Quantum matter
without quasiparticles

Blackboard talk
Kavli Institute for Theoretical Physics,
University of California, Santa Barbara,
September 18, 2017

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Talk online: sachdev.physics.harvard.edu
A quasiparticle is an “excited lump” in the many-electron state which responds just like an ordinary particle.
The quasiparticle idea is the key reason for the many successes of quantum condensed matter physics:

- Fermi liquid theory of metals, insulators, semiconductors
- Theory of superconductivity (pairing of quasiparticles)
- Theory of disordered metals and insulators (diffusion and localization of quasiparticles)
- Theory of metals in one dimension (collective modes as quasiparticles)
- Theory of the fractional quantum Hall effect (quasiparticles which are ‘fractions’ of an electron)
Quantum matter without quasiparticles

Resistivity $\sim \rho_0 + AT^\alpha$

$\text{BaFe}_2(\text{As}_{1-x}\text{P}_x)_2$

Strange Metal

Superconductivity

AF + nematic


Quantum matter without quasiparticles

Strange metal

Entangled electrons lead to “strange” temperature dependence of resistivity and other properties

Figure: K. Fujita and J. C. Seamus Davis
Quasiparticles are additive excitations:
The low-lying excitations of the many-body system can be identified as a set \( \{n_\alpha\} \) of quasiparticles with energy \( \varepsilon_\alpha \)

\[
E = \sum_\alpha n_\alpha \varepsilon_\alpha + \sum_\alpha,\beta F_{\alpha\beta} n_\alpha n_\beta + \ldots
\]
Quasiparticles eventually collide with each other. Such collisions eventually lead to thermal equilibration in a chaotic quantum state, but the equilibration takes a long time. In a Fermi liquid, this time is of order \( \frac{\hbar E_F}{(k_B T)^2} \) as \( T \to 0 \), where \( E_F \) is the Fermi energy.
Quantum matter without quasiparticles:

- No quasiparticle decomposition of low-lying states
- Rapid thermalization
Quantum Ising models

Qubits with states $|\uparrow\rangle_i$, $|\downarrow\rangle_i$, on the sites, $i$, of a regular lattice.

$$
\sigma^z |\uparrow\rangle = |\uparrow\rangle, \quad \sigma^z |\downarrow\rangle = -|\downarrow\rangle
$$

$$
\sigma^x |\uparrow\rangle = |\downarrow\rangle, \quad \sigma^x |\downarrow\rangle = |\uparrow\rangle
$$

$$
H = -J \left( \sum_{\langle ij \rangle} \sigma^z_i \sigma^z_j + g \sum_i \sigma^x_i \right)
$$

For $g = 0$, ground state is a ferromagnet:

$$
|G\rangle = |\cdots \uparrow \uparrow \uparrow \uparrow \cdots \rangle \quad \text{or} \quad |\cdots \downarrow \downarrow \downarrow \downarrow \cdots \rangle
$$

For $g \gg 1$, unique ‘paramagnetic’ ground state:

$$
|G\rangle = |\cdots \rightarrow \rightarrow \rightarrow \rightarrow \cdots \rangle
$$

where

$$
|\rightarrow\rangle = \frac{1}{\sqrt{2}} (|\uparrow\rangle + |\downarrow\rangle), \quad |\leftarrow\rangle = \frac{1}{\sqrt{2}} (|\uparrow\rangle - |\downarrow\rangle)
$$
In one dimension, quasiparticles exist even at the quantum critical point: there is a non-local transformation from the qubits to a system of free fermions.
In two dimensions, the “quantum critical” region provides us the first example of a system without a quasiparticle description. This is described by a strongly-coupled conformal field theory (CFT) in 2+1 dimensions, and dynamic properties cannot be computed accurately.
A simple model of a metal with quasiparticles

Pick a set of random positions
A simple model of a metal with quasiparticles

Place electrons randomly on some sites
A simple model of a metal with quasiparticles

Electrons move one-by-one randomly
A simple model of a metal with quasiparticles

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\[ H = \frac{1}{(N)^{1/2}} \sum_{i,j=1}^{N} t_{ij} c_i^\dagger c_j + \ldots \]

\[ c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij} \]

\[ \frac{1}{N} \sum_i c_i^\dagger c_i = Q \]

\( t_{ij} \) are independent random variables with \( \overline{t_{ij}} = 0 \) and \( \overline{|t_{ij}|^2} = t^2 \)

Fermions occupying the eigenstates of a \( N \times N \) random matrix
A simple model of a metal with quasiparticles

Let $\varepsilon_\alpha$ be the eigenvalues of the matrix $t_{ij}/\sqrt{N}$. The fermions will occupy the lowest $NQ$ eigenvalues, up to the Fermi energy $E_F$. The density of states is $\rho(\omega) = (1/N) \sum_\alpha \delta(\omega - \varepsilon_\alpha)$. 
A simple model of a metal with quasiparticles

There are $2^N$ many-body levels with energy

$$E = \sum_{\alpha=1}^{N} n_\alpha \varepsilon_\alpha,$$

where $n_\alpha = 0, 1$. Shown are all values of $E$ for a single cluster of size $N = 12$. The $\varepsilon_\alpha$ have a level spacing $\sim 1/N$. 

Many-body level spacing $\sim 2^{-N}$

Quasiparticle excitations with spacing $\sim 1/N$
The Sachdev-Ye-Kitaev (SYK) model

Pick a set of random positions
The Sachdev-Ye-Kitaev (SYK) model

Place electrons randomly on some sites
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Entangle electrons pairwise randomly
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This describes both a strange metal and a black hole!
The Sachdev-Ye-Kitaev (SYK) model

(See also: the “2-Body Random Ensemble” in nuclear physics; did not obtain the large $N$ limit; T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. 53, 385 (1981))

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij; k\ell} c_i^\dagger c_j^\dagger c_k c_\ell - \mu \sum_i c_i^\dagger c_i$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j c_i^\dagger = \delta_{ij}$$

$$Q = \frac{1}{N} \sum_i c_i^\dagger c_i$$

$J_{ij; k\ell}$ are independent random variables with $\overline{J_{ij; k\ell}} = 0$ and $|\overline{J_{ij; k\ell}}|^2 = J^2$

$N \to \infty$ yields critical strange metal.

S. Sachdev and J. Ye, PRL 70, 3339 (1993)

The Sachdev-Ye-Kitaev (SYK) model

There are $2^N$ many body levels with energy $E$, which do not admit a quasiparticle decomposition. Shown are all values of $E$ for a single cluster of size $N = 12$. The $T \to 0$ state has an entropy $S_{GPS} = N s_0$ with

$$s_0 = \frac{G}{\pi} + \frac{\ln(2)}{4} = 0.464848\ldots$$

where $G$ is Catalan’s constant, for the half-filled case $Q = 1/2$.

GPS: A. Georges, O. Parcollet, and S. Sachdev, PRB 63, 134406 (2001)

W. Fu and S. Sachdev, PRB 94, 035135 (2016)
The Sachdev-Ye-Kitaev (SYK) model

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$$s_0 = \frac{G}{\pi} + \frac{\ln(2)}{4} = 0.464848 \ldots < \ln 2$$

Non-quasiparticle excitations with spacing $\sim e^{-N s_0}$

Many-body level spacing $\sim 2^{-N} = e^{-N \ln 2}$

No quasiparticles!

$$E \neq \sum_{\alpha} n_{\alpha} \varepsilon_{\alpha} + \sum_{\alpha, \beta} F_{\alpha \beta} n_{\alpha} n_{\beta} + \ldots$$

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W. Fu and S. Sachdev, PRB 94, 035135 (2016)
The Sachdev-Ye-Kitaev (SYK) model

- Low energy, many-body density of states
  
  \[ \rho(E) \sim e^{N s_0} \sinh(\sqrt{2(E - E_0)}N \gamma) \]  
  
  (sinh factor is for Majorana version)

  A. Georges, O. Parcollet, and S. Sachdev, PRB 63, 134406 (2001)
  D. Stanford and E. Witten, 1703.04612
  A. M. Garica-Garcia, J.J.M. Verbaarschot, 1701.06593
  D. Bagrets, A. Altland, and A. Kamenev, 1607.00694
The Sachdev-Ye-Kitaev (SYK) model

• Low energy, many-body density of states
  \[ \rho(E) \sim e^{Ns_0} \sinh(\sqrt{2(E-E_0)}N\gamma) \]  
  (sinh factor is for Majorana version)

• Low temperature entropy
  \[ S = Ns_0 + N\gamma T + \ldots \]

A. Kitaev, unpublished
J. Maldacena and D. Stanford, 1604.07818
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- \( T = 0 \) fermion Green’s function
  \[ G(\tau) \sim \tau^{-1/2} \] at large \( \tau \).

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- \( T > 0 \) Green’s function has conformal invariance
  \[ G \sim \left( \frac{T}{\sin(\pi k_B T \tau / \hbar)} \right)^{1/2} \]

A. Georges and O. Parcollet PRB 59, 5341 (1999)
The Sachdev-Ye-Kitaev (SYK) model

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The SYK model is holographically dual to black holes with AdS2 horizons, which share the properties above.

S. Sachdev, PRL 105, 151602 (2010)
The SYK model has "dual" description in which an extra spatial dimension, $\zeta$, emerges. The curvature of this "emergent" spacetime is described by Einstein's theory of general relativity.
The BH entropy is proportional to the size of $\mathbb{T}^2$, and hence the surface area of the black hole. Mapping to SYK applies when temperature $\ll 1/(\text{size of } \mathbb{T}^2)$. 

$$S = \int d^4x \sqrt{-\hat{g}} \left( \hat{R} + 6/L^2 - \frac{1}{4} \hat{F}_{\mu\nu} \hat{F}^{\mu\nu} \right)$$
AdS$_2 \times \mathbb{T}^2$
\[ ds^2 = \frac{(d\zeta^2 - dt^2)}{\zeta^2} + d\vec{x}^2 \]

SL(2,R) is the isometry group of AdS$_2$: 
\[ ds^2 = \frac{(d\tau^2 + d\zeta^2)}{\zeta^2} \]

is invariant under

\[ \tau' + i\zeta' = \frac{a(\tau + i\zeta) + b}{c(\tau + i\zeta) + d} \]

with $ad - bc = 1$. 

SYK and black holes
Equilibrium dynamics described by a theory with SL(2,R) invariance, and effective Schwarzian action, $S[h(\tau)]$, of a time reparameterization $\tau \rightarrow h(\tau)$. 

Thermalization

• If we start the SYK model from a random initial state, it reaches thermal equilibrium in a time of order $\hbar/(k_B T)$. Note that this time is independent of the coupling energies in the Hamiltonian.

A. Georges and O. Parcollet PRB 59, 5341 (1999)
A. Eberlein, V. Kasper, S. Sachdev, and J. Steinberg, arXiv:1706.07803
If we start the SYK model from a random initial state, it reaches thermal equilibrium in a time of order $\hbar/(k_B T)$. Note that this time is independent of the coupling energies in the Hamiltonian.

If we perturb a black hole, its quasi-normal modes “ring”, and decay to thermal equilibrium in a time of order $\hbar/(k_B T_H)$, where $T_H$ is the Hawking temperature.
Thermalization

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• If we perturb a black hole, its quasi-normal modes “ring”, and decay to thermal equilibrium in a time of order $\hbar/(k_B T_H)$, where $T_H$ is the Hawking temperature.

• This relaxation/thermalization time, $\tau_\varphi$, was conjectured to be the shortest possible among all many-body quantum systems

$$\tau_\varphi > C \frac{\hbar}{k_B T}$$

S. Sachdev, Quantum Phase Transitions, Cambridge (1999)
Many-body quantum chaos

- In classical chaos, we measure the sensitivity of the position at time $t$, $q(t)$, to variations in the initial position, $q(0)$, i.e. we measure

$$\left( \frac{\partial q(t)}{\partial q(0)} \right)^2 = (\{q(t), p(0)\}_{\text{P.B.}})^2$$
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• By analogy, we define $\tau_L$ as the **Lyapunov time** over which the wavefunction of a quantum system is scrambled by an initial perturbation. This scrambling can be measured by

$$\left\langle \left| [\hat{A}(x, t), \hat{B}(0, 0)] \right|^2 \right\rangle \sim \exp \left( \frac{1}{\tau_L} \left[ t - \frac{|x|}{v_B} \right] \right),$$

where $v_B$ is the ‘butterfly velocity’. This time $\tau_L$ was argued to obey lower bound

$$\tau_L \geq \frac{1}{2\pi} \frac{\hbar}{k_B T}.$$  

There is no analogous bound in classical mechanics.

A. I. Larkin and Y. N. Ovchinnikov, JETP **28**, 6 (1969)

Many-body quantum chaos

- The SYK model, and black holes in Einstein gravity, saturate the bound on the Lyapunov time

\[ \tau_L = \frac{\hbar}{2\pi k_B T} \]

S. Shenker and D. Stanford, 1306.0622
A. Kitaev, unpublished
J. Maldacena and D. Stanford, 1604.07818
Quantum matter without quasiparticles:

- No quasiparticle decomposition of low-lying states:
  \[ E \neq \sum_{\alpha} n_{\alpha} \varepsilon_{\alpha} \]
  \[ + \sum_{\alpha, \beta} F_{\alpha \beta} n_{\alpha} n_{\beta} + \ldots \]

- Thermalization and many-body chaos in the shortest possible time of order \( \hbar/(k_B T) \).
The basic features can be determined by a simple power-counting. Considering for simplicity mentioned heavy fermion systems. We assume physics, here as we vary the temperature, two distinctive "SYK and thermal conductivities completely governed by different fermion "pair hopping" interactions. They obtained electrical tended the SYK model to higher spatial dimensions by coupling with other\]

\[ H = \sum_{x} \sum_{i<j,k<l} U_{ijkl} x c_{i}^{\dagger} c_{j}^{\dagger} c_{x} c_{lx} + \sum_{\langle xx' \rangle} \sum_{i,j} t_{ij,xx'} c_{i,x}^{\dagger} c_{j,x'} 

\[ |U_{ijkl}|^2 = \frac{2U^2}{N^3} \quad \quad |t_{ij,x,x'}|^2 = \frac{t_0^2}{N}. \]
Fermi liquid

\[ R = R_0 + AT^2 \]

for \( T \ll E_c \)

Crossover from heavy FL to strange metal

- Small coherence scale \( E_c = t^2/U \)
- Heavy mass \( \gamma \sim m^*/m \sim U/t \)
- Small QP weight \( Z \sim t/U \)
- Kadowaki-Woods \( A/\gamma^2 = \) constant
- Linear in \( T \) resistivity and \( T/\kappa \), \( R \sim (h/e^2)(T/E_c) \)
- Lorenz ratio crosses over from FL to NFL value
Fermi surface coupled to a gauge field

\[ \mathcal{L}[\Psi, a] = \Psi^\dagger \left( \partial_\tau - i a_\tau - \frac{(\nabla - i \vec{a})^2}{2m} - \mu \right) \Psi + \frac{1}{2g^2} (\nabla \times \vec{a})^2 \]
Fermi surface coupled to a gauge field

\[ \mathcal{L}[\psi_{\pm}, a] = \psi_+^\dagger \left( \partial_\tau - i \partial_x - \partial_y^2 \right) \psi_+ + \psi_-^\dagger \left( \partial_\tau + i \partial_x - \partial_y^2 \right) \psi_- \]

\[ -a \left( \psi_+^\dagger \psi_+ - \psi_-^\dagger \psi_- \right) + \frac{1}{2g^2} (\partial_y a)^2 \]

Fermi surface coupled to a gauge field

Compute out-of-time-order correlator to diagnose quantum chaos

\[
f(t) = \frac{1}{N^2} \theta(t) \sum_{i,j=1}^{N} \int d^2 x \ \text{Tr} \left[ e^{-\beta H/2} \{ \psi_i(x, t), \psi_j^{\dagger}(0) \} \right] \\
\quad \times e^{-\beta H/2} \{ \psi_i(x, t), \psi_j^{\dagger}(0) \}^{\dagger} \\
\sim \exp \left( \frac{(t - x/v_B)}{\tau_L} \right)
\]
Fermi surface coupled to a gauge field

Compute out-of-time-order correlator to diagnose quantum chaos

![Diagram with fermionic operators](image)

Strongly-coupled theory with no quasiparticles and fast scrambling:

\[ \tau_L \approx \frac{\hbar}{2.48 \, k_B T} \]
\[ v_B \approx 4.1 \frac{NT^{1/3}}{e^{4/3}} \frac{v_F^{5/3}}{\gamma^{1/3}} \]

\[ D_T = \frac{\text{thermal conductivity}}{\text{specific heat at fixed density}} \approx 0.42 \, v_B^2 \tau_L \]

\( N \) is the number of fermion flavors, \( v_F \) is the Fermi velocity, \( \gamma \) is the Fermi surface curvature, \( e \) is the gauge coupling constant. More generally, we find \( D_T \sim v_B^2 \tau_L \) in a large number of holographic models.

A. A. Patel and S. Sachdev, PNAS 114, 1844 (2017); M. A. Blake, R. A. Davison, and S. Sachdev 1705.07896
Entangled quantum matter without quasiparticles

- Is there a connection between strange metals and black holes?
  Yes, the SYK model leads to an explicit duality mapping.

- Why do they have the same local equilibration time $\sim \hbar/(k_B T)$?
  Strange metals don’t have quasiparticles and thermalize rapidly;
  General relativity leads to black hole quasi-normal modes,
  whose decay time $\sim \hbar/(k_B T_H)$,
  where $T_H$ is the Hawking temperature”.

- Theoretical predictions for strange metal transport in graphene agree well with experiments