Oscillation Spectrum of Rapidly Rotating Stars: Wave Chaos and Regular Modes

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F. L. and B.G., Phys. Rev. E **78**, 016215 (2008) and A&A **500**, 1173 (2009), M.P., B.G., F.L. and D.R. Phys. Rev. Lett. **107**, 121101 (2011)

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Asteroseismology of rapidly rotating stars

⇒ New observations: space missions Corot, Kepler

⇒ An asymptotic theory is important for mode identification and interpretation

⇒ For slowly rotating stars (e.g. the sun): an asymptotic theory has been built (Tassoul 1980, Deubner and Gough 1984, Roxburgh and Vorontsov 2000)

⇒ Requires approximate spherical symmetry

⇒Cannot be used for rapidly rotating stars, not spherically symmetric

⇒ Focus of this talk: build an asymptotic theory for acoustic waves (p-modes) in rapidly rotating stars using acoustic ray dynamics

⇒ Results will be checked by comparisons with modes obtained by numerical simulations of a polytropic model

Ray limit of acoustic waves

$\Rightarrow$ As for many other waves, the propagation of short-wavelength acoustic waves can be described by rays

eikonal equation:

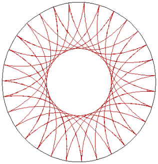
$$\omega^2 = \omega_c^2 + c_s^2 k^2 \quad (1)$$

c_s is the sound speed and ω_c is the cut-off frequency whose sharp increase in the outermost layers of the star provokes the back reflection of acoustic waves.

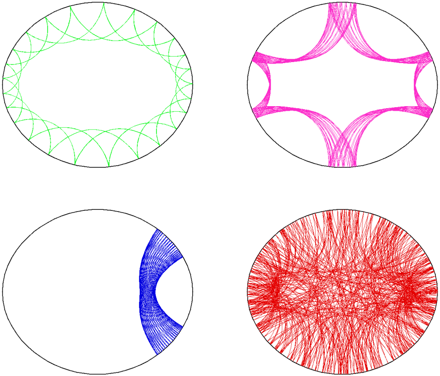
$\Rightarrow$ Acoustic ray: trajectory tangent to the wave vector **k** at the point **x** $\rightarrow$ Hamiltonian classical equations of motion (Lighthill 78, Gough 93)

$\Rightarrow$ Should enable to construct acoustic wave dynamics at high frequency, in the same way as quantum mechanics for $\hbar \to 0$ can be built from classical mechanics

New types of ray trajectories in rotating stars



$\Omega = 0$



$\Omega = 0.6\Omega_K$

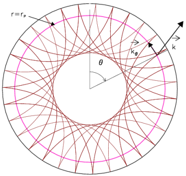
Phase space structure

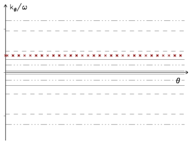
⇒ Poincaré Surfaces of Section give a global view of the ray dynamics properties

⇒ At $\Omega = 0$ the system is integrable (stable and localized trajectories)

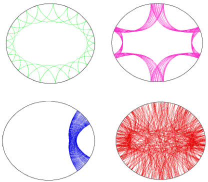
⇒ At high rotation, integrable and chaotic zones (mixed systems).

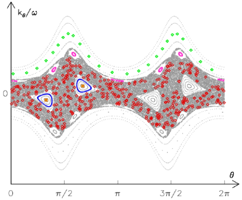
$\Omega = 0$





$\Omega = 0.6\Omega_K$

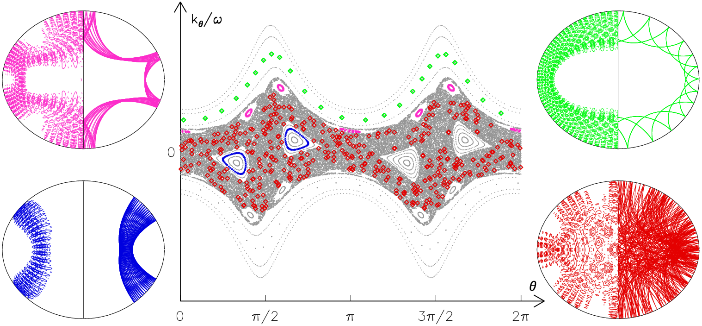




Asymptotic mode classification

⇒ Predictions of the ray-based theory (or quantum chaos theory): modes are constructed on phase space structures

⇒ Sucessfully confronted with numerically computed modes



Consequences for spectra

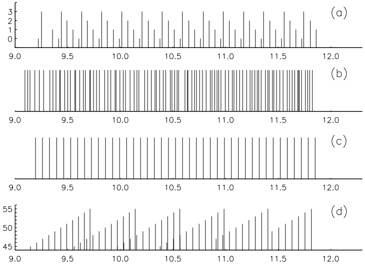
$\Rightarrow$ Prediction (quantum chaos theory): Spectrum should be divided into well-defined subspectra

$\Rightarrow$ Near-integrable regions produce regular sub-spectra $\omega = f_i(n_i, \ell_i, m)$

$\Rightarrow$ The chaotic region produces an irregular sub-spectrum with specific statistical properties

Frequency sub-spectra of four classes of modes :
(a) 2-period island modes
(b) chaotic modes
(c) 6-period island modes
(d) some whispering gallery modes

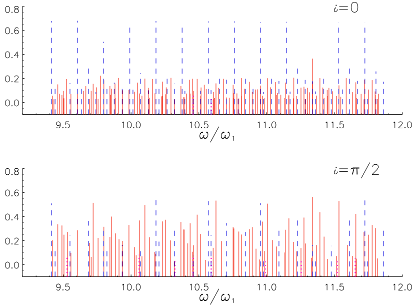
For sub-spectra (a) and (d), height of the vertical bar specifies one of the two quantum numbers.



Visibility of the modes

- At high rotation, whispering gallery modes are very strongly cancelled
- The (spatially irregular) chaotic modes are weakly cancelled

Frequency spectra with amplitude given by the visibility for a star seen pole-on $i = 0$ and equator-on $i = \pi/2$: 2-period island modes (blue), chaotic modes (red), 6-period island modes (magenta)



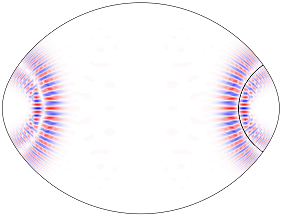
$\Rightarrow$ At high rotation, the spectrum is dominated by the 2-period island modes and chaotic modes

Regular spectrum: 2-period island modes

$\Rightarrow$ Largest group of near-integrable modes

$\Rightarrow$ Built around a central periodic orbit

$\Rightarrow$ Can be built systematically using parabolic equation method (Babich)



Example of mode

Asymptotic formula

Result gives closed formula for 2-period island modes:

$$\omega_{n,\ell,m} = \frac{1}{\oint_\gamma \frac{ds}{\tilde{c}_s}} \left[2\pi(n + \frac{1}{2}) + \left(\ell + \frac{1}{2}\right)(2\pi N_r + \alpha) \right] . \qquad (2)$$

$\Rightarrow$ Equation valid asymptotically for n large and $\ell \ll n$.

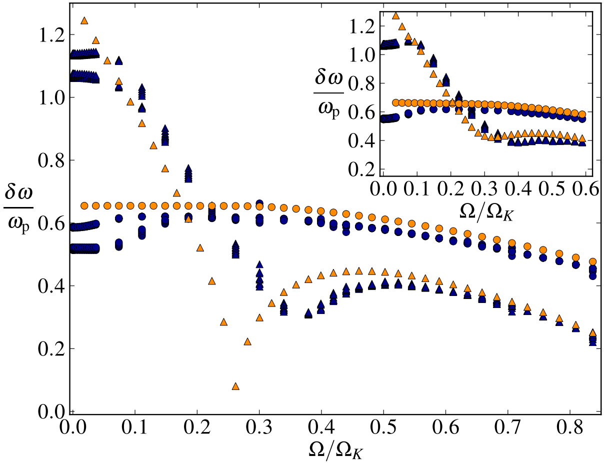
$\Rightarrow$ s is the curvilinear coordinate along the central periodic orbit γ

$\Rightarrow$ n and ℓ correspond to the number of nodes in the directions parallel and transverse to the orbit.

$\Rightarrow$ $\omega_{n,\ell,m}$ essentially described by two quantities, $\delta n = \frac{2\pi}{\oint_\gamma \frac{ds}{\tilde{c}_s}}$ and $\delta \ell = \frac{2\pi N_r + \alpha}{\oint_\gamma \frac{ds}{\tilde{c}_s}}$ (which depend on m)

$\Rightarrow$ The quantities δn and $\delta \ell$ probe the sound velocity along the path of the periodic orbit and its transverse derivatives.

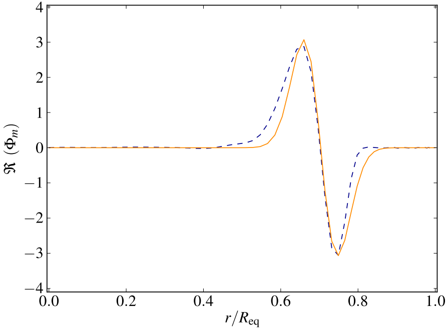
Comparison with numerical modes: spectrum



Comparison between actual regularities of regular modes and theoretical predictions for $m = 0$ and different values of Ω/Ω_K Inset: Same for $m = 1$.

Comparison with numerical modes: amplitude distribution

The same theory enables to construct the amplitude distribution of the modes in terms of transverse Hermite polynomials modulated by the longitudinal coordinate.



Amplitude distributions on the equator for a theoretical and a numerical mode.

Irregular (chaotic) spectrum

⇒ No simple asymptotic formula for chaotic modes

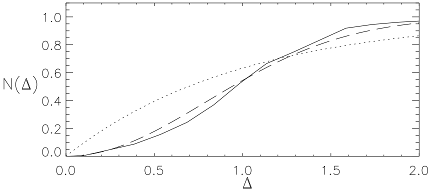
⇒ Conjecture (Bohigas-Giannoni-Schmit): level spacing statistics of chaotic modes should follow Random Matrix Theory

⇒ Verified by the numerical acoustic stellar modes

Integrated spacing distribution $N(\Delta)$ of chaotic modes (full line).

Dashed line: Random Matrix Theory

Dotted line: Poisson distribution typical of integrable systems.



Conclusion

- Dynamics of acoustic rays shows a transition from integrable to mixed system when rotation increases
- For sufficiently large rotation, the spectrum should be divided into well-defined regular or irregular subsets.
- This picture holds for numerical modes computed from a polytropic star model.
- The regular and irregular modes have both high visibility.
- First results of COROT: some regularity seems to be detected in δ scuti stars $\rightarrow$ more work to connect to observed spectra.
- Identification of the spectra should lead to better understanding of the star interior.
- Extensions: more refined numerical models, stratification, inertial modes, etc...