Binary Black Holes Simulations:

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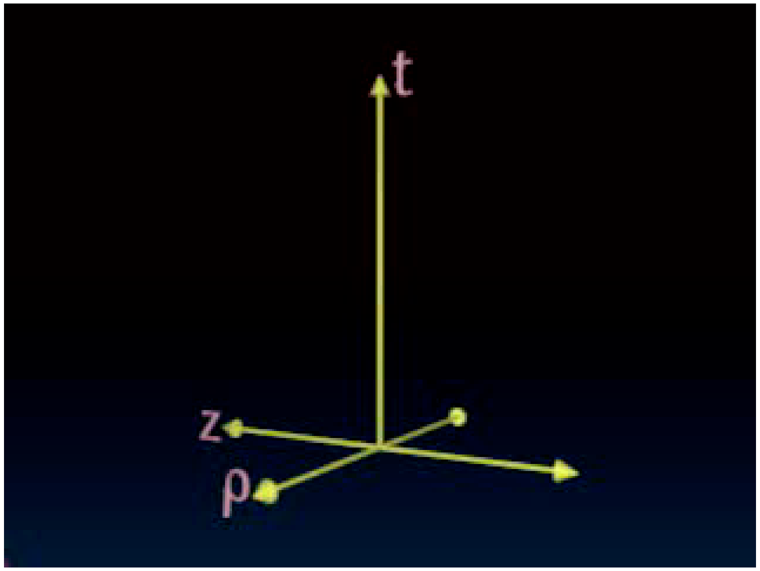
This talk …

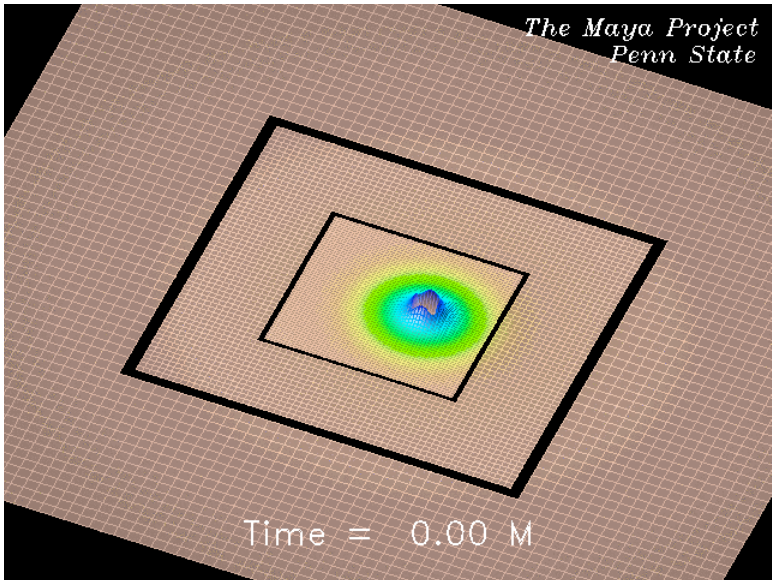
- Before 2004
- First Orbits
- What made it possible?
- Gravitational Wave Astrophysics:
  - Kicks
  - Spins
  - Gravitational Waves
- What next?

Before 2004

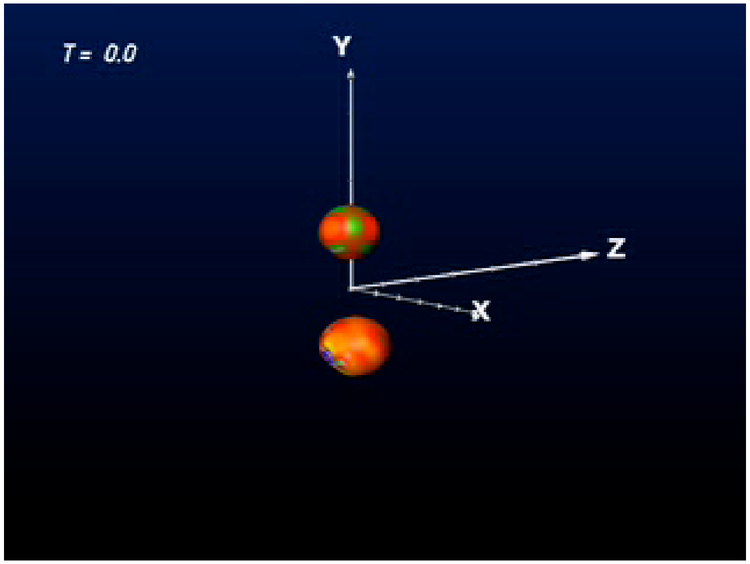
1975-1980 - The Pioneer years
1994 - 2D Head-on Collisions
1995 - 3D Single Black Hole
1997 - Boosted BH
1999 - 3D Head-on Collisions
2000 - Grazing Collisions
2004 - Plunge Collisions



BBH Alliance

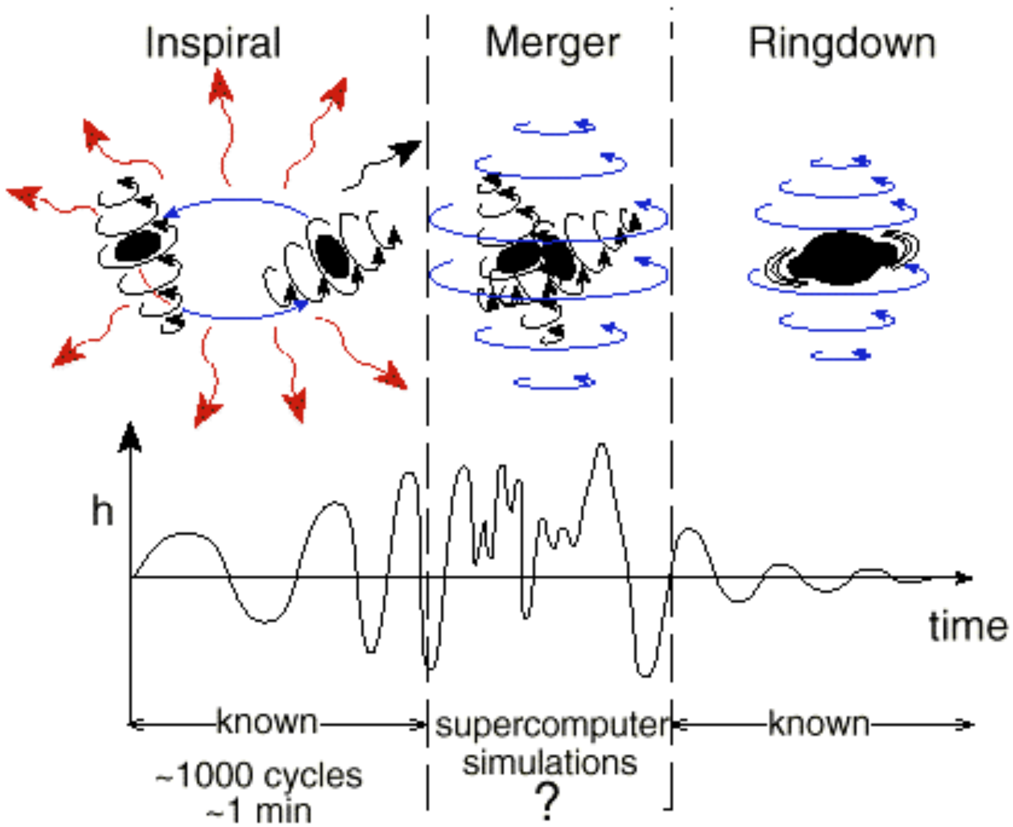


Penn State

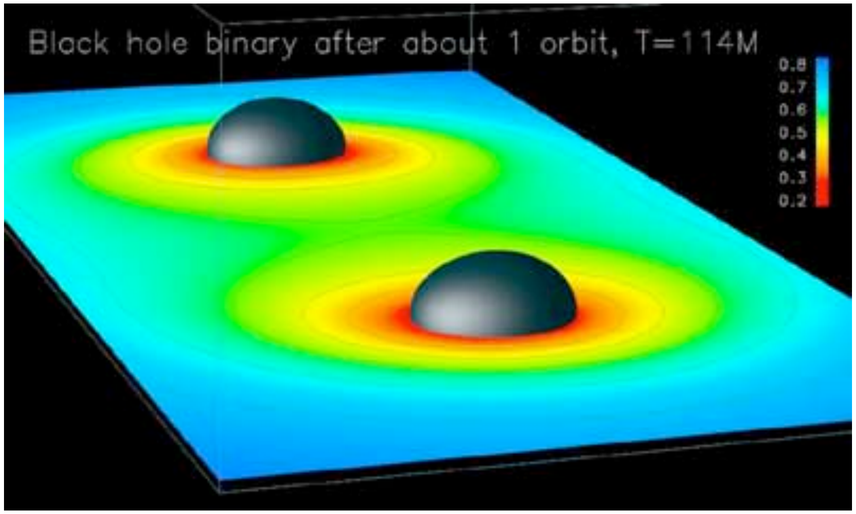


Albert Einstein Institute

The Ultimate Goal

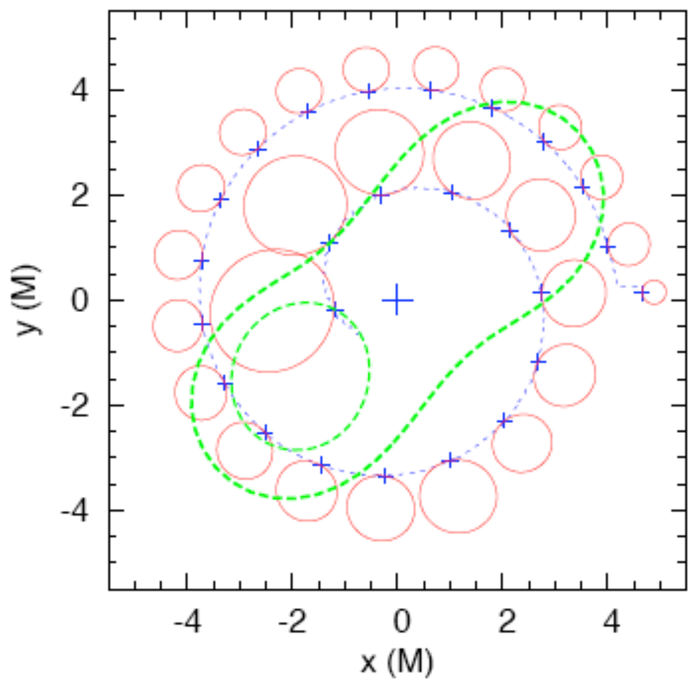


First Orbit



Bruegmann, Tichy, Jansen
PRL 92 (2004) 211101

- Co-rotating coordinate frame
- Evolution lasted 145 M
- Initial separation 6 M, Period ~ 114 M
- No waveforms were computed

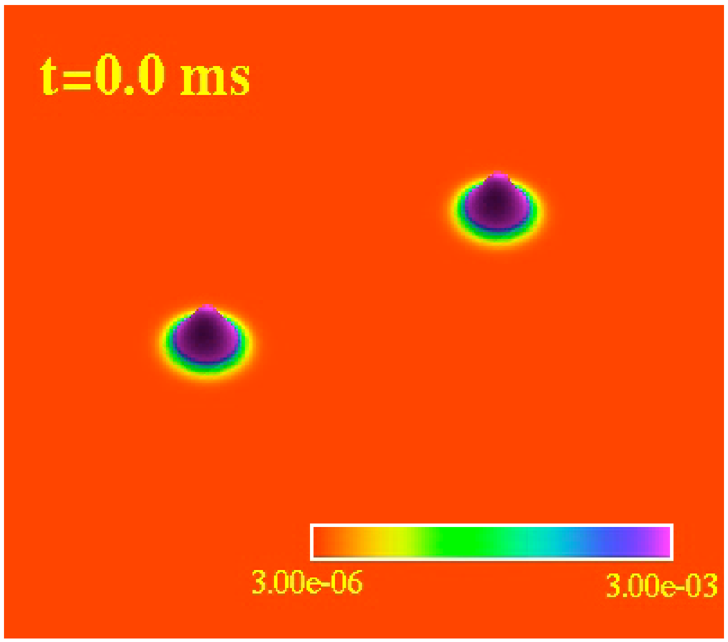


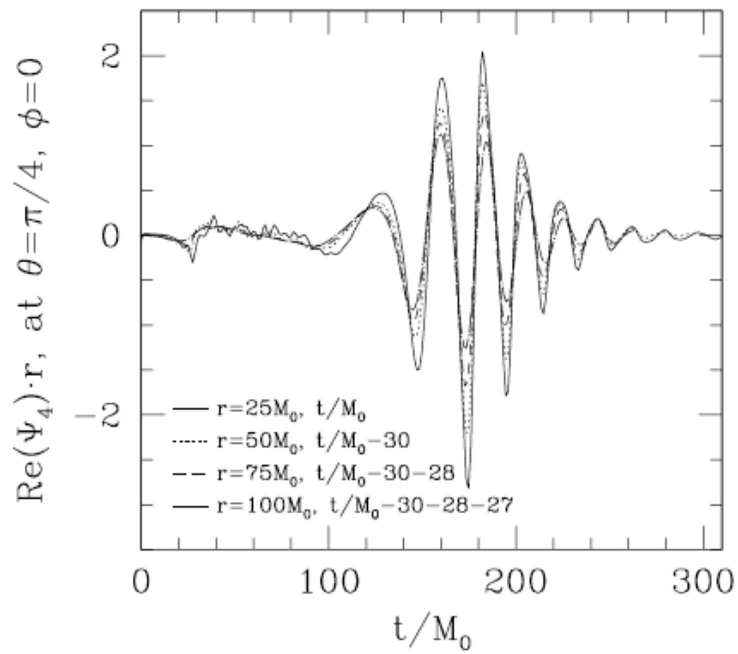
Later Reproduced by:

Diener et al PRL 96 (2006) 121101
AEI-LSU
Main issue: Resolution < M/40

First Orbits, Merger and Ring-down

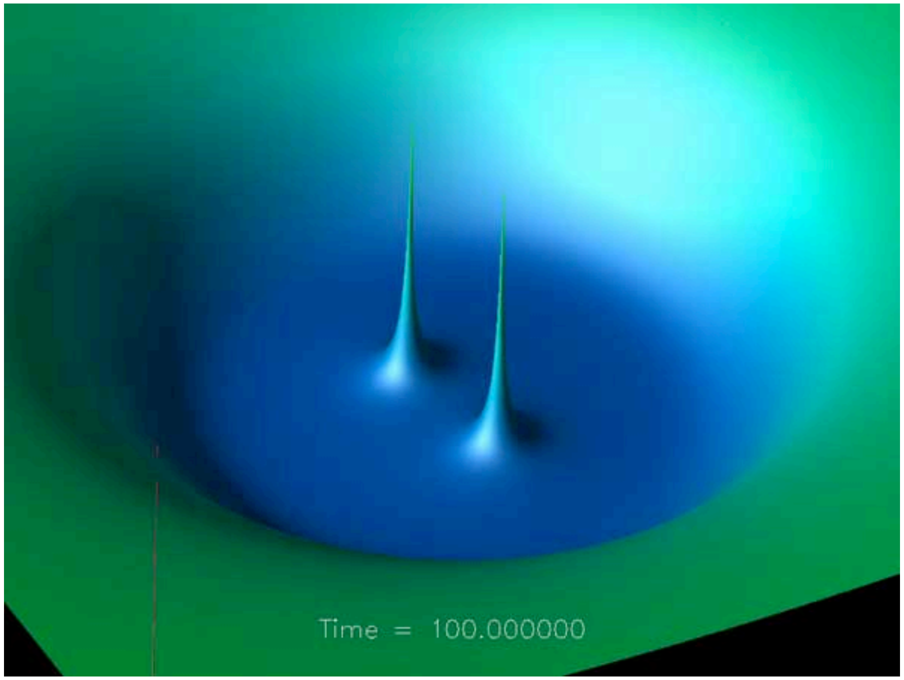
F. Pretorius (U. Alberta)
PRL 95 (2005) 121101



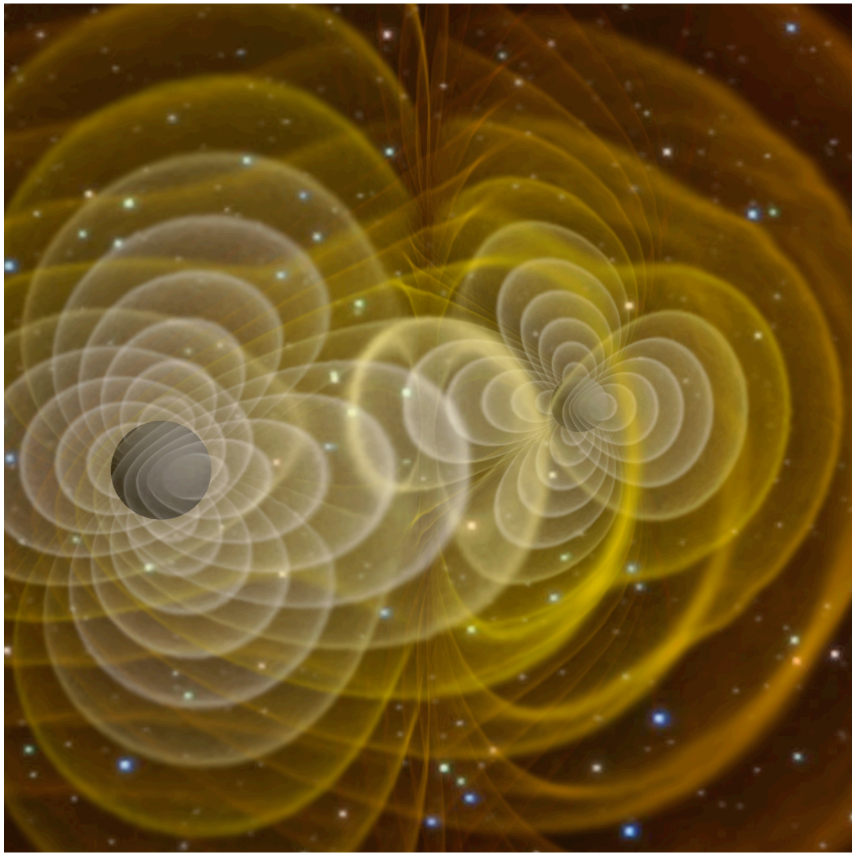


- Initial data from unstable scalar field “stars”
- Initial separation ~ 16 M
- Eccentricity < 0.2
- Final BH spin a ~ 0.7

The Last Orbit:
The Moving Puncture Recipe

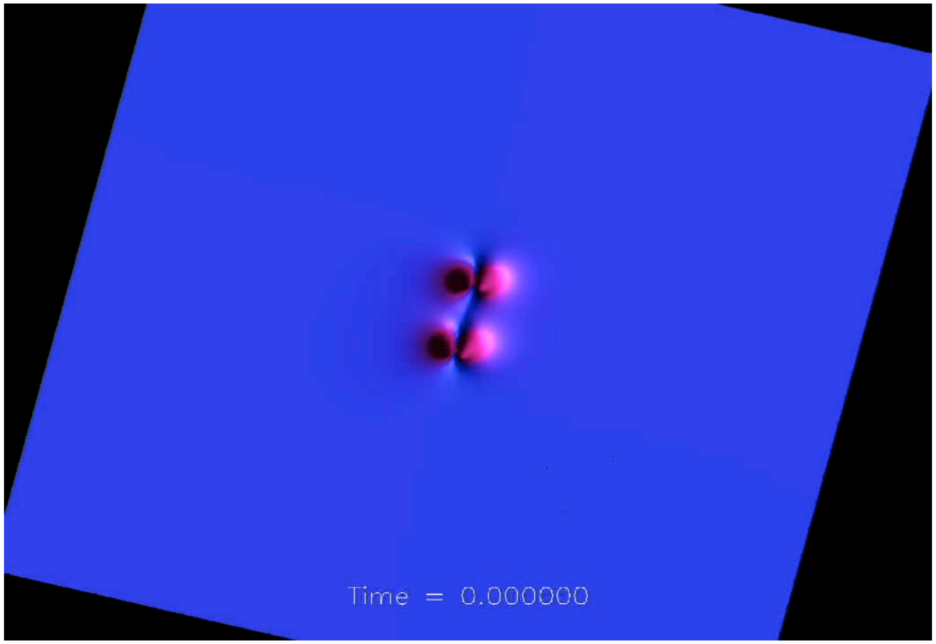


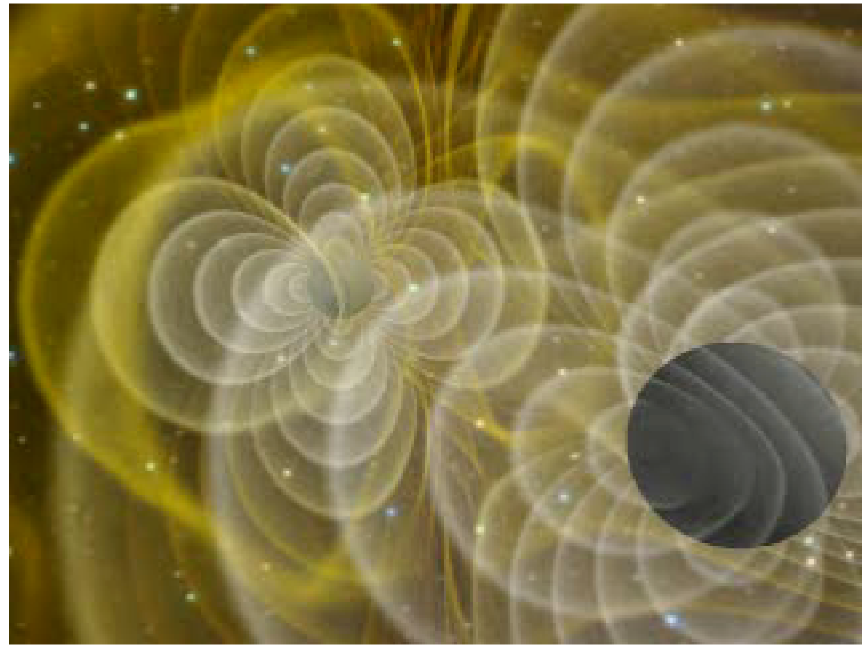
University of Texas at Brownsville
Campanelli, Louto, Zlochower
Phys.Rev.Lett. 96 (2006) 111101



NASA-GSFC
Baker, Centrella, Choi, Koppitz, van Meter
Phys.Rev.Lett. 96 (2006) 111102

The Movies



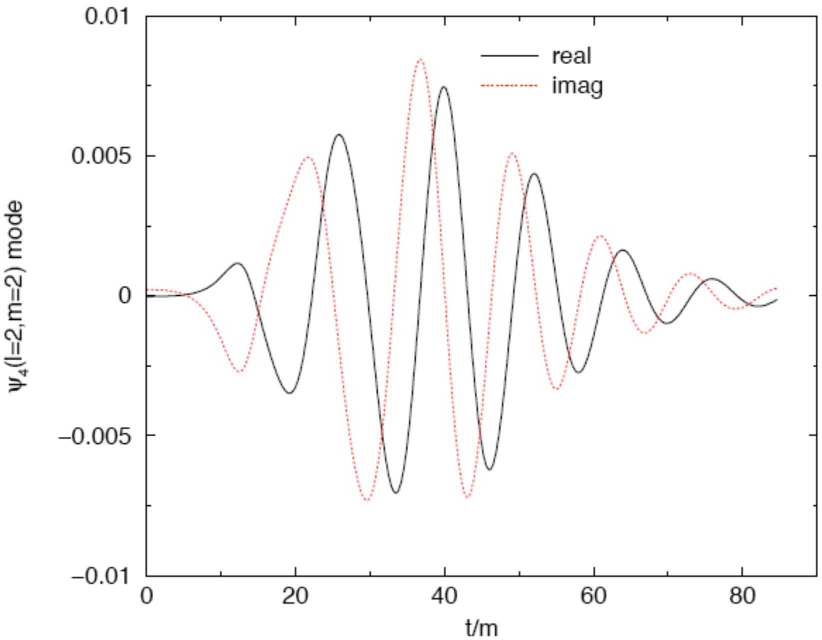


UTB NASA-GSFC

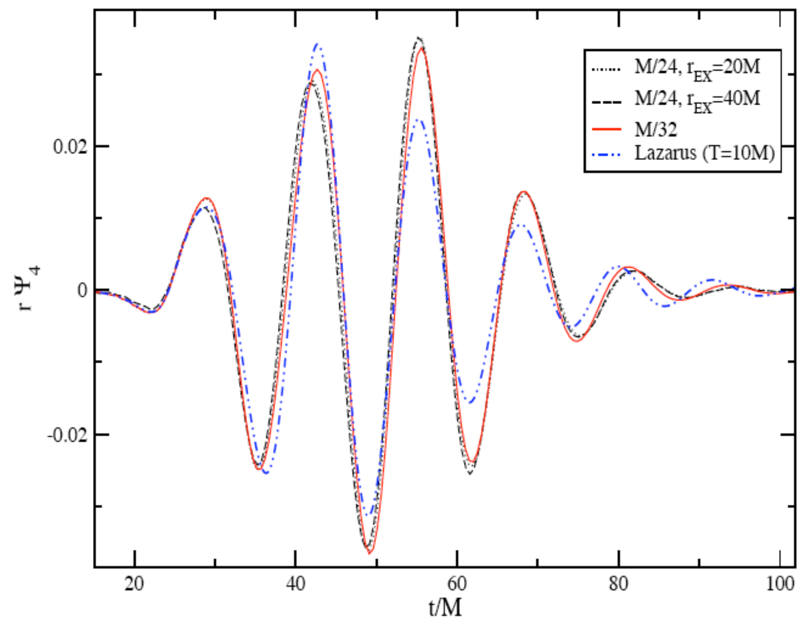
The Moving Puncture Recipe seems robust.
Penn State, AEI-LSU, Jena-FAU groups have successfully adopted.

Caltech-Cornell is also having success with the Generalized Harmonic Recipe

The Physics



UTB



NASA-GSFC

Energy radiated ~ 3%
Ang. Mom. radiated ~ 15%
Final BH spin parameter ~ 0.7

What made it possible?

- A “good” set of equations
- A “good” set of coordinate conditions
- Adaptive Mesh Refinement
- “Creative Engineering”
- Stop worrying too much about singularities!

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BSSN Equations

Baumgarte & Shapiro PRD 59, 024007 (1999)
Shibata & Nakamura PRD 52, 5428 (1995)

$$
\begin{aligned}
\partial_o \Phi &= -\frac{1}{6}\alpha\, K \\
\partial_o \hat{g}_{ij} &= -2\,\alpha\,\hat{A}_{ij}
\end{aligned}
$$

$$
\begin{aligned}
\partial_o K &= -\nabla_i \nabla^i \alpha + \alpha \left(\hat{A}_{ij}\hat{A}^{ij} + \frac{1}{3}\,\hat{K}^2 \right) \\
\partial_o \hat{A}_{ij} &= e^{-4\Phi}[-\nabla_i \nabla_j \alpha + \alpha R_{ij}]^{TF} + \alpha\,(K\,\hat{A}_{ij} - 2\,\hat{A}_{ik}\hat{A}^k{}_j)
\end{aligned}
$$

$$
\begin{aligned}
\partial_o \hat{\Gamma}^i &= \hat{g}^{jk}\partial_{jk}\beta^i + \frac{1}{3}\hat{g}^{ij}\partial_{jk}\beta^k - 2\,\hat{A}^{ij}\partial_j \alpha \\
&+ 2\,\alpha \hat{\Gamma}^i{}_{jk}\,\hat{A}^{jk} + 12\,\alpha\,\hat{A}^{ij}\,\partial_j \Phi - \frac{4}{3}\,\alpha\,\hat{g}^{ij}\partial_j K
\end{aligned}
$$

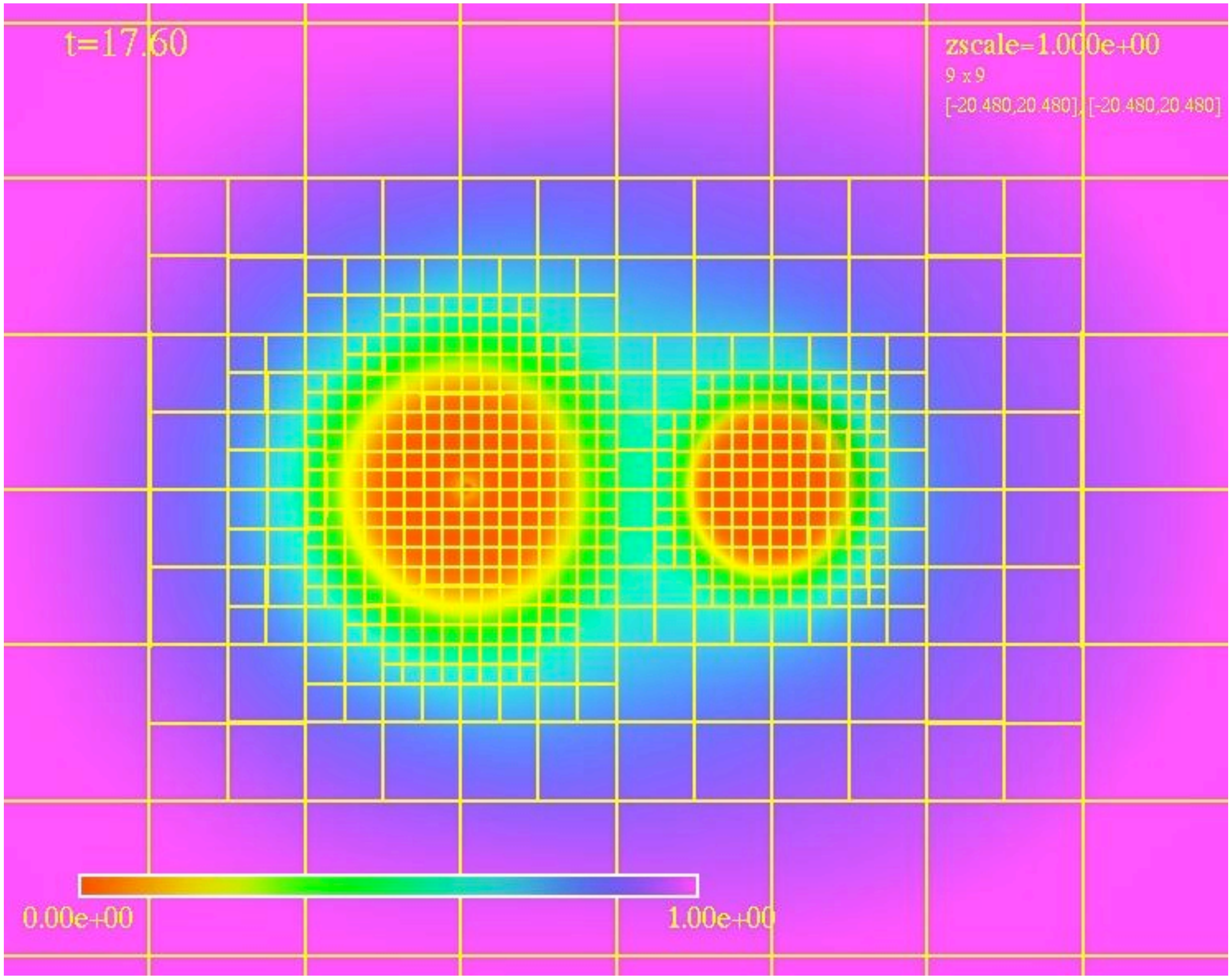
Generalized Harmonic Equations

Pretorius, CQG 22 (2005) 425

$$g^{\delta\gamma} g_{\alpha\beta,\gamma\delta} + g^{\gamma\delta}{}_{,\beta} g_{\alpha\delta,\gamma} + g^{\gamma\delta}{}_{,\alpha} g_{\beta\delta,\gamma} + 2H_{(\alpha,\beta)}$$
$$-2H_\delta \Gamma^\delta_{\alpha\beta} + 2\Gamma^\gamma_{\delta\beta}\Gamma^\delta_{\gamma\alpha} = -8\pi\left(2T_{\alpha\beta} - g_{\alpha\beta}T\right)$$

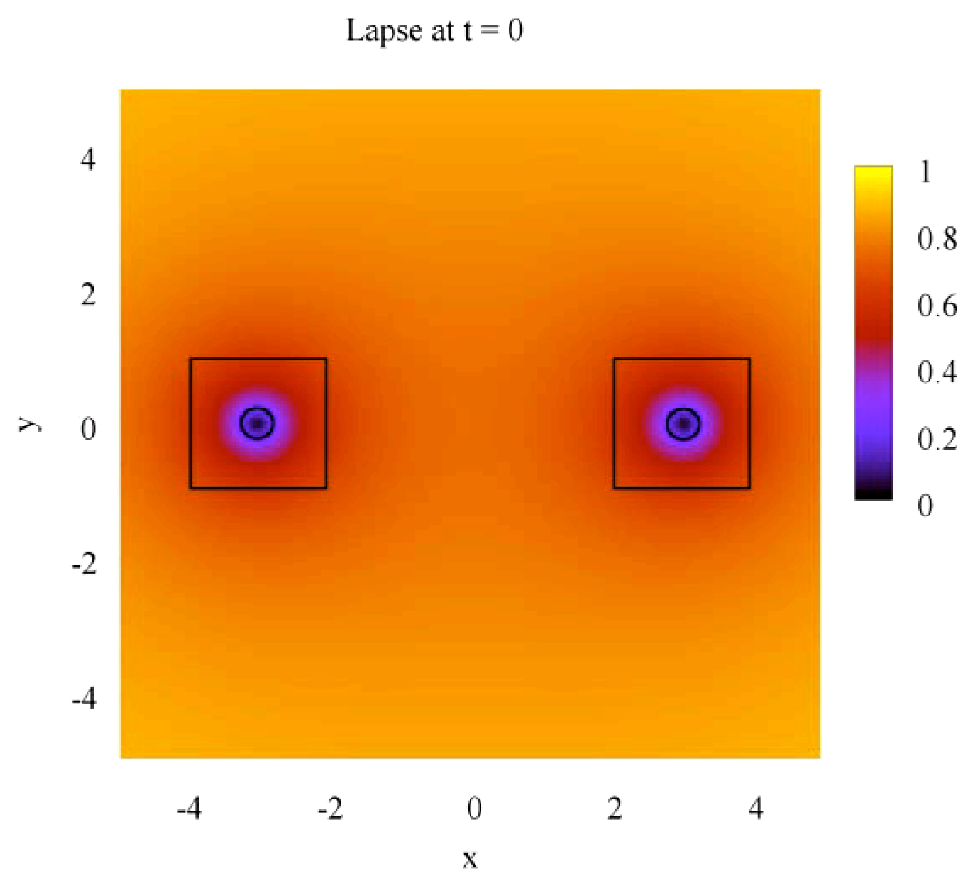
$$H^\mu \equiv \Box x^\mu$$

Adaptive Mesh Refinement



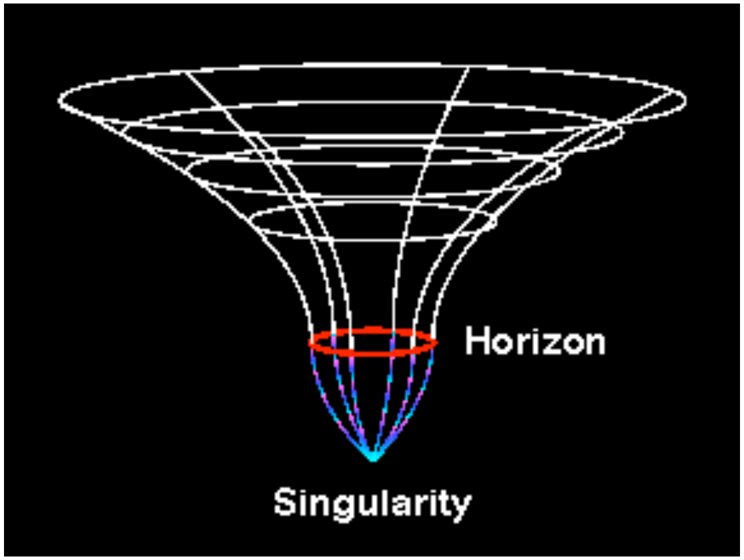
Dale Choi (NASA-GSFC)

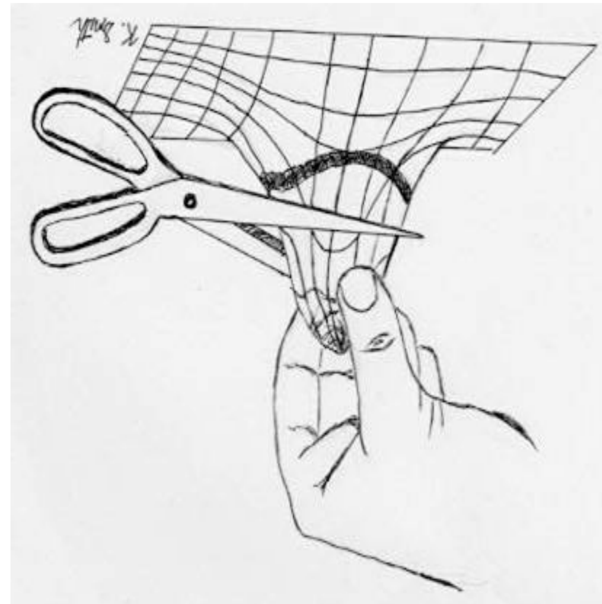
Fixed Mesh Refinements



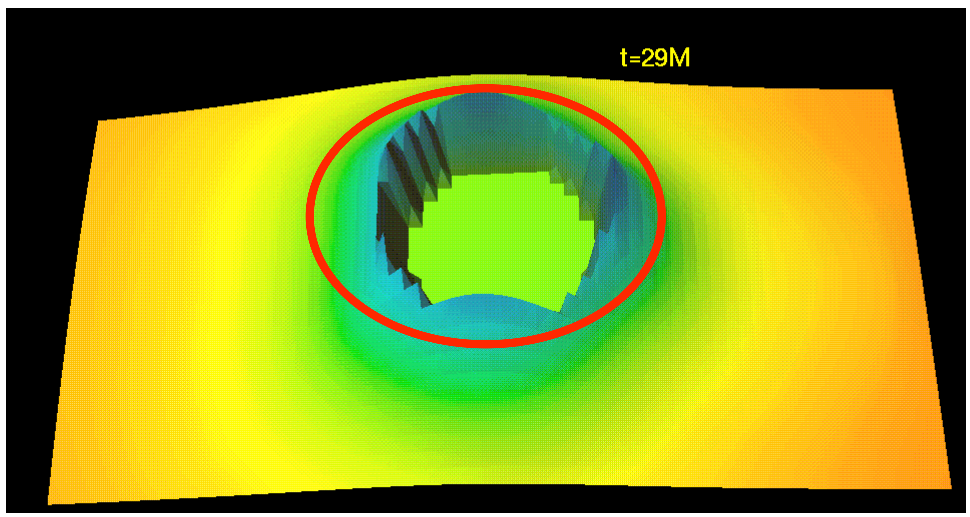
Herrman, Hinder, Shoemaker, Laguna

Black Hole Singularity: Excision

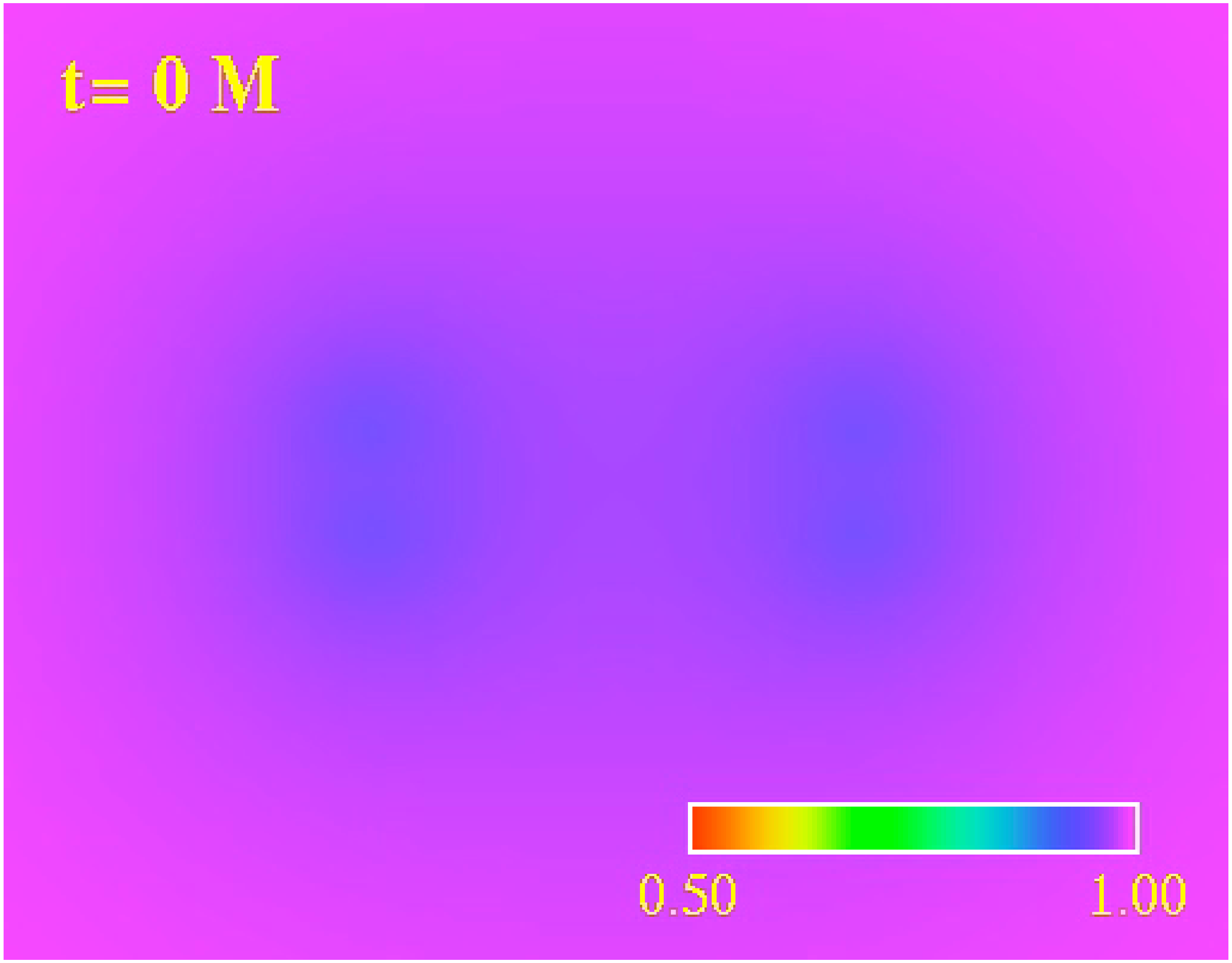




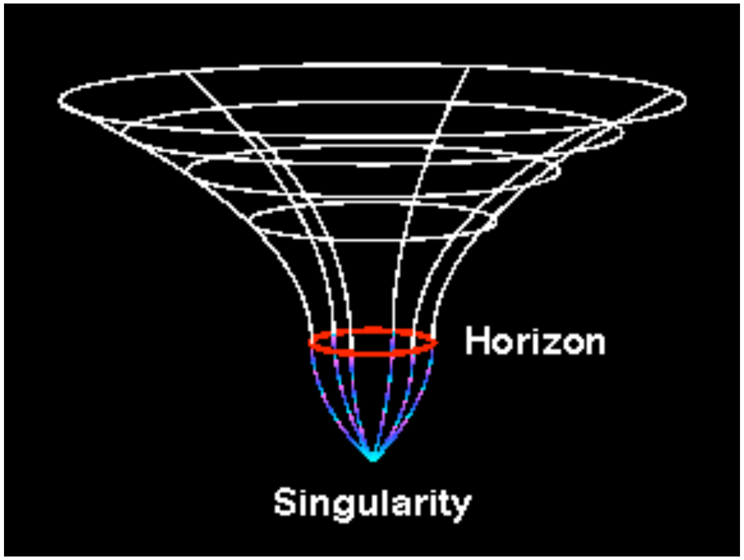
A. Hamilton



Pretorius



Black Hole Singularity: Punctures



$$g_{ij} = \left(1 + \frac{M_1}{|\vec{r} - \vec{r_1}|} + \frac{M_2}{|\vec{r} - \vec{r_2}|}\right)^4 h_{ij}$$

Old View:

Explicitly hard-code the divergences

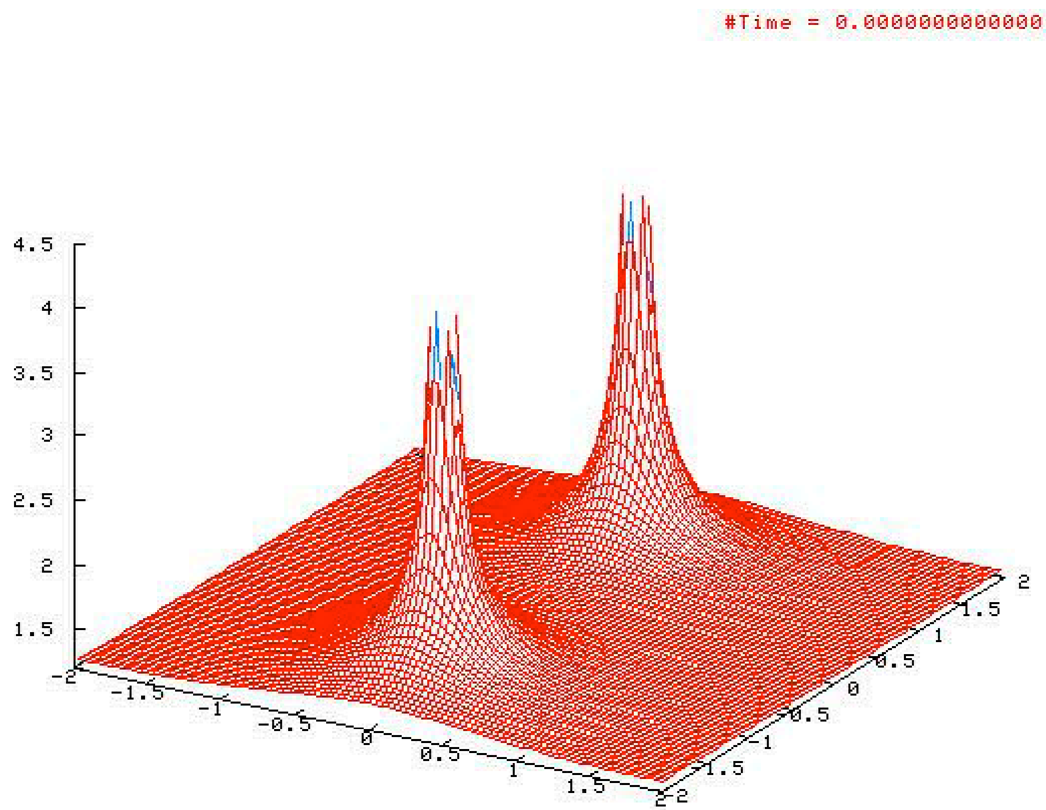
$$g_{ij} = \left(1 + \frac{M}{2\,r}\right)^4 \eta_{ij}$$

Flat Metric

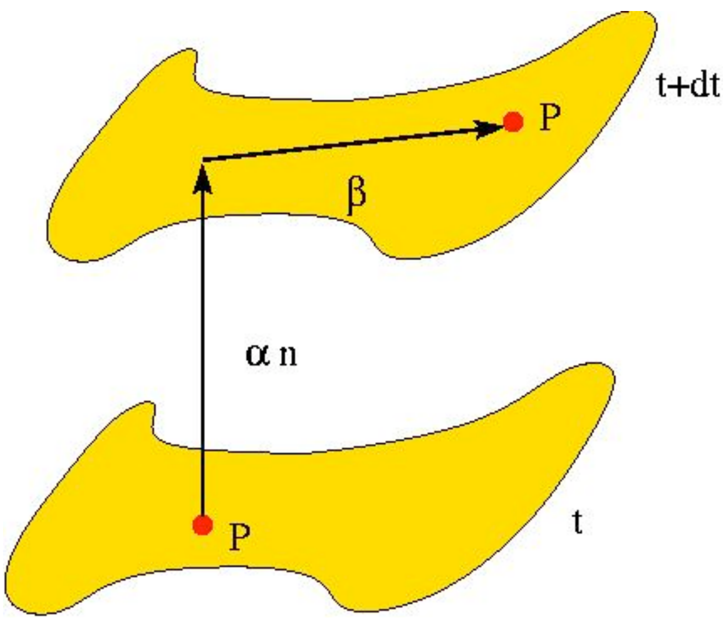
New View:

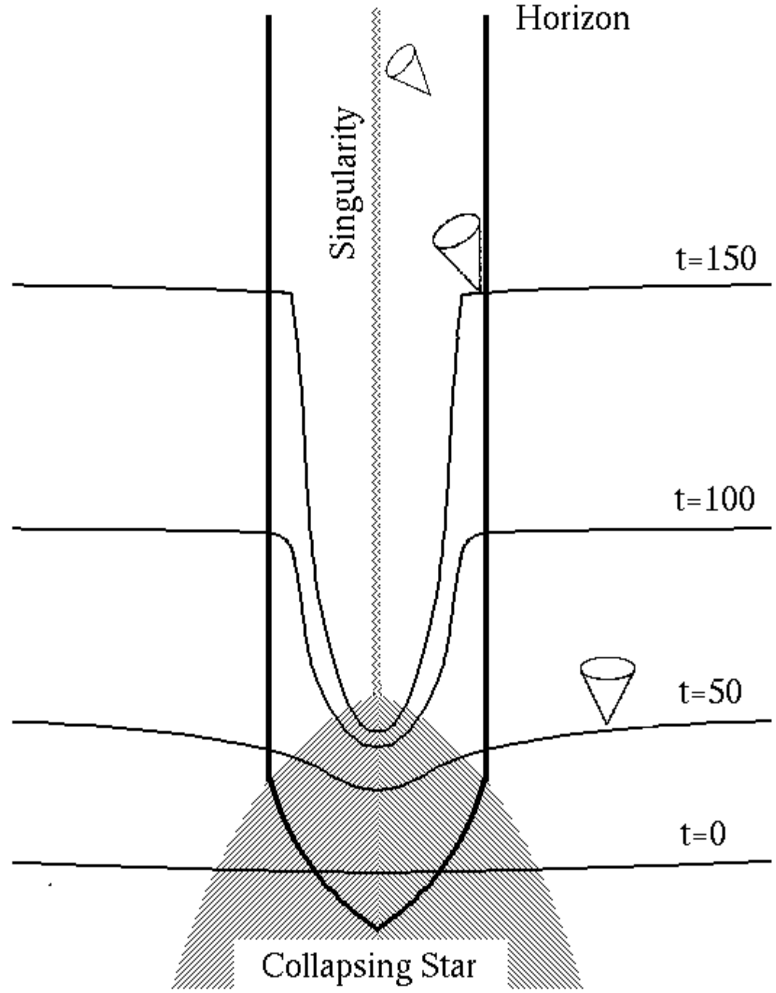
Don't worry, be happy.
Numerical smoothing will handle the singularities

UTB

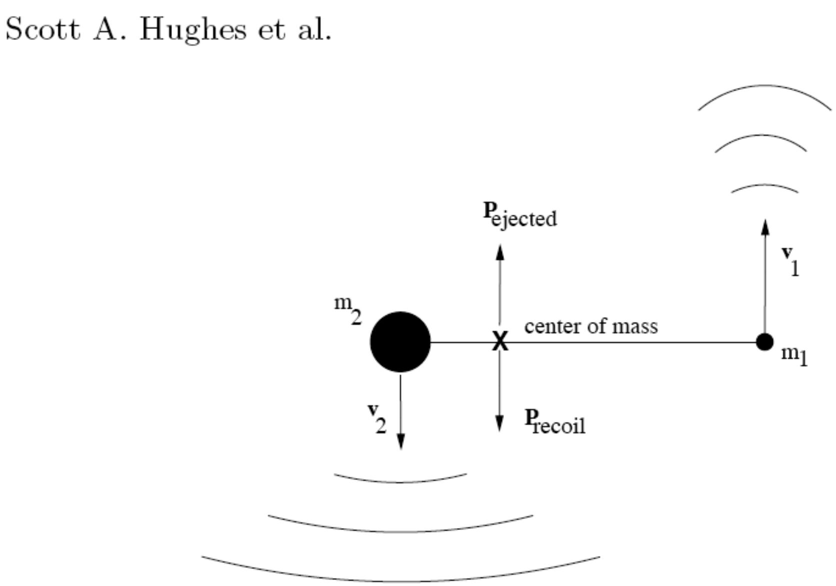

#Time = 0.0000000000000
4.5
4
3.5
3
2.5
2
1.5
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2

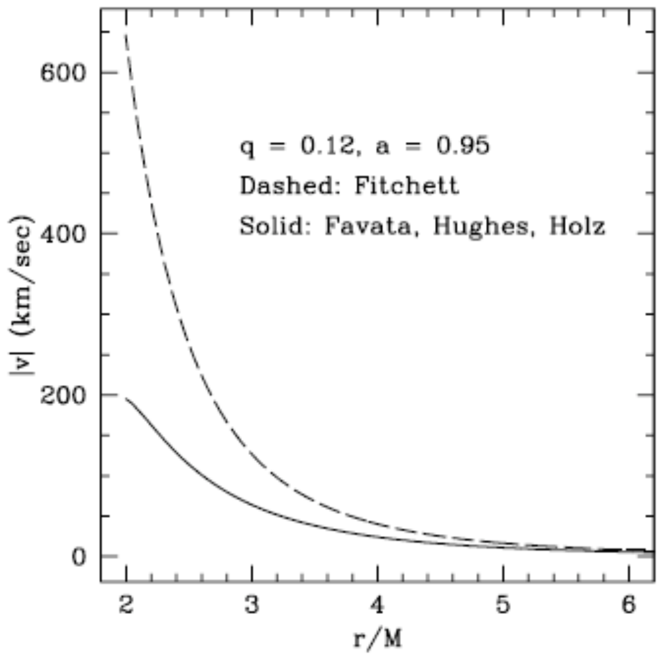
“Good” Coordinate Conditions





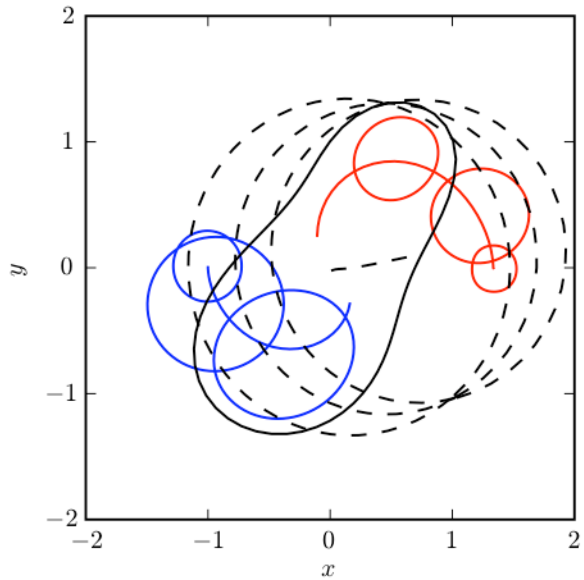
Gravitational Radiation Recoil: Black Hole Kicks

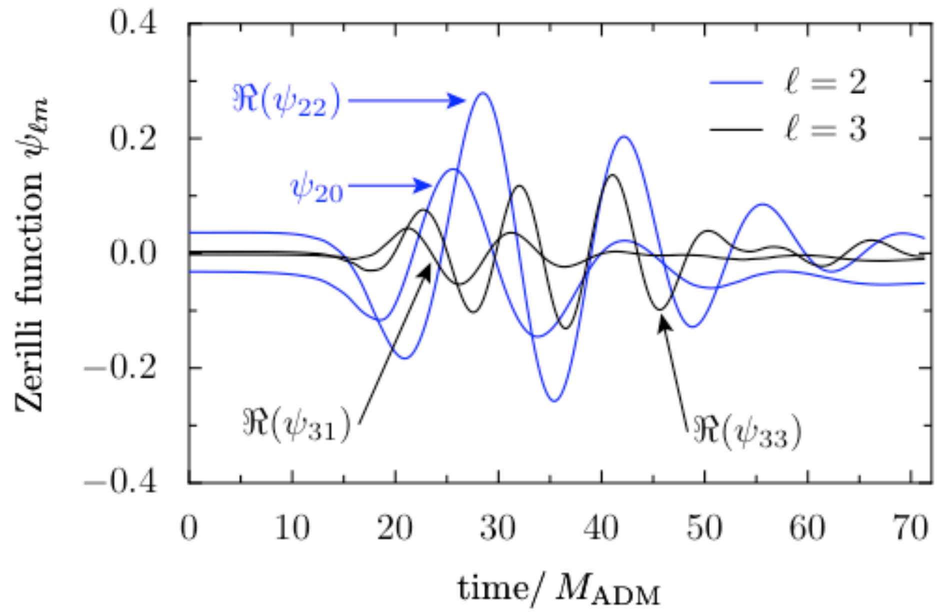




Kick Estimates

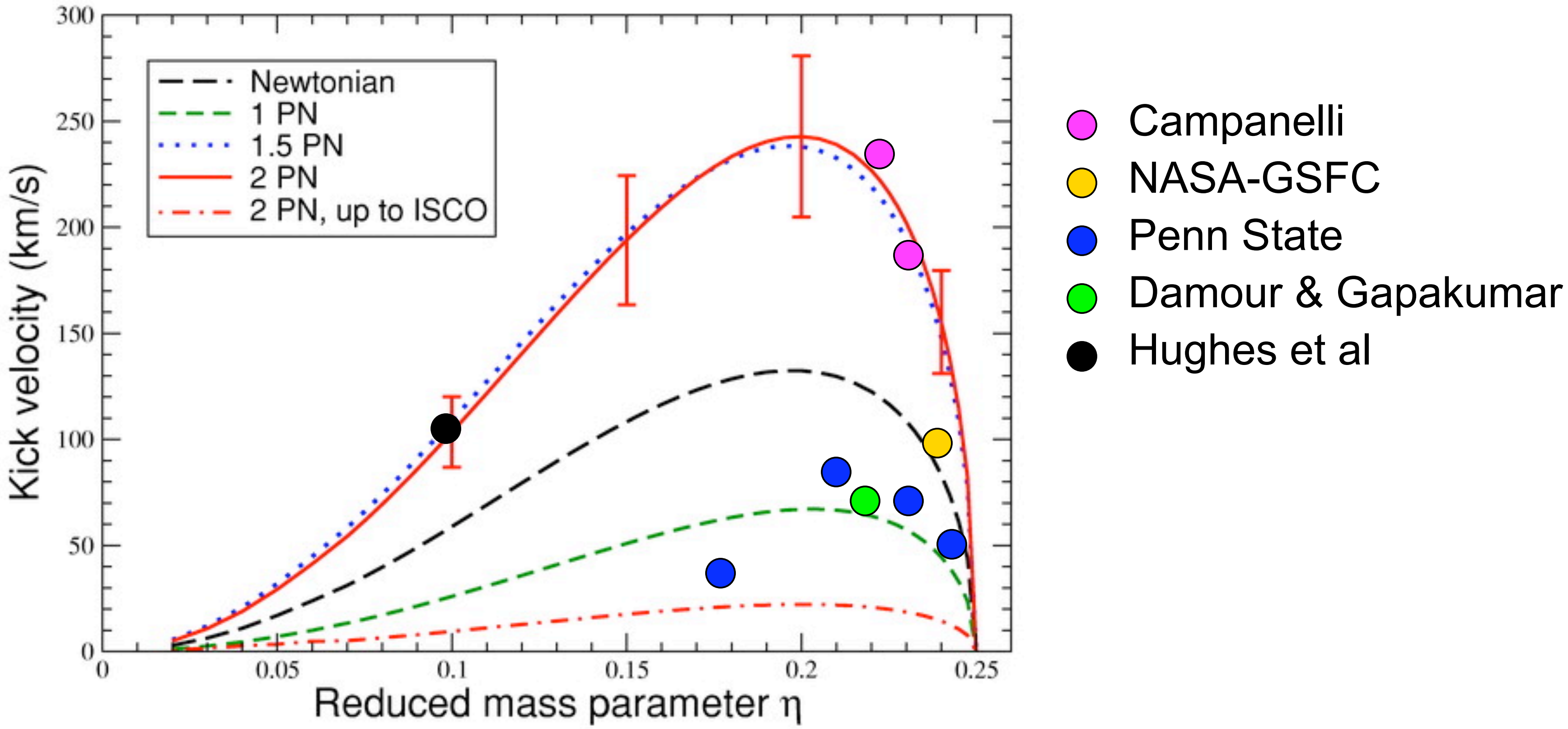
Herrmann, Hinder, Shoemaker, Laguna gr-qc/0601026





q	ΔE	ΔJ	V (km/s)
1.00	2.7	15	0
0.85	1.7	10	49
0.78	1.1	7.4	69
0.55	0.4	2.6	82
0.32	0.05	0.4	25

Kick Estimates



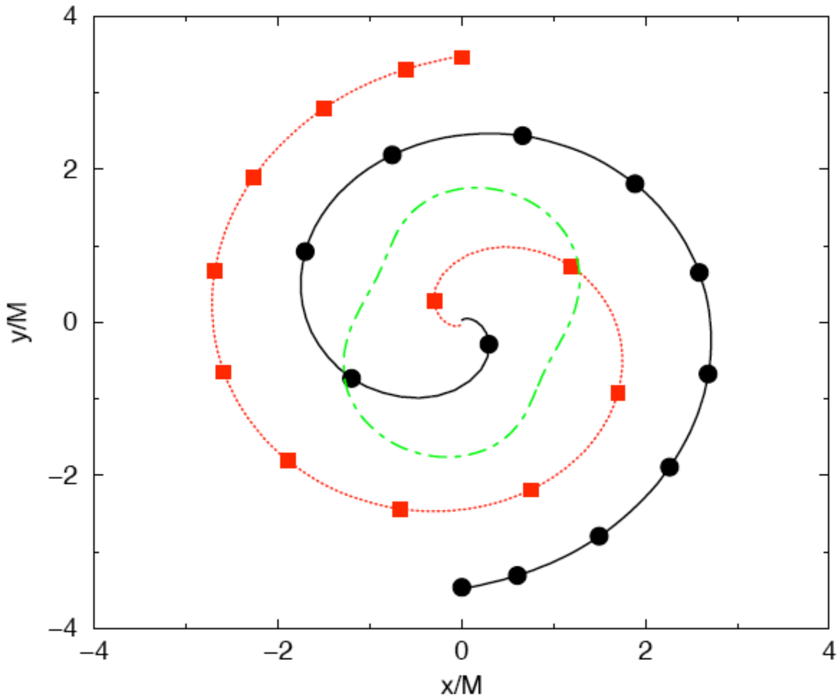
$$\eta = M_1\, M_2/(M_1 + M_2)^2$$

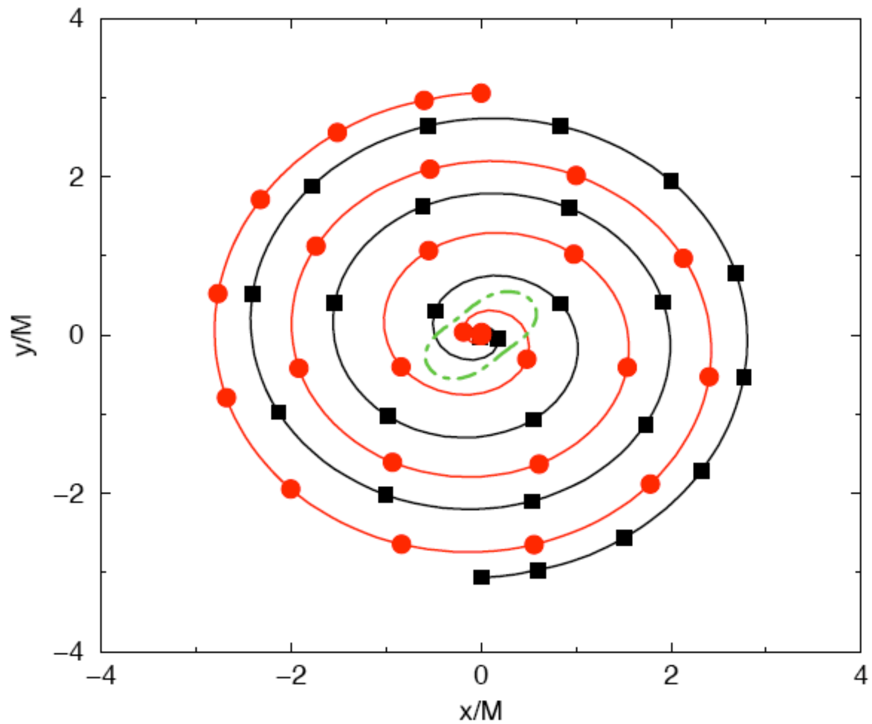
Blanchet, Qualiasah & Will (2005)

$$\frac{V}{c} = 0.043\eta^2 \sqrt{1-4\eta}\left(1+\frac{\eta}{4}\right)$$

BBH and Spins

Campanelli, Lousto, Zlochower gr-qc/0604012





Spins Counter-aligned
Initial Spin= 0.75
Time to merge ~ 105.5 M
Energy Radiated ~ 2%
Ang. Mom. Radiated ~ 26%
Final Spin ~ 0.44

Spins Aligned
Initial Spin= 0.75
Time to merge ~ 224.5 M
Energy Radiated ~ 6%
Ang. Mom. Radiated ~ 33%
Final Spin ~ 0.9

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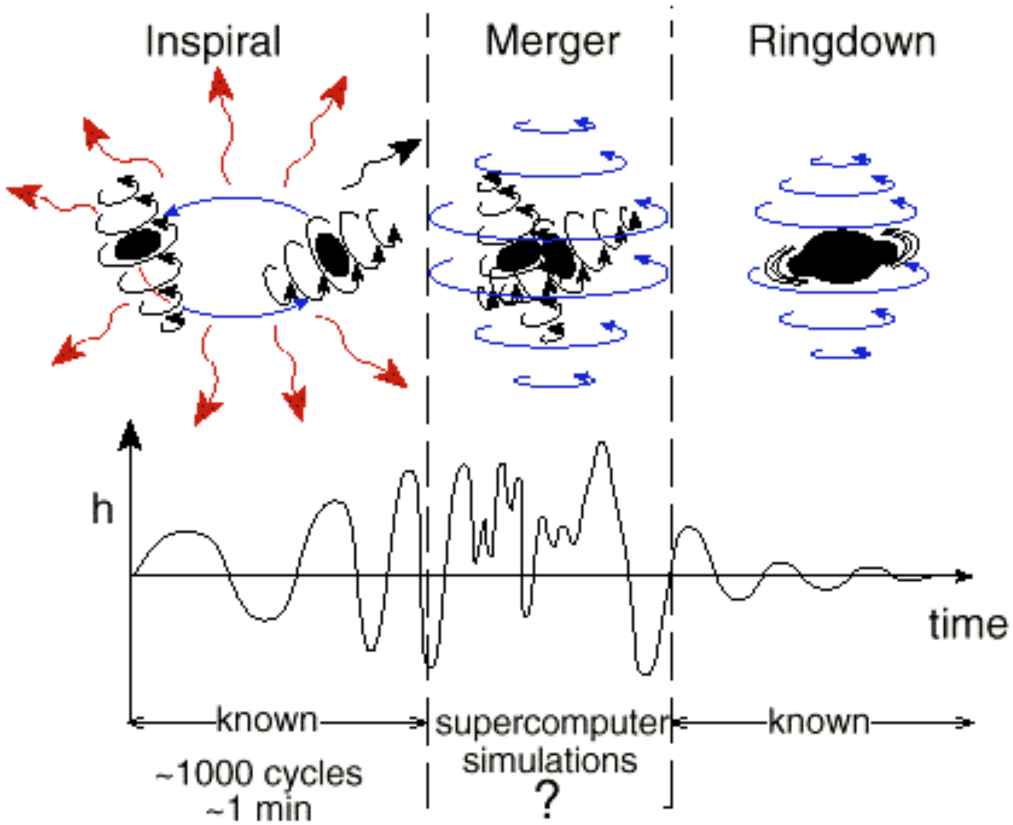
Gravitational Waves: Degree of Complexity



Is this the right picture?

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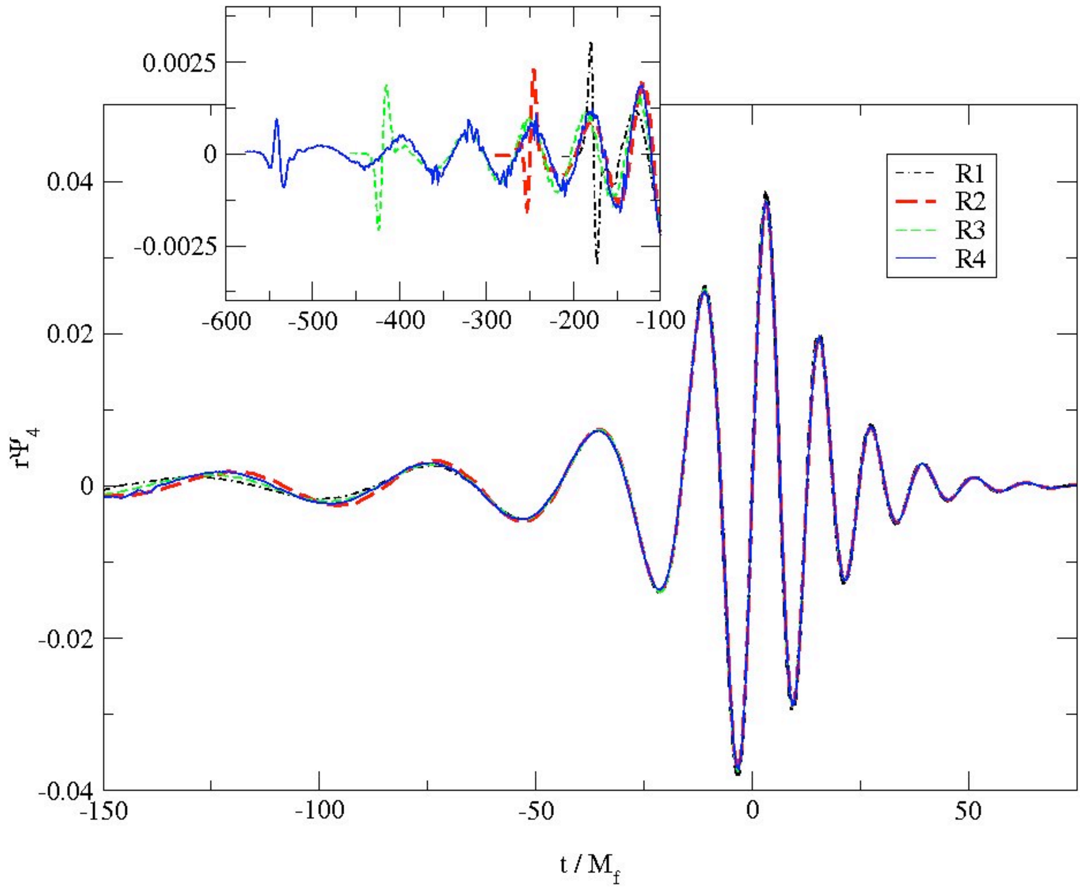
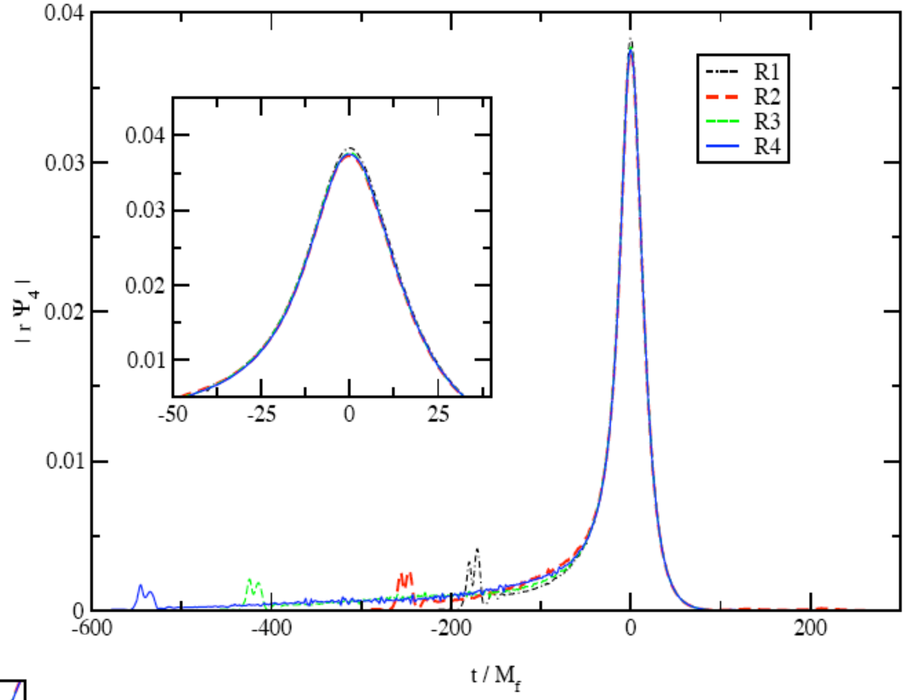
So far, it seems not!

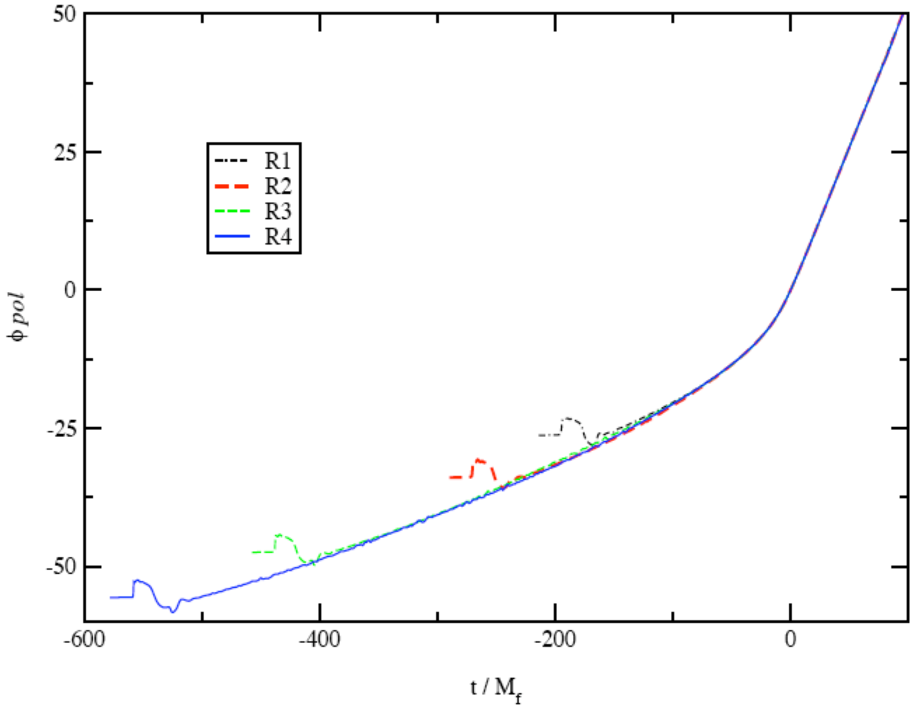


Initial separations:
R1 = 6.5 M
R2 = 7.6 M
R3 = 8.5 M
R4 = 9.6 M

NASA-GSFC
Baker, Centrella, Choi, Koppitz, van Meter
Phys.Rev. D73 (2006) 104002

Universality?





$$r\,\Psi_4 = A\,\exp\left(-i\,\phi\right)$$

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What Next?

- Better Initial Data
- Better Computational Infrastructure
- Access to faster and larger hardware
- Connection with Post-Newtonian results
- Connection with Data Analysis
- More on Spins and Un-equal Mass Binaries
- BH-NS Binaries
- Why is the moving puncture recipe working?

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In Conclusion,

Almost, and if you stop asking us, we'll get there much faster!