"Accretion Disc Turbulence, Or:

How I learned to stop worrying and love the photon"

Aristotle Socrates
Dept. of Physics
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w/ O. Blaes

Outline

- Back ground
  - Shakura-Sunyaev α-Disk
  - MRI
- Problems w/ α-Disk (some of them)
  - Thermal instability
  - Too much physics is burried in "α."
- Dynamical stability analysis of radiation pressure supported zone (inner)
  - Concepts of radiation hydrodynamics
  - EOM
  - Description of equilibrium
  - MHD waves
  - Any affect to the MRI?
  - Radiative instabilities (3)
  - Diffusion modified convective modes
- Possible application to proto-neutron stars
- Conclusions

WHY STUDY ACCRETION DISKS?

- ONE OF THE MOST COMMON STRUCTURES IN THE UNIVERSE
  ~1 FOR EVERY STAR,
- BY FAR THE MOST LUMINOUS OBJECTS IN THE UNIVERSE. (BH'S AND NS'S)
  - DEEP POTENTIAL WELL

ROUGHLY,

$$m_B c^2 \sim \text{GeV}$$

vs. $\sim 10^1 \frac{\text{MeV}}{\text{BARYON}}$ FOR FUSION PROCESS.

- PROTESTELLAR DISKS
- T-TAURI STARS
- WHITE DWARF BINARIES
- NS XRB (LMXB, HMXB)
- BH XRB
- AGN

} OUR PARTICULAR FOCUS.

ALSO,

PHYSICALLY - FASCINATING OBJECTS (CLASSICALLY)

- GR → SOLVED!*
- MHD/PLASMA PHYSICS
- RADIATIVE TRANSFER
- TURBULENCE

* BLACK HOLE ELECTRODYNAMICS, STILL ????

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THIN DISK - SHAKURA & SUNYAEV (1973)

- IN ORDER TO ACCRETE → TRANSPORT ANGULAR MOMENTUM OUTWARDS!

∴ NEED A TORQUE / STRESS.

- "MOLECULAR" VISCOSITY → NOT ENOUGH

LET $\boxed{\tau_{R\varphi} \simeq \alpha P}$ w/ $\boxed{\alpha \lesssim 1}$

NEED "ANAMOLOUS" SOURCE OF VISCOSITY

- DISK PROBABLY MAGNETIZED (LIKE SUN)

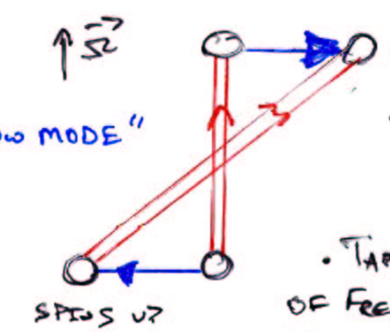
→ MAGNETO ROTATIONAL INSTABILITY (MRI)

(Velikhov, 1959 & CHANDRASEKHAR, 1960)

BALBUS & HAWLEY 1991 ApJ

- DIFFERENTIAL ROTATION → $\boxed{\frac{d\Omega}{dR} < 0}$
- SUBTHERMAL POLOIDAL $\vec{B}$

"SLOW MODE"



- PURE INSTABILITY
  Re [ω] = 0 !
  Im [ω] ~ Ω
- TAPS ENORMOUS RESERVOIR OF FREE ENERGY → DIFFERENTIAL ROTATION.

MHD TURBULENCE

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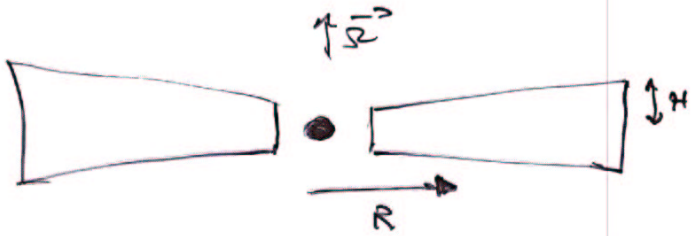

$$\Omega = \Omega(R) = \sqrt{\frac{GM}{R^3}} \quad \text{; KEPLERIAN.}$$

$$L_{EDD} = \frac{4\pi G M m_p c}{\sigma_T} \quad \text{OR} \quad L_{EDD} = \eta \dot{M} c^2$$

$$\dot{M}_{EDD} = \frac{4\pi G M m_p}{\sigma_T c \eta} \sim \frac{10^{-9}}{\eta} (10^{-4} s) \frac{M}{M_\odot} \; M_\odot$$

$$\dot{M} = 2\pi R \Sigma v_R \quad \{\text{CONTINUITY}\}$$

$$4\pi R^2 h \tau_{R\phi} = \dot{M}\sqrt{GMR}\left(1 - \sqrt{\frac{R_0}{R}}\right) \quad \{\text{MOMENTUM}\}$$

$$F(R) = \frac{3GM\dot{M}}{8\pi R^3}\left(1 - \sqrt{\frac{R_0}{R}}\right)$$

$$F(R) \sim T_{eff}^4 \rightarrow T \sim R^{-3/4}$$

$$T(R) \sim R^{-3/4} M^{-1/4} \dot{M}^{1/4} \eta^{-1/4} \alpha^{-1/4}$$

$$\Sigma \sim \dot{M}^{-1} R^{3/2} \alpha^{-1}$$

$$\rho \sim m^{-1} \dot{M}^{-2} R^{3/2}$$

MRI...

- INCOMPRESSIBLE MODE. (SLOW MODE)
- HYDRODYNAMIC SHEAR FLOW.

$$\frac{dL^2}{dR} \geq 0 \rightarrow \text{R-STABLE}$$

$L = R^2 \Omega(R)$ = SPEC. ANG. MOMENTUM.

KEPLERIAN DISK $L \sim R^{1/2} \rightarrow$ R-STABLE

- INTRODUCE POLOIDAL $\vec{B}$

$$\frac{d\Omega^2}{dR} > 0 \rightarrow \text{C-STABLE}$$

- ANY KEPLERIAN ($\Omega \sim R^{-3/2}$) DISK IS C-UNSTABLE.

FULL NUMERICAL MHD CALCULATION

(HAWLEY, GAMMIE, STONE, BALBUS, etc.)

- VIGOROUS MHD TURBULENCE
- LARGE SATURATION AMPLITUDES

$\rightarrow$ $\boxed{\alpha = .5 - 10^{-2} \; !!}$

- STILL DON'T UNDERSTAND TURBULENT CASCADE $\rightarrow$ DON'T UNDERSTAND DISSIPATION

i.e., HOW ARE PHOTONS MADE?

PROBLEM w/ α-DISC

$\boxed{\text{• THERMALLY UNSTABLE}}$ (LIGHTMAN & EARDLEY 1974, 75)

OR

$$Q^+ > Q^-$$

$Q^+ \equiv$ HEATING RATE

$Q^- \equiv$ COOLING RATE.

IF $P_{RAD} > P_{GAS}$ → THERMAL INSTABILITY.

$$Q^+ = -R\frac{d\Omega}{dR}\frac{(2h\,\tau_{R\phi})}{\Sigma/2h} \sim T^6$$

$$Q^- = \frac{F(R)}{\Sigma} \sim T^4$$

ALSO A "VISCOUS" INSTABILITY, BUT A FACTOR OF $\left(\frac{h}{R}\right)^2$ WEAKER. (TIMESCALE)

· POSSIBLE SOLUTION → GAMMIE (1998) "Photon Bubbles" (slow mode) w/out ROTATION.

· MRI IS A SLOW MODE TOO? → WE INVESTIGATE.

$\boxed{\text{WHAT EXACTLY IS THE NATURE OF THE MHD TURBULENCE?}}$

· CAN A THEORY BE CONSTRUCTED?

→ DYNAMICS → ANALYTICALLY FIND α

→ DISSIPATION → P^+ ORE^{-1} HEATING?

↳ OUTSIDE THE REALM OF NUMERICAL WORK.

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(BLAES & SOCRATES ApJ 2001)

$\boxed{\text{STABILITY ANALYSIS OF RADIATION PRESSURE SUPPORTED ACCRETION DISKS}}$

· LINEAR ANALYSIS OF RMHD EQUATIONS.
· LOCAL ANALYSIS → WKB WAVES (EASIEST)

· HOW DOES ONE MODEL A RADIATING, MAGNETIZED, CONDUCTING FLUID (THEORETICALLY)?

ANS: BOLTZMANN THEORY.
AKA KINETIC THEORY.

WE MUST COUPLE

GAS + PHOTONS

VIA SOME COLLISIONAL PROCESS....

$$e + \gamma \rightleftharpoons e + \gamma$$ COMPTON SCATTERING

IGNORE ABSORPTION PROCESS

$\boxed{K_a \ll K_s}$ ⇝ $E_\gamma \simeq 10^{-2} - 10^{2}$ keV

* PHOTONS AND PLASMA CAN ONLY EXCHANGE MOMENTUM. WE ARE NOT CONSIDERING EXCHANGE OF ENERGY.

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BASICS OF RADIATION HYDRODYNAMICS*

TECHNICALLY COUPLE 2 DISTRIBUTIONS. $I_\nu \rightleftarrows f(\vec{x}, \vec{v}; t)$

IN PRACTICE (PHYSICALLY) WE ARE COUPLING

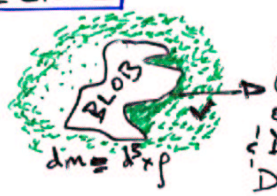
$\vec{F}(\vec{x}, t) \rightleftarrows \vec{v}(\vec{x}, t)$

...AND OTHER MOMENTS.

$\boxed{\text{PHYSICAL PICTURE}}$

→ Photons

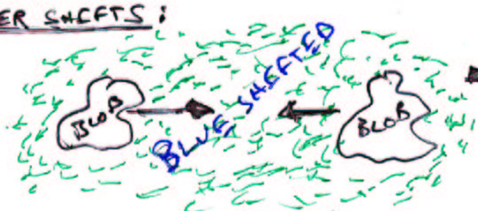
EFFECTS # Density. ←



ADVECTION:

RESULTS IN AN OBSERVED (FLUID FRAME) INCREASE OF PRESSURE UPSTREAM & DECREASE OF PRESSURE DOWNSTREAM.

DOPPLER SHIFTS:



→ EFFECTS ν AND THUS SPECTRAL DISTRIBUTION

• THESE TWO BASIC EFFECTS PRODUCE ALL OF THE INTERESTING COUPLING BETWEEN $\vec{F}$ AND $\vec{v}$

NOTE: THESE EFFECTS ARE $\mathcal{O}\left(\frac{v}{c}\right)$

*WARNING: RHD MAY NOT BE = RMHD

- TALK MORE ABOUT THIS LATER

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$\boxed{\text{BASIC EQUATIONS}}$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0,$$

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v}\right) = -\nabla p - \rho\nabla\Phi + \frac{1}{4\pi}(\nabla\times\mathbf{B})\times\mathbf{B} + \frac{\kappa_{\rm es}\rho}{c}\mathbf{F},$$

$$\frac{\partial p}{\partial t} + \mathbf{v}\cdot\nabla p = \frac{\gamma p}{\rho}\left(\frac{\partial \rho}{\partial t} + \mathbf{v}\cdot\nabla\rho\right),$$

$$\frac{\partial E}{\partial t} + \mathbf{v}\cdot\nabla E + \frac{4}{3}E\nabla\cdot\mathbf{v} = -\nabla\cdot\mathbf{F},$$

$$\mathbf{F} = -\frac{c}{3\kappa_{\rm es}\rho}\nabla E,$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla\times(\mathbf{v}\times\mathbf{B}).$$

w/ $\nabla\cdot\mathbf{B} = 0$

NOTE:

- DIFFUSION LIMIT
- ELECTRON SCATTERING ONLY
- IDEAL MHD
- NO SELF GRAVITATION
- ADIABATIC EOS

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- WE ARE PERFORMING A LINEAR ANALYSIS

$$Q \rightarrow Q + \delta Q$$

- ANALYSIS IS LOCAL

$$\lambda << H$$

→ WKB LIMIT

$$\delta Q = \delta Q\, e^{i[k \cdot x - \omega t]}$$

- REDUCES TO A LINEAR EIGENVALUE PROBLEM

→ STABILITY ANALYSIS ←

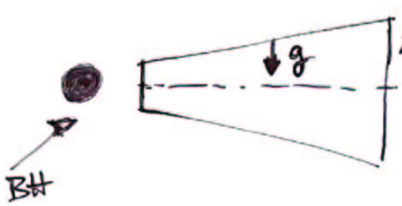
EQUILIBRIUM

$$v_\varphi^{(0)} = R\Omega(R) \rightarrow \text{SHEAR} \rightarrow \frac{d\Omega}{dR} \neq 0$$

$$\Omega(R) \sim R^{-3/2} \rightarrow \text{KEPLERIAN}$$

VERTICAL STRUCTURE

$$-\frac{1}{\rho}\frac{\partial P}{\partial z} + \frac{K_{es}}{c} F_z = g$$



"thin α DISK"

$$-\frac{1}{\rho}\frac{\partial}{\partial z}\left[P + \frac{E}{3}\right] = g$$

$$\frac{E}{3} = P_r$$

(10)

$$\boxed{\vec{B} = B_z \hat{e}_z + B_\varphi \hat{e}_\varphi} \rightarrow \text{TIME INDEPENDANT BACKGROUND!}$$

WAVES TO EXPECT: $\boxed{P_r \sim P_m >> P_g}$

$\vec{B} = B\hat{b}$

ALFVEN WAVES (SHEAR)

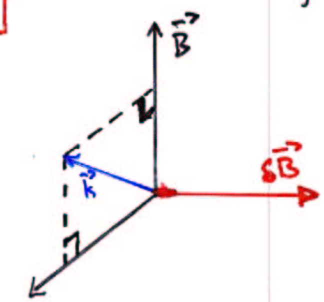
$$\omega_k^A = \pm \frac{k \cdot B}{\sqrt{4\pi\rho}} \qquad \delta v = \mp \delta b$$

EXACT SOLUTIONS!

TRAVEL ALONG $\vec{B}$ w/ $\vec{v}_g = v_A \hat{b} = \frac{B}{\sqrt{4\pi\rho}}\hat{b}$

MAGNETIC TENSION

$$\hat{e} = \frac{\hat{k} \times \hat{b}}{|\hat{k} \times \hat{b}|}$$



COMPLETELY INCOMPRESSIBLE

$\nabla \cdot v = 0$

MAGNETOSONIC WAVES (FAST/SLOW)

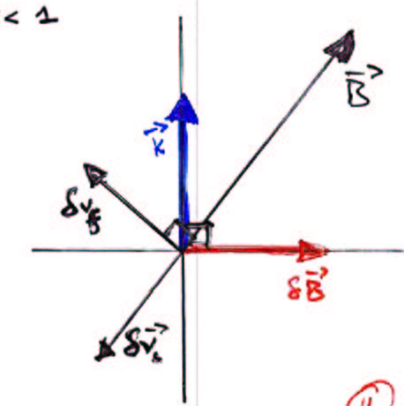
Low β PLASMA → $\frac{P_g}{P_m} << 1$

$$\omega_k^f = \pm k\sqrt{v_{Az}^2 + v_{A\varphi}^2}$$

$$\omega_k^s = \pm c_g \frac{k \cdot B}{|B|}$$

COMPRESSION

* lim SLOW → PSEUDO ALFVEN
$\nabla \cdot v = 0$



(11)

OTHER WAVES TO EXPECT

INERTIAL WAVES ($B \to 0$)

$\omega = \pm 2 \frac{(\vec{k} \cdot \vec{\Omega})}{k} \rightleftarrows$ $\boxed{\text{CORIOLIS FORCE.}}$

- ELLIPTICALLY POLARIZED
- LIFTS DEGENERACY BETWEEN SHEAR AND PSEUDO ALFVEN MODES.

g-MODES ($B \to 0$)

$\omega^2 = \frac{k_\perp}{k^2} N^2 \rightleftarrows$ $\boxed{\text{BUOYANCY}}$

N = BRUNT-VAISALLA FREQUENCY

$N^2 = -\vec{g} \cdot \nabla S$; S = SPECIFIC ENTROPY

IF $N^2 < 0 \to$ CONVECTION

- ENTROPY GRADIENTS IN EITHER THE GAS OR RADIATION MAY PROVIDE OSCILLATIONS AND CONVECTION.

NOTE: CONSIDERED ONLY $\boxed{\text{AXI-SYMMETRIC}}$ MODES ONLY. $\to \vec{k} = k_z \hat{z} + k_R \hat{R}$

- ALLOWS MODES TO BE PURELY LOCAL.

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Q: WHAT DOES A WHOPPING RADIATIVE FLUX DO TO ALL THESE MODES???

RESULTS ....

- OBTAINED A 7th ORDER DISPERSION RELATION..

MRI:

- REMAINS VIRTUALLY UNSCATHED
- IF $B_\varphi \gg B_z \to$ MODES BECOME MORE COMPRESSIBLE (w/ Quick Diffusion)

MRI GROWTH RATE $\sim \Omega \frac{c_g}{v_{A\varphi}}$ INSTEAD OF JUST $\sim \Omega$

- MAKES SENSE! $\to$ CAN HAVE INCOMPRESSIBLE MODES IN A COMPRESSIBLE MEDIUM.
- RADIATIVE DIFFUSION REDUCES THE GROWTH RATE, BUT THE MRI STILL PERSISTS.

$\to$ CONFIRMED BY A 2-D RMHD SIMULATION (TURNER et. al ApJ 2002) $\leftarrow$

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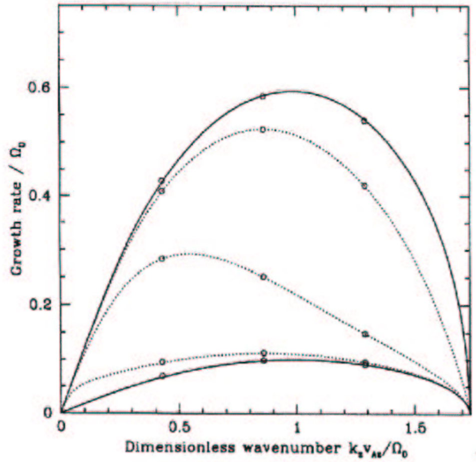
DIFFUSION ONLY!

2-D CALCULATION. → NO STRATIFICATION.



TURNER et. AL. ApJ 2002.

$B_\varphi \neq 0$

- AGREEMENT BETWEEN LINEAR THEORY & THE FULL CALCULATION.
- RADIATIVE DIFFUSION CAN REDUCE GROWTH RATE IF $B_\varphi > B_z$.

. INCREASING COMPRESSIBILITY

PROVIDES ADDITIONAL RESTORING FORCE (COMPRESSION), CHANGES FORCE BALANCE.

STRATIFICATION? i.e, $\vec{F} \neq 0$

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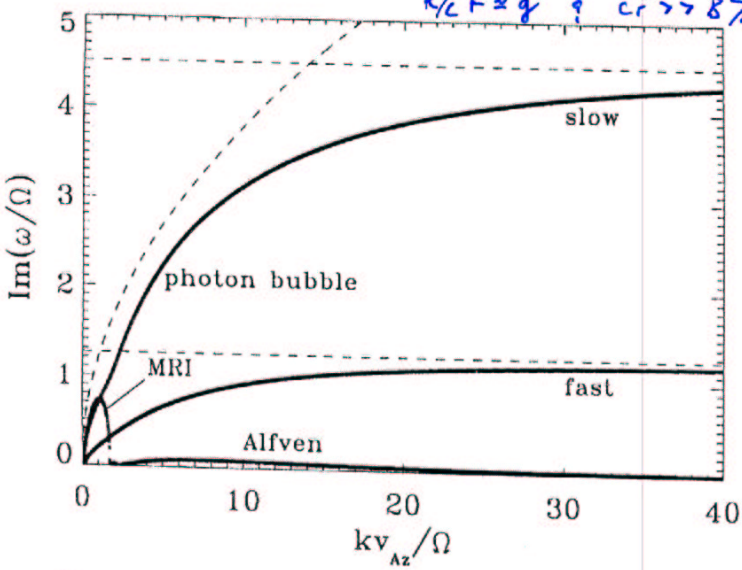
$K/c\, F \approx g$ & $c_r^2 \gg B^2/4\pi$



. RADIATIVE FLUX, $\vec{F}$, MAKES THE COMPRESSIBLE MODES UNSTABLE.

SLOW MODE/PHOTON BUBBLE

- PHOTON BUBBLE (ARONS '92, GAMMIE '98)
  - SLIGHTLY AFFECTED BY $\frac{d\Omega}{dR}$

$\omega \sim k^{1/2}$ → PHOTON BUBBLE

↳ $\omega^2 = -i g \left[\frac{k_z k_R^2}{k^2}\right]\left(\frac{B_z}{B}\right)^2$ AGREES w/ GAMMIE '98

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ASYMPTOTIC FORM ($k \to \infty$)

$$\omega \simeq \pm c_g k_z \left(\frac{B_z}{B}\right) \mp i \frac{k_R^2}{2k^2} \frac{\kappa_{es}}{c} \frac{F_z}{c_g} \left(\frac{B_z}{B}\right)$$

FAST-MODE INSTABILITY (NEW!!)

STABILITY CRITERIA (ROUGH)

$$F_z > \rho c_r^2 v_A \times \left\{ \begin{array}{c} \text{GEOMETRICAL} \\ \text{FACTOR} \end{array} \right\}$$

OR ~

$$\boxed{F > E v_A}$$

NOTE, THAT IN THE FREE STREAMING LIMIT,

$$F \simeq cE$$

FAST MODE AND SLOW MODE INSTABILITES PROPAGATE IN OPPOSITE DIRECTION!?

ALSO,

ALFVENIC INSTABILITY (NEW)

DUE TO FREQUENCY SPLITTING (ROTATION?)

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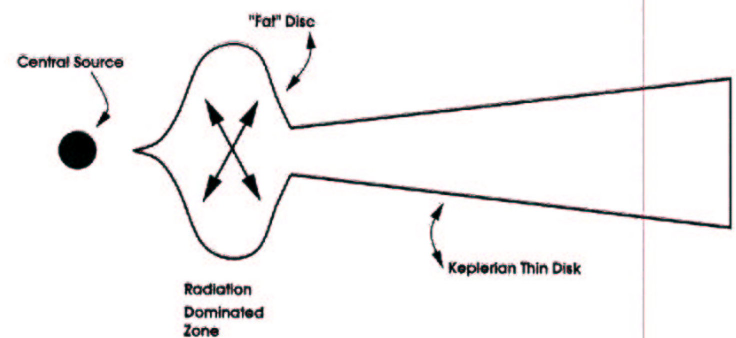
MORE EXTENDED ANALYSIS:



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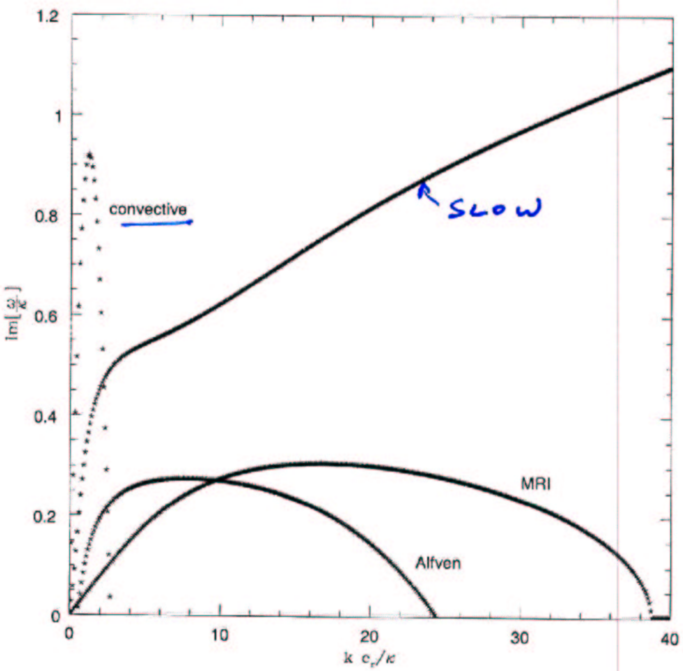
$B_\varphi = 10 B_z$

$N_{Rad} = i\Omega$

$\frac{k_z}{k} = 0.1$



· CONVECTIVE-TYPE INSTABILITY, FROM RADIAL RADIATION ENTROPY GRADIENTS.

· NOTICE THAT MRI IS NOT THE FASTEST GROWING MODE.

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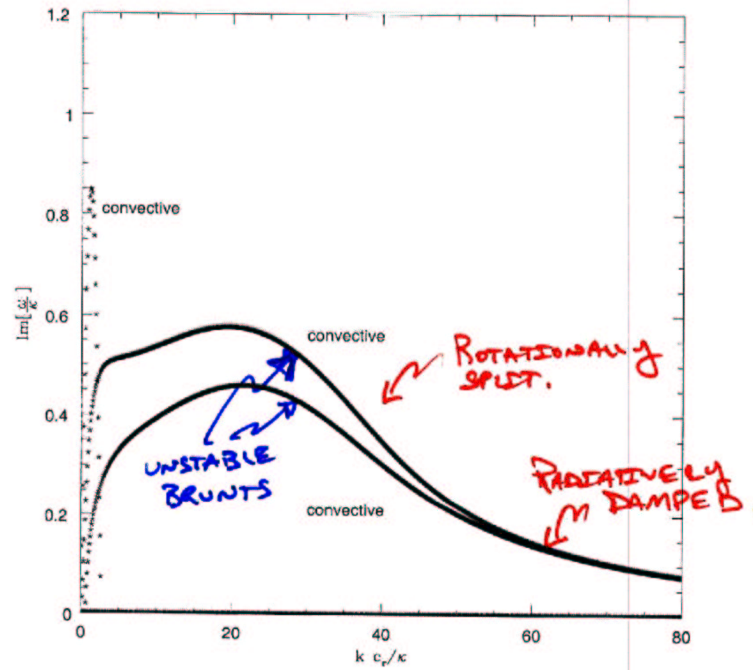
PURELY HYDRODYNAMIC.



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OTHER WORKERS

BEGELMAN et al. (2001, 2002 ApJ) {ANALYTIC}

- FOUND A 1-D SOLUTION TO THE PHOTON BUBBLE USING FULLY NON-LINEAR SOLUTION.
- RESULTS AGREE w/ LINEAR THEORY.
- BEGELMAN'S SOLUTION PRODUCES SUPER-EDDINGTON LUMINOSITIES! (COULD DISCREDIT CLAIMS OF INTERMEDIATE M BH'S!)

TURNER et al (2002 ApJ) {NUMERICAL}

- 2-D MESH CODE w/ RADIATIVE DIFFUSION BUT NO RADIATIVE FLUX.
- NOTICED SMALL-MEDIUM SCALE DENSITY INHOMOGENEITIES
- THE RELEVANT PRESSURE IS STILL TOTAL PRESSURE

MAINTAIN $B^2 \sim \alpha P_{RAD} \sim \alpha P_{TOT}$

$w/ \alpha \simeq 0.1 - 0.01$

APPLY TO OTHER SYSTEMS

PROTO-NEUTRON STARS (1ST 10s POST CORE-COLLAPSE)

npe PLASMA @ ν-Sphere

- INTERACTIONS ARE

$$\nu + N \rightarrow \nu + N \quad \text{SCATTERING}$$

$L_\nu = L_\nu + L_{\bar{\nu}} \simeq 10^{52}\,\mathrm{erg\,s^{-1}}$

OR $L_\nu \simeq 3\times10^{-2} L_{EDD}$ (IN NEUTRINOS)

SAME PHYSICS AS ACCRETION DISK.

- "NEUTRINO BUBBLE"/SLOW INSTABILITY.

$$\omega \simeq \pm k_z v_{Az} \mp i \frac{k_R^2}{k^2}\left(\frac{v_{Az}}{c_g}\right) \frac{\kappa_\nu F_\nu}{c_g c}$$

- GROWTH RATE $\propto |\vec{B}|$ AND DEPENDS ON ORIENTATION

Kick Mechanism? $\longrightarrow$ $B \gtrsim 3\times10^{13}$ G

- ONLY NEED ANISOTROPY OF A FEW % IN L_ν

- INSTABILITY IS INHERENTLY ANISOTROPIC.

CONCLUSIONS

- TO LINEAR ORDER, MRI ROUGHLY UNAFFECTED
- PRESENCE OF RADIATIVE FLUX GENERALLY MAKES A MAGNETIZED FLOW UNSTABLE.
- RADIATIVE INSTABILITIES ARE COMPRESSIVE IN NATURE.
  - PHOTON BUBBLE /SLOW
  - FAST
  - ALFVENIC
- CONVECTION WRT TO THE RADIATION FLUID IS AFFECTED BY ROTATION AND DAMPED BY DIFFUSION.

SPECULATE

- SLOW & FAST INSTABILITIES MAY ACT AS AN ANAMOLOUS THERMAL TRANSPORT MECHANISM.

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