Polymer physics meets cell biology

- Polymers (filamentous proteins) in the cell?
- Outline of topics and problems
- Properties and phase behavior in bulk
  - Model polymers, biomaterials, …novel rheology
- Single filament properties
  - dynamics & response --- a toy problem
- Active gels & force propagation

Biopolymers

DNA

Filamentous proteins
(e.g., structural elements of cells)

G-actin, a Globular protein of
MW=43k

Stryer, 1995

200 nm

>20 μm

2 nm
Plant and animal cells have a cytoskeleton, consisting largely of polymer, for structure, organization, and transport.

Why biopolymers?

- **SIZE**: can visualize basic polymer physics in an optical microscope!

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<table>
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<th>J Kas, P Janmey, 1995</th>
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Viruses (fd, tmv, …)

Elasticity of *semiflexible* polymers

\[
E_{\text{bend}} = \frac{1}{2} \kappa \int \left( \frac{\partial \theta}{\partial s} \right)^2 ds
\]

\[
\langle \cos(\theta(s) - \theta(s')) \rangle = e^{-|s-s'|/\ell_p}, \quad \text{where} \quad \ell_p = \kappa / kT
\]

Expect: \( \kappa \approx Ea^4 \)

For \( E \approx 10^9 \) Pa and \( a \approx 3 \) nm, \( \ell_p \approx 10 \) \( \mu \)m

\[
a \approx 3 \text{ nm, } \ell_p \approx 10 \text{ nm} \\
a \approx 10 \text{ nm, } \ell_p \approx 1 \text{ mm} \\
a \approx 0.2 \text{ nm, } \ell_p \approx 100 \text{ nm}
\]

- Actin
- Microtubules
- DNA
Topics

- Phase behavior (liquid crystalline phases)
  order from entropy
- Rheology (flow properties, viscoelastic response)
  in Bulk
  micro rheology (theory and experiment)
  composites of biopolymer + membrane
- Dynamics & response of single filaments
- Geometry of twist and writhe
- Collapse and condensation of biopolymers
- Active gels! (with molecular motors)
- Force generation, transmission *in vivo*

When should we have a nematic?

Onsager:

excluded volume of one rod

total concentration for nematic

\[ v \equiv L^2 a \]
\[ \phi \equiv a / L \equiv a / \ell_p \]  
(Khokhlov,Semenov)
Some Liquid Crystalline Phases

When should we have a nematic?

Onsager:

\[ v \equiv \frac{L^2 a}{\phi} \]

\[ \phi \equiv \frac{a}{L} \equiv a / \ell_p \]  
(Khokhlov, Semenov)

Isotropic, flexible solution?

\[ \phi \leq \left( \frac{a / \ell_p}{2} \right)^2 \]

e.g.,

DNA \( c = 1-10 \) mg/ml

F-actin \( c = (0.01) - 2 \) mg/ml
Viscoelasticity of (bio)polymers

The Shear Modulus depends on frequency and time

\[ G \approx \omega^{3/4} \]

Experiments (actin): Gittes, ...; Schnurr, ... '97; Xu, ... '98; Gisler, Weitz '99
Theory: Morse '98; Gittes, FCM '98; Pasquali ... '01

Single-filament dynamics/response: a toy problem

fluctuations

\[ E_{\text{rand}} = \frac{kT}{2} \int (\nabla^2 u) \, dx \Rightarrow \langle u_x^2 \rangle = \frac{1}{2 \ell_s q^4} \]

dynamics

\[ \kappa \nabla u + \xi u = \text{noise} ; \quad \xi = 4 \pi \eta / \log(0.6 \lambda / a) \]

\[ \Rightarrow \langle u, \ell(t) \rangle = \langle u_x^2 \rangle e^{-\omega t} ; \quad \omega = \kappa q^4 / \xi \]

projected length fluctuations

\[ \langle \delta x(t) - \delta x(0) \rangle^2 = \frac{1}{2} \sum q^4 \left( \langle u_x^2 \rangle^2 - \langle u_x^2 \rangle^2 \right) = \frac{1}{\ell_s^2} \left( \frac{\kappa}{\xi} \right)^{1/4} \]

response function and modulus

\[ \langle \delta u_x \rangle = \frac{2kT}{\omega} \Im(\alpha_\omega) \Rightarrow G(\omega) = \rho \frac{\ell}{15 \sigma_\omega} - i \omega \eta = \rho kT \left( \frac{2}{15} \frac{2i \xi \omega}{\kappa} \right)^{1/4} - i \omega \eta \]

Morse '98; Gittes ... '98; Pasquali ... '01
High rheology

Experiments
Gittes, ...; Schnurr, ... '97
Xu, ... '98;
Gisler, Weitz '99

Theory
Morse '98; Gittes ... '98;
Pasquali ... '01

Tension propagation in the toy problem?

\[ \ell_T(t) \approx \sqrt{\ell_p \ell_\perp(t)} \approx t^{1/8}, \]
where \( \ell_\perp(t) = (t \kappa / \zeta)^{3/4} \)

\[ \delta \ell^2 \approx t^{7/8} \]
\[ \delta \ell^2 \approx t^{3/4} \]
Simulations

Everaers … ’99

\[ \frac{\lambda_{1,2}}{\kappa^2} \]

\[ t^{3/4} \quad t^{7/8} \quad t^{1/2} \]

\[ 10^{-12} \quad 10^{-10} \quad 10^{-8} \quad 10^{-6} \quad 10^{-4} \quad 10^{-2} \quad 1 \]

\[ t/\kappa^3 \]

Topics

- Phase behavior (liquid crystalline phases)
  - order from entropy
- Rheology (flow properties, viscoelastic response)
  - in Bulk
  - microrheology (theory and experiment)
  - composites of biopolymer + membrane
- Dynamics & response of single filaments
- Geometry of twist and writhe
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- Force generation, transmission in vivo