Burst Oscillations, Nonradial Modes, and Neutron Star Crusts

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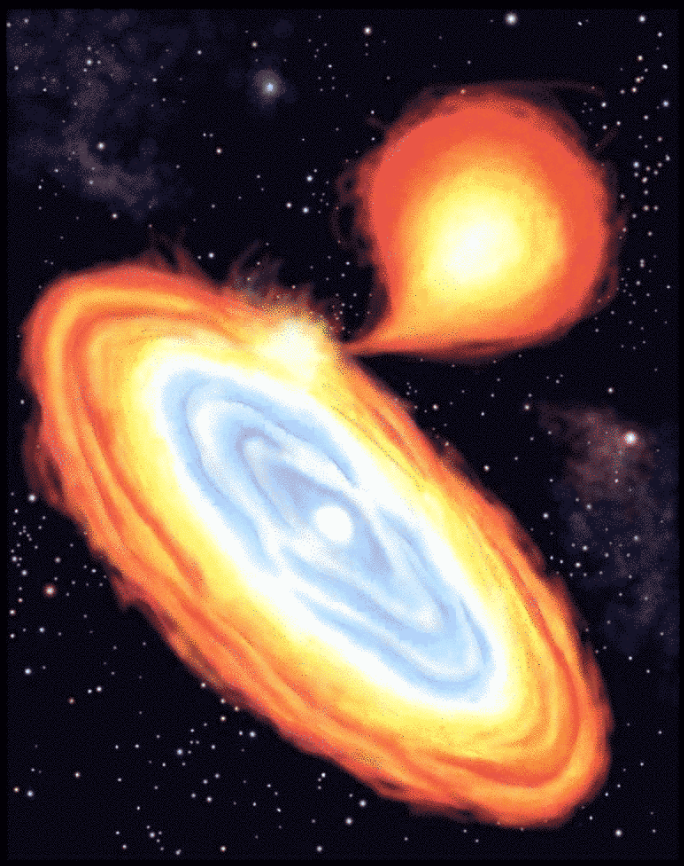


Talk Outline

Introduction

- Review of accreting neutron stars, X-ray bursts, and burst oscillations
- Comparisons between magnetic and non-magnetic neutron stars

Theory of Surface Modes on Neutron Stars

- Review of shallow gravity waves
- Bursting neutron star surfaces and the resulting modes
- Predictions, comparisons with observations, and constraints on the properties of neutron star crusts

Conclusion

- Lingering mysteries, future work, and other comparisons with observations

Low Mass X-ray Binaries (LMXBs)

A "typical" neutron star is 1.4 solar masses and 10 km. Orbital periods of these binaries are a few hours to half a day.

The neutron star accretes from its donor star at

$$\dot{M} \sim 10^{-10} - 10^{-8} M_\odot \text{ yr}^{-1}$$

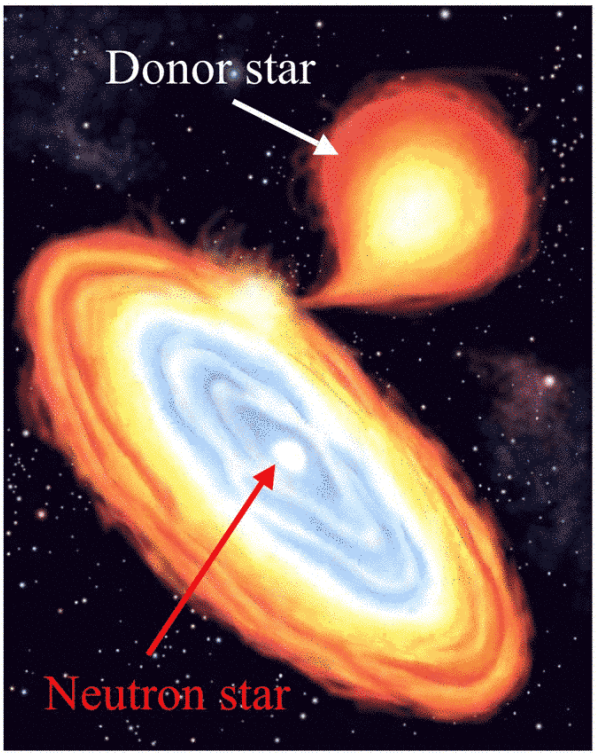
$$L \approx \frac{GM\dot{M}}{R} \approx 10^{36} - 10^{38} \text{ erg s}^{-1}$$

$$\frac{GMm_p}{R} \approx 200 \text{ MeV nucleon}^{-1}$$

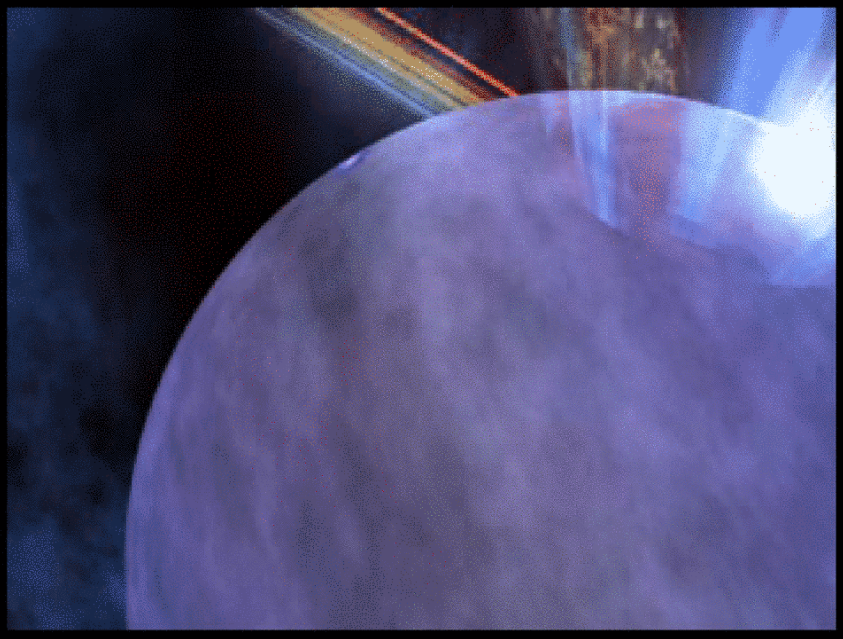
Nuclear burning on its surface releases

$$\approx 5 \text{ MeV nucleon}^{-1}$$

This can never be seen…right?



Yes…when the nuclear energy is released all at once!



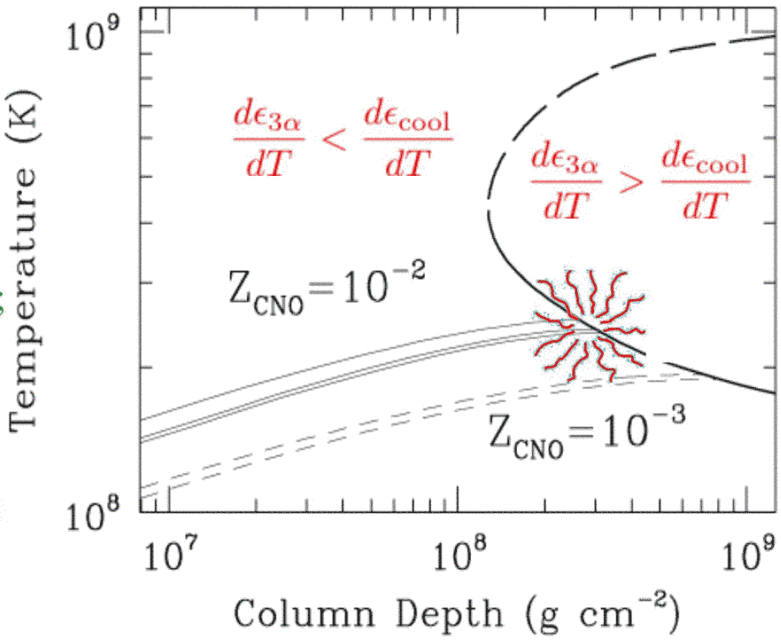
Type I X-ray Bursts

Nuclear fuel is stored up and then burns rapidly!

- Unstable Helium ignition (triple-alpha) as predicted by Hansen & van Horn '75
- Observed by Belian et al '76; Grindlay et al '76 and identified by Woosley & Taam '76; Maraschi & Cavaliere '77; Joss '77, '78; Lamb and Lamb '78
- Bursts repeat every few hours to days (timescale to accrete an unstable column)
- Energy release:

$$E_{\rm burst} \approx 5 \times 10^{39} \text{ ergs}$$

Bildsten 1998



X-ray Bursts and Surface Composition

- Burst length and α-value indicate composition of bursting fuel

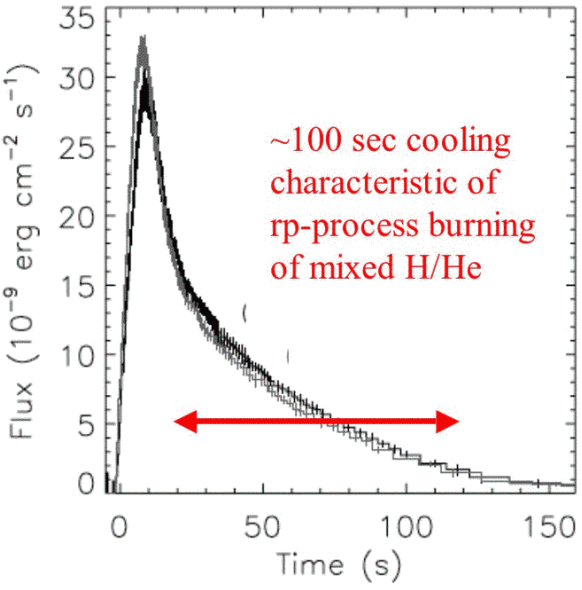
$$\alpha \equiv \frac{\langle L_{\rm acc} \rangle}{\langle L_{\rm burst} \rangle}$$

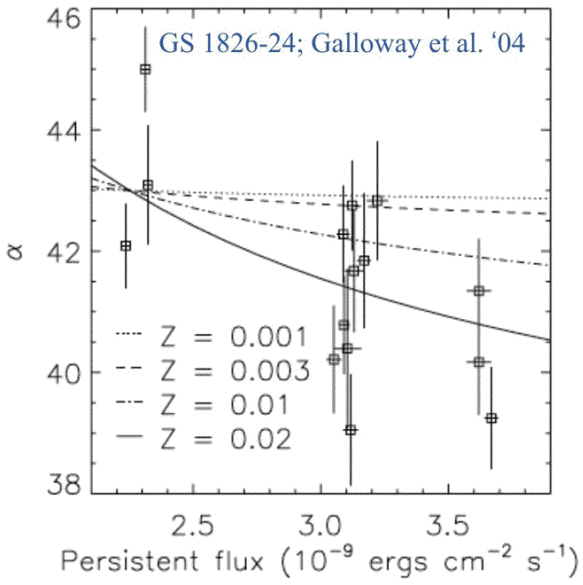
Mixed H/He burns to A~100 heavy ashes

$$\alpha \approx \frac{200 \text{ MeV}}{5 \text{ MeV}} \approx 40$$

Pure He burns to α-elements like ^{28}Si, ^{40}Ca, ^{64}Zn, etc.

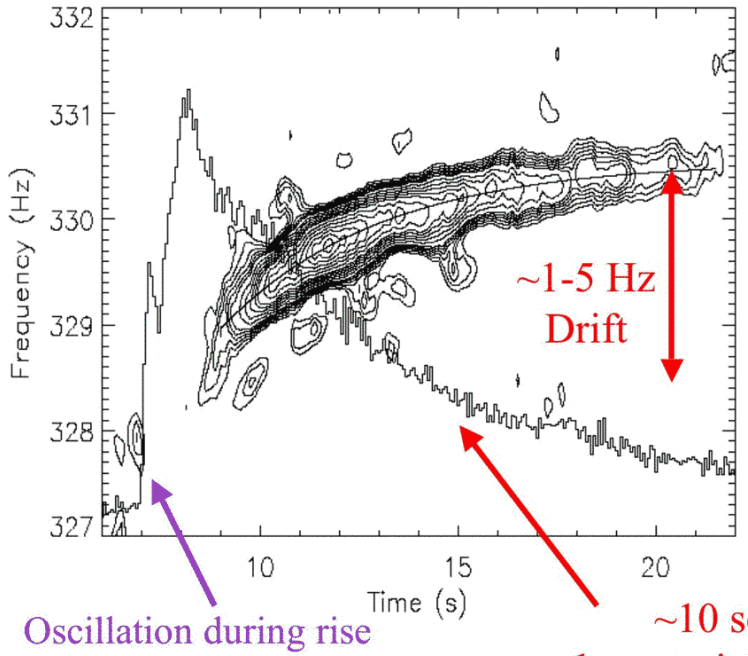
$$\alpha \approx \frac{200 \text{ MeV}}{1.6 \text{ MeV}} \approx 100$$





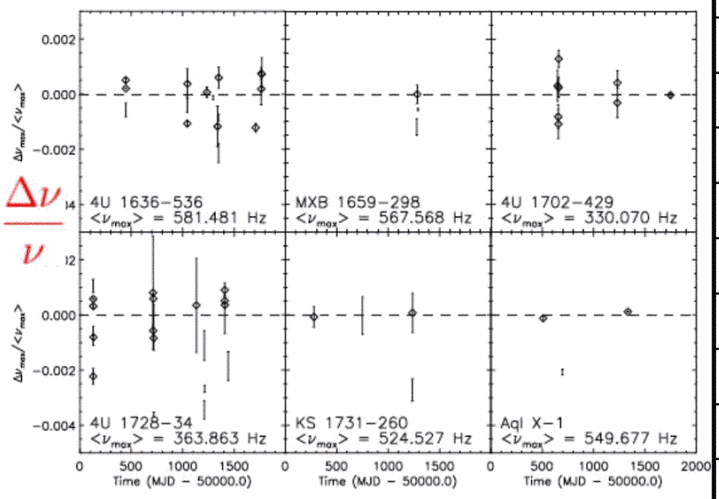
Burst Oscillations from LMXBs

4U 1702-429; Strohmayer & Markwardt '99



- Frequency and amplitude during rise are consistent with a hot spot spreading on a rotating star (Strohmayer et al. '97)
- Angular momentum conservation of surface layers (Strohmayer et al. '97) underpredicts late time drift (Cumming et al. '02)

The asymptotic frequency is characteristic to each object

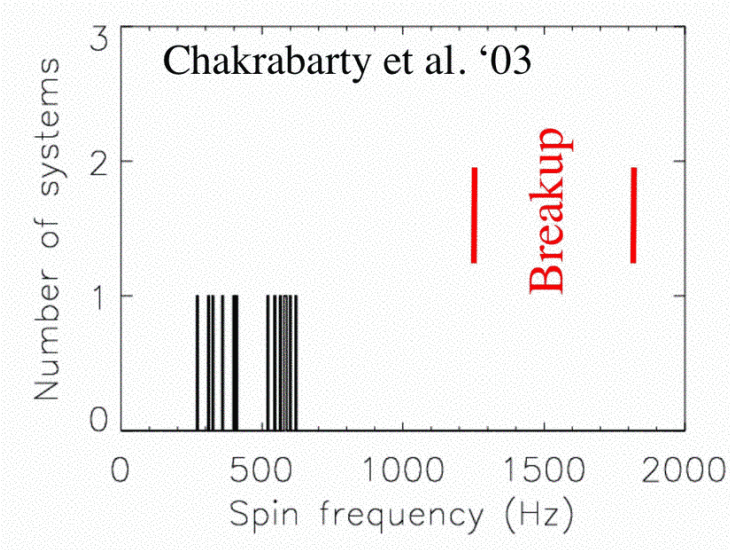


- Frequency stable over many observations (within 1 part in 1000; Muno et al. '02)

It must be the spin…right?

Source	Asymptotic Freq. (Hz)
4U 1608-522	620
SAX J1750-2900	600
MXB 1743-29	589
4U 1636-536	581
MXB 1659-298	567
Aql X-1	549
KS 1731-260	524
SAX J1748.9-2901	410
SAX J1808.4-3658	401
4U 1728-34	363
4U 1702-429	329
XTE J1814-338	314
4U 1926-053	270
EXO 0748-676	45

Neutron Stars Speed Limit?

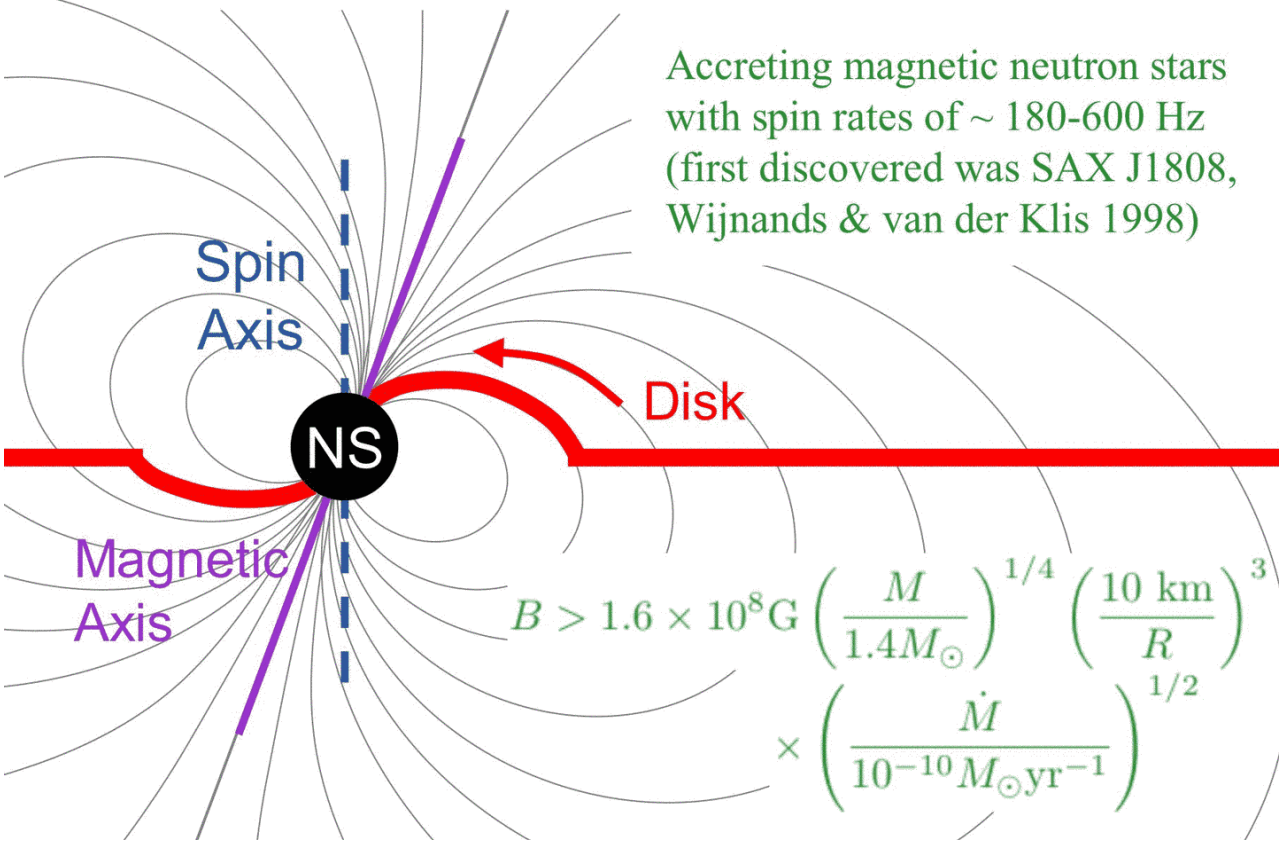


- Enough time and accretion to spin the neutron star up to the breakup ~ 1200 Hz
- No observational bias against fast or slow rotators so small range is meaningful
- Chakrabarty et al. '03 found highest frequency < 730 Hz at 95% confidence!

Probably interesting physics behind explanation. Magnetic spin equilibrium (Ghosh & Lamb '79)? Accretion torques balanced by gravitational waves (Bildsten '98, Wagoner '84, Andersson '98)?

Accreting Millisecond Pulsars

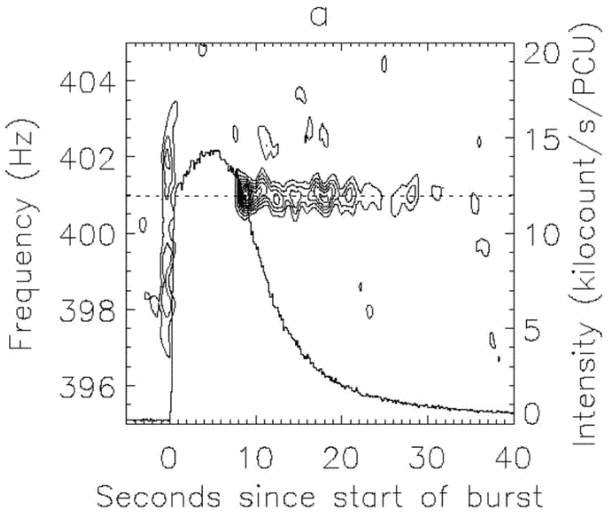
Accreting magnetic neutron stars with spin rates of ~ 180-600 Hz (first discovered was SAX J1808, Wijnands & van der Klis 1998)



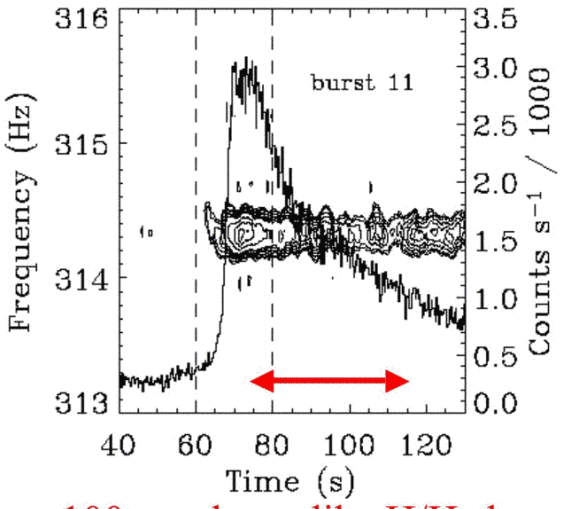
$$B > 1.6 \times 10^8 \mathrm{G} \left(\frac{M}{1.4 M_\odot}\right)^{1/4} \left(\frac{10\ \mathrm{km}}{R}\right)^3 \times \left(\frac{\dot{M}}{10^{-10} M_\odot \mathrm{yr}^{-1}}\right)^{1/2}$$

Burst Oscillations from Pulsars

SAX J1808.4-3658; Chakrabarty et al. '03



XTE J1814-338; Strohmayer et al. '03



~ 100 sec decay like H/He burst!

- Burst oscillation frequency = spin!
- No frequency drift, likely due to large B-field (Cumming et al. 2001)

What Creates Burst Oscillations in Non-pulsar Neutron Stars?

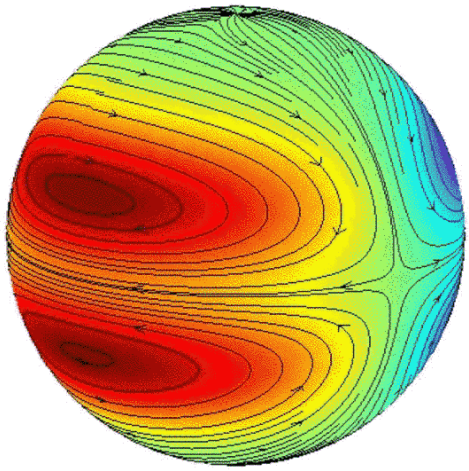
Important differences:

- Non-pulsars only show oscillations in short (~ 2-10 s) bursts, while pulsars have shown oscillations in longer bursts (~ 100 s)
- Non-pulsars show frequency drifts often late into cooling tail, while pulsars show no frequency evolution after burst peak
- Non-pulsars have highly sinusoidal oscillations (Muno et al. '02), while pulsars show harmonic content (Strohmayer et al. '03)
- The pulsed amplitude as a function of energy different between the two types of objects (Muno et al. '03; Watts & Strohmayer '04)

These differences support the hypothesis that a different mechanism may be acting in the case of the non-pulsars.

Perhaps Nonradial Oscillations?

Initially calculated by McDemott & Taam (1987), BEFORE burst oscillations were discovered. Hypothesized by Heyl (2004).



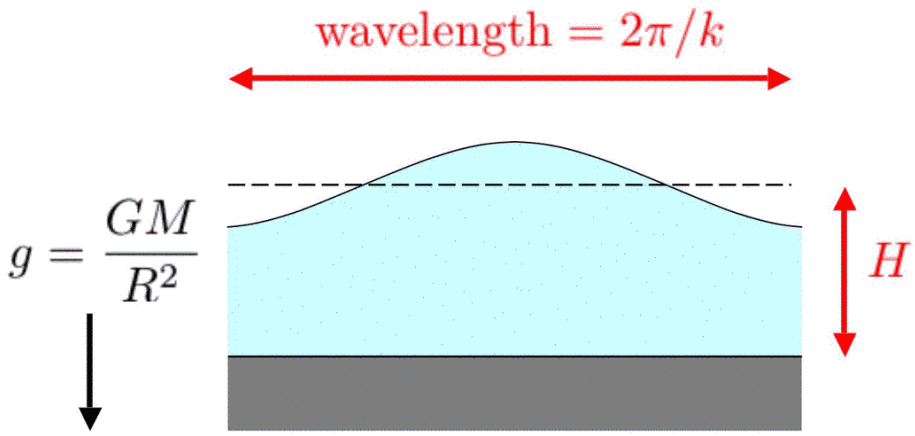
Graphic courtesy of G. Ushomirsky

- It's the most obvious way to create a late time surface asymmetry in a liquid.
- It is supported by the HIGHLY sinusoidal nature of oscillations
- The angular and radial eigenfunctions are severely restricted by the main characteristics of burst oscillations.
- Heyl (2004) identified that the angular structure must be an $m = 1$ buoyant r-mode (we'll come back to this later)

Let's first consider radial part…

Surface Gravity Waves

Consider a liquid layer with depth H above a rigid floor



The frequency of the oscillation is given by the dispersion relation:

$$\omega^2 = gk \tanh kH$$

Deep and Shallow Limits

The general dispersion relation is $\omega^2 = gk \tanh kH$ but it has important deep and shallow limits.

Deep layer, $kH \gg 1, \tanh kH \to 1$

$$\omega^2 = gk \qquad v_p = \frac{\omega}{k} = \sqrt{\frac{g}{k}} \qquad v_g = \frac{d\omega}{dk} = \frac{1}{2}\sqrt{\frac{g}{k}}$$

Dispersive!

Shallow layer, $kH \ll 1, \tanh kH \to kH$

$$\boxed{\omega^2 = gHk^2} \qquad v_p = v_g = \sqrt{gH}$$

Important formula!

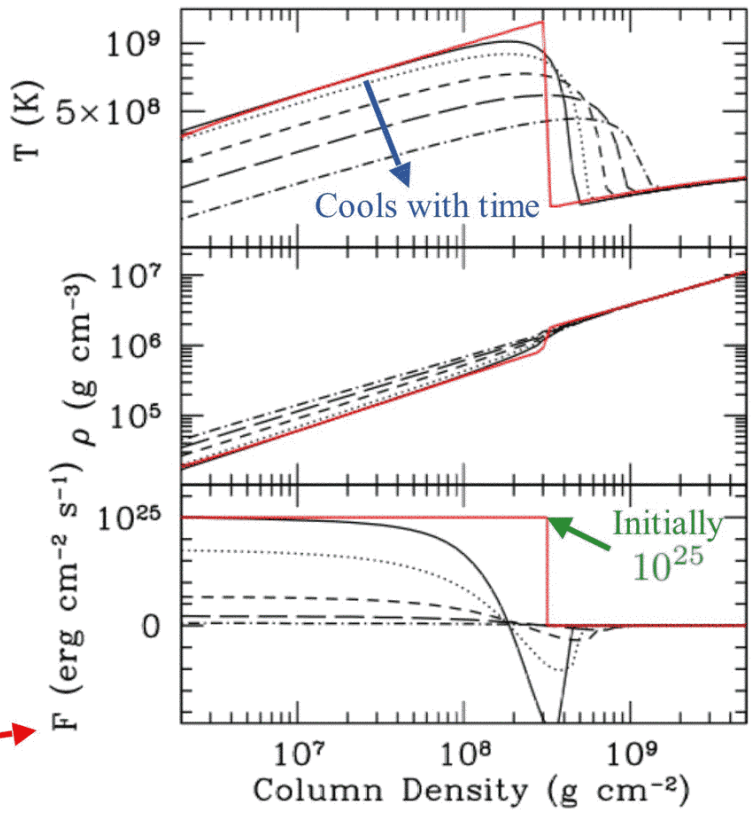
No dispersion

Just like a Tsunami!

Cooling Neutron Star Surface

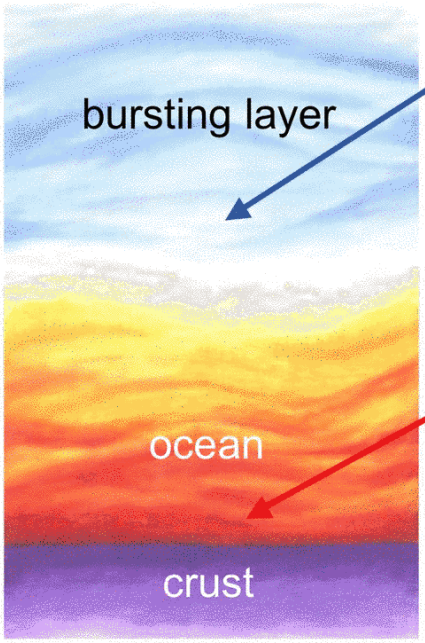
- We construct a simple cooling model of the surface layers
- The composition is set from the He-rich bursts of Woosley et al. '04
- Profile is evolved forward in time using finite differencing (Cumming & Macbeth '04)

Time steps of 0.1, 0.3, 1, 3, & 10 seconds



Modes On Neutron Star Surface

Depth	Density
< 1 m	10^4 g cm^{-3}
$H_b \approx 2$ m	10^6 g cm^{-3}
$H_c \approx 20$ m	10^9 g cm^{-3}



Shallow surface wave

$$\omega_s^2 = g H_b k^2 \frac{\Delta\rho}{\rho}$$

$$k^2 = \frac{\lambda}{R^2}$$

Crustal interface wave
(Piro & Bildsten '05)

$$\omega_c^2 = g H_c k^2 \frac{\mu}{P}$$

$$\frac{\mu}{P} \approx 10^{-2}$$

Shallow Surface Wave

$$\omega_s^2 = g H_b k^2 \frac{\Delta\rho}{\rho}$$

$$H_b = \frac{k_B T_b}{\mu_b m_p g} \qquad k^2 = \frac{\lambda}{R^2} \qquad \frac{\Delta\rho}{\rho} = 1 - \frac{T_c}{T_b}\frac{\mu_b}{\mu_c}$$

If the neutron star is not rotating then $\lambda = l(l+1)$. We instead use $\lambda = 1/9 \approx 0.11$ which we later explain from the effects of rotation.

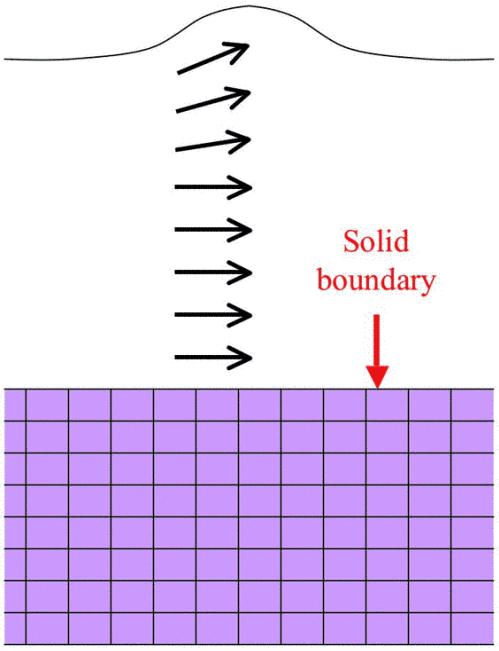
$\omega_s \propto T_b^{1/2}$

$$\frac{\omega_s}{2\pi} = 10.8 \text{ Hz} \left(\frac{2Z_b}{A_b}\frac{T_b}{10^9 \text{ K}}\frac{\lambda}{0.11}\right)^{1/2} \left(\frac{10 \text{ km}}{R}\right) \left(1 - \frac{T_c}{T_b}\frac{\mu_b}{\mu_c}\right)^{1/2}$$

(This mode was studied by McDermott & Taam 1987.)

Crustal Interface Wave

McDermott, Van Horn & Hansen '88; Piro & Bildsten '05

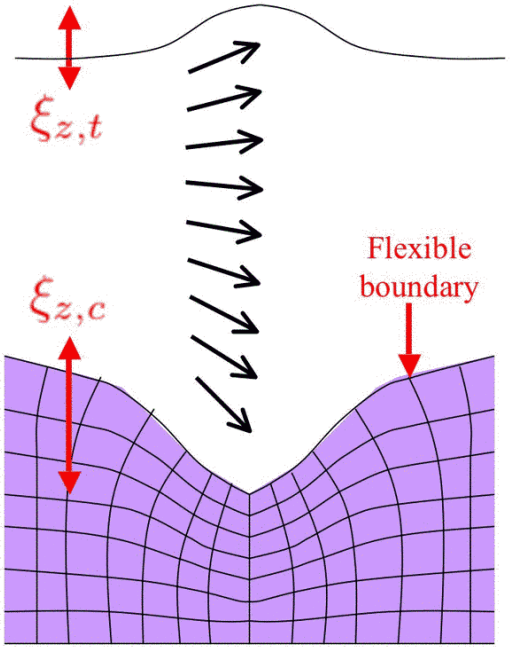


Easy case!…
a solid bottom boundary
results in a frequency:

$$\omega^2 = g H_c k^2$$

Crustal Interface Wave

McDermott, Van Horn & Hansen '88; Piro & Bildsten '05



A non-zero bottom displacement
decreases frequency dramatically!

$$\omega^2 \approx g H_c k^2 \left|\frac{\xi_{z,t}}{\xi_{z,c}}\right| \approx g H_c k^2 \frac{\mu}{P}$$

$$\left|\frac{\xi_{z,t}}{\xi_{z,c}}\right| \approx \frac{\mu}{P} \sim 10^{-2}$$

$$\frac{\omega_c}{2\pi} = 4.3 \text{ Hz} \left(\frac{64}{A_c}\frac{T_{c,8}}{3}\frac{\lambda}{0.11}\right)^{1/2} \times \left(\frac{10 \text{ km}}{R}\right)$$

The First 3 Radial Modes

(using $\lambda = 0.11$)

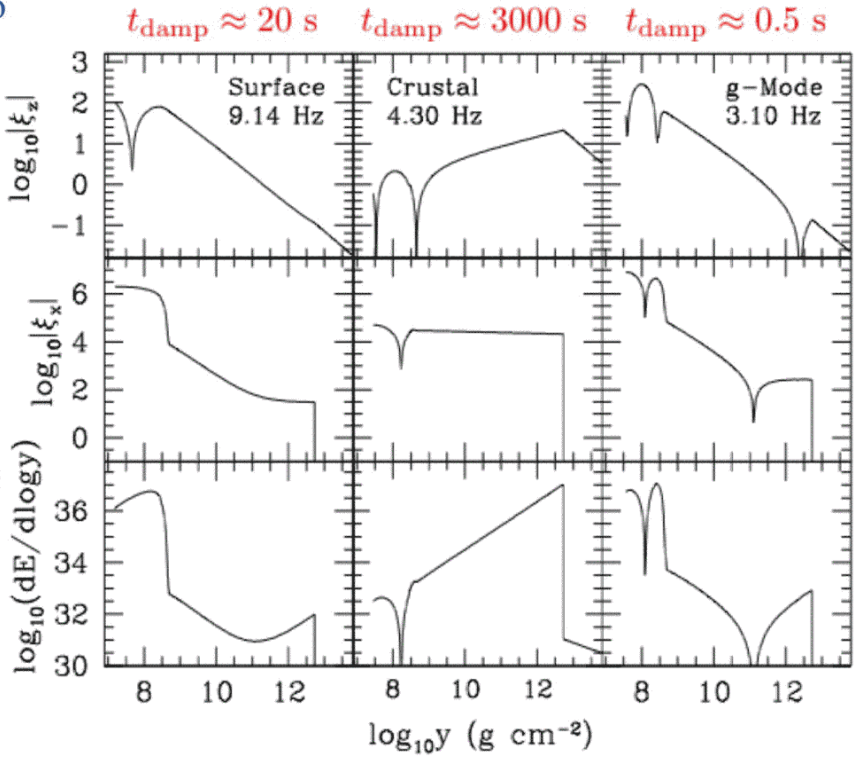
- Mode energy is set to

  5×10^{36} ergs

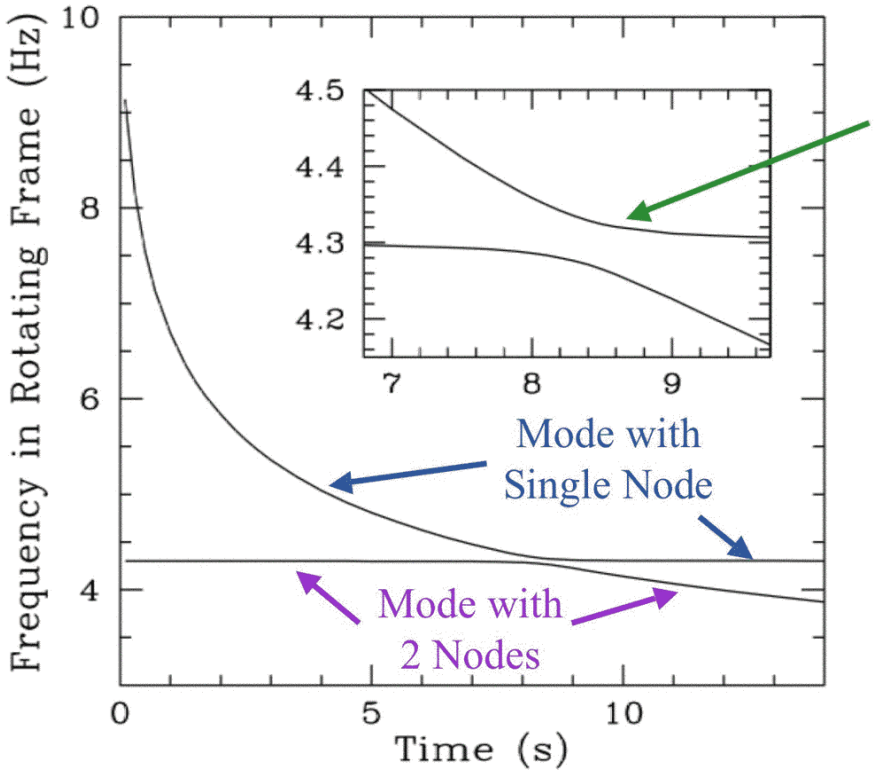
  10^{-3} of the energy in a burst (Bildsten '98)

- Estimate radiative damping time using "work integral" (Unno et al. '89)

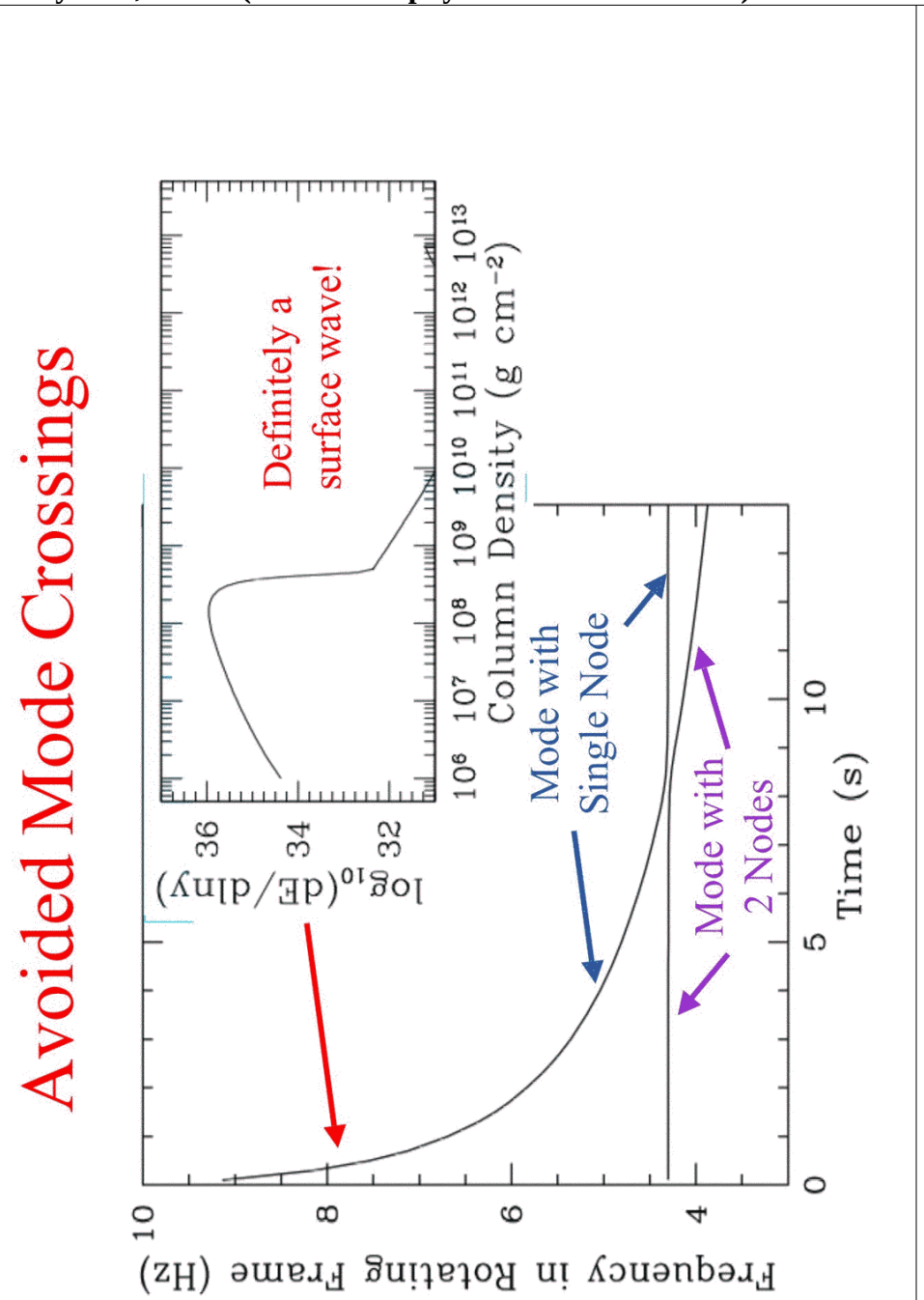
- Surface wave (single node) has best chance of being seen (long damping time + large surface amplitude)

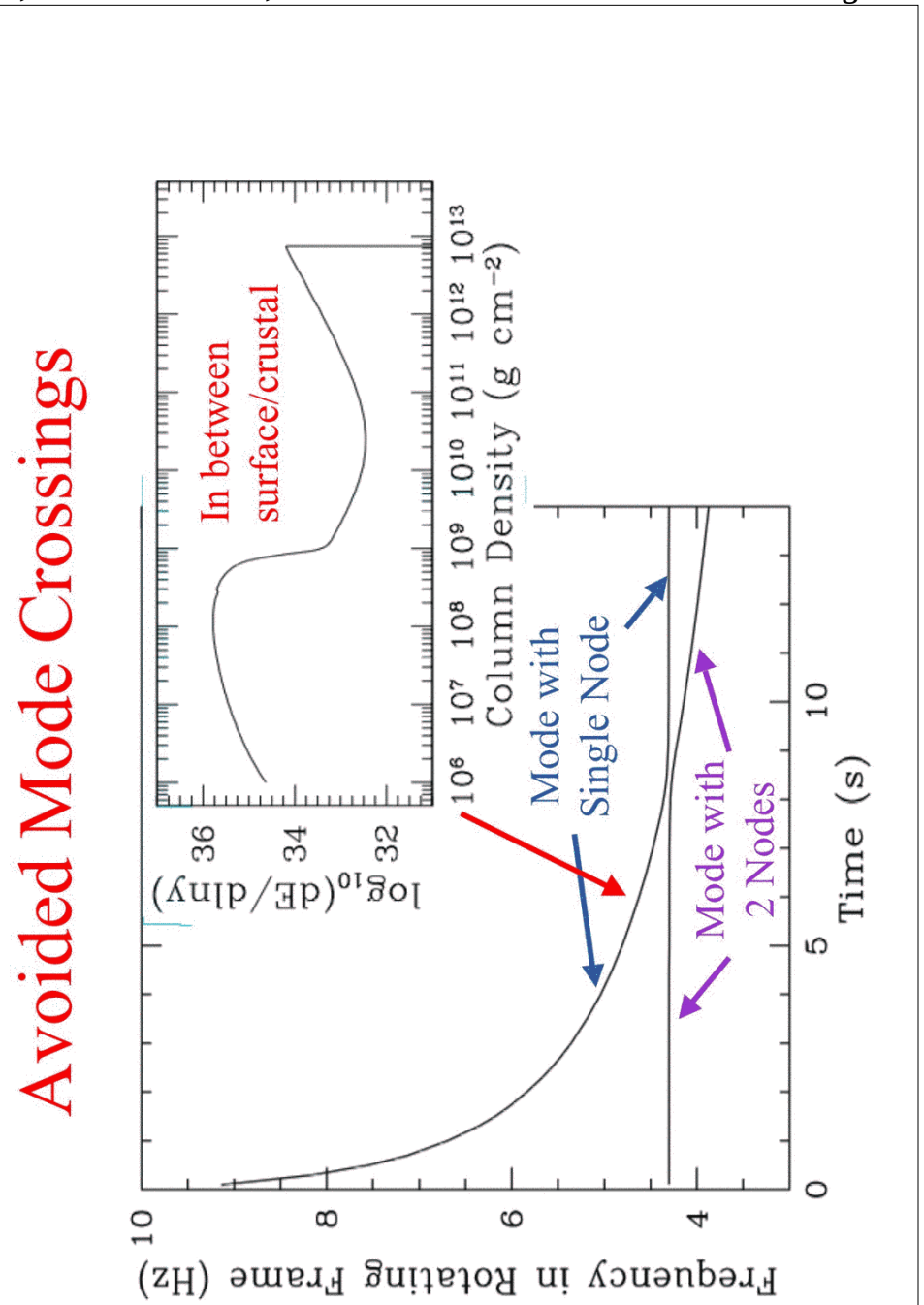


Avoided Mode Crossings

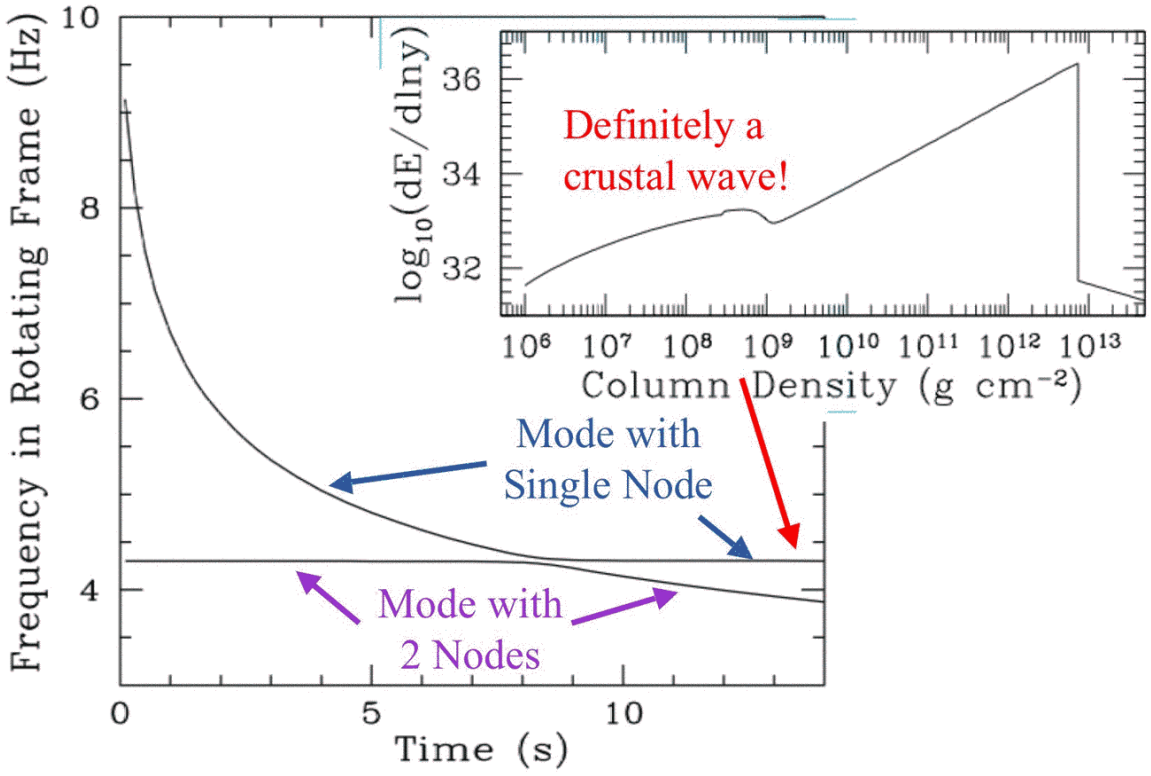


The two modes meet at an avoided crossing


Avoided Mode Crossings
Definitely a surface wave!
Mode with Single Node
Mode with 2 Nodes
Frequency in Rotating Frame (Hz)
Time (s)
Column Density (g cm^{-2})
$\log_{10}(dE/dlny)$


Avoided Mode Crossings
In between surface/crustal
Mode with Single Node
Mode with 2 Nodes
Frequency in Rotating Frame (Hz)
Time (s)
Column Density (g cm^{-2})
$\log_{10}(dE/dlny)$

Avoided Mode Crossings



What Angular Eigenfunction?

Heyl ('04) identified crucial properties:

- Highly sinusoidal nature (Muno et al. '02) implies $m = 1$ or $m = -1$
- The OBSERVED frequency is

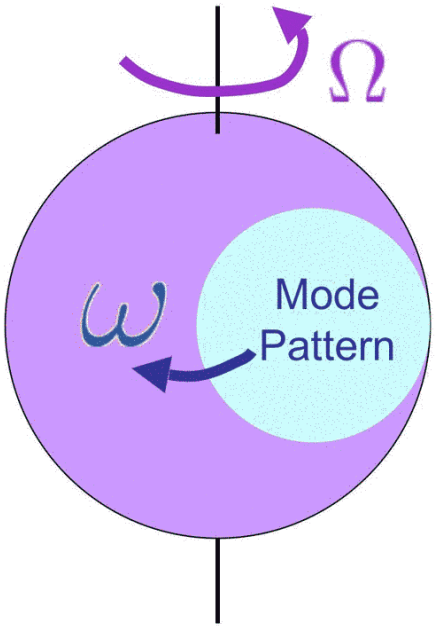
$$\omega_{\rm obs} = |m\Omega - \omega|$$

If the mode travels RETROGRADE ($m = -1$) a DECREASING frequency is observed

$$\omega_{\rm obs} = \Omega + \omega$$

If the mode travels PROGRADE ($m = 1$) an INCREASING frequency is observed

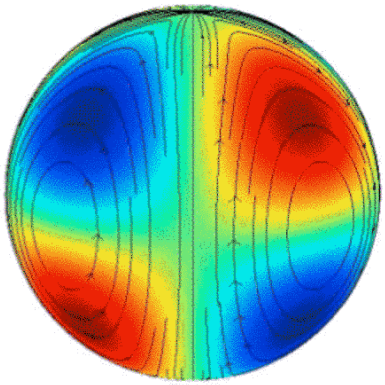
$$\omega_{\rm obs} = \Omega - \omega$$



Rotational Modifications

Since layer is thin and buoyancy is very strong, Coriolis effects ONLY alter ANGULAR mode patterns and latitudinal wavelength (through λ) and NOT radial eigenfunctions! (Bildsten et al. '96)

Inertial R-modes



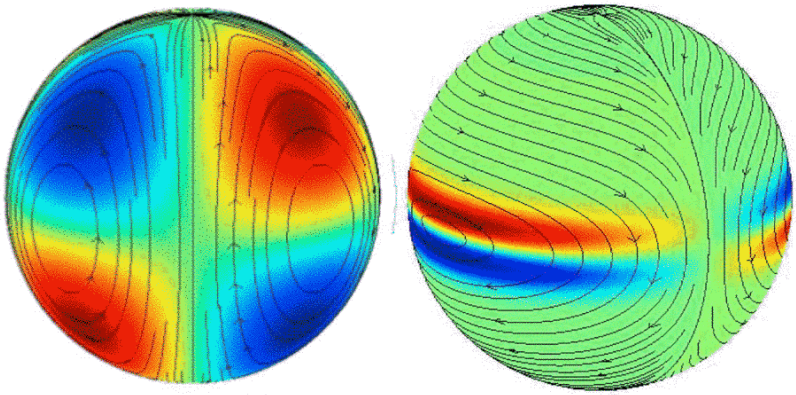
$$\omega = \frac{2m\Omega}{l(l+1)}$$

Only at slow spin.
Not applicable.

Rotational Modifications

Since layer is thin and buoyancy is very strong, Coriolis effects ONLY alter ANGULAR mode patterns and latitudinal wavelength (through λ) and NOT radial eigenfunctions! (Bildsten et al. '96)

Inertial R-modes $\quad l = m$, Buoyant R-modes



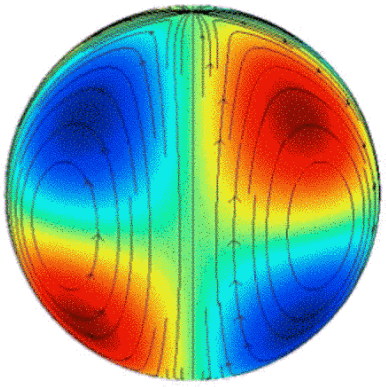
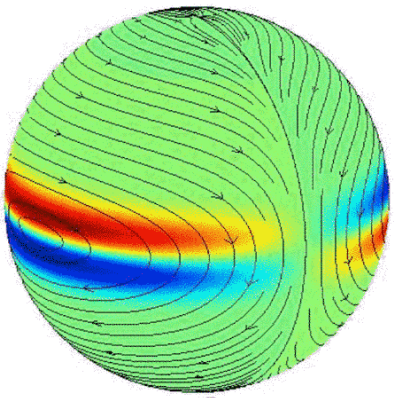
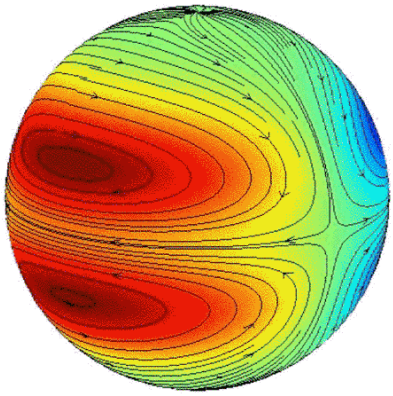
$$\omega = \frac{2m\Omega}{l(l+1)} \qquad \lambda \sim \left(\frac{2\Omega}{\omega}\right)^2 \sim 10 - 10^3$$

Only at slow spin.
Not applicable.

Too large of drifts
and hard to see.

Rotational Modifications

Since layer is thin and buoyancy is very strong, Coriolis effects ONLY alter ANGULAR mode patterns and latitudinal wavelength (through λ) and NOT radial eigenfunctions! (Bildsten et al. '96)

Inertial R-modes	$l = m$, Buoyant R-modes	$l = 2$, $m = 1$ Buoyant R-mode
		
$\omega = \frac{2m\Omega}{l(l+1)}$	$\lambda \sim \left(\frac{2\Omega}{\omega}\right)^2 \sim 10 - 10^3$	$\lambda = 0.11$
Only at slow spin. Not applicable.	Too large of drifts and hard to see.	Just right. Gives drifts as observed and nice wide eigenfunction

Observed Frequencies

400 Hz neutron star spin

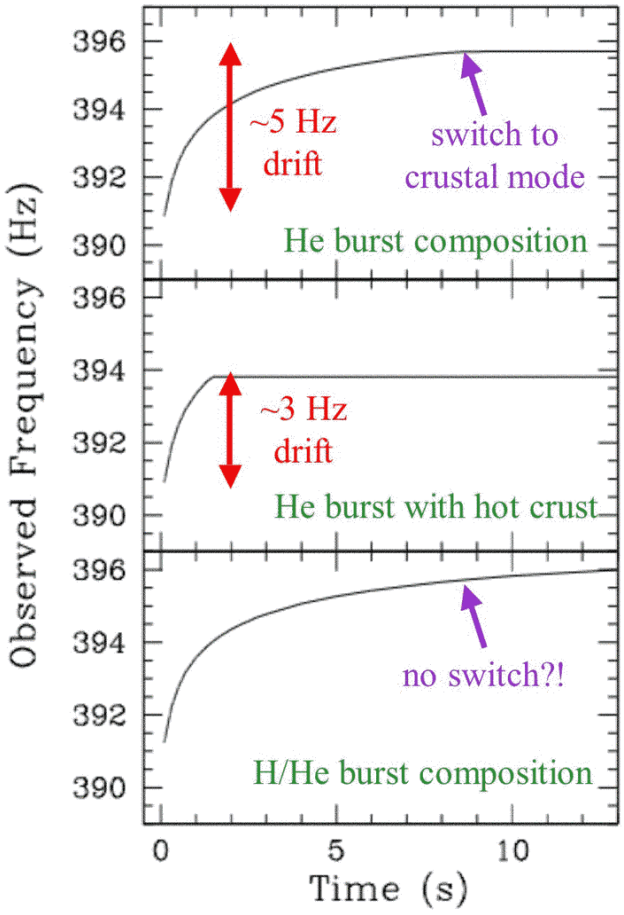
$$\omega_{\rm obs} = |m\Omega - \omega|$$

- Lowest order mode that matches burst oscillations is the $l = 2$, $m = 1$, r-mode

$$\lambda \approx 1/9 \approx 0.11$$

- Neutron star still spinning close to burst oscillation frequency (~ 4 Hz above)

All sounds nice…but can we make any predictions?



Comparison with Drift Observations

• The observed drift is just the difference of

$$\frac{\omega_s}{2\pi} \approx 9.5 \text{ Hz}$$

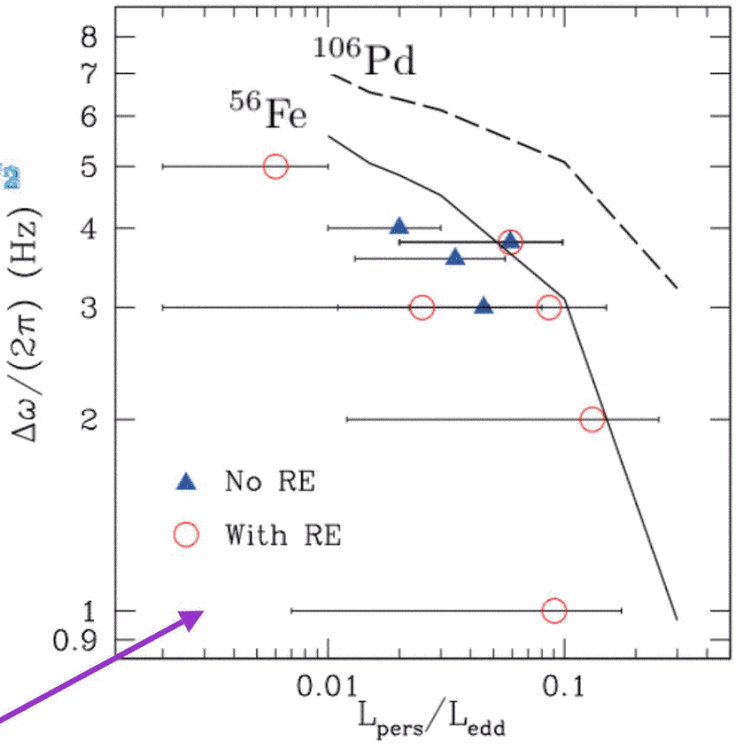
$$\frac{\omega_c}{2\pi} \approx 4.3 \text{ Hz} \left(\frac{64}{A_c}\frac{T_{c,8}}{3}\right)^{1/2}$$

Hot crust = small drift!

• Use T_c from crust models courtesy of E. Brown.

• We compared these with the observed drifts and persistent luminosity ranges.

• Comparison favors a Fe-like crust, consistent with He-rich bursts.

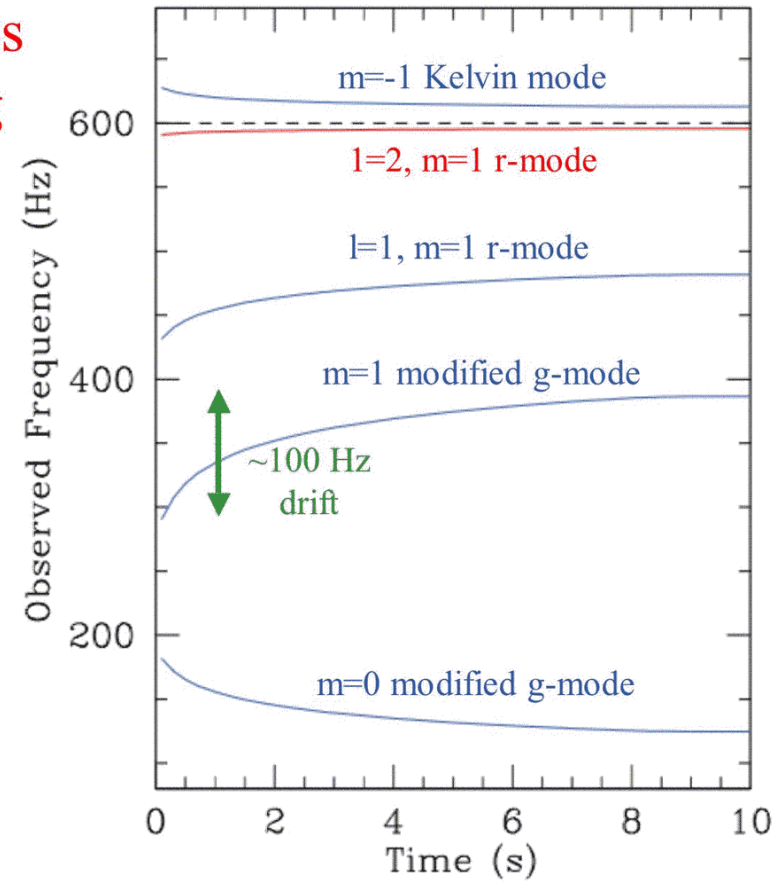


$L_{\rm Edd} \approx 3 \times 10^{38} \rm erg\ s^{-1}$

Could other modes be present during X-ray bursts?

• Nothing precludes the other low-angular order modes from also being present.

• Such modes would show 15-100 Hz frequency drifts, so they may be hidden in current observations.



Conclusions and Discussions

• We propose a surface wave transitioning into a crustal interface wave as the burst oscillations. Only ONE combination of radial and angular eigenfunctions gives the correct properties!

• This is the first explanation for burst oscillations that fits both the frequencies and the drifts, and provides testable predictions.

• Why short (~ 2-10 sec) bursts only?

• Why the $m = 1$ buoyant r-mode? Need to understand excitation mechanism!

What can we now learn?

• Provides constraints on ocean/crust compositions (using drifts)

• Shows that neutron stars are indeed spinning at ~ 270-620 Hz (4-5 Hz ABOVE burst oscillation frequency)