(Non-)Fermi Liquids and Emergent Quantum Criticality from gravity

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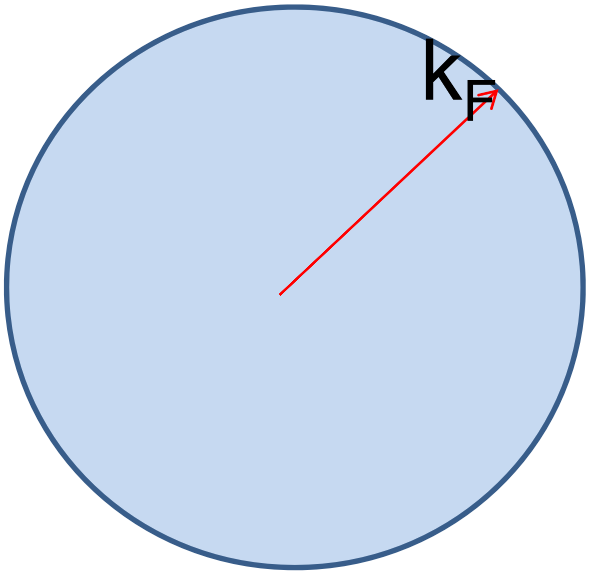
Thanks to Senthil for many patient and inspiring discussions

Fermi Liquids: RG perspective

Polchinski, Shankar



Landau Fermi Liquid: fixed point of LEEF around a Fermi surface.

stable, modular BCS instability

Free fermion CFT at each point of the Fermi surface.

RG: non-Fermi liquids (with other nontrivial fixed points) inevitable.

Theory: Luttinger liquid (2d), coupling to gauge field(s), …….

Experiment: normal state of high Tc cuprates, heavy fermions …

Quasi-particles

Key concept in Landau theory:

Low energy excitations near a Fermi surface

Weakly interacting quasi-particles

→ Thermodynamics, kinetic theory (transport)

appear as poles in the single-particle Green function:

$$G_R(t,\vec{x}) = i\theta(t)\langle\{\psi(t,\vec{x}),\psi(0,0)\}\rangle$$

$$G_R(\omega,\vec{k}) = \frac{Z}{\omega - v_F k_\perp + i\Gamma} + \cdots, \quad \Gamma \propto \omega^2$$

$$A(\omega,\vec{k}) \equiv \mathrm{Im}\, G_R(\omega,\vec{k}) \overset{k_\perp \to 0}{\sim} Z\delta(\omega - v_F k_\perp)$$ with Z finite

Non-Fermi liquids

A sharp Fermi surface still exists.

But quasi-particle picture breaks down generically

Example: normal state of optimally doped cuprates

Anomalous thermodynamic and transports properties

In the phenomenological marginal Fermi liquid description

Z vanishes as $\frac{1}{|\log\omega|}$ as the Fermi surface is approached

Example: at the critical point for a continuous metal-insulator transition

Z has to vanish on Fermi surface

What is the basic principle for NFL?

Suppose LEEF near a FS is controlled by a nontrivial fixed point:

What do we need to know about the fixed point to characterize:

nature of low energy excitations,
spectral functions,
thermodynamics
transport

What should be the organizing principle for NFL?

Can AdS/CFT help?

Can we find new examples of non-Fermi liquids?

If yes, can it yield clues to an organizing principle?

Note:

While it would be nice to find gravity description of real-life systems,

it might be difficult, if possible at all, in short term.

This might not be necessary.

AdS/CFT correspondence

Maldacena (1997), Gubser, Klebanov, Polyakov, Witten

Certain d-dimensional conformal field theory ⟺ A string theory in (d+1)-dimensional anti-de Sitter spacetime

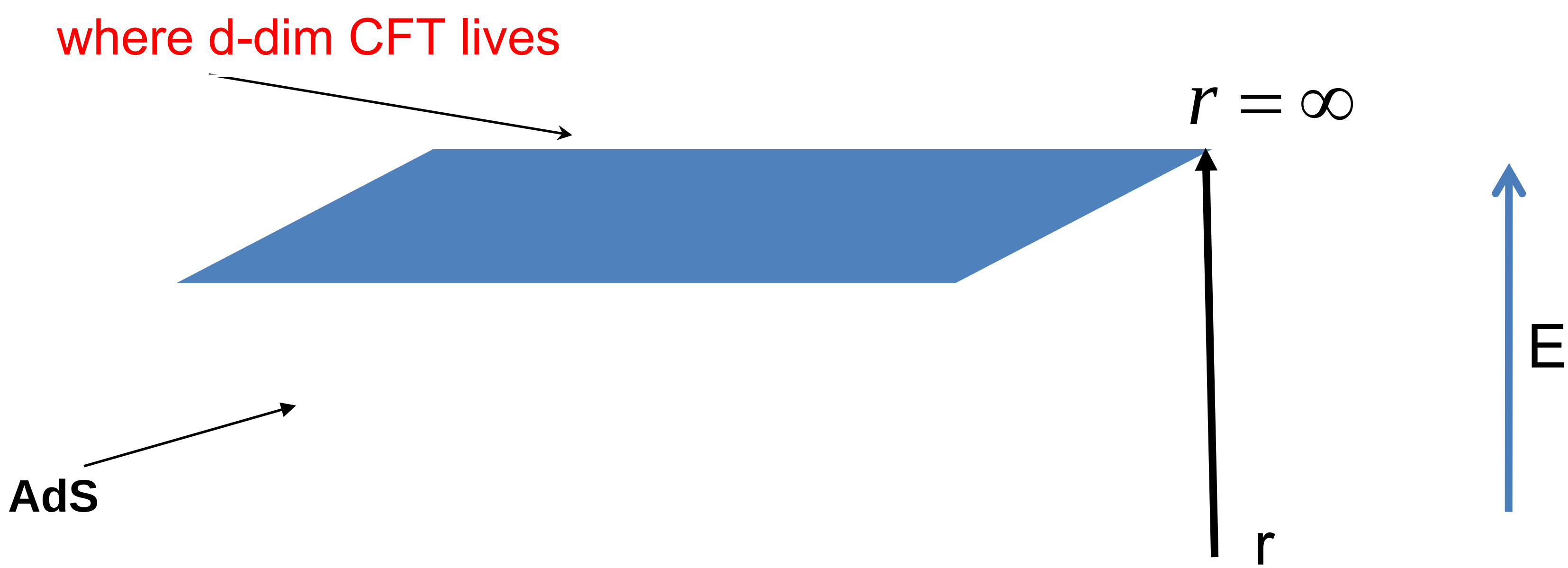
Many examples in different dimensions are known including non-conformal ones.

Conformal symmetries ⟺ AdS isometries

global symmetries ⟺ gauge symmetries

Large N, strongly coupling ⟺ classical gravity

IR/UV connection

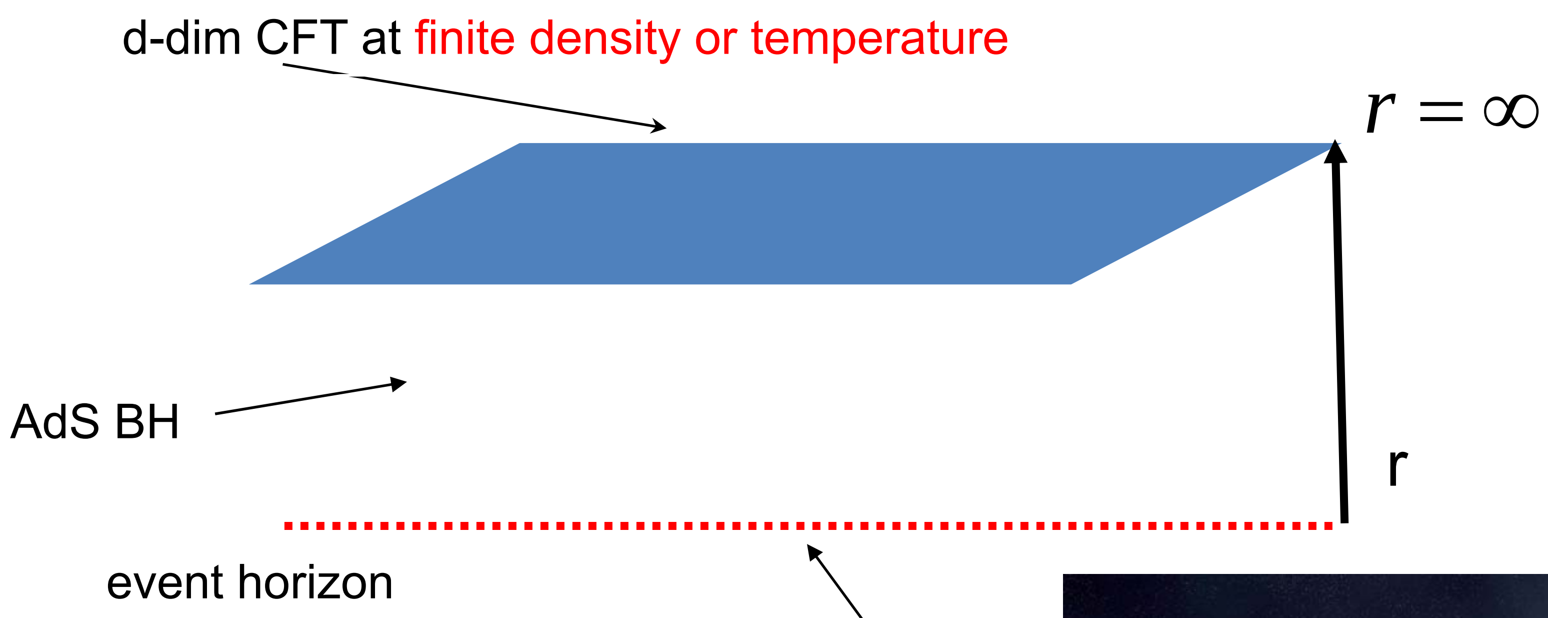


CFT lives at the boundary of AdS.

Near the boundary
reflects UV physics of the boundary field theory

Deep in the interior:
reflects IR physics of the boundary field theory

Finite density/temperature



Putting a black hole in the center of AdS

Gravity paradigm for many-body physics

1. Many body → single or few body problem in BH

2. Highly QM, strong coupling phenomena → geometry

Thermodynamics and transport without using quasi-particles,

But from geometry

3. Large N, strong coupling limit, all CFT ⟺ A universal sector of string theory: Einstein gravity plus matter fields

Search for (non)-Fermi liquids from gravity

Strategy

Take a theory with a gravity dual, fermions and a U(1) global symmetry. Put it at a finite charge density.

At T=0:

Gravity side: extremal charged BH in AdS_4

$$ds^2 = r^2(-f dt^2 + dx_1^2 + dx_2^2) + \frac{dr^2}{r^2 f}$$

$$f = 1 + \frac{3}{r^4} - \frac{4}{r^3}, \qquad A_t = \mu \left(1 - \frac{1}{r}\right)$$

μ: chemical potential horizon: $r = 1$

What kind of quantum liquid is that ?
Fermi surface? (non)-Fermi liquid?

To look for a Fermi surface, we search for sharp features at finite momentum in fermionic Green functions.

S-S Lee

$O \Longleftrightarrow \psi$

$\Delta \Longleftrightarrow \frac{3}{2} + m$ (standard quantization)

q $\Longleftrightarrow$ q

Two-point retarded function for O $\Longleftrightarrow$ Solving Dirac equation for ψ, extracting boundary values

Universality of 2-point functions:

do not depend on which specific theory and operator we use. Results will only depend on charge q and dimension Δ .

Still a few words on the type of theories we study:

1. Many known, yet many many many more believed to exist, but not known explicitly. Examples:

   d=3: M2 brane theory, ABJM . ….

   d=4: $\mathcal{N}$=4 SYM, Klebanov-Witten, ….

2. contain both elementary bosons and fermions coupled to non-Abelian gauge fields (classical gravity: N→∞ limit)

3. Non-vanishing ground state entropy in the large N limit

4. Very large, complicated system

   We are probing a tiny part of it.

Spinor retarded functions from gravity

Son and Starinets
Iqbal, HL

Solve Dirac equation for the corresponding bulk spinor field in the BH geometry.

Impose that at the horizon, the solution is an infalling wave.

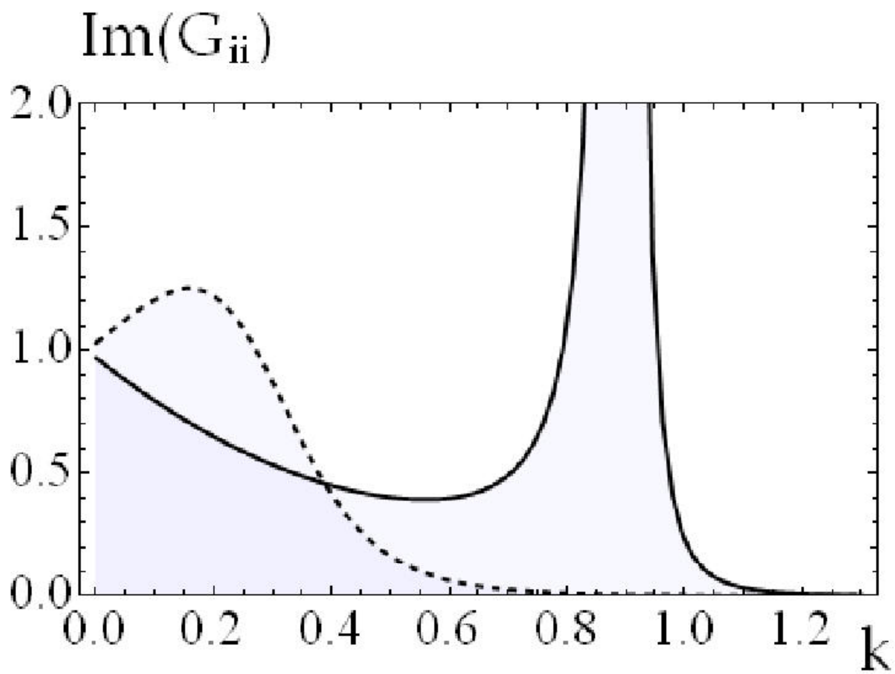
Expand the solution at the boundary

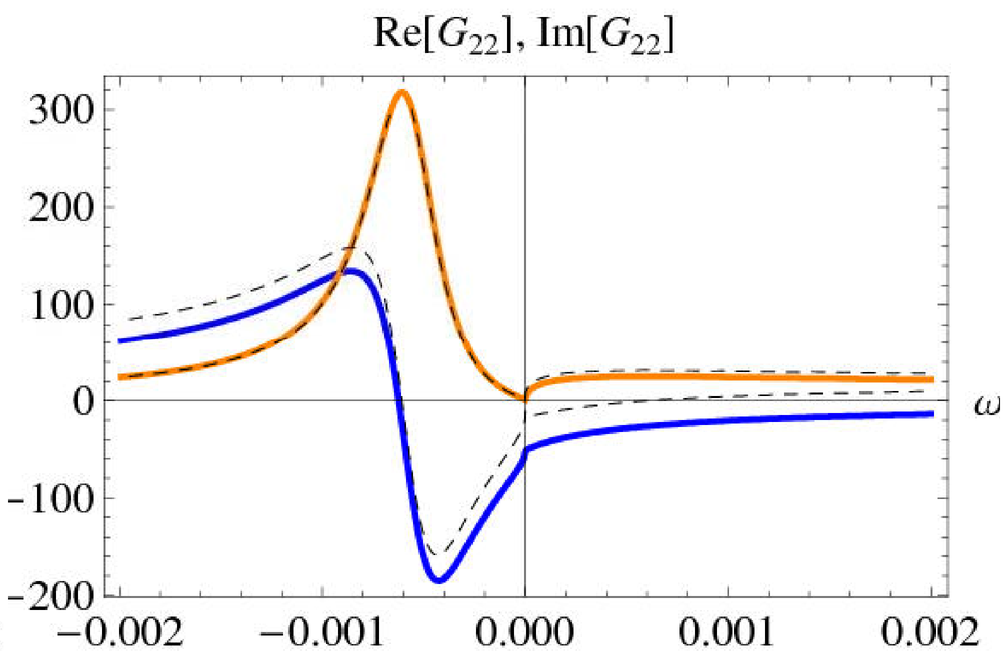
$$\Phi_\alpha \overset{r\to\infty}{\approx} a_\alpha r^m \begin{pmatrix} 0 \\ 1 \end{pmatrix} + b_\alpha r^{-m} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \alpha = 1, 2$$

$$G_\alpha(\omega, k) = \frac{b_\alpha}{a_\alpha}, \quad \alpha = 1, 2$$

two independent eigenvalues of boundary functions

Fermi surfaces





MDC: Plot G(ω,k) as function of k
for ω =-0.001 (for q=1, Δ=3/2)

EDC: k=0.9, one indeed finds
a quasi-particle-like peak

$$k_F \approx 0.918528499$$

Non-Fermi liquids

The peak moves with a dispersion relation $\omega \sim k_\perp^z$

with

$$z = 2.09\,(\,q = 1,\ \Delta = 3/2)$$

$$z = 5.32\,(\,q = 0.6,\ \Delta = 3/2)$$

Scaling behavior: $G_R(\lambda k_\perp, \lambda^z \omega) = \lambda^{-\alpha} G_R(k_\perp, \omega)$ $\alpha = 1$

Landau Fermi liquid: $z = \alpha = 1$

Non-Fermi liquids !

At this stage:

AdS/CFT is like a black box which just spits out numbers (or consistent spectral functions).

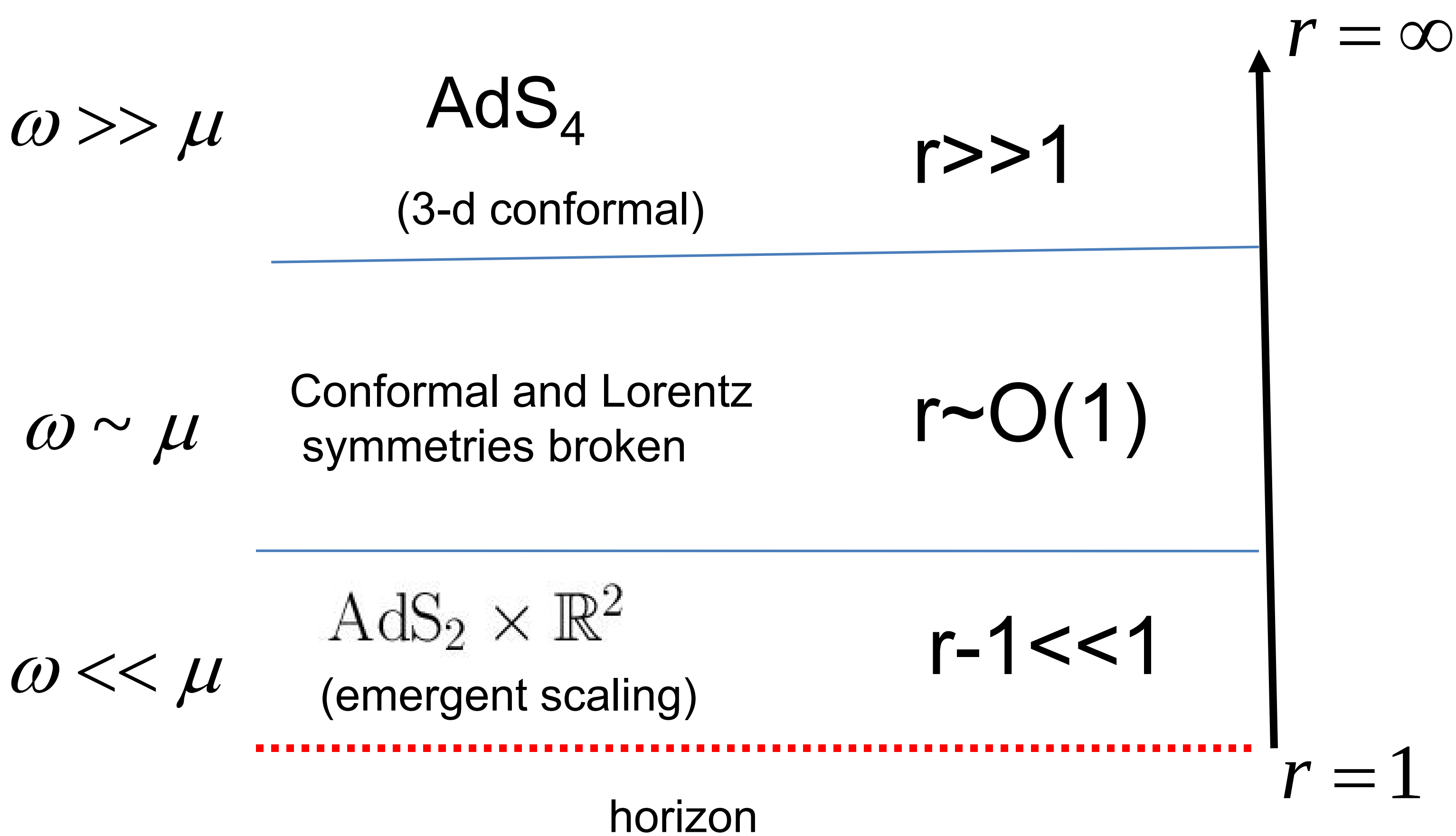
What controls these exponents?

We have to dissect this black box.

Search for an organizing principle for these exponents

Black hole geometry revisited

$$ds^2 = r^2(-f dt^2 + dx_1^2 + dx_2^2) + \frac{dr^2}{r^2 f} \qquad f = 1 + \frac{3}{r^4} - \frac{4}{r^3}$$



An emergent IR CFT

One can in fact define a scaling limit to decouple the AdS_2 region from the rest of geometry

$$r - 1 = \lambda \frac{R_2^2}{\zeta}, \quad t = \lambda^{-1}\tau, \quad \lambda \to 0 \quad \text{with} \quad \zeta, \tau \ \text{finite}$$

$$\Longrightarrow \qquad ds^2 = \frac{R_2^2}{\zeta^2}\left(-d\tau^2 + d\zeta^2\right) + d\vec{x}^2, \quad R_2 = \frac{1}{\sqrt{6}}$$

this is a long time limit, i.e. low frequency limit

Standard lore of AdS/CFT: $\Longrightarrow$

Gravity in the AdS_2 region $\Longleftrightarrow$ a (0+1)-d CFT

An emergent IR CFT

At low frequencies, the parent theory should be controlled by an emergent IR CFT !

Power of AdS/CFT ! (from geometry)

Not much is known about this theory:

1. It should have a zero temperature entropy

2. It may have a single copy of Virasoro algebra with a nontrivial central charge.

Lu, Mei, Pope, Vazquez-Poritz

Likely a chiral sector of a (1+1)-d CFT

Correlation functions in IR CFT

AdS_2 gravity → Operator dimensions
correlation functions

Each operator O in the parent theory becomes a family of operators $O_{\vec{k}}$

$\vec{k}$: momentum in transverse spatial directions

Conformal dimensions (in IR CFT):

$$\delta_{\vec{k}} = \frac{1}{2} + \nu_{\vec{k}}, \quad \nu_{\vec{k}} = \frac{1}{\sqrt{6}}\sqrt{\left(\Delta - \frac{3}{2}\right)^2 + k^2 - \frac{q^2}{2}}$$

$$\langle \mathcal{O}_{\vec{k}}(t)\mathcal{O}_{\vec{k}'}(0)\rangle \propto \delta(\vec{k} - \vec{k}')t^{-2\delta_k} \qquad \mathcal{G}_k(\omega) = c(k)\omega^{2\nu_k}$$

complex

Small frequency expansions (I)

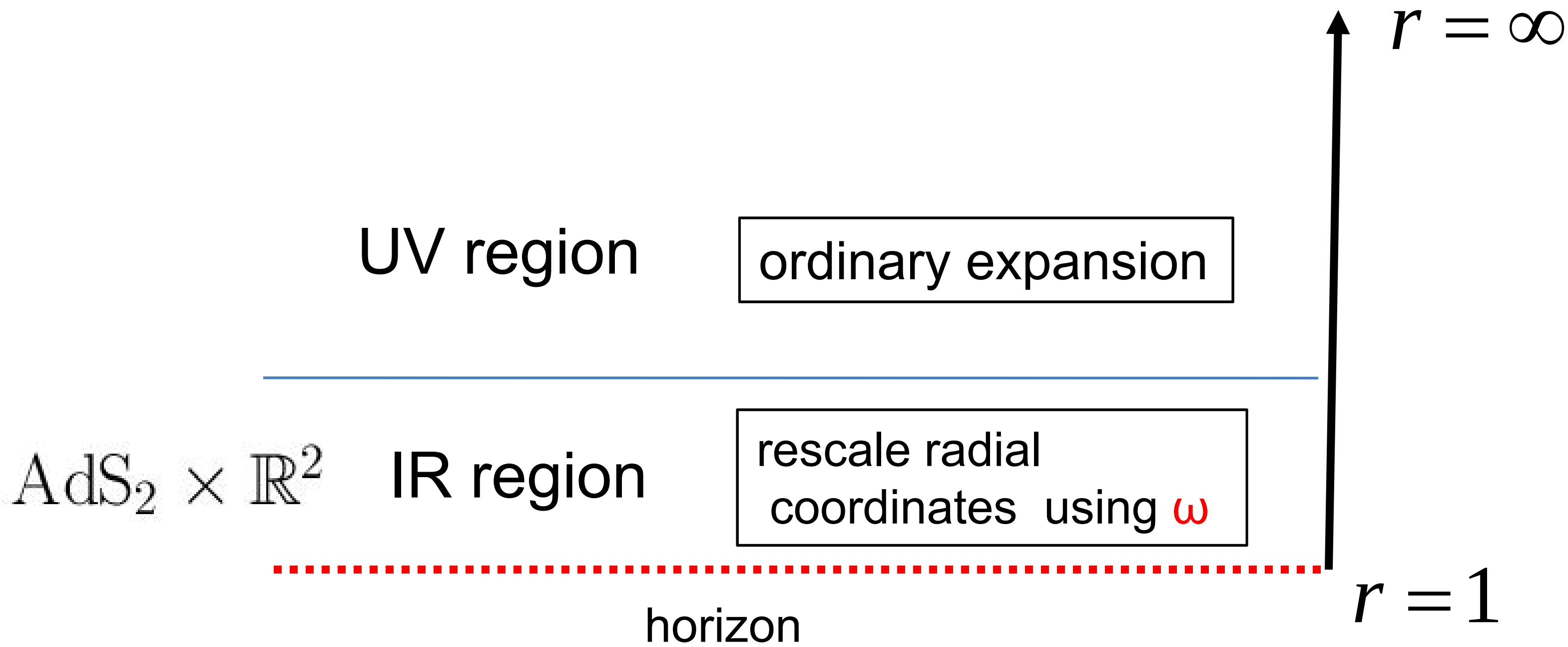
To understand the scaling around Fermi surfaces, need to study the low frequency behavior of the correlation functions.

For ordinary BH with a non-degenerate horizon: this can be done directly, a reflection that at a finite T G is analytic in small ω.

For extremal BH (T=0), this cannot be done straightforwardly:

ω –dependent terms in the Dirac equation always become singular at the horizon. (small ω expansion cannot be done Uniformly.)

Small frequency expansions (II)



1. Separate the BH spacetime into two regions: IR , UV
2. Perform small ω expansions in each region separately
3. Match them at the overlapping region.

Reminiscent of the standard RG picture

Small frequency expansions (III)

$$G_R(\omega, k) = \frac{b_+^{(0)} + \omega b_+^{(1)} + O(\omega^2) + \mathcal{G}_k(\omega)\left(b_-^{(0)} + \omega b_-^{(1)} + O(\omega^2)\right)}{a_+^{(0)} + \omega a_+^{(1)} + O(\omega^2) + \mathcal{G}_k(\omega)\left(a_-^{(0)} + \omega a_-^{(1)} + O(\omega^2)\right)}$$

$\mathcal{G}_k(\omega)$: retarded function for $O_{\vec{k}}$ in the IR CFT, depending only on the AdS_2 region. (IR data)

$a_\pm^{(0)}, a_\pm^{(1)}, b_\pm^{(0)}, b_\pm^{(1)}$ all k-dependent and from solving the Dirac equation in the UV region perturbatively.

They are sensitive to the metric of the outer region.

(UV data)

Generic k

$$G_R(\omega,k)=\frac{b_+^{(0)}+\omega b_+^{(1)}+O(\omega^2)+\mathcal{G}_k(\omega)\left(b_-^{(0)}+\omega b_-^{(1)}+O(\omega^2)\right)}{a_+^{(0)}+\omega a_+^{(1)}+O(\omega^2)+\mathcal{G}_k(\omega)\left(a_-^{(0)}+\omega a_-^{(1)}+O(\omega^2)\right)}$$

$a_+^{(0)}(k)\neq 0$ (a's and b's all real) ⇒ Small ω expansion:

$$G_R(\omega,k)=\frac{b_+^{(0)}}{a_+^{(0)}}+f_1(k)\omega+f_2\mathcal{G}_k(\omega)+\cdots$$

$$\mathcal{G}_k(\omega)=c(k)\omega^{2\delta_k-1}$$

Non-analytic behavior and dissipation are controlled by the IR CFT. (clear from geometry)

Fermi surfaces (I)

$$G_R(\omega, k) = \frac{b_+^{(0)} + \omega b_+^{(1)} + O(\omega^2) + \mathcal{G}_k(\omega)\left(b_-^{(0)} + \omega b_-^{(1)} + O(\omega^2)\right)}{a_+^{(0)} + \omega a_+^{(1)} + O(\omega^2) + \mathcal{G}_k(\omega)\left(a_-^{(0)} + \omega a_-^{(1)} + O(\omega^2)\right)}$$

Suppose at some k_F $\quad a_+^{(0)}(k_F) = 0$ outer region equation has a bound state at ω=0.

Near k_F, small ω

$$G_R(k, \omega) = \frac{h_1}{k_\perp - \frac{1}{v_F}\omega - h_2\,\mathcal{G}_{k_F}(\omega)} + \cdots$$

$$\mathcal{G}_k(\omega) = c(k)\,\omega^{2\delta_k - 1}$$

k_F, h_1, h_2, v_F: real (UV data)

precisely giving rise to the quasi-particle peak we saw earlier.

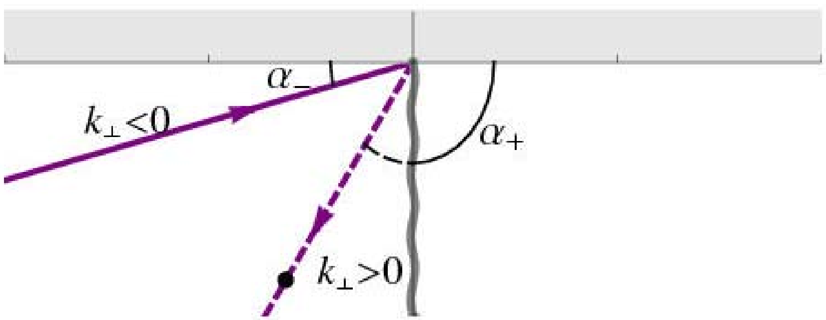
(now we know where the scaling exponents come from)

Relevant operator: singular FL

Suppose At Fermi momentum $O_{\vec{k}}$ is relevant $\delta_{k_F} < 1$

$$G_R(k,\omega) = \frac{h_1}{k_\perp - h_2\, c(k)\omega^{2\nu_{k_F}}} + \cdots \qquad \delta_k = \frac{1}{2} + \nu_k$$

$$\omega_*(k) \sim k_\perp^z, \quad z = \frac{1}{2\nu_{k_F}} > 1, \quad \frac{\Gamma(k)}{\omega_*(k)} = \text{const}$$



$$Z \propto k_\perp^{\frac{1-2\nu_{k_F}}{2\nu_{k_F}}} \to 0, \quad k_\perp \to 0$$

Quasi-particle-like peak , never stable, zero residue at the Fermi surface.

No quasi-particle description   singular FL

Irrelevant operator: FL

Suppose $O_{\vec{k}_F}$ is irrelevant $\delta_{k_F} > 1 \quad (\nu_{k_F} > 1/2)$

$$G_R(k,\omega) = \frac{h_1}{k_\perp - \frac{1}{v_F}\omega - h_2 c(k_F)\omega^{2\nu_{k_F}}} + \cdots$$

In the limit $k_\perp \to 0$

$$\omega_*(k) = v_F k_\perp + \cdots , \qquad \frac{\Gamma(k)}{\omega_*(k)} \propto k_\perp^{2\nu_{k_F}-1} \to 0, \qquad Z = h_1 v_F$$

Linear dispersion relation, the quasi-particle becomes stable, non-vanishing residue at the Fermi surface.

Quasi-particle picture applies, Fermi liquids. (v.s. Landau FL)

Luttinger theorem should apply , may have different thermodynamic and transport properties compared to Landau FL.

Marginal operator: "Marginal Fermi liquids"

Suppose $O_{\vec{k}_F}$ is marginal: $\delta_{k_F} = 1 \quad (\nu_{k_F} = 1/2)$

$$G_R(k,\omega) = \frac{h_1}{k_\perp - \frac{1}{v_F}\omega - h_2 c(k_F)\omega^{2\nu_{k_F}}} + \cdots$$

v_F goes to zero and $c(k_F)$ has a pole

$$G_R \approx \frac{h_1}{k_\perp + \tilde{c}_1 \omega \log \omega + c_1 \omega}$$

$\tilde{c}_1$: real

c_1 : complex

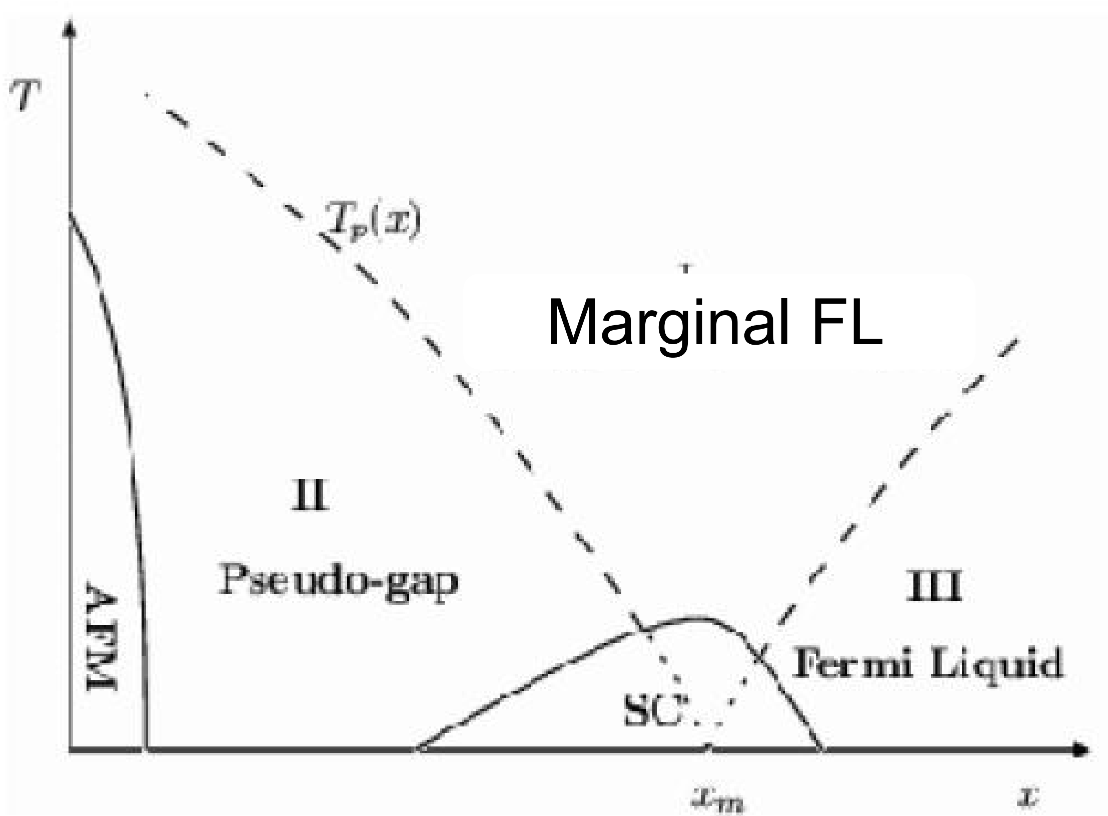
Singularity of G_R : branch point, rather than a pole

$$k_\perp \to 0, \quad Z \sim \frac{1}{|\log \omega_*|} \to 0$$

``Marginal Fermi liquid'' for high Tc cuprates near optimal doping.

Varma et al (1989)

$$G_R \approx \frac{h_1}{k_\perp + \tilde{c}_1 \omega \log \omega + c_1 \omega}$$



Landau Fermi liquid?

For $\delta_{k_F} = 2$ (require fine tuning of parameters)

$$G_R(\omega, k) \approx \frac{h_1}{k_\perp - \frac{1}{v_F}\omega + \tilde{c}_2\omega^2 \log\omega + c_2\omega^2}$$

$\tilde{c}_2$: real

c_2 : complex

not quite Landau Fermi liquid, logarithmic term leads to a particle-hole asymmetry

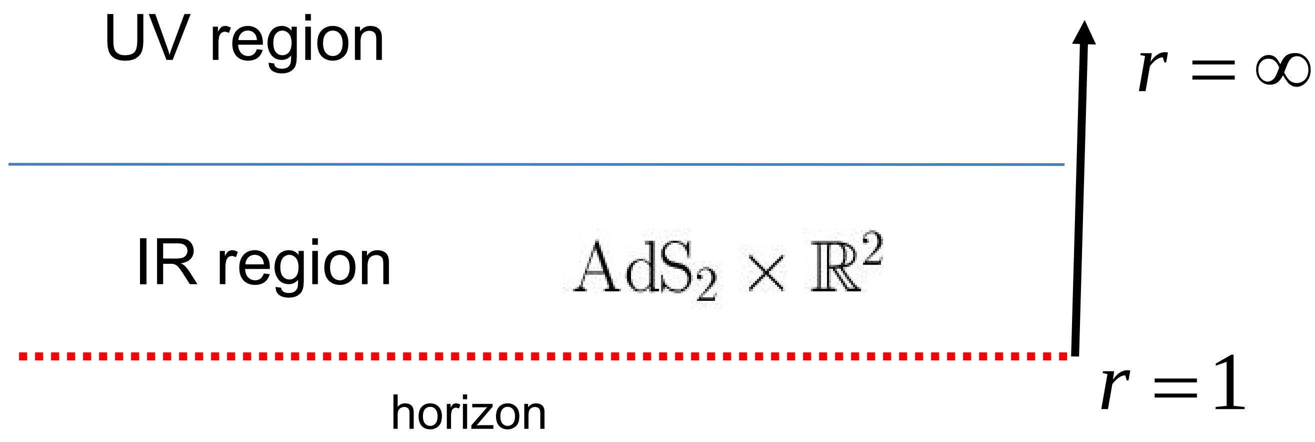
$$\Gamma(\omega_* < 0) - \Gamma(\omega_* > 0) = \pi\tilde{c}_2\omega_*^2$$

Summary: IR data

Operator dimensions in the IR CFT	↔	Scaling exponents near the Fermi surface
relevant operator	↔	Singular Fermi liquid
irrelevant operator	↔	Fermi liquid
Marginal operator	↔	Marginal Fermi liquid

Landau Fermi liquid never arises in our story

A phenomenological description



Our G_R can be written as

$$G_R(\omega,\vec{k}_\perp) = \frac{1}{\Sigma(\omega,\vec{k}_\perp) + D^2(\omega,\vec{k}_F)\mathcal{G}_{k_F}(\omega)}$$

Real, UV data, analytic in ω

Separate O into UV and IR part

$$S = \int \underbrace{d\omega d\vec{k}\, \bar{\mathcal{O}}_U\, \Sigma(\omega, \vec{k}_\perp)\, \mathcal{O}_U}_{\text{UV physics}} + \int_{FS} \underbrace{D(\omega, \vec{k}_F) \bar{\mathcal{O}}_U(\omega, \vec{k}_F)^\dagger \mathcal{O}_I(\omega, \vec{k}_F)}_{\text{mixing between UV/IR}} + h.c.$$

$$\left\langle \mathcal{O}_I(\omega, \vec{k}_F)^\dagger \mathcal{O}_I(\omega', \vec{k}'_F) \right\rangle_{IR} = \mathcal{G}_{k_F}(\omega)\, \delta_{\vec{k}_F, \vec{k}'_F}\, \delta(\omega - \omega')$$

G_R =

$$\Rightarrow \quad G_R(\omega, \vec{k}_\perp) = \frac{1}{\Sigma(\omega, \vec{k}_\perp) + D^2(\omega, \vec{k}_F)\, \mathcal{G}_{k_F}(\omega)}$$

Other NFLs may be understandable in this language.

Imaginary exponent

$$G_R(\omega,k) = \frac{b_+^{(0)} + \omega b_+^{(1)} + O(\omega^2) + \mathcal{G}_k(\omega)\left(b_-^{(0)} + \omega b_-^{(1)} + O(\omega^2)\right)}{a_+^{(0)} + \omega a_+^{(1)} + O(\omega^2) + \mathcal{G}_k(\omega)\left(a_-^{(0)} + \omega a_-^{(1)} + O(\omega^2)\right)}$$

$$\mathcal{G}_k(\omega) = c(k)\omega^{2\nu_k} \qquad \delta_k = \frac{1}{2} + \nu_k$$

$\nu_k = -i\lambda_k$ is pure imaginary for small enough k when

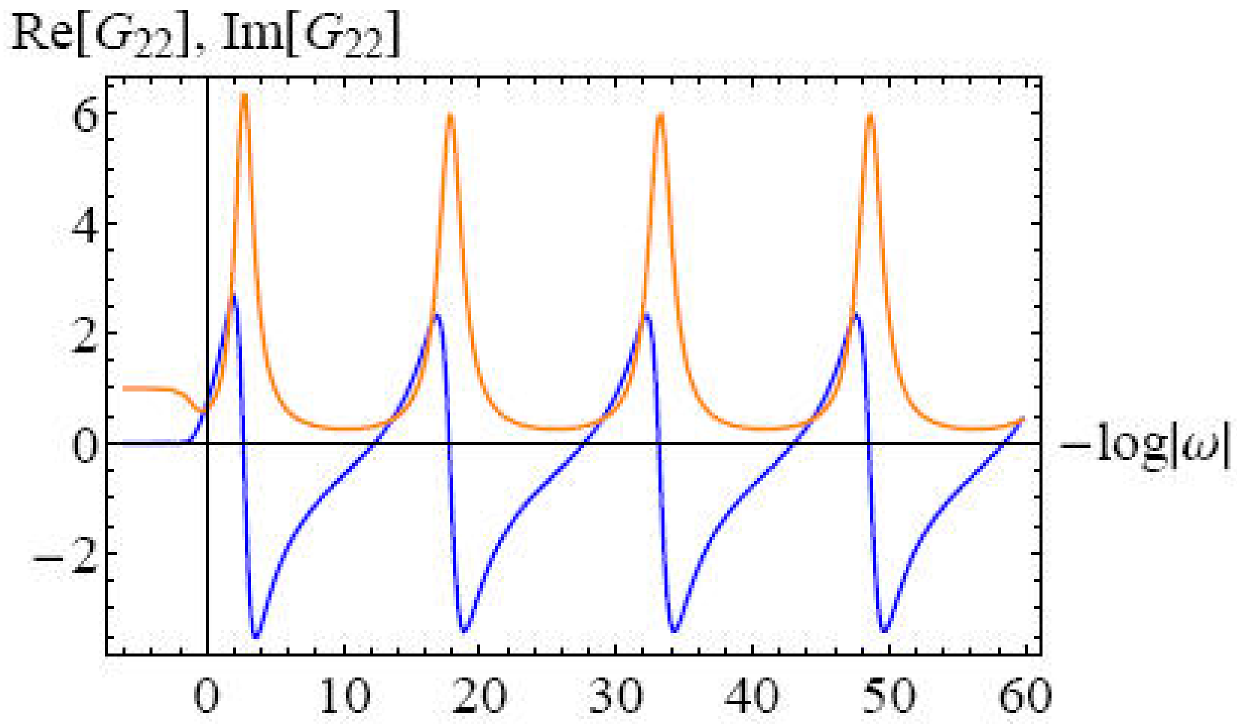
$$\Delta < \frac{|q|}{\sqrt{2}} + \frac{d}{2}$$

$$G_R(\omega,k) \approx \frac{b_+^{(0)} + b_-^{(0)} c(k)\omega^{-2i\lambda_k}}{a_+^{(0)} + a_-^{(0)} c(k)\omega^{-2i\lambda_k}} + O(\omega)$$

Note: no instability

Log-periodic behavior

This leads to a discrete scaling symmetry and



So far:

Suppose at some k_F $$a_+^{(0)}(k_F) = 0$$ outer region equation has a bound state at $\omega=0$.

→ A Fermi surface

what could in principle happen near the Fermi surface given the analytic structure of the correlation function.

UV data: Fermi momentum

For what values of q and Δ, are Fermi surfaces allowed?
How does k_F depend on q and Δ ?

$$\Delta < \frac{|q|}{\sqrt{3}} + \frac{d}{2}$$

For Δ = 3/2

$$\frac{1}{\sqrt{6}} \leq \frac{k_F}{q\mu} \leq 1$$

It always lies inside the region which allows log-periodic behavior

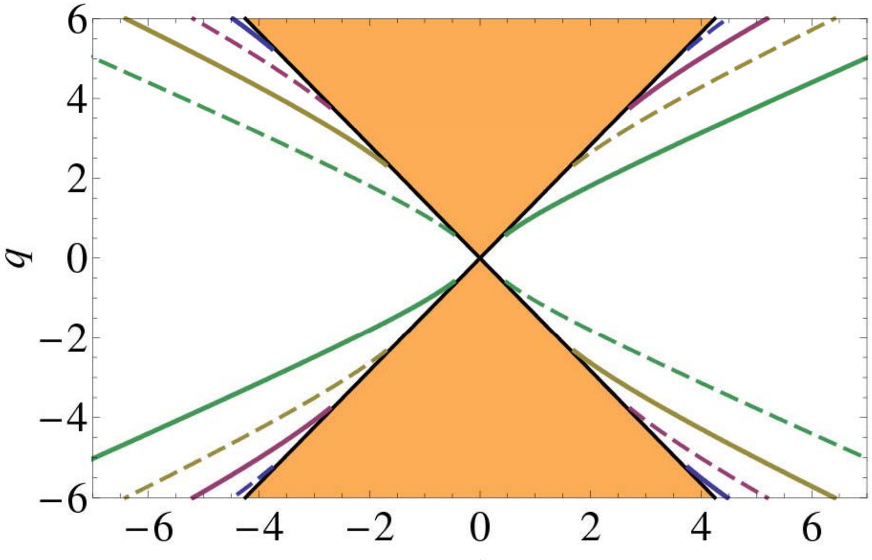
$$\Delta < \frac{|q|}{\sqrt{2}} + \frac{d}{2}$$

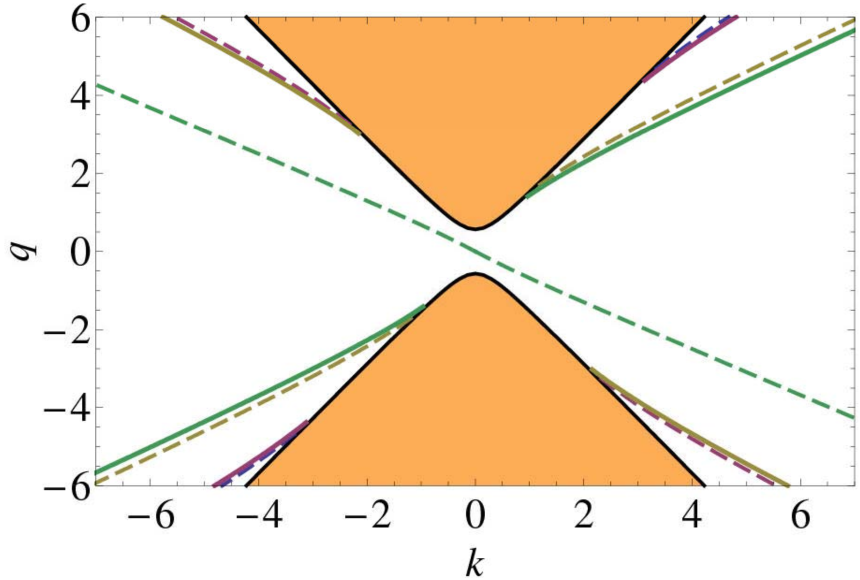
Except for

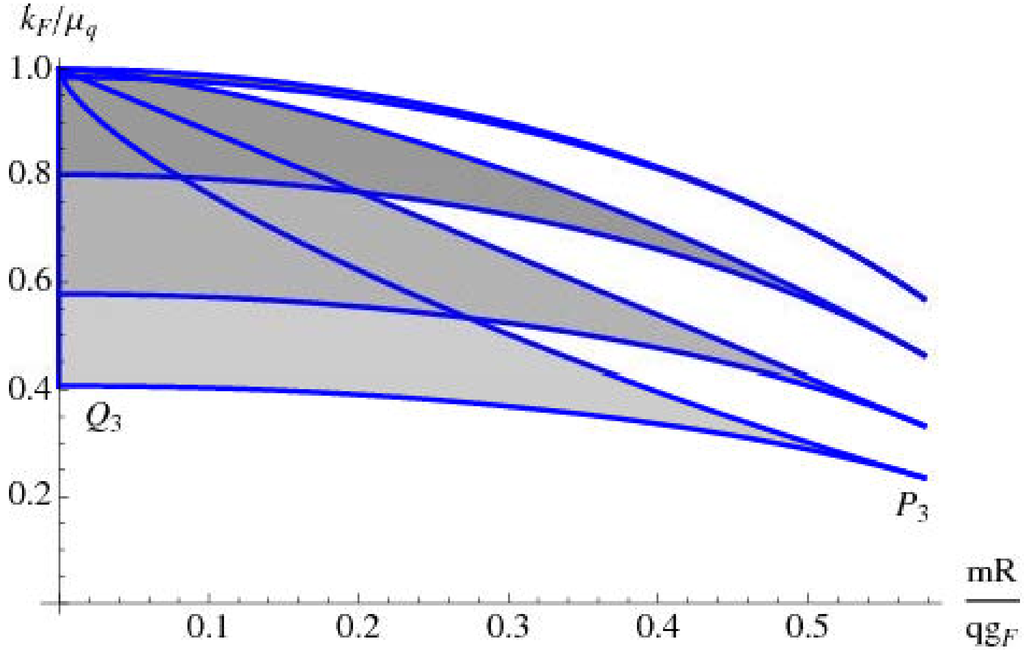
$$\frac{d-1}{2} < \Delta < \frac{d}{2} - \frac{|q|}{\sqrt{2}}$$

(alternative quantization)

Note: the upper limit (which applies to any Δ) is saturated by free relativistic fermions. (suggests repulsive interactions)





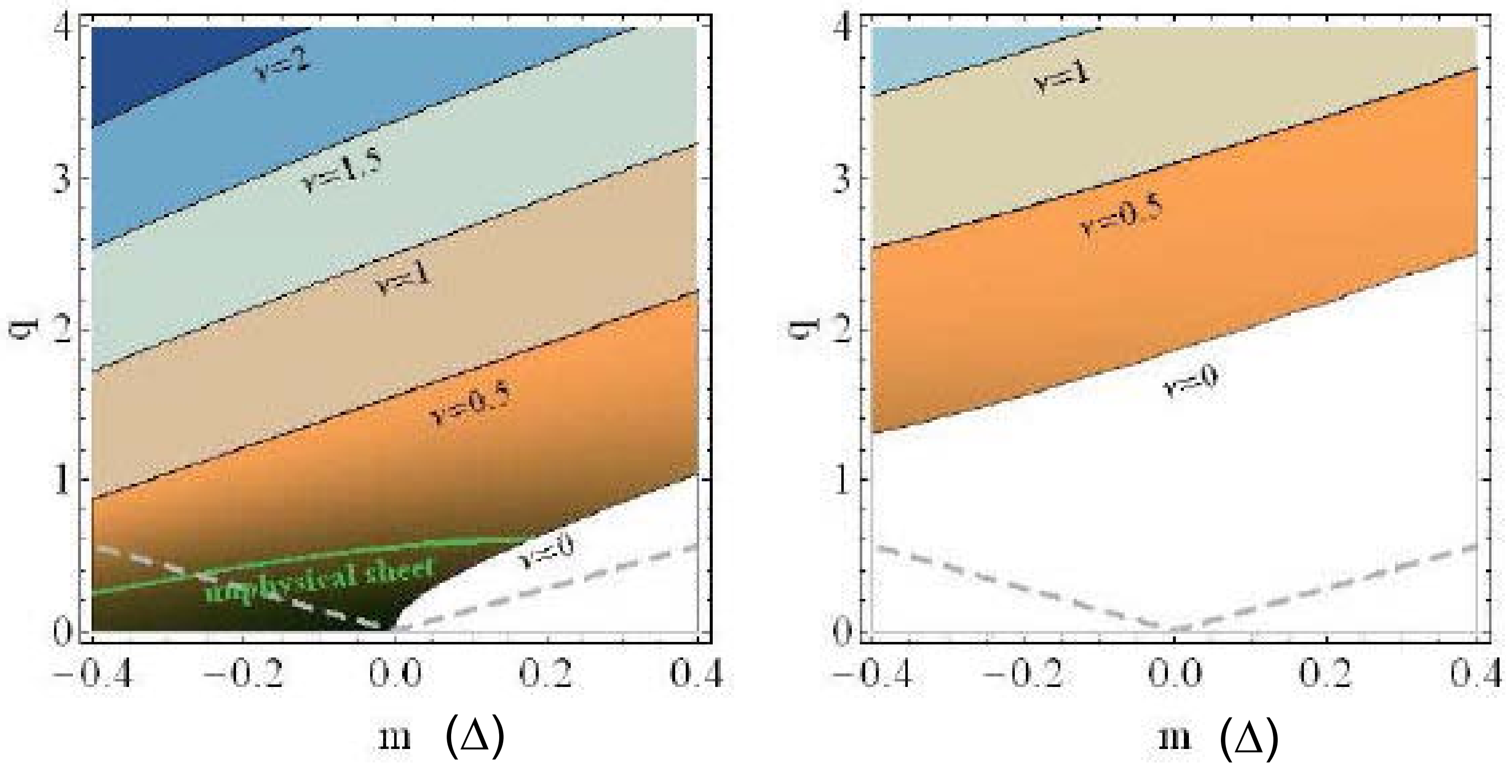


For fixed Δ, k_F increases with q.

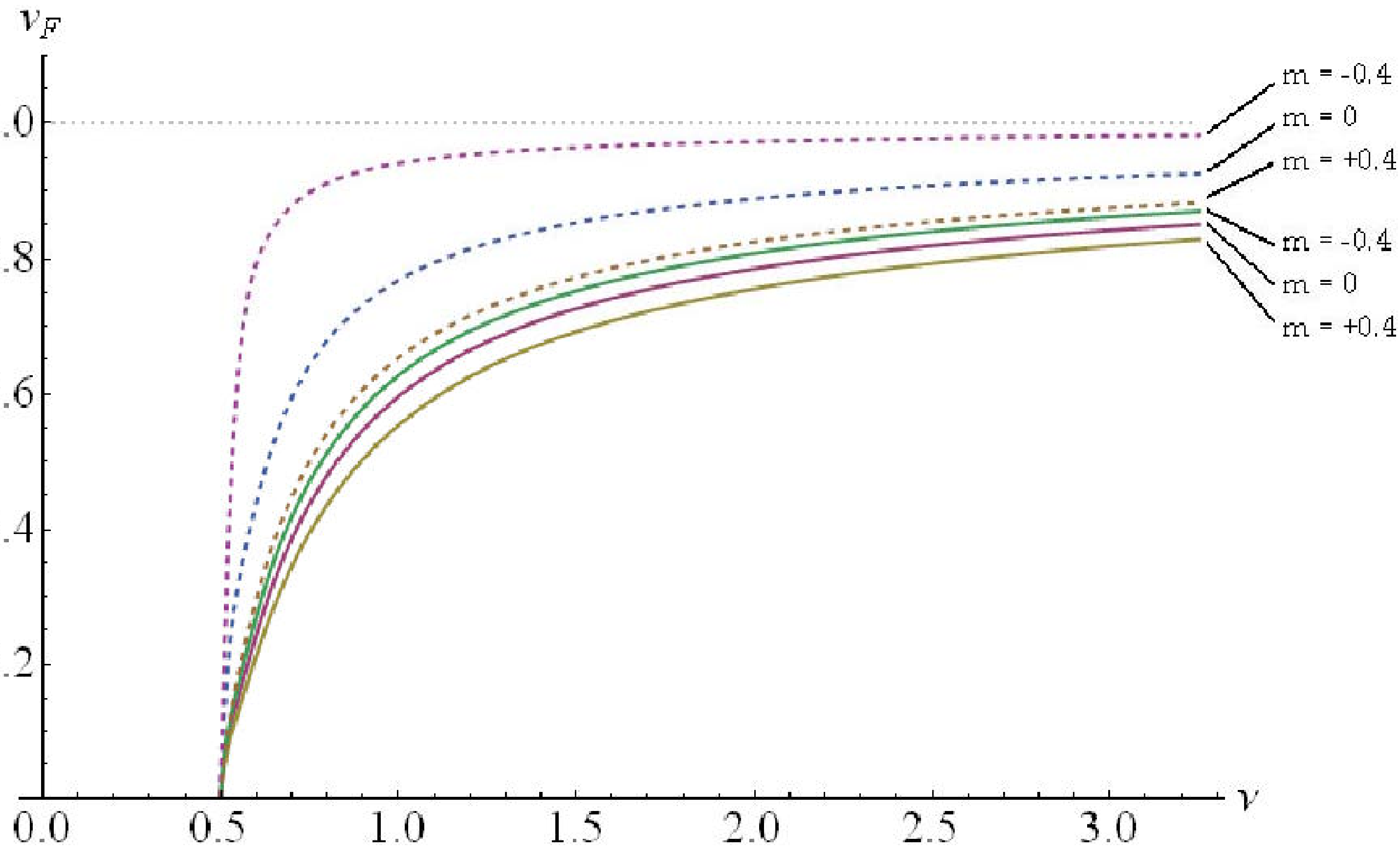
For fixed q, k_F decreases with Δ.

UV data: Distribution of δ_{k_F}

$$\delta_{\vec{k}} = \frac{1}{2} + \nu_{\vec{k}}, \quad \nu_{\vec{k}} = \frac{1}{\sqrt{6}}\sqrt{m^2 + k^2 - \frac{q^2}{2}}$$



UV data: Fermi Velocity



Fermi velocity goes to zero as the marginal limit is approached, so does the residue.

Summary

Operator dimensions
in the IR CFT

Scaling exponents
near the Fermi surface

We have mapped out the landscape of non-(Fermi) liquids in the landscape of theories with AdS dual.

Question:

Here we found an (0+1)-d IR CFT, while naively one would expect a (1+1)-d CFT?

Any thought?

Many other interesting aspects I have not time to cover:

Particle-hole asymmetry,

formula for Fermi velocity,

Disappearance of Fermi surfaces under relevant deformation of the parent theory

Free fermion limit

Story for a charged bosons (new instability)

statistics and instability

Finite temperature

Future questions

1. Density of states, Thermodynamic properties
2. Scattering of quasi-particles
3. transport: conductivity …

All in principle calculable, much more complicated

With all these data, plus insights from the bulk geometry

an organizing principle for NFLs.