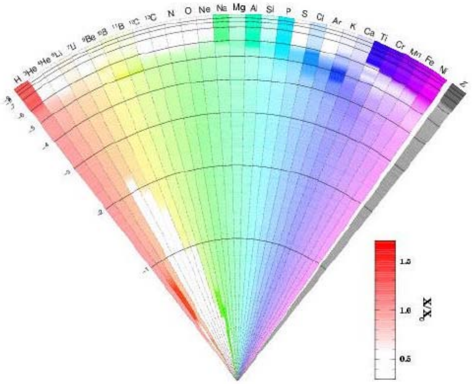
Diffusion in Main Sequence Stars

Atomic diffusion, mixing, thermohaline convection, links with asteroseismology

Sylvie Vauclair

Institut de Recherches en Astrophysique et Planétologie, OMP, CNRS,
Université de Toulouse, Institut universitaire de France

The Impact of Asteroseismology across Stellar Astrophysics, KITP, Santa Barbara, 24-28 October 2011



Element Diffusion in Stars

(Stars are non-uniform multi-component gases)

Basics of stellar physics : two kinds of processes in competition

- « microscopic processes » (atomic diffusion)
- « macroscopic processes » (mixing, mass loss, etc.)

Importance of precise microphysics for stellar structure and evolution

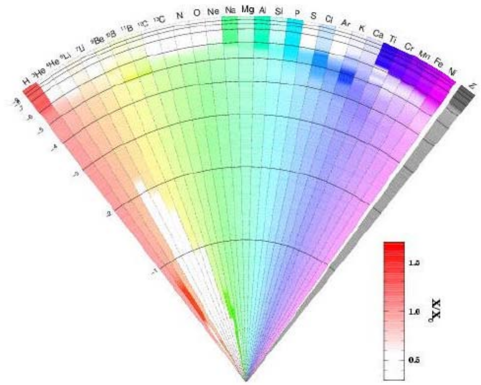
1) gravitational settling
2) thermal diffusion
3) concentration gradients
4) radiative accelerations

-Large data basis on atomic physics, in relation with opacity projects:
OPAL , OP…

-Helio and asteroseismic tests

e.g. helium gradients below convective zones and many other consequences

The development of knowledge



Beginning of 20th century :

- basics of stellar physics including discussions on atomic diffusion…
  Eddington 1916, 1926; Chapman 1917; etc
- and macroscopic processes
  Von Zeipel 1924; Vogt 1925; Eddington 1929; Sweet 1950; etc.

1950 to 1970:

- discussions of diffusion in white dwarfs, and in the Sun
  Schatzman 1944, 1945, 1958; Aller and Chapman 1960, etc.

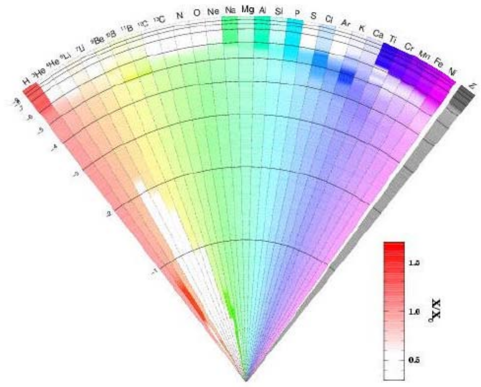
In the 1970s:

- first explanations of chemically peculiar stars by atomic diffusion…
  F. Praderie 1967; G. Michaud 1970; W.D. Watson 1971
- and competition with macroscopic processes: MCV2; V2SM; V2M, etc.
  also: instabilities INDUCED by atomic diffusion??? (Ed. Spiegel)

Now:

- Helio and asteroseismic tests
- detailed studies of the induced instabilities

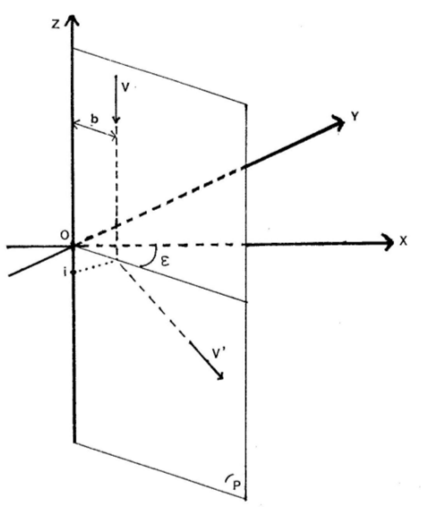
Chapman-Enskog description



Treatment of collisions (Boltzmann integro-differential equation):

$$[f(\mathbf{c}+\boldsymbol{F}dt,\ \boldsymbol{r}+\mathbf{c}dt,\ t+dt) - f(\mathbf{c},\boldsymbol{r},t)] = (\text{collision term})$$

f (c,r,t) is the distribution function of the particles, i.e. the number of particles in the volume element (**r**, **r+dr**) with velocities in the range (**c**, **c+dc**) at time t



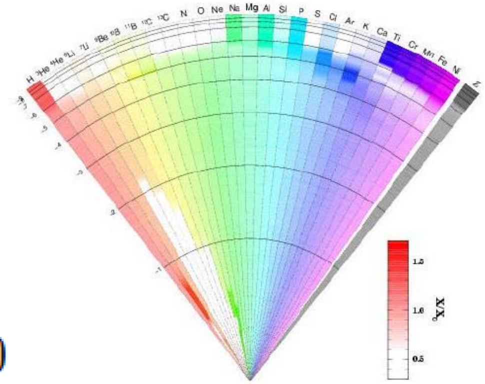
$$\left(\frac{\partial f_i}{\partial t}\right)_{col} = \iiint \left(f_i' f_j' - f_i f_j\right) \boldsymbol{v}\, b\, db\, d\varepsilon\, \mathbf{dc_j}$$

Maxwellian ($\left(\frac{\partial f_i}{\partial t}\right)_{col} = 0$) : $f_i' f_j' = f_i f_j$

in the presence of gradients ($\left(\frac{\partial f_i}{\partial t}\right)_{col} \neq 0$) : $f_i' f_j' \neq f_i f_j$

Compute the collision integrals as a development in terms of small knudsen number $k = l/L = t_{mic}/t_{mac}$

Results:



Diffusion equation:

$$\frac{\partial(\rho c_i)}{\partial t} + div(\rho c_i v_i) = 0$$

for test atoms:
(including mixing coefficient Dth)

$$v_i = D_i\left(-\frac{D_i + D_{th}}{D_i}\nabla \ln c + k_P \nabla \ln P + k_T \nabla \ln T + \frac{m_i g_i}{kT}\right)$$

Approximate expressions (not used in codes but interesting for physical discussions):

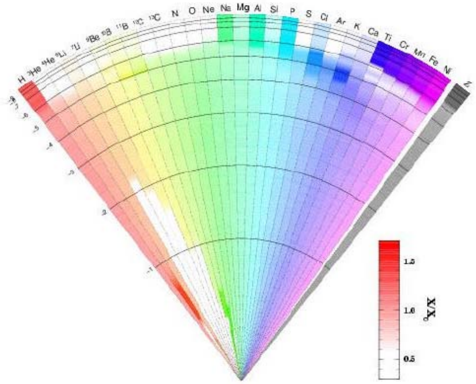
$$v_d = -D\left[\frac{1}{c}\frac{\partial c}{\partial r} - \frac{m(g_R - g_{GT})}{kT}\right]$$

with:

$$D = \tfrac{1}{3} l C_M = \tfrac{1}{3} t_{col} C_M{}^2 = t_{col}\frac{kT}{m}$$

$$\longrightarrow \; v_d = t_{col}\, g_{eff}$$

Burgers diffusion equations (see Hu 2011)



$$\frac{\mathrm{d}p_i}{\mathrm{d}r} + \rho_i(g - g_{\mathrm{rad},i}) - n_i \bar{Z}_i e E =$$

$$\sum_{j\neq i}^{N} K_{ij}(w_j - w_i) + \sum_{j\neq i}^{N} K_{ij} z_{ij} \frac{m_j r_i - m_i r_j}{m_i + m_j},$$

including the heat flow equations,

$$\frac{5}{2} n_i k_{\mathrm{B}} \nabla T = \frac{5}{2} \sum_{j\neq i}^{N} z_{ij} \frac{m_j}{m_i + m_j}(w_j - w_i) - \frac{2}{5} K_{ii} z''_{ii} r_i$$

$$- \sum_{j\neq i}^{N} \frac{K_{ij}}{(m_i + m_j)^2}(3m_i^2 + m_j^2 z'_{ij} + 0.8 m_i m_j z''_{ij}) r_i$$

$$+ \sum_{j\neq i}^{N} \frac{K_{ij} m_i m_j}{(m_i + m_j)^2}(3 + z'_{ij} - 0.8 z''_{ij}) r_j.$$

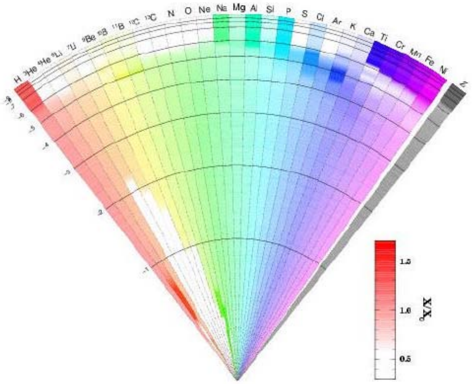
In addition, we have two constraints, current neutrality,

$$\sum_i \bar{Z}_i n_i w_i = 0$$

and local mass conservation,

$$\sum_i m_i n_i w_i = 0.$$

Orders of magnitude



- Atomic diffusion is a very slow but efficient process

- V_d increases and t_d decreases towards the surface

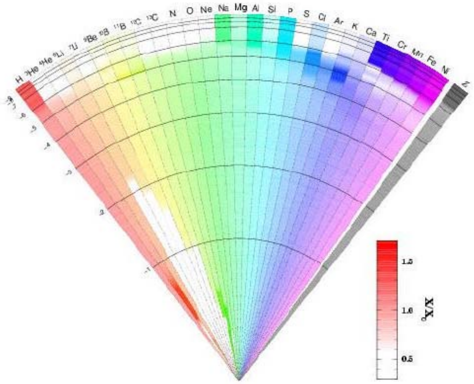
- Below the solar convective zone: $t_d \sim 10^{10}$ yrs

- In atmospheres: $t_d \sim 10^2 \sim 10^4$ yrs

- At the bottom of the second convective zone in Am stars:
$D_m \sim 100$ cm^2/sec , $t_d \sim 10^6$ yr, $v_d \sim 10^{-6}$ cm/sec, $t_{col} \sim 10^{-10}$ sec

- Competition with macroscopic motions : selective vs homogenizing processes

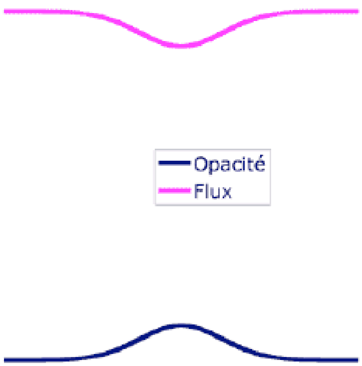
Radiative accelerations :



$$g_{\mathrm{rad}}(A) = \frac{1}{4\pi r^2}\frac{L_r^{\mathrm{rad}}}{c}\frac{\kappa_R}{X_A}\int_0^\infty \frac{\kappa_u(A)}{\kappa_u(total)}\mathcal{P}(u)du$$

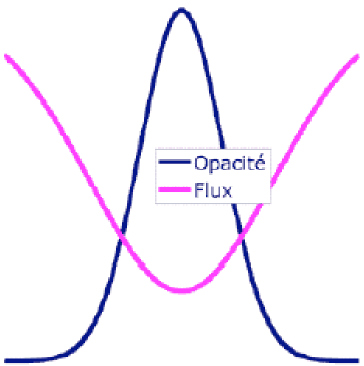
$$\mathcal{P}(u) \equiv \frac{15}{4\pi^4}\frac{u^4 e^u}{(e^u-1)^2} \qquad u \equiv h\nu/kT$$

unsaturated line



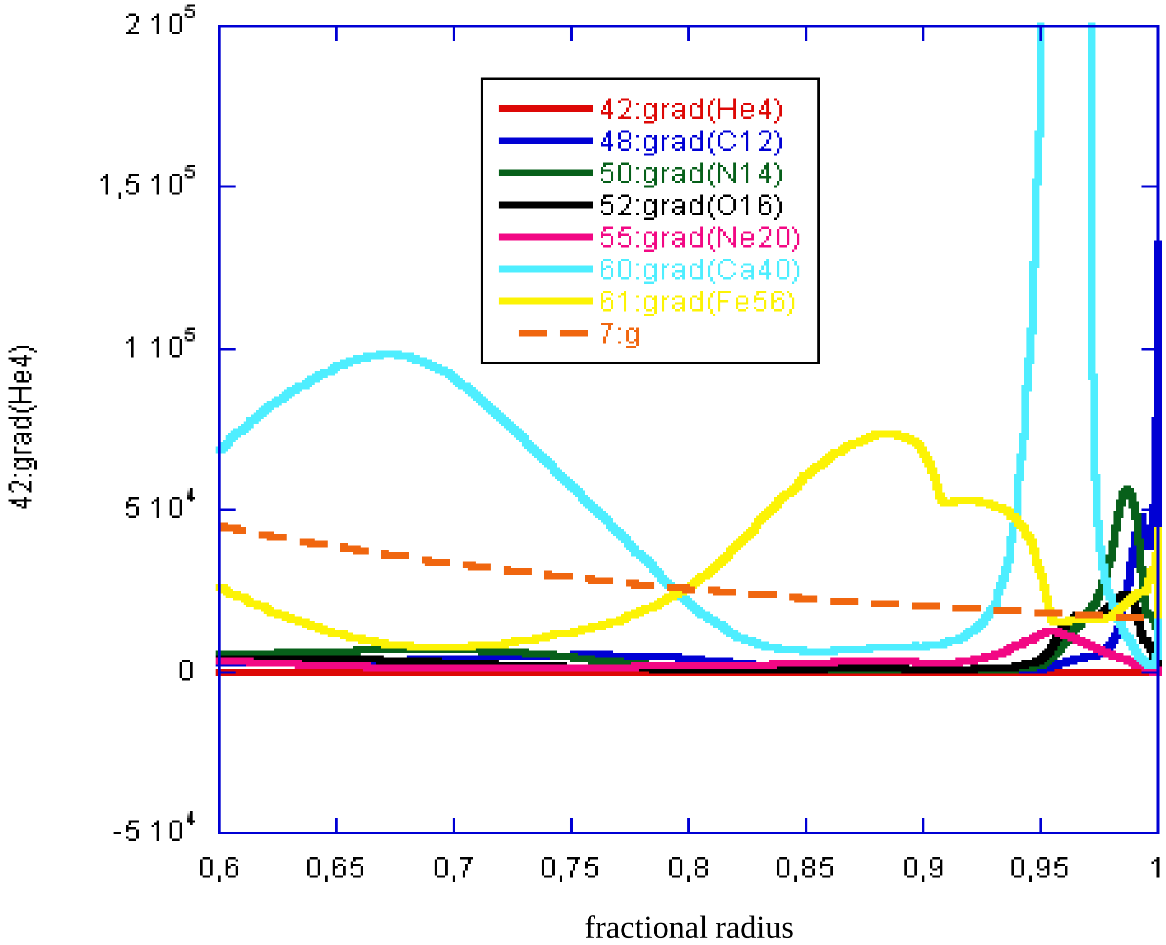
g_{rad} independent of N

saturated line

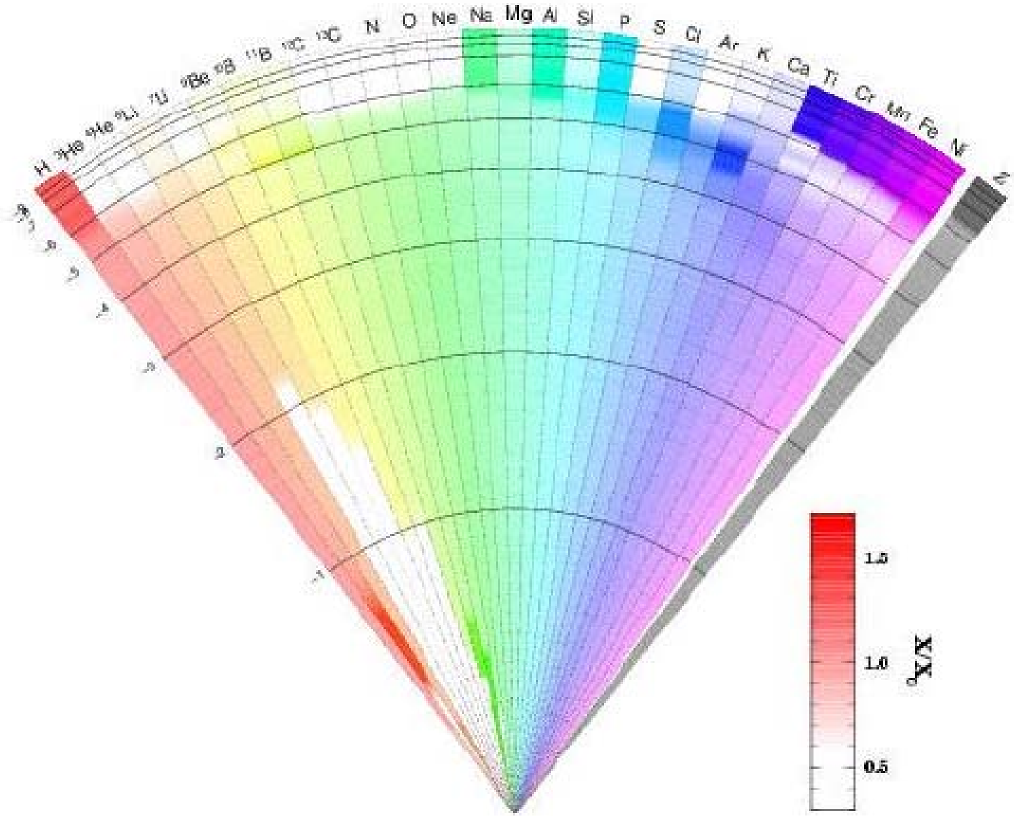


g_{rad} ↑ when N↓

Radiative accelerations in an A star, 1.7Msun, 403 Myr



(TGEC code, Théado & Vauclair, in prep.)



Examples of color intensity coded elements concentrations after pure diffusion : 3Msun, 70Myr (Richer et al. 1999)

Possible macroscopic consequences of atomic diffusion:
(other than observed abundances)

- dynamical convection
- thermohaline convection
- stellar oscillations

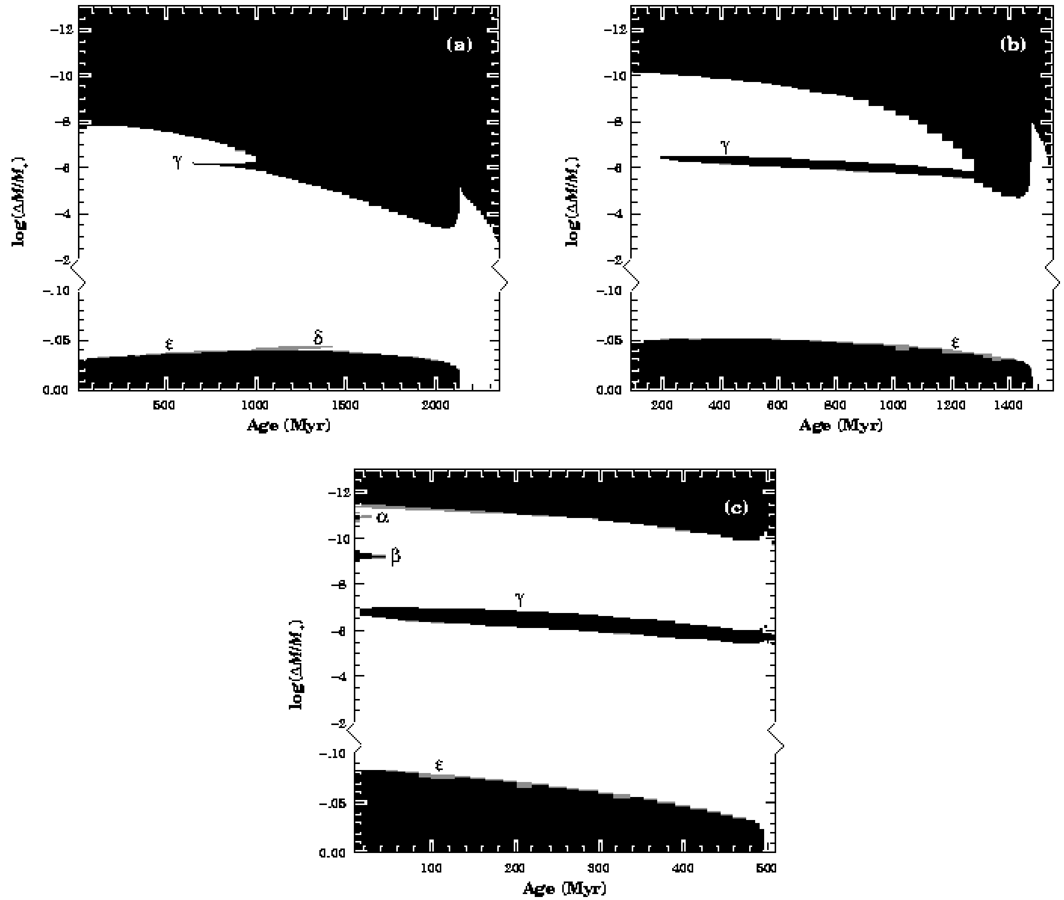


FIG. 8.—Convection and semiconvection zones in three models with turbulence parameterized by 5.3D50-3: (*a*) 1.5 $M_{\odot}$, (*b*) 1.7 $M_{\odot}$, and (*c*) 2.5 $M_{\odot}$. The radiative zones are in white, the convection zones in black, and the semiconvection zones in gray. The convection zones α and β, due to He I and II, rapidly disappear because of He settling. The convection zone γ is the Fe convection zone. Close to the central convective core, there appear semiconvection zones, ε and δ.

Richard, Michaud, Richer 2001

Thermohaline convection: the ocean case

Define the density anomaly ratio as :

$$R_\rho = \alpha \nabla T / \beta \nabla S$$

where :

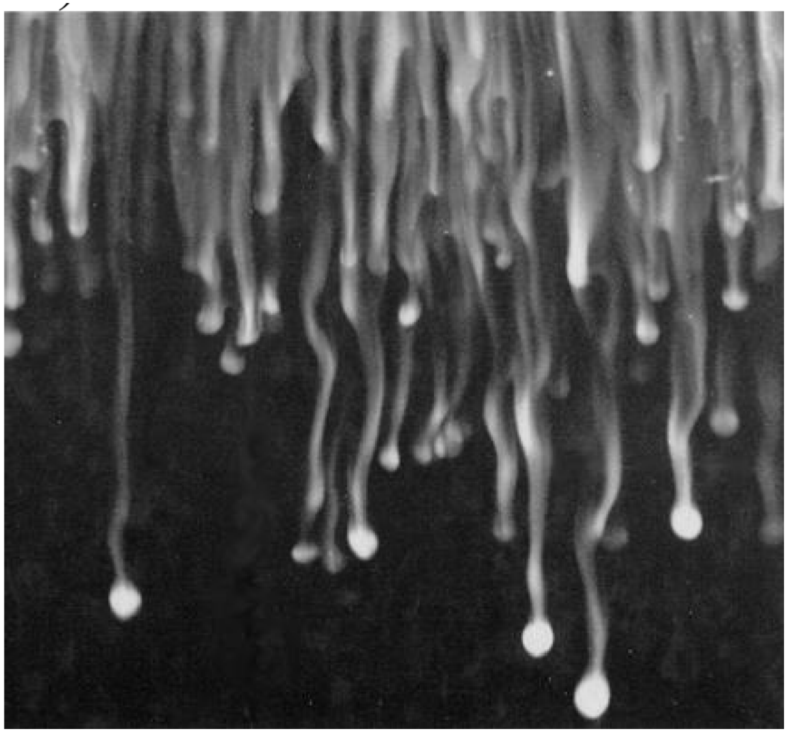
$$\alpha = (1/\rho)(\partial\rho/\partial T)_{S,P}$$
$$\beta = (1/\rho)(\partial\rho/\partial S)_{T,P}$$

and the lewis number :

$$\tau = \kappa_S/\kappa_T = t_T/t_S$$

salt fingers can grow if :

$$1 \leq R_\rho \leq \tau^{-1}$$



Stern 1960, Kato 1966, Veronis 1965, Turner 1973, Turner and Veronis 2000, Wells 2001, Piacsek and Toomre 1980, Shen and Veronis 1997, Yoshida and Nagashima 2003, Gargett and Ruddick 2003

The stellar case

∇_μ = dlnμ/dlnP plays the role of the salinity gradient;

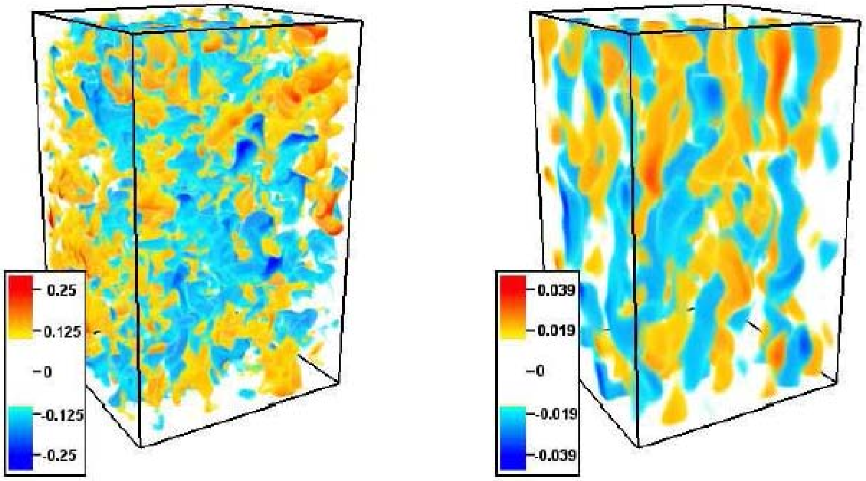
∇_{rad}- ∇ plays the role of the temperature gradient

« fingers » form if :

$$1 < R_0 < \frac{1}{\tau}$$

with: $$R_0 = \frac{\nabla_{ad} - \nabla_{rad}}{\nabla_\mu}$$

and $\tau = \kappa_\mu / \kappa_T = \tau_T / \tau_\mu$



For R_0<1, dynamical convection
For R_0=1/τ, dissipation

Simulations by Traxler, Garaud, Stellmach 2011
R_0 = 1.45 and 9.1 ; 1/τ = 10

Thermohaline diffusion coefficients

Ulrich 1972, Kippenhahn et al. 1980

$$D_{th} = C_t \frac{4acT^3}{3c_p\kappa\rho^2} \frac{H_p}{\nabla_{ad} - \nabla} \left|\frac{d\ln\mu}{dr}\right|$$

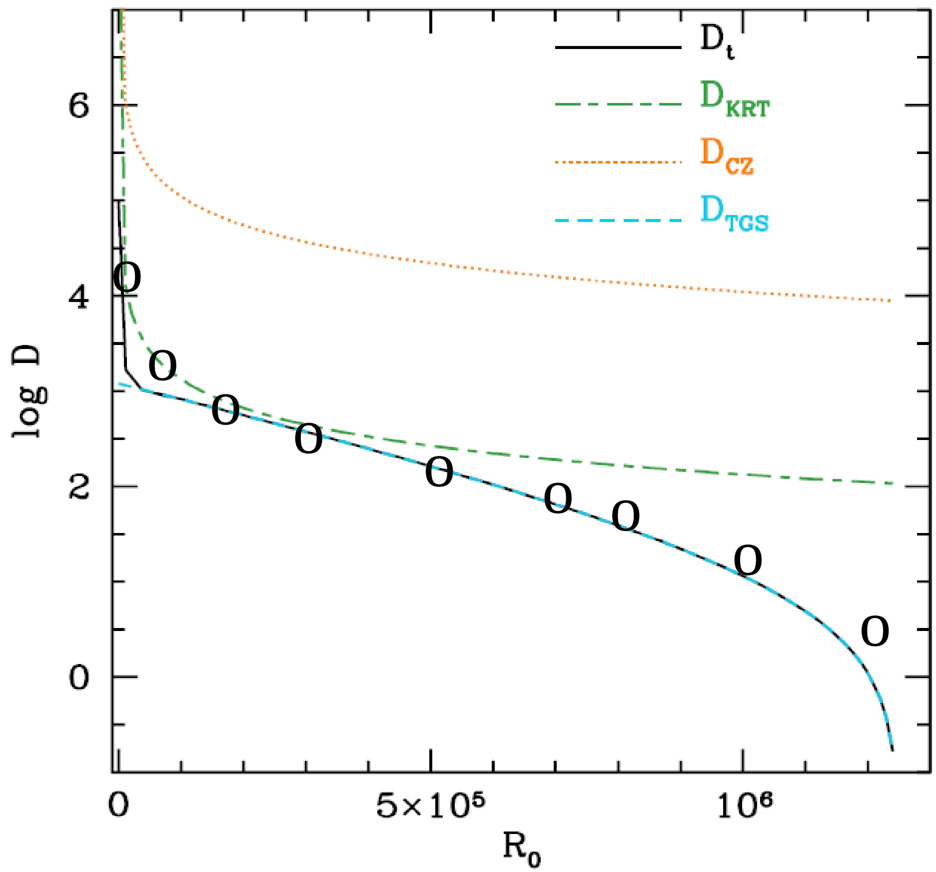
$$D_{th} = C_t \kappa_T R_0^{-1}$$

Denissenkov 2010

$$D_{th} = C_t \kappa_T (R_0 - 1)^{-1} (1 - R_0 \tau)$$

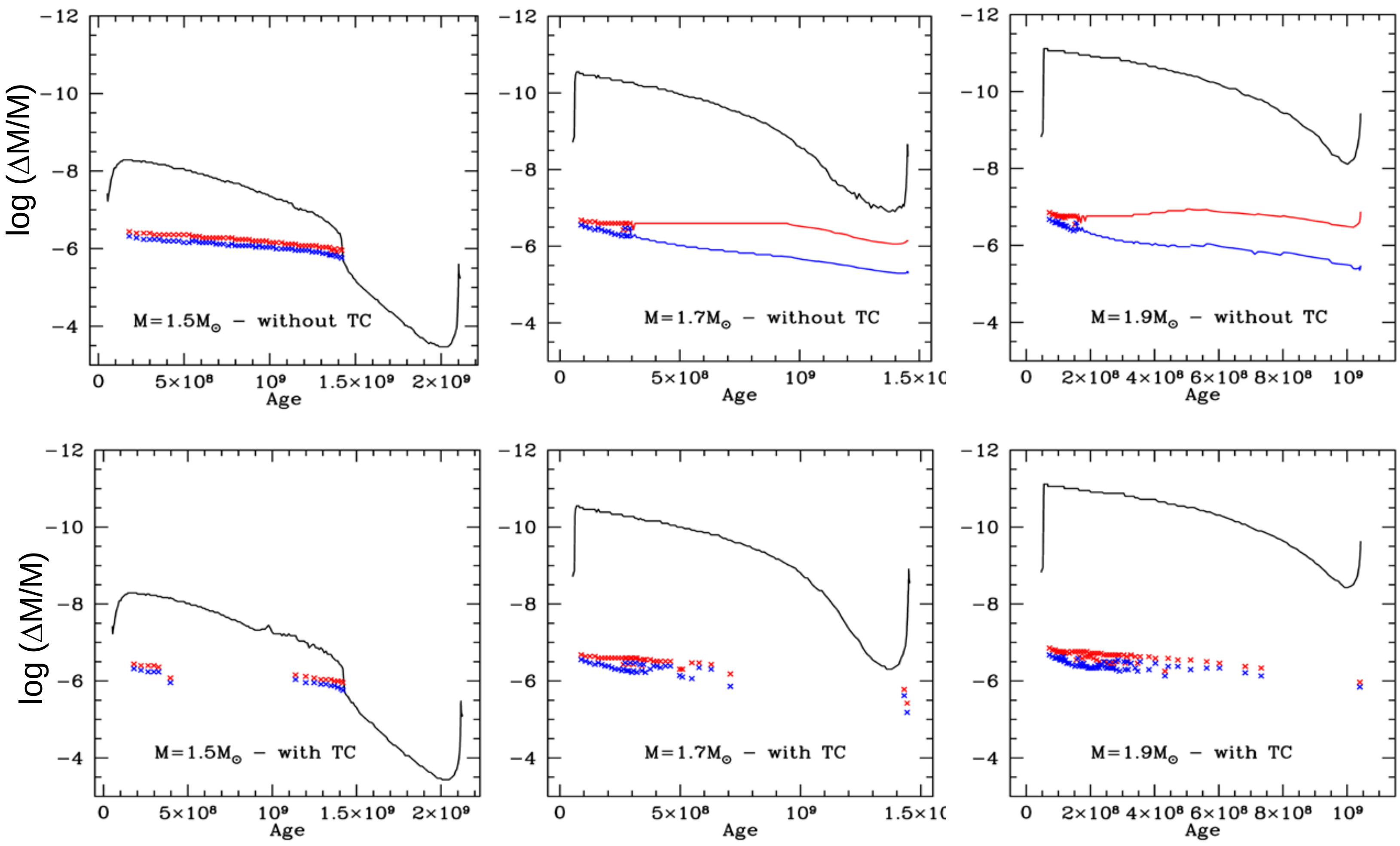
Traxler et al. 2011

$$D_{th} = 101\sqrt{\kappa_\mu \nu}\,\exp(-3.6r)(1-r)^{1.1}$$



see Théado & Vauclair 2011, arXiv 1109.4238

Théado, Vauclair, Alecian, Leblanc 2009



many applications: SPB/βCeph, sdB, γ Dor, etc.

Extra-effects on thermohaline

In case of horizontal turbulence :

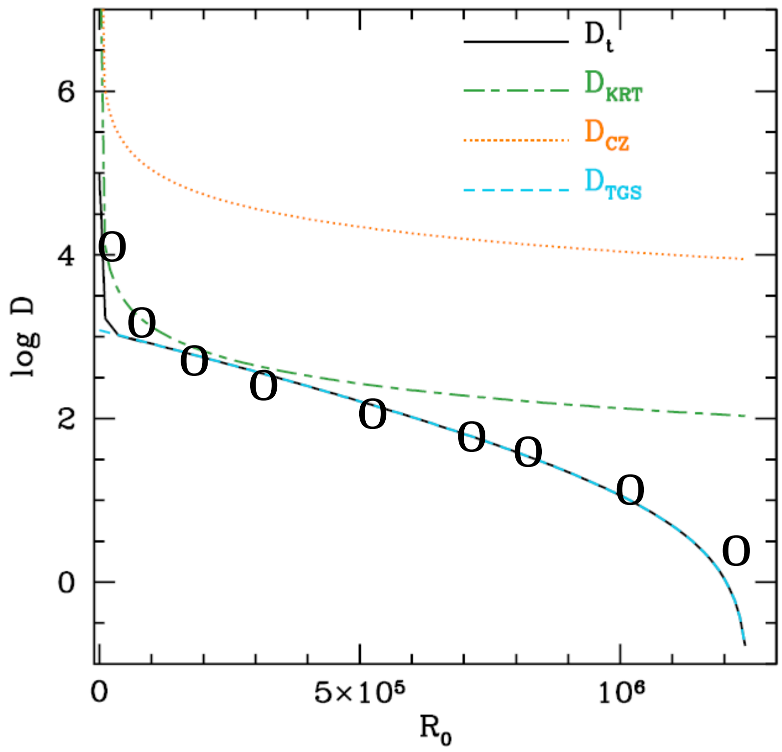
$1/\tau$ replaced by κ_T/κ_{mix}

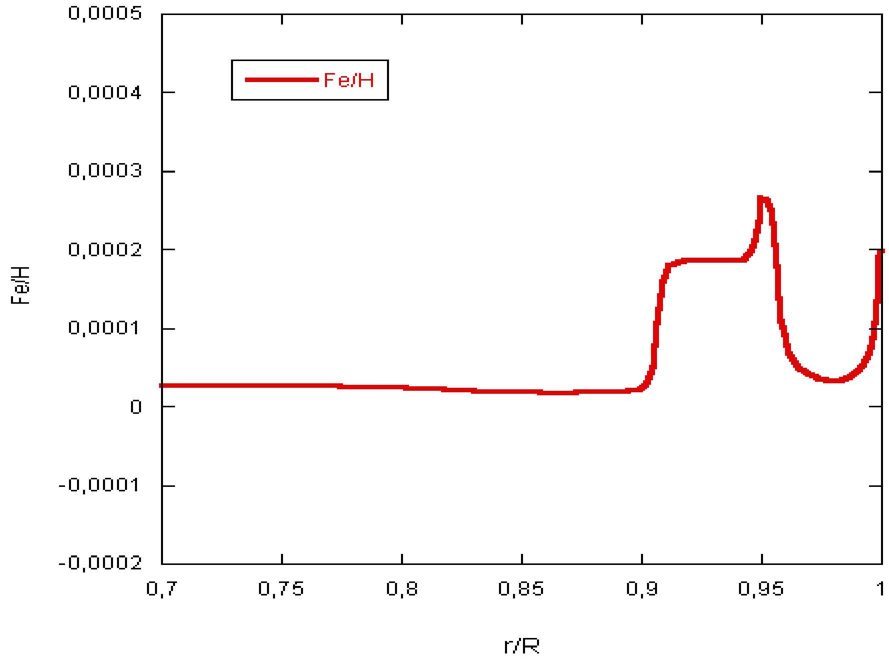
(see Denissenkov and Pinsonneault 2008)

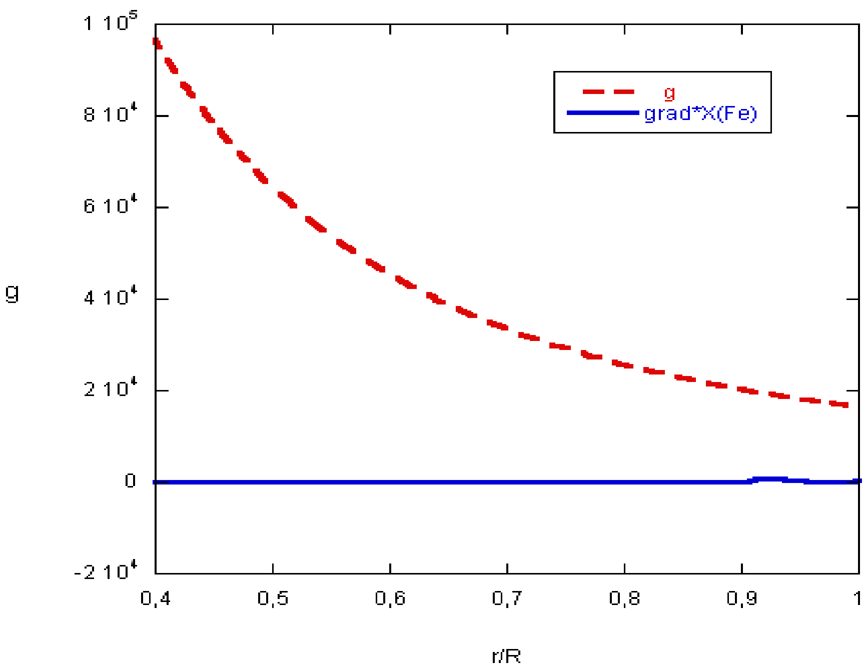
Influence of radiative acceleration:

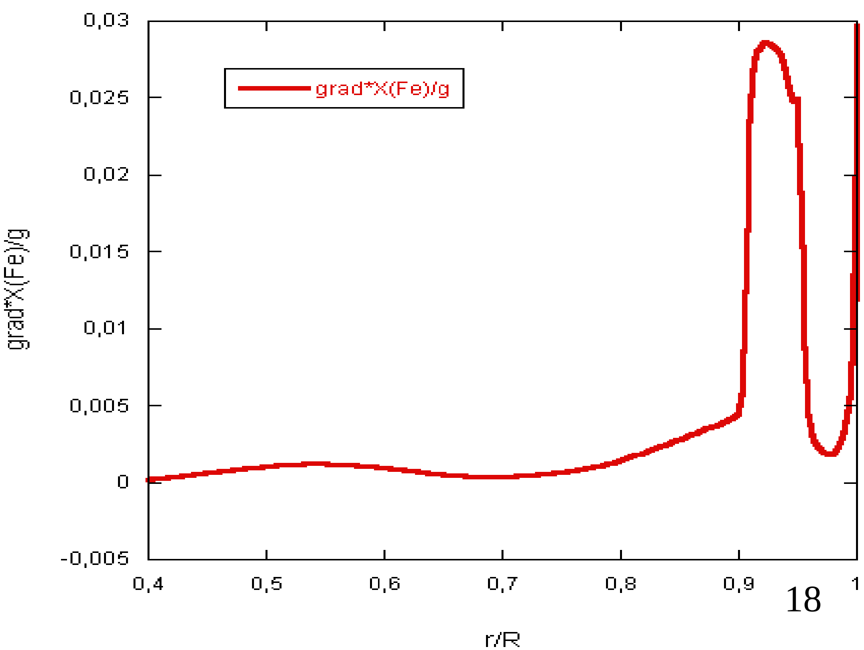
$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}, \nabla)\mathbf{v} = \frac{\delta\rho}{\rho_0}(g - g_R) + \nu\nabla^2\mathbf{v},$$
$$\frac{\partial T}{\partial t} + (\mathbf{v}, \nabla)T = k_T\nabla^2 T,$$
$$\frac{\partial \mu}{\partial t} + (\mathbf{v}, \nabla)\mu = \nabla[k_\mu(\nabla\mu + f(g, g_R))]$$

limit smaller than $1/\tau$ (see talk Haili Hu))

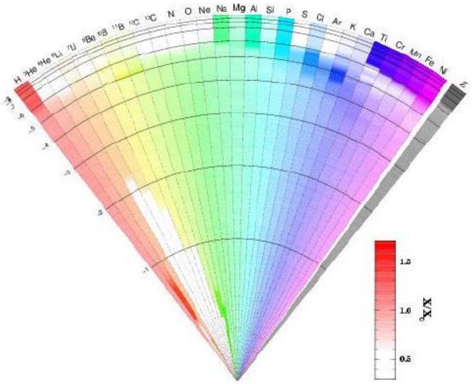



Fe/H
0,0005
0,0004
0,0003
0,0002
0,0001
0
-0,0001
-0,0002
0,7
0,75
0,8
0,85
0,9
0,95
1
r/R


g
grad*X(Fe)
1 10^5
8 10^4
6 10^4
4 10^4
2 10^4
0
-2 10^4
0,4
0,5
0,6
0,7
0,8
0,9
1
r/R


grad*X(Fe)/g
0,03
0,025
0,02
0,015
0,01
0,005
0
-0,005
0,4
0,5
0,6
0,7
0,8
0,9
1
r/R

Conclusions



- Importance of chemical stratification for stellar structure and evolution, and for helio and asteroseismology

- Abundances modifications inside stars can trigger or prevent oscillations

- Competition between micro and macroscopic motions studied for many years

- But :importance of the instabilities induced by the stratification itself: still lot of work to do !