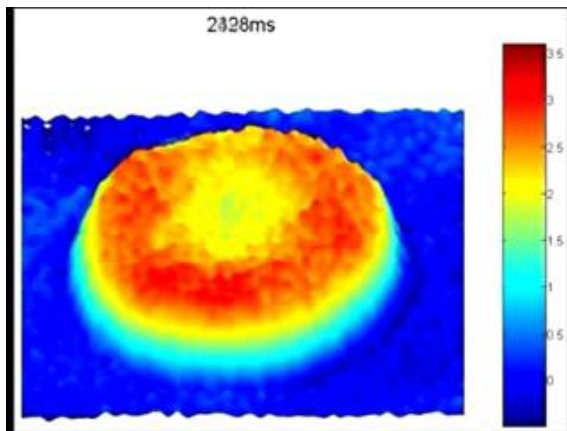


KITP, Santa-Barbara
June 21, 2011

Effective Temperature of Active systems: Red Blood Cell Membrane Fluctuations



Nir Gov



מכון ויצמן למדע
WEIZMANN INSTITUTE OF SCIENCE

FACULTY OF CHEMISTRY
CHEMICAL PHYSICS Department

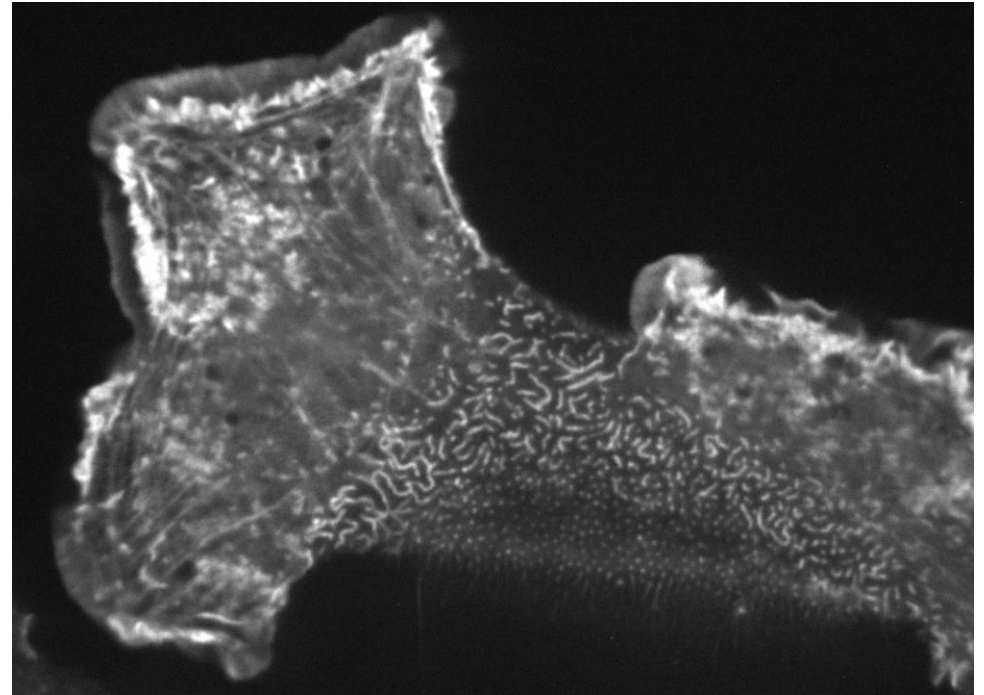
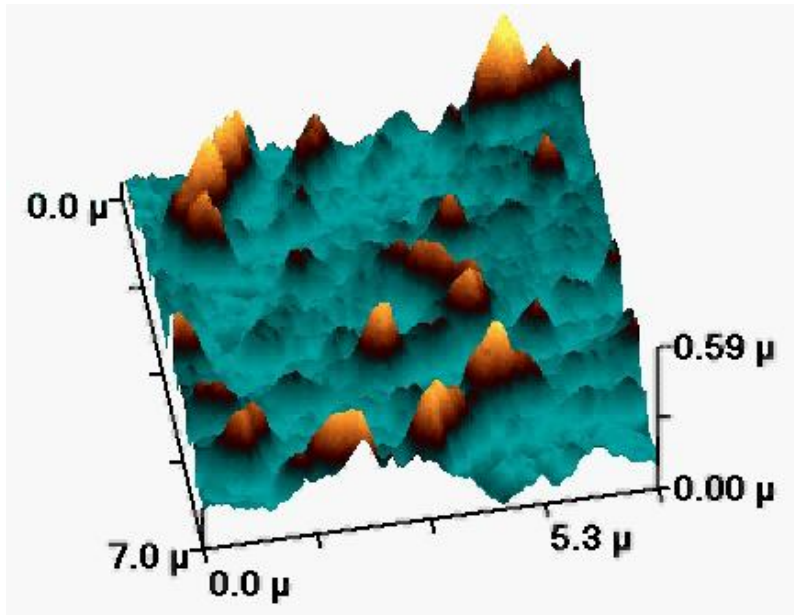
Outline:

- Motivation: using “effective temperature” in an “active”-thermodynamics scheme to describe patterns in biological systems.

- Red-blood cell: recent experiments showing non-thermal nature of the fluctuations.
- Comparing to a simple quantitative model.

- Actin-driven membrane clusters.

Motivation:



Active Microvilli on the upper surface of cells
(Movie curtsey of Bechara Kachar)

Our treatment of this system in terms of “thermodynamic” phase transitions driven by “effective temperature”:

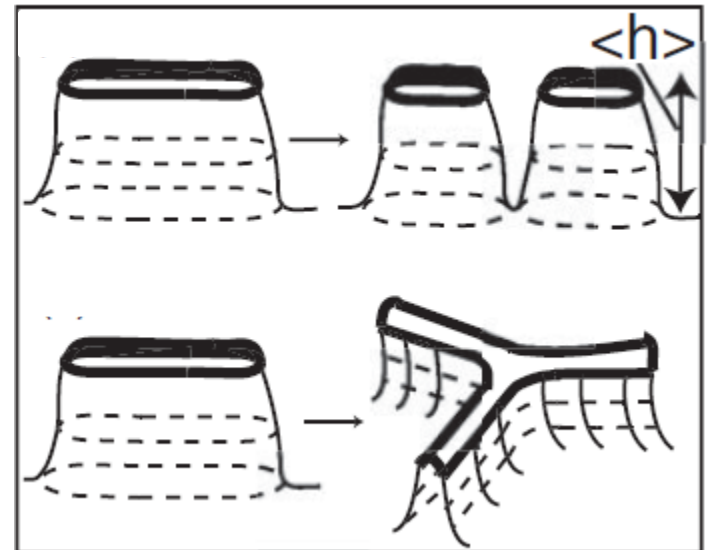
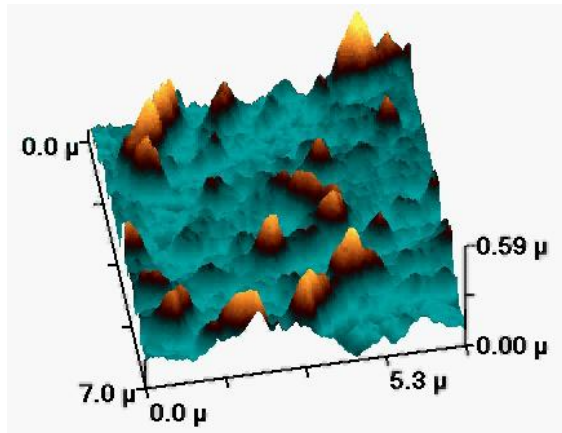
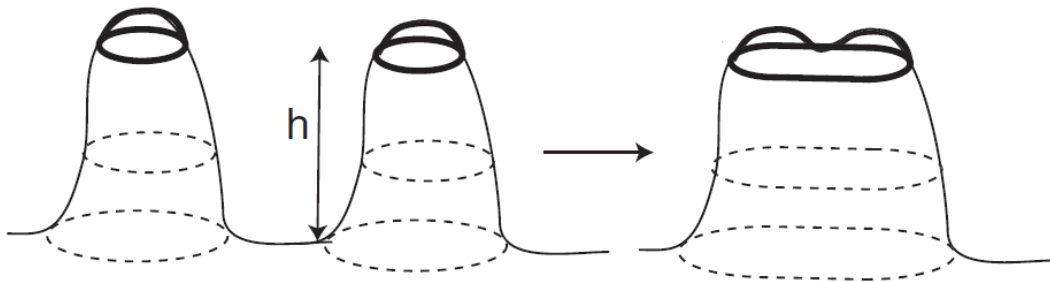
PRL **97**, 018101 (2006)

PHYSICAL REVIEW LETTERS

week ending
7 JULY 2006

Dynamics and Morphology of Microvilli Driven by Actin Polymerization

Nir S. Gov

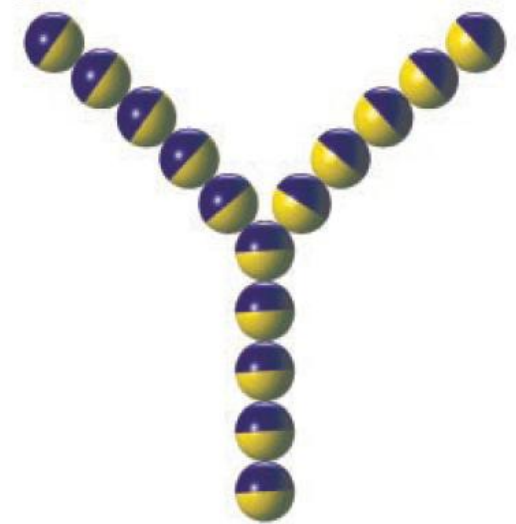


J. Gorelik *et al.*, Proc. Natl. Acad. Sci. U.S.A. **100**, 5819 (2003).

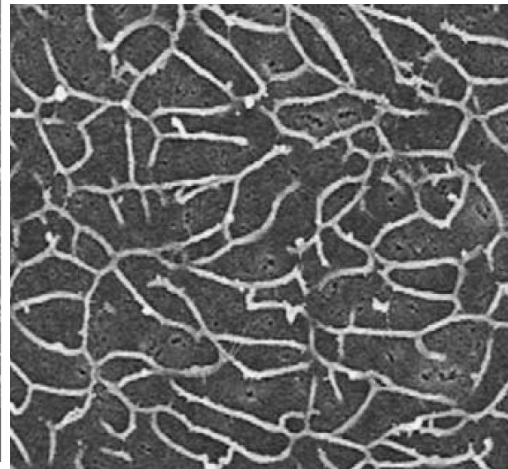
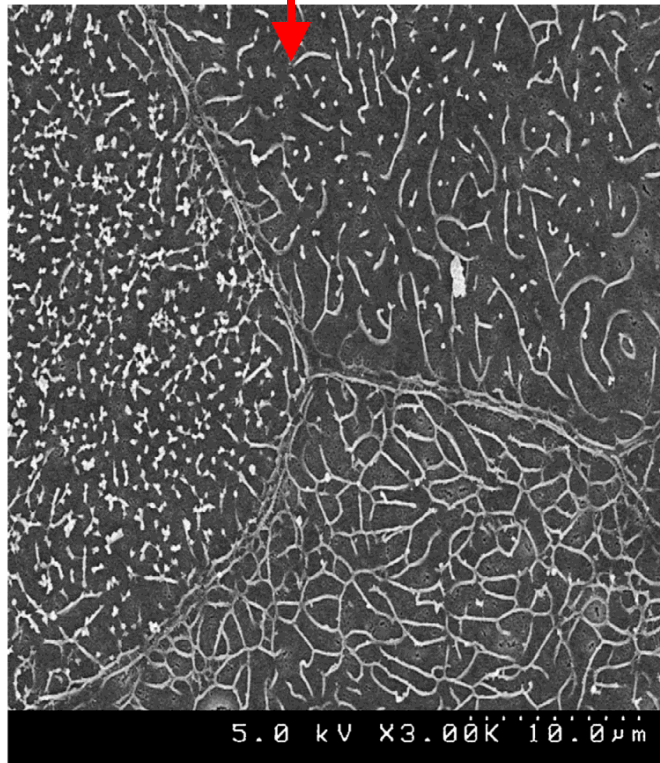
J. Gorelik *et al.*, Molec. Cell. Endocrin. **217**, 101 (2004).

Microvilli: Spatial distribution/patterns

- Linear aggregates due to positive spontaneous curvature of tip complex



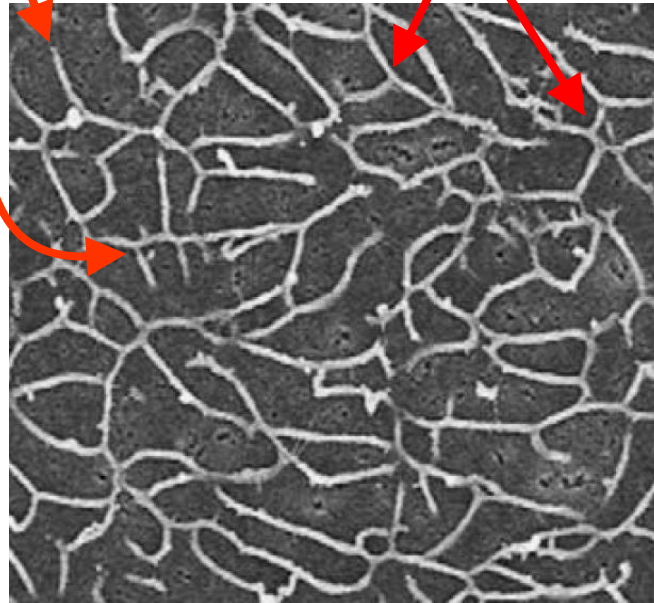
T. Tlusty & S. Safran; *Science* 290 (2000) 1328



2D dipolar fluid,
network of
worm-like
micelles etc.

Microvilli: Spatial distribution/patterns

- Assume single height of MV: $\langle h \rangle$
- Excluded volume interactions
- Defects: free ends and 3-fold junctions



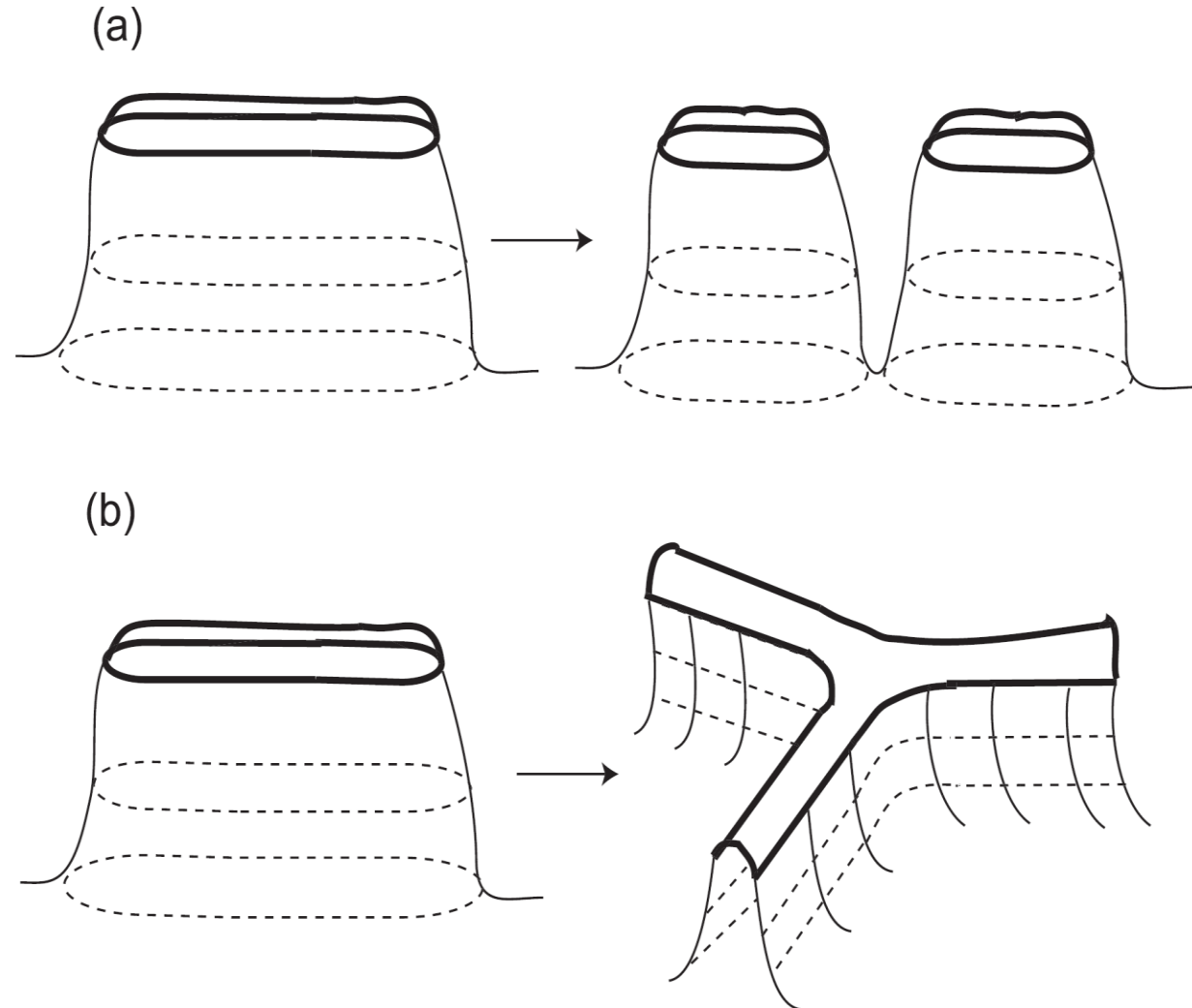
Microvilli: Energy of defects

$$E_{end} \approx \frac{2\pi\kappa h}{R} - \frac{\pi\kappa}{8}$$

Increases
with h

$$E_{junct} \approx \frac{\pi\kappa h}{6R} + \frac{2\pi\kappa}{15}$$

Increases
more slowly
with h



Free energy of gas of defects

$$F(\phi)/k_B T_{eff} = (1 - \phi) \ln(1 - \phi) + \phi_e (\ln \phi_e - 1) + \phi_j (\ln \phi_j - 1) \\ + \phi_e \epsilon_e + \phi_j \epsilon_j - \frac{1}{2} \phi_e \ln \phi - \frac{3}{2} \phi_j \ln \phi$$

ϕ is the area fraction of the MV.

ϕ_e, ϕ_j are the area fraction of the ends and 3-fold junctions respectively.

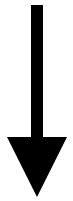
Minimize with respect to independent defects' concentrations:

$$\phi_j = \phi^{3/2} e^{-\epsilon_j}$$

$$\phi_e = \phi^{1/2} e^{-\epsilon_e}$$

Formation of networks

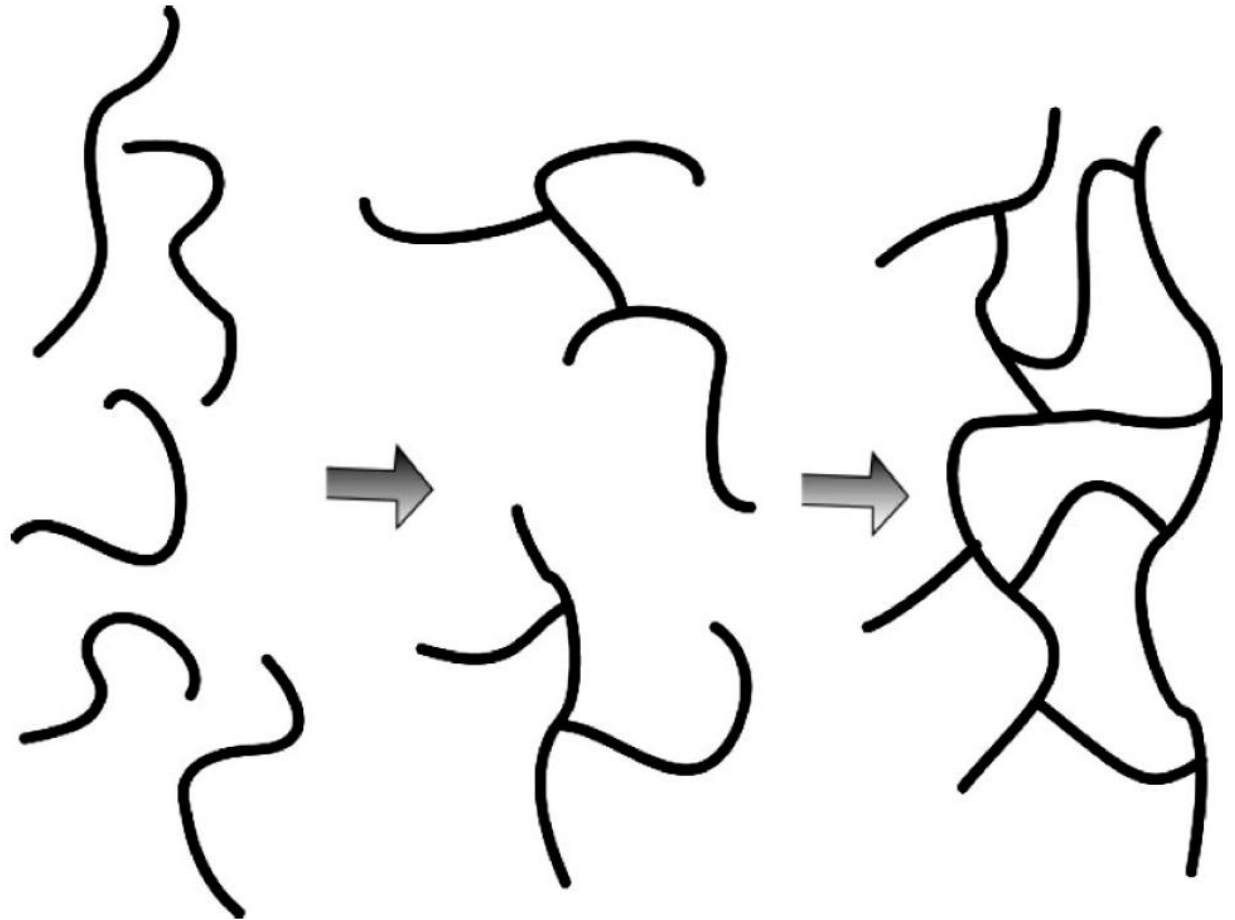
If the MV height increases, junctions multiply over ends:



Phase transition to a connected network:

$$\partial^2 F / \partial \phi^2 = 0$$

Spinodal

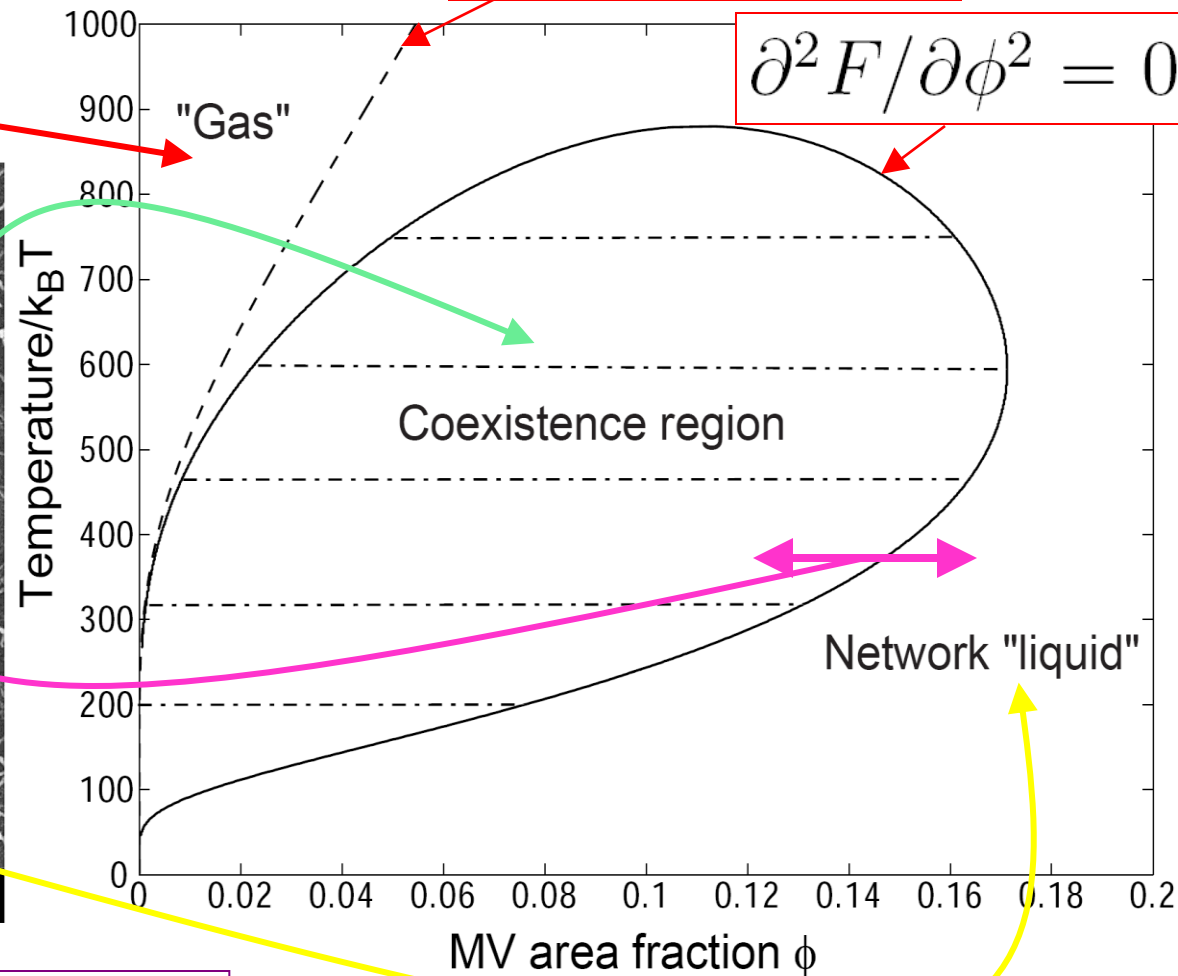
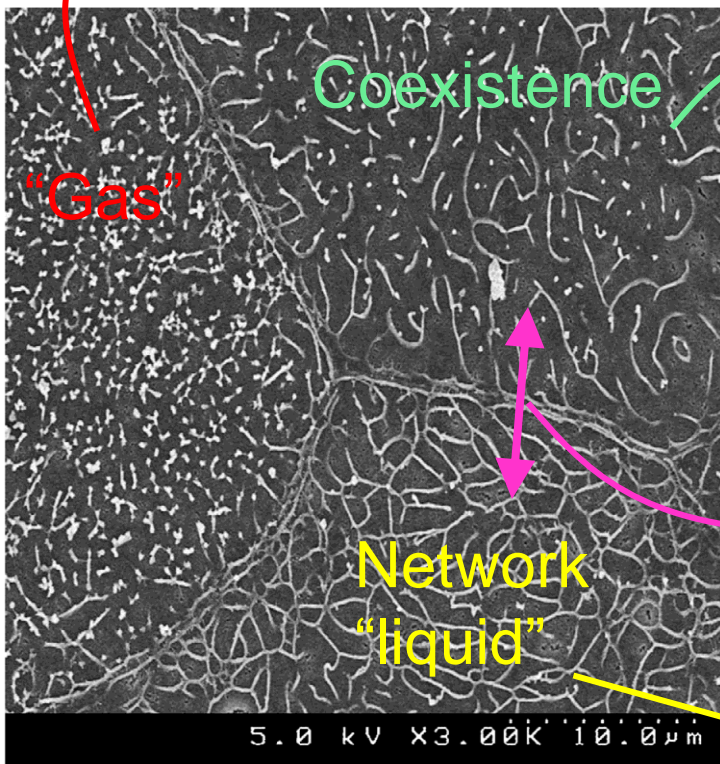


Microvilli: Spatial distribution/patterns

Using: $\kappa=10 k_B T$, $h=400\text{nm}$

Percolation line

$$\partial^2 F / \partial \phi^2 = 0$$

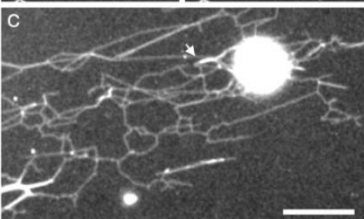
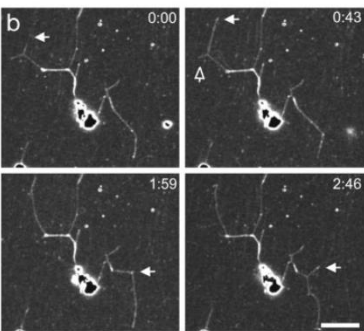
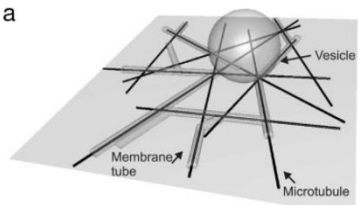
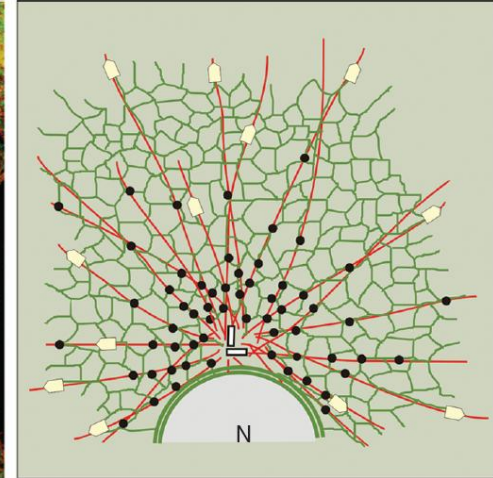
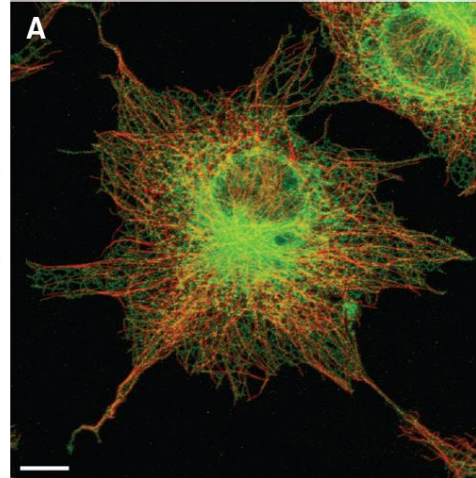
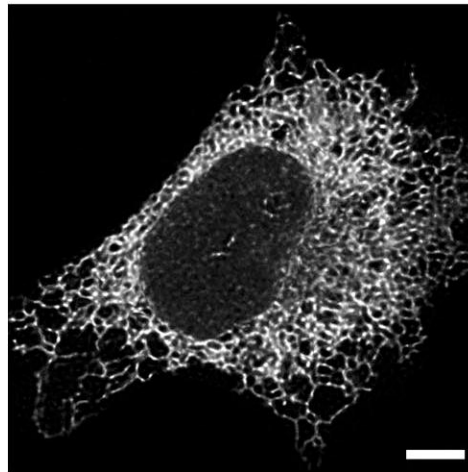


Note the large "effective temperature"

Motivation: Dynamic morphology of motor-driven membrane tubules (ER, golgi)

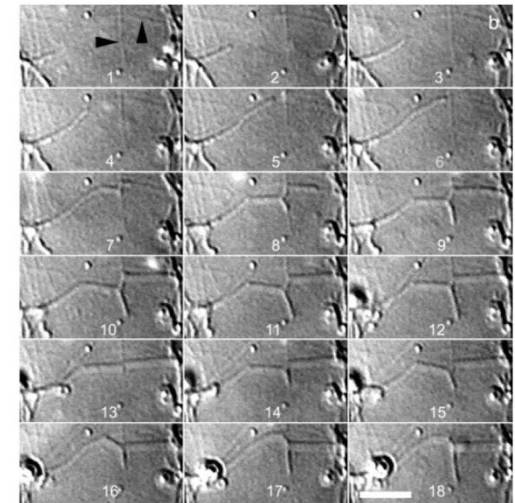
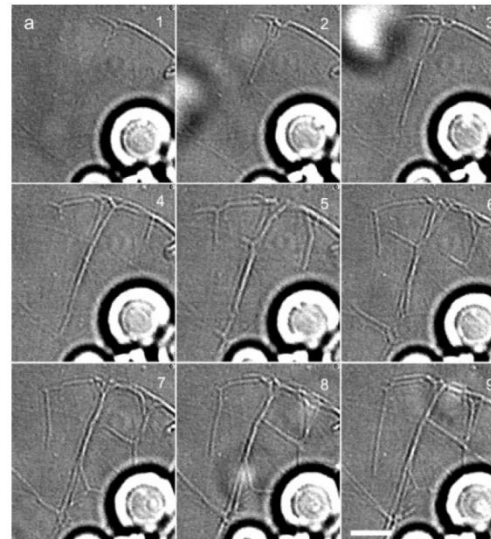
In-vivo:

Vedrenne 2006;
Jokitalo 2007



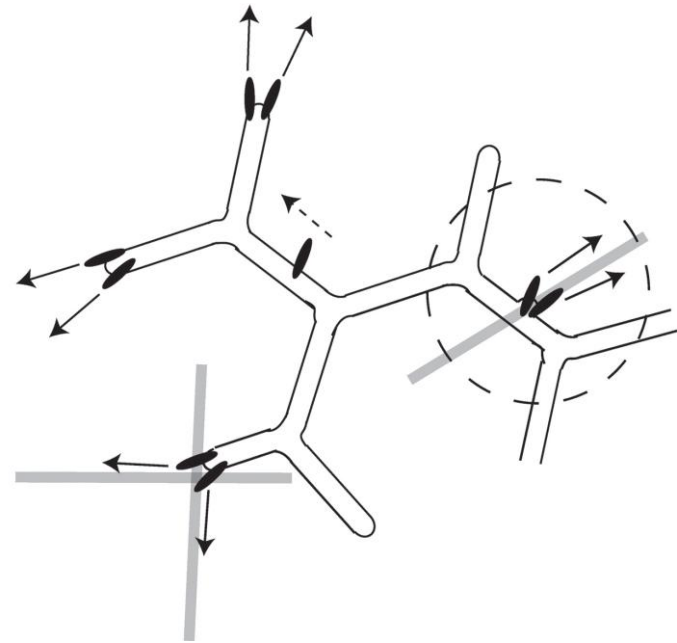
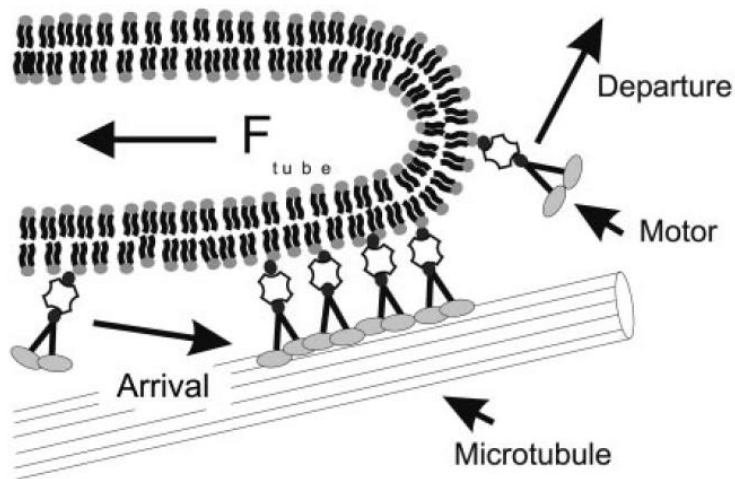
In-vitro:

Bassereau
2002,2004;
Dogterom, 2003



Our model: network phase transitions with “hotter” tips

Motors accumulate at the tubules tips \rightarrow Higher effective temperature



Motors pull the tubules tips \rightarrow random motion on the background network of MTs

PAPER

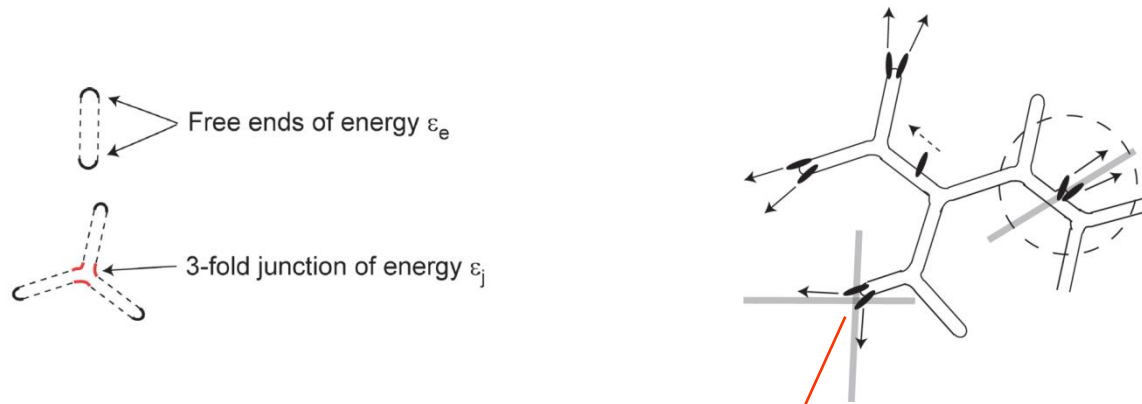
www.rsc.org/softmatter | Soft Matter

Phases of membrane tubules pulled by molecular motors†

N. S. Gov*

Soft Matter, (2009) 5, 2431-2437

Our model: network phase transitions with “hotter” tips



$$\tilde{F}(\phi)/k_B T = (1 - \phi)\ln(1 - \phi) + T_e \phi_e (\ln \phi_e - 1) + \phi_j (\ln \phi_j - 1) + \phi_e \varepsilon_e + \phi_j \varepsilon_j - \frac{1}{2} \phi_e \ln \phi - \frac{3}{2} \phi_j \ln \phi$$

- Activity is limited to just one specie (structure).
- Different “effective temperatures” in different parts of the network.
- Do not equilibrate.

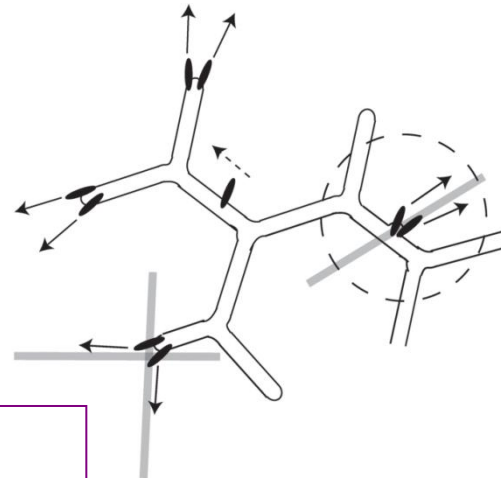
The “effective temperature” depends on the number of motors:

$$T_e \propto \langle v^2 \rangle \propto \left(1 - \frac{F_0}{F_m}\right)^2$$

$$F_m = F_s n_b$$

F_s is the stall force of each motor

n_b is the average number of motors at the tips

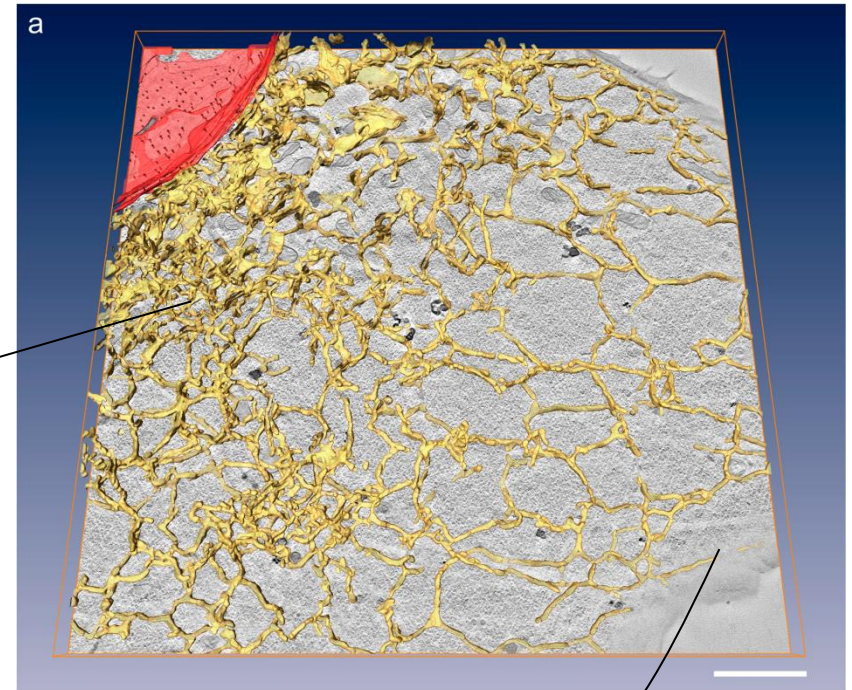
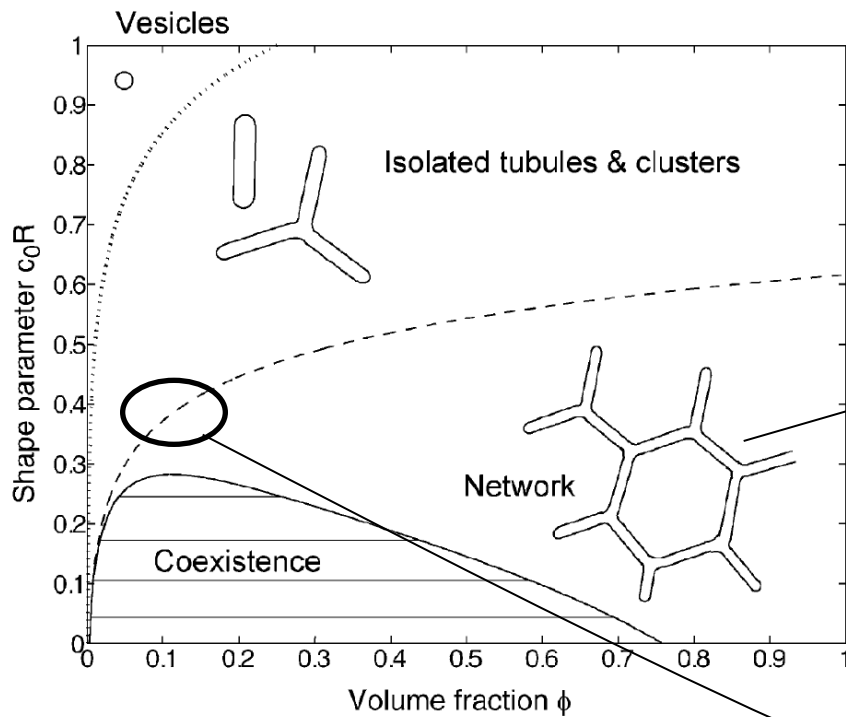


We can treat two cases:

- Constant n_b and T_e (saturation of motors).
- Average number of motors gets diluted as a function of the density of ends.

Our model: network phase transitions with “hotter” tips

M. Puhka, H. Vihinen, M. Joensuu and E. Jokitalo, *J. Cell Biol.*, 2007, **179**, 895.

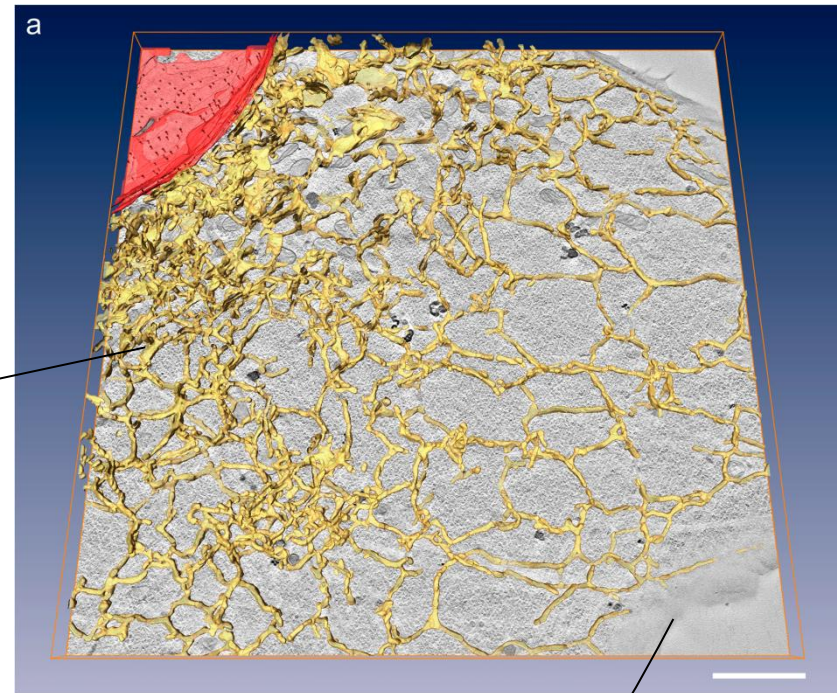
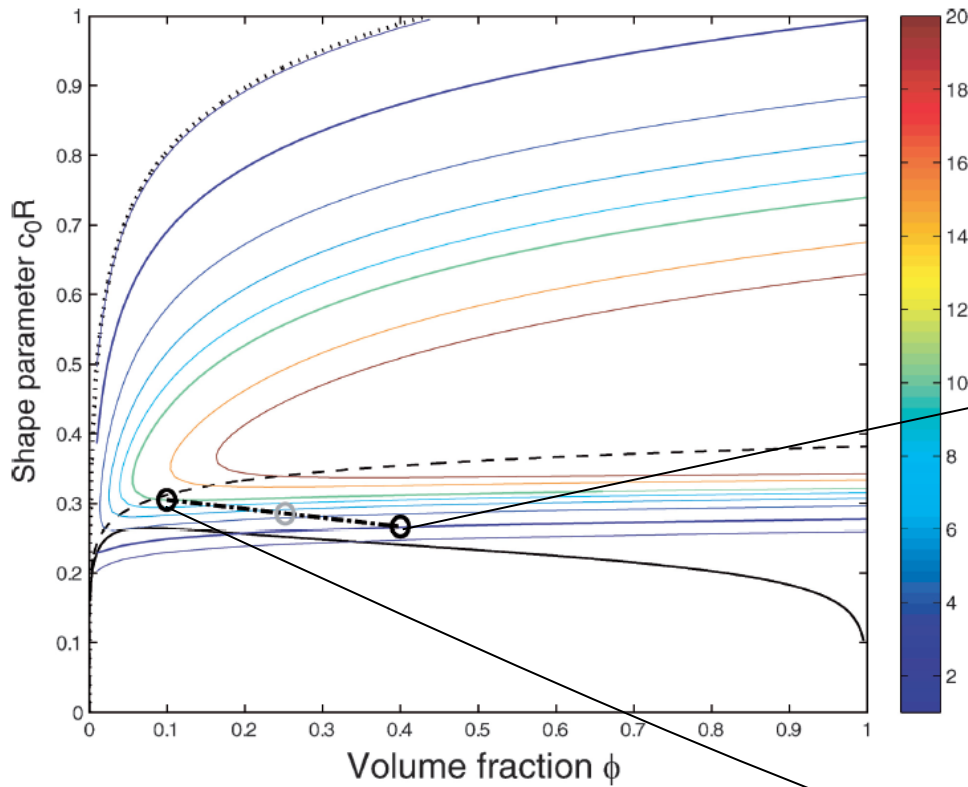


Comparing to observations:

Mean segment length:

$$\frac{\bar{L}}{R} = \frac{\phi}{\frac{1}{2}(\phi_e + 3\phi_j)}$$

$$R \approx 50 \text{ nm}$$



$$T_{\text{eff}} \sim 3k_B T$$

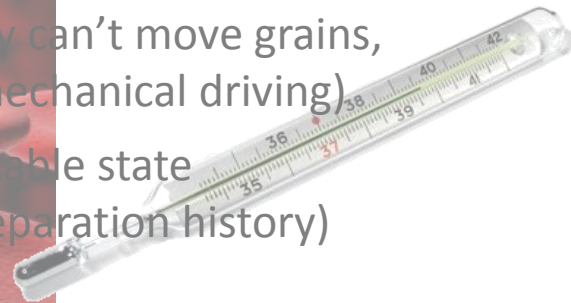
Can effective temperature describe non-equilibrium steady state ?

Live matter:

Molecular motors consume ATP and generate fluctuations that can be much larger than thermal fluctuations
→ far from thermodynamic equilibrium

Dead non-equilibrium systems:

- Granular (thermal energy can't move grains, usually have mechanical driving)
- Glass (frozen in metastable state reminiscent of preparation history)
- And many more...

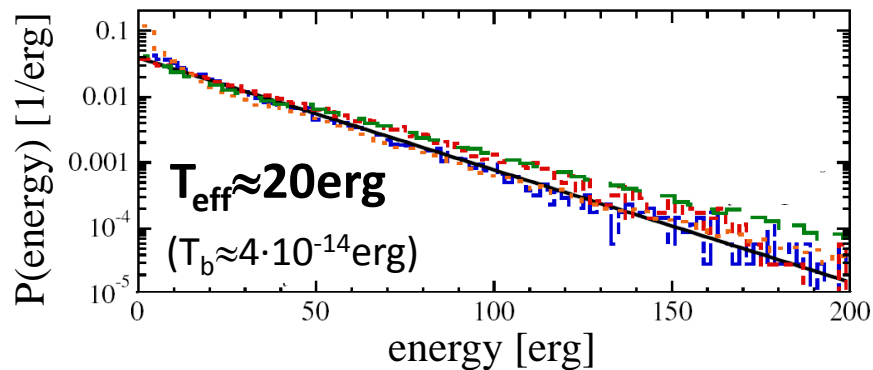


Fluctuations are **not thermal**,
but are there situations in which they are **thermal-like?**
(=equivalent to those of some elevated effective temperature)

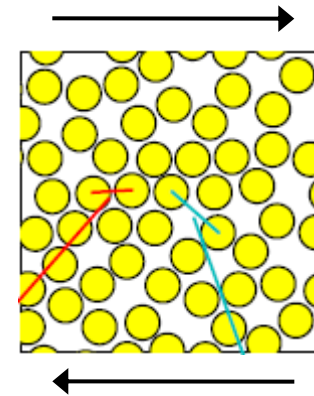
Thermal-Like (1):

$$P(E) \approx \exp(-E/T_{\text{eff}})$$

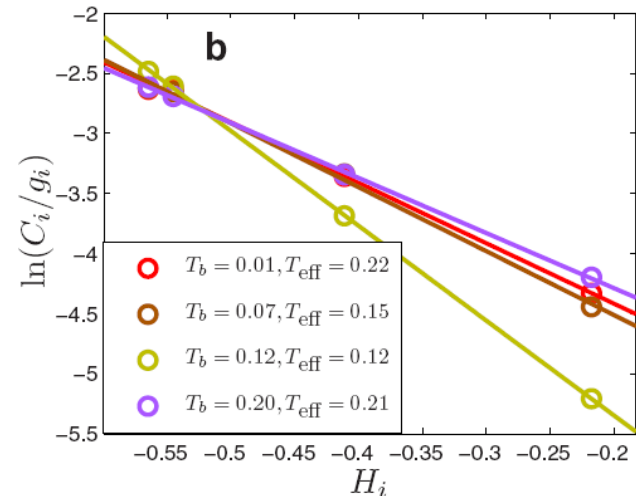
Air fluidized ping-pong balls
(model for granular matter)



Sheared amorphous solid



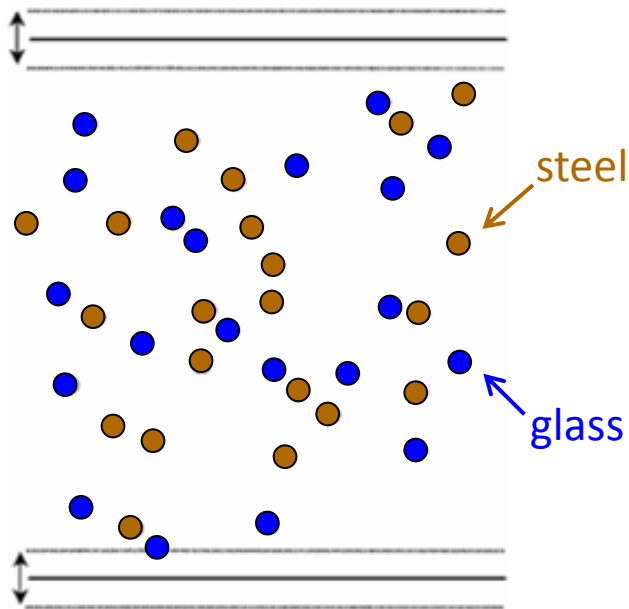
concentration of quasispecies:



Thermal-Like (2):

T_{eff} equilibrates at contact ?

Vertically shaken box of grains



$$\langle E_{\text{steel}} \rangle > \langle E_{\text{glass}} \rangle$$

$\langle E \rangle$ not useful as effective temperature

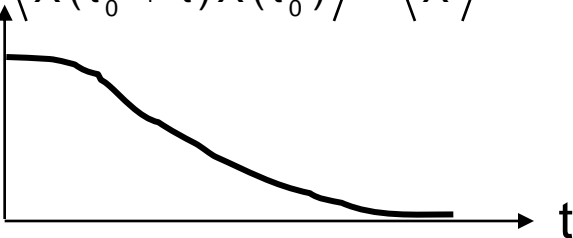
Feitosa & Menon (2002)

But sometimes can identify operational temperature that controls direction of energy flow and eventually equilibrates

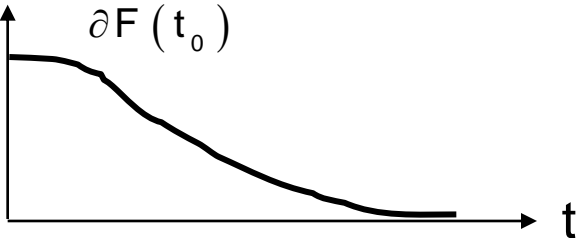
Shokef, Shulkind, Levine (2007)

Thermal-Like (3): Fluctuation-Dissipation Relations

Correlation (fluctuation):

$$C(t) \equiv \langle X(t_0 + t)X(t_0) \rangle - \langle X \rangle^2$$


Response (dissipation):

$$R(t) \equiv \frac{\partial \langle X(t_0 + t) \rangle}{\partial F(t_0)}$$


- **Equilibrium:** Correlation = Temperature x Response

Callen & Welton (1951)

- **Far from equilibrium:**

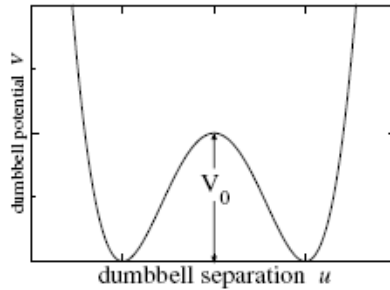
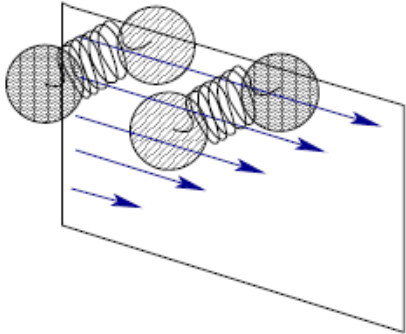
- Define $T_{\text{eff}} = \frac{\text{Correlation}}{\text{Response}}$
- Does T_{eff} depend on: observable ? waiting time (measurement frequency) ?
- Is T_{eff} related to other effective “temperatures” ?

Cugliandolo, Kurchan, Peliti (1997)

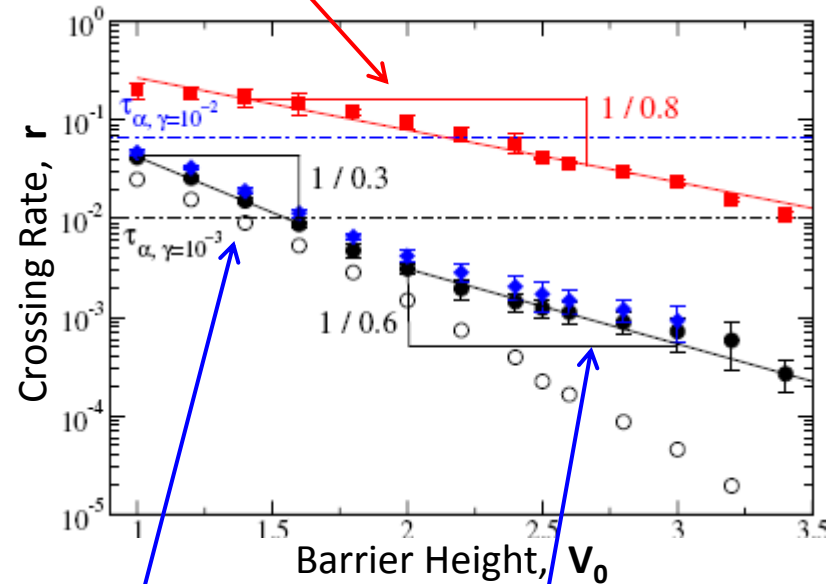
Thermal-Like (4):

T_{eff} Determines Reaction Rates

Double-well dumbbell in sheared glassy fluid



$T_B=0.8$ Thermal: $r \propto \exp(-V_0/T_B)$



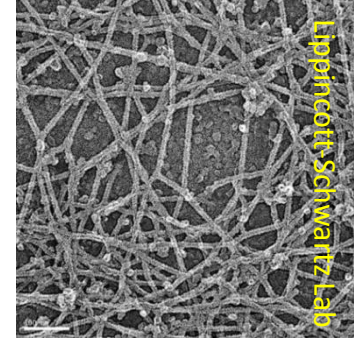
$T_B=0.3$ Glassy:

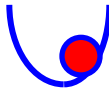
small V_0 :
bath dominates:
 $r \propto \exp(-V_0/T_B)$

high V_0 :
driving dominates:
 $r \propto \exp(-V_0/T_{\text{eff}})$, $T_{\text{eff}}=0.6$

(FD gives $T_{\text{eff}}=0.65 \approx 0.6$)

In Vitro Acto-Myosin Network



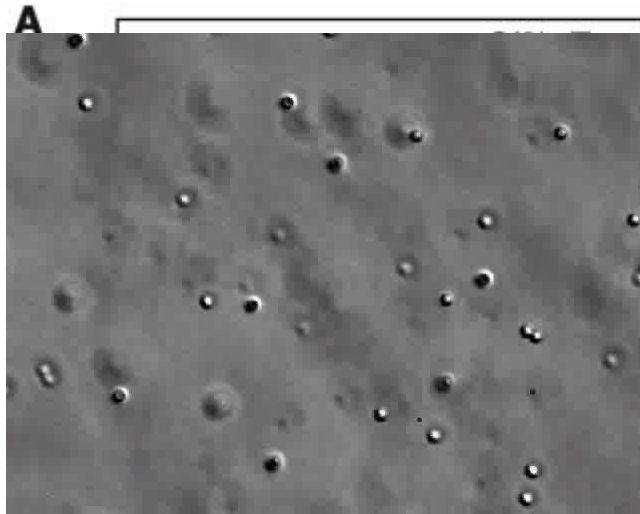
Micro-rheology (μm bead + optical tweezers): 

Passive: measure spontaneous fluctuations \rightarrow power spectrum $C(\omega)$

Active: apply periodic force, measure resulting displacement \rightarrow response function $\alpha(\omega)$

Only actin (thermal)

6.8hr after introduction of myosin ("live")



Beads hardly move
 Frequency (Hz) 10^{-1} 10^0 10^1 10^2 10^3 10^4

Beads move much more

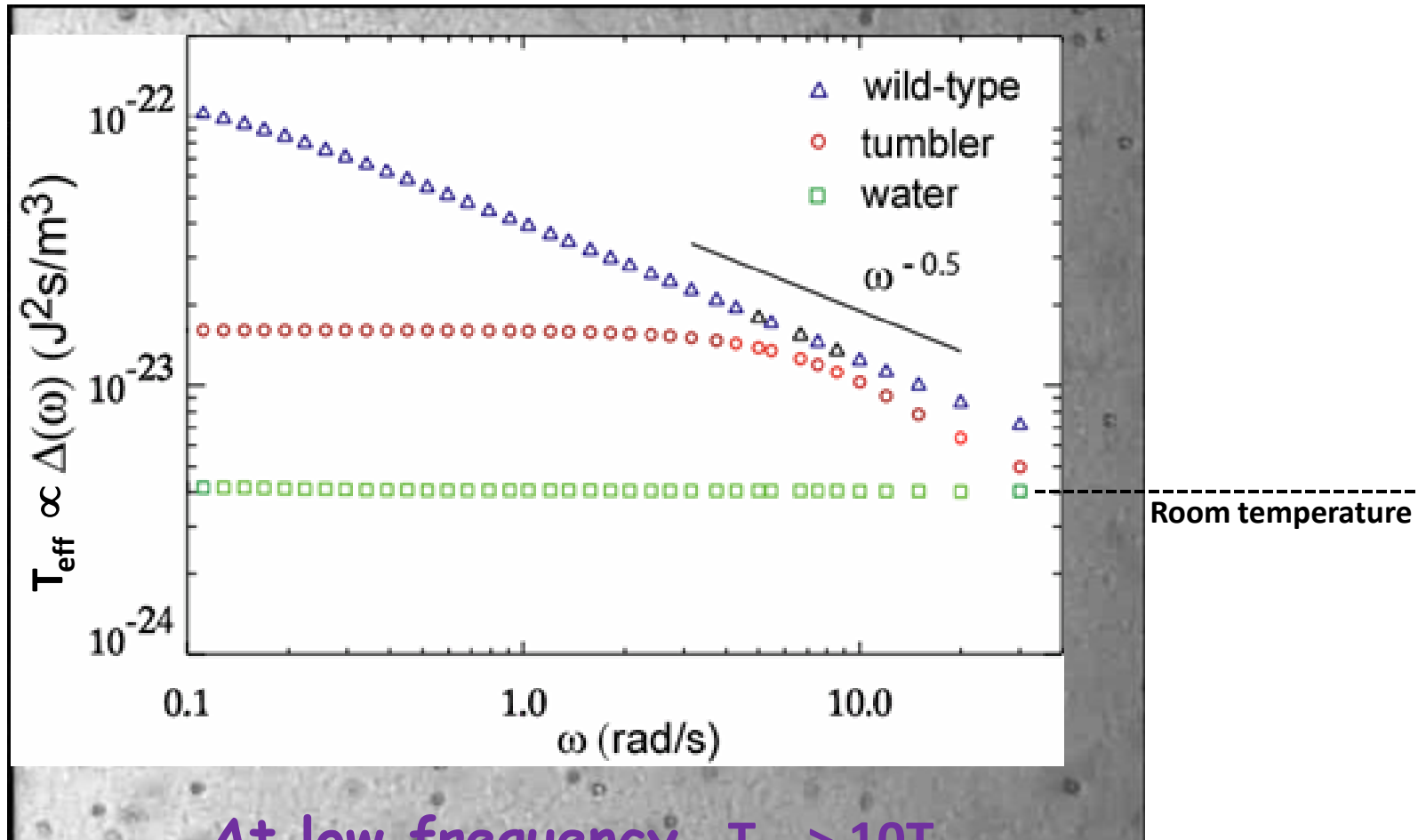
Fluctuation-dissipation theorem works: $\omega C(\omega) = 2k_B T \alpha''(\omega)$

Enhanced fluctuations at low frequency: $C_{\text{non-eq}}(\omega) > 10 \times C_{\text{eq}}(\omega)$
Response hardly effected:

$$\alpha_{\text{non-eq}}(\omega) \approx \alpha_{\text{eq}}(\omega)$$

Bacterial Bath

Two-point microrheology ($\sim 1\mu\text{m}$ beads) in solution with E. Coli ($\sim 3\mu\text{m}$)



At low frequency, $T_{\text{eff}} > 10T_B$

Even Genes do it....

84

Biophysical Journal Volume 91 July 2006 84–94

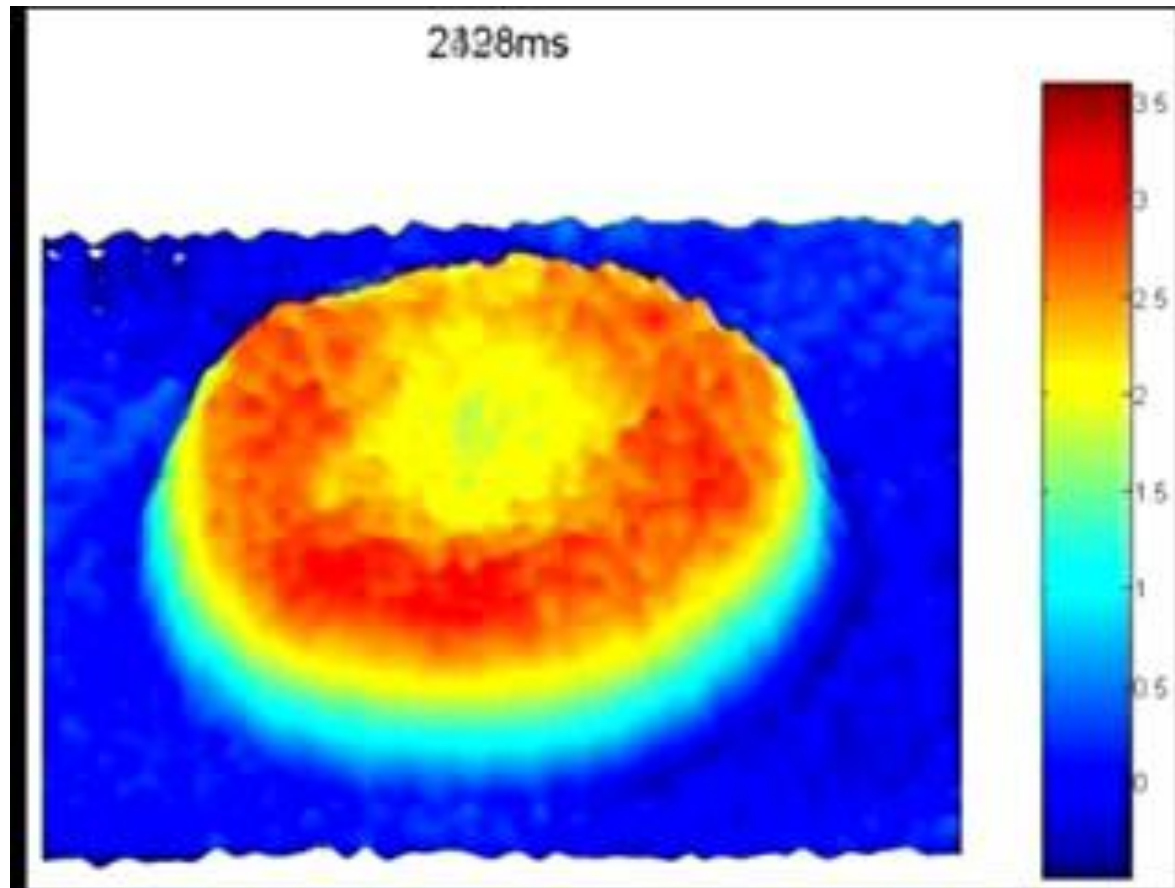
Effective Temperature in Stochastic Kinetics and Gene Networks

Ting Lu,^{*§} Jeff Hasty,[†] and Peter G. Wolynes^{*‡§}

**...But lets get back to biological
matter**

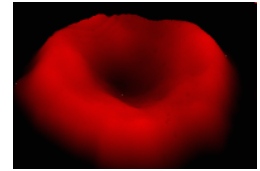
Red Blood Cell Membrane Fluctuations

Cell Waltz:

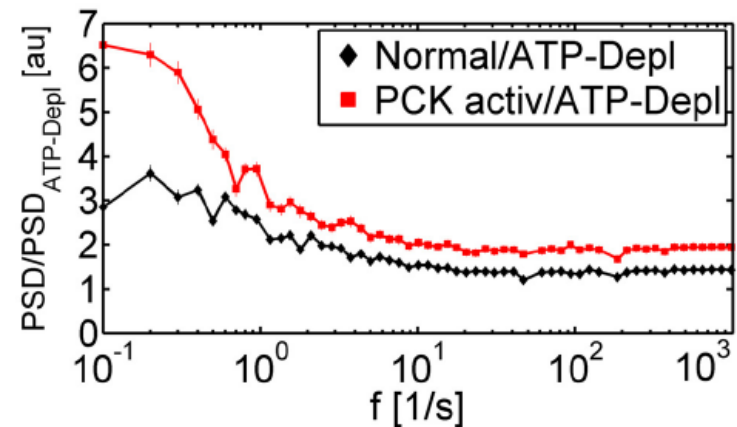
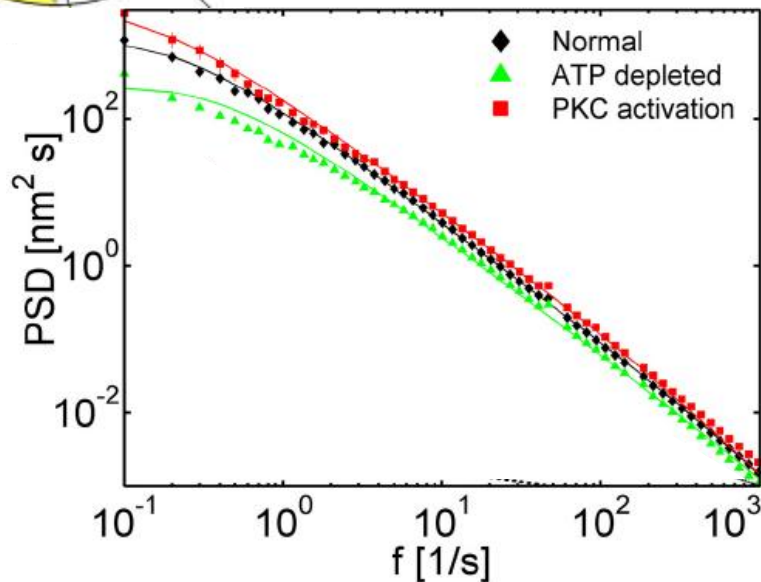
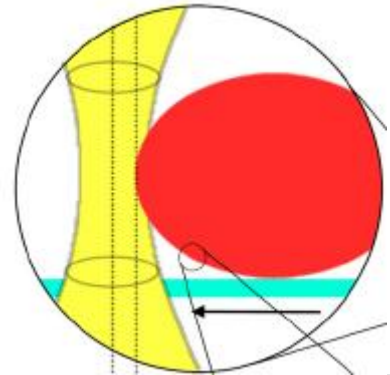


Gabriel Popescu
Quantitative Light Imaging Laboratory
[University of Illinois at Urbana-Champaign](http://light.ece.illinois.edu/)
Department of Electrical and Computer Engineering
<http://light.ece.illinois.edu/>

Red Blood Cell Edge Fluctuations: Recent experiments

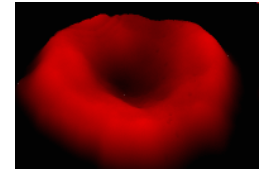


Position: $y(t)$ \rightarrow Power spectrum: $\text{PSD}(\omega) = \int \langle y(t)y(0) \rangle e^{i\omega t} dt$

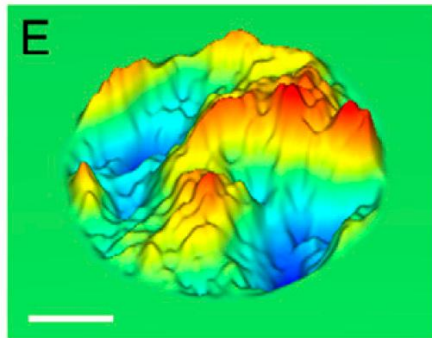
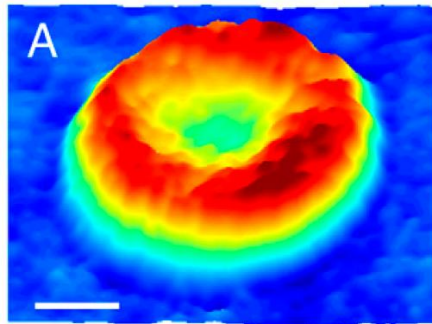


Fluctuations larger than thermal by 3(normal)-6(PKC activated),
but cannot infer T_{eff} without response measurement...

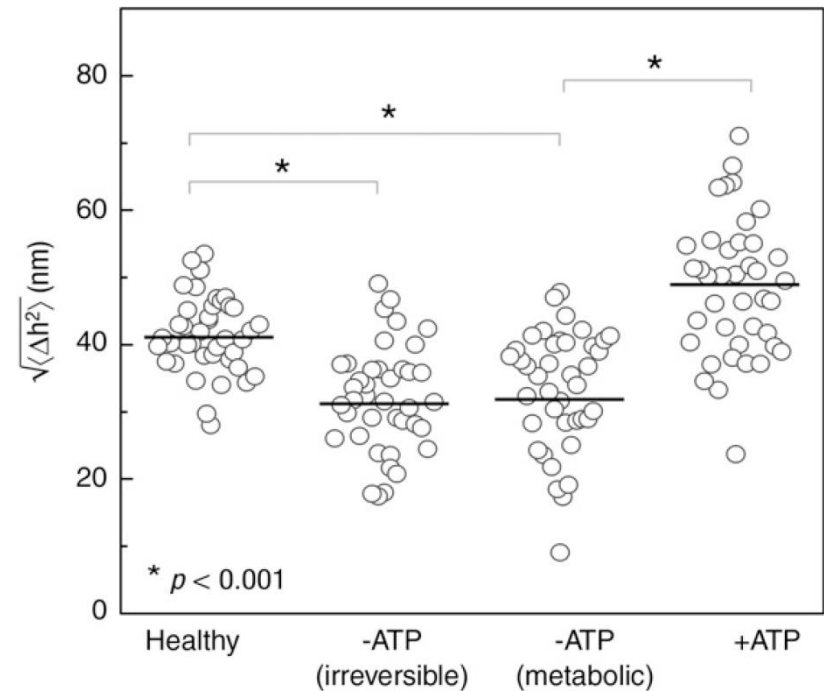
Red Blood Cell Edge Fluctuations: Recent experiments



High resolution optical measurements of the membrane displacement field:

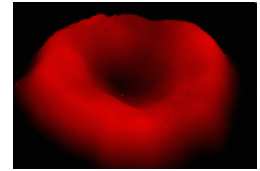


Amplitude of the displacement field depends on the ATP content:



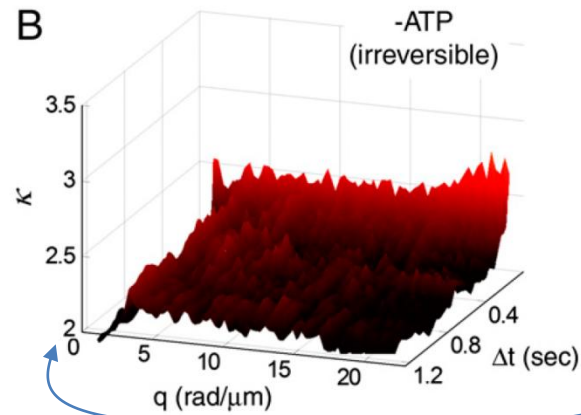
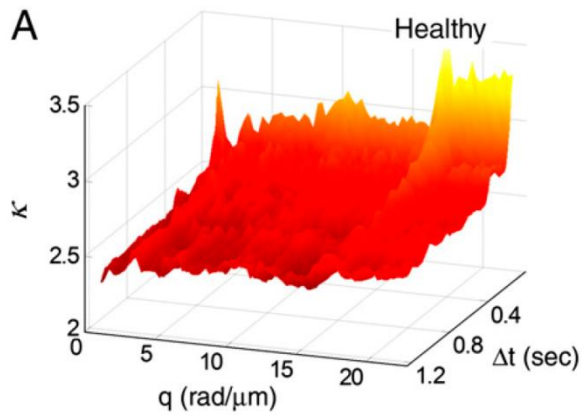
But how do we know that they are non-thermal ?
(i.e. not due to changes to the elastic constants)

Red Blood Cell Edge Fluctuations: Recent experiments

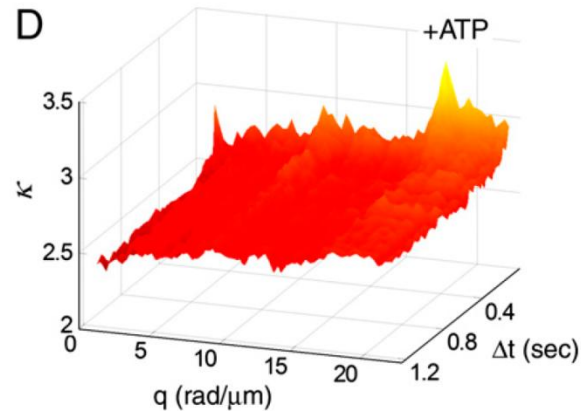
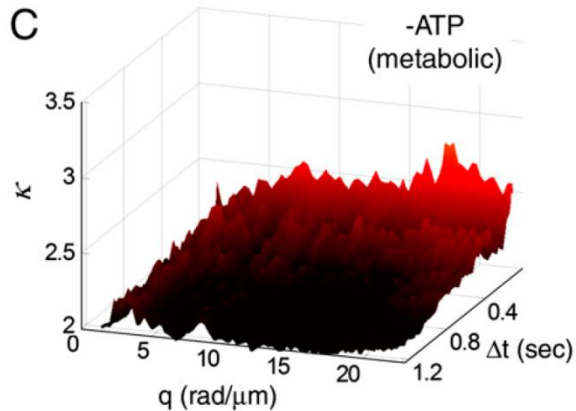


Measure the Kurtosis, non-Gaussianity of the fluctuation distribution:

$$\kappa = \langle |h(q, \Delta t) - h(q, 0)|^4 \rangle / \langle |h(q, \Delta t) - h(q, 0)|^2 \rangle^2$$



For a (thermal) Gaussian:
 $\kappa=2$



But what is the
elusive motor in the
RBC ?

Can we explain these observations using a simple theoretical model ?

Yes, and its just out:

PRL **106**, 238103 (2011)

PHYSICAL REVIEW LETTERS

week ending
10 JUNE 2011

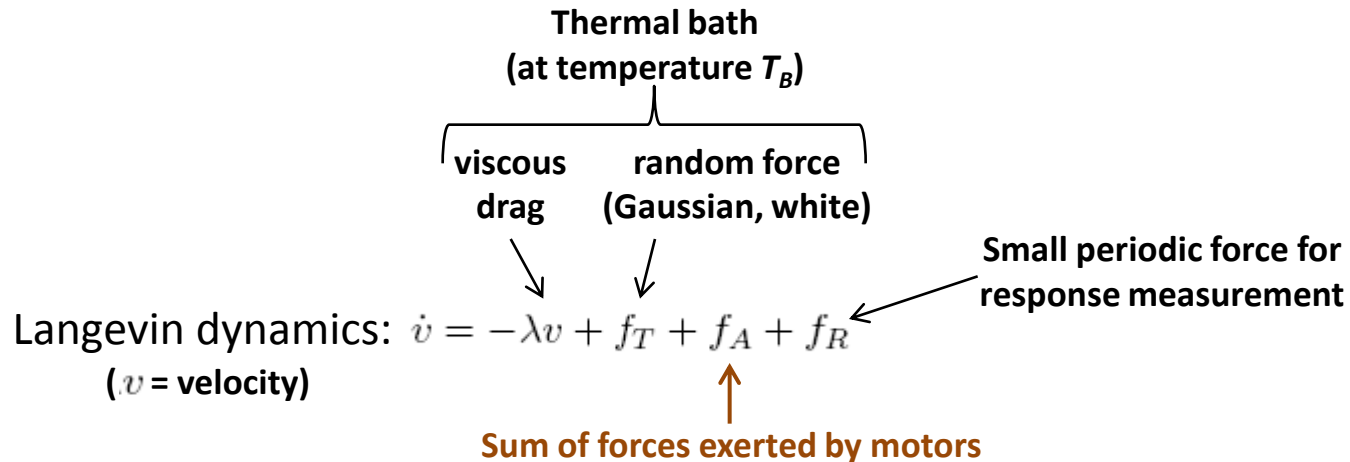
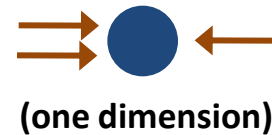
Effective Temperature of Red-Blood-Cell Membrane Fluctuations

Eyal Ben-Isaac,¹ YongKeun Park,² Gabriel Popescu,³ Frank L. H. Brown,⁴ Nir S. Gov,^{1,*} and Yair Shokef^{5,†}

Minimal Model for Active Fluctuations (of Red Blood Cell Membranes)

“Particle” (= Degree of freedom, spatial mode, ...)

N_m “Motors” (= Elements that occasionally apply force)



Each motor:

- turns on as Poisson process with average waiting time τ
- exerts constant force f_0 in random direction
- turns off after time $\Delta\tau$ (typically constant)
- uncorrelated from other motors (in direction and timing)

Minimal Model for Active Fluctuations (of Red Blood Cell Membranes)



#1: **Linear** differential equation: $\dot{v} = -\lambda v + f_T + f_A + f_R$



Solution is **superposition**: $v(t) = v_T(t) + v_A(t) + v_R(t)$ of solutions to:

$$\dot{v}_T = -\lambda v_T + f_T$$

$$\dot{v}_A = -\lambda v_A + f_A$$

$$\dot{v}_R = -\lambda v_R + f_R$$

Consequences for FD measurement:

i) Response independent of bath & motors: $f_R = F_0 e^{i\omega t} \rightarrow \langle \delta x(t) \rangle = \chi_{xx}(\omega) F_0 e^{i\omega t}$
with: $\chi_{xx}(\omega) = \frac{1}{\omega(i\lambda - \omega)}$

ii) v_T & v_A uncorrelated $\rightarrow \langle v(t)v(0) \rangle = \langle v_T(t)v_T(0) \rangle + \langle v_A(t)v_A(0) \rangle$

$$= \langle v_T^2 \rangle e^{-\lambda t}$$

(f_T Gaussian & white)

We're left with calculating active part of fluctuations



#2: **Direction of force uncorrelated (different motors & different pulses of same motor)**

$$\langle v_A(t)v_A(0) \rangle = \frac{N_m \Delta \tau}{\tau + \Delta \tau} \langle v_p(t)v_p(0) \rangle$$

Average number of pulses per unit time

$v_p(t)$ = velocity change following single pulse

And after not that much algebra...

$$T_{eff}(\omega) = T_B + \frac{N_m v_0^2 \lambda [1 - \cos(\omega \Delta \tau)]}{(\tau + \Delta \tau) \omega^2}$$

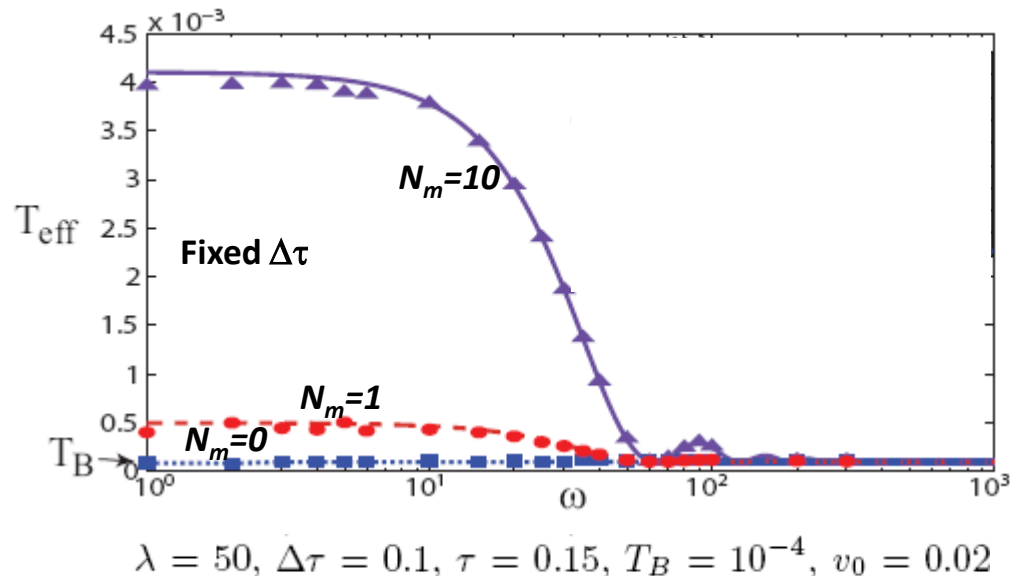
($v_0 \equiv f_0/\lambda$)

Effective Temperature

$$T_{eff}(\omega) = T_B + \frac{N_m v_0^2 \lambda [1 - \cos(\omega \Delta \tau)]}{(\tau + \Delta \tau) \omega^2}$$

Straightforward to generalize to variable pulse length $T_{eff}(\omega) = T_B + \frac{N_m v_0^2 \lambda \langle 1 - \cos(\omega \Delta \tau) \rangle}{(\tau + \langle \Delta \tau \rangle) \omega^2}$

$P(\Delta \tau) = \text{Poisson} \rightarrow f_A(t) = \text{shot noise}$ [Gov 2004] and: $T_{eff}(\omega) = T_B + \frac{N_m v_0^2 \lambda}{(\tau + \langle \Delta \tau \rangle) \left(\omega^2 + \frac{1}{\langle \Delta \tau \rangle^2} \right)}$



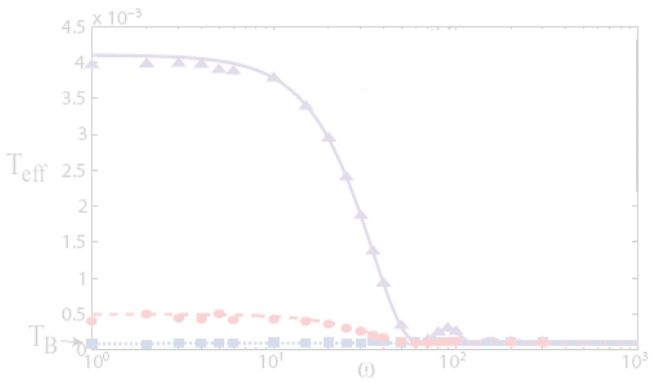
Are Active Fluctuations Thermal-Like?

$$\langle v^2 \rangle = \langle v_T^2 \rangle + \langle v_A^2 \rangle = T_B + \frac{N_m v_0^2 (\lambda \Delta \tau - 1 + e^{-\lambda \Delta \tau})}{\lambda (\tau + \Delta \tau)}$$

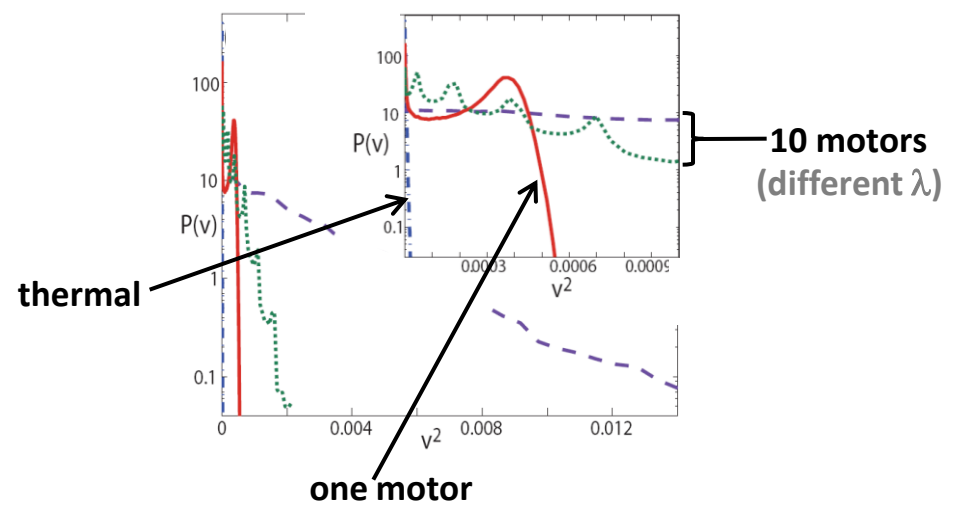
Velocity fluctuations $\left\{ \begin{array}{l} \text{amplitude} > \text{thermal} \\ \text{is their nature thermal-like? (is the system effectively in equilibrium at } T_{\text{eff}} = \langle v^2 \rangle \text{?)} \end{array} \right.$

1) Fluctuation-dissipation ratio:

depends on frequency: $T_{\text{eff}}(\omega)$
 $T_{\text{eff}}(\omega) = \text{const.}$ only in thermal limit
 but:
 $T_{\text{eff}}(\omega) \rightarrow \text{const.}$ for small ω



2) Velocity distribution, $P(v) \approx \exp(-v^2/T_{\text{eff}})$?



Quantify deviation from Gaussian by Kurtosis $\kappa \equiv \frac{\langle v^4 \rangle}{\langle v^2 \rangle^2}$
 ($\kappa_{\text{Gaussian}} = 3$)

Kurtosis ($\kappa \equiv \frac{\langle v^4 \rangle}{\langle v^2 \rangle^2}$)

2nd moment: $v(t) = v_T(t) + v_A(t)$, v_T & v_A uncorrelated $\rightarrow \langle v_T v_A \rangle = 0 \rightarrow \langle v^2 \rangle = \langle v_T^2 \rangle + \langle v_A^2 \rangle$

$v_A(t) = \sum \sigma_i v_p(t-t_i)$, directions uncorrelated ($\sigma_i \sigma_j = \delta_{ij}$) $\rightarrow \langle v_A^2 \rangle \propto \langle v_p^2 \rangle$

sum over pulses \uparrow direction of pulse i \uparrow time pulse i started

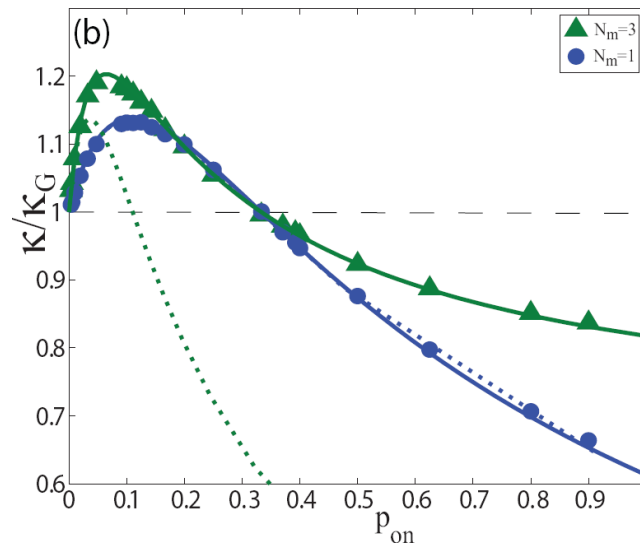


4th moment: $\langle v^4 \rangle = \langle v_T^4 \rangle + 6\langle v_T^2 \rangle \langle v_A^2 \rangle + \langle v_A^4 \rangle$, but now $\langle v_A^4 \rangle \propto \langle v_p^4 \rangle + \langle v_p^2(t-t_i) v_p^2(t-t_j) \rangle$



Solution: Assume $v_p \approx 0$ by the time next pulse starts (\rightarrow neglect cross term)

Valid only for $N_m=1$ & $\lambda\tau \gg 1$:



Kurtosis ($\kappa \equiv \frac{\langle v^4 \rangle}{\langle v^2 \rangle^2}$)

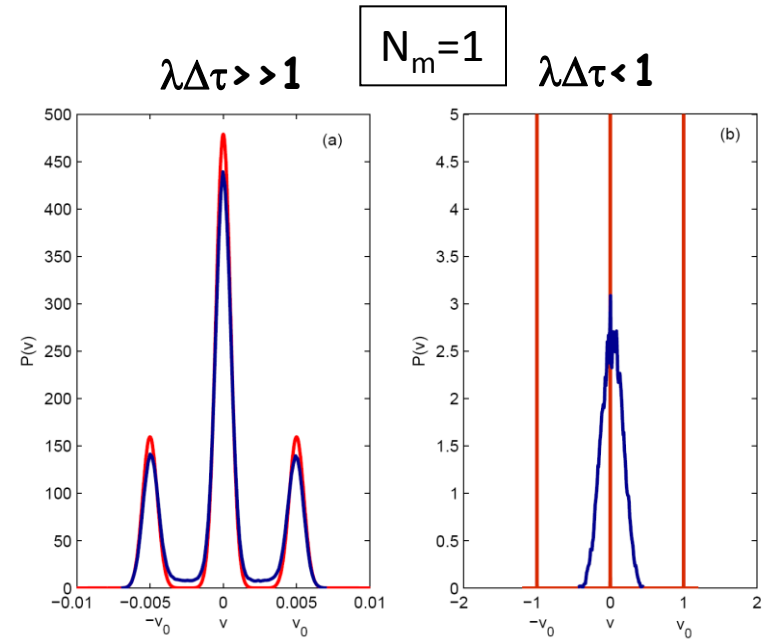
For multiple motors, we use another description:

Assume each motor immediately generates $v_0 = f_0/\lambda$ ($\rightarrow \lambda\Delta\tau \gg 1$)



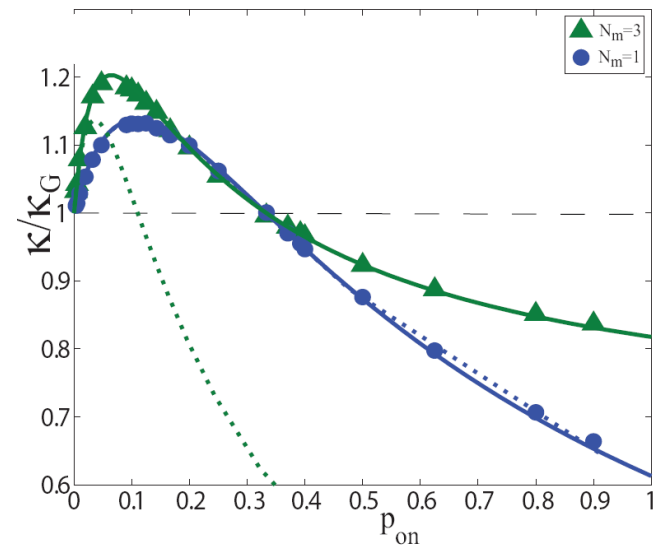
$P(v)$ is sum of shifted thermal Gaussians (weighted according to combinatoric probability that a given number of motors are simultaneously on)

Probability of each motor to be on: $p_{on} = \frac{\Delta\tau}{\tau + \Delta\tau}$

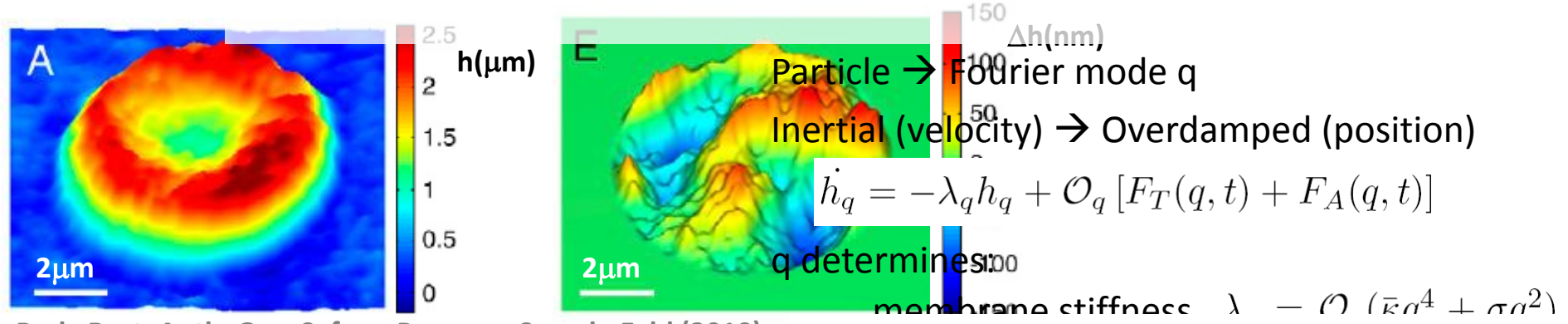


- Perfect agreement with simulations
- Non-monotonic dependence of κ on activity (p_{on})
- Can retain $\kappa_{Gaussian}$ even when far from equilibrium

$\lambda\Delta\tau \gg 1$

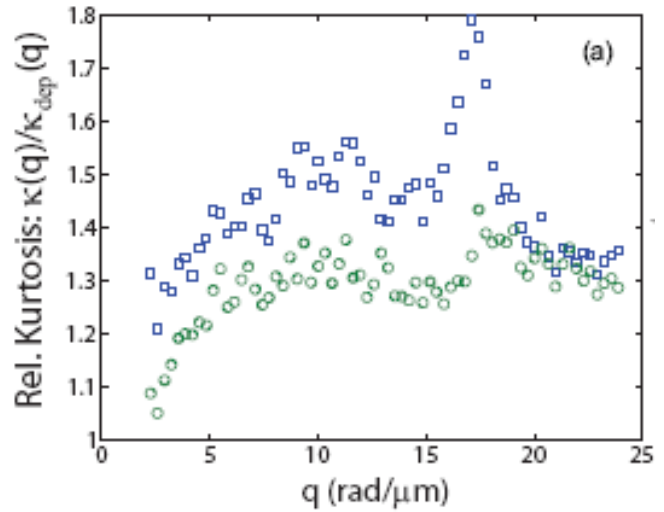


Minimal Model for Active Fluctuations of Red Blood Cell Membranes



Park, Best, Auth, Gov, Safran, Popescu, Suresh, Feld (2010)

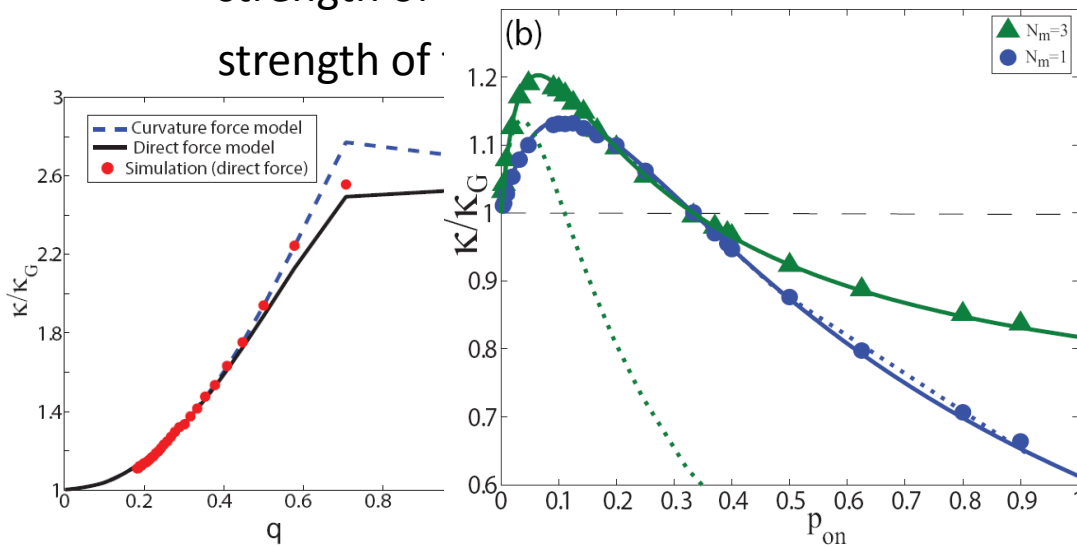
New experimental measurements of kurtosis with vs without ATP



$\kappa(\text{w/ATP}) > \kappa(\text{w/o ATP})$
 $\rightarrow \kappa$ grows with activity
 \rightarrow we're in the low p_{on} region

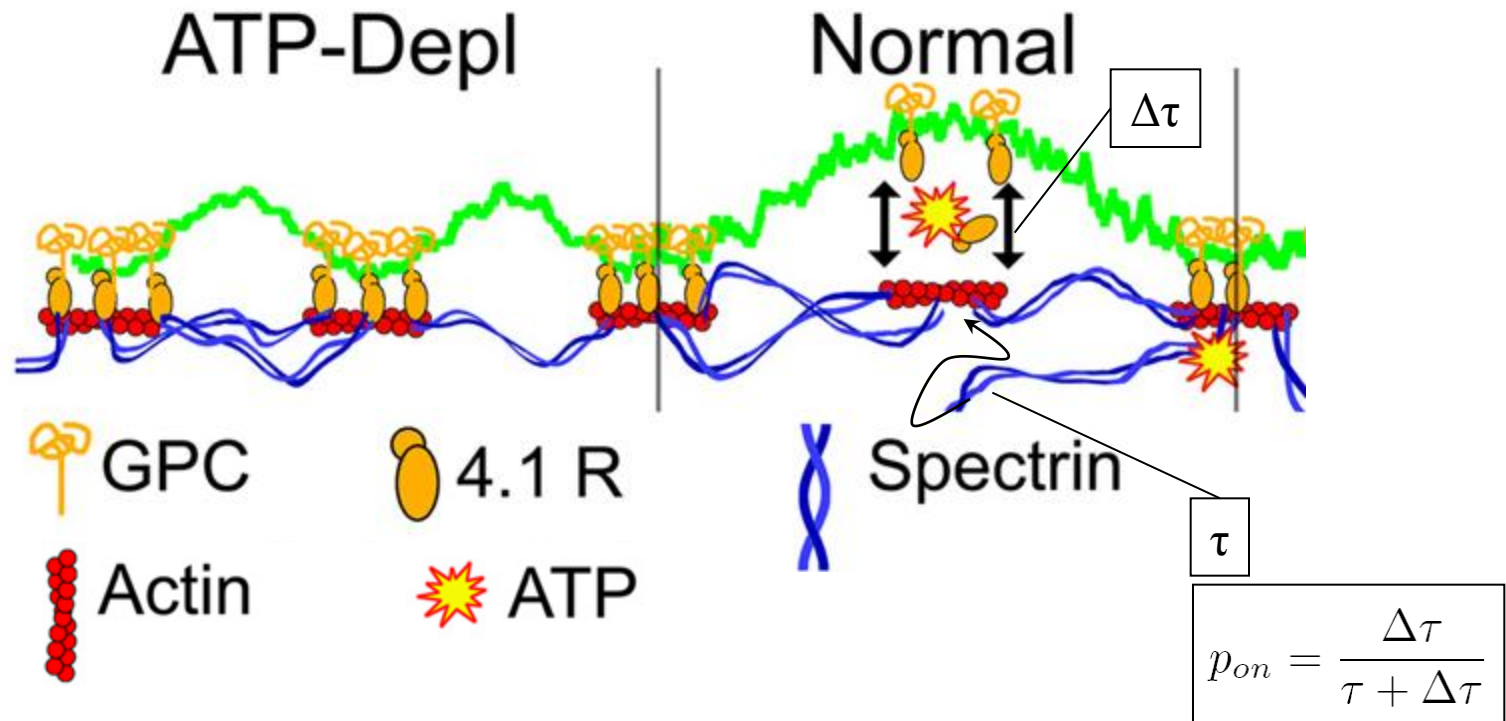
strength of active force $F_A \propto a^0 \cdot a^2$

strength of



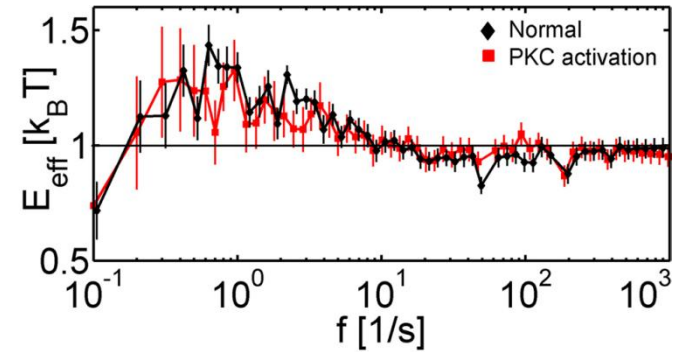
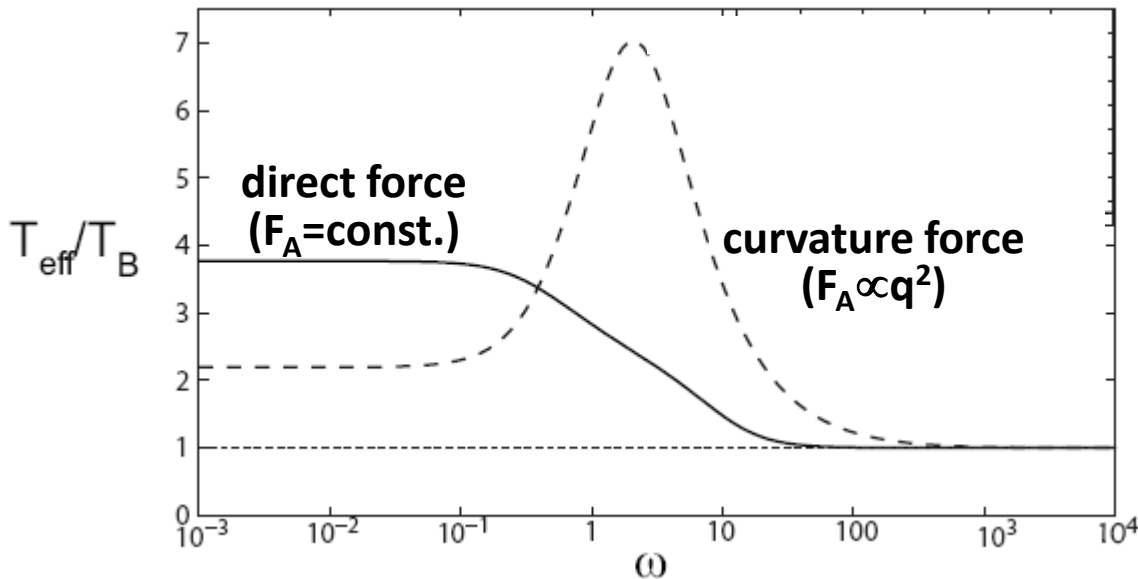
The analysis can teach us about the physical properties of the elusive motor:

- Low p_{on} \rightarrow the RBC membrane motor has a long recovery time
- Supports a model of the ATP-driven activity through filament dissociations



Minimal Model for Active Fluctuations of Red Blood Cell Membranes

(Local) fluctuation-dissipation measurement \rightarrow integrate over q
(here $P(\Delta\tau)=\text{Poisson}$)



Betz, Lenz, Joanny, Sykes (2009)

- Calls for experiments that would measure also response and not only fluctuations in RBC
- Would be interesting to see if indeed $T_{\text{eff}}(\omega)$ is non-monotonic? (preliminary data indicate monotonic behavior).

Take-Home Messages on the Non-Equilibrium Nature of Active Fluctuations

1. When using fluctuations to quantify deviation from equilibrium, need to compare to appropriate response (via the fluctuation-dissipation formalism)
2. Kurtosis may be misleading in characterizing deviation from equilibrium: Gaussian values may result from large N_m effects; in such cases $T_{\text{eff}}(\omega)$ still shows strong frequency dependence
3. **Kurtosis** vs. **activity** is non-monotonic
4. $T_{\text{eff}}(\omega)$ vs. **activity** and **frequency** may be **non-monotonic**

Suggestions for our experimental collaborators...



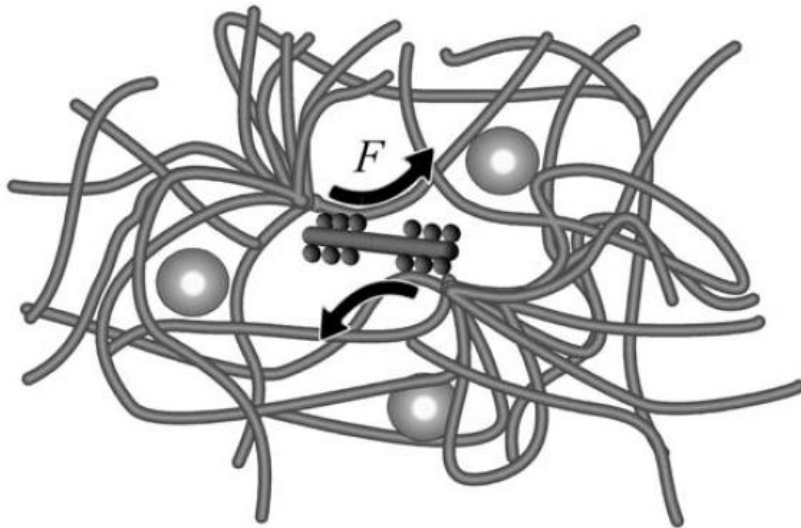
Timo Betz (Curie): Measure response of red blood cell membrane to deduce $T_{\text{eff}}(\omega)$ and see if it's non-monotonic



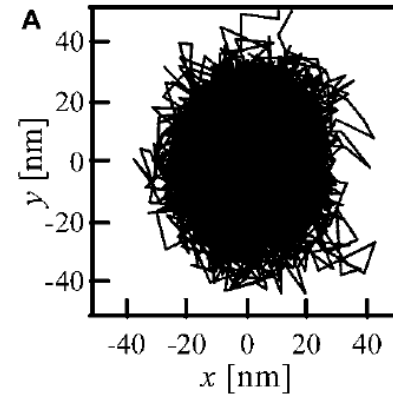
Paul Park (KAIST): Increase activity or decrease wavevector to identify situations in which kurtosis decreases with activity

Recent observation of non-monotonous kurtosis:

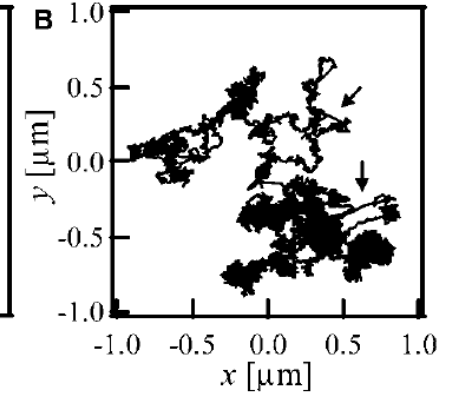
In-vitro actin-myosin gel:



“Dead”:

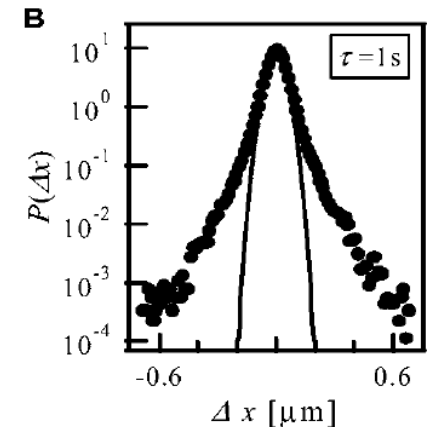
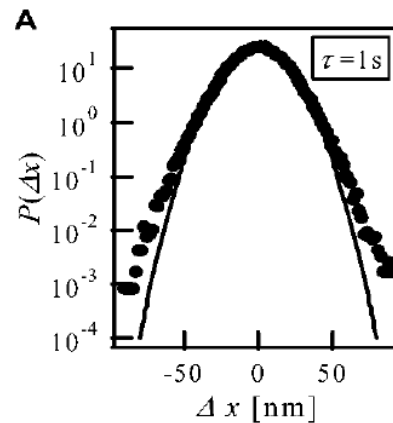


“Alive”:



Van-Hove correlations:

$$P(\Delta x(\tau)), \Delta x(\tau) = x(t + \tau) - x(t)$$



Cite this: *Soft Matter*, 2011, **7**, 3234

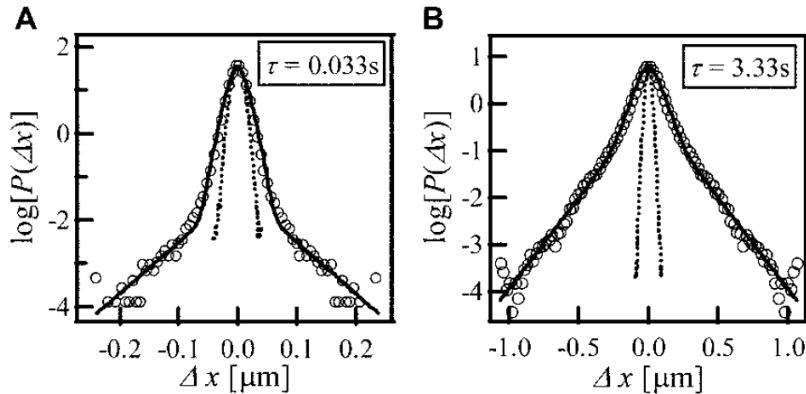
www.rsc.org/softmatter

Non-Gaussian athermal fluctuations in active gels†

Toshihiro Toyota,^a David A. Head,^b Christoph F. Schmidt^c and Daisuke Mizuno^{*a}

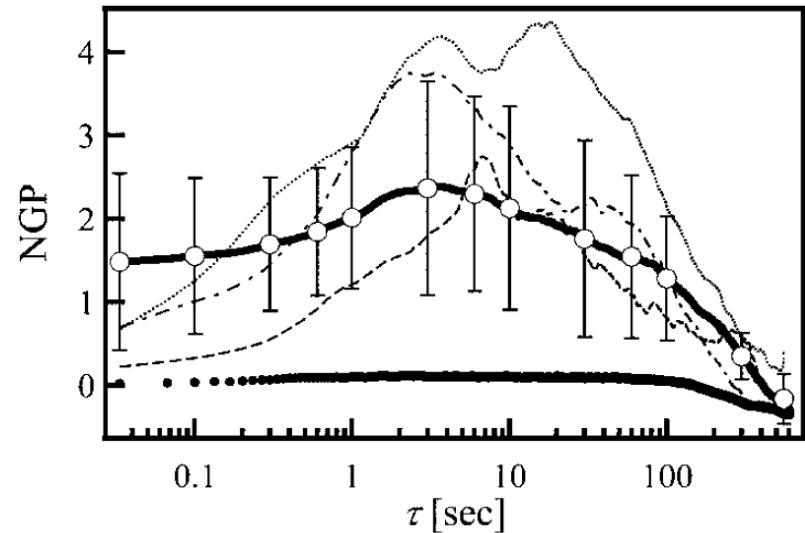
Recent observation of non-monotonous kurtosis:

Distribution of velocity increment depend on the lag-time:



Non-Gaussianity Parameter:

$$\text{NGP} = \frac{\langle \Delta x(\tau)^4 \rangle}{3 \langle \Delta x(\tau)^2 \rangle^2} - 1,$$



Cite this: *Soft Matter*, 2011, **7**, 3234

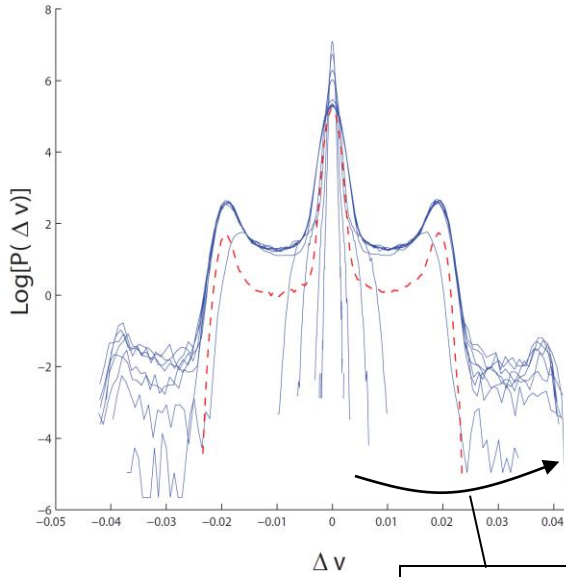
www.rsc.org/softmatter

Non-Gaussian athermal fluctuations in active gels†

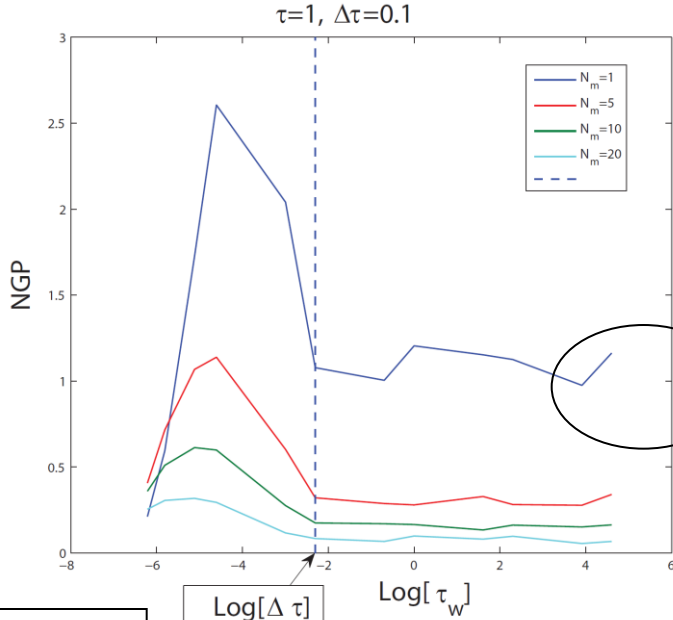
Toshihiro Toyota,^a David A. Head,^b Christoph F. Schmidt^c and Daisuke Mizuno^{*a}

Comparing to our simple model:

Single motor, $p_{on} \sim 0.1$:



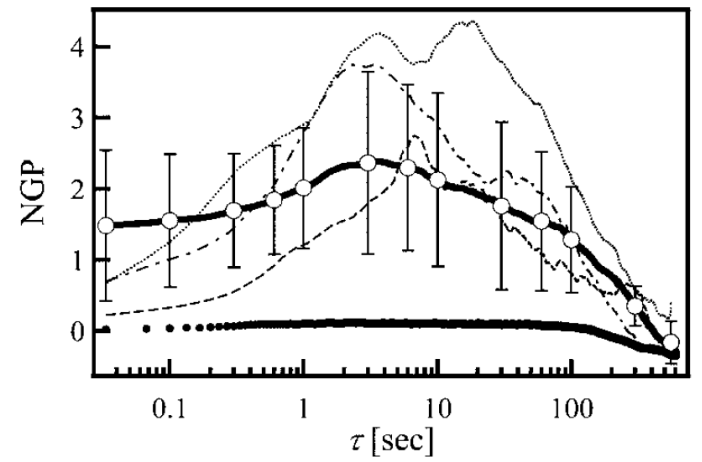
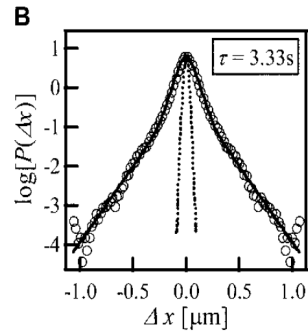
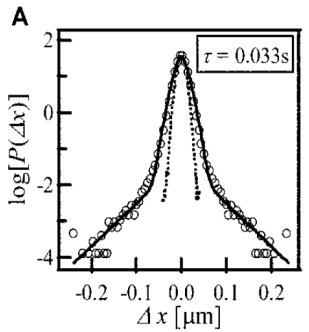
Increasing τ_w



The limit of $\tau_w \rightarrow \infty$ we have:

$$P(\Delta v(\tau_w)) \rightarrow \int_{-\infty}^{\infty} P(v)P(v + \Delta v)dv$$

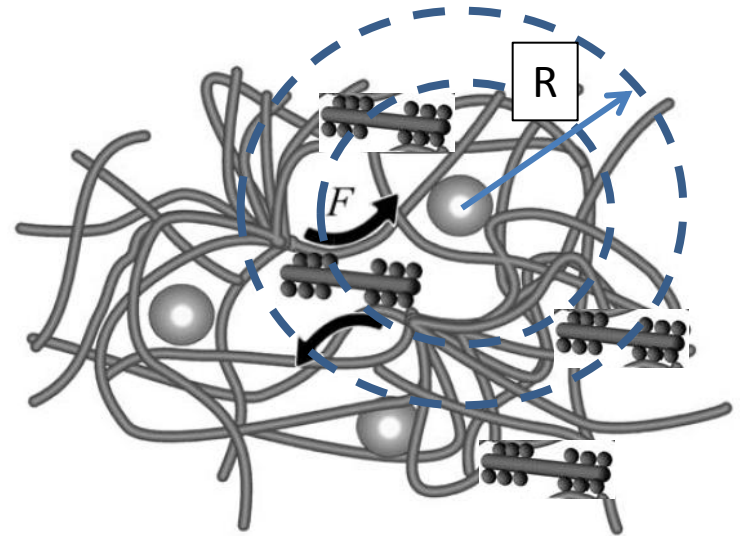
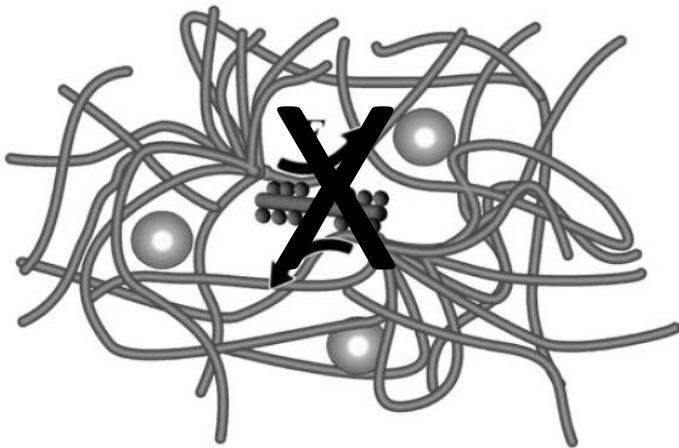
which is not necessarily a Gaussian !



Comparing to our simple model:

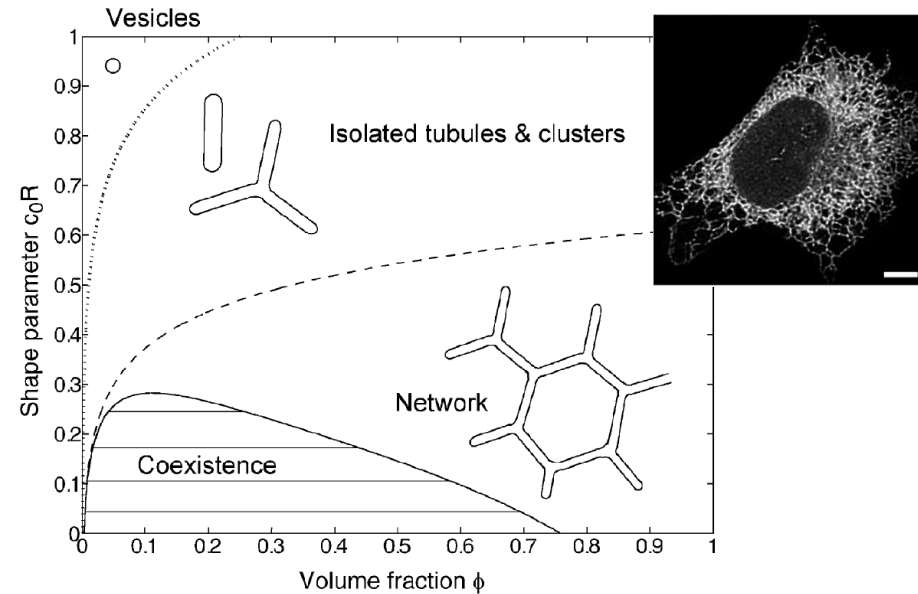
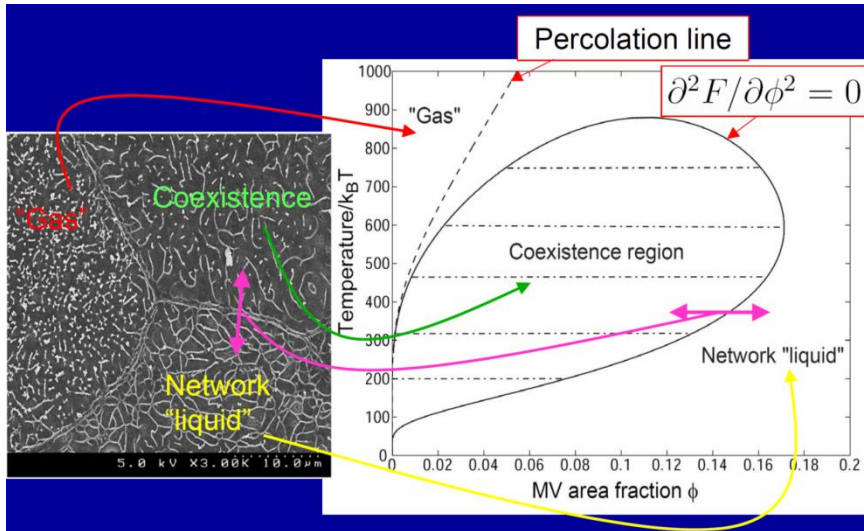


- The bead is NOT affected by only one proximal motor, but by the whole spatial distribution of motors:
- Numerous motors: $N_m \sim R^2$
- But weaker: $f_0 \sim 1/R^2$



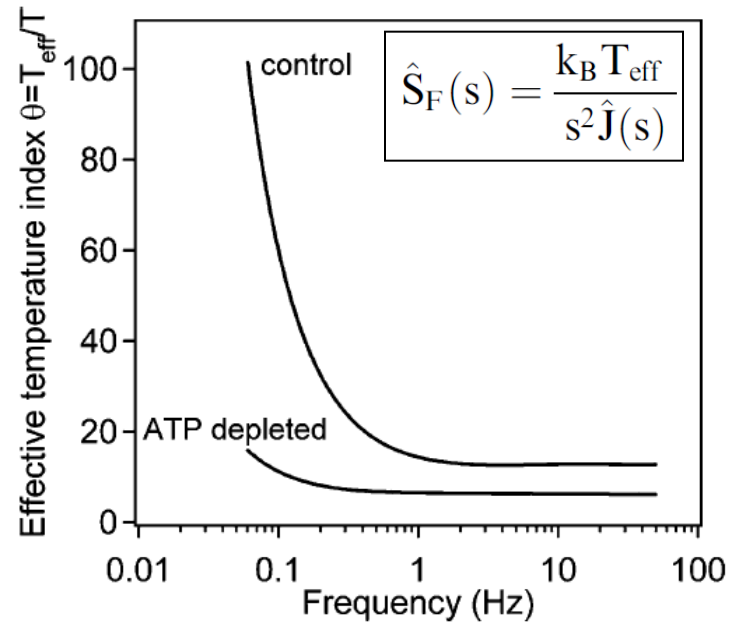
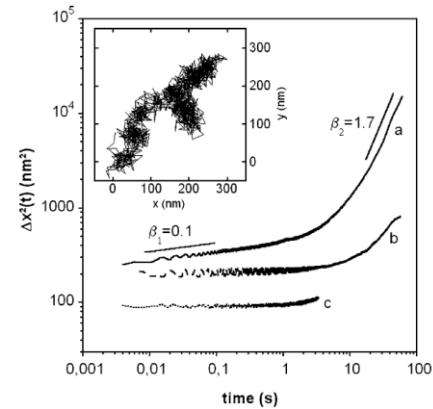
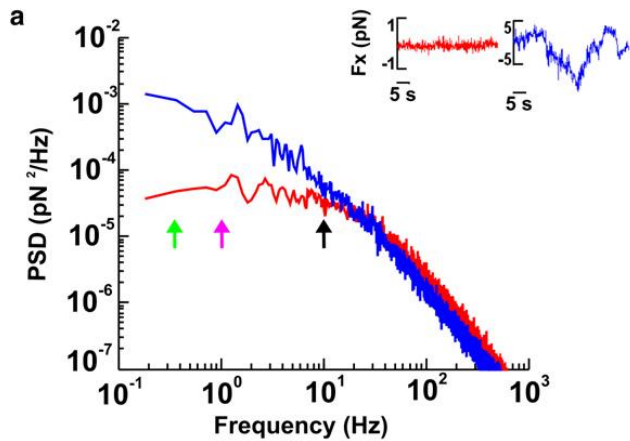
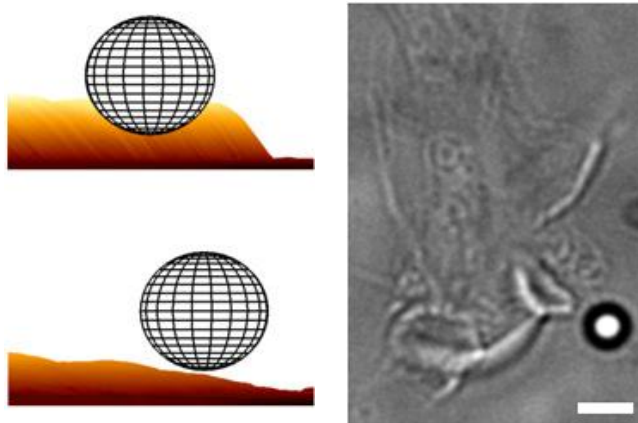
We're now calculating this spatially-extended problem

The future: how to describe phase transitions with active fluctuations ?



- Is there a critical effective temperature for a phase transition, especially when the components have different effective temperatures?
- What are the critical exponents near such a transition (active-RG) ?

Actin-myosin-driven cellular shape fluctuations



Biophysical Journal Volume 98 March 2010 979–988

Force Generation in Lamellipodia Is a Probabilistic Process with Fast Growth and Retraction Events

Rajesh Shahapure,[†] Francesco Difato,^{††} Alessandro Laio,[†] Giacomo Bisson,[†] Erika Ercolini,^{†§} Ladan Amin,[†] Enrico Ferrari,[‡] and Vincent Torre^{††*}

Power spectrum of out-of-equilibrium forces in living cells: amplitude and frequency dependence

François Gallet,^{*} Delphine Arcizet,[‡] Pierre Bohec and Alain Richert

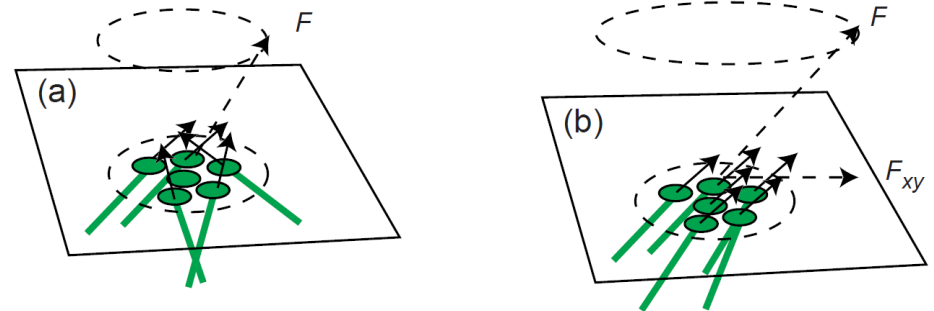
Soft Matter, 2009, 5, 2947–2953

Detailed model of the Active Fluctuations of actin-driven membrane patches and protrusions

Motion of actin patches in yeast cell



Model: random and bundled actin



Smith, Swamy, Pon (2001)

- Each patch has different number of pushing actin filaments, i.e. number of motors N

C.M. mean-square velocity for patch of mass M (shot-noise force correlations, τ):

$$\langle F'(t)F'(t') \rangle = \frac{Ncf^2}{M^2} e^{-|t-t'|/\tau}$$

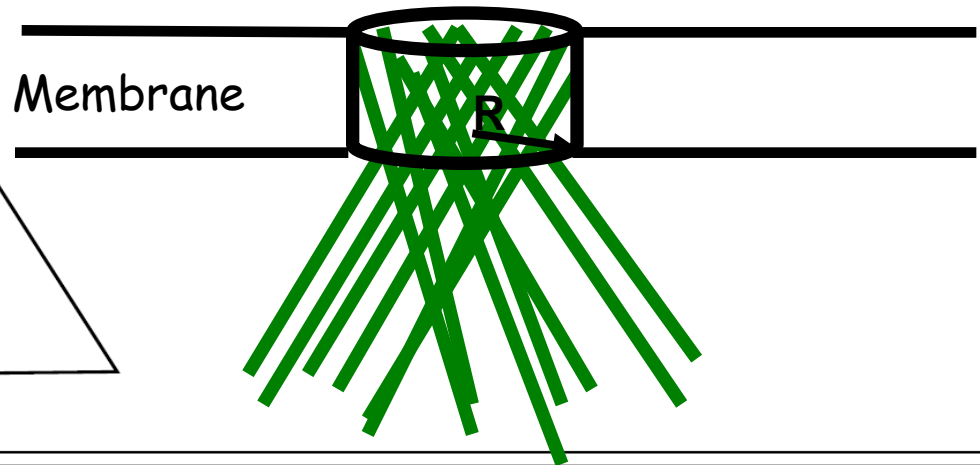
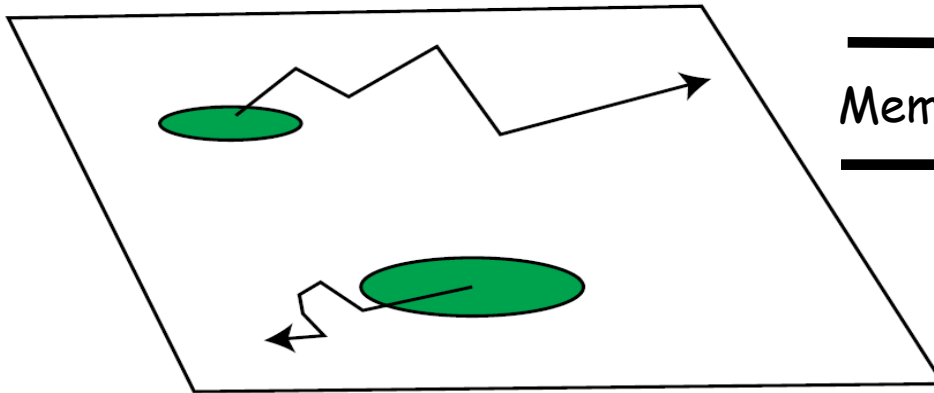
$$\langle v_{cm}^2 \rangle_{t \rightarrow \infty} = \frac{cf^2\tau}{2M\lambda(1 + \lambda\tau/M)}$$

$$\langle F'(t)F'(t') \rangle_{thermal} = \frac{4k_B T \lambda}{M^2} \delta(t-t')$$

Thermal forces depend on the friction

Effective temperature of actin-driven membrane patches

Different forms of patch friction give different forms of effective temperature:



- Hydrodynamics in 2D (Saffman-Delbruck)
- Stokes-like behavior ($D \sim 1/R$)
- Stick-slip with underlying cytoskeleton

Friction λ	δ -correlations Shot-Noise $\tau \ll 1$	or Shot-Noise correlations $\tau \gg 1$
Const.	$\langle v^2 \rangle \propto N^{-1}$	$\langle v^2 \rangle \propto Const.$
$\propto \sqrt{N}$	$\langle v^2 \rangle \propto N^{-3/2}$	$\langle v^2 \rangle \propto N^{-1}$
$\propto N$	$\langle v^2 \rangle \propto N^{-2}$	$\langle v^2 \rangle \propto N^{-2}$

Very different behaviors:

$$k_B T_{eff,N} = M \langle v_{cm}^2 \rangle / 2$$

$$T_{eff}(N) \sim N$$

$$T_{eff}(N) \sim 1/N$$

Size-distribution of actin-driven membrane patches (active thermodynamics)

Interacting patches of different effective temperatures:

$$F = \sum_N \rho_N \left[k_B T_{eff}(N) \ln(\rho_N) - \mu N + \gamma \sqrt{N} \right]$$

Minimizing this free energy with respect to the cluster size distribution:

$$\rho_N = A e^{-\left[\frac{-\mu N + \gamma \sqrt{N}}{k_B T_{eff}(N)} \right]}$$

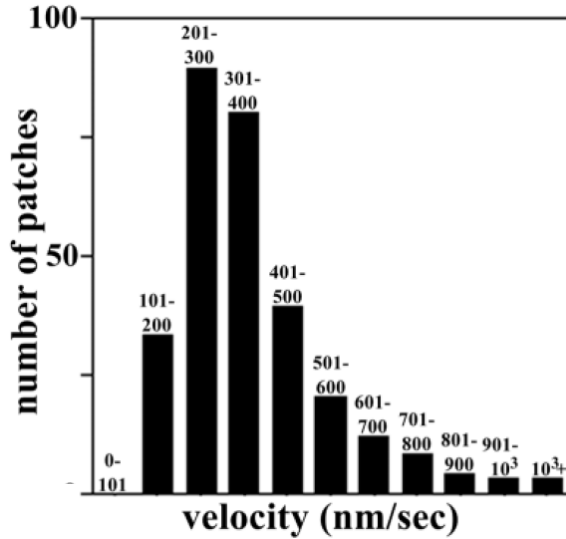
for dilute systems such that $\mu \ll 0$ and the system is far from phase separation (phase transition).

But is this a “kosher” procedure ? Doe this actually happen ?

Work in progress...

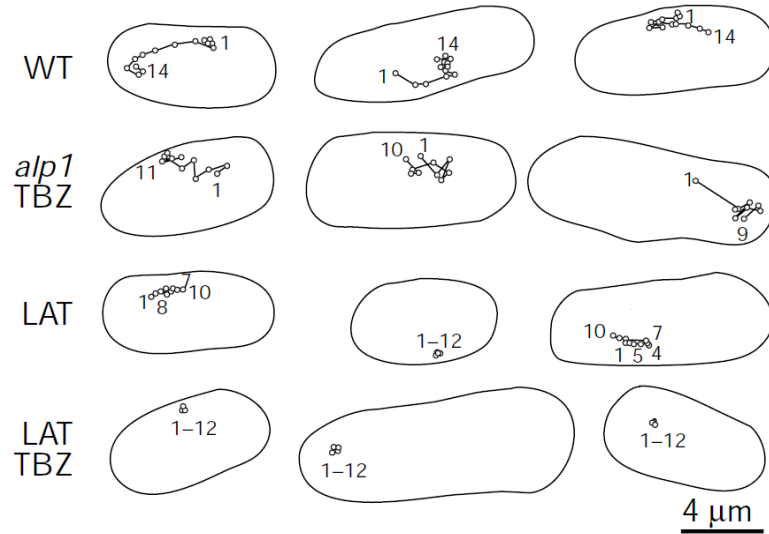
Comparison to observations:

Velocity distribution of patches:



Smith, Swamy, Pon (2001)

Actin drives the random motion:



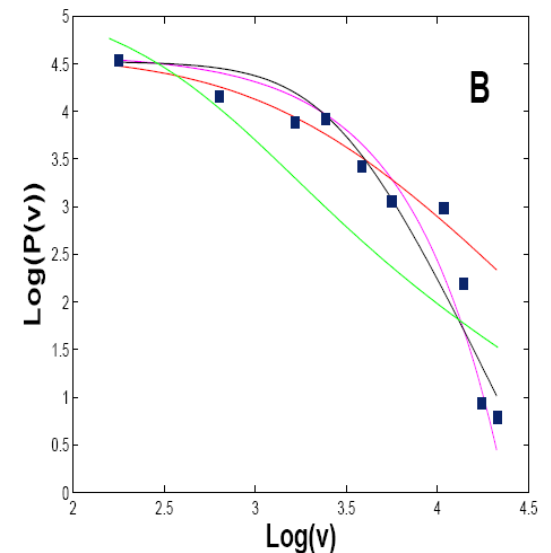
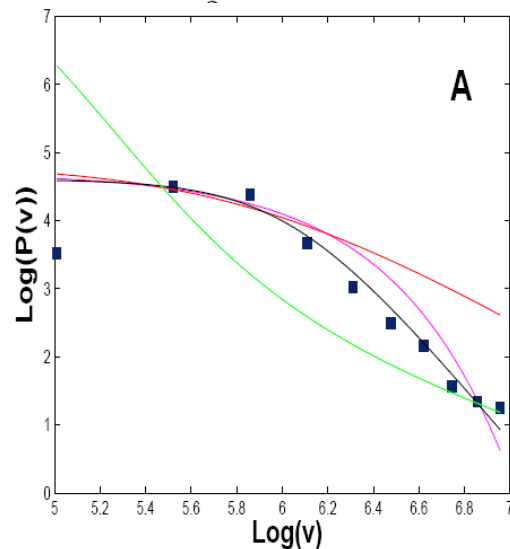
Chang (1999)

Comparison to observations:

Thermal case: $P(v) \propto (a + v^2)^{-1}$ (red)

Active cases: $T_{eff}(N) \propto N, N^2$ we get $P(v) \propto \exp(-v^2)$ (purple)

$T_{eff}(N) \propto \sqrt{N}$ we get $P(v) \propto (a + v^2)^{-2}$ (black)



- An indication that the clusters are active
- May furthermore indicate what is the dominant friction mechanism

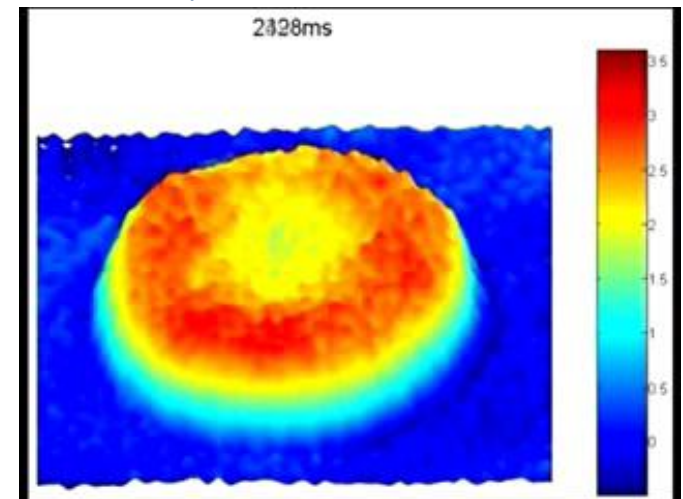
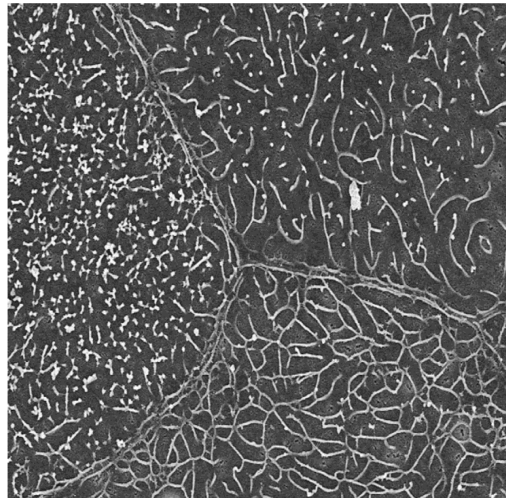
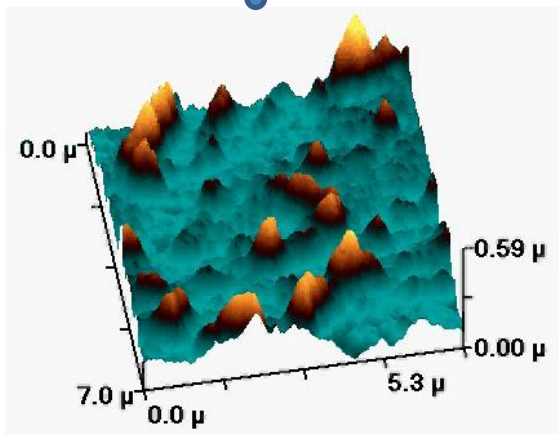
Conclusion:

- Active motion in living systems, driven by a variety of molecular motors, affects pattern formation
- The random motion may be treated as “effective temperature” ?

- Example where Biological Physics motivates research into new non-equilibrium systems.
- We are just beginning to quantify and ask the questions regarding these active phase-transitions.

Its moving...

Its alive !



Acknowledgements

Group



Tomer Bar-On
(post-doc)



Eyal Ben-Isaac
(PhD)



Claudia Vartash
(Post-doc)



Elisabeth Fischer-Friedrich
(Post-doc)

Collaborations

Yair Shokef
(WIS)

Frank Brown
(UCSB)

Gabi Popescu
(UIUC)

Paul Park
(KAIST)

Grants:

