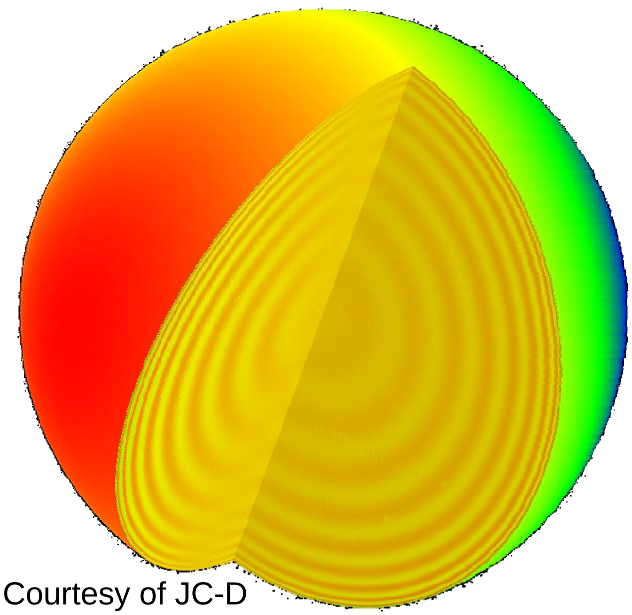
# On the seismic age and heavy-element abundance of the Sun

**Günter Houdek**

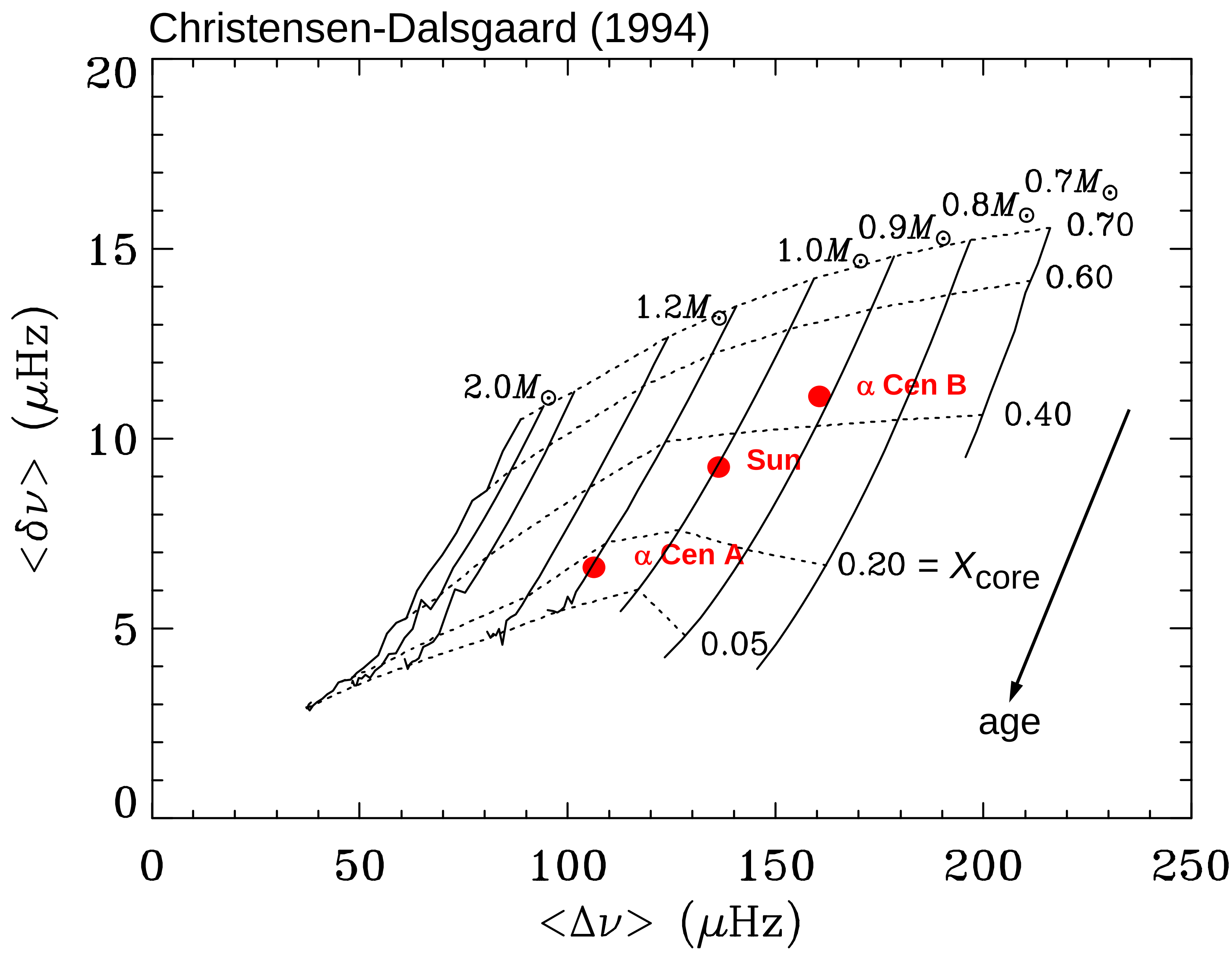
In collaboration with
Douglas Gough



Courtesy of JC-D

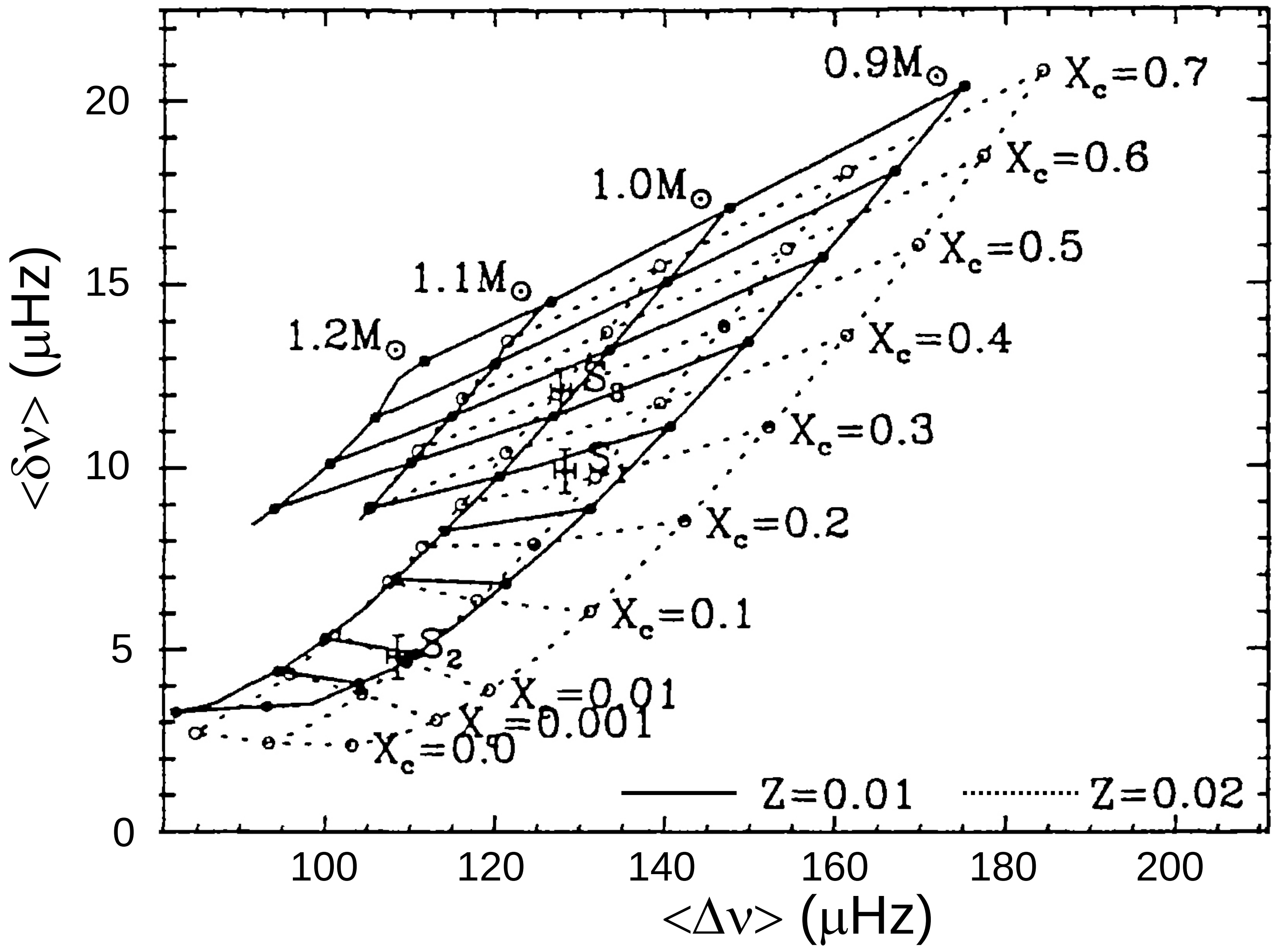
**KITP** – UCSB – November 2011

# Asteroseismic (Δν,δν) diagram



# Asteroseismic (Δν,δν) diagram

Monteiro et al. (2002)



# Points to consider in an age calibration procedure

- match seismic signatures of theoretical frequencies to observed frequencies.

- signatures are chosen to reflect principally the properties of energy-generating core.

- but such core signatures are also susceptible to e.g. zero-age chemical abundances and are contaminated by contributions produced by the surface layers.

- we need additional diagnostic to measure abundance (e.g., helium) independently and to separate the surface from the core signatures.

- here we use abrupt variation of the first adiabatic exponent $\gamma_1$ induced by He ionization.

# Age-sensitive diagnostics of the stellar structure

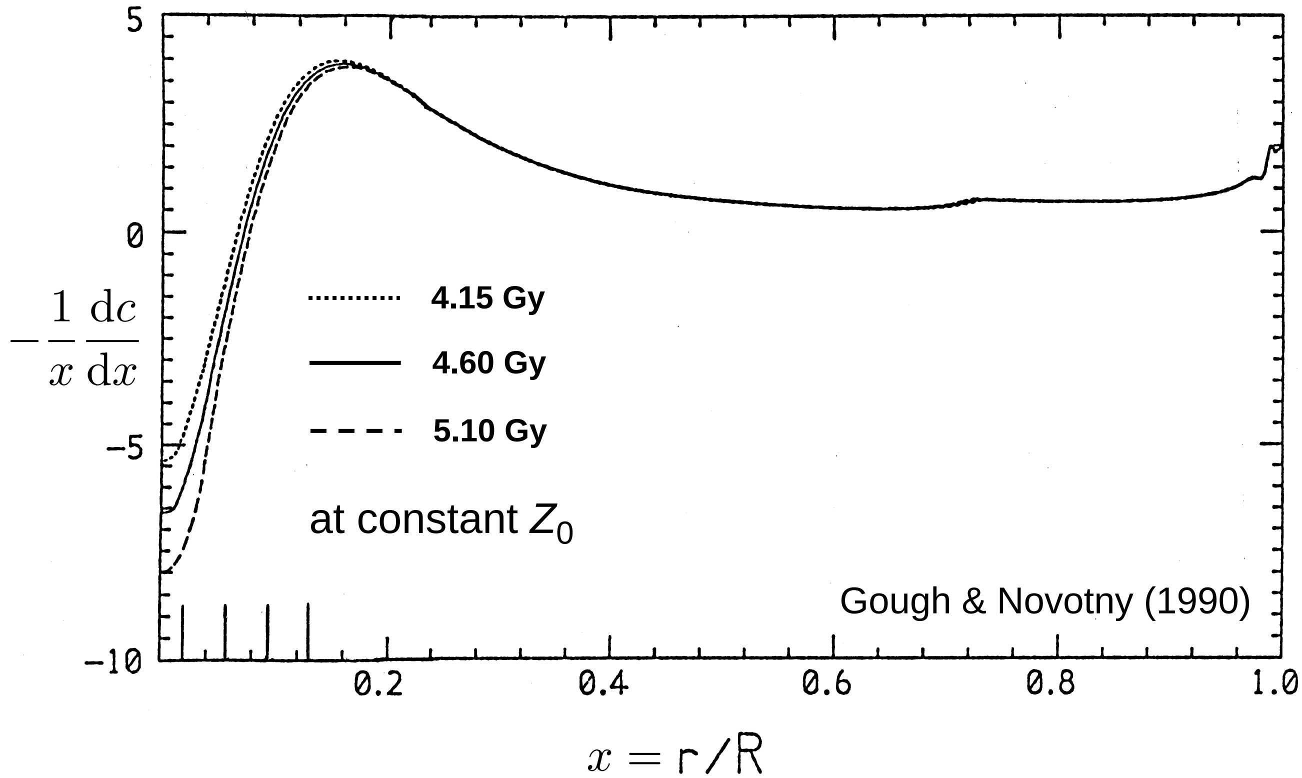
-asymptotic p-mode frequency behaviour (*n*>>*l*): $[L^2 = l(l+1)]$

$$\nu \simeq (n + \frac{1}{2}l + \epsilon)\nu_0 - \frac{A\,L^2 - B}{\nu}\,\nu_0^2$$

$$A = \frac{1}{4\pi^2\nu_0}\left[\frac{c(R)}{R} - \int_0^R \boxed{\frac{1}{r}\frac{\mathrm{d}c}{\mathrm{d}r}}\mathrm{d}r\right]$$

- evolutionary computations depend on 3 initial parameters: e.g., $Y_0$, $Z_0$ and $\alpha_c$

- Calibrated (*L*, *R*) models: ➡ $Z_0(Y_0,\alpha_c)$ @ any $t_\star$ ➡ 2-parameter set of models $(Z_0,t_\star)$



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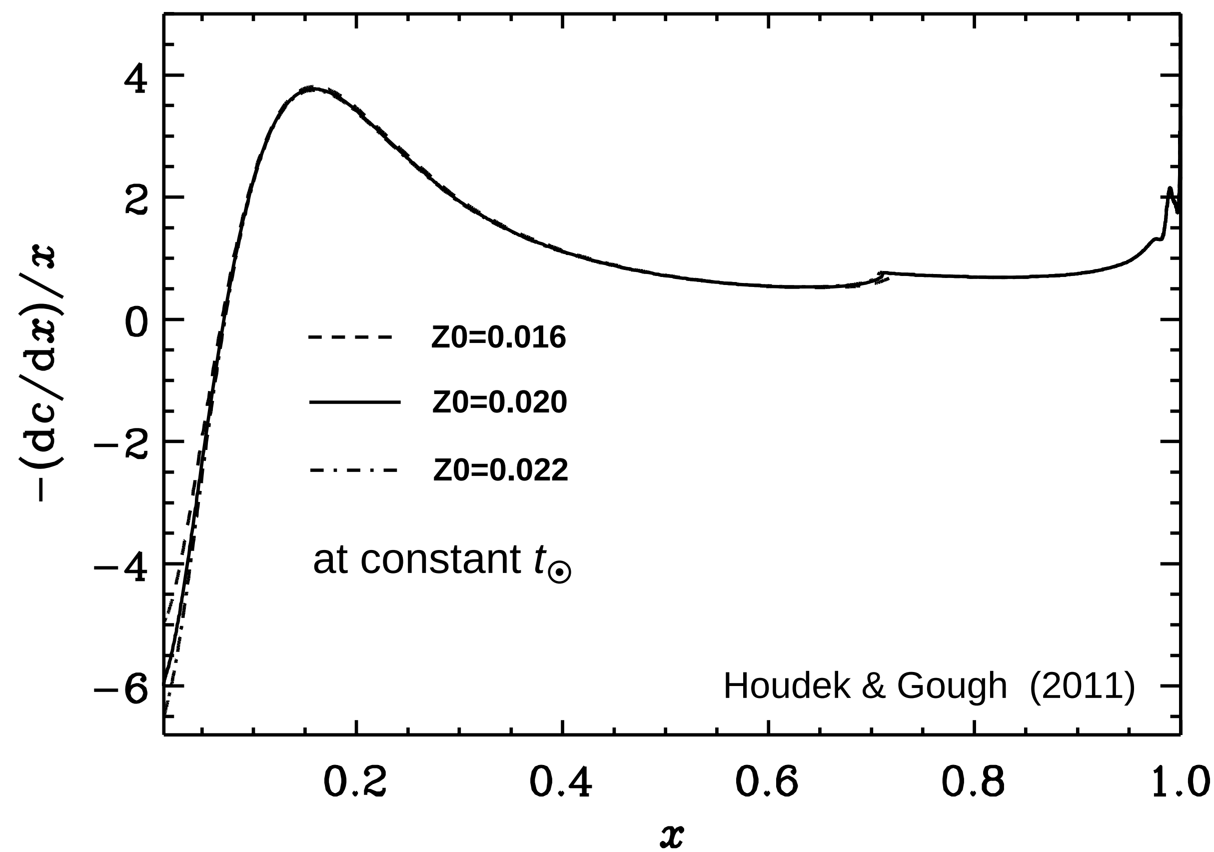

# Age-sensitive diagnostics of the stellar structure

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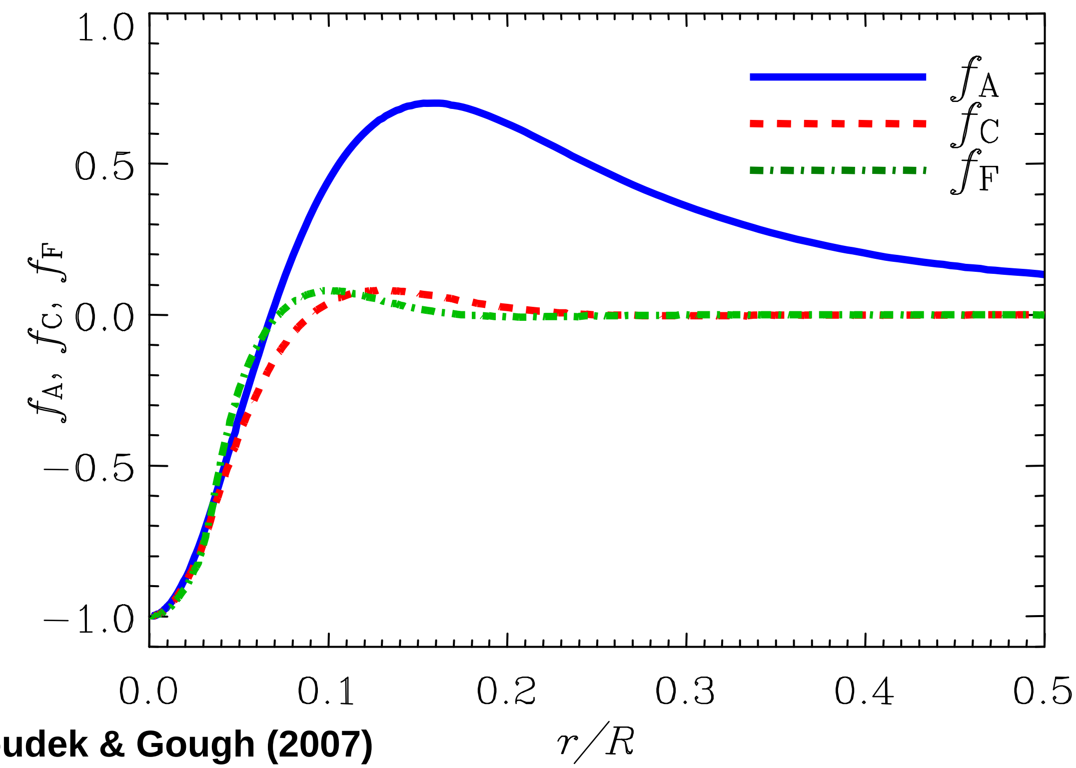



**Houdek & Gough (2007)**

# Age-sensitive diagnostics of the stellar structure

-asymptotic p-mode frequency behaviour (*n*>>*l*):

$$\nu_{\rm s} \simeq (n+\frac{1}{2}l+\epsilon)\nu_0 - \frac{A\,L^2-B}{\nu_{\rm s}}\,\nu_0^2 - \frac{C\,L^4-D\,L^2+E}{{\nu_{\rm s}}^3}\,\nu_0^4 - \frac{F L^6-GL^4+HL^2-I}{{\nu_{\rm s}}^5}\,\nu_0^6$$

- calibration using combinations of the seismically determined parameters

$$\xi_\alpha = (\underbrace{\widehat{A},\widehat{C},\widehat{F}}_{\text{age}},\underbrace{-\delta\gamma_1/\gamma_1}_{Y}), \qquad \alpha = 1,2,3,4\,, \qquad \widehat{A} = \nu_0 A\,,$$

$$-\delta\gamma_1/\gamma_1 \propto Y \qquad \widehat{C} = \nu_0^3 C\,,$$

$$\widehat{F} = \nu_0^5 F$$

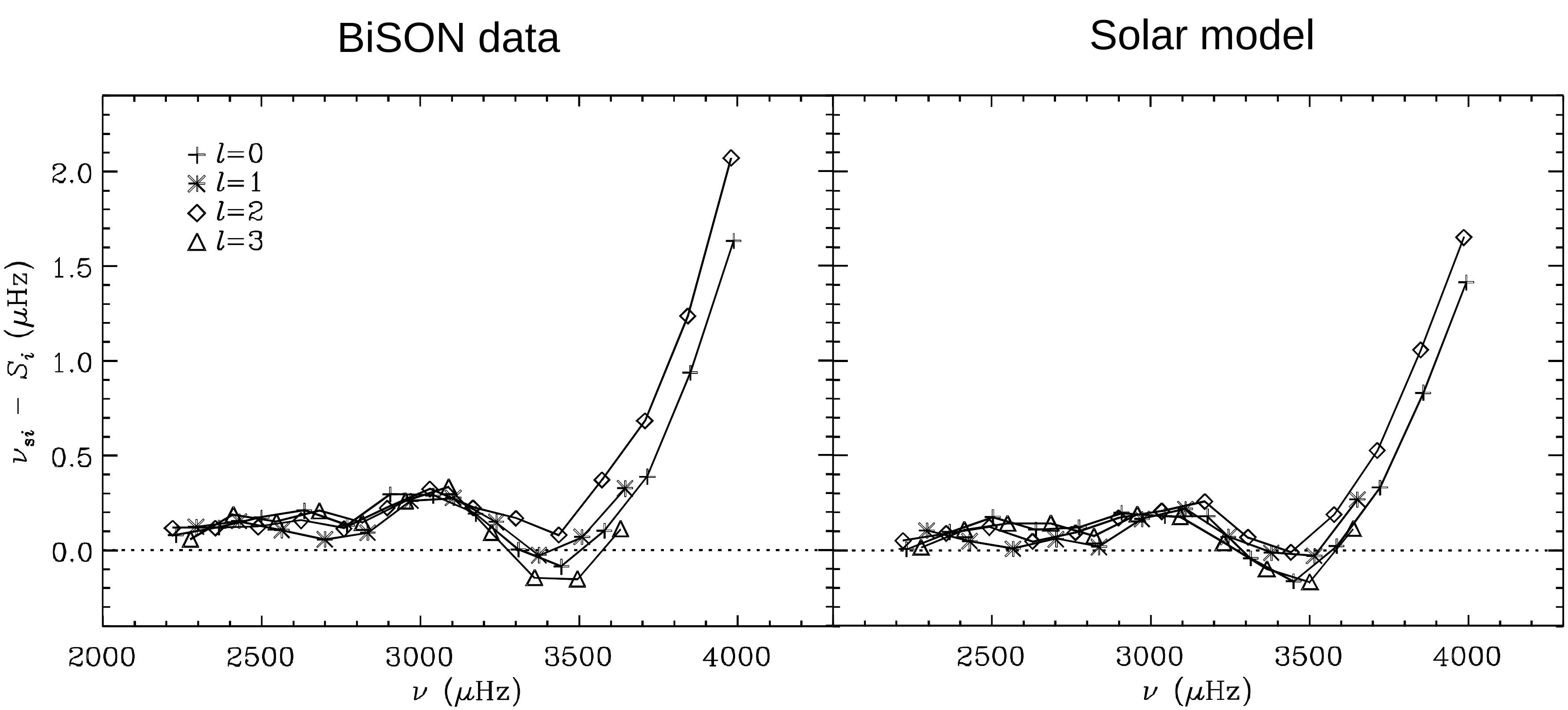
- asymptotic formula approximates adiabatic $\nu_S$ only if scale height $H \gg k_{\rm v}^{-1}$

→ glitch-free $\nu_S$ are frequencies of a “smoothed” stellar model

**→ we need a diagnostics for the acoustic glitch contributions to estimate $-\delta\gamma_1/\gamma_1$ and to construct $\nu_S$**

# Age-sensitive diagnostics of the stellar structure

Glitch-free BiSON data / model frequencies – asymptotic expression ($S_i$)



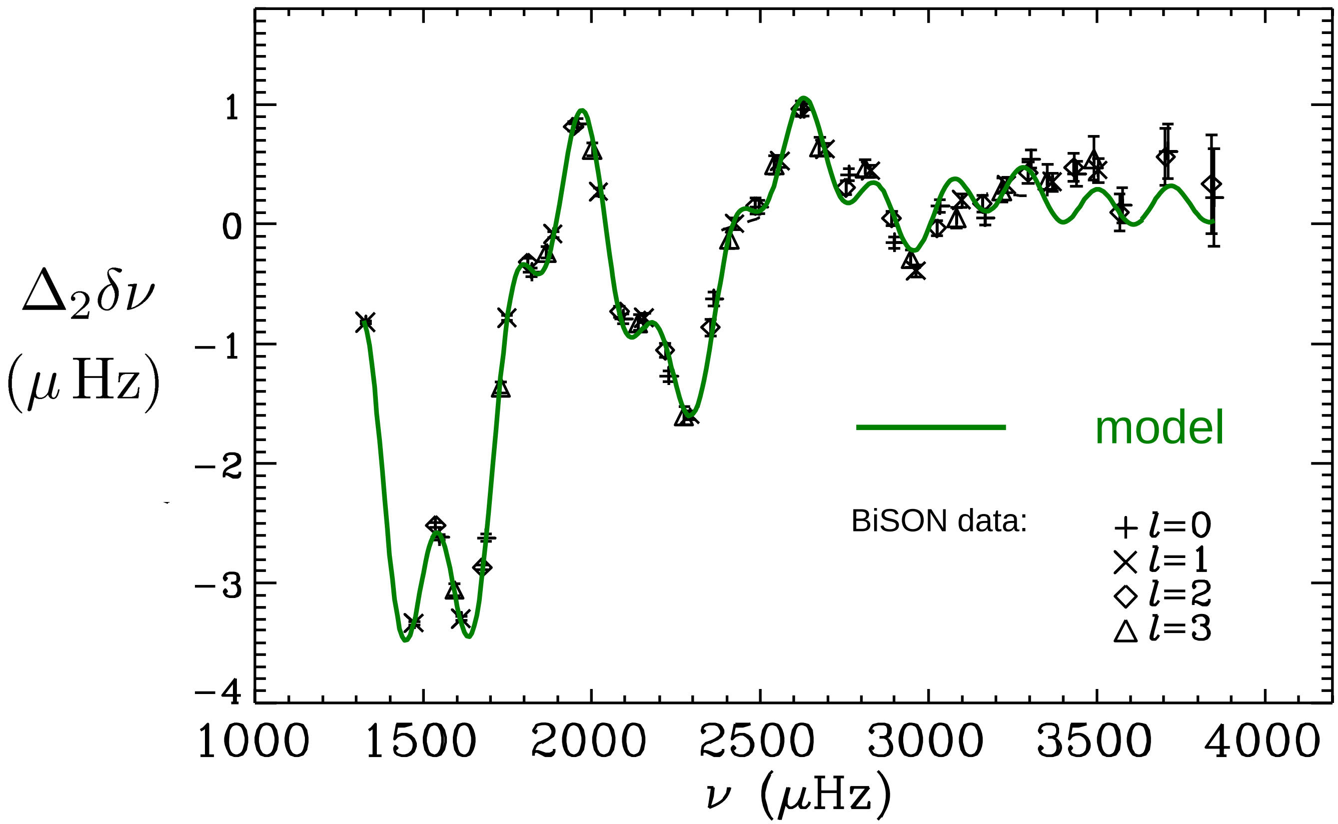
Glitch contributions $\delta\nu = \nu - \nu_{\rm s}$

( for $H \ll k_{\rm v}^{-1}$)

# Glitch contributions

$$\delta\nu = \nu \; - \; \nu_{\rm s}$$

Gough (1990): $2^{\rm nd}$ frequency differences : $\Delta_2\nu \equiv \nu_{n-1} - 2\nu_n + \nu_{n+1}$



# A model for glitch contributions
# (A seismic diagnostics)

# Seismic diagnostics: variational principle in (nonrotating) stars

Linearized, adiabatic, wave equation:

$$\omega^2 \boldsymbol{\xi} = \mathcal{L}(\boldsymbol{\xi})$$

operator $\rho_0 \mathcal{L}$ is hermitian for $\nabla p_0 = 0$ at boundary:

$$\delta\omega \simeq \frac{\delta_\gamma \mathcal{K}}{2\omega I}$$

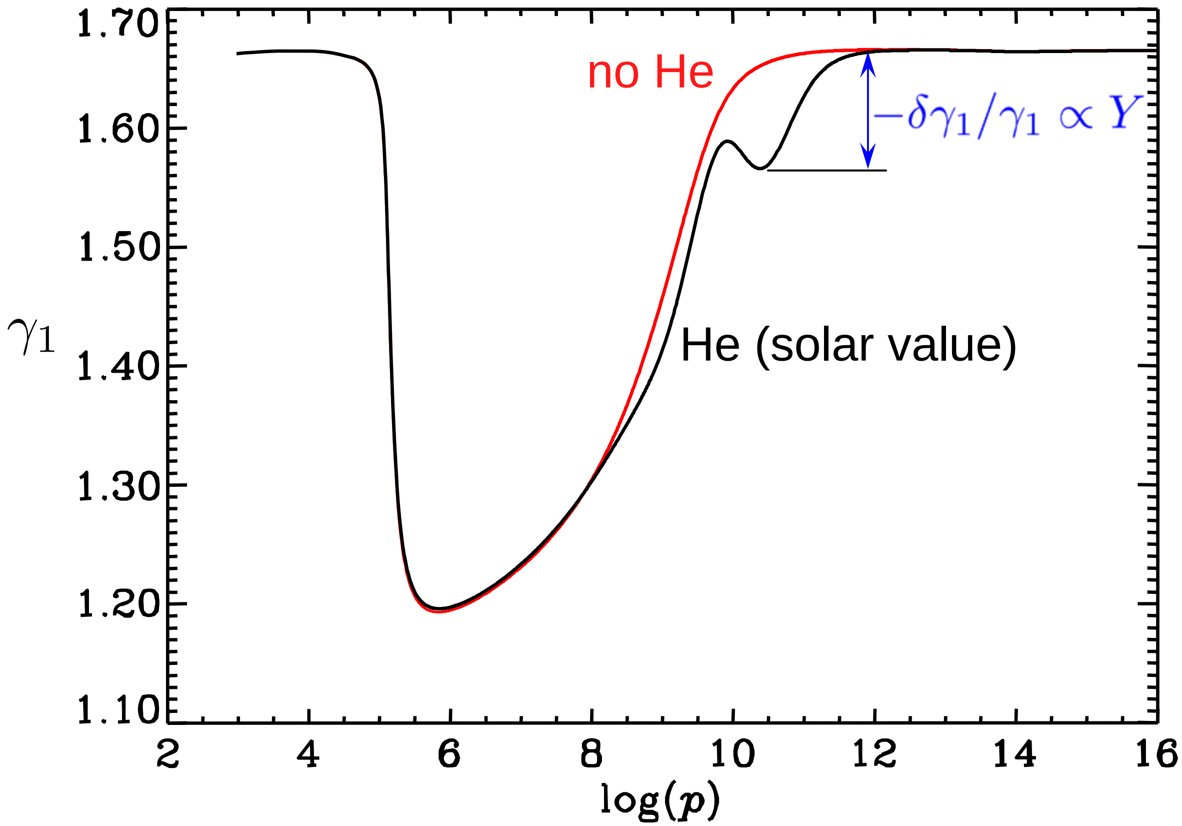
asymptotic limit (JWKB):

$$\delta_\gamma \mathcal{K} \simeq \pi\omega^3 \int \kappa^{-1} \boxed{\frac{\delta\gamma_1}{\gamma_1}} |x|^{1/2} |\mathrm{Ai}(-x)|^2 \, \mathrm{d}\tau$$

# Seismic diagnostics

Squared adiabatic sound speed $c^2 = \gamma_1\, p/\rho$ $\qquad \gamma_1 = (\partial \ln p / \partial \ln \rho)_s$

Adiabatic exponent $\gamma_1$



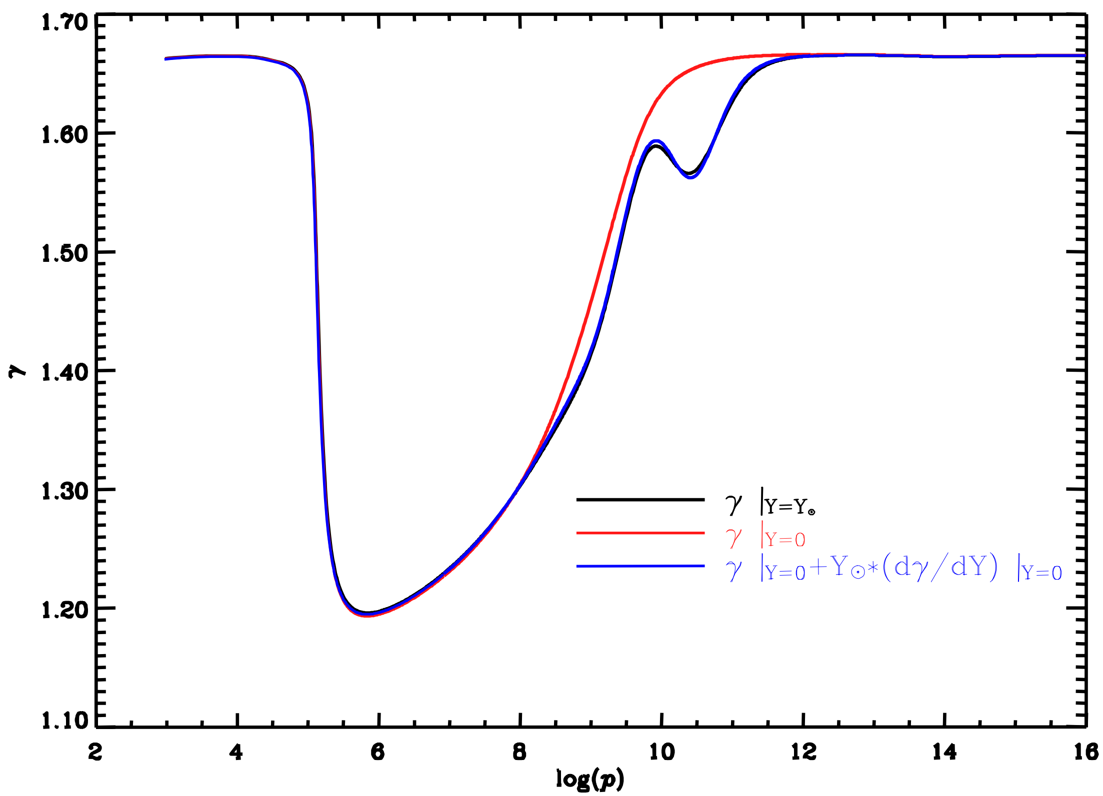
# Seismic diagnostics

Expand γ with respect to *Y* about *Y* = 0: $\gamma \simeq (\gamma)_{Y=0} + (\partial\gamma/\partial Y)_{Y=0}\, Y$ .

Glitches may then be written: $\delta\gamma \simeq (\partial\gamma/\partial Y)_{Y=0}\, Y$

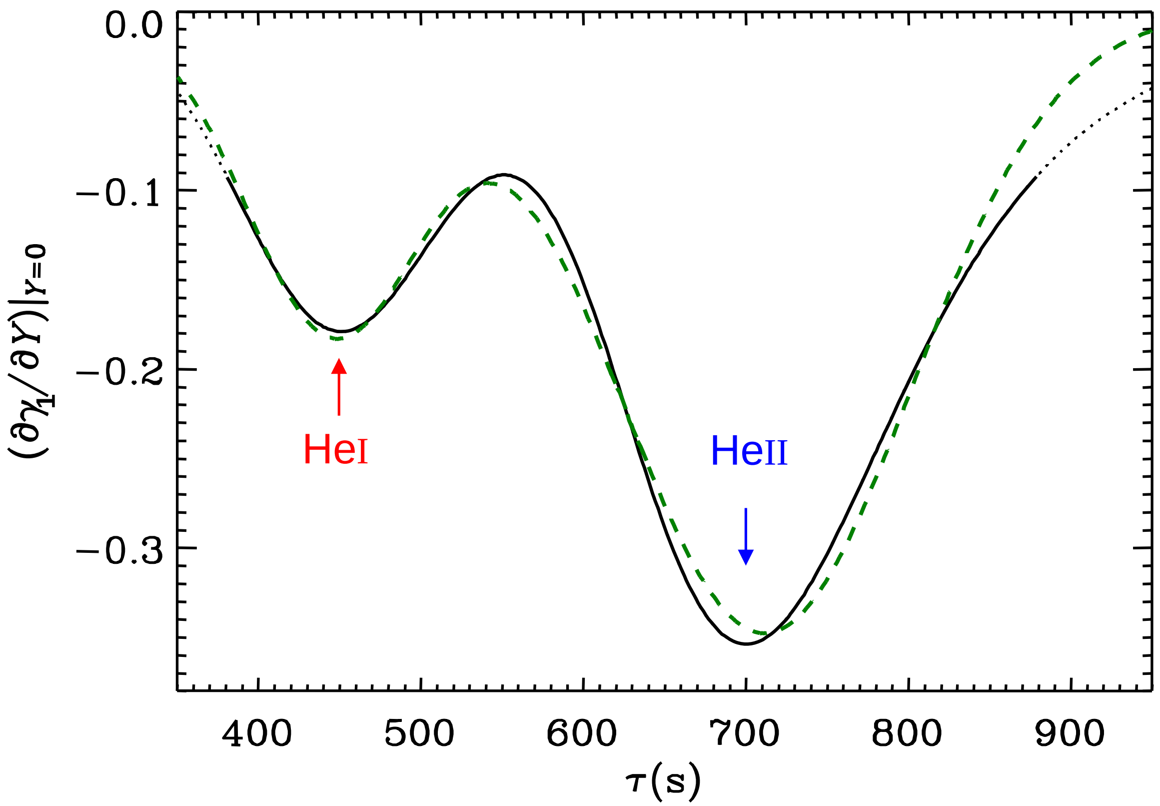
Adiabatic exponent $\gamma_1$

$Y$ …. helium abundance



# Seismic diagnostics

Glitches may be written as: $\delta\gamma \simeq (\partial\gamma/\partial Y)_{Y=0}\, Y$




$Y$ .... helium abundance

$$\frac{\partial\gamma}{\partial Y} \simeq -\frac{\gamma}{\sqrt{2\pi}Y}\left[\frac{\Gamma_{\rm I}}{\Delta_{\rm I}}e^{-(\tau-\tau_{\rm I})^2/2\Delta_{\rm I}^2} + \frac{\Gamma_{\rm II}}{\Delta_{\rm II}}e^{-(\tau-\tau_{\rm II})^2/2\Delta_{\rm II}^2}\right].$$

# Seismic diagnostic

$$\delta\nu \simeq \boxed{\delta_\gamma \nu} + \boxed{\delta_{\rm c}\nu} + \boxed{\delta_{\rm s}\nu}$$

He     BCZ     surface term

$$\boxed{\delta_\gamma \nu} = -\sqrt{2\pi} A_{\rm II} \Delta_{\rm II}^{-1} \left[\nu + \tfrac{1}{2}(m+1)\nu_0\right]$$

$$\times \left[\mu\beta \int_0^T \kappa_{\rm I}^{-1} {\rm e}^{-(\tau-\eta\tau_{\rm II})^2/2\mu^2\Delta_{\rm II}^2} |x|^{1/2} |{\rm Ai}(-x)|^2 \, {\rm d}\tau \right.$$ ..... HeI

$$\left. + \int_0^T \kappa_{\rm II}^{-1} {\rm e}^{-(\tau-\tau_{\rm II})^2/2\Delta_{\rm II}^2} |x|^{1/2} |{\rm Ai}(-x)|^2 \, {\rm d}\tau \right]$$ ..... HeII

$$\boxed{\delta_{\rm c}\nu} \simeq A_{\rm c} \nu_0^3 \nu^{-2} \left(1 + 1/16\pi^2 \tau_0^2 \nu^2\right)^{-1/2}$$

$$\times \left\{\cos[2\psi_{\rm c} + \tan^{-1}(4\pi\tau_0\nu)] - (16\pi^2 \tilde{\tau}_{\rm c}^2 \nu^2 + 1)^{1/2}\right\}$$ ..... BCZ

$$\boxed{\delta_{\rm s}\nu} \simeq \widehat{A} + \widehat{B}\nu + \left[a_0\nu^2/2 + a_1\nu(\ln\nu - 1) - a_2 \ln\nu + a_3/2\nu\right]/h^2$$ ..... Surface

( H, $\nabla$-$\nabla_{\rm ad}$ )

$x = f(\omega\tau, \epsilon, m)$     $\psi_{\rm c} = f(\omega\tau_{\rm c}, \epsilon_{\rm c})$     $h = (\nu_{n+1} - \nu_{n-1})/2$

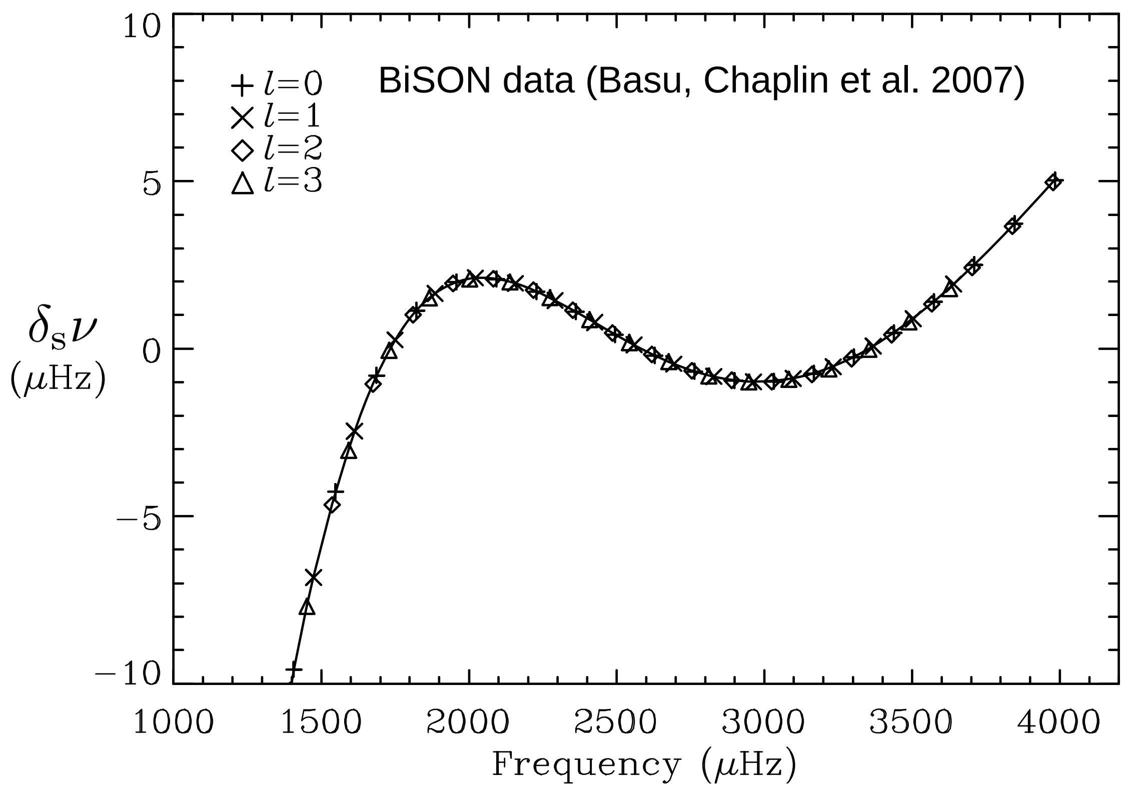
# Seismic diagnostics: surface term $\delta_{\rm s}\nu$

$$\delta_{\rm s}\nu \equiv \Delta_2^{-1}\sum_{k=0}^{3} a_k\nu^{-k}$$

$$\simeq \tilde{A} + \tilde{B}\nu + \left[a_0\nu^2/2 + a_1\nu(\ln\nu - 1) - a_2\ln\nu + a_3/2\nu\right]/h^2$$

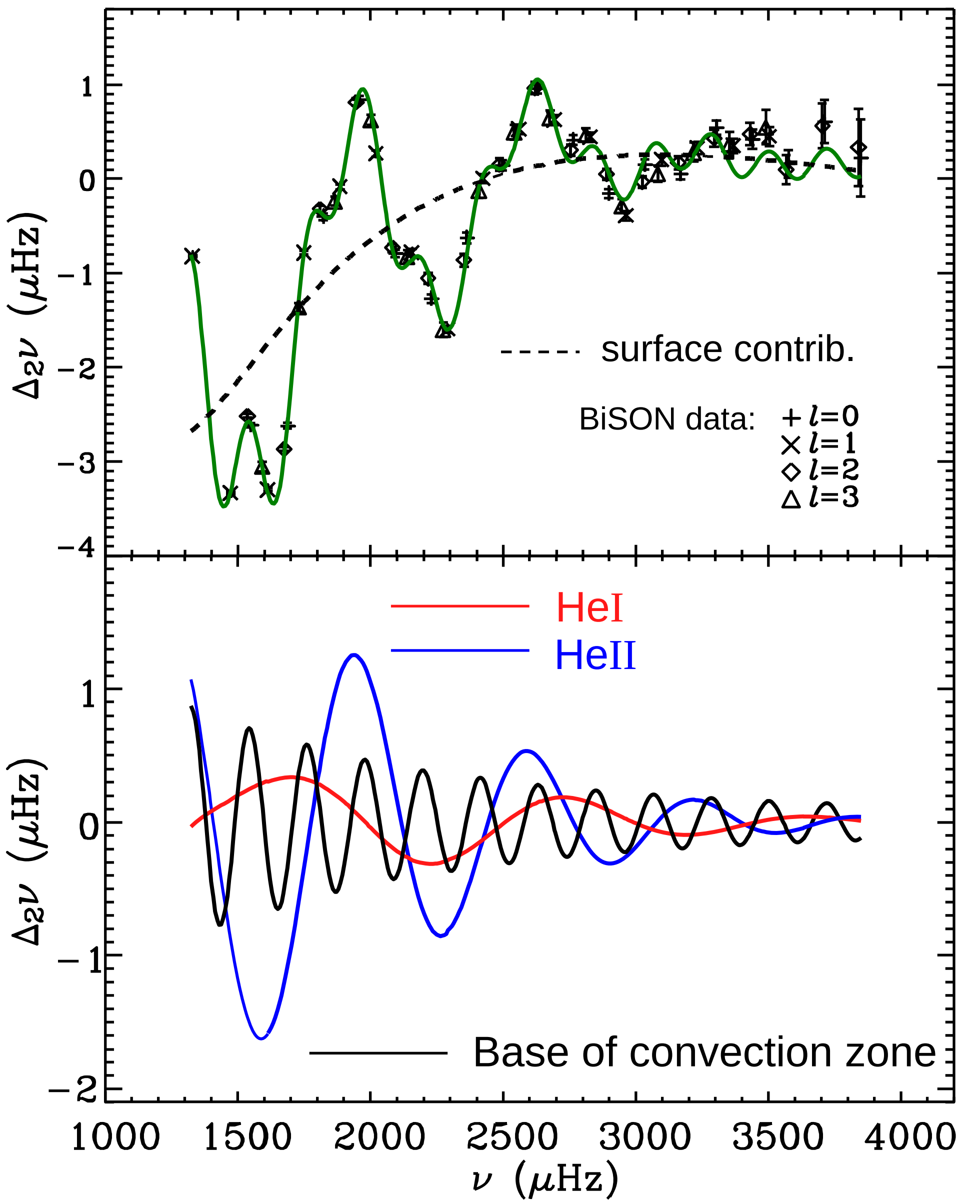
$\tilde{A}$ and $\tilde{B}$ are two undetermined constants of summation of 3rd - order polynomial

Choose $\tilde{A}$ and $\tilde{B}$ by minimizing: $E \equiv ||\delta_{\rm s}\nu||_2 = \sum_n (\tilde{A}+\tilde{B}\nu+F_{\rm s})^2$



Applying the seismic diagnostic to the Sun and simulated data (SONG)

# Applying the seismic diagnostic to low-degree p modes: Sun



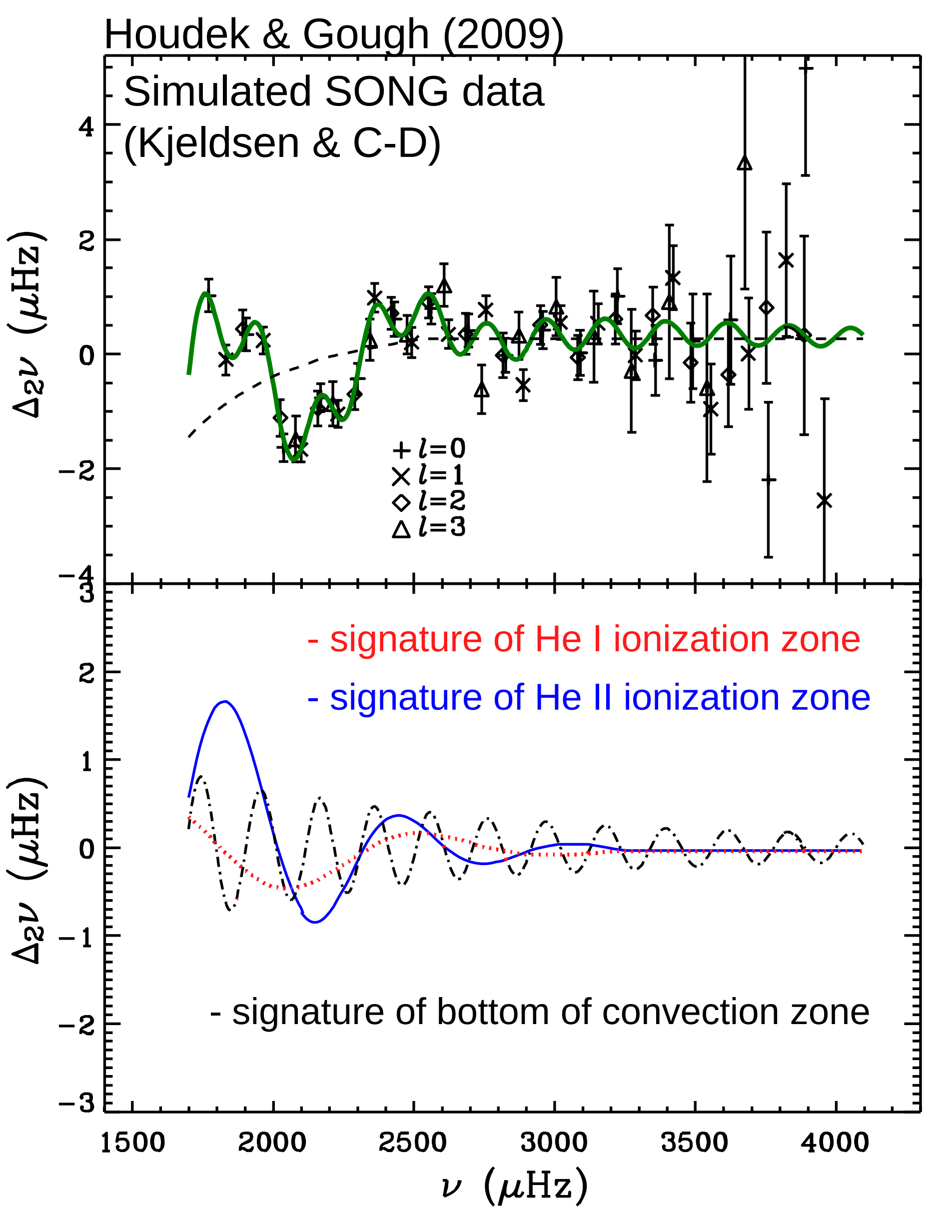

Seismic diagnostic

$$\delta\nu = \underbrace{\delta_{\rm I}\nu + \delta_{\rm II}\nu + \delta_{\rm c}\nu}_{\text{glitch contributions}} + \underbrace{\delta_{\rm s}\nu}_{\text{surface contribution}}$$

$$-\frac{\delta\gamma_1}{\gamma_1} \propto Y$$ …helium abundance

| For BiSON data: | For Model S: |
|---|---|
| $-\delta\gamma_1/\gamma_1\|_{\tau_{\rm II}} = 0.043$ | $0.045$ |
| $\tau_{\rm II} = 819\,{\rm s}$ | $815\,{\rm s}$ |
| $\tau_{\rm c} = 2310\,{\rm s}$ | $2270\,{\rm s}$ |

# Applying the seismic diagnostic to a solar-like star




| Results for SONG **data:** | for **model**: |
|---|---|
| $-\delta\gamma_1/\gamma_1\|_{\tau_{II}} = 0.086$ | $0.062$ |
| $\tau_{II} = 818\,\mathrm{s}$ | $792\,\mathrm{s}$ |
| $\tau_c = 2402\,\mathrm{s}$ | $2330\,\mathrm{s}$ |

$$\gamma_1 = (\partial \ln p / \partial \ln \rho)_s$$

$-\delta\gamma_1/\gamma_1|_{\tau_{II}}$ … rel. depression of $\gamma_1$ in He II ionization zone

$\tau_{II}$ … acoustic depth of HeII ionization zone

$\tau_c$ … acoustic depth of bottom of convection zone

# Solar/stellar age calibration

# Solar/stellar age calibration

-asymptotic p-mode frequency behaviour (*n*>>*l*):

$$\nu_s \simeq (n + \frac{1}{2}l + \epsilon)\nu_0 - \frac{A\,L^2 - B}{\nu_s}\nu_0^2 - \frac{C\,L^4 - D\,L^2 + E}{{\nu_s}^3}\nu_0^4 - \frac{F L^6 - G L^4 + H L^2 - I}{{\nu_s}^5}\nu_0^6$$

- calibration using combinations of the seismically determined parameters

$$\xi_\alpha = (\underbrace{\widehat{A}, \widehat{C}, \widehat{F}}_{\text{age}}, \underbrace{-\delta\gamma_1/\gamma_1}_{Y}), \qquad \alpha = 1,2,3,4\,, \qquad -\delta\gamma_1/\gamma_1 \propto Y$$

$$\widehat{A} = \nu_0 A\,, \quad \widehat{C} = \nu_0^3 C\,, \quad \widehat{F} = \nu_0^5 F$$

- approximate solar value $\xi_\alpha^\odot$ by a two-term expansion about reference value $\xi_\alpha^{\rm r}$

$$\xi_\alpha^\odot = \xi_\alpha^{\rm r} + \left(\frac{\partial \xi_\alpha}{\partial t_\odot}\right)_Z \Delta t + \left(\frac{\partial \xi_\alpha}{\partial Z}\right)_{t_\odot} \Delta Z - \epsilon_{\xi\alpha}\,.$$

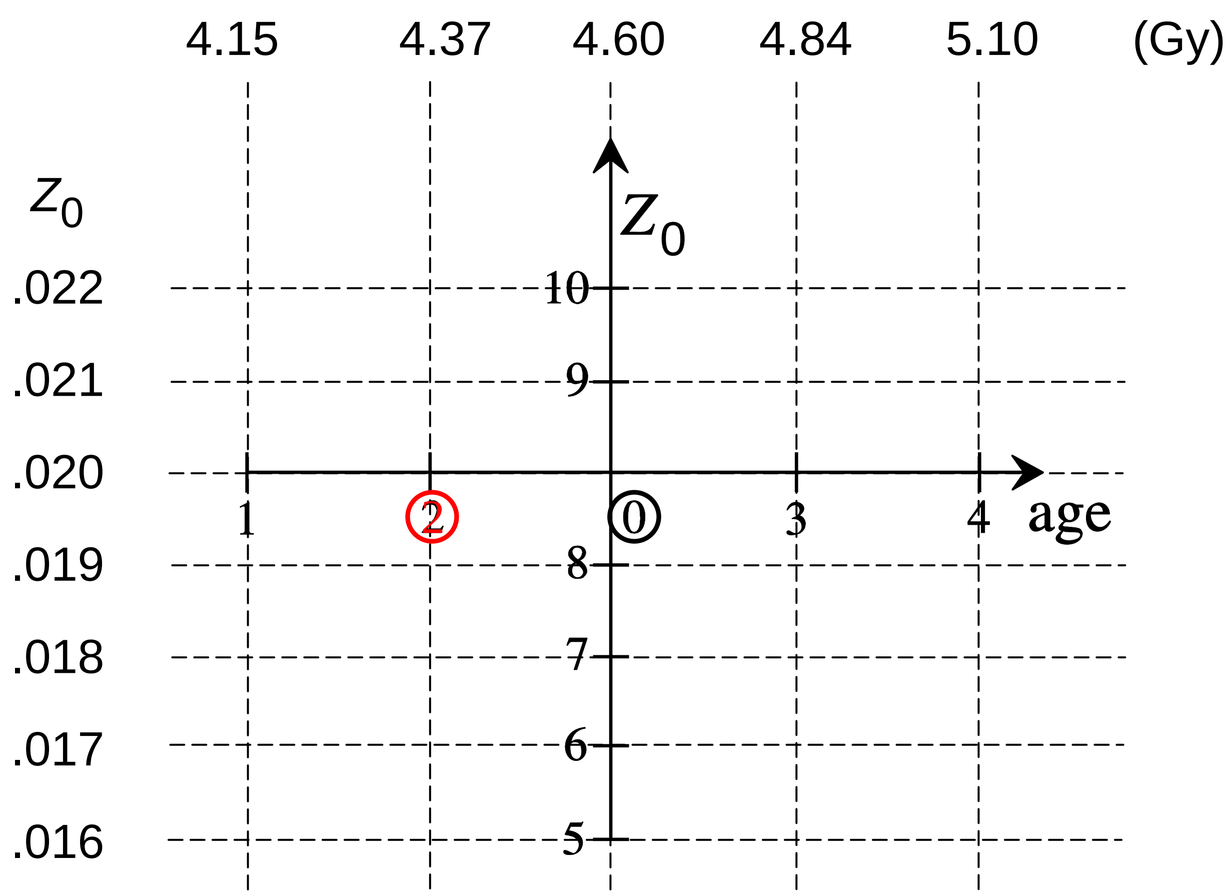
- and the **solution is**:

$$t_\odot = t_{\rm ref} + \Delta t$$
$$Z_\odot = Z_{\rm ref} + \Delta Z$$

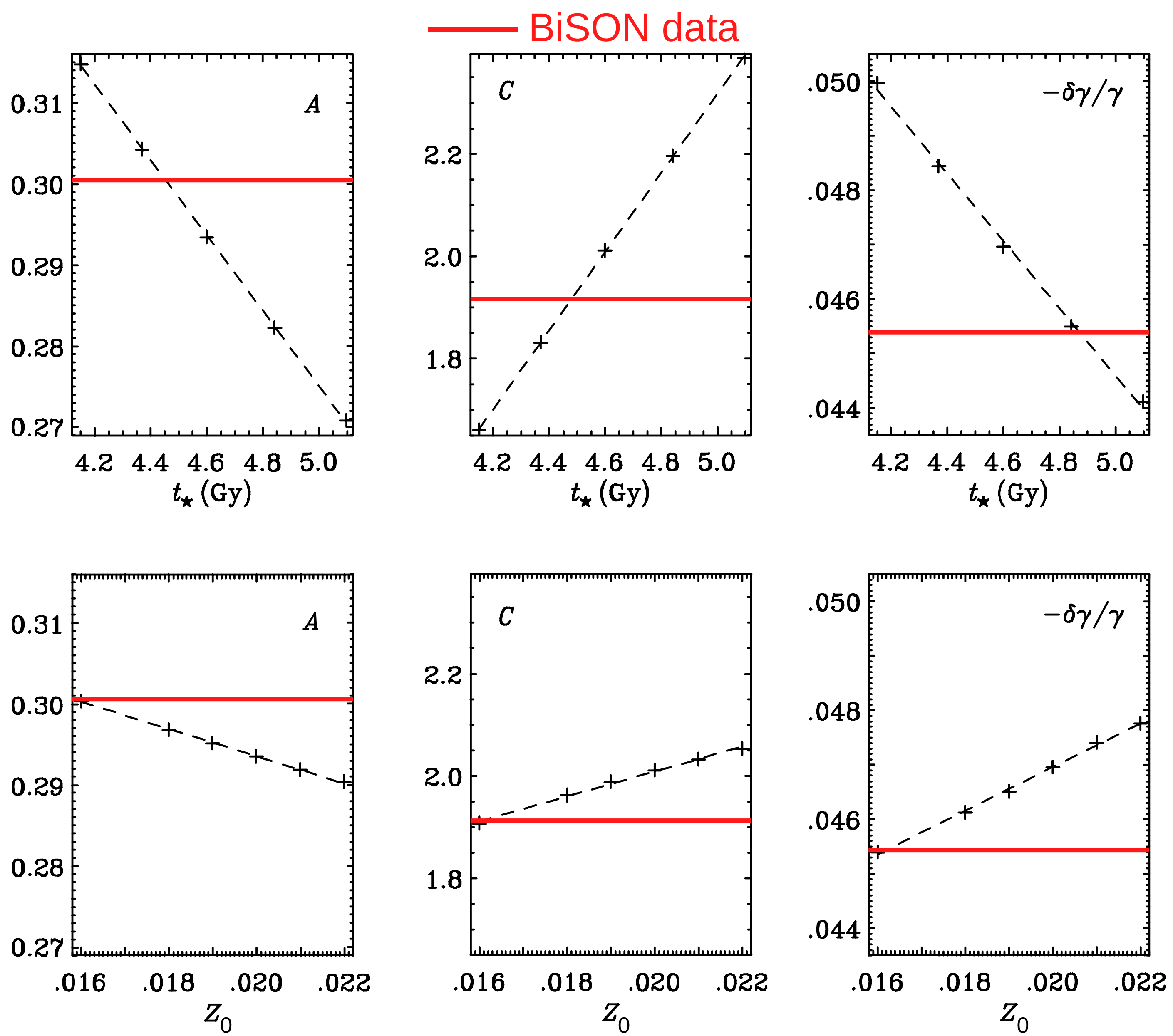
from reference model

# Solar/stellar age calibration

<u>Eleven models calibrated to solar luminosity and radius</u>



# Solar/stellar age calibration



# Solar/stellar age calibration

-asymptotic p-mode frequency behaviour (*n*>>*l*):

$$\nu_s \simeq (n + \frac{1}{2}l + \epsilon)\nu_0 - \frac{A\,L^2 - B}{\nu_s}\nu_0^2 - \frac{C\,L^4 - D\,L^2 + E}{\nu_s{}^3}\nu_0^4 - \frac{F L^6 - G L^4 + H L^2 - I}{\nu_s{}^5}\nu_0^6$$

- calibration using combinations of the seismically determined parameters

$$\xi_\alpha = (\widehat{A}, \widehat{C}, \widehat{F}, -\delta\gamma_1/\gamma_1), \qquad \alpha = 1, 2, 3, 4\,, \qquad -\delta\gamma_1/\gamma_1 \propto Y$$

| $\xi_\alpha$ | $t_\odot$ (Gy) | $Z_0$ | $Y_0$ | $t_\odot$ (Gy) | $Z_0$ | $Y_0$ |
|---|---|---|---|---|---|---|
| $\hat{A}, \hat{C}, \hat{F}, -\delta\gamma_1/\gamma_1$ | 4.592 | 0.0156 | 0.252 | 4.597 | 0.0155 | 0.251 |
| $\hat{A}, \hat{F}, -\delta\gamma_1/\gamma_1$ | 4.580 | 0.0157 | 0.252 | 4.582 | 0.0156 | 0.251 |
| $\hat{C}, \hat{F}, -\delta\gamma_1/\gamma_1$ | 4.591 | 0.0157 | 0.252 | 4.595 | 0.0155 | 0.251 |
| $\hat{A}, \hat{C}, -\delta\gamma_1/\gamma_1$ | 4.597 | 0.0160 | 0.254 | 4.603 | 0.0160 | 0.253 |
| $\hat{A}, \hat{C}, \hat{F}$ | 4.619 | 0.0153 | 0.252 | 4.632 | 0.0151 | 0.248 |
| $\hat{A}, \hat{C}$ | 4.638 | 0.0147 | 0.246 | 4.654 | 0.0143 | 0.245 |

Referenz model 0:
4.60 Gy, $Z_0$=0.02

Referenz model 2:
4.37 Gy, $Z_0$=0.02

# Solar/stellar age calibration

-asymptotic p-mode frequency behaviour (*n*>>*l*):

$$\nu_s \simeq (n + \frac{1}{2}l + \epsilon)\nu_0 - \frac{A\,L^2 - B}{\nu_s}\nu_0^2 - \frac{C\,L^4 - D\,L^2 + E}{\nu_s{}^3}\nu_0^4 - \frac{F L^6 - G L^4 + H L^2 - I}{\nu_s{}^5}\nu_0^6$$

- calibration using combinations of the seismically determined parameters

$$\xi_\alpha = (\widehat{A}, \widehat{C}, \widehat{F}, -\delta\gamma_1/\gamma_1), \qquad \alpha = 1, 2, 3, 4\,, \qquad -\delta\gamma_1/\gamma_1 \propto Y$$

| $\xi_\alpha$ | $t_\odot$ (Gy) | $Z_0$ | $C_{\Theta 11}^{1/2}$ | $-(-C_{\Theta 12})^{1/2}$ | $C_{\Theta 22}^{1/2}$ |
|---|---|---|---|---|---|
| $\hat{A}, \hat{C}, \hat{F}, -\delta\gamma_1/\gamma_1$ | 4.592 | 0.0156 | 0.039 | 0.0013 | 0.0005 |
| $\hat{A}, \hat{F}, -\delta\gamma_1/\gamma_1$ | 4.580 | 0.0157 | 0.045 | 0.0016 | 0.0006 |
| $\hat{C}, \hat{F}, -\delta\gamma_1/\gamma_1$ | 4.591 | 0.0157 | 0.044 | 0.0004 | 0.0005 |
| $\hat{A}, \hat{C}, -\delta\gamma_1/\gamma_1$ | 4.597 | 0.0160 | 0.045 | 0.0036 | 0.0008 |
| $\hat{A}, \hat{C}, \hat{F}$ | 4.619 | 0.0153 | 0.095 | 0.0104 | 0.0013 |
| $\hat{A}, \hat{C}$ | 4.638 | 0.0147 | 1.049 | 0.1791 | 0.0306 |

Referenz model 0:
4.60 Gy, $Z_0$=0.02

error co-variance matrix

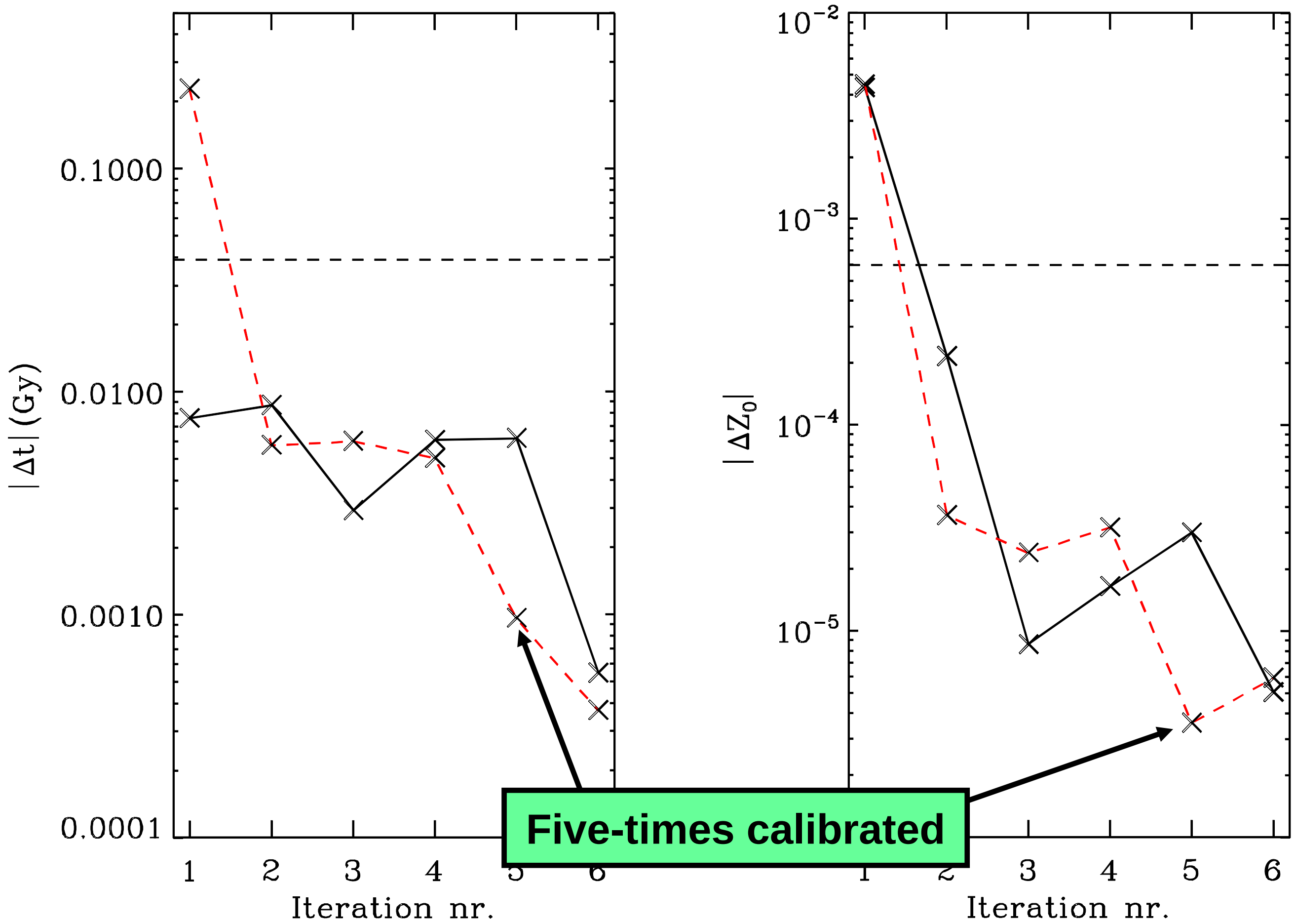
Note: Model S: age=4.60 Gy, $Z_0$=0.0196

# Solar/stellar age calibration

Referenz model⓪: 4.60 Gy, Z=0.02

Referenz model②: 4.37 Gy, Z=0.02

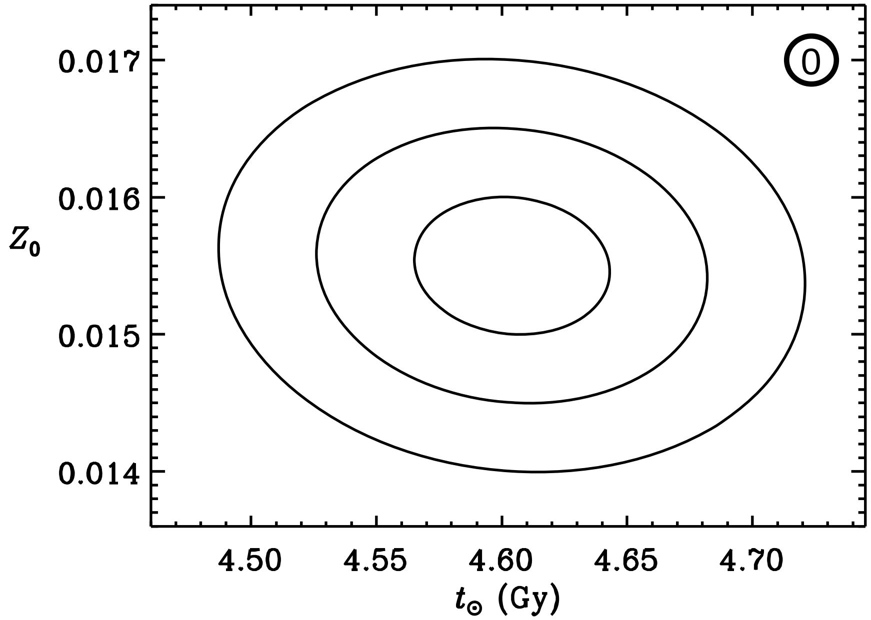
$(\widehat{A}, \widehat{C}, \widehat{F}, -\delta\gamma_1/\gamma_1)$



# Solar/stellar age calibration

Results for
five-times calibrated reference models
using BiSON data & $(\widehat{A}, \widehat{C}, \widehat{F}, -\delta\gamma_1/\gamma_1)$

| | Reference model | $t_\odot$ (Gy) | $Z_0$ | $Y_0$ |
|---|---|---|---|---|
| 0 | 4.60 Gy/$Z_0$=0.02 | 4.604 | 0.0155 | 0.250 |
| 2 | 4.37 Gy/$Z_0$=0.02 | 4.603 | 0.0155 | 0.250 |



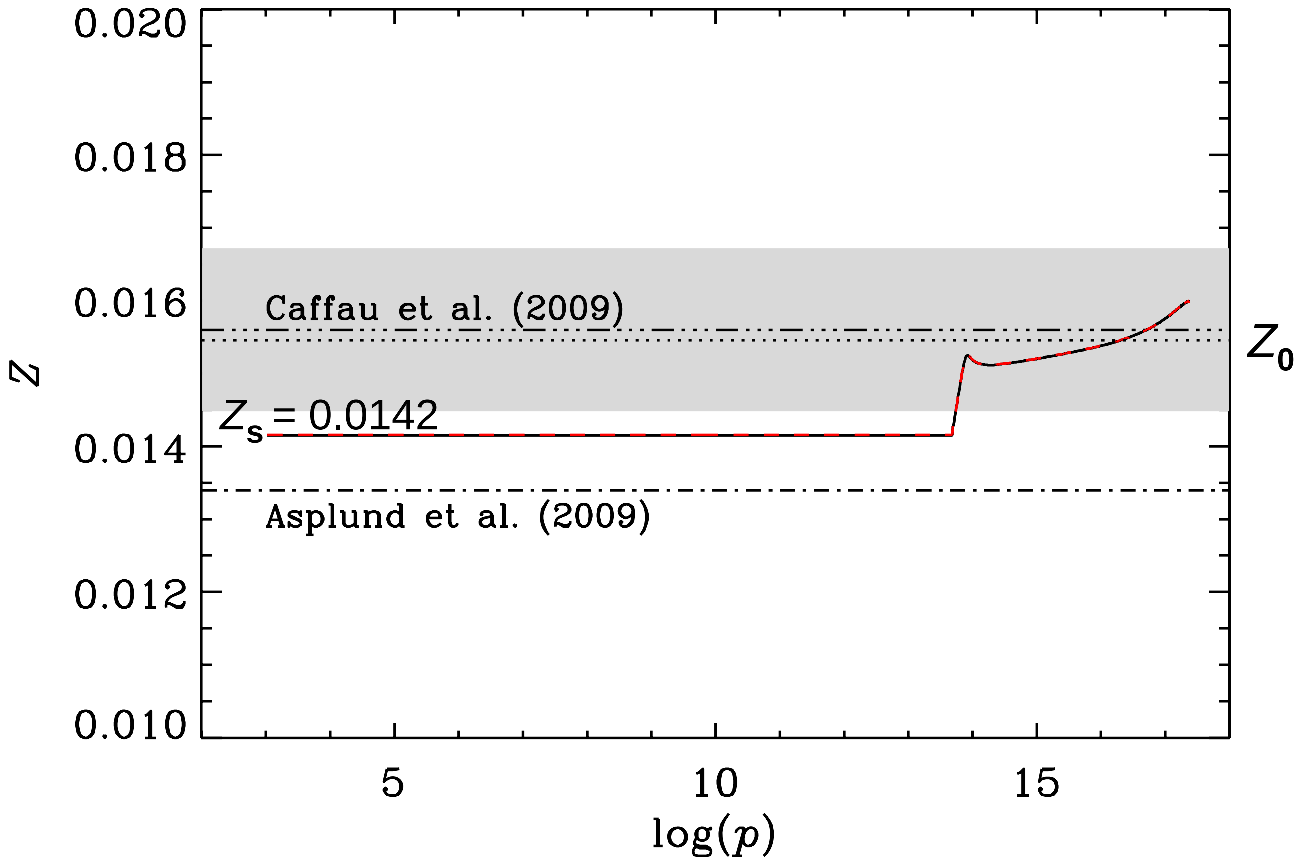

Note: Model S: age = 4.60 Gy, $Z_0$ = 0.0196

# Solar/stellar age calibration

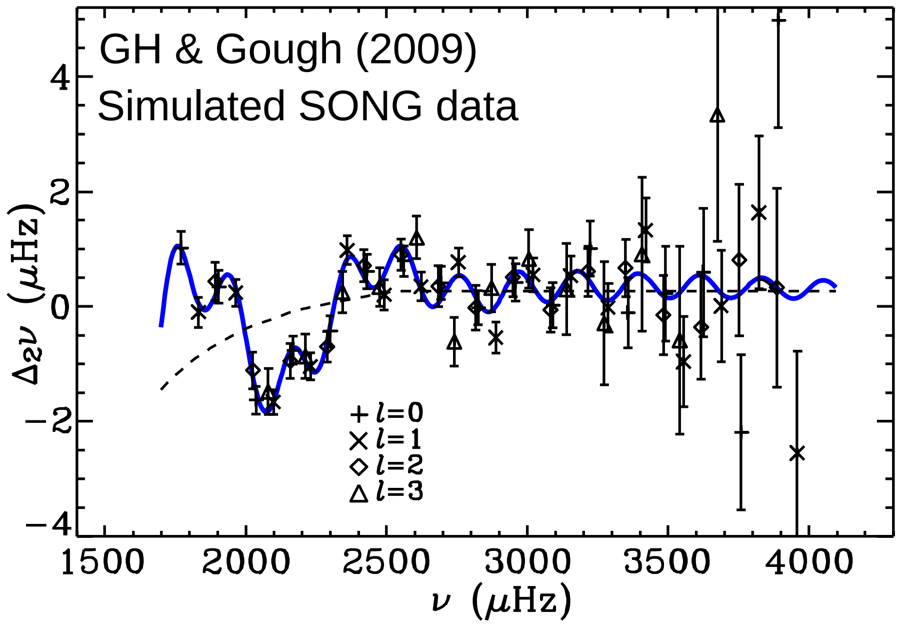
Five-times calibrated

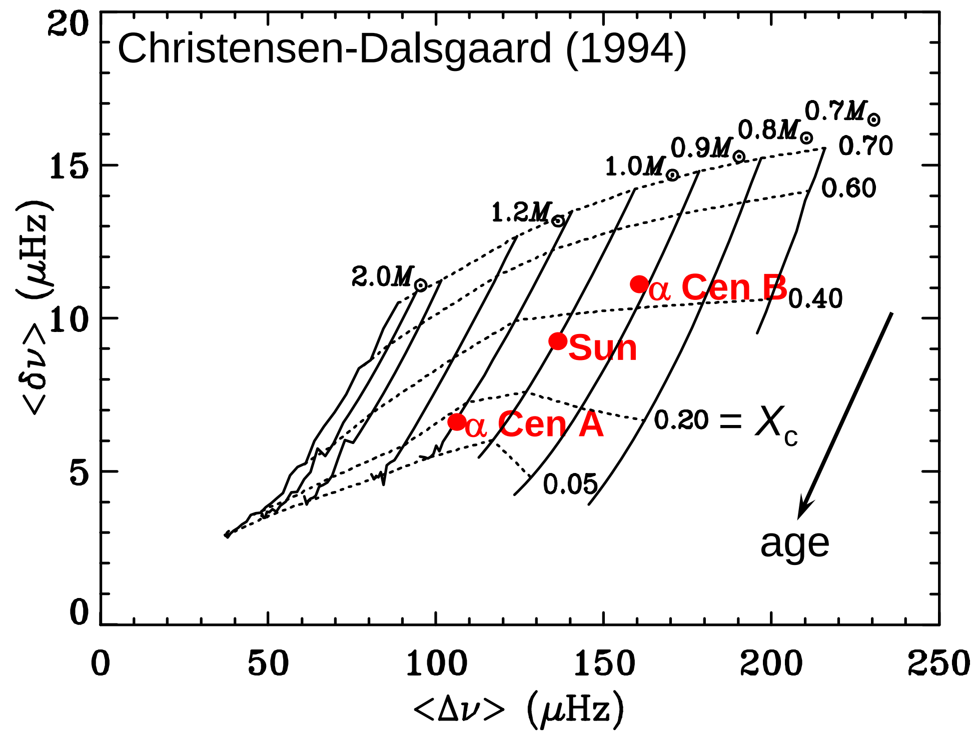
Referenz model⓪: 4.60 Gy, $Z_0$=0.02

Referenz model②: 4.37 Gy, $Z_0$=0.02



# Solar/stellar age calibration




$$\widehat{A}, \widehat{C}, \widehat{F}\ , -\delta\gamma_1/\gamma_1$$

$$t_\star(\pm 5\%)$$

Kjeldsen et al. (2008) :

$$\langle\Delta\rangle(\pm 0.5\%)\ , \langle\delta\nu\rangle(\pm 10\%)\ , T_{\rm eff}(\pm 2\%)$$

$$t_\star(\pm 10\%)$$

# Summary/conclusion

- Seismic diagnostics (2nd frequency differences) of low-degree modes can be used to estimate gross properties (*Y* , DCZ) of solar-type stars.

- Removing the seismic signature of rapid variations in the background state from the frequencies could substantially improve the calibration of stellar ages and abundances.

- This seismic calibration procedure can be applied to data from CoRoT, Kepler and planned observing campaigns (SONG).

- The values of *Z* should not be regarded strictly as statements for initial heavy-element abundance, but rather as a measure of the opacity in the radiative interior.