DYNAMICAL TIDES IN KOI-54 AND RESONANCE LOCKING

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Jim Fuller and Dong Lai, 2011, MNRAS, in press, (arXiv:1107.4594)

Observations: Welsh et al. 2011, ApJ

See also Burkart et al. 2011, (arXiv:1108.3822)

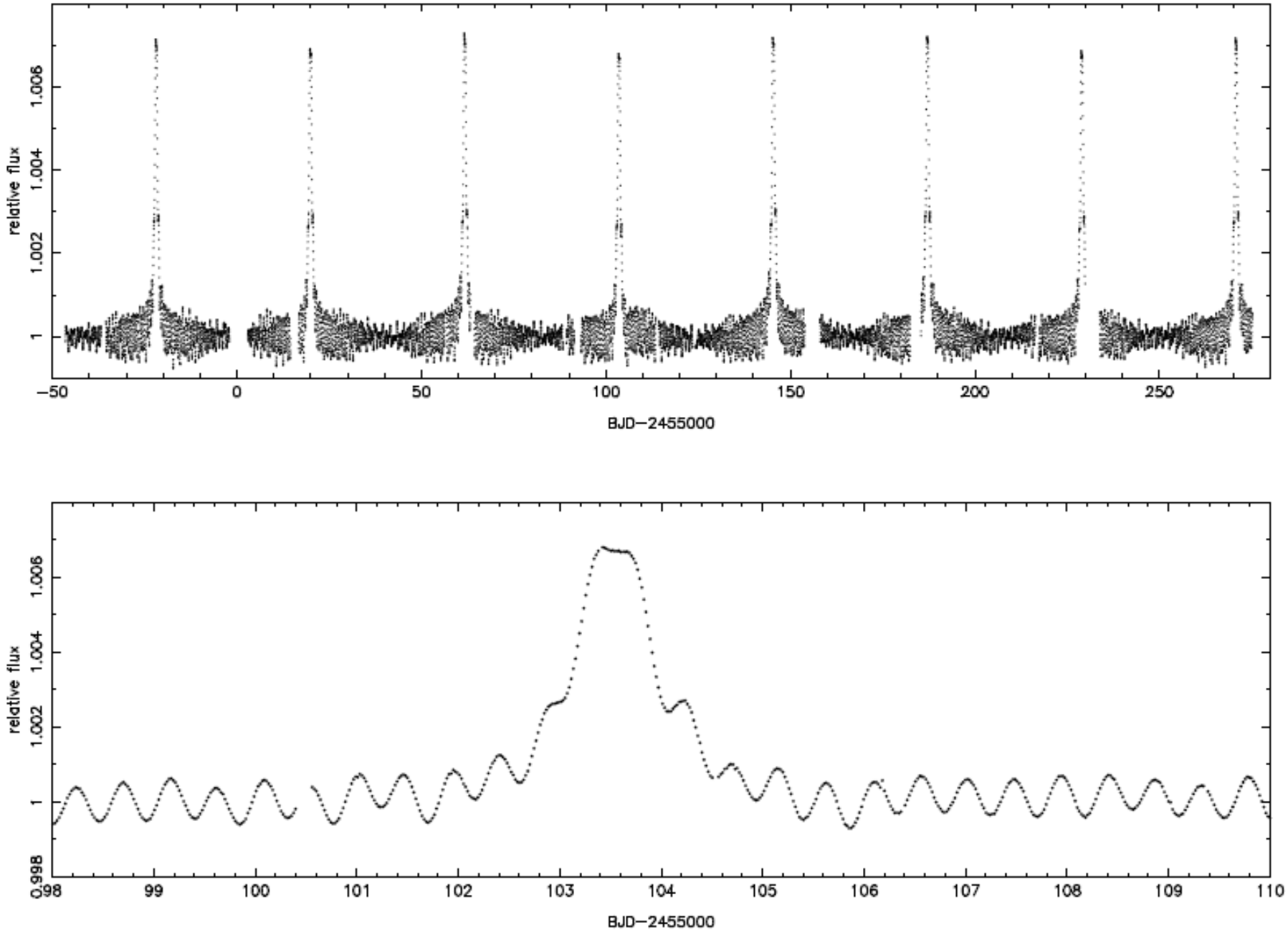


FIG. 1.— *upper:* The detrended and normalized *Kepler* light curve of KOI-54. *lower:* A detailed view of a brightening event.

TABLE 3
THIRTY LARGEST KOI-54 PULSATIONS

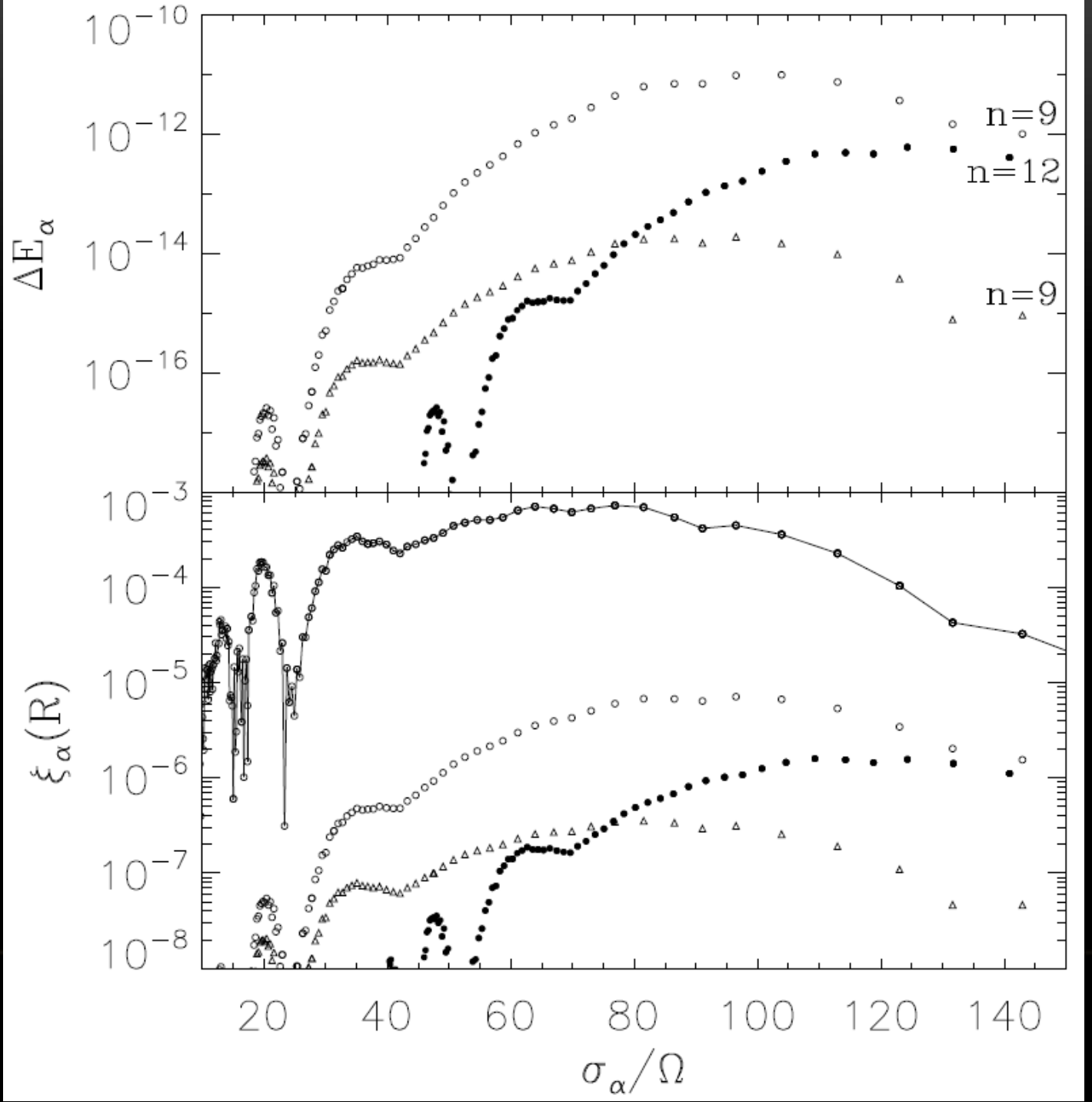
ID	frequency (d^{-1})	frequency (μHz)	amplitude (μmag)	f/f_{orbit}	nearest harmonic
F1	2.1529	24.917	297.7	90.00	90
F2	2.1768	25.195	229.4	91.00	91
F3	1.0525	12.182	97.2	44.00	44
F4	0.9568	11.074	82.9	40.00	40
F5	0.5363	6.207	82.9	22.42	—
F6	1.6405	18.988	49.3	68.58	—
F7	1.7222	19.933	30.2	72.00	72
F8	1.5087	17.462	17.3	63.07	63
F9	1.3773	15.941	15.9	57.58	—
F10	0.6697	7.751	14.6	28.00	28
F11	1.2678	14.673	13.6	53.00	53
F12	1.1241	13.011	13.4	46.99	47
F13	0.9329	10.798	12.5	39.00	39
F14	1.4349	16.608	11.6	59.99	60
F15	0.8851	10.244	11.5	37.00	37
F16	1.6983	19.656	11.4	71.00	71
F17	0.6183	7.156	11.1	25.85	—
F18	1.8178	21.039	9.8	75.99	76
F19	0.8574	9.924	9.3	35.84	—
F20	0.6458	7.475	9.1	27.00	27
F21	1.0284	11.903	8.4	42.99	43
F22	1.0765	12.460	8.3	45.01	45
F23	1.5092	17.467	8.1	63.09	63
F24	0.8610	9.965	6.9	35.99	36
F25	1.4452	16.726	6.8	60.42	—
F26	1.2439	14.397	6.4	52.00	52
F27	1.0078	11.664	6.3	42.13	—
F28	0.7894	9.137	5.9	33.00	33
F29	0.6937	8.028	5.8	29.00	29
F30	1.1483	13.290	5.7	48.00	48

NOTE. — Formal uncertainty in amplitudes is 0.3 μmag. Orbital frequency f_{orbit} was found via least-squares fit to best match the harmonics: $f_{orbit} = 0.0239205\ d^{-1} = 0.276857\ \mu Hz$.

TIDALLY FORCED OSCILLATIONS

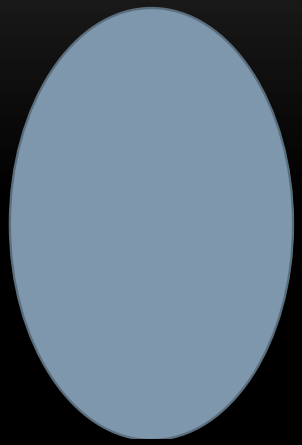
- Mode amplitudes can be calculated directly:

$$c_\alpha(t) = \frac{GM'W_{lm}Q_\alpha}{2\varepsilon_\alpha a^{l+1}} \sum_{N=-\infty}^{\infty} \frac{F_{Nm}e^{-i(N\Omega - m\Omega_s)t}}{(\sigma_\alpha - N\Omega) - i\gamma_\alpha}$$

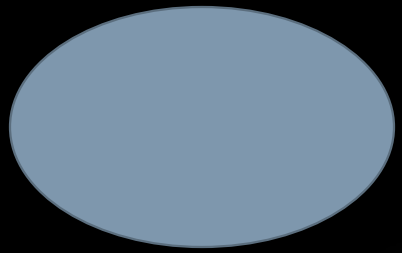

10^{-10}
10^{-12}
10^{-14}
10^{-16}
10^{-3}
ΔE_α
n=9
n=12
n=9
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
$\xi_\alpha(R)$
20
40
60
80
100
120
140
σ_α/Ω

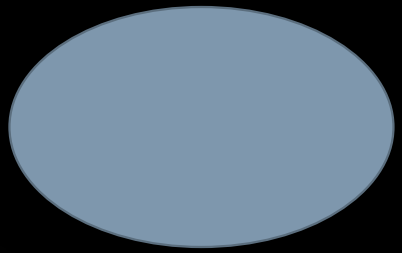
MODE VISIBILITY

$$|\Delta \mathrm{mag}|_\alpha \propto \left[3\cos^2(\theta) - 1\right]\xi_r(R) \qquad (\text{for } m = 0)$$



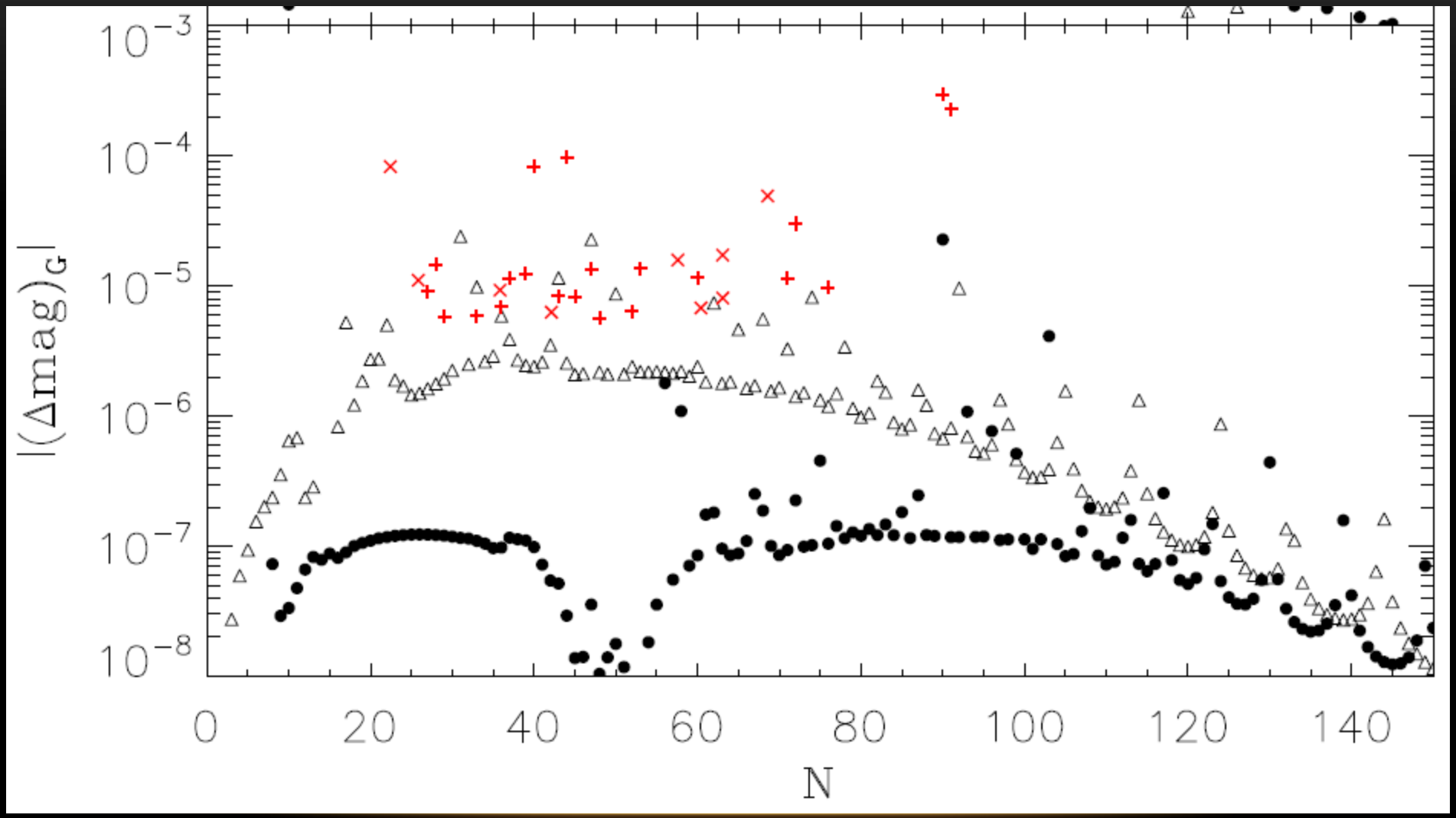
$$|\Delta \mathrm{mag}|_\alpha \propto \sin^2(\theta)\xi_r(R) \qquad (\text{for } m = 2)$$





Face-on ($\theta=0$)

Edge-on ($\theta=90$)


10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
$|(\Delta mag)_G|$
0
20
40
60
80
100
120
140
N

WHY N=90,91?

What is probability of seeing high amplitude modes?

- Consider mode near resonance: $\omega_\alpha = (N + \epsilon)\Omega_{\rm orb}$

- Then by chance

$$P_{{\rm res},\alpha} = \frac{\Delta t_{\rm res}}{\Delta t_{\rm nonres}} \simeq 2\epsilon$$

- If mode dominates energy transfer

$$P_{{\rm res},\alpha} \approx \frac{8\pi^2}{3}|\epsilon|^3$$

RESONANCE LOCKING

- As modes are excited and damp, they spin up star, changing the mode frequency in the inertial frame

$$\omega_\alpha = \omega_\alpha^{(0)} + m(1 - C_\alpha)\Omega_s$$

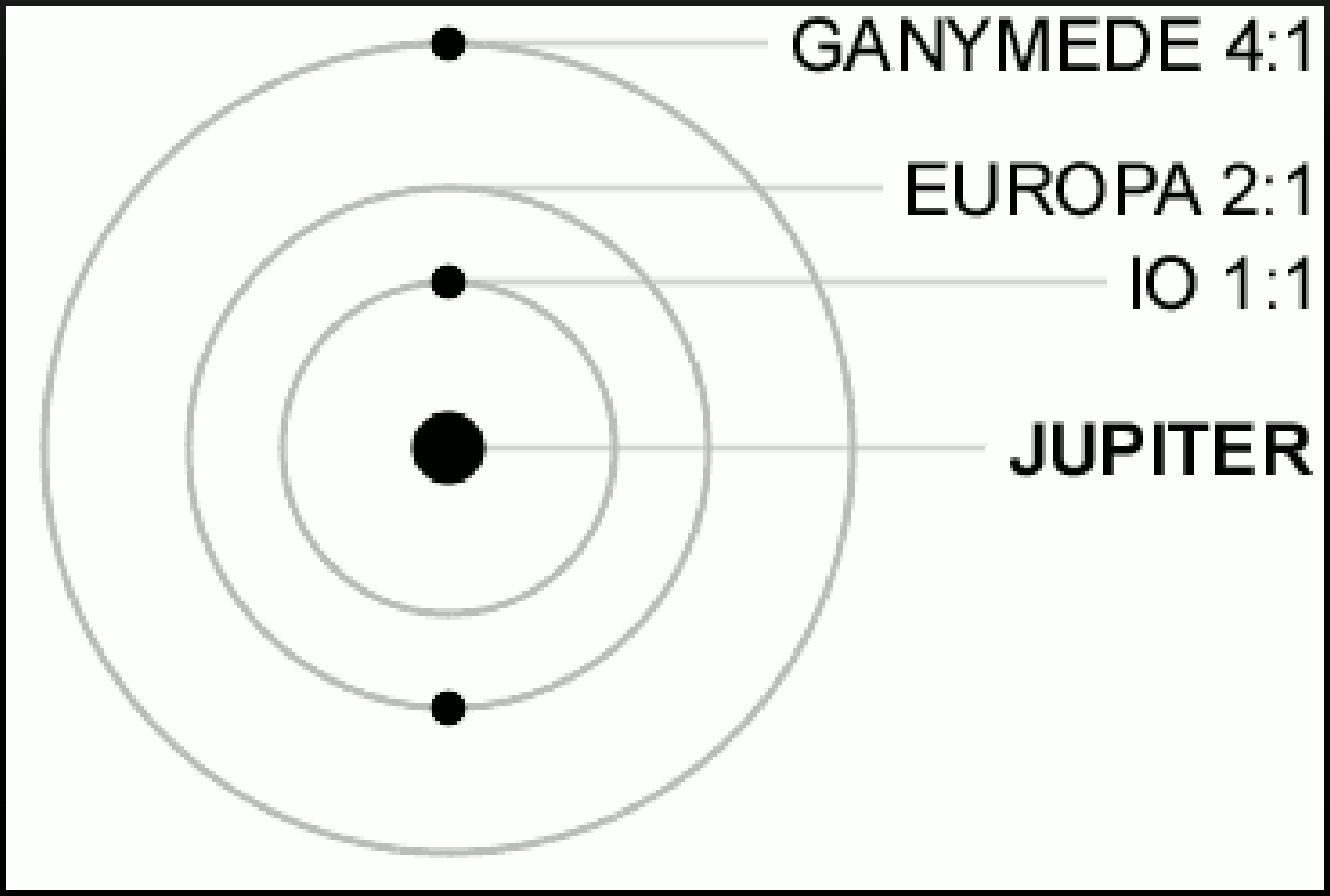
- At resonance,

$$\frac{\omega_\alpha}{\Omega_{\rm orb}} = N$$

- Mode can stay in resonance if

$$\frac{d}{dt}\left(\frac{\omega_\alpha}{\Omega_{\rm orb}}\right) = 0$$

RESONANCE LOCKING IN SOLAR SYSTEM



RESONANCE LOCKING CONTINUED

- Mode can lock into resonance if

$$\frac{\omega_\alpha}{\Omega_{\rm orb}} = N_c$$

where

$$N_c = m\left[\frac{(1-C_\alpha)\mu a^2}{3I}\right]^{1/2} \simeq 135$$

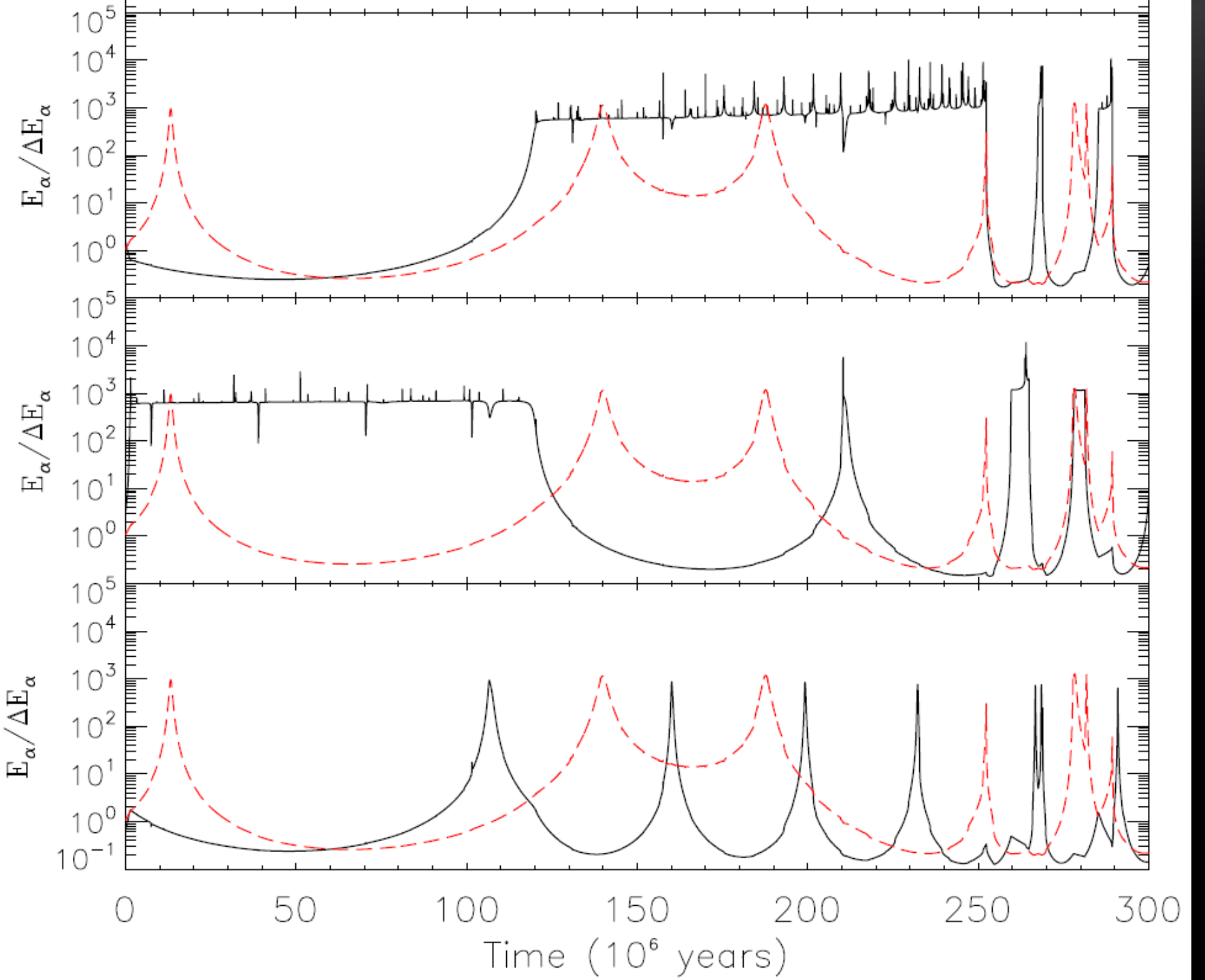
- If star is spinning down, mode can lock into resonance if

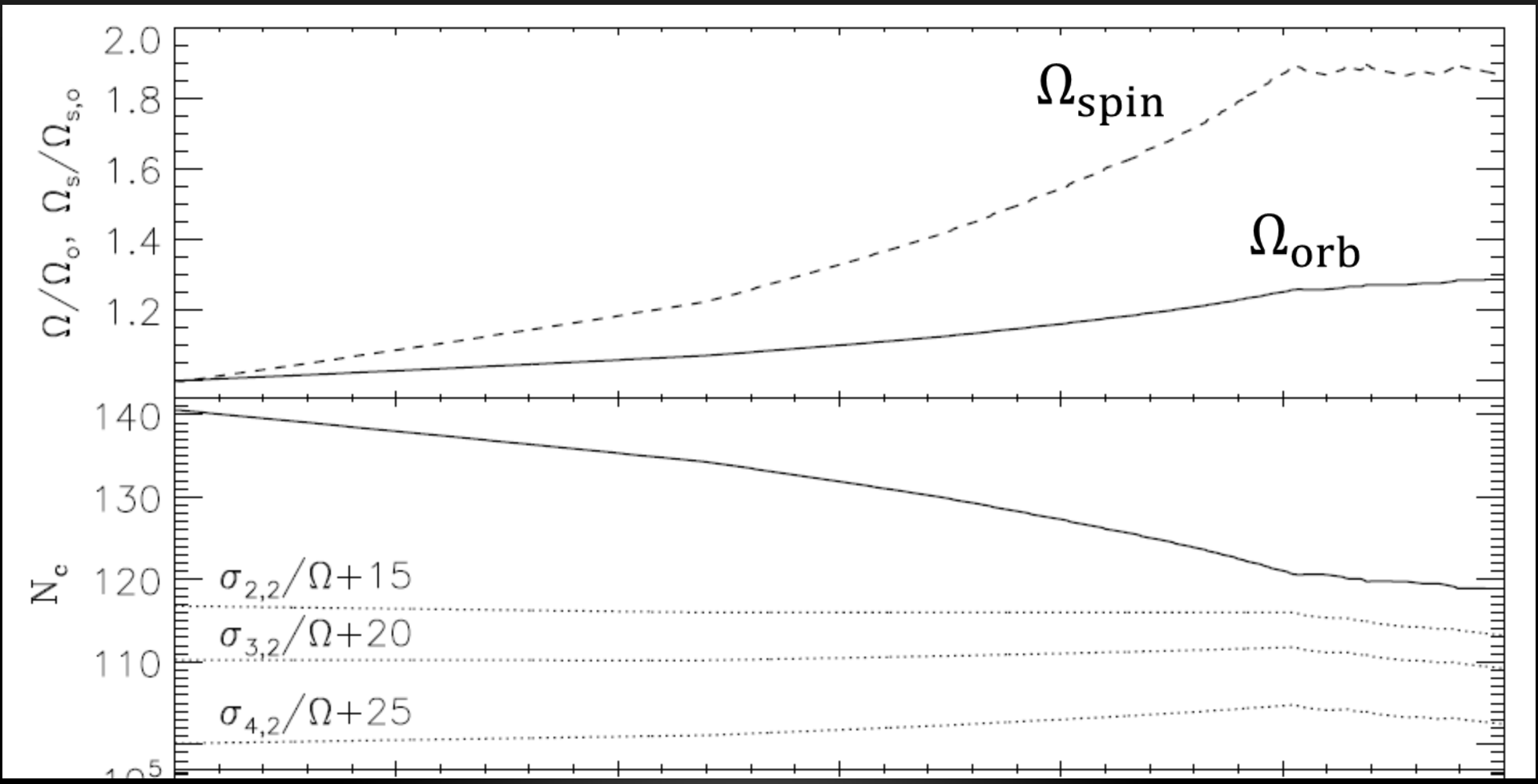
$$\frac{\omega_\alpha}{\Omega_{\rm orb}} < N_c$$

NUMERICAL EXAMPLES

- Compute coupled evolution of mode amplitudes and orbital parameters for KOI-54 system, including
  - Orbital Frequency
  - Orbital Eccentricity
  - Spin Frequency
  - Mode Frequency
  - Mode Energy

- Include spin down term at rate consistent with stellar evolution


$E_\alpha/\Delta E_\alpha$
10^5
10^4
10^3
10^2
10^1
10^0
10^{-1}
0
50
100
150
200
250
300
Time (10^6 years)


2.0
1.8
1.6
1.4
1.2
Ω/Ω_o, $\Omega_s/\Omega_{s,o}$
Ω_{spin}
Ω_{orb}
140
130
120
110
N_c
$\sigma_{2,2}/\Omega+15$
$\sigma_{3,2}/\Omega+20$
$\sigma_{4,2}/\Omega+25$

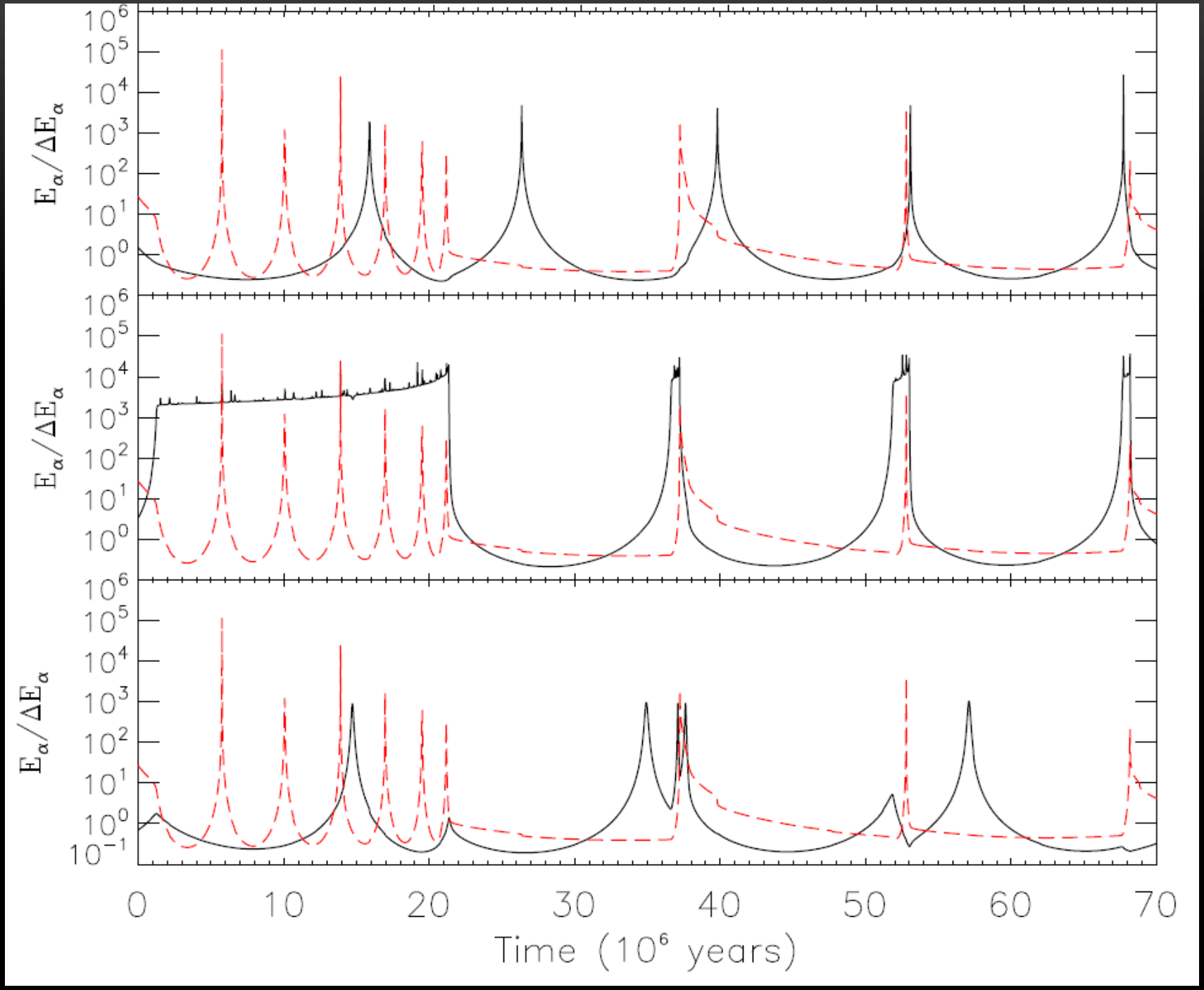
EVEN MORE RESONANCE LOCKING

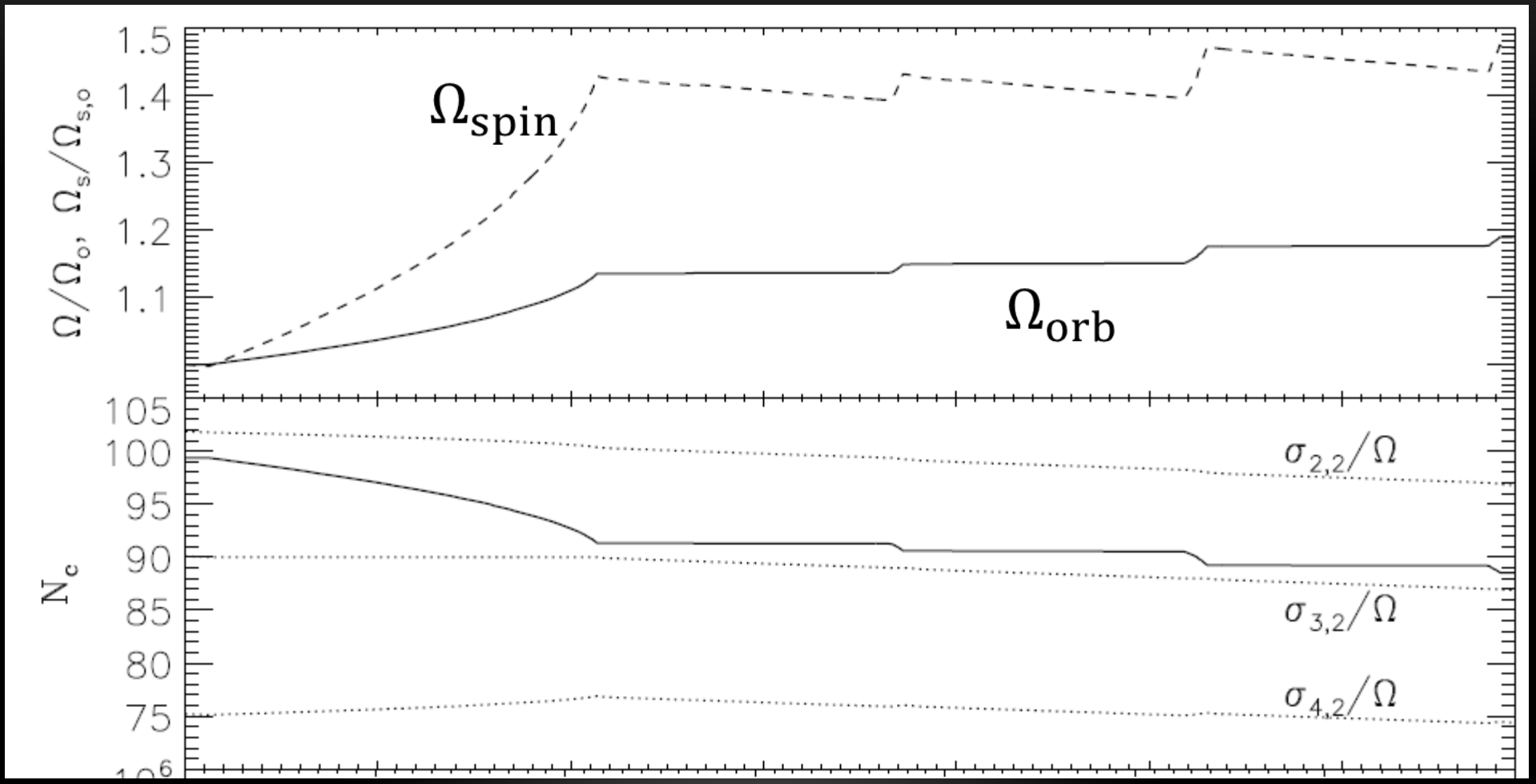
- Can modes be locked simultaneously in both stars?

- For equal tidal dissipation in both stars,

$$N_c \simeq 95$$

- Could explain presence of observed N=90,91 oscillation modes


$E_\alpha/\Delta E_\alpha$
10^{-1}
10^0
10^1
10^2
10^3
10^4
10^5
10^6
0
10
20
30
40
50
60
70
Time (10^6 years)


1.5
1.4
1.3
1.2
1.1
Ω/Ω_o, $\Omega_s/\Omega_{s,o}$
Ω_{spin}
Ω_{orb}
105
100
95
90
85
80
75
N_c
$\sigma_{2,2}/\Omega$
$\sigma_{3,2}/\Omega$
$\sigma_{4,2}/\Omega$

NON-LINEAR MODE COUPLING

- Several modes detected at non-integer multiples of orbital frequency
  - Likely the result of non-linear coupling to daughter modes
- Occurs when

$$\omega_p \simeq \omega_{d1} + \omega_{d2}$$

- In KOI-54,

$$\frac{\omega_2}{\Omega_{\rm orb}} = 91.00 \qquad \frac{\omega_6}{\Omega_{\rm orb}} = 68.58 \qquad \frac{\omega_5}{\Omega_{\rm orb}} = 22.42$$

- Other non-integer modes likely due to non-linear coupling in which one of the daughter modes is invisible

FUTURE PROSPECTS

- Identify modes using spectral techniques
- Find more of them!
- Measure orbital decay?
- Tidal asteroseismology