Rotational effects on pulsation

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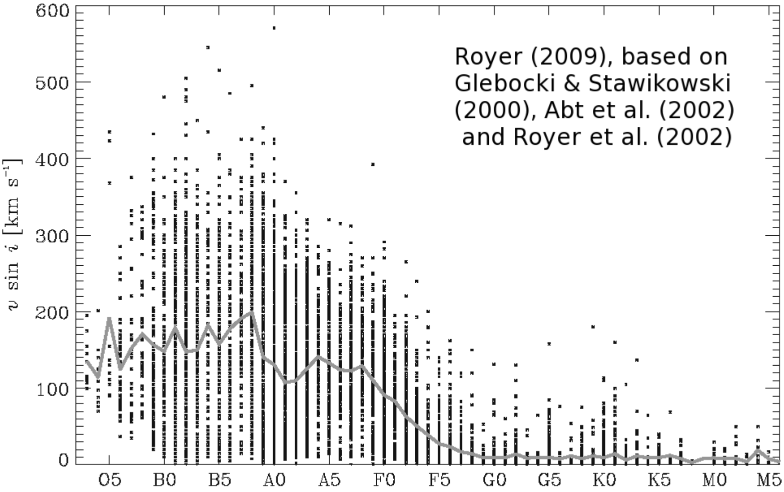
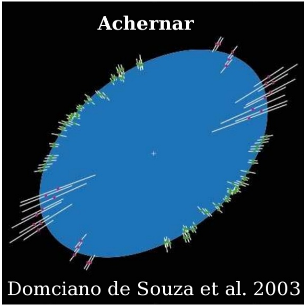
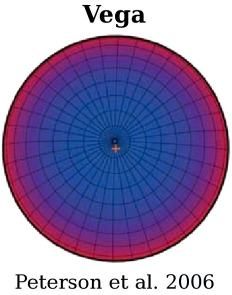
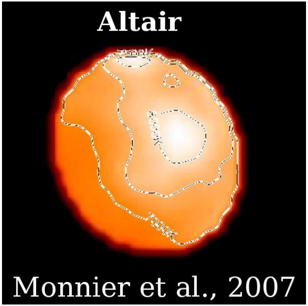
Introduction

- spectroscopic studies shows that many stars rotate rapidly









- interferometry reveals the drastic effects of rapid rotation

Effects of rotation

Effects on stars

- short term structural effects (centrifugal deformation, gravity darkening)
- long term evolutionary effects (mixing, transport, stellar lifetime)
- detailed review given in talks by F. Espinosa Lara, M. Pinsonneault and J.-P. Zahn

Effects on stellar pulsations

- many new challenges which need to be addressed
- interpreting stellar pulsations is crucial to gaining a better understanding of rapidly rotating stars

Outline

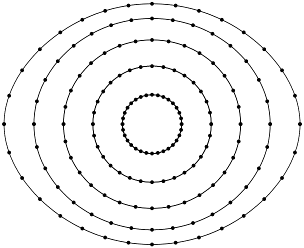
Inertial forces

- stellar rotation introduces 2 inertial forces
  - the centrifugal force
  - the Coriolis force
- neither respects spherical symmetry

  $\Rightarrow$ two-dimensional eigenvalue problem
  - pulsation modes are no longer described by a single spherical harmonic

The centrifugal force

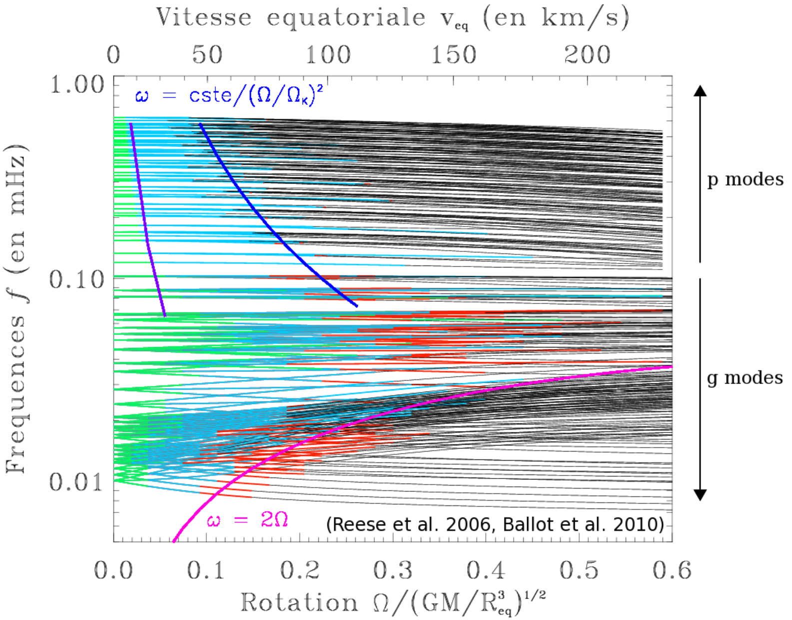
- stellar deformation $= \epsilon \propto \frac{\Omega^2 R_{\mathrm{eq}}^3}{GM}$
- the outer layers are the most deformed
- effect on acoustic modes $\propto \frac{\epsilon}{\lambda} \propto \omega\Omega^2$
  - $\lambda =$ mode's wavelength, $\omega =$ mode's frequency
- smaller effect on gravito-inertial modes which tend to be deeper inside the star



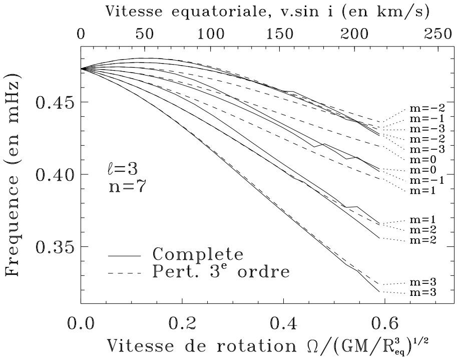
(Roxburgh 2004)

The Coriolis force

- conservation of angular momentum
- intervenes directly in the oscillatory movements
- scales as $2\Omega/\omega$
  - strongest effect on low frequency modes ⇒ gravito-inertial modes
  - inertial modes (incl. r modes) owe their existence to the Coriolis force (e.g. Papaloizou & Pringle, 1978, Lee 2006, Rieutord et al. 2001, Dintrans et al. 1999)


Vitesse equatoriale v_{eq} (en km/s)
0 50 100 150 200
1.00
0.10
0.01
Frequences f (en mHz)
$\omega = cste/(\Omega/\Omega_K)^2$
$\omega = 2\Omega$
(Reese et al. 2006, Ballot et al. 2010)
0.0 0.1 0.2 0.3 0.4 0.5 0.6
Rotation $\Omega/(GM/R^3_{eq})^{1/2}$
p modes
g modes

A multiplet



$$\omega = \omega_0 - m(\underbrace{1}_{\text{geometric}} \underbrace{-C}_{\text{Coriolis}})\Omega + \underbrace{\omega_2 \Omega^2}_{\text{centrifugal \& Coriolis}} \quad \dots$$

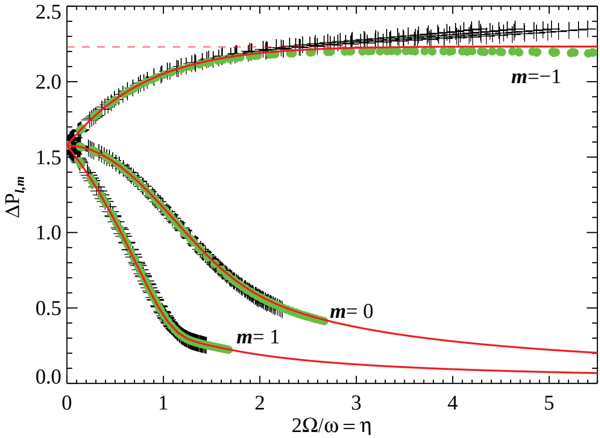
Effects on gravito-inertial modes

Period spacing

- in the non-rotating case, g-modes are evenly spaced out in period, based on Tassoul's asymptotic formula (Tassoul 1980):

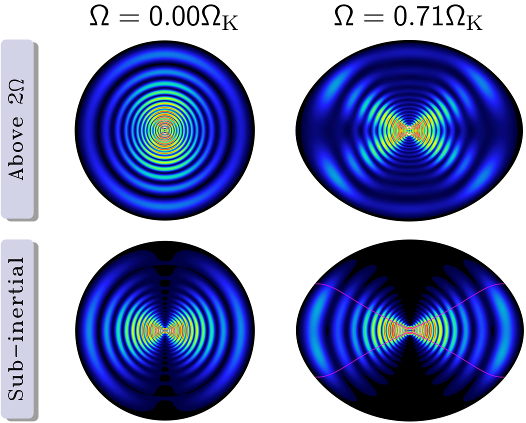
$$P \simeq (n + \alpha_{\ell,g})\Delta P \qquad \text{where} \qquad \Delta P = \frac{2\pi^2}{\sqrt{\ell(\ell+1)}} \left(\int_{r_1}^{r_2} N \frac{dr}{r} \right)^{-1}$$

- in the rotating case, the period spacing becomes dependent on $\eta = 2\Omega/\omega$



(Ballot et al. 2011)

- agrees well with the traditional approximation (Berthomieu et al. 1978, Lee & Saio 1987)

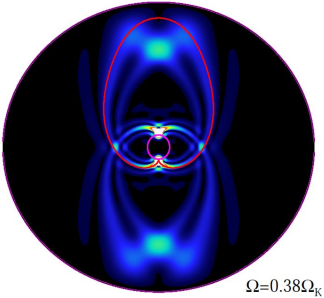


(Ballot et al. 2010)

- critical surface based on Dintrans & Rieutord 2000
- similar confinement also in traditional approximation (e.g. Townsend 2003)

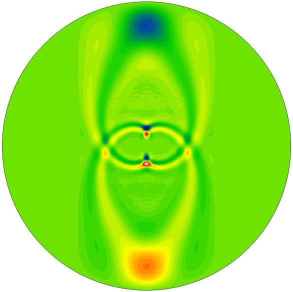
Rosetta modes

With deformation



(Ballot et al. 2011)

Without deformation



- in specific frequencies ranges, so far, above 2Ω
- closely follows underlying ray path
- no need for centrifugal force

Acoustic modes

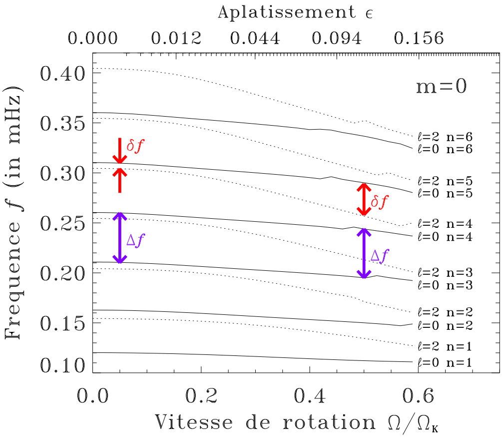
Frequency spacing

- in the non-rotating case, p-modes are evenly spaced out in frequency, based on Tassoul's asymptotic formula (Tassoul 1980):

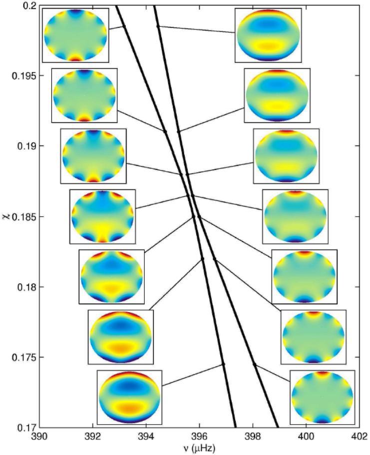
$$\nu \simeq \Delta\nu(n + \frac{\ell}{2} + \epsilon) \qquad \text{where} \qquad \Delta\nu = \left[2\int_0^R \frac{dr}{c}\right]^{-1}$$

- $\Delta\nu_n = \nu_{n,\ell} - \nu_{n-1,\ell} =$ large frequency separation
- $\delta\nu_n = \nu_{n,\ell} - \nu_{n-1,\ell+2} =$ small frequency separation (from higher order terms in the asymptotic formula)

The large and small frequency separations



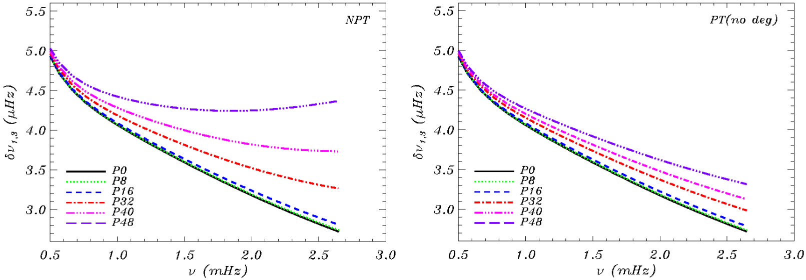
- $\Delta\nu_n$ survives, but not $\delta\nu_n$



(Espinosa et al. 2004)

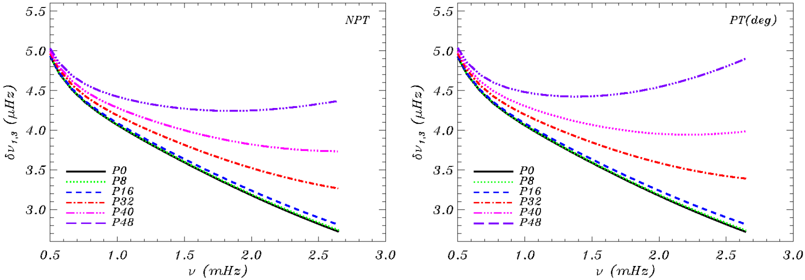
Avoided crossings

- rotation causes avoided crossings between:
  - coupled p modes
  - low order p and g modes
- a hindrance to mode labeling (according to M. Takata's talk, it's already a challenge in 1D)



(Suárez et al. 2010)

- mode degeneracy affects the small frequency separation, even at small rotation rates
- also see Ouazzani et al. 2008



(Suárez et al. 2010)

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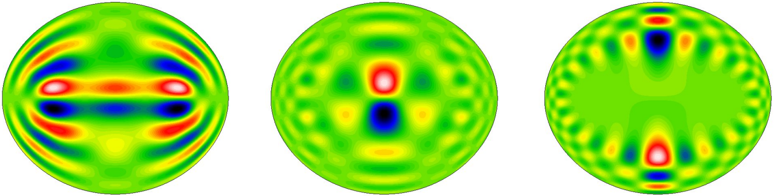
New mode classification

Island	**Chaotic**	**Whispering gallery**
low $\ell - \lvert m \rvert$	medium $\ell - \lvert m \rvert$	high $\ell - \lvert m \rvert$



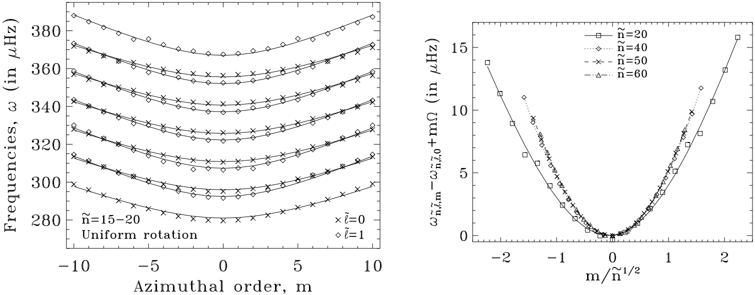
- based on ray dynamics, Lignières & Georgeot (2008, 2009) found different classes of modes:
  - separate geometry
  - separate frequency organization
- more on this in B. Georgeot's talk

Island modes

- new quantum numbers:

$$\begin{aligned} \tilde{n} &= 2n + \varepsilon, \\ \tilde{\ell} &= \frac{\ell - |m| - \varepsilon}{2}, \\ \varepsilon &\equiv \ell + m \; [2] \end{aligned}$$

Island modes

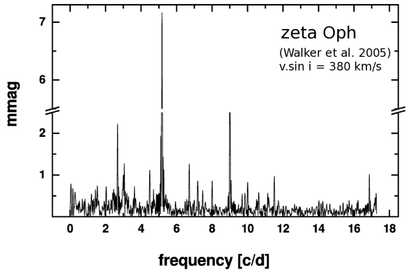


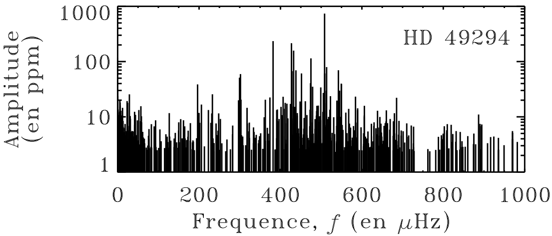
(Reese et al., 2009)

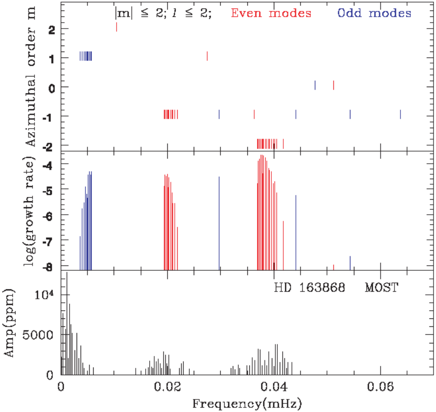
$$\omega_{\tilde{n},\tilde{\ell},\tilde{m}} \simeq \tilde{n}\Delta_{\tilde{n}} + D_{\tilde{m}}(\tilde{\ell})\sqrt{\frac{\tilde{m}^2}{\tilde{n}} + \mu(\tilde{\ell})} - \tilde{m}\Omega + \alpha(\tilde{\ell})$$

- $\Delta_{\tilde{n}}$ and $\Delta_{\tilde{\ell}} = \omega_{\tilde{\ell}+1} - \omega_{\tilde{\ell}}$ can be calculated from travel time integrals (B. Georgeot's talk)

Interpreting asteroseismic data



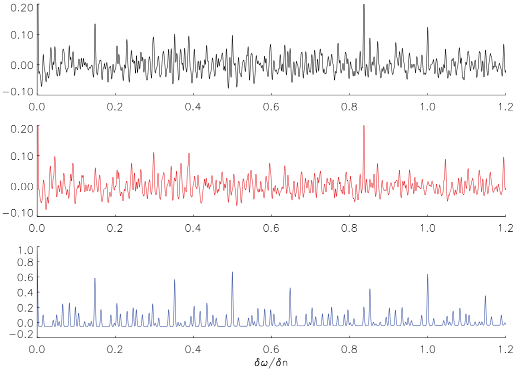




(Walker et al. 2008)

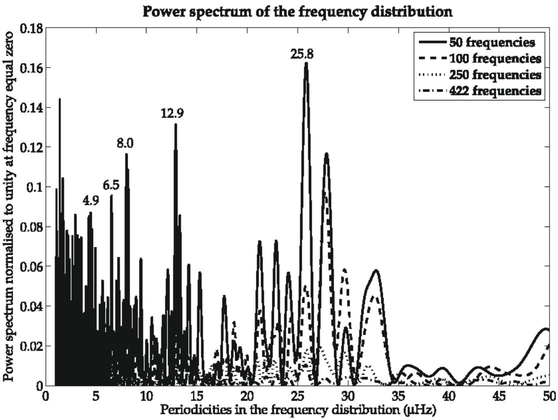
- $\nu_{\text{inert.}} = |\nu_{\text{corot.}} - m\Omega|$
- also see Dziembowski et al. (2007), Savonije (2007), Saio et al. (2007), Cameron et al. (2008)

Periodic structures in frequency spectrum



(Lignières et al. 2010)

- auto-correlation function of synthetic spectra
- both $\Delta_{\tilde{n}}$ and Ω can stand out as peaks



(García Hernández et al. 2009)

- power spectrum of frequency subsets for HD 174936 (observed by CoRoT)
- periodicity which matches $\Delta_{\tilde{n}}$

Mode identification

Importance

- needed to confirm global approach (avoid confusion between $\Delta_{\tilde{n}}$ and Ω)
- more detailed modeling through direct comparison and inverse methods

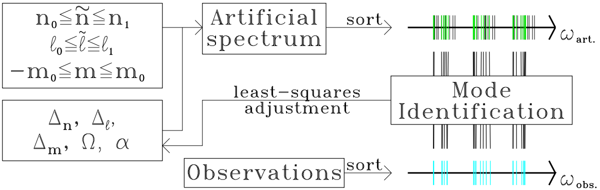
Difficulties

- difficulties caused by new mode organization
- avoided crossings
- chaotic modes

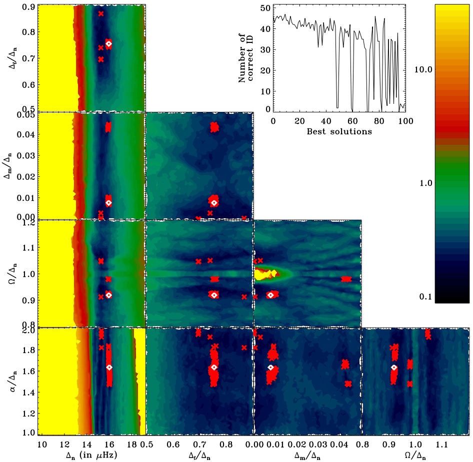
Searching for frequency patterns

- compare observed frequencies to spectra based on a simplified asymptotic formula
- systematic search through parameter space for closest fit

$$\omega = \tilde{n}\Delta_{\tilde{n}} + \tilde{\ell}\Delta_{\tilde{\ell}} + m^2\Delta_{\tilde{m}} - m\Omega + \tilde{\alpha}$$



(Reese et al. 2009b)



(Reese et al. 2009b)

- works well only for high frequency and no chaotic modes
- chaotic modes are very likely to be visible (Lignières & Georgeot, 2009)

Photometric and spectroscopic mode identification

- multi-color photometric and spectroscopic mode identification commonly used in slowly rotating case
- before doing identification, need for theoretical studies on the forward problem

Photometric signatures of modes

Previous studies

- Lignières et al. (2006), Lignières & Georgeot (2009)
  - 2D pulsation calculations
  - only temperature fluctuations
  - no limb or gravity darkening
- Daszyńska-Daszkiewicz et al. (2002, 2007), Townsend (2003)
  - perturbative approach or traditional approximation
  - effects of avoided crossings included
  - amplitude ratios and phase differences depend on m and i (the inclination)

Results presented here

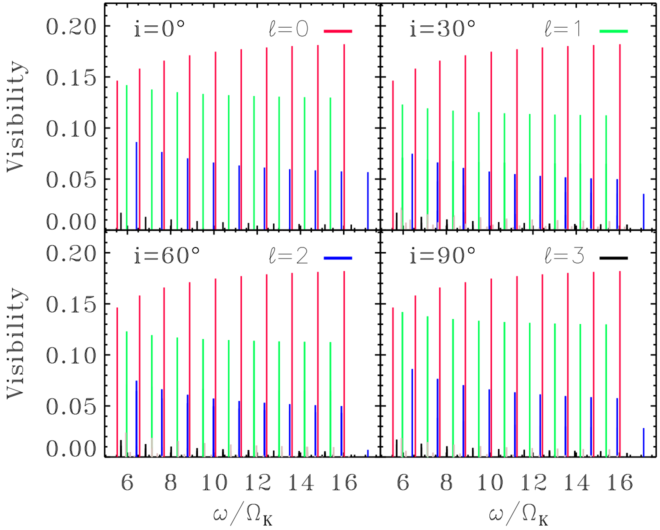
What it includes

- 2D p-modes of deformed SCF models (Jackson et al. 2005, MacGregor et al. 2007)
- latitude dependent intensities based on Kurucz atmospheres (calculated by C. Barban)
    - limb darkening
    - gravity darkening
- geometric distortion to surface from pulsations

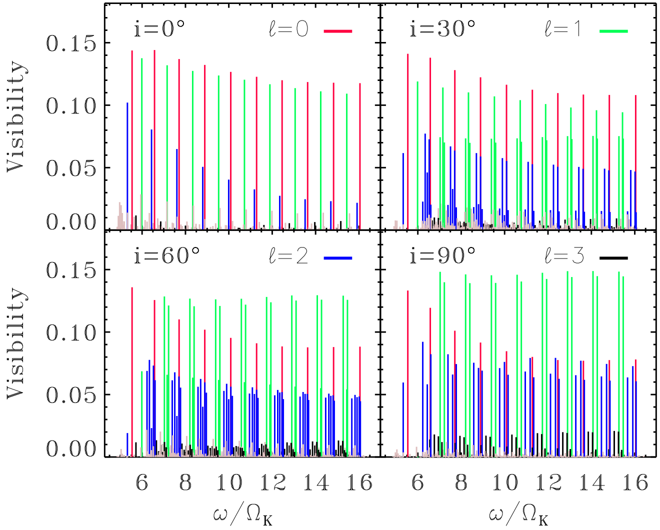
What it lacks

- non-adiabatic effects
    - $\delta T_{\text{eff}}/T_{\text{eff}}$ approximated by $\delta T/T$
- calculations in 1 band for now

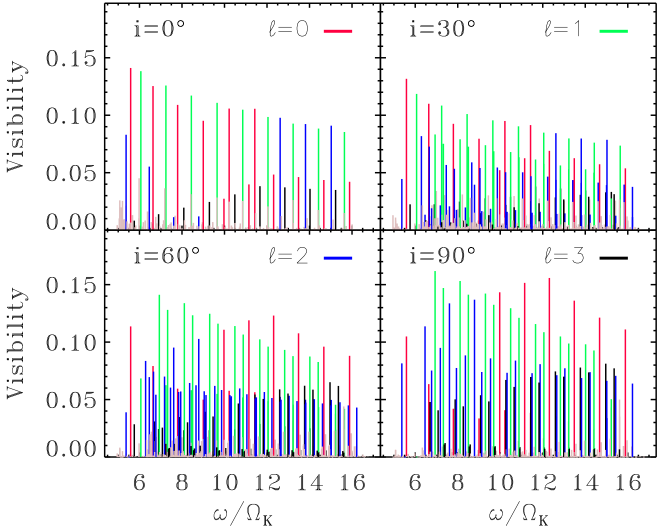
$\Omega = 0.0\ \Omega_C$



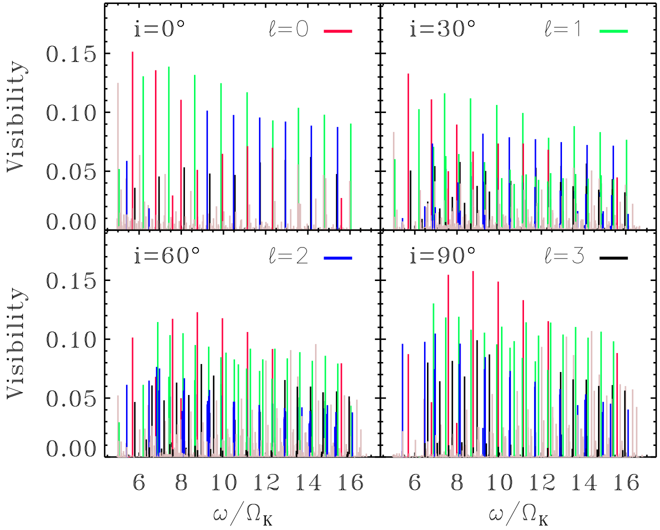
$\Omega = 0.1\ \Omega_C$



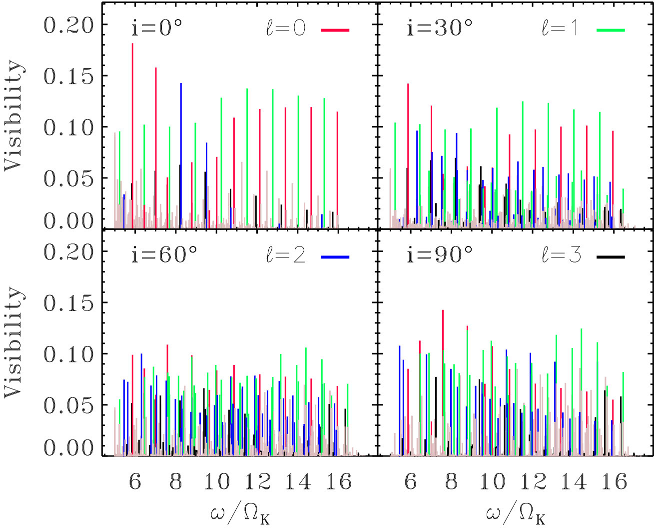
$\Omega = 0.2\ \Omega_C$



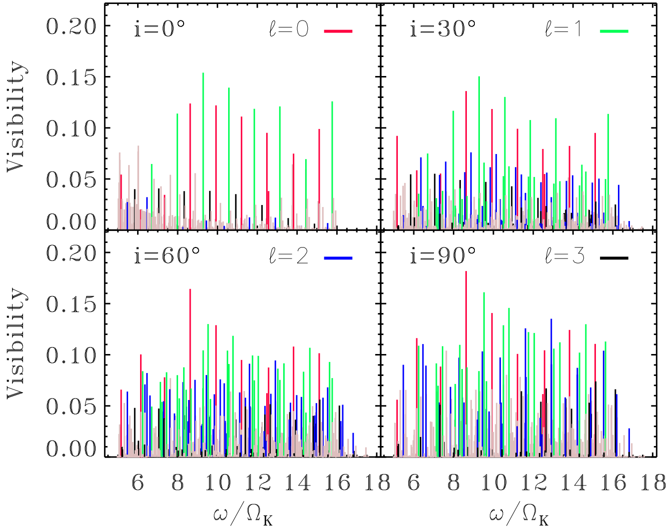
$\Omega = 0.3\ \Omega_C$



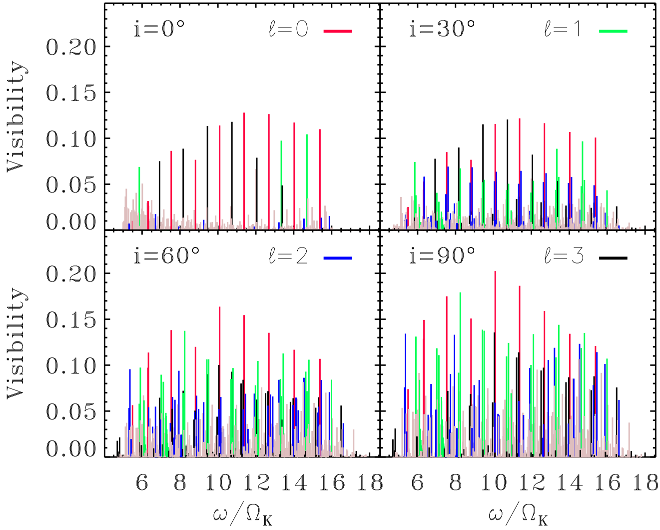
$\Omega = 0.4\ \Omega_C$



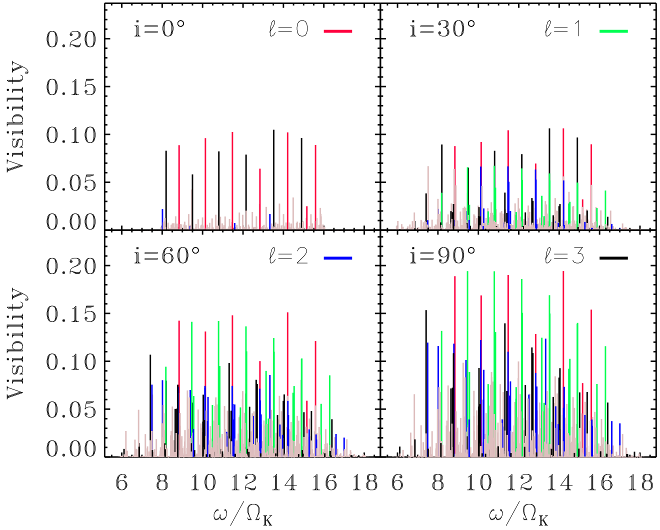
$\Omega = 0.5\ \Omega_C$



$\Omega = 0.6\ \Omega_C$

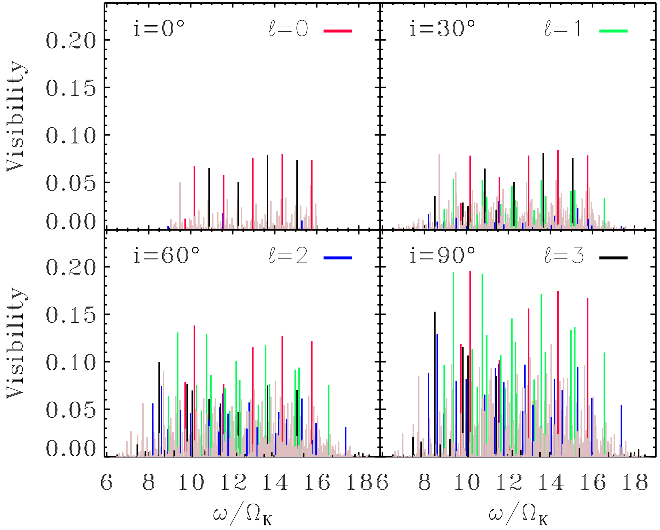


$\Omega = 0.7\ \Omega_C$



(Reese et al., in preparation)

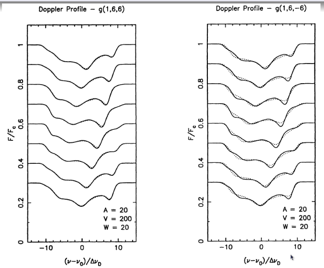
$\Omega = 0.8\ \Omega_C$



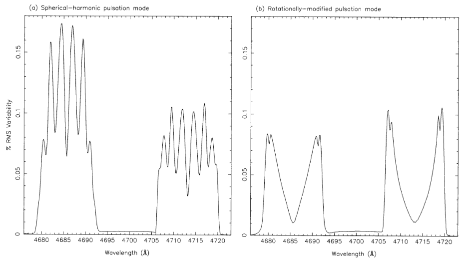
(Reese et al., in preparation)

Line profile variations (LPVs)

- most current methods treat the effects of rotation perturbatively (up to 1st for the eigenfunctions)
  - for example : Schrijvers et al. (1997), FAMIAS (Zima 2008)
- exceptions :
  - Clement (1994) uses 2D calculations
  - Lee & Saio (1990) and Townsend (1997) use the traditional approximation



Clement, 1994

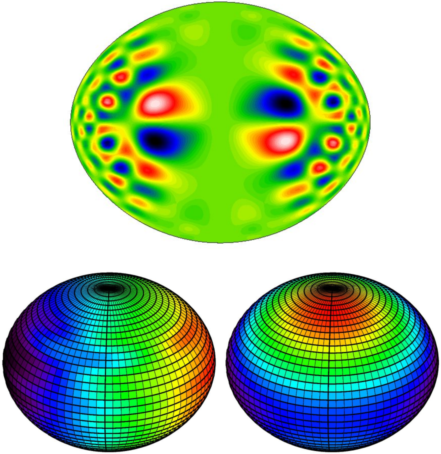


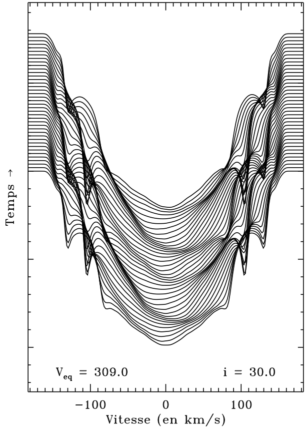
Townsend, 1997

Results presented here

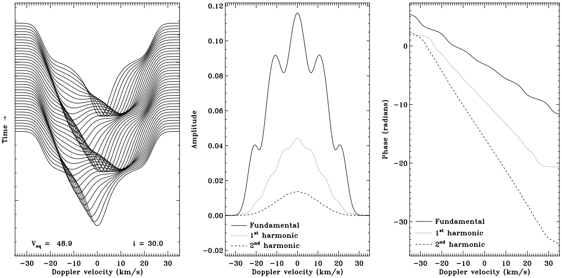
- 2D p-modes of deformed SCF models (Jackson et al. 2005, MacGregor et al. 2007)
- Planck's black-body spectrum
- Claret (2000) limb-darkening law (no latitude dependence)
- simple Gaussian absorption profiles
- no deformation from pulsations

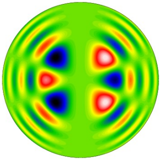
Example



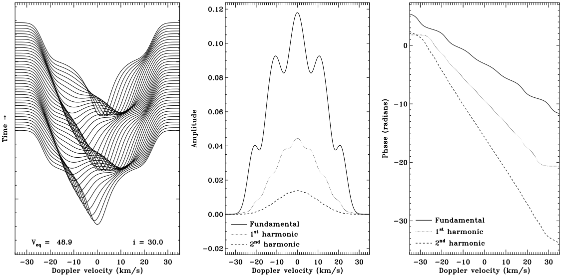


Complete



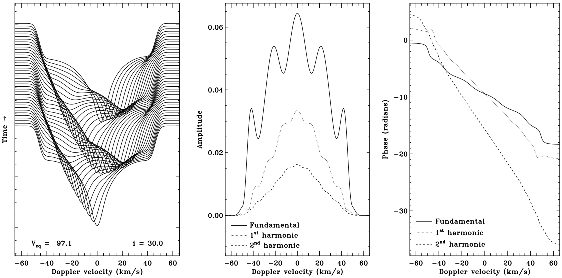


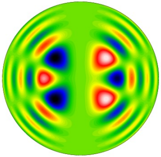
Perturbative



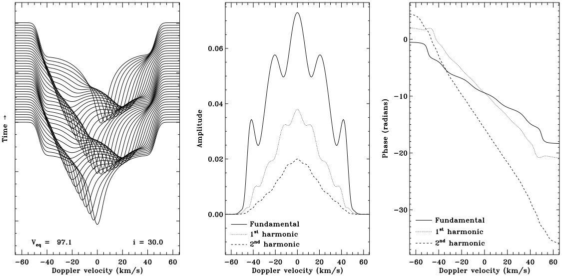
$$n = 6$$
$$\ell = 5$$
$$m = 3$$
$$\Omega = 0.1\Omega_{\mathrm{C}}$$

Complete



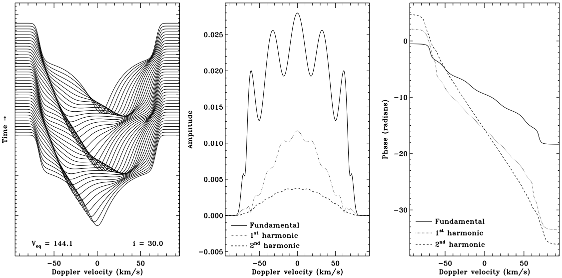


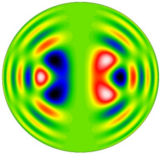
Perturbative



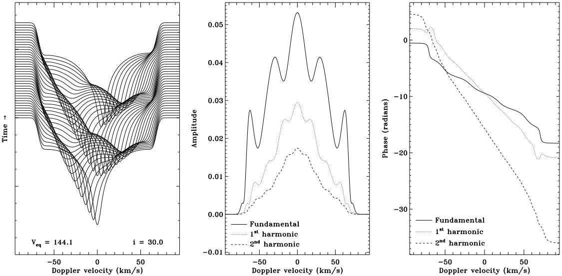
$$n = 6$$
$$\ell = 5$$
$$m = 3$$
$$\Omega = 0.2\Omega_C$$

Complete



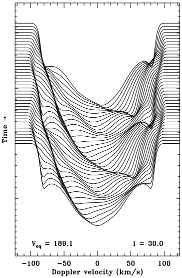


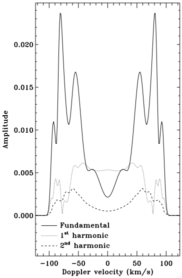
Perturbative

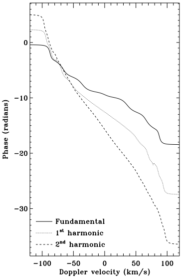


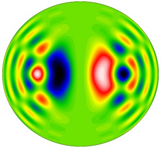
$$n = 6$$
$$\ell = 5$$
$$m = 3$$
$$\Omega = 0.3\Omega_C$$

Complete

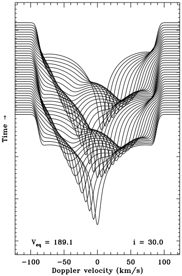


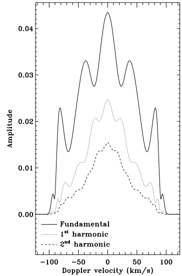


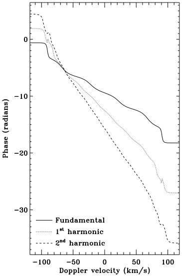




Perturbative

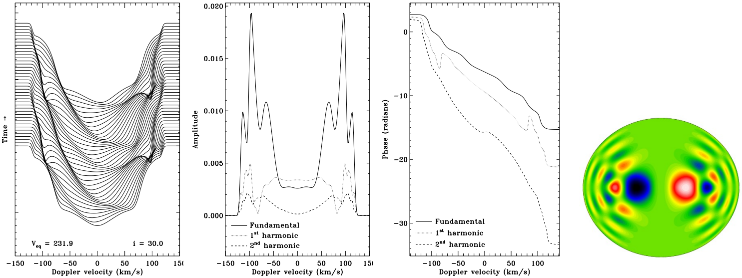




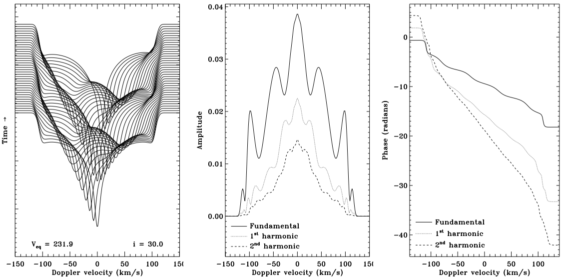


$$n = 6$$
$$\ell = 5$$
$$m = 3$$
$$\Omega = 0.4\Omega_C$$

Complete

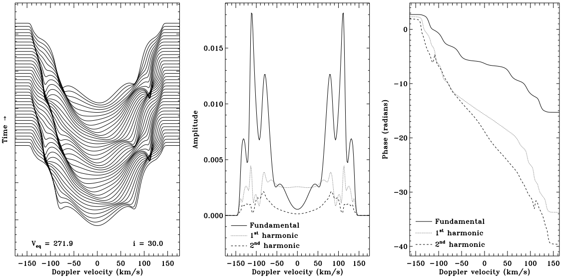


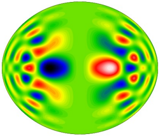
Perturbative



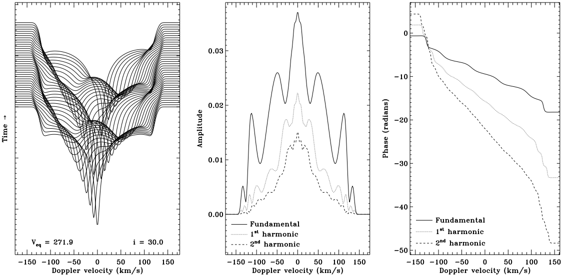
$$n = 6$$
$$\ell = 5$$
$$m = 3$$
$$\Omega = 0.5\Omega_C$$

Complete



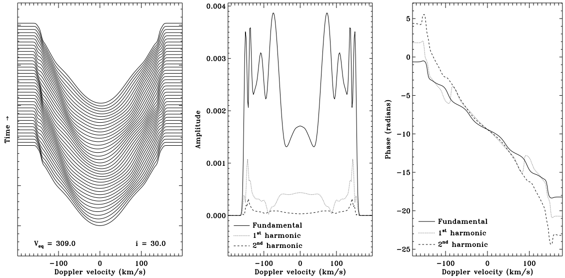


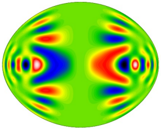
Perturbative



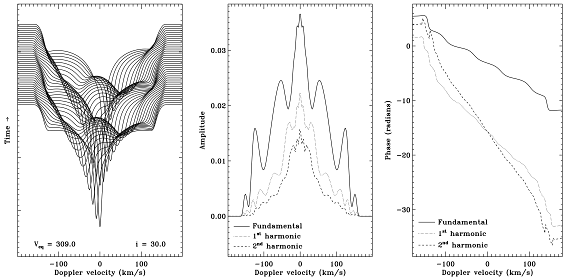
$$n = 6$$
$$\ell = 5$$
$$m = 3$$
$$\Omega = 0.6\Omega_C$$

Complete



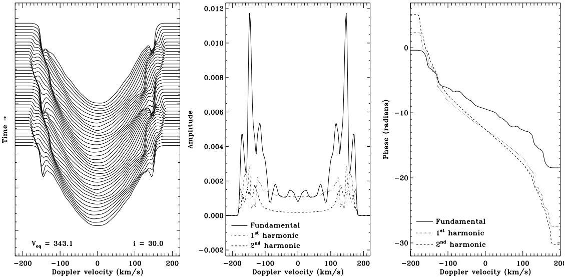


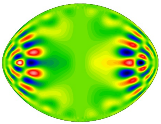
Perturbative



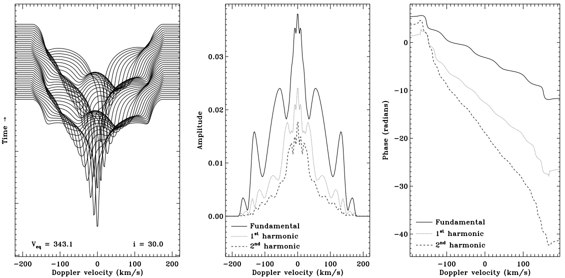
$$
\begin{aligned}
n &= 6 \\
\ell &= 5 \\
m &= 3 \\
\Omega &= 0.7\Omega_{\mathrm{C}}
\end{aligned}
$$

Complete



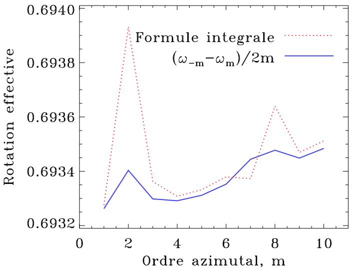


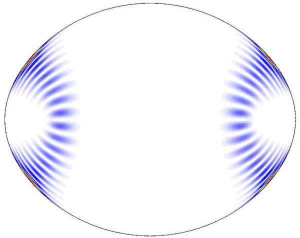
Perturbative



$$n = 6$$
$$\ell = 5$$
$$m = 3$$
$$\Omega = 0.8\Omega_C$$

Inversions





- if mode identification succeeds, it may be possible to invert for Ω, using the generalized splitting:

$$\frac{\omega_{-m}-\omega_m}{2m} \simeq \frac{\Omega^{\rm eff}_m+\Omega^{\rm eff}_{-m}}{2} + \frac{\mathcal{C}_m+\mathcal{C}_{-m}}{2}$$

$$\Omega_{\rm eff} = \frac{\int_V \Omega\rho_o\|\vec{\xi}\|^2 dV}{\int_V \rho_o\|\vec{\xi}\|^2 dV} \qquad \mathcal{C} = \frac{i}{m}\frac{\int_V \rho_o\vec{\Omega}\cdot\left(\vec{\xi}^*\times\vec{\xi}\right) dV}{\int_V \rho_o\|\vec{\xi}\|^2 dV}$$

Conclusion

Needs

- interpreting pulsation data remains a challenge
  - computational aspects
  - mode identification
- need for further observational constraints
  - global parameters
  - multicolor and spectroscopic data
  - easier stars (such as pole-on)

Prospects

- better grasp of global stellar properties
- better grasp of physical processes (differential rotation, mixing, transport)