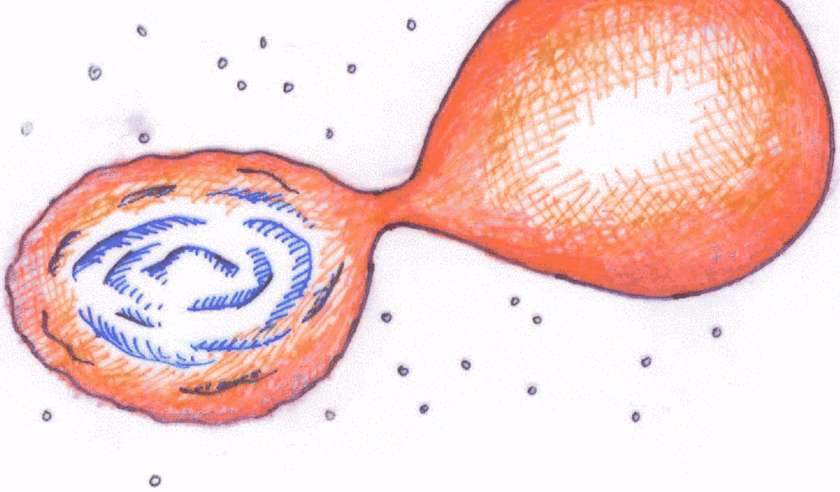
The Spreading of Accreted Material on White Dwarfs

Anthony Piro
Advisor: Lars Bildsten
UCSB



Why is transition region important?

How bright?

Similar to disk!

$$L = \frac{1}{2}\dot{M}R^2\Omega^2 = \frac{GM\dot{M}}{2R} \approx 10^{32} - 10^{34}\ \frac{\text{erg}}{\text{s}}$$

How hot?

$$4\pi R H \sigma T_{\text{eff}}^4 = \frac{GM\dot{M}}{2R}$$

Soft X-rays or EUV

using H for thin disk, $T_{\text{eff}} \approx 10^5\,\text{K}$

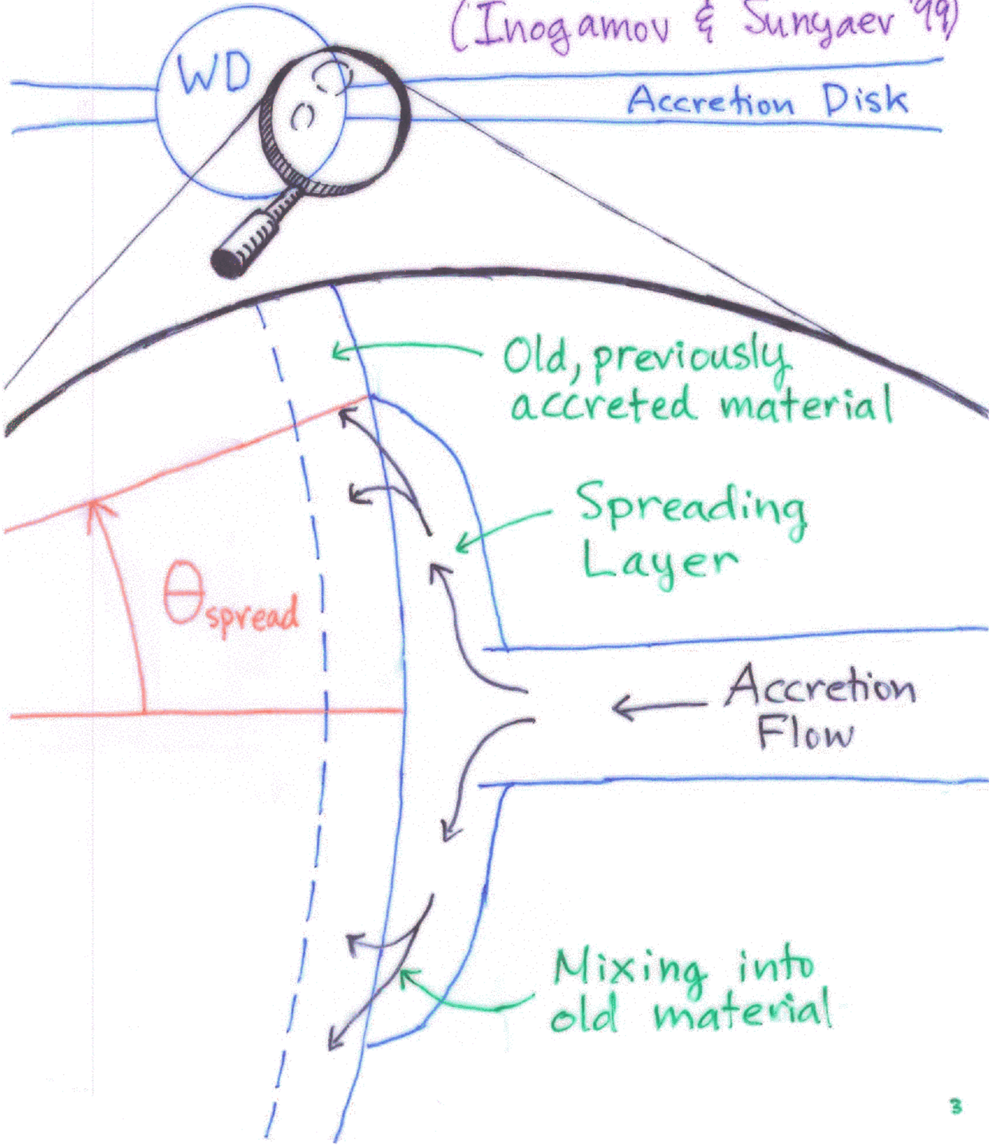
Boundary Layer Models

Standard models follow radial coordinate and make assumptions about vertical structure (Popham & Narayan '95) or use 2-d numerical simulations (Kley 1989)

Is there another way?...

The "Spreading Layer"

(Inogamov & Sunyaev '99)



Constructing the "Spreading Layer"

Not a new idea!

➡ Use conservation equations by Inogamov & Sunyaev ('99)

Game Plan:

① One-zone model for radial structure

② Write conservation equations for fluid confined to sphere

③ Take steady state limit

④ Set boundary conditions and integrate

⑤ Rinse and repeat!

Radial Properties of SL

Construct a one-zone model in plane parallel geometry (h<<R)

$$g_{eff} = \frac{GM}{R^2} - \frac{v_\theta^2}{R} - \frac{v_\varphi^2}{R}$$

Hydrostatic balance

⇨ $P = g_{eff}\, y \xleftarrow{} g/\mathrm{cm}^2$

Radiative transfer

⇨ $F = \frac{4acT^3}{3\kappa}\frac{dT}{dy}$

where $\kappa = \kappa_{es} = 0.34\ \mathrm{cm}^2/\mathrm{g}$

& F is independent of radius

Equation of state

⇨ $P = \frac{\rho k T}{\mu m_p} + \frac{aT^4}{3}$

Integrating results in...

$$T = \left(\frac{3\kappa F}{ac}\right)^{1/4} y^{1/4}$$

$$\rho = \frac{\mu m_p}{kT}\left(g_{eff} - g_{rad}\right) y$$

where

$$g_{rad} \equiv \frac{\kappa F}{c}$$

Notice that

$g_{eff} = g_{rad}$ ⇨ Eddington Limit

(useful trick for remembering Eddington Limit, $\dot{M}_{Edd} = 4\pi Rc/\kappa$)

Conservation Equations

Want to follow radially integrated conserved fluxes (fluxing in θ-direction)

Mass conservation

$$\frac{\partial}{\partial t}(2\pi R\cos\theta\, y) + \frac{1}{R}\frac{\partial}{\partial\theta}(2\pi R\cos\theta\, y v_\theta) = 0$$

θ-momentum

$$\frac{\partial}{\partial t}(2\pi R\cos\theta\, y v_\theta) + \frac{1}{R}\frac{\partial}{\partial\theta}(2\pi R\cos\theta\, y v_\theta^2)$$

$$+ 2\pi R\cos\theta\, y\frac{v_\varphi^2}{R}\tan\theta$$

$$= -2\pi R\cos\theta\frac{1}{R}\frac{\partial}{\partial\theta}\left(\int_0^h P\,dz\right) - 2\pi R\cos\theta\,\tau_\theta$$

$$\int_0^h P\,dz = \frac{4}{5}\frac{kT}{\mu m_p}\frac{g_{eff}\, y}{g_{eff}-g_{rad}}$$

viscous stress!

Also φ-momentum and energy conservation!

Additional Conservation Equations

φ-momentum

$$\frac{\partial}{\partial t}(2\pi R\cos\theta\, y v_\varphi) + \frac{1}{R}\frac{\partial}{\partial\theta}(2\pi R\cos\theta\, y v_\theta v_\varphi)$$

$$- 2\pi R\cos\theta\, y\frac{v_\theta v_\varphi}{R}\tan\theta = -2\pi R\cos\theta\,\tau_\varphi$$

Energy

$$\frac{\partial}{\partial t}(2\pi R\cos\theta\, E_{tot})$$

$$+ \frac{1}{R}\frac{\partial}{\partial\theta}\left[\cos\theta\, v_\theta\left(E_{tot} + \int_0^h P\,dz\right)\right]$$

$$= -R\cos\theta\, F$$

$$E_{KE} = \frac{1}{2}(v_\theta^2 + v_\varphi^2)\,y$$

$$E_{grav} = \int_0^h \rho g_{eff} z\,dz = \frac{4}{5}\frac{kT}{\mu m_p}\frac{g_{eff}}{g_{eff}-g_{rad}}\,y$$

$$E_{gas} = \frac{3}{2}\int_0^h \frac{\rho kT}{\mu m_p}\,dz = \frac{6}{5}\frac{kT}{\mu m_p}\,y$$

$$E_{rad} = \int_0^h aT^4\,dz = \frac{12}{5}\frac{kT}{\mu m_p}\frac{g_{rad}}{g_{eff}-g_{rad}}\,y$$

How do we deal with viscous stress?

Use standard cheat

$$\boxed{\tau = \alpha \rho v^2}$$

Lower Limits on α...

Ion viscosity

$$\tau = \nu_i \rho \frac{\partial v}{\partial z} \approx \nu_i \rho \frac{v}{h}$$

Using Spitzer (1965) ⇨ $\alpha \approx 10^{-10}$

Radiative viscosity

$$\tau = \nu_r \rho_r \frac{\partial v}{\partial z} \approx \nu_r \rho_r \frac{v}{h}$$

$$\nu_r = \lambda c, \quad \lambda = 1/(\rho \kappa)$$

⇨ $\alpha = \frac{\rho_r}{\rho}\frac{\lambda}{h}\frac{c}{v} \approx 10^{-6}$

Both too small!

Taking Steady State Limit

Need a sink!

When $v_\varphi \approx 0$ & $T_{eff} = (1-3)\times 10^4$K spreading material becomes part of star

(Townsley & Bildsten '03
Sion 1999)

Mass conservation:

$$\frac{\partial}{\partial t}\cancel{(2\pi R\cos\theta\, y)} + \frac{1}{R}\frac{\partial}{\partial \theta}(2\pi R \cos\theta\, y\, v_\theta) = 0$$

⇨ $\boxed{\frac{1}{2}\dot{M} = 2\pi R \cos\theta\, y\, v_\theta}$

θ-momentum:

$$\boxed{y v_\theta \frac{dv_\theta}{d\theta} + y v_\varphi^2 \tan\theta = -\frac{4}{5}\frac{d}{d\theta}\left[\frac{kT}{\mu m_p}\frac{g_{eff}}{g_{eff}-g_{rad}}\right] - R\tau_\theta}$$

+ φ-momentum + energy

= 3 equations for v_θ, v_φ, T

Analytic Estimates

Balancing pressure gradient with Coriolis force...

$$\frac{1}{R}\frac{dP}{d\theta} = -2\Omega v_\varphi \rho \underbrace{\sin\theta}_{\approx\theta}$$

setting $\Omega = v_\varphi/R$,

$$R\theta_{SL} \sim R\frac{(P/\rho)^{1/2}}{v_\varphi} \sim R\frac{c_s}{v_\varphi}$$

Using φ-momentum...

$$\frac{1}{R}\frac{d}{d\theta}(y v_\varphi v_\theta) = -\tau_\varphi = -\alpha\rho v_\varphi^2$$

$$\Rightarrow \frac{1}{R\theta_{SL}} y v_\varphi v_\theta \approx \alpha\rho v_\varphi^2$$

$$\Rightarrow \boxed{R\theta_{SL} \approx \frac{kT}{\mu m_p g_{eff}} \frac{\rho v_\varphi v_\theta}{\alpha\rho v_\varphi^2}}$$

Time to Integrate!

But what boundary conditions?...
Assume Shakura & Sunyaev ('73) disk

$$\theta_{disk} = 1.5\times10^{-2}\,\alpha_{disk}^{-1/10}\,\dot{M}_{18}^{3/20}\,M_1^{-3/8}\,R_9^{1/8}$$

$\dot{M}_{18} = \dot{M}/10^{18}\,g/s$ $M_1 = M/M_\odot$

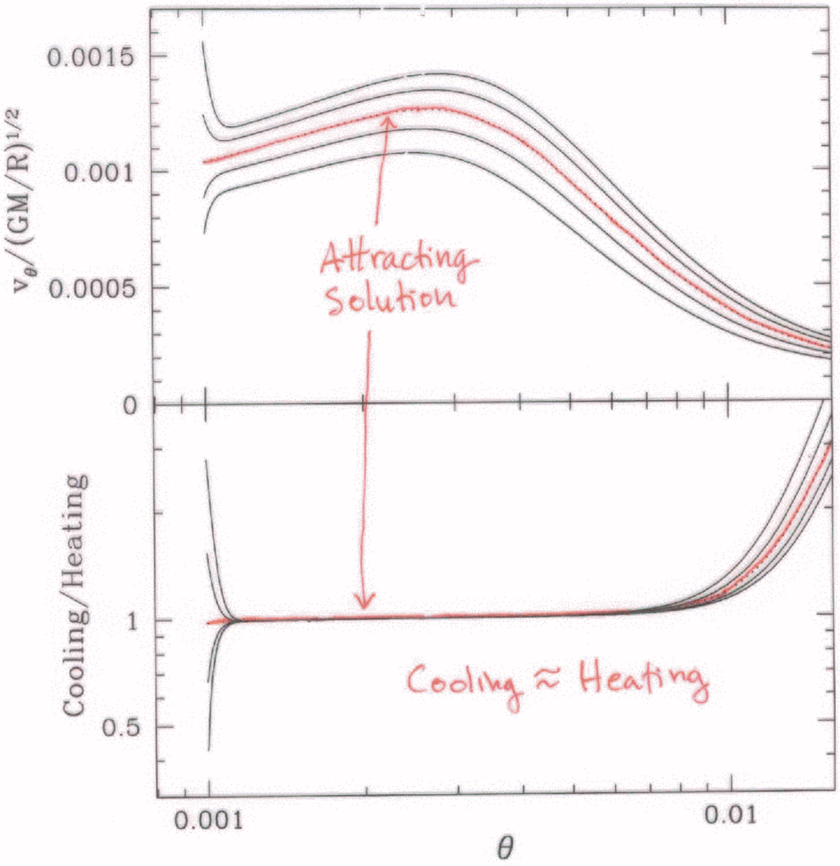
$R_9 = R/10^9\,cm$

use $\theta_0 \approx 10^{-3}$.

$$T_{disk} = 3.6\times10^5\,K\,\alpha_{disk}^{-1/5}\,\dot{M}_{18}^{3/10}\,M_1^{1/4}\,R_9^{-3/4}$$

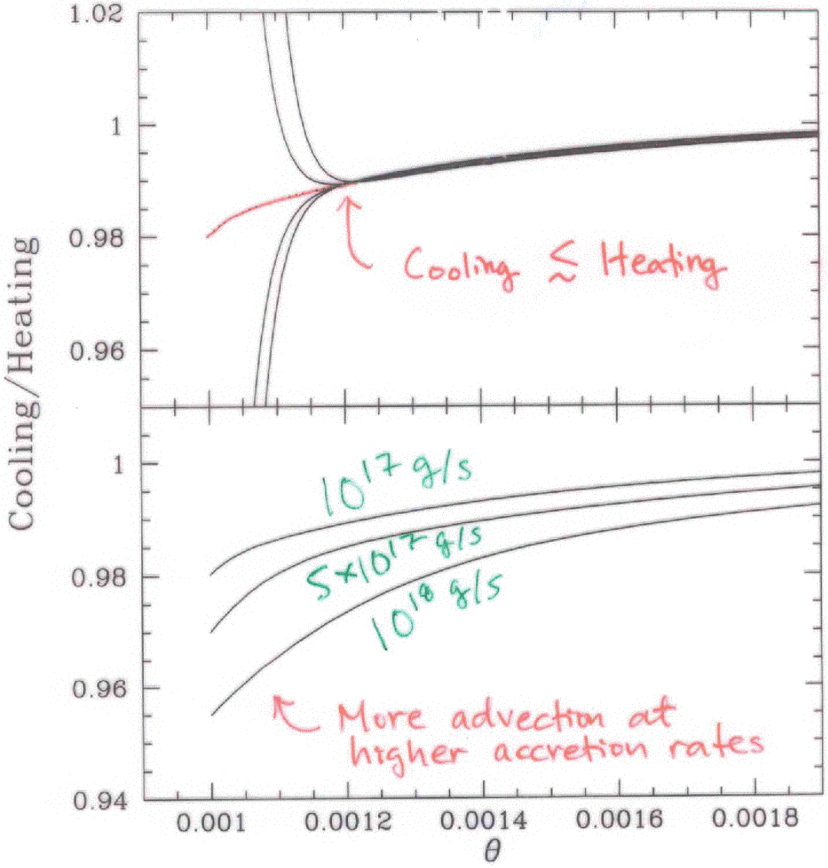
$$v_{\varphi,0} = 0.99\,(GM/R)^{1/2}$$

What about $v_{\theta,0}$?



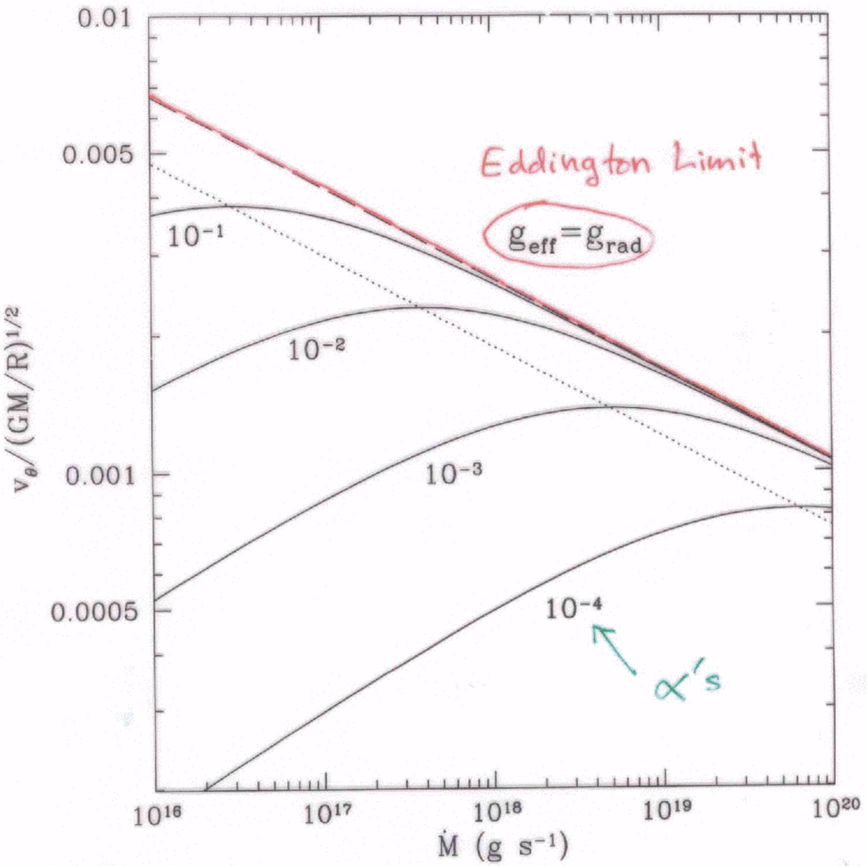
$M = 0.6 M_\odot$, $\alpha = 10^{-3}$, $\alpha_{disk} = 0.1$
$\dot{M} = 10^{17}$ g/s

Advection in the Spreading Layer



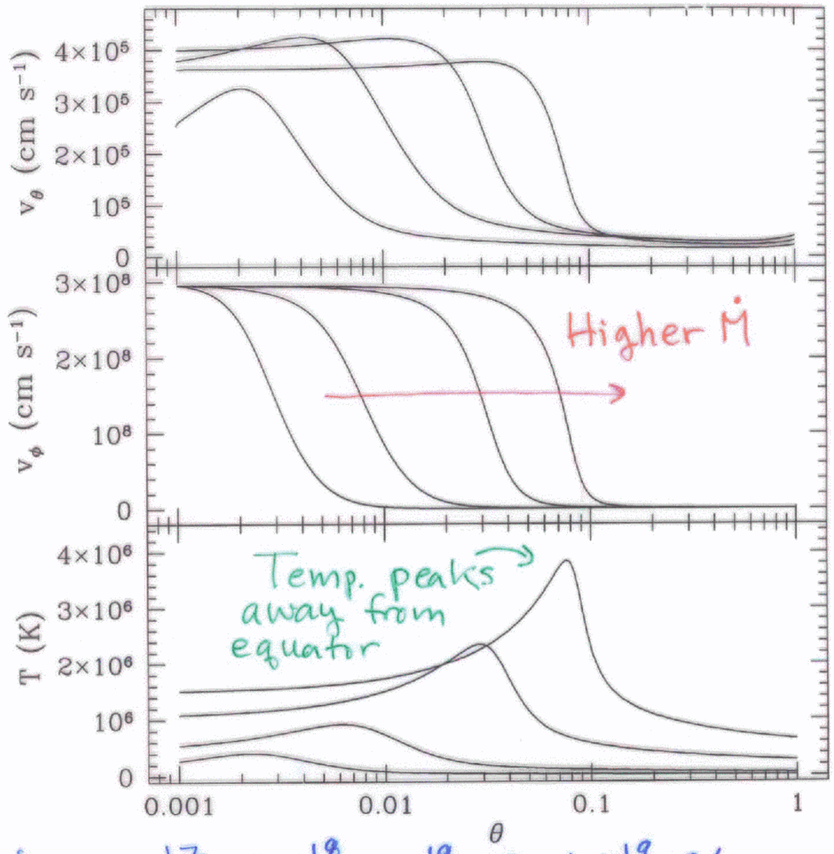
$M = 0.6 M_\odot$, $\alpha = 10^{-3}$, $\alpha_{disk} = 0.1$

Setting the $v_{\theta,0}$ Boundary



WD Spreading Layer typically far from Eddington Limit

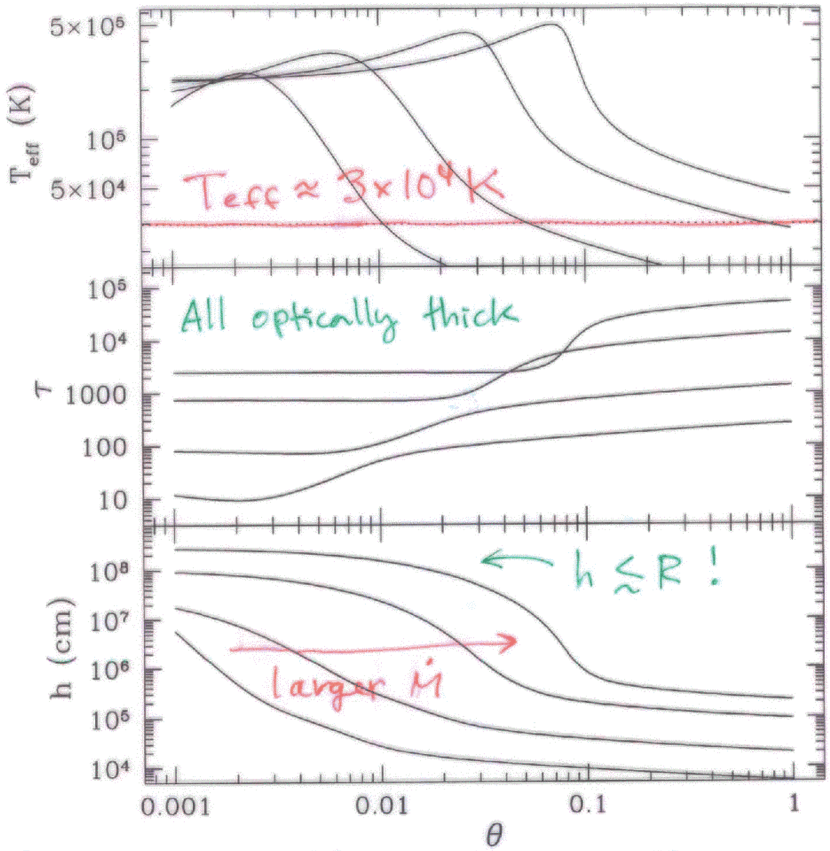
Spreading Layer Properties



$\dot{M} = 10^{17}, 10^{18}, 10^{19}, 3\times10^{19}$ g/s

$M = 0.6 M_\odot$, $\alpha=10^{-3}$, $\alpha_{disk} = 0.1$

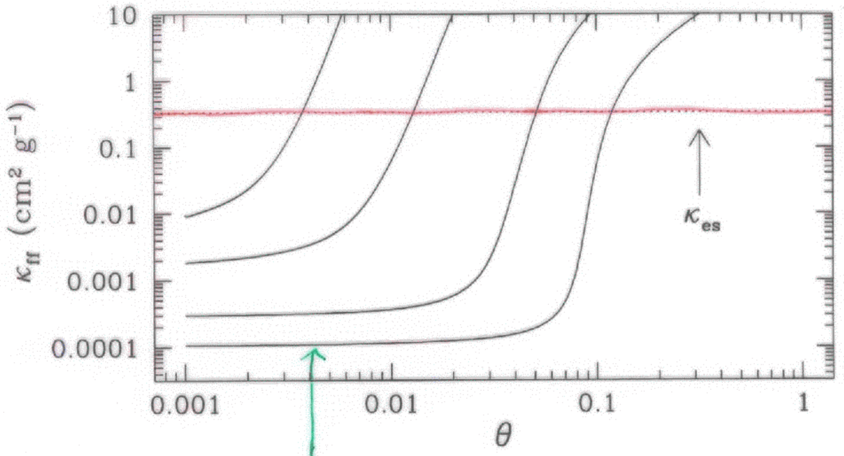
More Properties



$\dot{M} = 10^{17}, 10^{18}, 10^{19}, 3\times10^{19}$ g/s

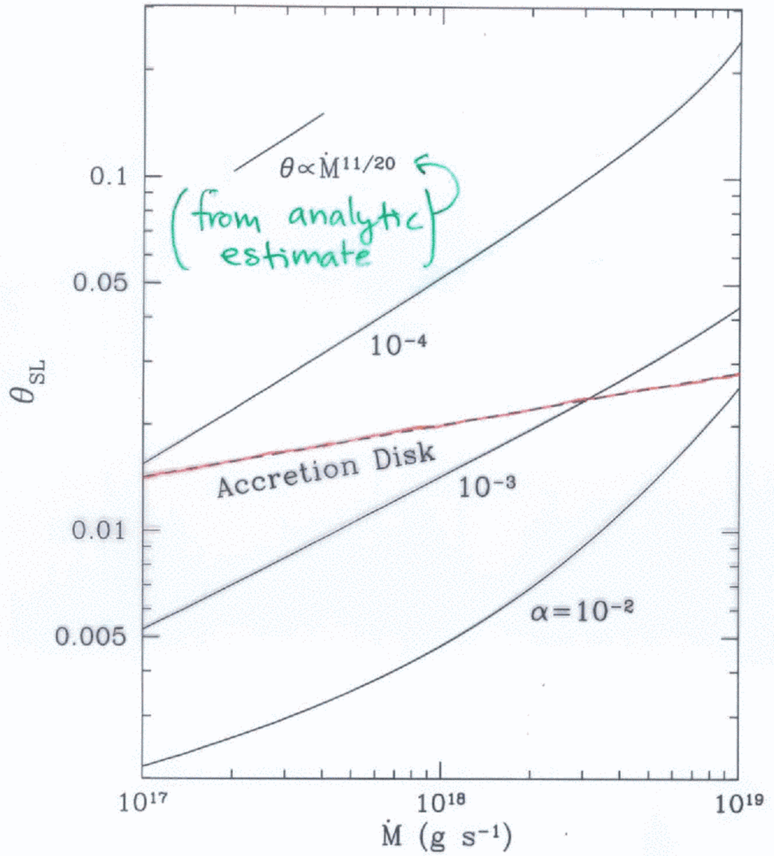
$M = 0.6 M_\odot$, $\alpha = 10^{-3}$, $\alpha_{disk} = 0.1$

Opacity Check



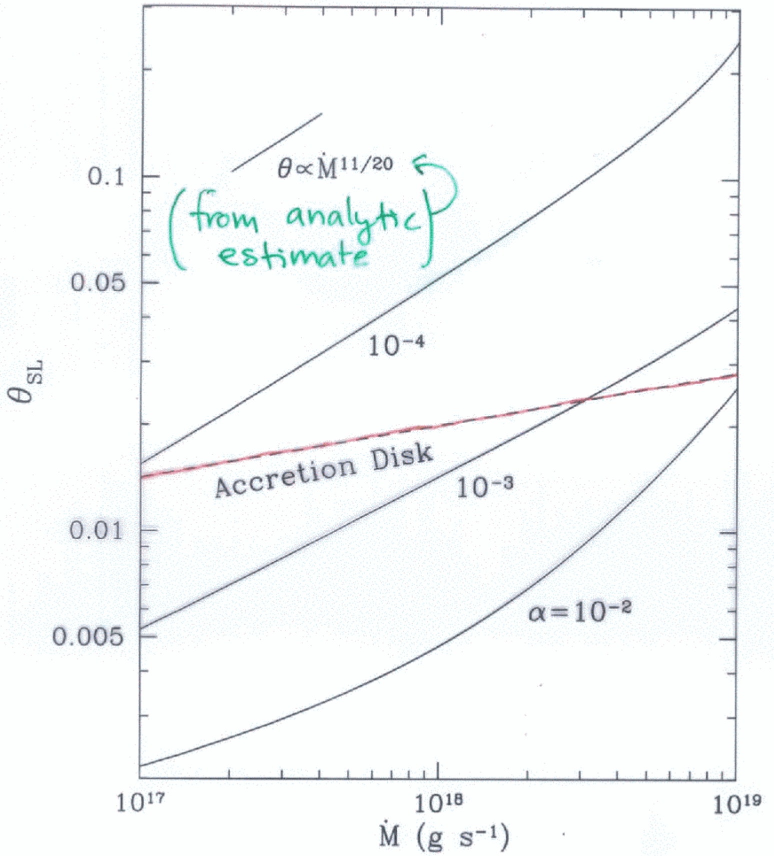
free-free opacity too low in area with most flux

Spreading Angle



Need high $\dot{M}$'s to see SL!

Spreading Angle



Need high $\dot{M}$'s to see SL!

17

Summary of SL Results

- Need high $\dot{M}$

$$\dot{M} \gtrsim 3.7\times10^{17}\frac{g}{s}\,\alpha_{disk}^{3/2}\,\alpha_3^{5/4}\,M_1^{15/8}\,R_9^{5/8}$$

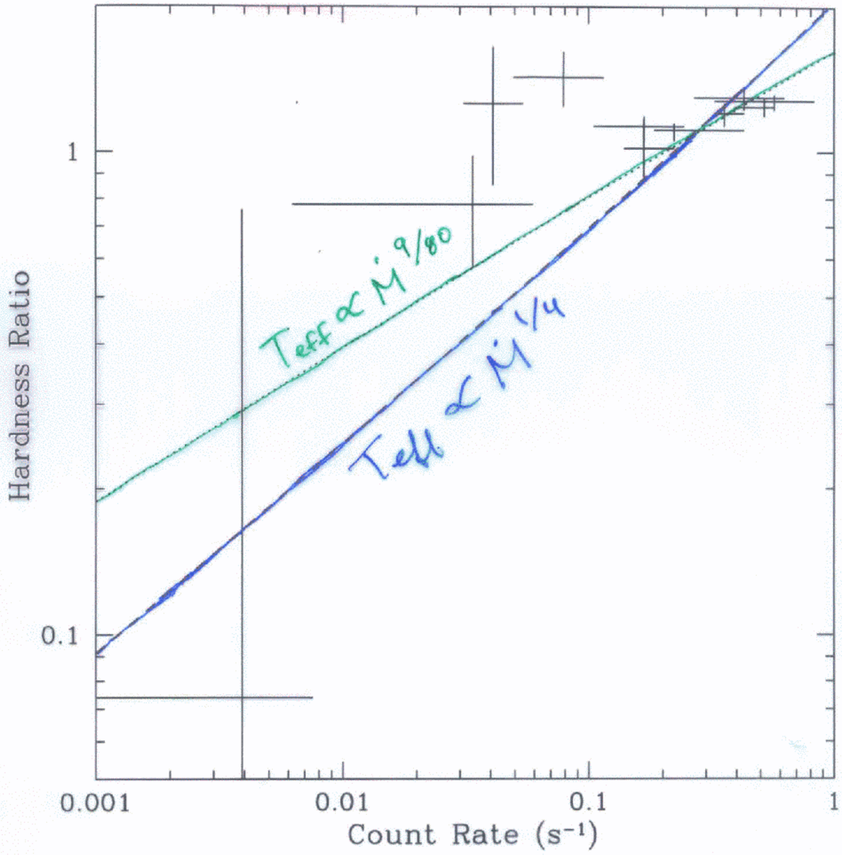
$$\alpha_3 \equiv \alpha/10^{-3}$$

- Possible observations
  - Symbiotic Binaries ($10^{17}-10^{20}$ g/s)
  - Supersoft sources ($10^{18}-10^{20}$ g/s)
  - Dwarf Nova Outbursts ($10^{16}-10^{18}$ g/s)
- Shallow $T_{eff}-\dot{M}$ scaling

$$T_{eff,SL}=2\times10^5\,K\,\alpha_{disk}^{-3/40}\,\alpha_3^{1/8}\,\dot{M}_{18}^{9/80}\,M_1^{13/32}\,R_9^{-23/32}$$

$$\boxed{T_{eff}\propto\dot{M}^{9/80}}$$

SS Cyg from Wheatley, Mauche & Mattei ('03)



$$\text{Hardness ratio} = \frac{72-95\,\text{Å}}{95-130\,\text{Å}}$$

Dwarf Nova Oscillations (DNOs)

Important Properties:

① Typical periods of $P \approx 5\text{-}40$ sec

② Only seen at high $\dot{M}$
(typically in dwarf nova outbursts)

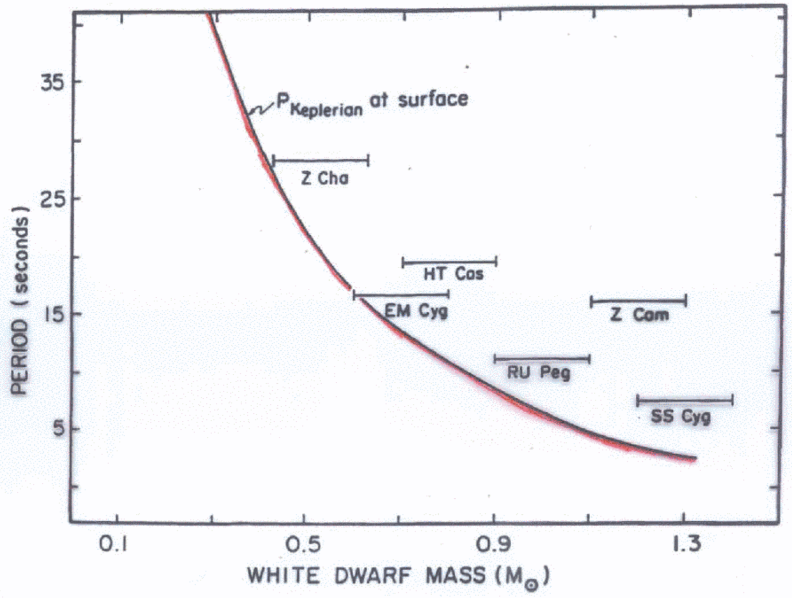
③ High $Q \equiv |dP/dt|^{-1} \approx 10^4 - 10^6$
and very sinusoidal
(but magnetic WDs have $Q \approx 10^{12}$)

④ Low amplitude ($\lesssim 0.5\%$) and
small radiating area ($\sim 10^{-2}$ of WD)

⑤ Monotonic scaling with accretion

$$P \propto \dot{M}^{-\beta} \quad \beta \approx 0.1 - 0.2$$

Patterson (1981)



DNOs know about WD surface gravity!

Nonradial Oscillations in SL?

Let's try simplest thing first...

① Start with shallow surface wave g-mode

$$\omega = (g h k^2)^{1/2}$$

$$k^2 = \ell(\ell+1)/R^2$$

② Calculate rotational modifications using Laplace's Tidal Equation

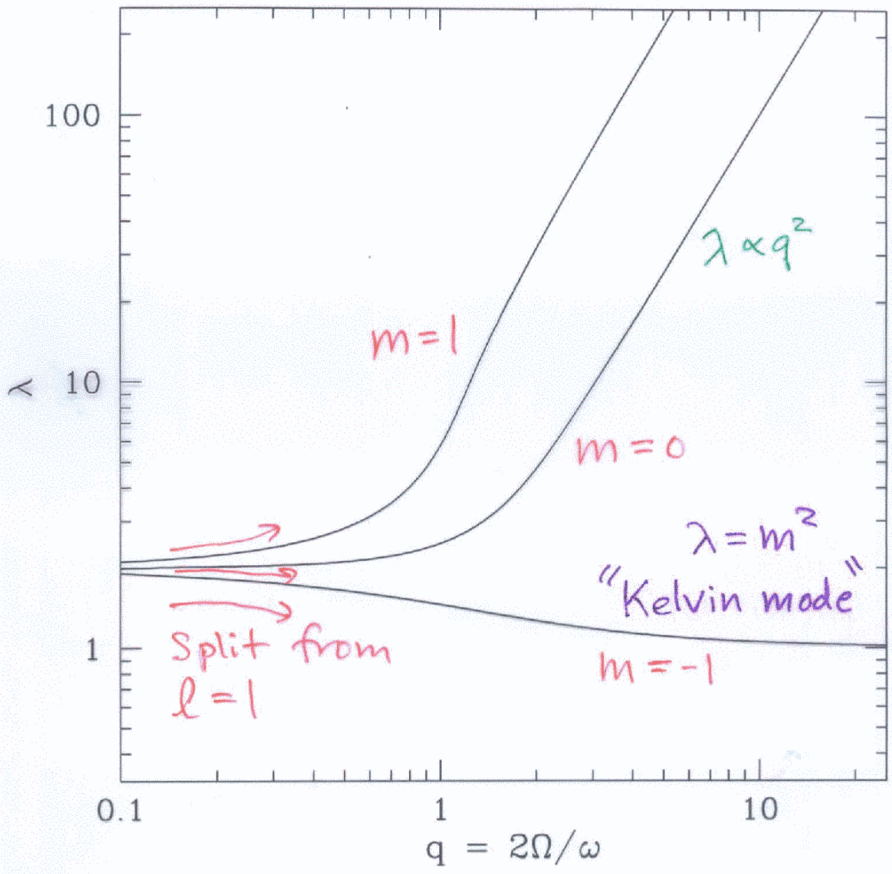
$$\ell(\ell+1) = 2 \Rightarrow \lambda \text{ with } \begin{matrix} m=1 \\ m=0 \\ m=-1 \end{matrix}$$

Bildsten, Ushomirsky, & Cutler ('96)
Piro & Bildsten (2004)

③ Calculate observed frequency

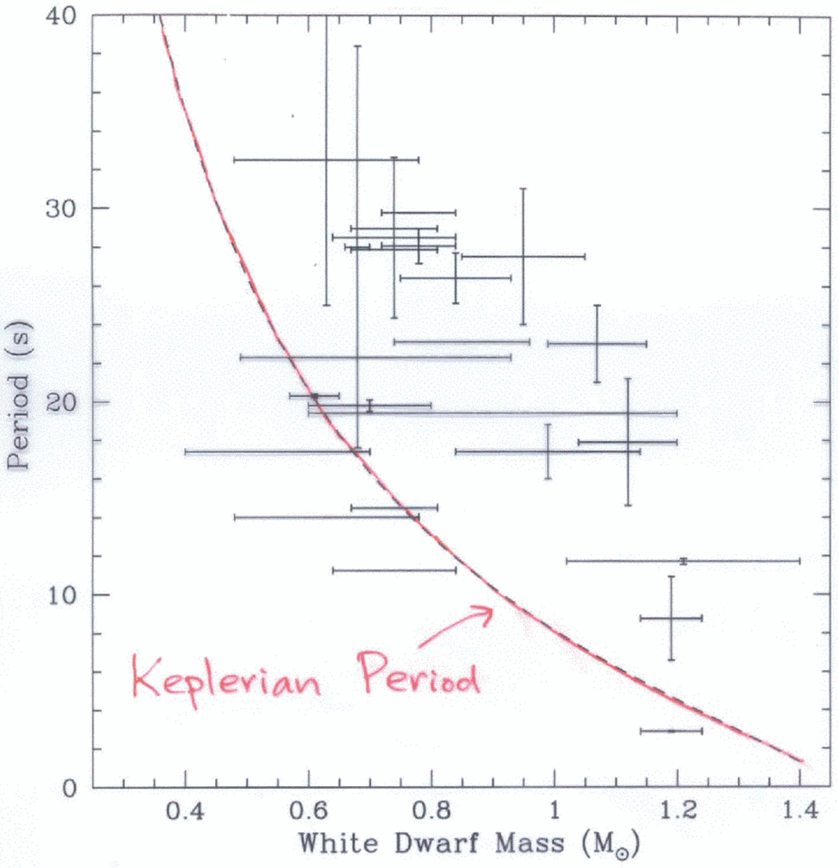
$$\omega_{obs} = \left| \omega \left[\frac{\lambda}{\ell(\ell+1)}\right]^{1/2} - m\Omega \right|$$

Rotationally Modified Wavenumber



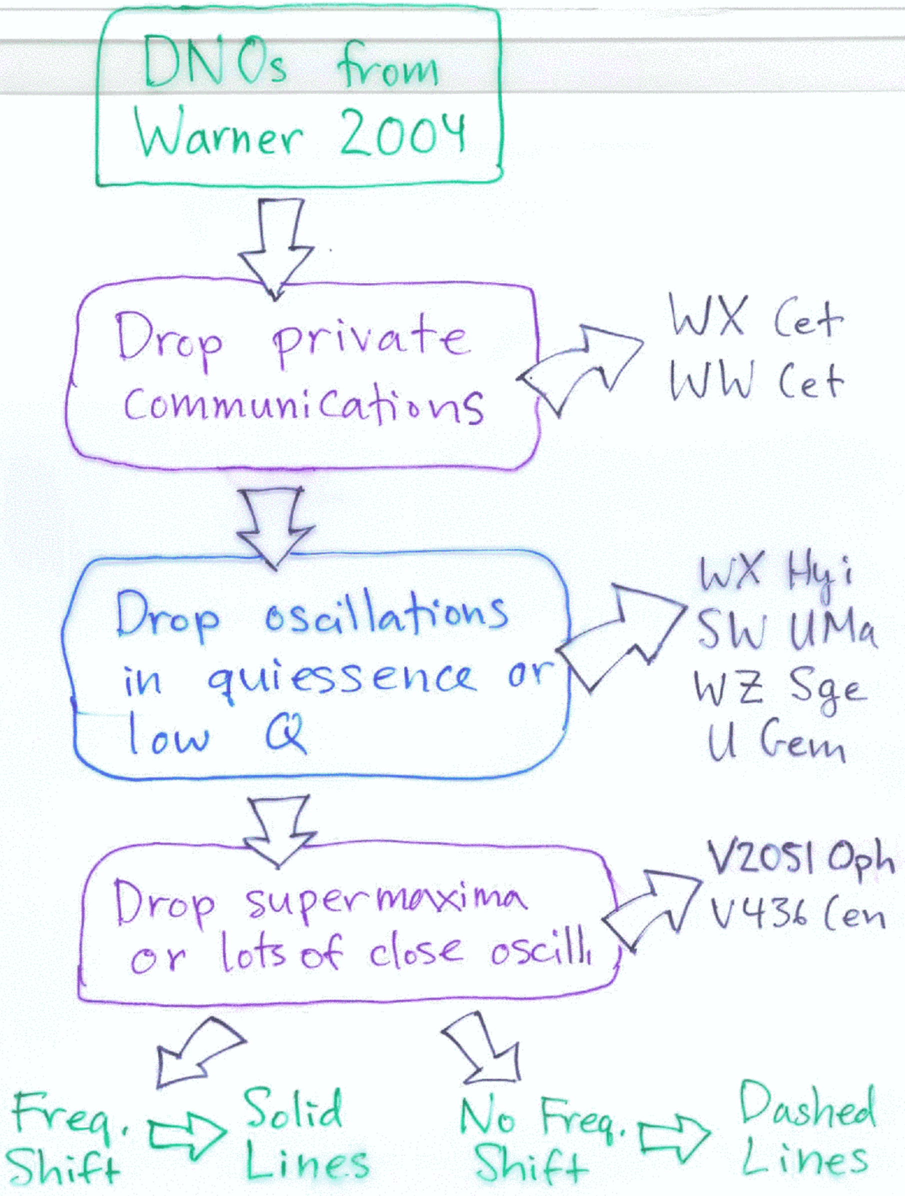
→ Faster Spin

Update of Patterson ('81)



Keplerian Period

Periods from Warner (2004)
Masses from Ritter & Kolb ('03)



DNOs from Warner 2004

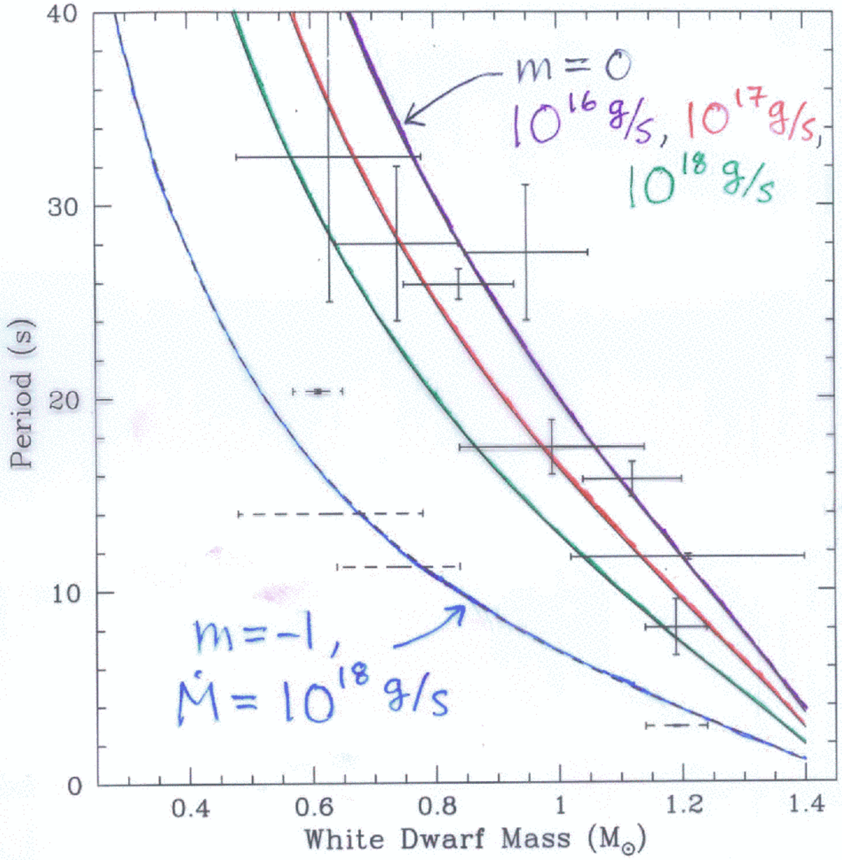
Drop private communications → WX Cet, WW Cet

Drop oscillations in quiessence or low Q → WX Hyi, SW UMa, WZ Sge, U Gem

Drop supermaxima or lots of close oscill. → V2051 Oph, V436 Cen

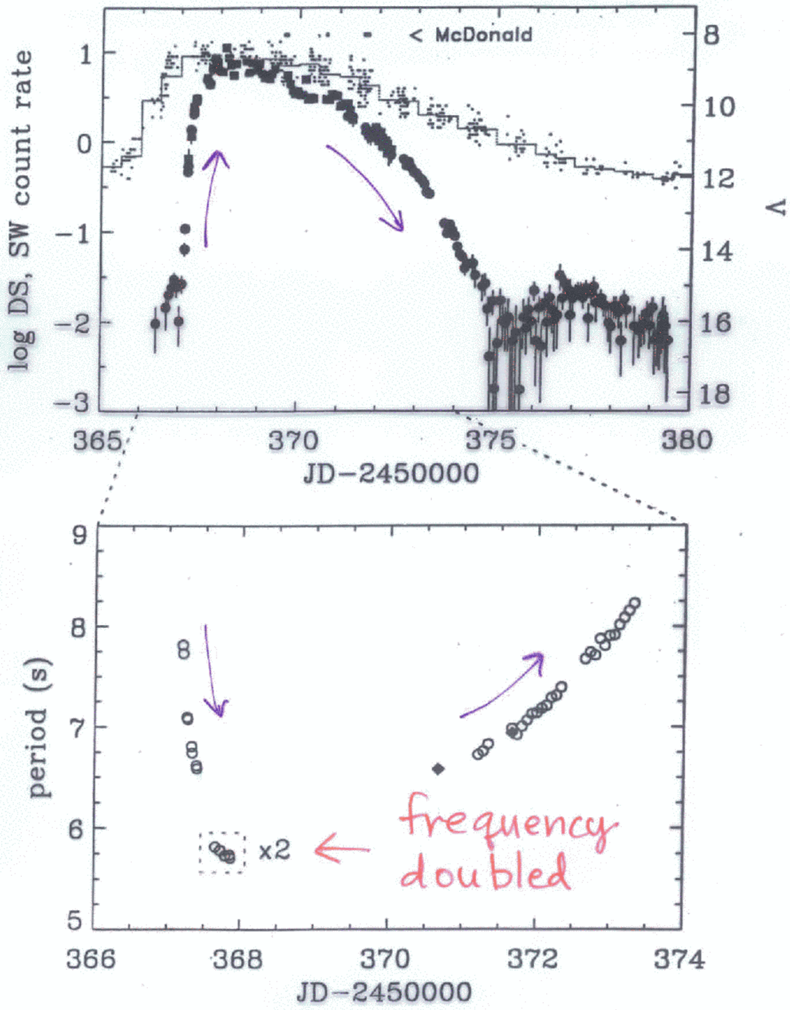
Freq. Shift → Solid Lines

No Freq. Shift → Dashed Lines

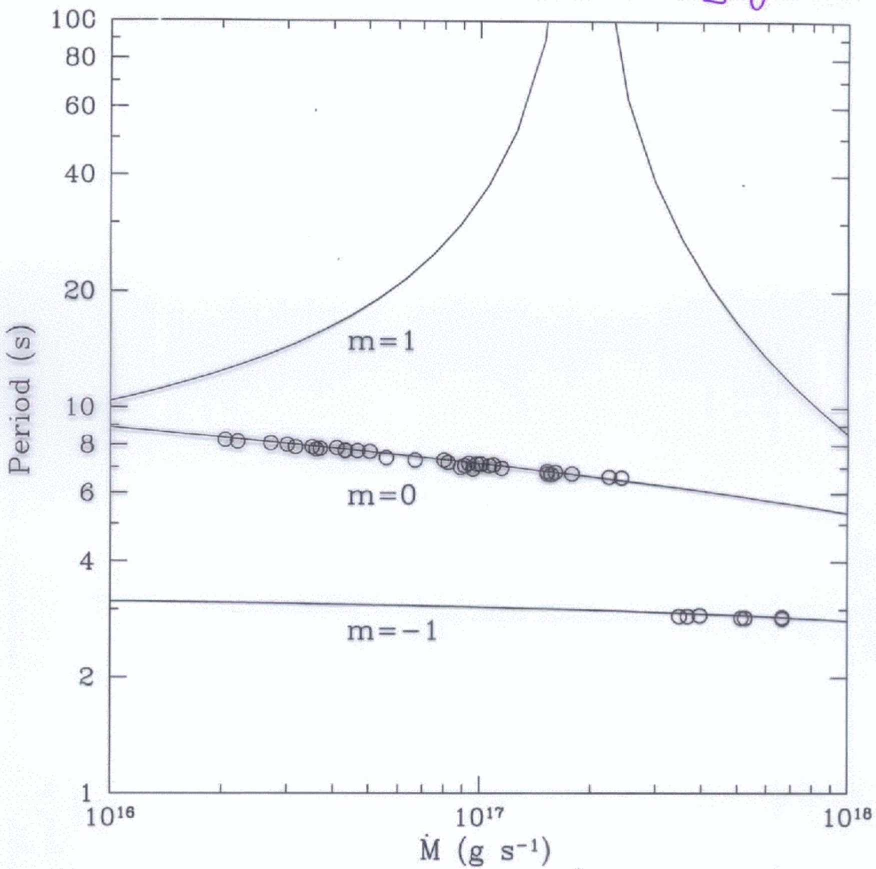


All using $\alpha = 10^{-4}$, $\alpha_{disk} = 0.1$

SS Cyg Outburst from Manche & Robinson (2001)



DNO from SS Cyg



Using $M = 1.25\,M_\odot$ & $\alpha = 10^{-4}$

Conclusions

- Important SL properties
  - High $\dot{M}$ systems
  - Radiating area changes with $\dot{M}$
  - First calculation of differential rotation profile
- Observations
  - Shallow $T_{eff} \propto \dot{M}^{9/80}$
  - Possible explanation for DNOs

Future Work

- Correctly calculate nonradial oscillations with differential rotation (maybe neutron stars?)
- Look at Symbiotic binaries / Supersoft Sources