Control of microbial locomotion by boundaries and flow gradients

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Collaborators

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- Marco Polin (DAMTP/Warwick)
- Francis Woodhouse (DAMTP)

- Julia Yeomans (Oxford)
- Sebastian Heidenreich (PTB Berlin)
- Markus Baer (PTB Berlin)

- Rik Wensink (CNRS Paris)
- Hartmut Loewen (Dusseldorf)

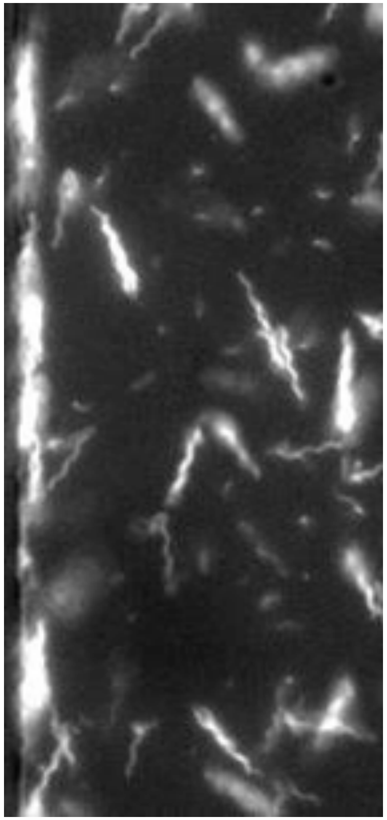
Knut Drescher (Princeton)

Hugo Wioland
(Cambridge)

Vasily Kantsler
(Warwick/SkolTech)

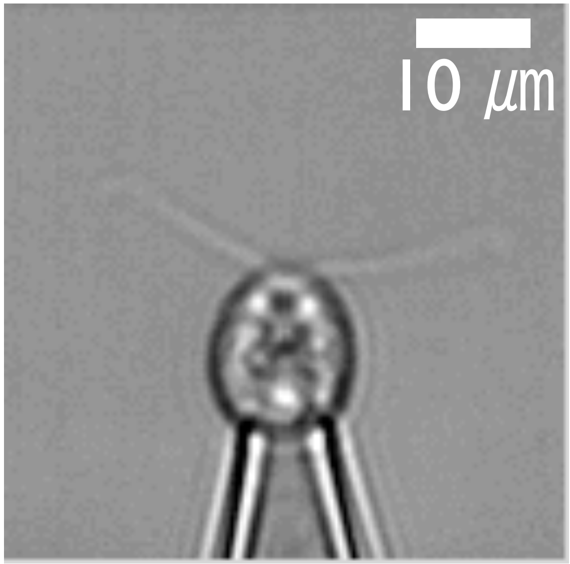
Spatio-temporal control of cell locomotion (swimming)

bacteria



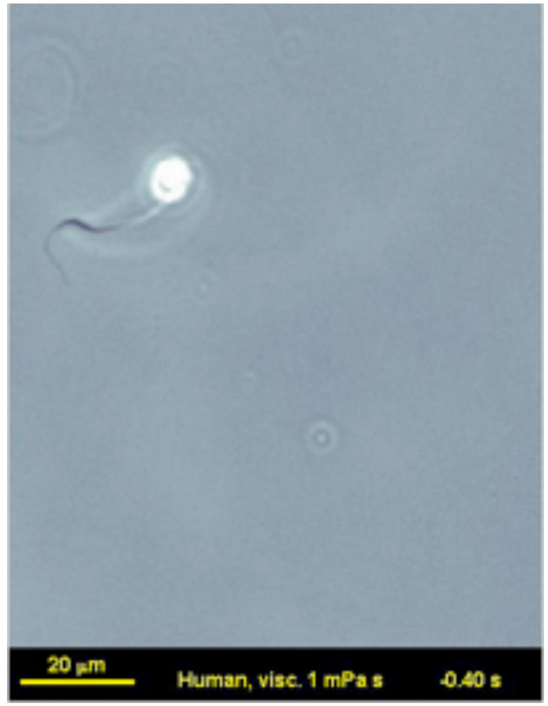
Drescher et al PNAS 2011
Dunkel et al PRL 2013
Wioland et al PRL 2014

algae



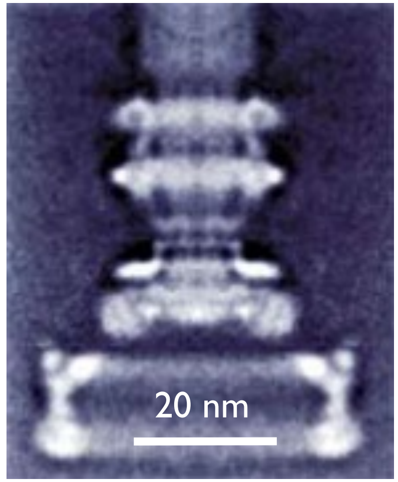
Kantsler et al 2013 PNAS

sperm



Kantsler et al 2014 (submitted)

Basic questions

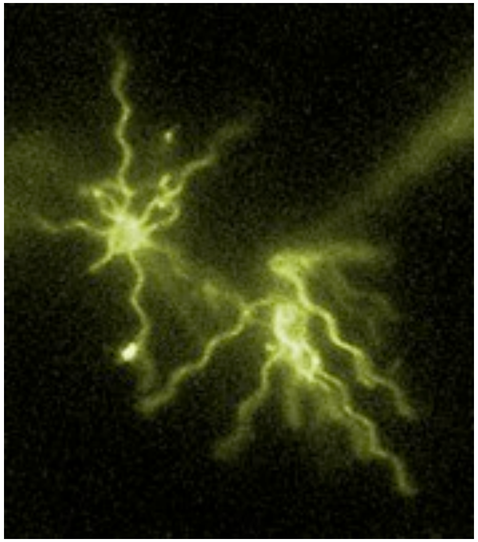


Berg (1999) Physics Today

- ✓ Swimming mechanisms ?
- Flow fields of individual organisms ?
- Collective dynamics ?
- Surface & flow interactions & control ?

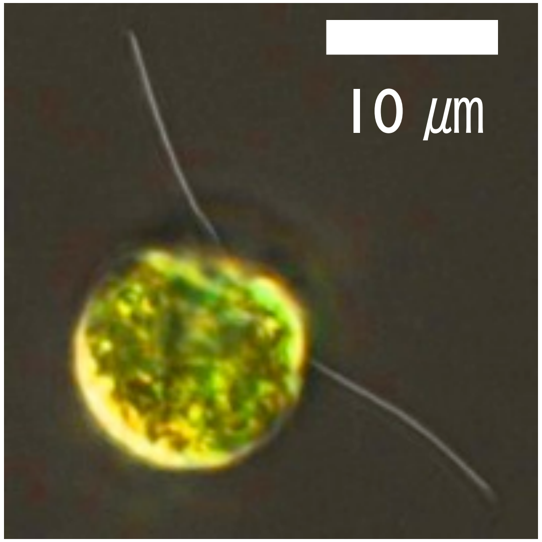
I. Flow fields of individual microorganisms

E coli



Drescher et al PNAS 2011

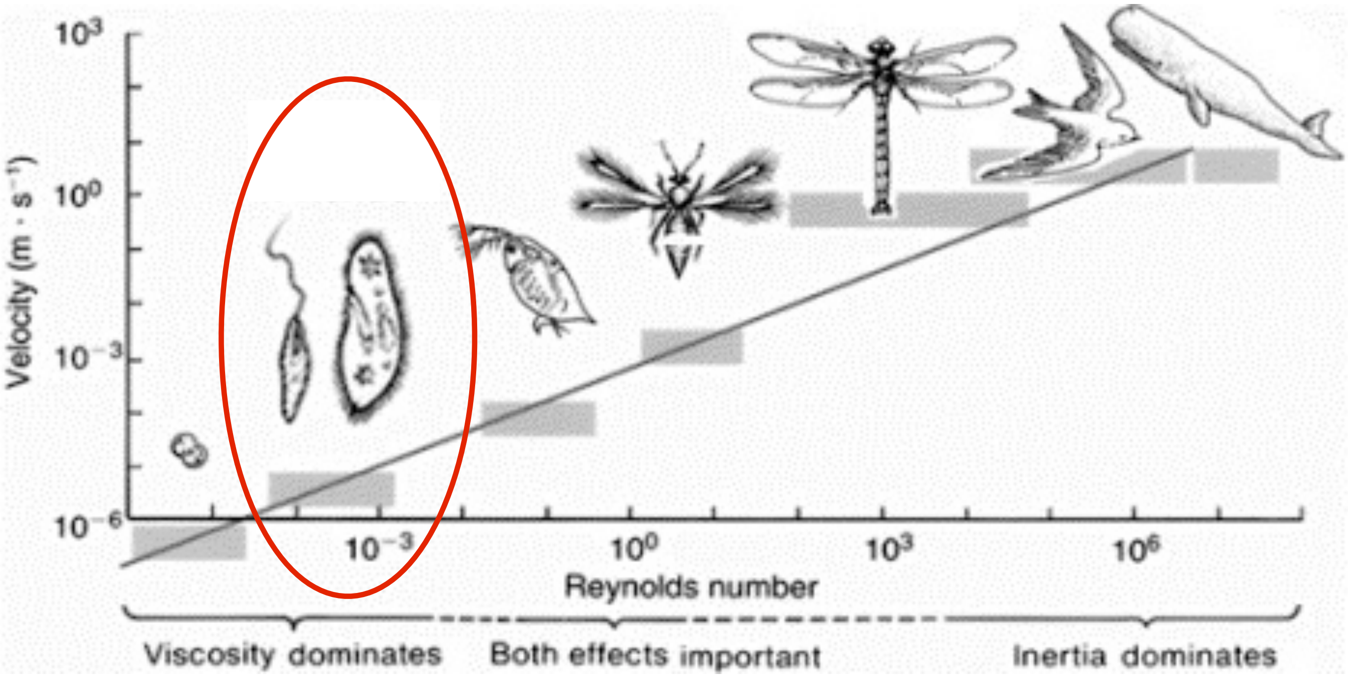
Chlamydomonas



Drescher et al PRL 2010
Guasto et al PRL 2010

Typical Reynolds numbers

$$Re = \frac{\rho U L}{\mu} = \frac{UL}{\nu}$$



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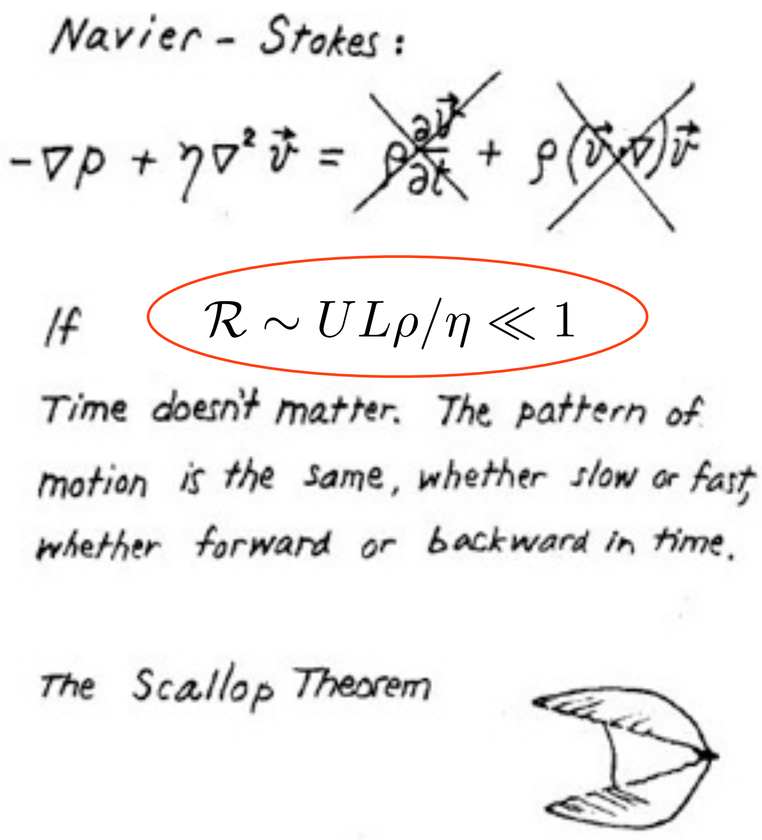
Swimming at low Reynolds number



American Journal of Physics, Vol. 45, No. 1, January 1977

Geoffrey Ingram Taylor

James Lighthill

$$
\begin{aligned}
0 &= \mu \nabla^2 \boldsymbol{u} - \nabla p + \boldsymbol{f}, \\
0 &= \nabla \cdot \boldsymbol{u}.
\end{aligned}
$$

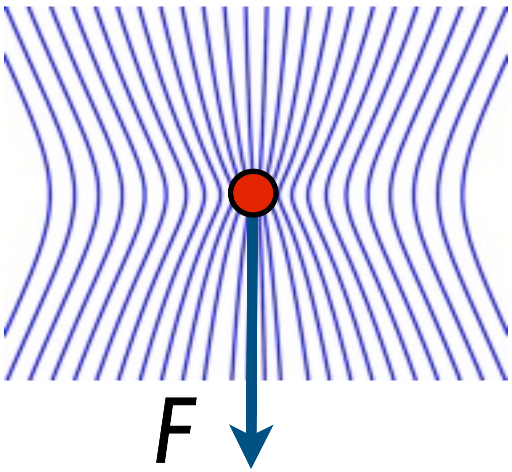
+ time-dependent BCs

Edward Purcell

Shapere & Wilczek (1987) PRL

Superposition of singularities

stokeslet

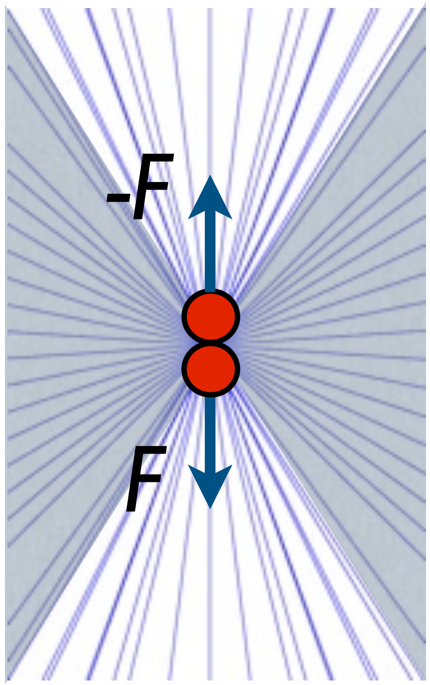


$$p(\boldsymbol{r}) = \frac{\hat{\boldsymbol{r}} \cdot \boldsymbol{F}}{4\pi r^2} + p_0$$

$$v_i(\boldsymbol{r}) = \frac{(8\pi\mu)^{-1}}{r}[\delta_{ij} + \hat{r}_i \hat{r}_j] F_j$$

flow ~ r^{-1}

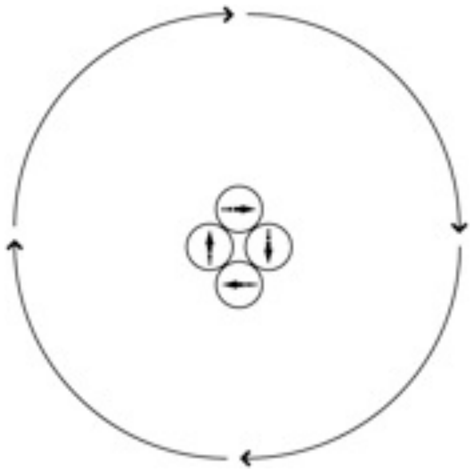
2x stokeslet = symmetric dipole



r^{-2}

'pusher'

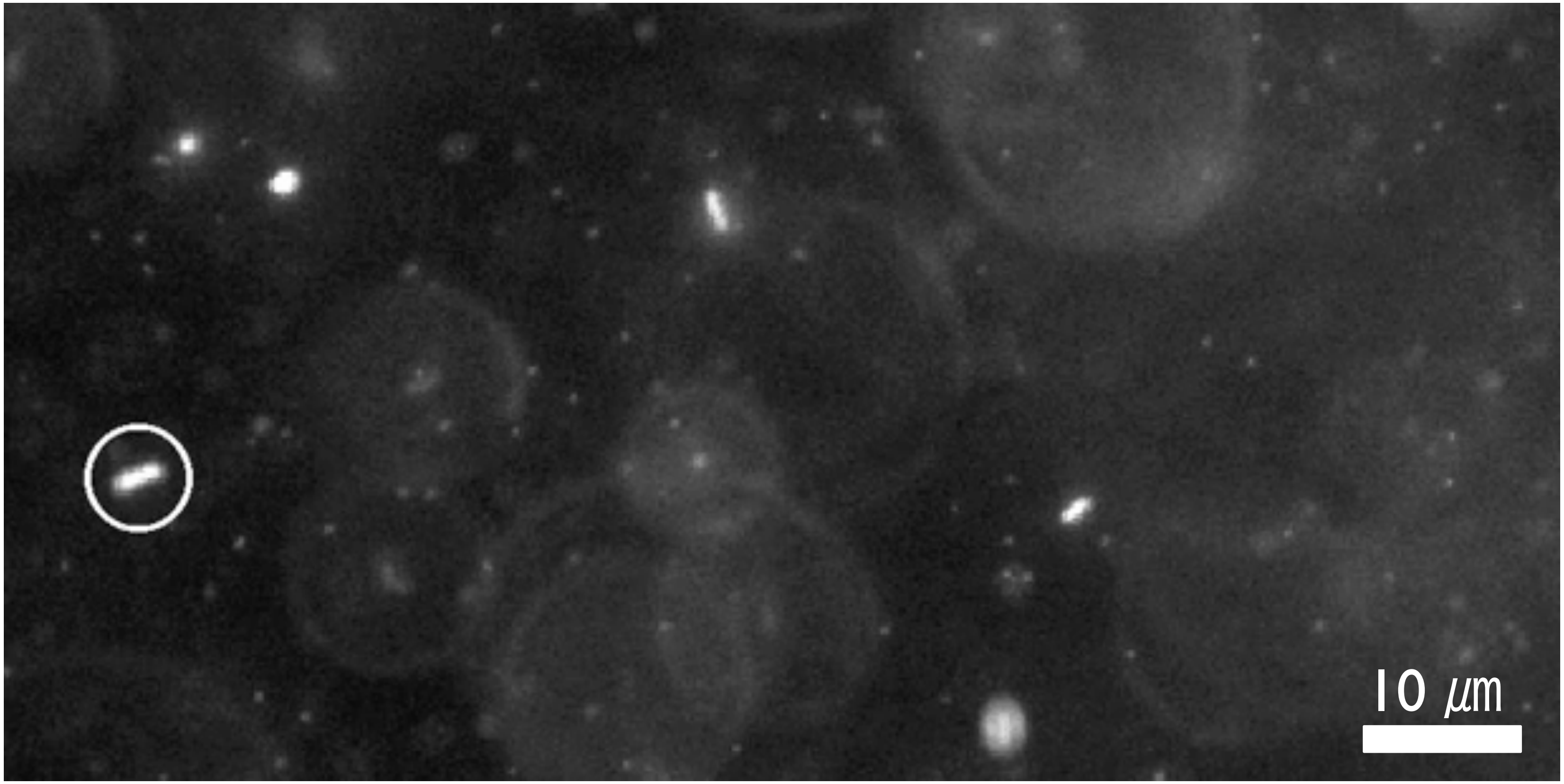
rotlet



r^{-2}

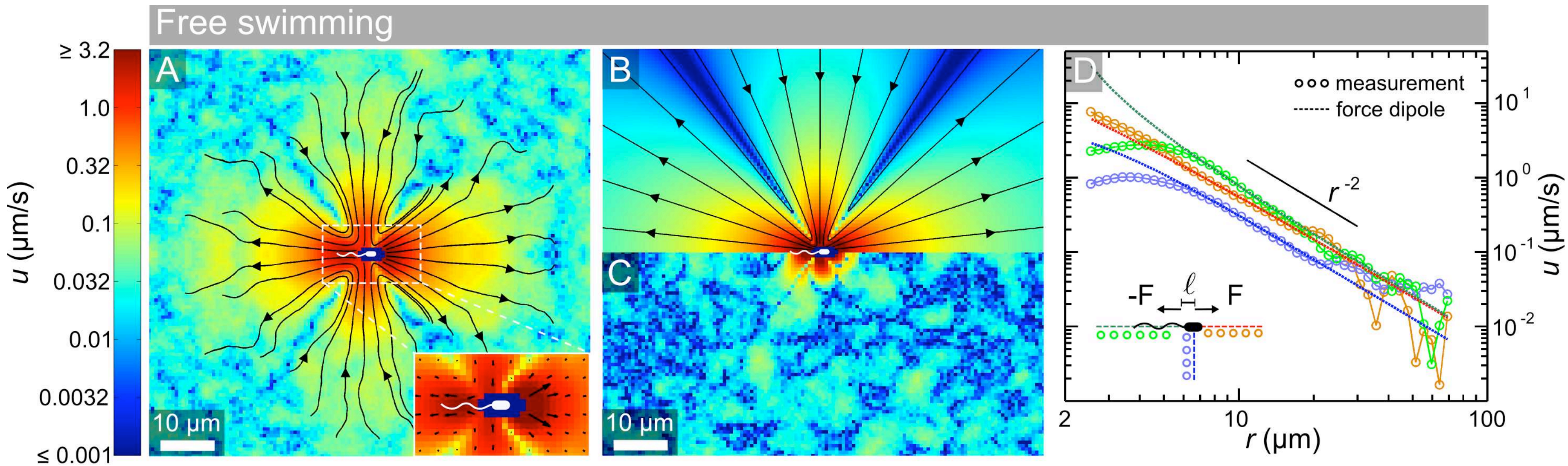
E. coli (non-tumling)

non-tumbling HCB 437



Drescher et al (2011) PNAS

E.coli (non-tumbling HCB 437)



$$\boldsymbol{u}(\boldsymbol{r}) = \frac{A}{|\boldsymbol{r}|^2}\left[3(\hat{\boldsymbol{r}}.\hat{\boldsymbol{d}})^2 - 1\right]\hat{\boldsymbol{r}}, \quad A = \frac{\ell F}{8\pi\eta}, \quad \hat{\boldsymbol{r}} = \frac{\boldsymbol{r}}{|\boldsymbol{r}|}$$

$$V_0 = 22 \pm 5 \ \ \mu\text{m/s}$$

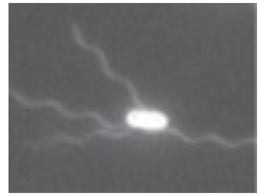
$$\ell = 1.9 \ \mu\text{m}$$

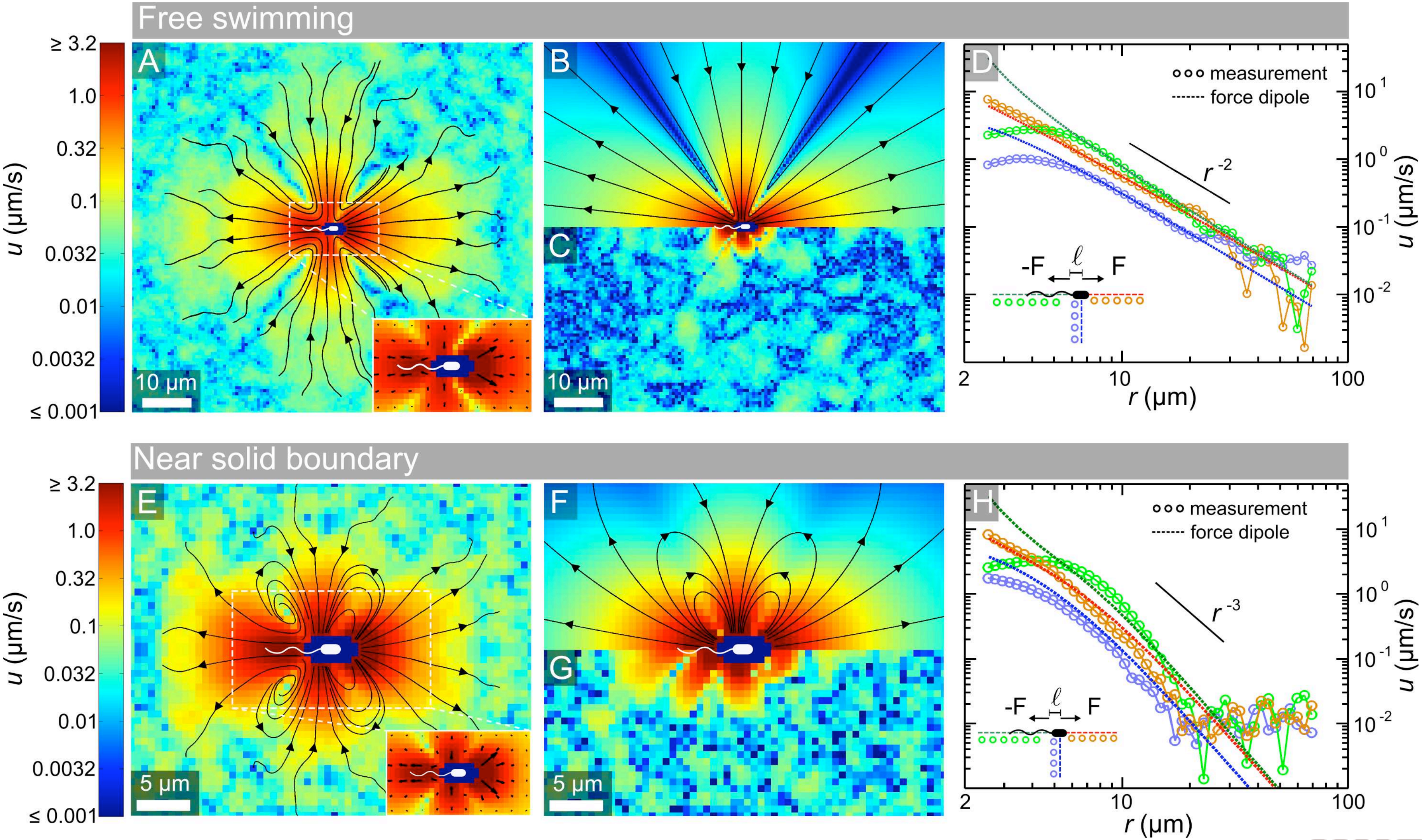
$$F = 0.42 \ \text{pN}$$

‘pusher’ dipole

Drescher et al (2011) PNAS

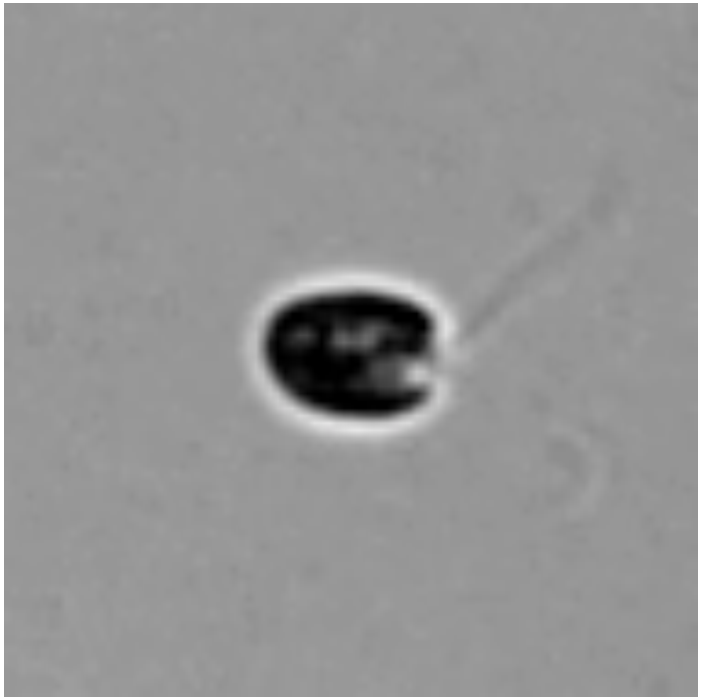
E.coli (non-tumbling HCB 437)



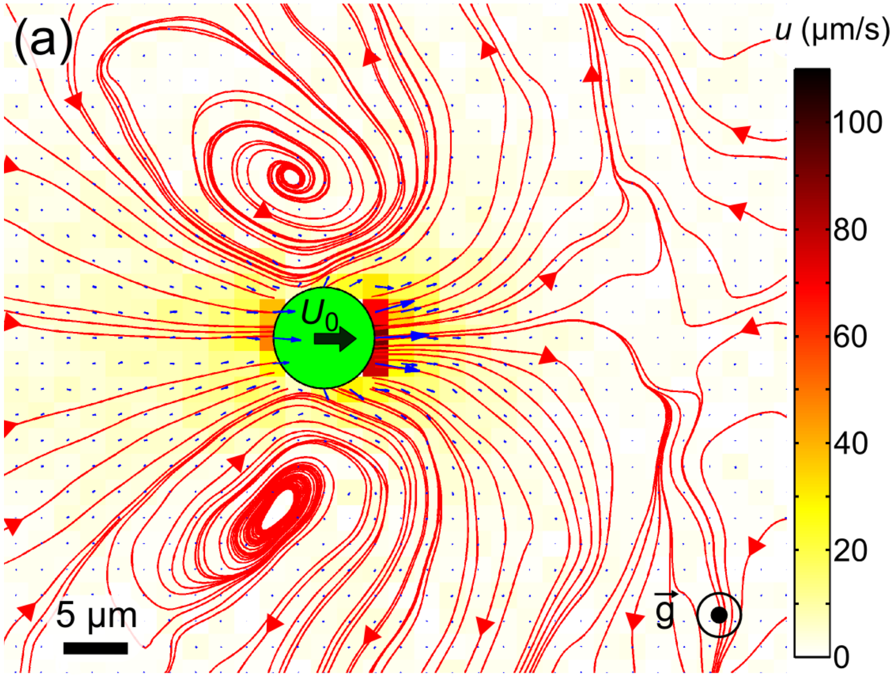


Drescher et al (2011) PNAS

Chlamydomonas



Movie: Jeff Guasto (TUFTS)

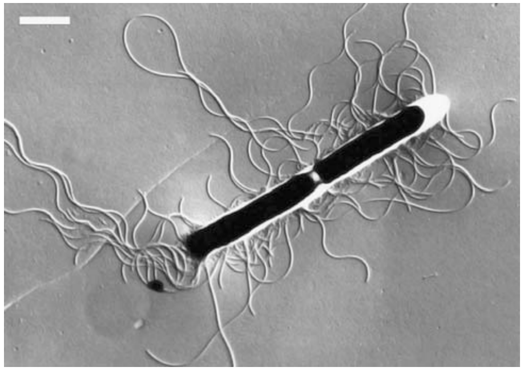


Drescher et al PRL 2010
Guasto et al PRL 2010

'puller'

size ~ 20μm
speed ~ 100μm/s
beat frequency ~30 Hz

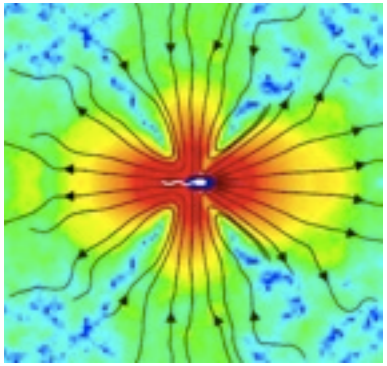
2. Control of collective bacterial swimming



Bacillus subtilis

Cisneros et al (2007) Exp Fluids

Relevant physical mechanisms



✓ hydrodynamic advection $\boldsymbol{v} \sim \frac{A}{r^2}$

✓ steric alignment

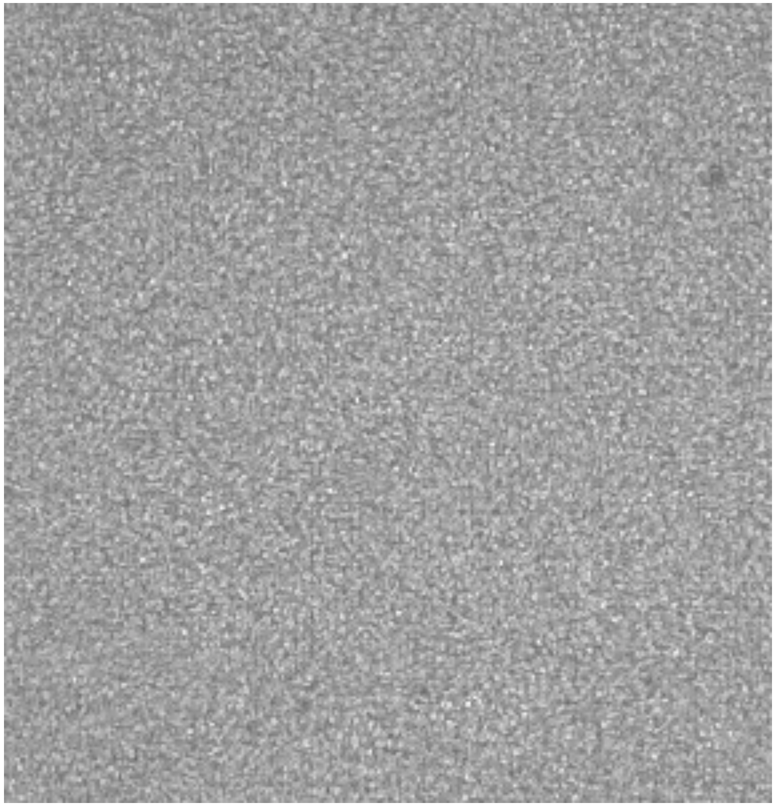
- hydrodynamic alignment less important $\omega = \nabla \times \boldsymbol{v} \sim \frac{A}{r^3}$

✓ intrinsic rotational noise
much larger than thermal noise $D_r = 0.057\ \mathrm{rad}^2/\mathrm{s}$

Drescher et al (2011) PNAS

Bacterial ‘turbulence’

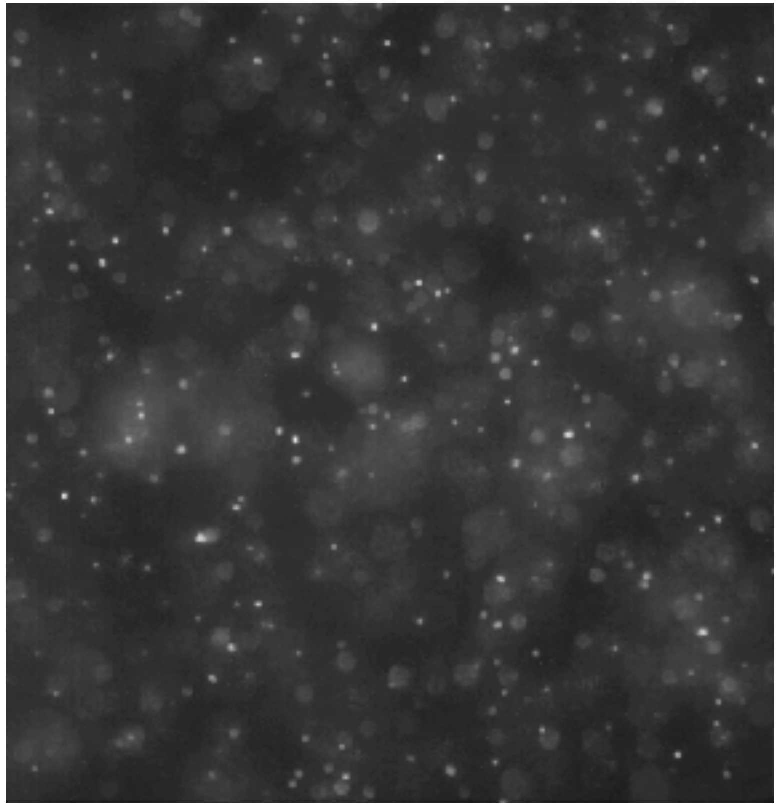
B. subtilis



bright field

Wensink et al PNAS 2012

tracers

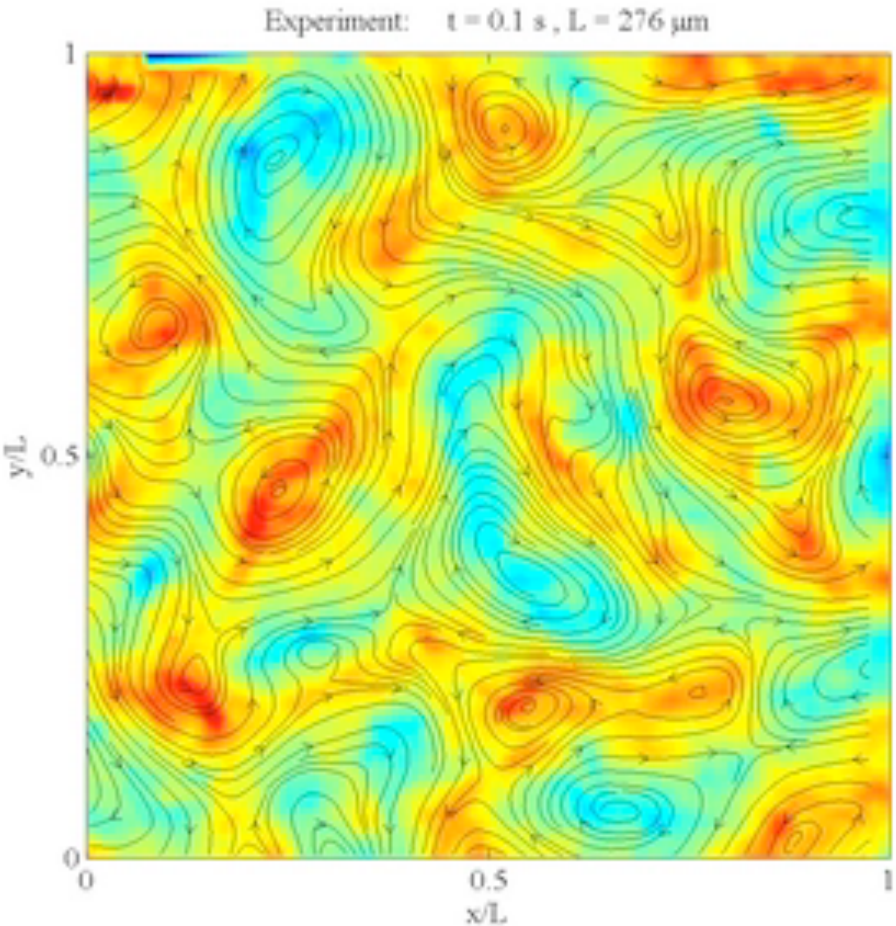


fluorescence

Dunkel et al PRL 2013

Bacterial 'turbulence'

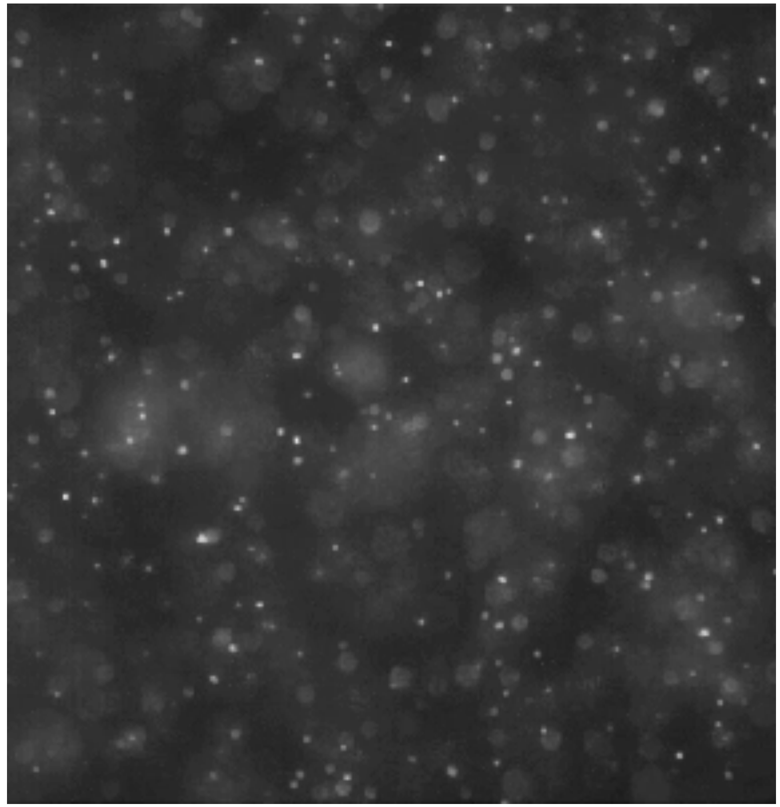
PIV



Vortex diameter ~ 70μm

Vortex life time ~ 1 sec

tracers



fluorescence

Dunkel et al PRL 2013

Minimal continuum theory for bacterial velocity field

incompressibility $\nabla \cdot \boldsymbol{v} = 0$

nematic stresses

polar alignment

$$(\partial_t + \lambda_0 \boldsymbol{v} \cdot \nabla)\boldsymbol{v} = -\nabla(p + \lambda_1 \boldsymbol{v}^2) - (\beta \boldsymbol{v}^2 + \alpha)\boldsymbol{v} +$$
$$+ \Gamma_0 \nabla^2 \boldsymbol{v} - \Gamma_2 (\nabla^2)^2 \boldsymbol{v}$$

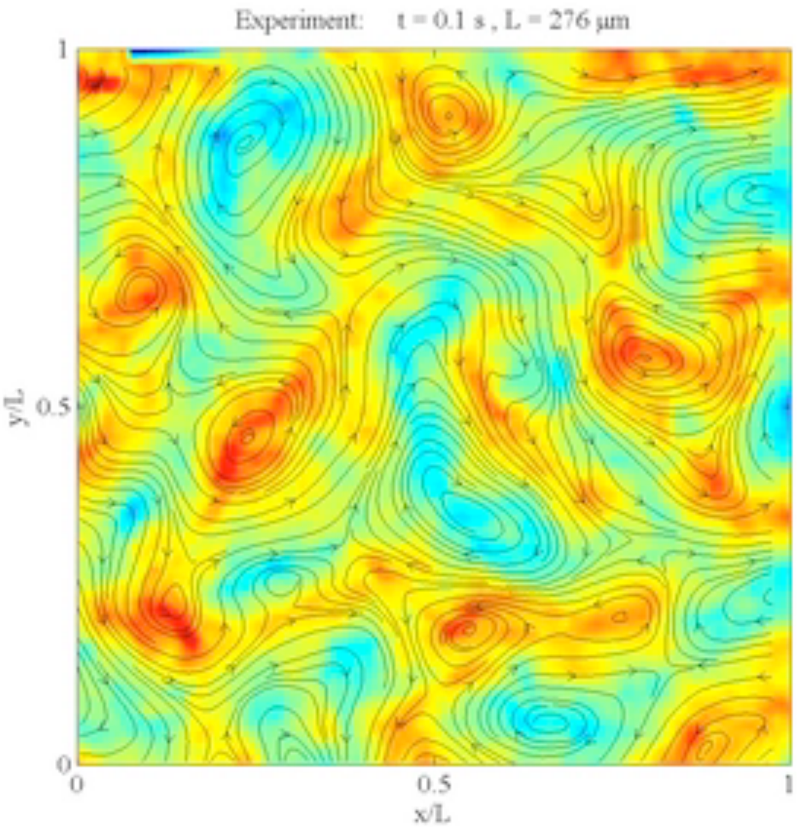
vortices

PNAS 2012

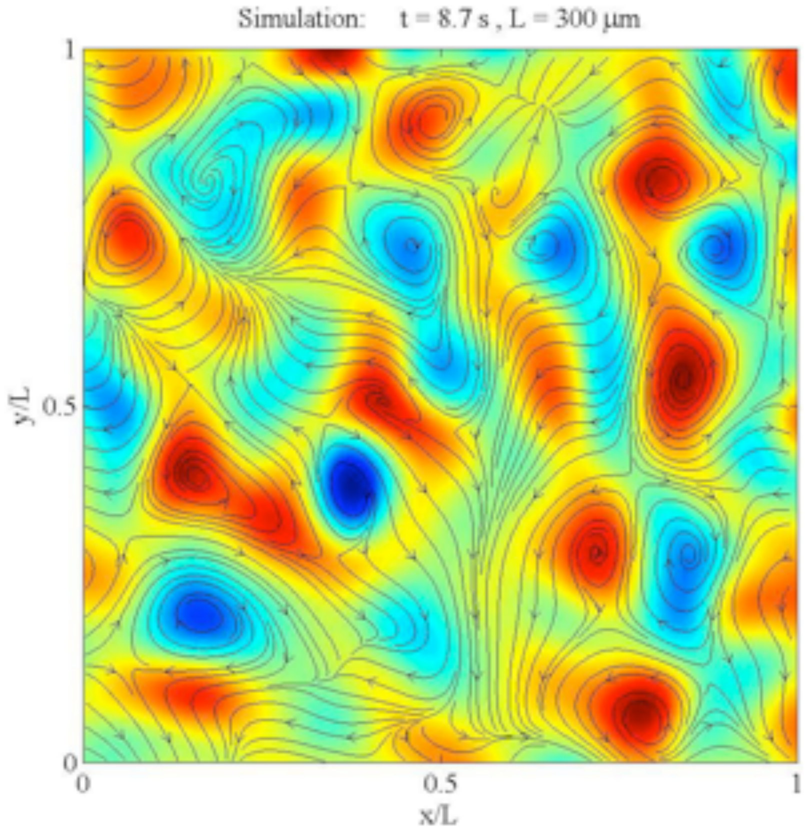
New J Phys 2013

PRL 2013

experiment *vs.* theory



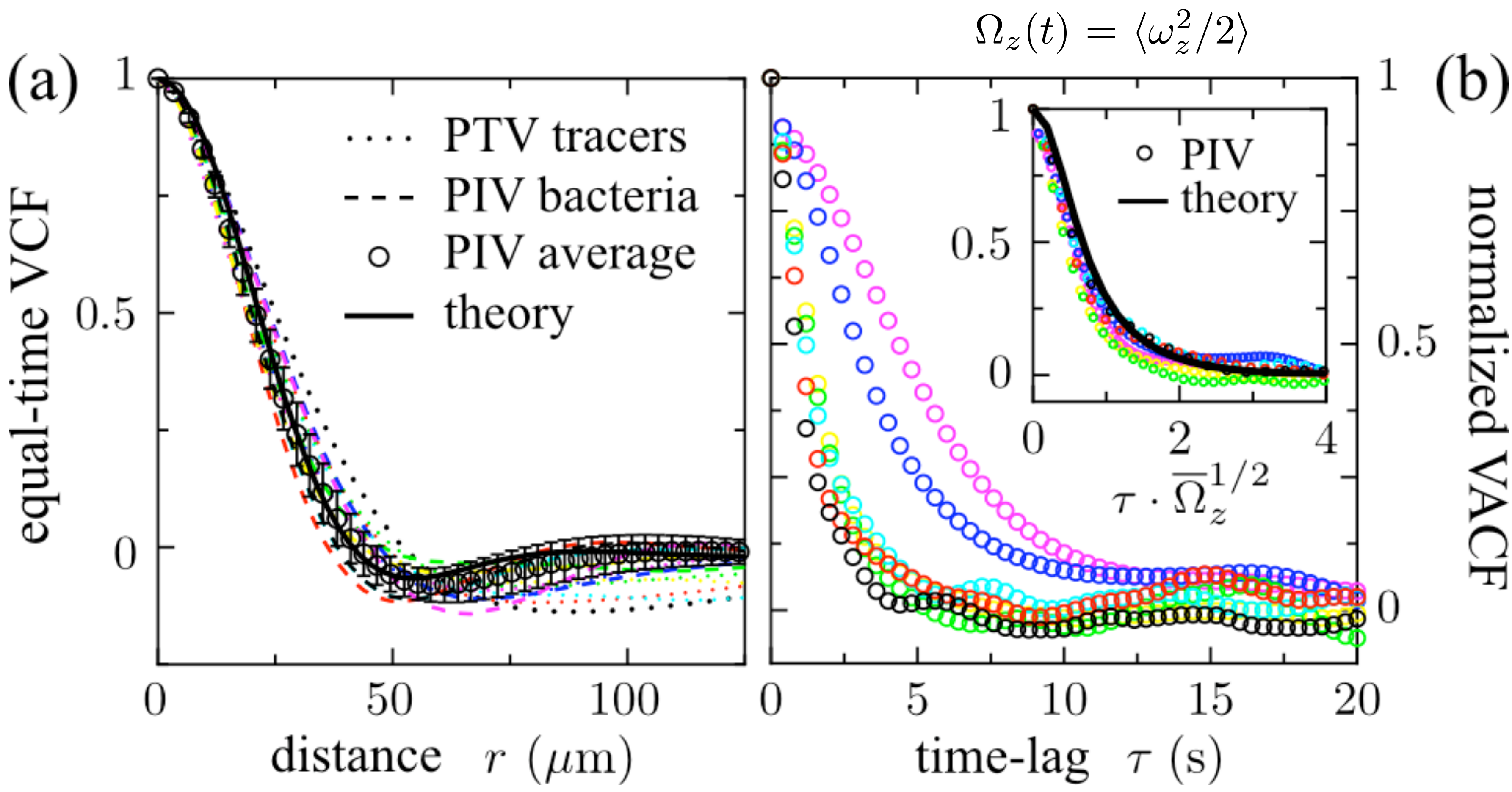
quasi-2D slice



2D slice
from 3D simulation

Dunkel et al PRL 2013

Velocity correlations



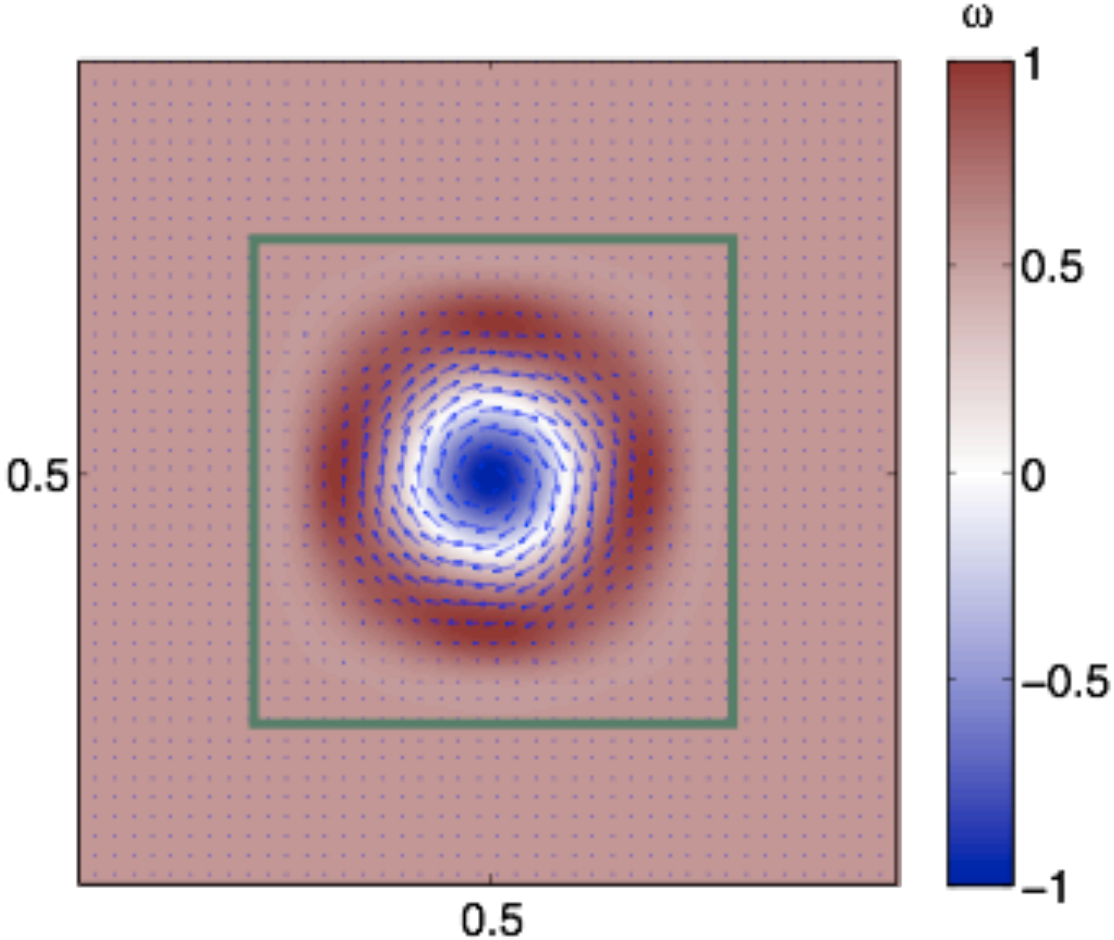
Vortex diameter ~ 70μm

Vortex life time ~ seconds

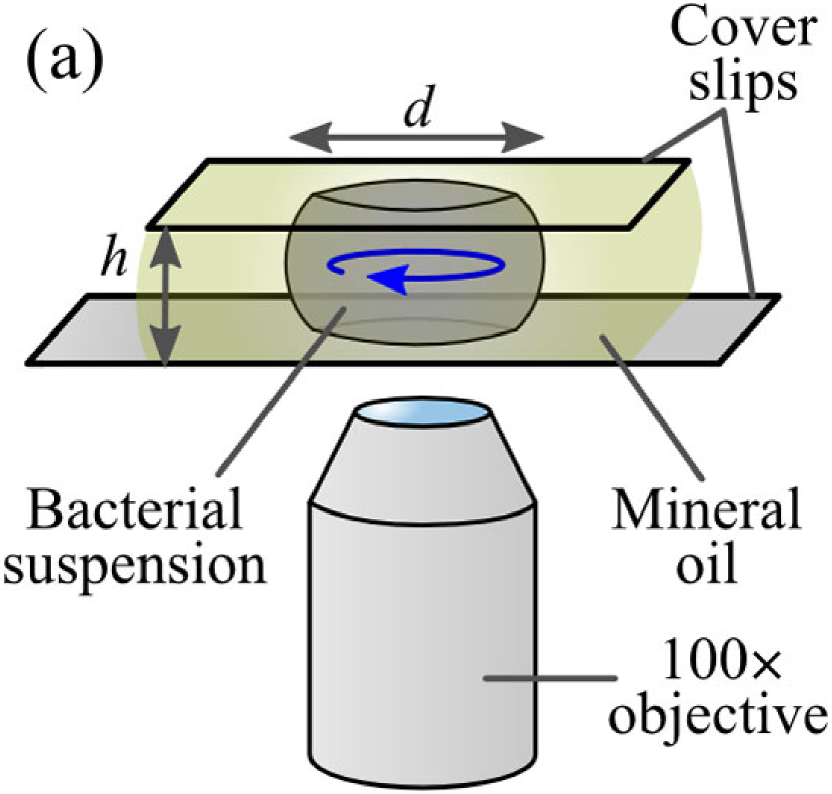
Dunkel et al PRL 2013

Can we stabilize vortices ?

Theoretical ‘prediction’ ... of many models

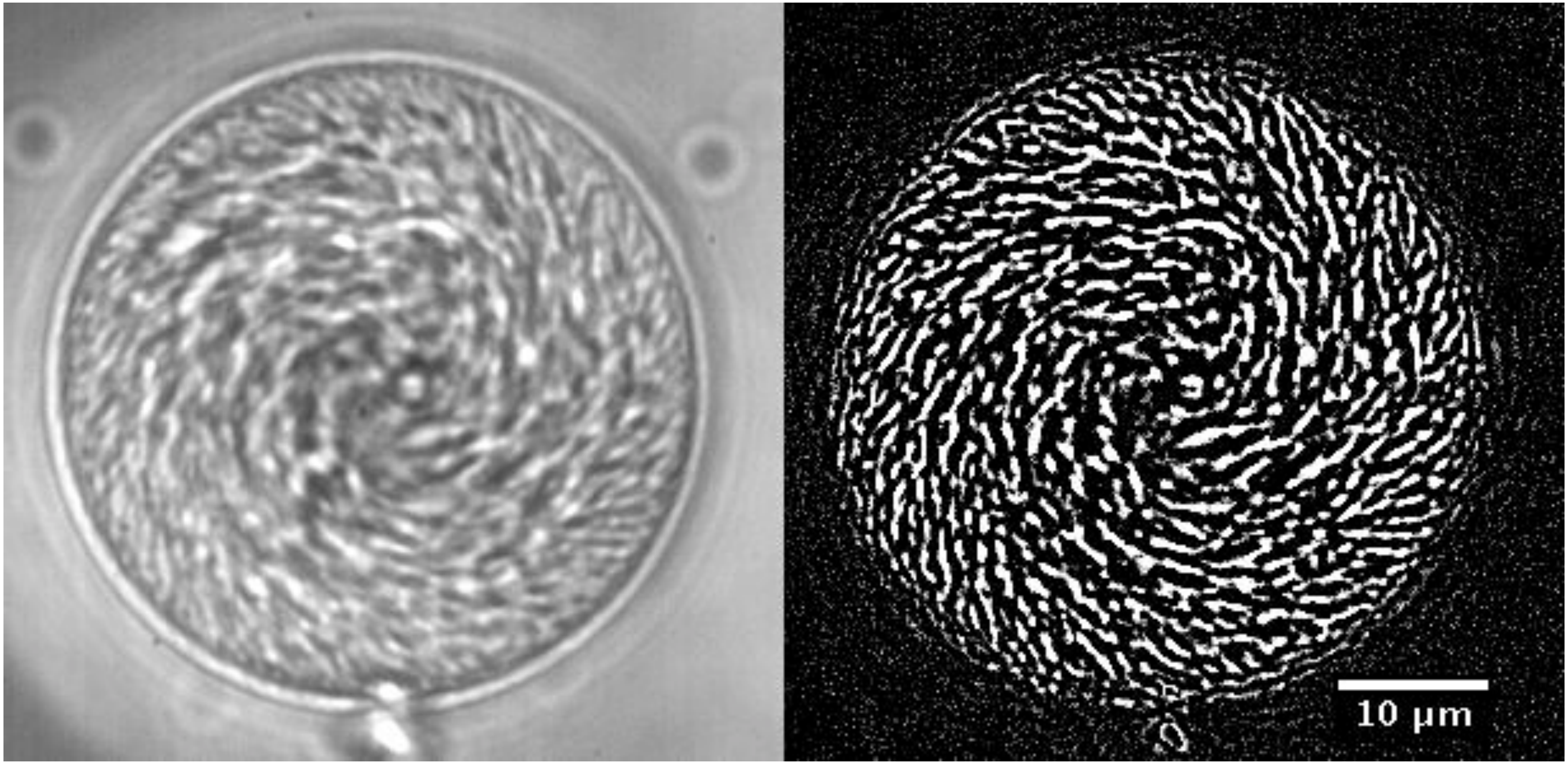


Experiment



Wioland et al (2013) PRL

Stable bacterial spiral vortex

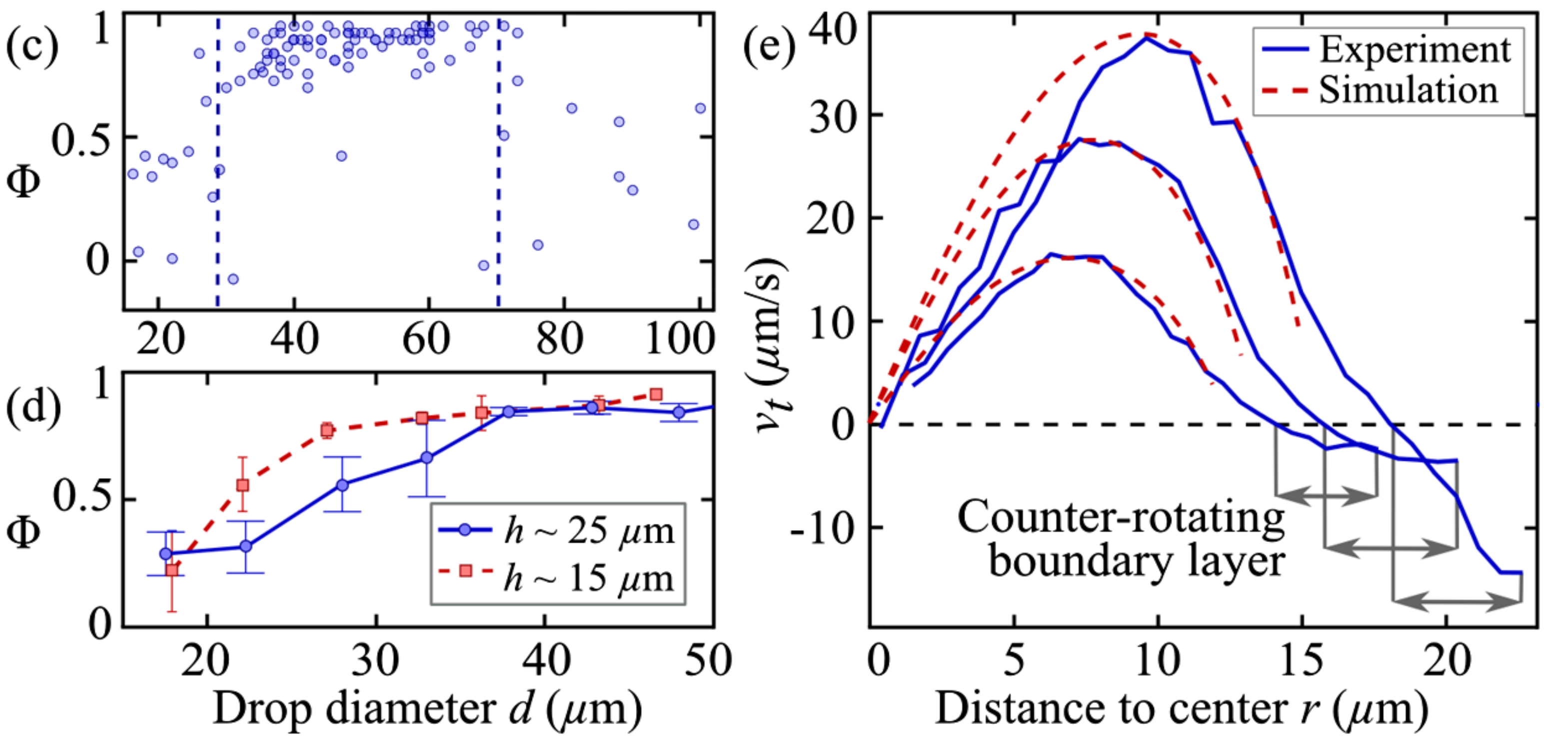


Vortex life time ~ minutes

Wioland et al (2013) PRL

Vortex characterization

$$\Phi = \frac{\sum_i |\mathbf{v}_i \cdot \mathbf{t}_i| / \sum_j \|\mathbf{v}_j\| - 2/\pi}{1 - 2/\pi}$$



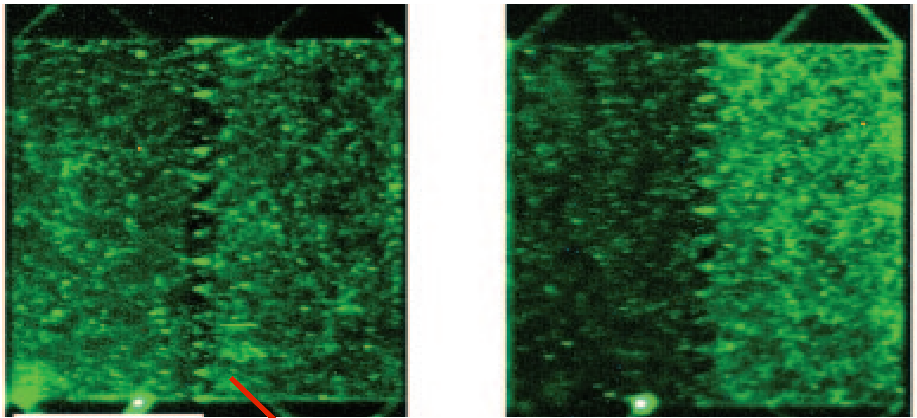
... bacteria create their own BCs !

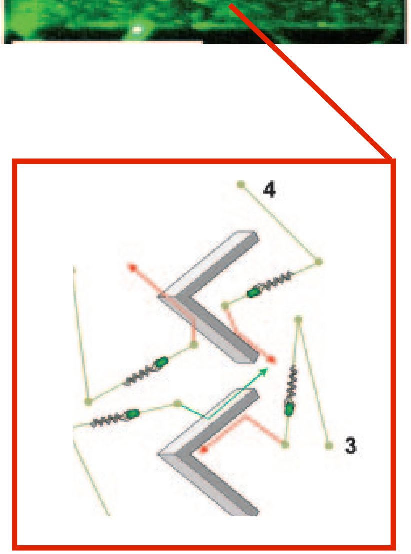
Wioland et al (2013) PRL

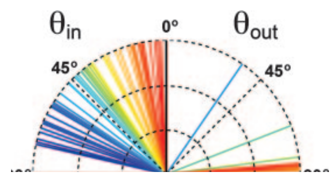
3. Control of eukaryotic locomotion

- rectification of algal swimming
- long distance navigation of sperm cells

Rectification of prokaryotic locomotion



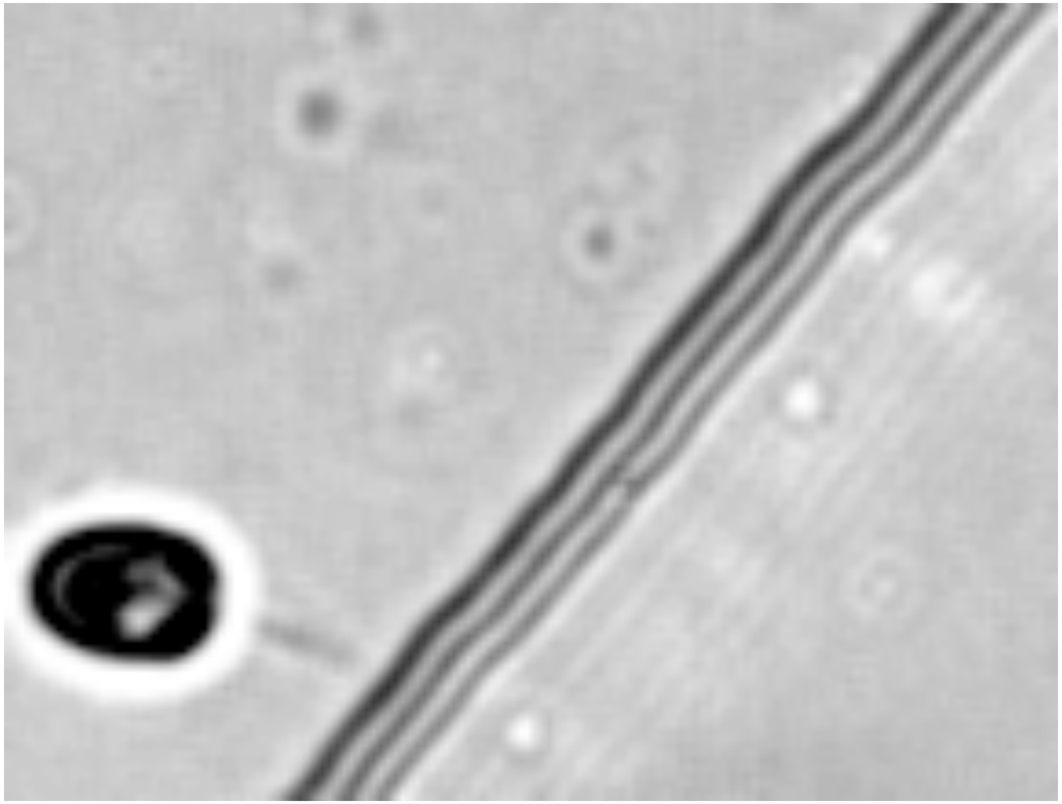


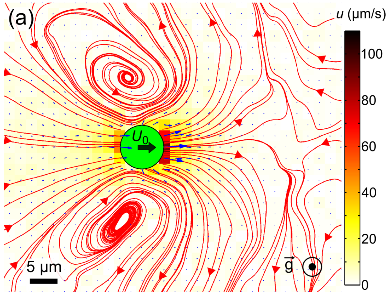


Galadja et al (2009)
J Bacteriology

Austin lab, Princeton, 2009

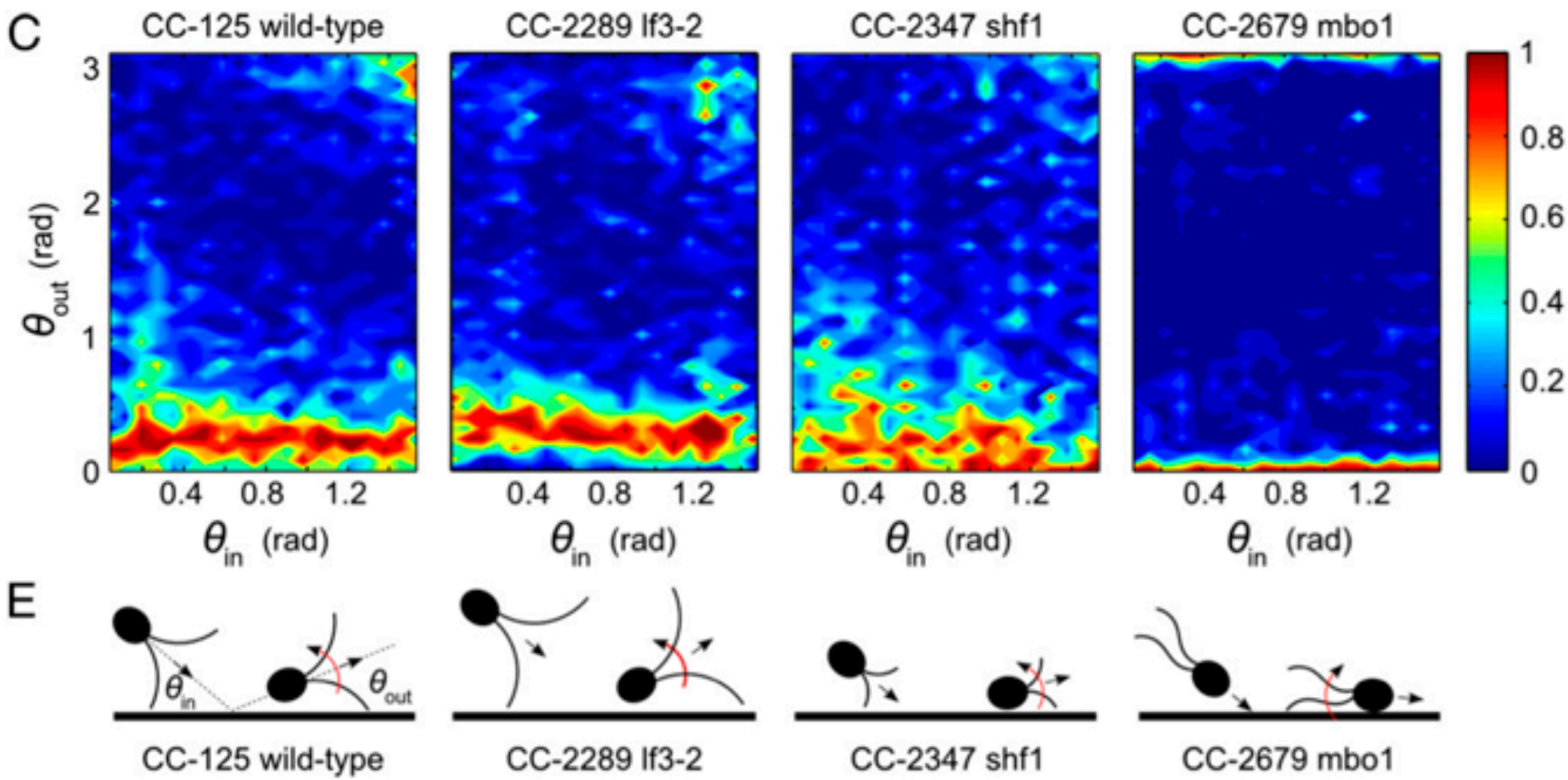
Mechanical control of algal locomotion





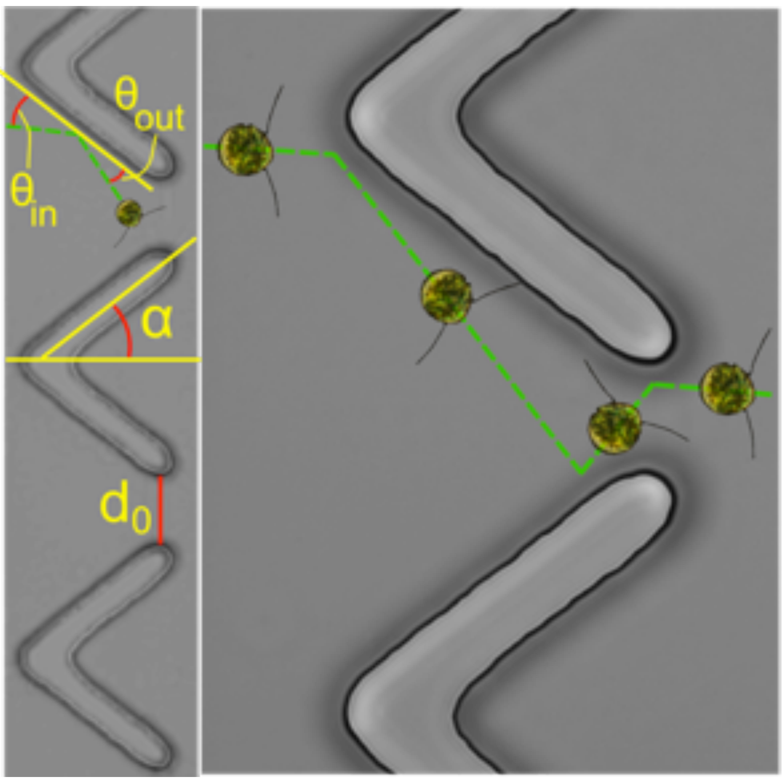
Kantsler, Dunkel, Polin, Goldstein (2012) PNAS

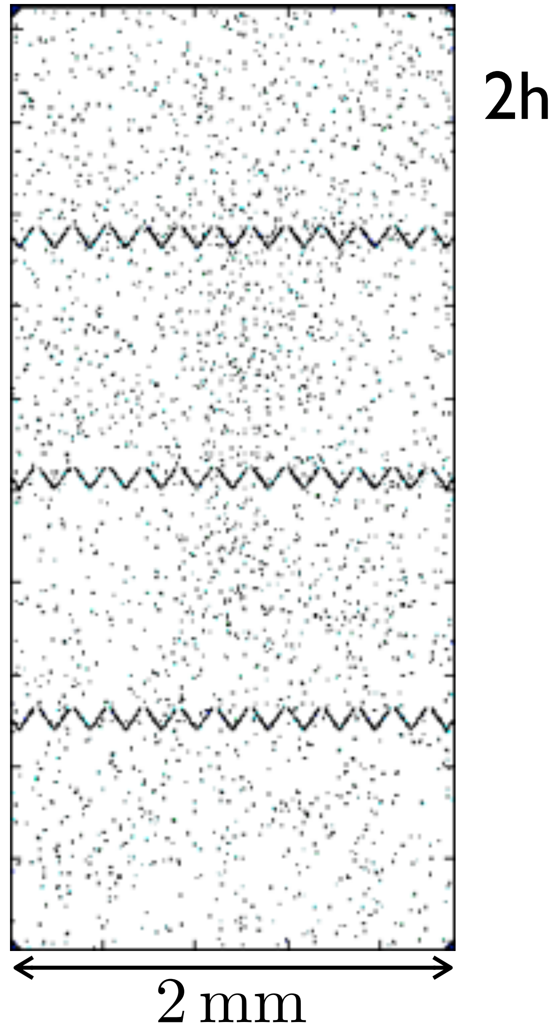
Surface scattering laws



Kantsler, Dunkel, Polin, Goldstein (2012) PNAS

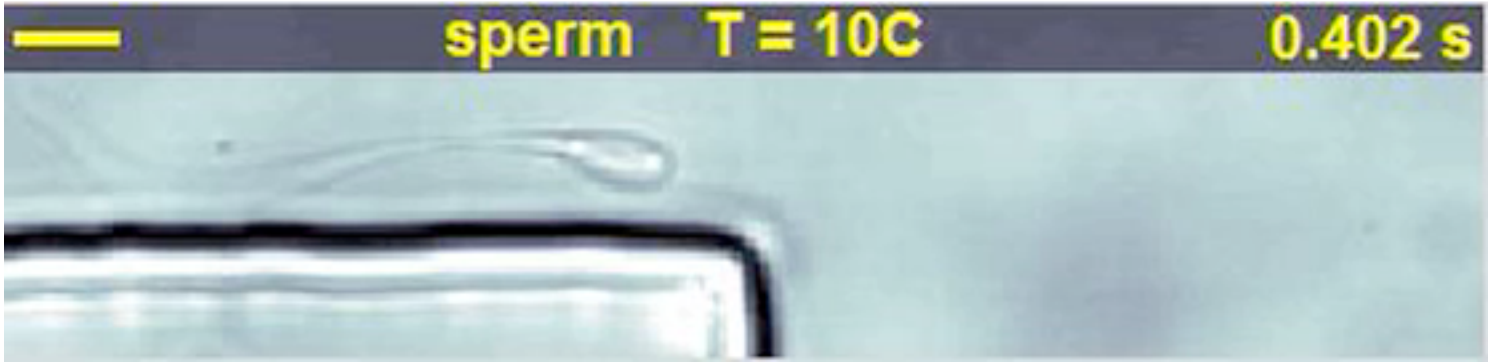
Control of algal locomotion

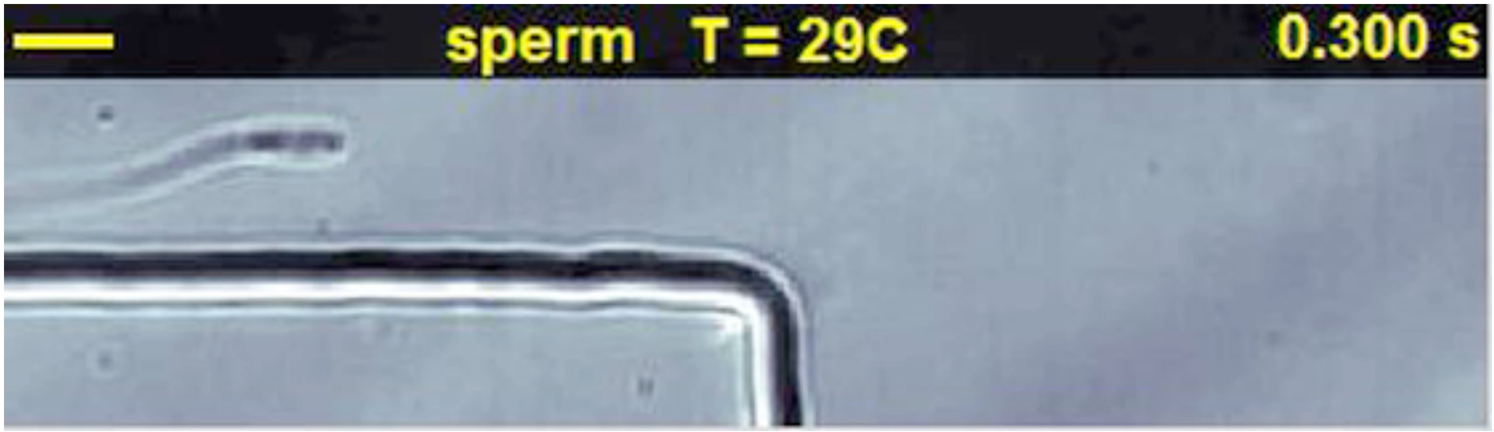


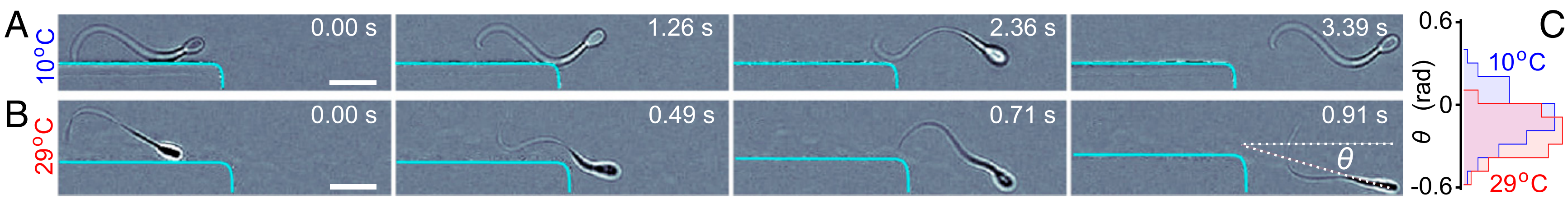


Kantsler, Dunkel, Polin, Goldstein (2012) PNAS

Sperm near surfaces

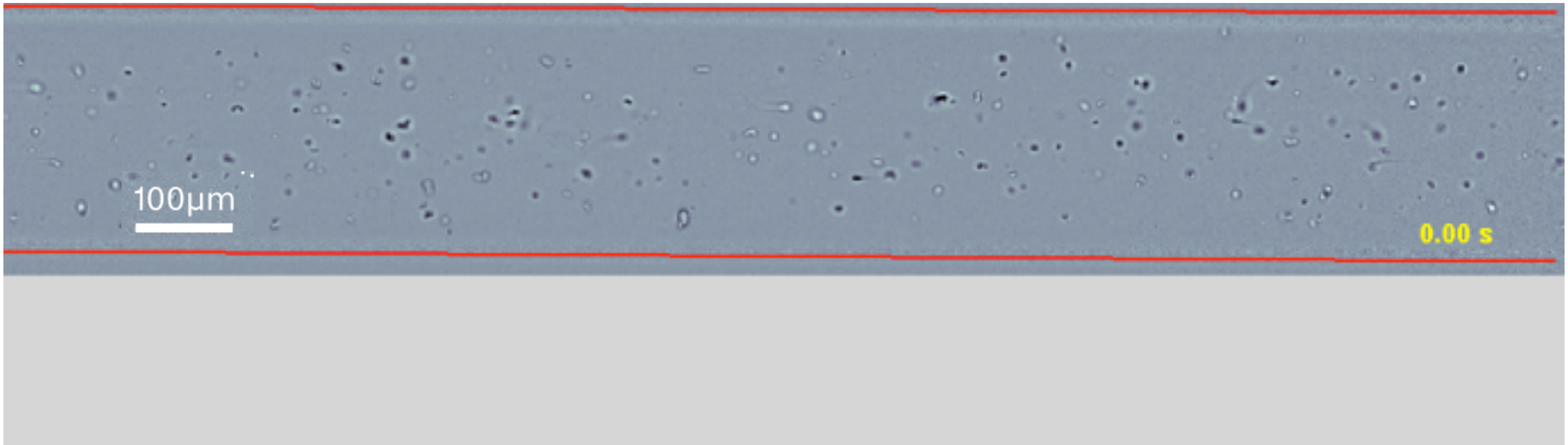






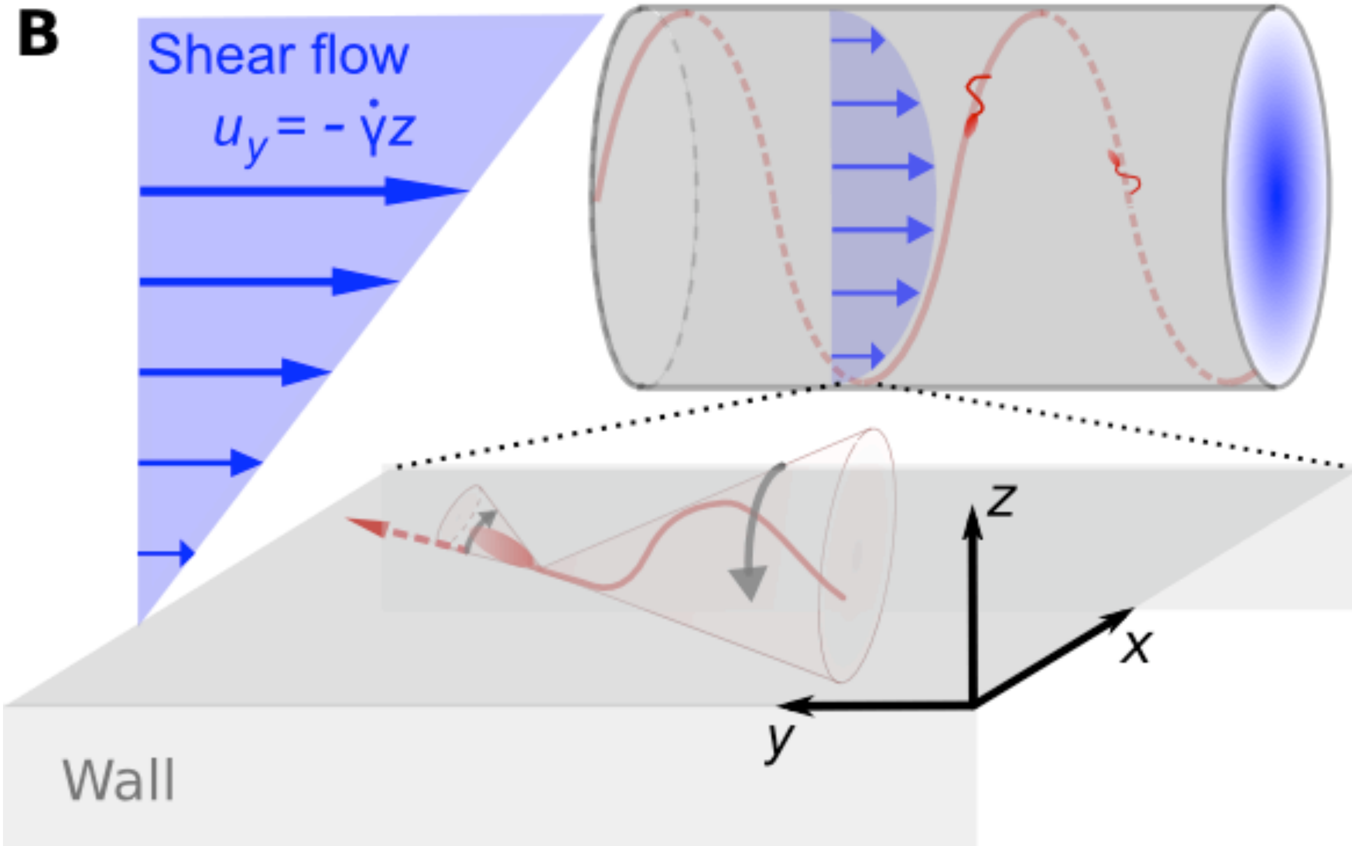
Kantsler, Dunkel, Polin, Goldstein (2012) PNAS

Surface + shear flow



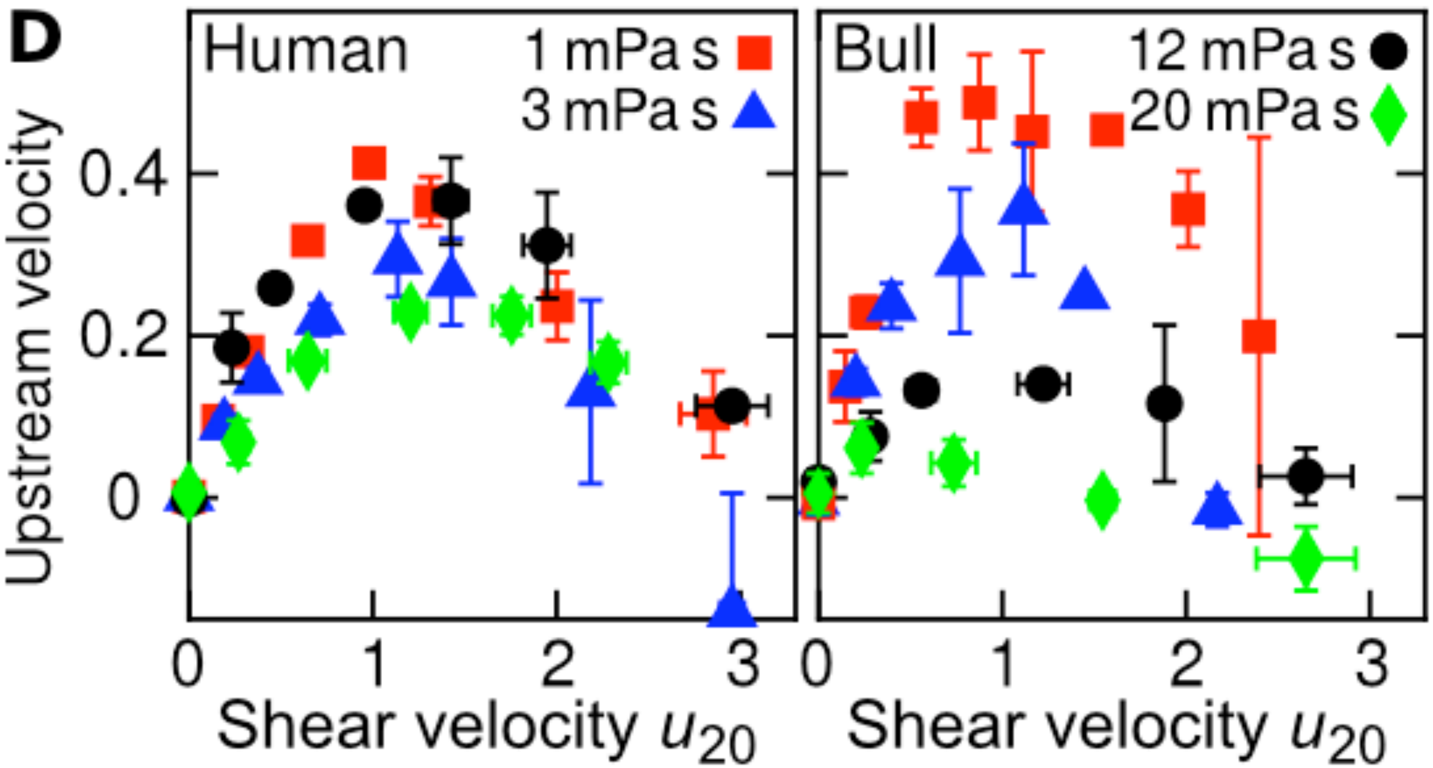
Kantsler et al 2014 (submitted)

Rheotaxis facilitates upstream navigation



Kantsler et al 2014 (submitted)

Viscosity & shear dependence



long distance navigation by rheotaxis ?

2D minimal model

Resistive force theory

$$0 = F_i = \int_0^S ds \left\| \frac{d\hat{\boldsymbol{C}}(s)}{ds} \right\| f_i(s),$$

$$0 = \tau_i = \int_0^S ds \left\| \frac{d\hat{\boldsymbol{C}}(s)}{ds} \right\| \epsilon_{ijk}[C_j(s) - X_j^*] f_k(s)$$

$$\begin{aligned} \boldsymbol{f}(s) &= \zeta_{||} \left\{ \left[\boldsymbol{u}(\boldsymbol{C}(s)) - \dot{\boldsymbol{C}}(s) \right] \cdot \boldsymbol{t}(s) \right\} \boldsymbol{t}(s) + \\ & \quad \zeta_{\perp} \left\{ \left[\boldsymbol{u}(\boldsymbol{C}(s)) - \dot{\boldsymbol{C}}(s) \right] \cdot [\boldsymbol{I} - \boldsymbol{t}(s)\boldsymbol{t}(s)] \right\} \end{aligned}$$

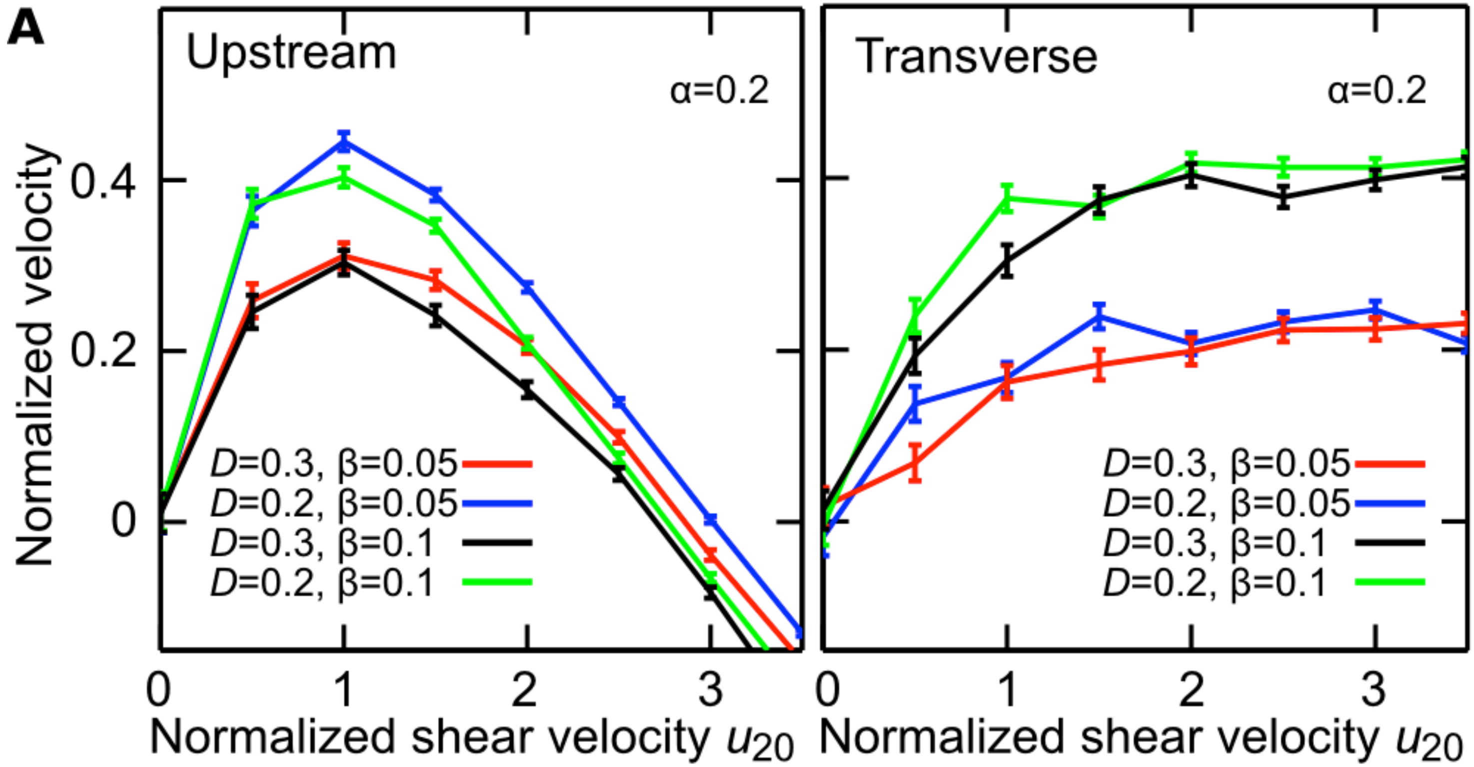
+ some approximations + noise gives to leading order

$$\dot{\boldsymbol{R}} = V\boldsymbol{N} + \sigma\overline{U}\boldsymbol{e}_y,$$

$$\dot{\boldsymbol{N}} = \sigma\dot{\gamma}\alpha \begin{pmatrix} N_x N_y \\ N_y^2 - 1 \end{pmatrix} + \sigma\dot{\gamma}\chi\beta \begin{pmatrix} N_x^2 - 1 \\ N_x N_y \end{pmatrix} + (2D)^{1/2}(\boldsymbol{I} - \boldsymbol{N}\boldsymbol{N}) \cdot \boldsymbol{\xi}(t).$$

Kantsler et al 2014 (submitted)

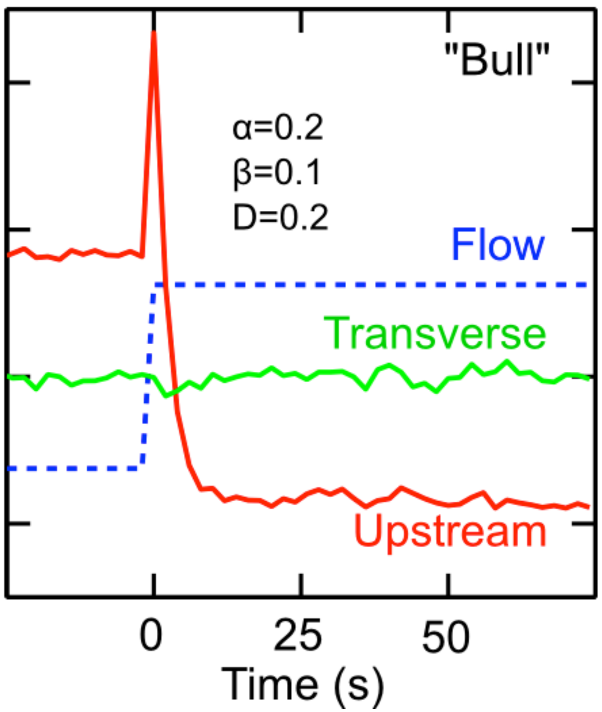
2D minimal model

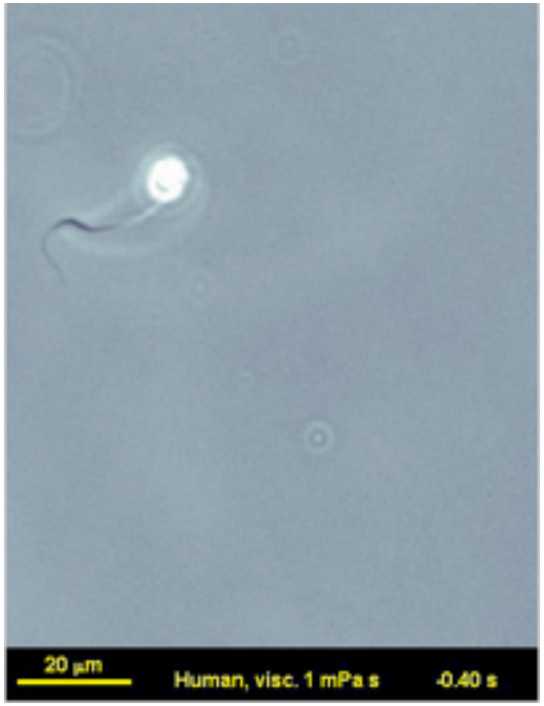


Kantsler et al 2014 (submitted)

Response to flow switch

model

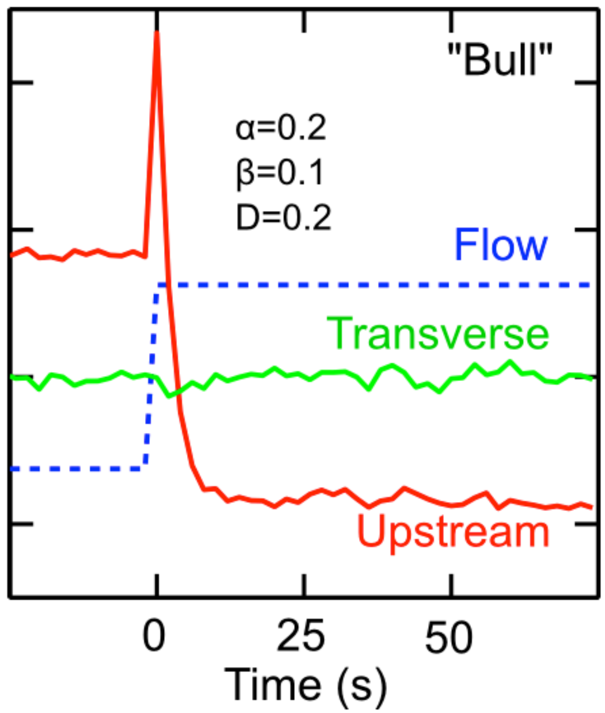




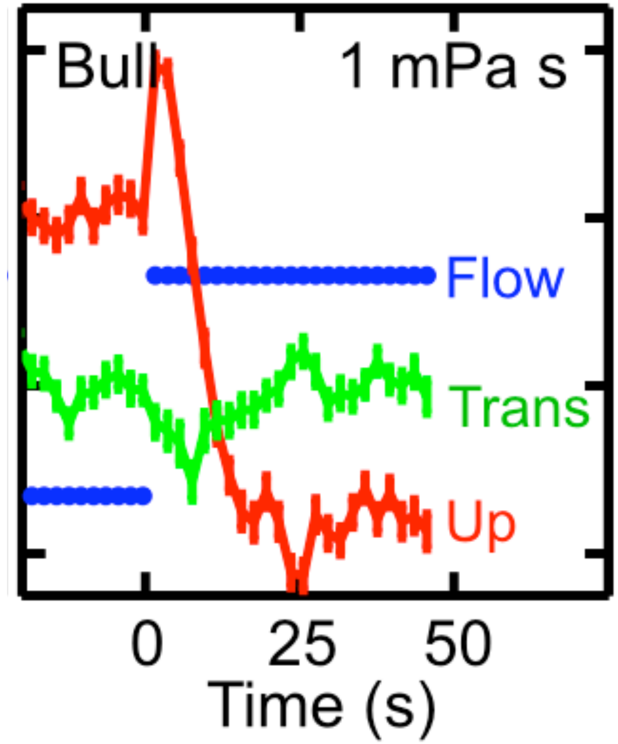
Kantsler et al 2014 (submitted)

Response to flow switch

model



experiment

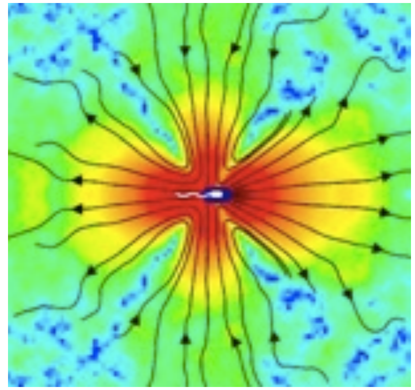


Summary

Bacteria

Drescher et al (2011) PNAS
Wensink et al (2012) PNAS
Dunkel et al (2013) PRL
Wioland et al (2013) PRL

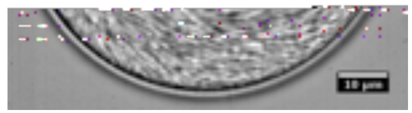
- weak dipole flows
- vortex stabilization by boundaries



Algae

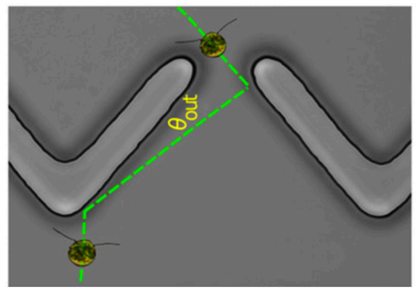
Kantsler et al (2013) PNAS

- inelastic ‘geometric’ scattering
- rectification in ratchets



Sperm

Kantsler et al (2014) submitted



- boundary interactions & rheotaxis
- swim upstream on spirals