-hair black hole (Faulkner et al., 911.340):



‘BCS’ Gap in fermion  
spectrum !!



‘Pseudogap’ Temperature dependence

# The top-down holographic superconductors

---



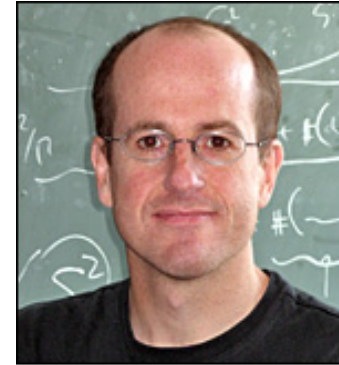
Erdmenger et al.:

D3/D7 brane  
intersections,  
(arXiv:0810.2316)



Gubser et al.:

type II sugra  
(arXiv:0907.3510)



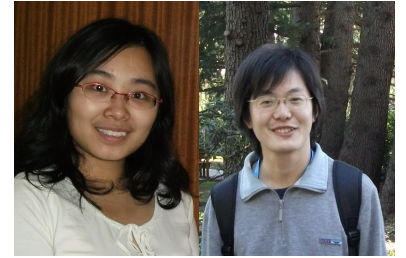
Professor Jerome Gauntlet

Gauntlett et al.:

M-theory, Sasaki-  
Einstein (arXiv:  
0907.3796).



# Holographic superconductivity: the equations



Sun

Liu

“**Double trace**”: Roberts, Faulkner, Horowitz, arXiv:1008.1581

Bulk action:

$$S = \int d^4x \sqrt{-g} \left[ R + \frac{6}{L^2} - \frac{1}{4} (1 + g_0 |\Psi|^2) F_{\mu\nu} F^{\mu\nu} - m^2 |\Psi|^2 - |\nabla^\mu \Psi - ie A^\mu \Psi|^2 \right]$$

R = Ricci scalar, L= AdS radius,  $F_{\mu\nu}$  Maxwell tensor,  $\Psi$  scalar field dual to pair field

Near boundary asymptotics:  $\Psi(r) \approx \frac{\psi_-}{r^{\Delta_-}} + \frac{\psi_+}{r^{\Delta_+}}, r \rightarrow \infty \quad \left( \Delta_{\pm} = \frac{3}{2} \pm \frac{\sqrt{9 + 4m^2}}{2} \right)$

**Pair susceptibility CFT:**  $\chi_{pair} = \frac{(\psi_- / \psi_+)}{1 - \kappa(\psi_- / \psi_+)}$  “**pair breaking**” interaction, like RPA!!

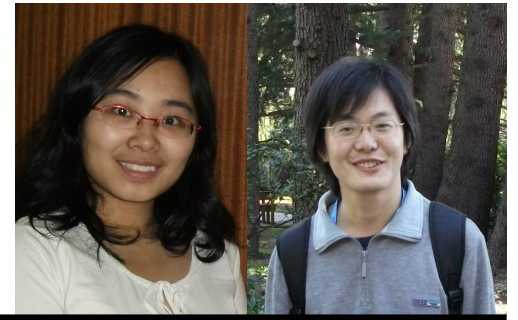
$$S_{int} \propto \kappa \int d^3x c^+ c^+ c c$$

“AdS<sup>4</sup>”:  $e \gg 1, \kappa = 0$  like “local pair SC”,  $T_c \approx \mu$

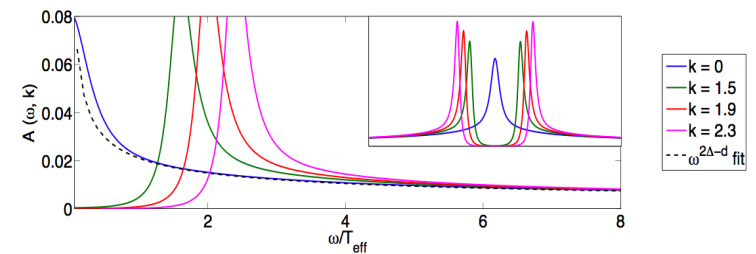
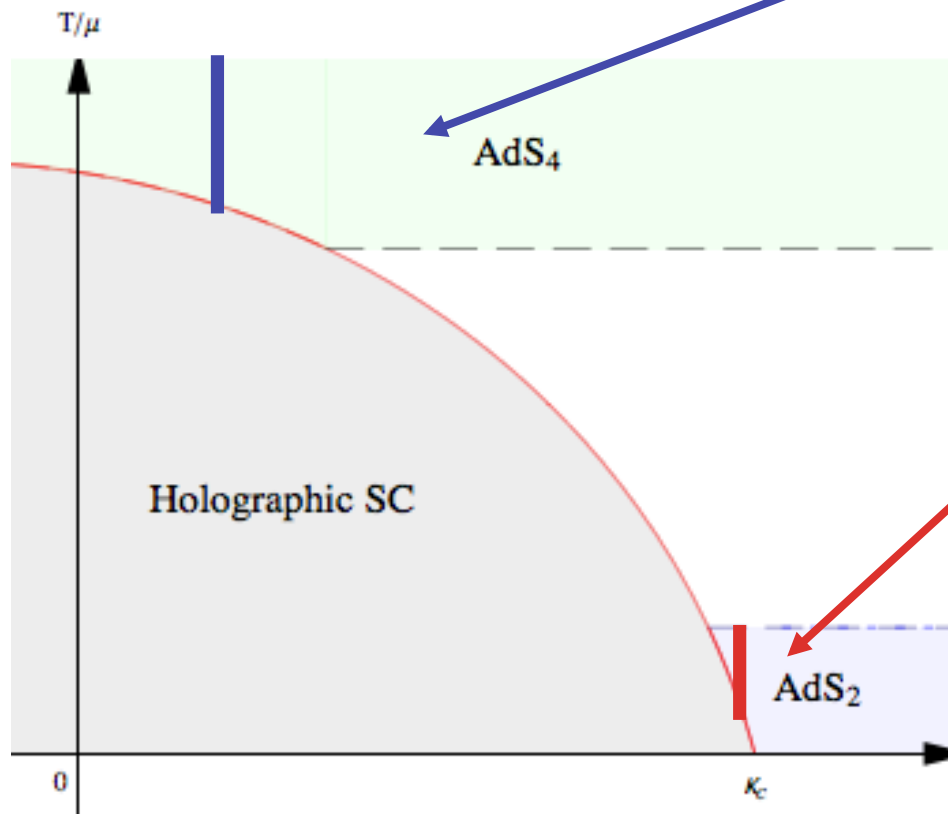
“AdS<sup>2</sup>”:  $e \ll 1, \kappa < 0$  “BCS from AdS<sup>2</sup> metal”,  $T_c \ll \mu$



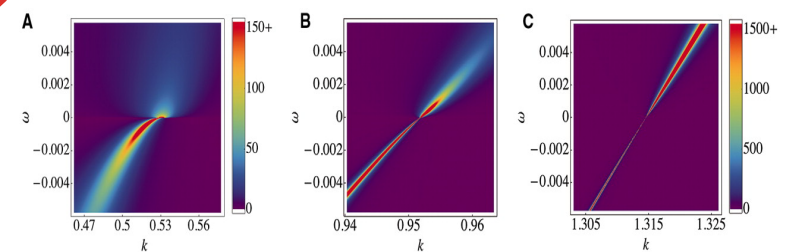
# “Double trace” Phase Diagram



This looks like “quantum critical graphene” at zero density

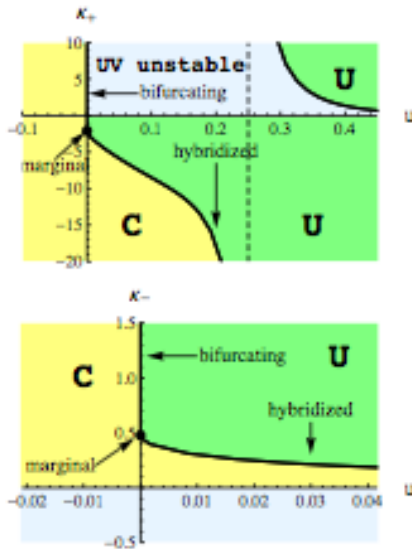


This is the “marginal Fermi-liquid” Liu style



# More fanciful: Iqbal, Liu, Mezei

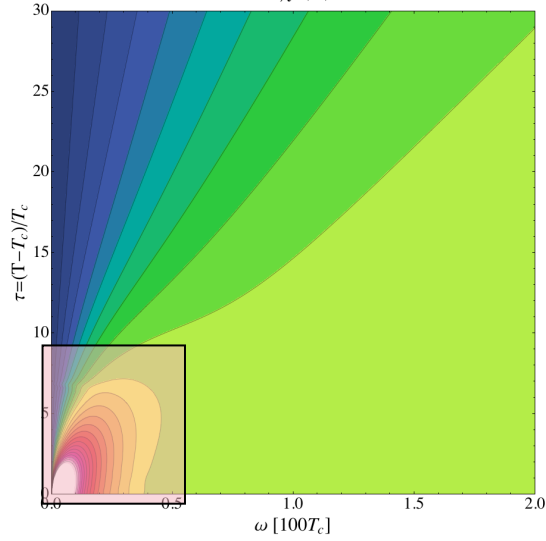
arXiv:1108.0425



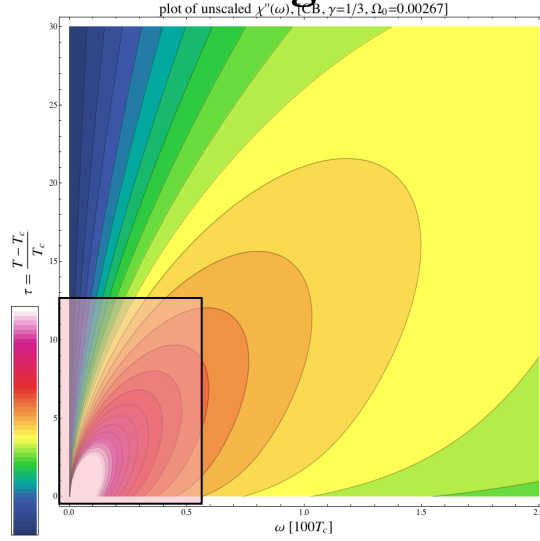
Quite different behaviors of the holographic quantum phase transitions by tuning the holographic SC down by mass or double trace deformation

FIG. 7. The full phase diagram of the system for a neutral scalar.  $C$  ( $U$ ) denotes regions with (without) IR instabilities. Top plot: standard quantization. For  $u < 0$ , i.e.  $m^2 R^2 < -\frac{1}{2}$  the system is always unstable in the IR with  $u = 0$  the critical line for a bifurcating QCP. For  $-\frac{1}{2} < m^2 R^2 < 0$ , i.e.  $0 < u < \frac{1}{4}$ , the system develops an IR instability for  $\kappa_+ < \kappa_c(m^2) < 0$  with  $\kappa_c(m^2)$  giving the critical line for hybridized QCP. The marginal critical point lies at the intersection for the critical lines for bifurcating and hybridized instabilities. The system has a vacuum UV instability for  $\kappa_+ > 0$ . For  $m^2 > 0$ , i.e.  $u > \frac{1}{4}$ , as discussed in the caption of Fig. 6, the vacuum instability is cured by finite density effect for sufficiently large  $\kappa_+$ . Bottom plot: phase diagram for the alternative quantization (for  $\nu_U \in (0, 1)$ , hence the limited range in  $u$  compared to the top plot,  $u < \frac{1}{2\nu_U}$ ), which can be obtained from that of the standard quantization by using the relation (3.4). In the vacuum, the system has an IR instability for  $\kappa_- < 0$ , i.e. with  $\kappa_- = 0$  the critical line. At a finite density the critical line is pushed into the region  $\kappa_- > 0$ .

### Standard BCS

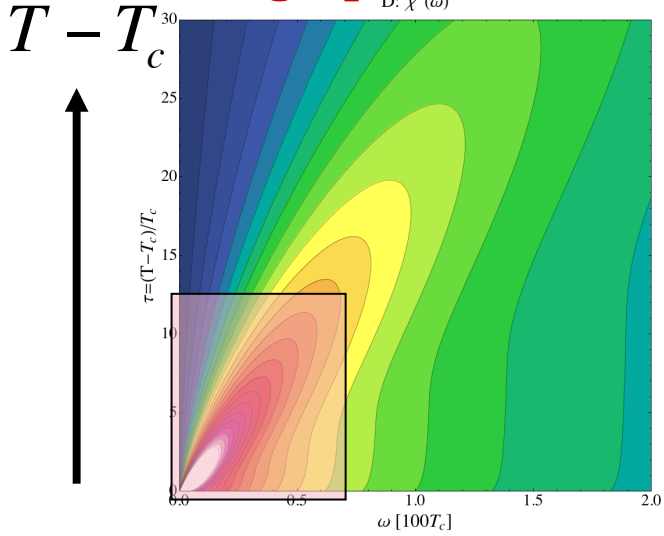


### “Critical glue”

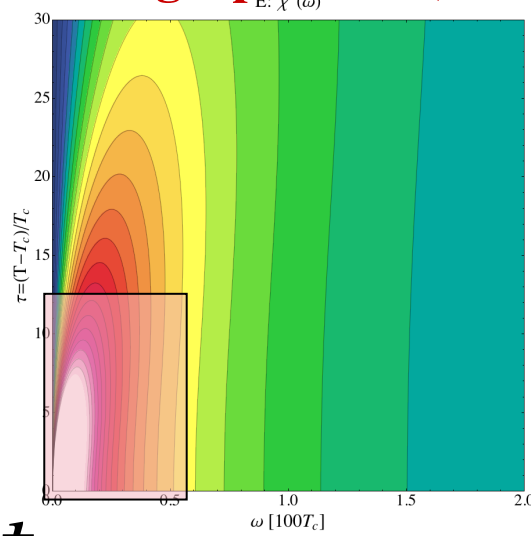


$$\chi''_p(\hbar\omega)$$

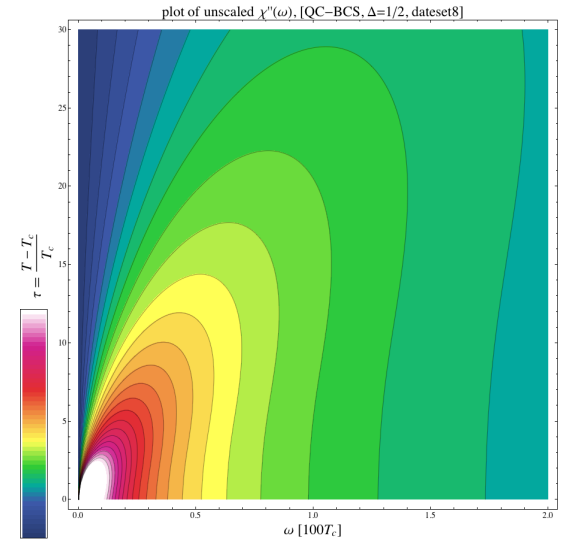
### Holographic SC (AdS4)



### Holographic SC (AdS2)



### QC-BCS



$T - T_c$

↑

→  $\hbar\omega$

# Plan

---

1. On susceptibility, quantum criticality and instability.
2. A template: BCS and Hertz-Millis-Chubukov.
3. Pair susceptibility versus holographic superconductivity.
- 4. Scaling toy model: quantum critical BCS.**
5. How to build the pairing telescope?

# Quantum Critical BCS

PRB80, 184518 (2009)



J.-H. She

Depart from BCS:

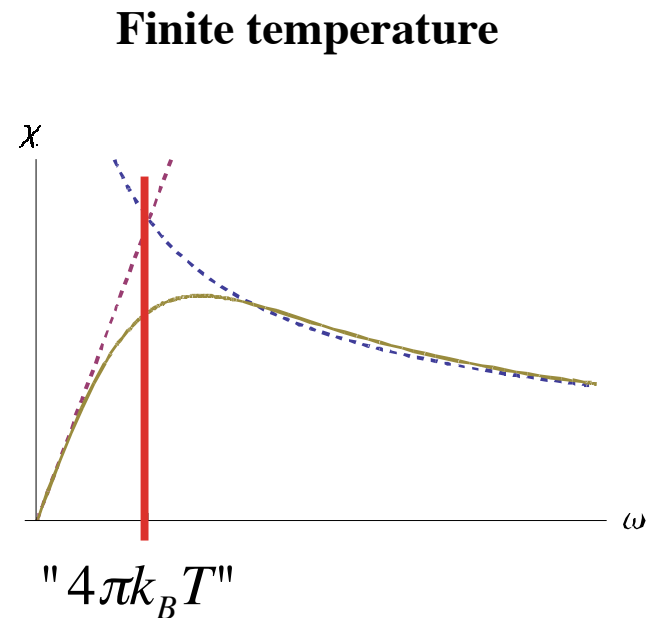
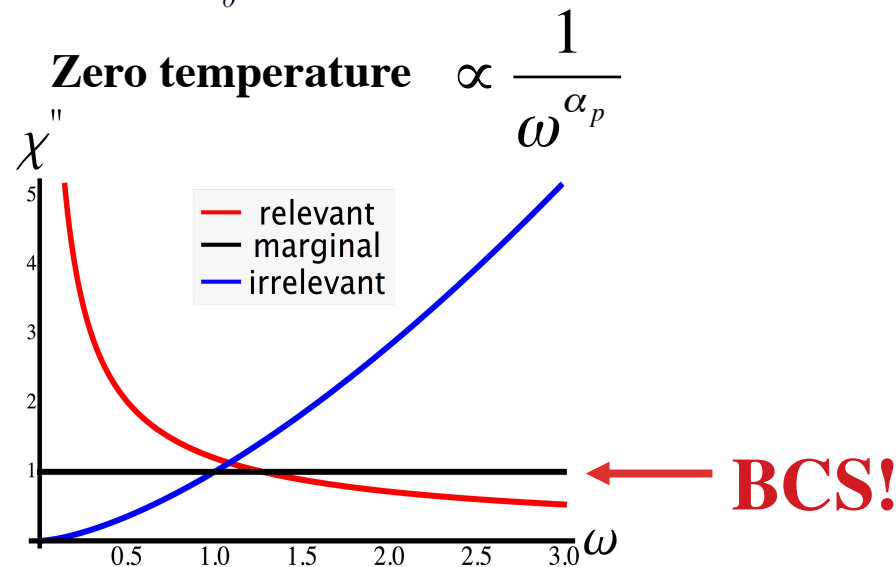
$$\chi_{\text{pair}}(\omega) = \frac{\chi_{\text{pair}}^{(0)}(\omega)}{1 - g\chi_{\text{pair}}^{(0)}(\omega)}$$

$$\lambda(i\Omega) = \frac{g}{A} \frac{\omega_b^2}{\omega_b^2 + \Omega^2}$$

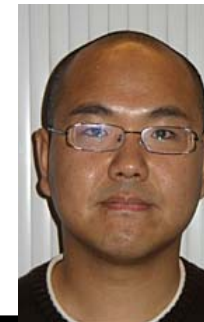
Conformal ansatz for

$$\chi_{\text{pair}}^{(0)}(\omega, T) = \frac{1}{T^{\alpha_p}} \mathcal{F}\left(\frac{\omega}{T}\right)$$

E.g. 1+1D scaling function + gymnastics to account for retardation



# Scaling versus the BCS gap



J.-H. She

Gap equation:

$$1 - \frac{g}{\omega_c} \int_{\tilde{\Delta}_0}^{2\hbar\tilde{\omega}_B} \frac{d\tilde{\omega}}{\tilde{\omega}^\alpha} = 0$$

Fermi-liquid:

$$\omega_c = E_F, \lambda = \frac{g}{E_F}, \alpha = 1 \Rightarrow \Delta_0 = 2\hbar\omega_B e^{-1/\lambda}$$

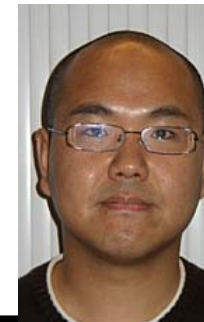
Critical case:

$$\lambda = \frac{g}{\omega_c}, \alpha = \frac{2 - \eta_{pp}}{z} \Rightarrow$$

**‘Huang’s equation’:**

$$\Delta_0 = 2\hbar\omega_B \left( \frac{\lambda}{\lambda + (2\omega_B/\omega_c)^{(2-z-\eta_{pp})/z}} \right)^{\frac{z}{2-z-\eta_{pp}}}$$

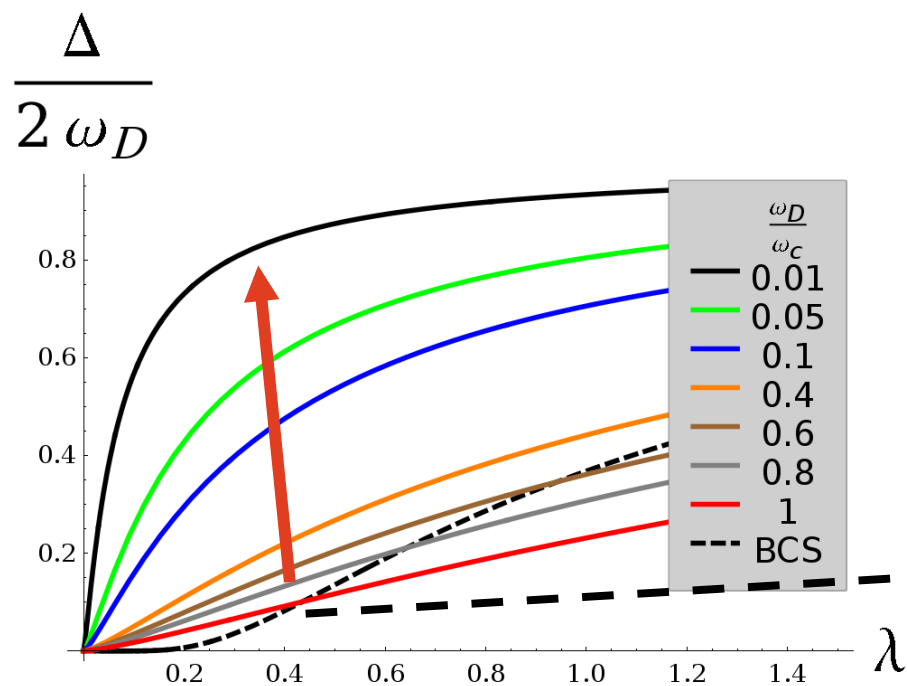
# Huang's equation at work



J.-H. She

$$\Delta_0 = 2\hbar\omega_B \left( \frac{\lambda}{\lambda + (2\omega_B/\omega_c)^{(2-z-\eta_{pp})/z}} \right)^{\frac{z}{2-z-\eta_{pp}}}$$

Strongly interacting critical state, e.g. 1+1D Ising:  $\eta_{pp} = 1/4$ ,  $z = 1$



**Increasing retardation:  
more bang for the bucks!**

Standard BCS



# Huang's equation versus high $T_c$



J-H She

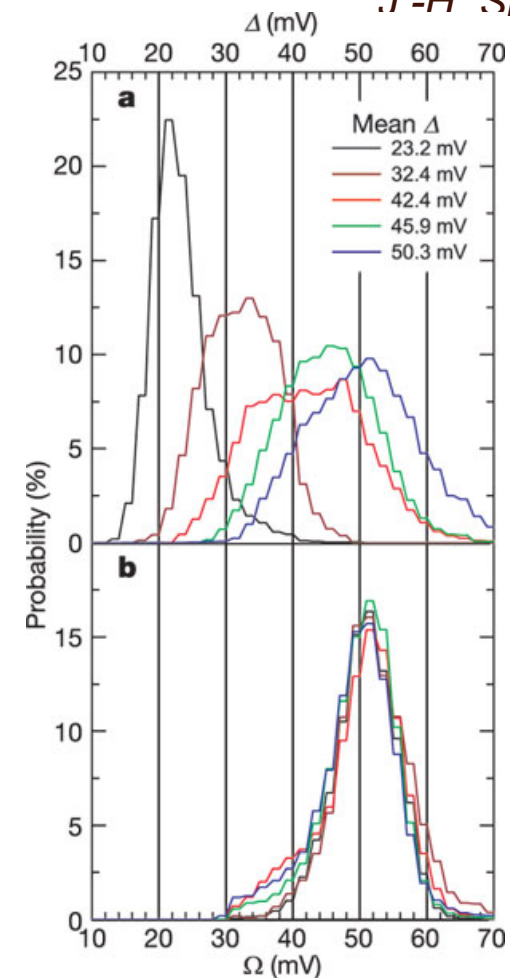
E.g. 1+1D Ising:  $\eta_{pp} = 1/4, z = 1$

Typical phonon-,  
cut-off energy:  $\frac{\omega_B}{\omega_c} = \frac{50 \text{ meV}}{500 \text{ meV}}$

Typical gap:  $\Delta_0 = 40 \text{ meV}$

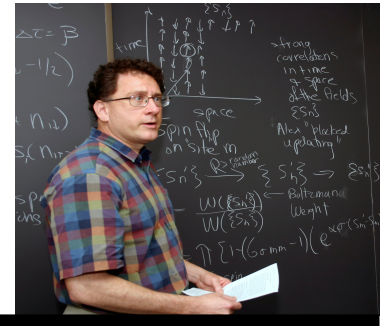
Fermi-liquid:  $\lambda \approx 1.1$

Critical case:  $\lambda \approx 0.3 !!!$

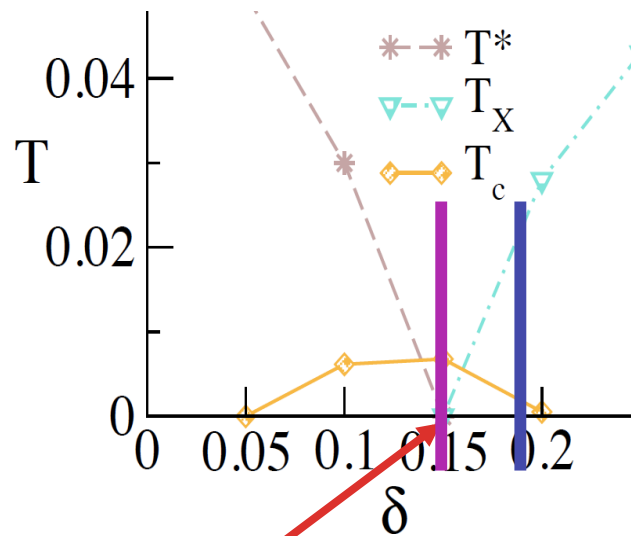


Davis, Balatsky

# Jarrell's DCA quantum criticality (PRL 106, 047004, 2011)

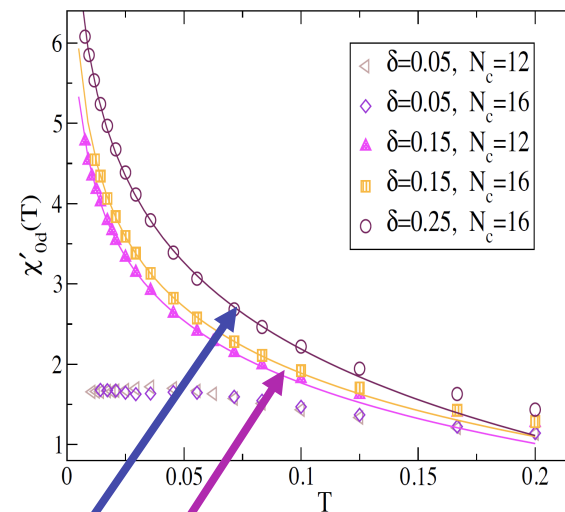


## 2D Hubbard model:



**Phase separation quantum critical end point**

## Real part "bare" pair susceptibility:



**Overdoped:**  $\chi'_{0d}(\omega = 0) = A \ln(\omega_c/T)$

**Optimally doped:**

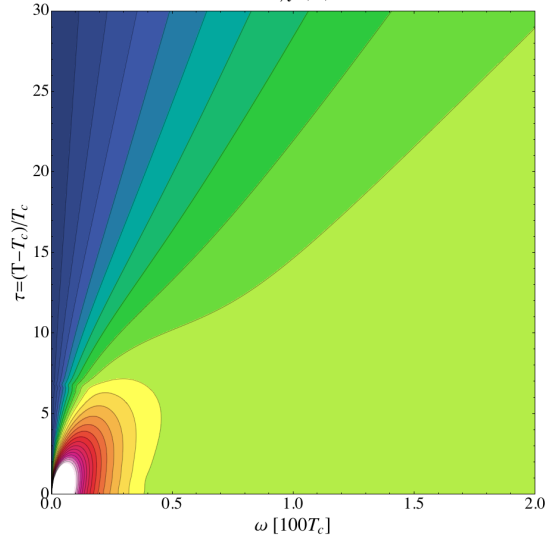
$$\chi'_{0d}(\omega = 0) = B/T^{0.5}$$

# Holographic smoking gun ...

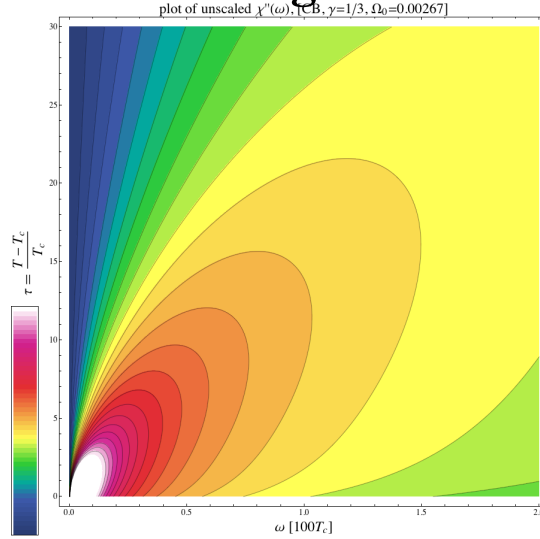
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**Check the “energy-temperature” (conformal) scaling properties of the pair susceptibility away from the superconducting transition!**

### Standard BCS

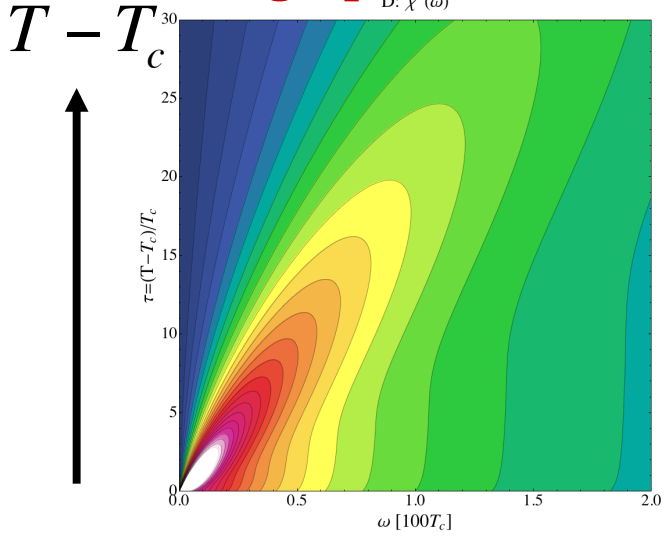


### “Critical glue”

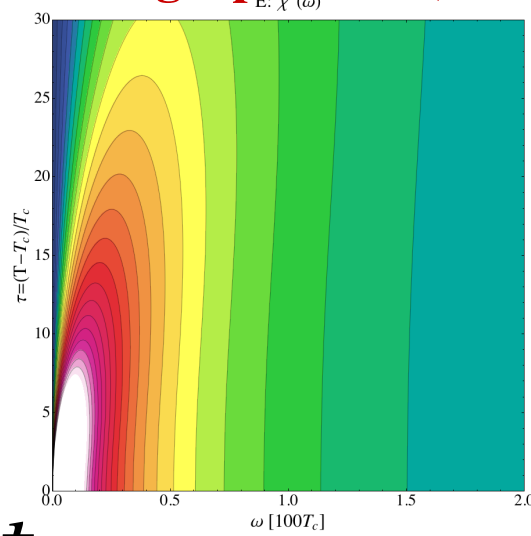


$$\chi_p''(\hbar\omega)$$

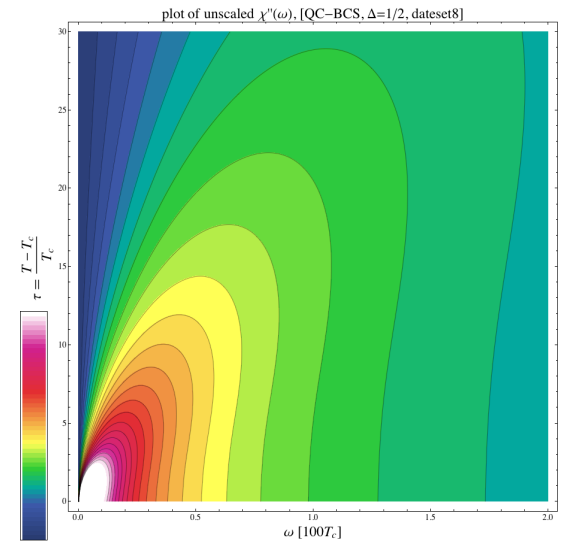
### Holographic SC (AdS4)



### Holographic SC (AdS2)



### QC-BCS



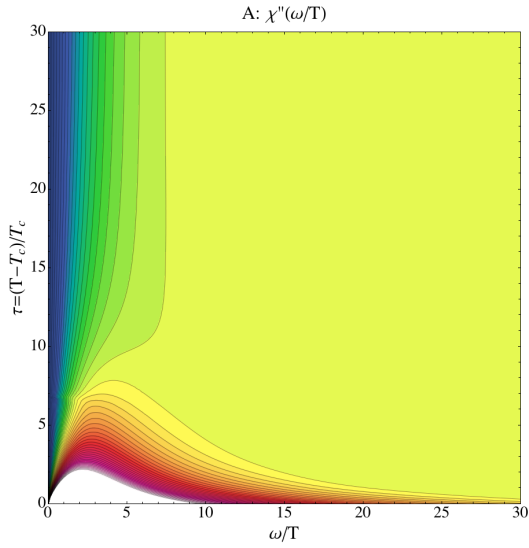
$T - T_c$

↑

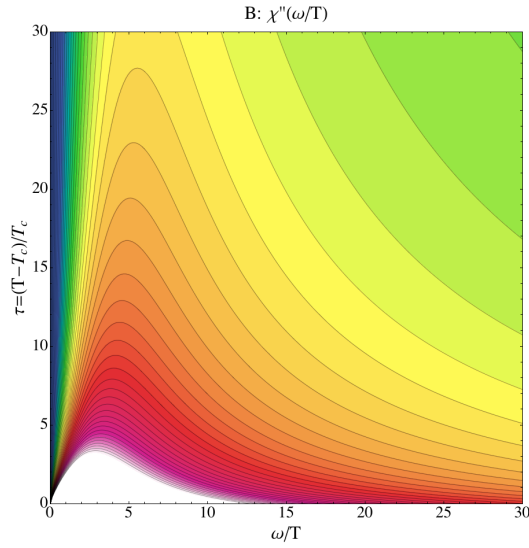
→

$\hbar\omega$

# Standard BCS

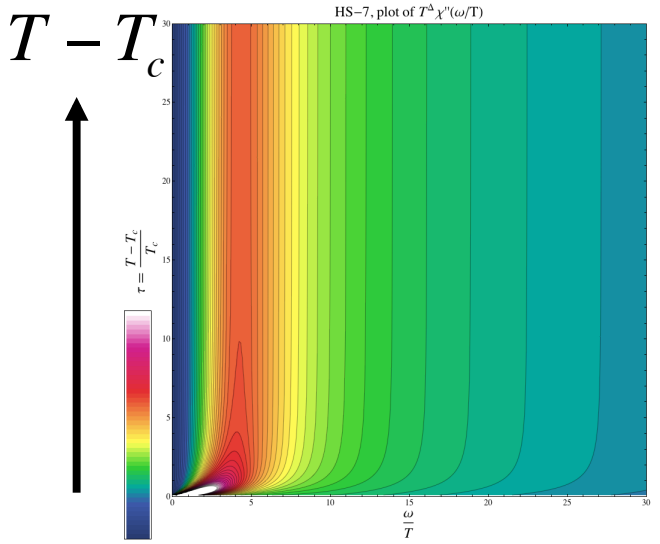


# “Critical glue”

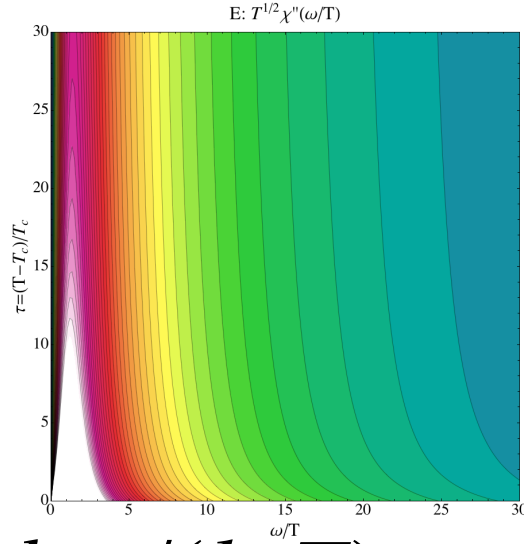


$$T^\Delta \chi_p'' \left( \frac{\hbar\omega}{k_B T} \right)$$

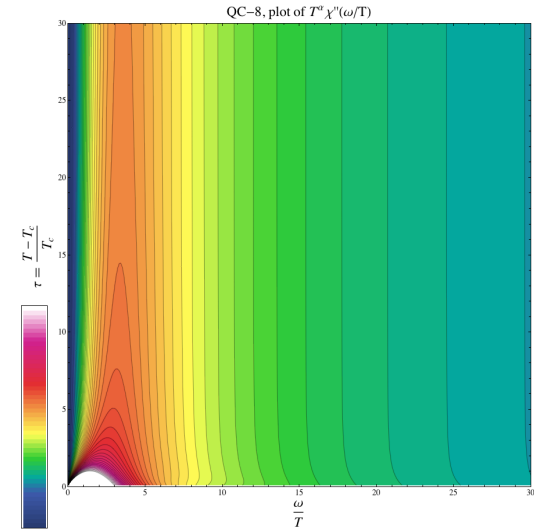
# Holographic SC (AdS4)



# Holographic SC (AdS2)



# QC-BCS



$T - T_c$



$\hbar\omega / (k_B T)$

# Holographic smoking gun ...

---

- Fermi gas shaken from below (“Hertz-Millis”): emergent conformal metal **only in deep IR**.  $T > T_c$ : non-conformal but interesting cross-over regime knowing about  $E_F$  (Metlitski talk, Chubukov, D.H. Lee, ...).

# Dress the BCS fermion loop with a marginal self energy.

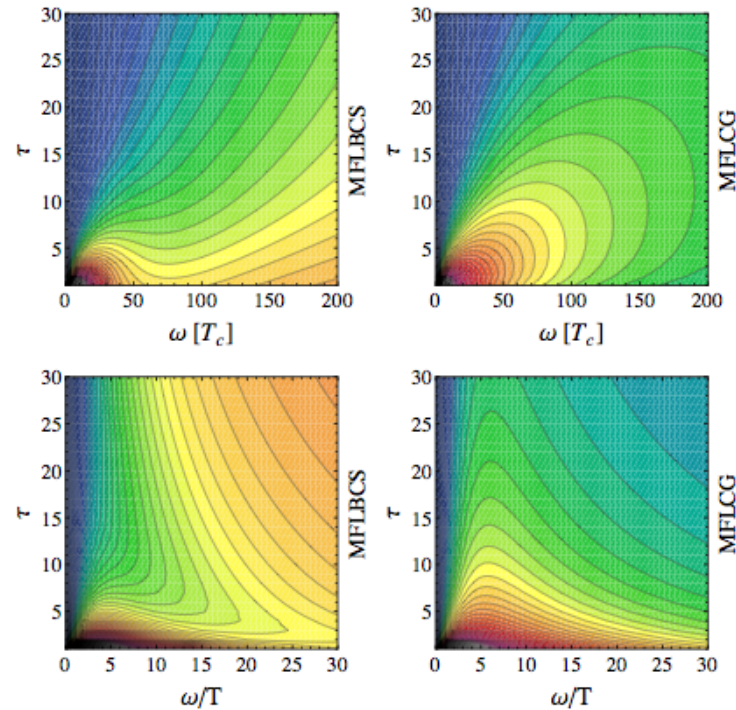


FIG. 8: (Color online) Marginal Fermi liquid pair susceptibility with smooth density of states. *Top*: False-color plot of the imaginary part of the pair susceptibility  $\chi''$  as function of frequency  $\omega$  (in units of  $T_c$ ) and reduced temperature  $\tau = (T - T_c)/T_c$ , for two different models: marginal Fermi-liquid with BCS pairing and marginal Fermi-liquid with critical glue. In both cases, the density of states is taken to be constant. *Bottom*: the same plot, but now the horizontal axis is rescaled by temperature while the magnitude is rescaled by temperature to a certain power: we are plotting  $T^a \chi''(\omega/T, \tau)$ , in order to show energy-temperature scaling at high temperatures. Here for both models  $T_c = 0.01$  and  $\delta = 0$ . The color scheme is the same as used in the main text. For MFLBCS, the parameters are  $a = 0.3, \omega_E = 1, g = 0.9627, \omega_b = 0.5$ . For MFLCG, we take  $a = 0.4, \omega_E = 0.2, \gamma = 1/3, \Omega_0 = 0.0134$ .



# Holographic smoking gun ...

---

- Fermi gas shaken from below (“Hertz-Millis”): emergent conformal metal **only in deep IR**.  $T > T_c$ : non-conformal but interesting cross-over regime knowing about  $E_F$  (Melitski talk, Chubukov, D.H. Lee, ...).
- Holographic superconductor: **robust “intermediate” conformal ( $AdS_2$ ) metal**, collective pair response (conformal !!) completely detached from single fermion (marginal FL) response. Favorite metaphor: Luttinger liquid.
- Glue or not glue? Quantum critical BCS as poor man’s double trace deformation ...

# Conformal metal versus the “quantum critical BCS” glue

The width of the relaxational peak knows about the **glue scale**:

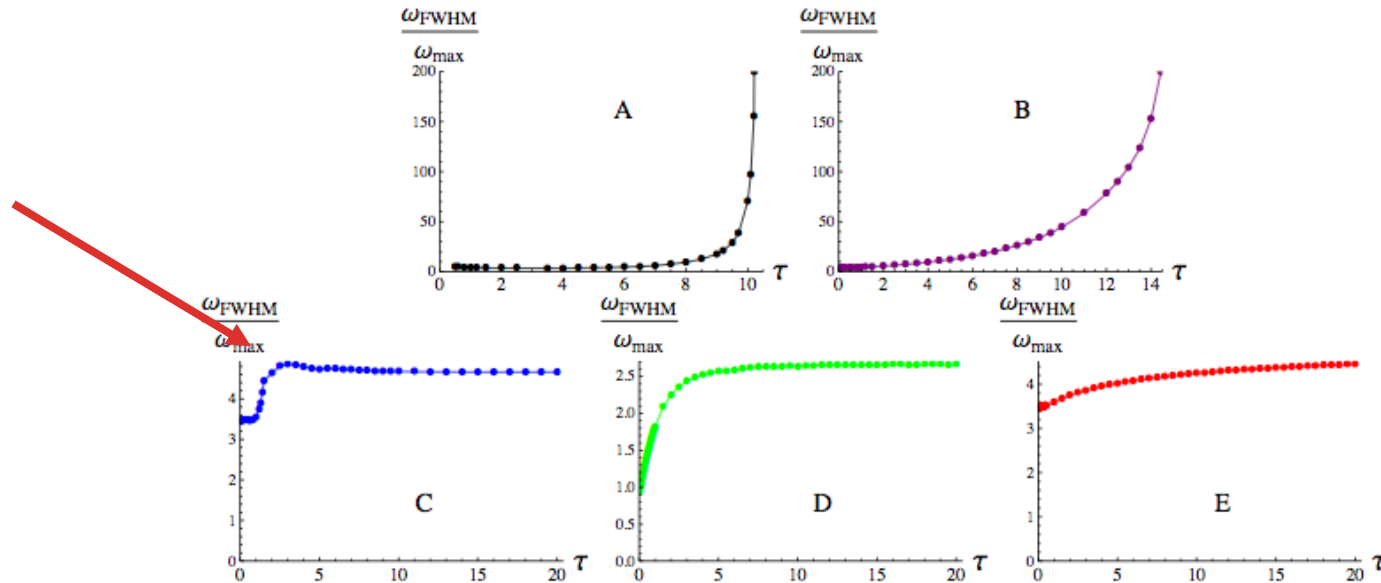


FIG. 7: (Color online) **Peak width crossover.** Evolution of the relative peak width, i.e., the ratio of the full width at half maximum (FWHM) of the peak and peak location  $\omega_{\text{max}}$ , as a function of reduced temperature  $\tau = (T - T_c)/T_c$  for the five different models. For FLBCS (A) and CGBCS (B), the ratio diverges at high temperature. For QCBCS (C) there is a sudden change from the low temperature relaxational behavior to the high temperature conformal field theory behavior. For the two holographic superconductors (D–E), the crossover from high temperature region to low temperature region is more smooth.

# Holographic smoking gun ...

---

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**Experiment can tell the difference!**

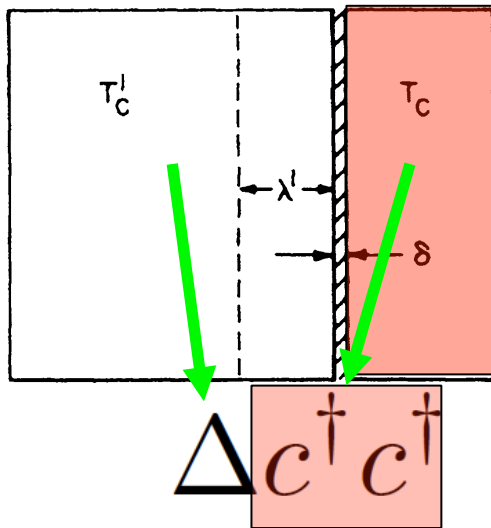
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- 5. How to build the pairing telescope?**

# Observing the origin of the pairing mechanism

SUPERCONDUCTOR 2 SUPERCONDUCTOR 1



$$T'_c > T > T_c$$

**2<sup>nd</sup> order Josephson effect**



Ferrell Scalapino

1969

1970

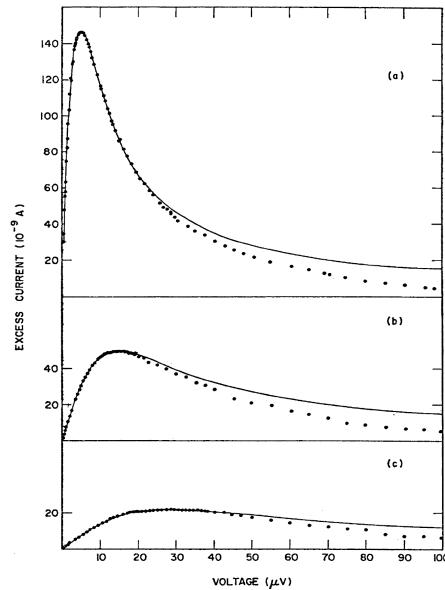
$$I_s(\mathbf{H}, V) \sim \frac{1}{R_N^2} \text{Im} \chi_{\text{pair}}(\mathbf{k}, \omega)$$

$$\omega = 2eV$$

# Proof of principle ....

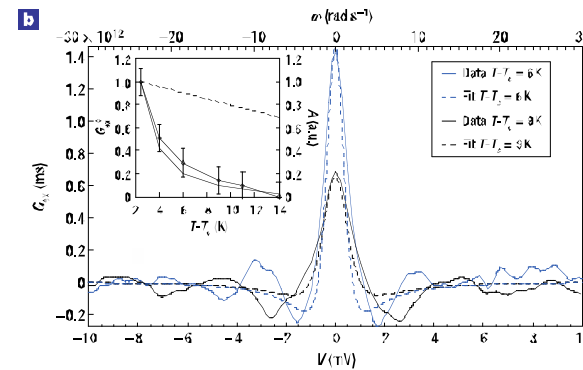
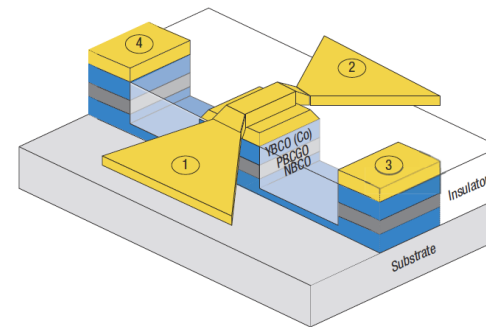


**Al-Pb junction: “Relaxational peak” Al near the BCS transition**



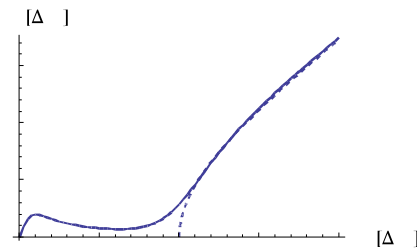
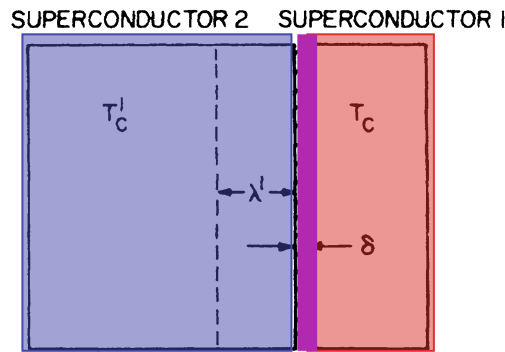
$$I_s(V) = \frac{4eA|C|^2}{dN_0\varepsilon} \frac{\omega/\Gamma_0}{1 + (\omega/\Gamma_0)^2}$$

**Recent: 60K-90K cuprate superconductors (Bergeal, Nature Physics 2008).**



Goldman  
1970's

# Why **Webb** Pairing telescope?



$$I_{tun}(V) = I_{qp}(V) + I_{pair}(V)$$

## **QC metal:**

Need large dynamical range:

$$T, \omega \propto 10 - 100 T_c$$

QC superconductor at ambient conditions with low  $T_c$ :

**CeIrIn<sub>5</sub>,  $T_c = 0.4K$**

## **Probe superconductor:**

High  $T_c$

Tunneling into d(?) -wave channel

**Cuprate ?**

Full gap to suppress QP current (?)

**MgB<sub>2</sub> ( $T_c=40K$ )?**

**Barrier is the challenge!**



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**Experiment can tell the difference!**

# Further reading

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## **AdS/CMT tutorials:**

J. Mc Greevy, arXiv:0909.0518; S. Hartnoll, arXiv:0909.3553

## **AdS/CMT fermions:**

Hong Liu et al., arXiv:0903.2477,0907.2694,1003.0010; M. Cubrovic et al. Science 325,429 (2009), arXiv:1011.xxxx; T. Faulkner et al., Science 329, 1043 (2010).

## **Condensed matter:**

**High Tc:** J. Zaanen et al., Nature 430, 512, Nature Physics 2, 138; C.M. Varma et al., Phys. Rep. 361, 267417

**Heavy Fermions:** J. Zaanen, Science 319, 1205; von Loehneisen et al, Rev. Mod. Phys. 79, 1015