SPIN FLUCTUATIONS AND MAGNETIC SUSCEPTIBILITY OF A NEARLY FERROMAGNETIC FERMI GAS

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Experimental realization of BEC-BCS crossover:

- one of the most important achievements of ultracold atomic physics

- valuable contribution to our understanding of (high Tc) superconductivity
Many-body features of a Fermi gas near a Feshbach resonance

- s-wave scattering length
- BEC regime of molecules
- unitary limit
- BCS regime
On the BEC side additional branch occurs.

What happens if value of $a$ is increased adiabatically starting from $a=0$?
Superfluid molecular BEC branch Eq of State now measured with high precision (see Salomon’s talk)

Metastable branch small $a$: Fermi gas of hard spheres (Lee-Yang 1957)

Possible scenarios for the fate (large $a$) of the repulsive branch

- **Solidification** (liquid-solid transition in He3)
- **Transition** to superfluid molecular BEC phase
- Itinerant **ferromagnetism** (Mit experiment)
- Metastable **combination** of FM and SF
First experimental evidence for repulsive branch (ENS 2003)

(interaction energy is positive !)
Recent experiments at MIT (Jo et al. Science, 2009) suggest occurrence of a transition of the repulsive branch to a \textit{ferromagnetic} phase when the scattering length is positive and exceeds a \textit{critical value}.

\textbf{However:} Spin domains are \textbf{not observed} above the transition.

Rich (and not exhausted) debate about nature of the FM phase in resonantly interacting Fermi gases (mainly on role of \textit{metastability} and \textit{adiabaticity}).

Ho, Conduit, Simons, Zhai, Altman, Demler …
Assuming equilibrium one can compare experiment with predictions of many-body theories (mean field Stoner model, Monte Carlo simulations).

Stoner model

based on mean field density functional

\[
E(M) = \int d\mathbf{r} \left[ \frac{3\hbar^2}{10m} \left(6\pi^2\right)^{2/3} (n_{\uparrow}^{5/3} + n_{\downarrow}^{5/3}) + \frac{4\pi\hbar^2a}{m} n_{\uparrow} n_{\downarrow} \right]
\]

Where \( \frac{n_{\uparrow} - n_{\downarrow}}{n_{\uparrow} + n_{\downarrow}} \) is gas magnetization
PREDICTIONS OF STONER MODEL

Magnetic instability (Ferromagnetism) occurs at

\[ k_F a = \pi / 2 \approx 1.6 \] (second order transition)

Physical quantities at constant pressure

Minimum in kinetic energy at the FM transition

E(M) at constant volume
Magnetic instability is associated with divergent behavior of magnetic susceptibility

\[ \chi = \chi_0 \frac{m^*}{m(1 + F_0^a)} \]

Stoner model predicts

\[ F_0^a = -\frac{2}{\pi} k_F a \]

\[ \chi_0 = \frac{3n}{2\varepsilon_F} \] is magnetic susceptibility of ideal Fermi gas

Question:
is Stoner model valid for large values of \( k_F a \)?
At the lowest temperature, kinetic energy exhibits pronounced minimum at

\[ k_F a \approx 2 \]

\[ k_F^0 = \left(24N\right)^{1/6} / a_{ho} \]

(Fermi momentum of ideal Trapped Fermi gas)
Recent **MC calculations** of equation of state (Pilati et al., arXiv 1004.1169, Chang et al. arXiv 1004.2680) reveal important **deviations** with respect to Stoner model ($k_Fa \approx 0.85$ vs $k_Fa = 1.6$) and even more compared to experiment ($k_Fa \approx 2$)
Magnetic susceptibility diverges at the FM transition

\[ \chi^{-1} = V \partial^2 A / \partial (N_\uparrow - N_\downarrow)^2 \]

Inverse susceptibility at T=0

\[ \frac{\chi_0}{\chi} = \left[ 1 - \frac{2}{\pi} k_F a - \frac{16(2 + \ln 2)}{15\pi^2} (k_F a)^2 \right] \]

Stoner (mean field)

Monte Carlo

Galitskii expansion
Questions

Can we probe directly the magnetic properties of the gas?

- Spin fluctuations (CORRELATIONS)

- Frequency of spin oscillations (DYNAMICS)
  (first exp at Jila 2002)

- Measurement of the equation of state of imbalanced Fermi gas (THERMODYNAMICS)
Spin fluctuations: potentially powerful tool to explore the nature of quantum phases:

Examples:

- **Fermi liquid** (Fermi gas with small $a>0$)
- **Itinerant ferromagnetism** (large $a>0$)
- **Gap** in Fermi superfluids (unitary Fermi gas)
- **AF and FM** in optical traps

At finite T spin fluctuations provide direct access to the **magnetic susceptibility**
Spin fluctuations and magnetic susceptibility

At finite temperature statistical mechanics provides general relationship between spin fluctuations and magnetic susceptibility:

\[
\frac{\Delta (N_{\uparrow} - N_{\downarrow})^2}{N} = k_B T \frac{\chi(T)}{n}
\]

Analog of result for density fluctuations in terms of isothermal compressibility

\[
\frac{\Delta N^2}{N} = k_B T n k_T
\]

N : number of atoms in small subvolume of whole cloud
At low $T$ thermal fluctuations are suppressed and quantum fluctuations become more and more important.

At zero temperature only quantum fluctuations survive. They exhibit weaker $N$ dependence with respect to thermal fluctuations.

(Astrakarchick, Combescot, Pitaevskii (PRA 2007))
For systems exhibiting static structure factor with linear $q$ behavior:

$$\frac{\Delta(N_{\uparrow} - N_{\downarrow})^2}{N} = 2\alpha \left( \frac{12}{\pi^4 N} \right)^{1/3} \ln(CN^{1/3})$$

- $\alpha$ defined by small $q$-behavior of static structure factor (FT of two-body correlation function)
- $C$: cut-off fixed by short range behavior of $S(q)$ (ideal Fermi gas: $\alpha = 3/2^{7/3}$; $C = 8.3$)
- Holds for normal Fermi liquids (example: repulsive gas before FM transition)
- For Fermi superfluids existence of gap implies and hence even weaker $N$ dependence

$S^a(q) \rightarrow \alpha q / q_F$

$S^a(q) \propto q^2$
Thermal vs. Quantum Fluctuations

\[ \frac{\Delta(N^\uparrow - N^\downarrow)^2}{N} = k_B T \frac{\chi(T)}{n} \]

\[ \frac{\Delta(N^\uparrow - N^\downarrow)^2}{N} = 2\alpha \left( \frac{12}{\pi^4 N} \right)^{1/3} \ln(CN^{1/3}) \]

Comparison shows that, for the ideal Fermi gas at \( T = 0.1T_F \), (where we can use \( \chi_0(T) \approx 3n / 2k_B T_F \) ), quantum fluctuations become important for \( N < 10^3 \).
Some questions

Is transition between thermal and quantum fluctuations measurable in actual experiments?

What is the behavior of quantum fluctuations near FM transition?
The simplest example: the ideal Fermi gas

N=100

Thermal & quantum fluctuations

Only thermal fluctuations

\( \sigma_N/N \) vs. \( T/T_F \)
At low $T$ one observes stronger deviations from linear $N$ dependence.

Fluctuations for smaller $N$ measured at Zurich (Mueller et al. arXiv 1005.0302)
Behavior of spin quantum fluctuations near FM transition?

First answer provided by Landau theory of Fermi liquids. Ignore Landau parameters with \( l \geq 1 \). From behavior of dynamic response function one extracts

\[
S^a(q) \rightarrow \alpha q / q_F
\]

with

\[
\alpha = \frac{3}{2^{4/3}} \int_0^1 \frac{\lambda d\lambda}{\left(1 - F_0^a \left(1 - \frac{\lambda}{2} \ln \frac{1+\lambda}{1-\lambda}\right)\right)^2 + \left(\lambda F_0^a / 2\right)^2}
\]
Near the FM transition quantum fluctuations diverge logarithmically

\[ \frac{\Delta (N_{\uparrow} - N_{\downarrow})^2}{N} \propto \ln \frac{1}{1 + F_0^a} \]

while thermal fluctuations (at low T) diverge as

\[ \frac{\Delta (N_{\uparrow} - N_{\downarrow})^2}{N} \propto \frac{1}{1 + F_0^a} \]

follows from

\[ \Delta (N_{\uparrow} - N_{\downarrow})^2 / N \propto \chi \; ; \; and \; \chi = \chi_0 \frac{m^*}{m(1 + F_0^a)} \]

\( F_0^a \to -1 \) at the FM transition
Questions

Can we probe directly the magnetic properties of the gas?

- Spin fluctuations (CORRELATIONS)
- Frequency of spin oscillations (DYNAMICS) (first exp at Jila 2002)
- Measurement of the equation of state of imbalanced Fermi gas (THERMODYNAMICS)
Spin dipole frequency of a trapped repulsive Fermi gas

(In analog of giant dipole resonance of nuclear physics)

In uniform Fermi liquids spin oscillation is Landau damped if $F_0^a < 0$

In harmonic trap dipole oscillation is not Landau damped because of absence of s.p excitations below oscillator frequency
Sum rule approach to the spin dipole frequency of a trapped repulsive Fermi gas

\[ \hbar^2 \omega_{\text{spin}}^2 = \frac{m_1(D)}{m_{-1}(D)} \]

Rigorous upper bound to the lowest frequency excited by the spin dipole operator

\[ D = \sum_{i \uparrow} z_i - \sum_{i \downarrow} z_i \]

\[ m_k(D) = \sum_n (E_n - E_0)^k |\langle 0|D|n \rangle|^2 \]

are k-moments of dipole dynamic structure factor
Energy-weighted sum rule is given by Thomas Reich-Kuhn sum rule

\[ m_1(D) = \frac{1}{2} < [D, [H, D]] > = N\hbar^2 / 2m \]

Inverse energy-weighted sum rule can be derived in the local density approximation (LDA) starting from energy functional

\[ E(n_{\uparrow}, n_{\downarrow}) = \int d\vec{r} \varepsilon(n_{\uparrow}, n_{\downarrow}) - \lambda \int d\vec{r} z(n_{\uparrow} - n_{\downarrow}) \]

to evaluate spin dipole polarizability. By expanding up to terms quadratic in \( (n_{\uparrow} - n_{\downarrow})^2 \) one finds

\[ m_{-1}(D) = \frac{1}{2\lambda} \int d\vec{r} z \delta(n_{\uparrow} - n_{\downarrow}) = \frac{1}{2} \int d\vec{r} z^2 \chi \]

with \( \chi^{-1} = \partial^2 \varepsilon(n_{\uparrow}, n_{\downarrow}) / \partial(n_{\uparrow} - n_{\downarrow})^2 \) inverse susceptibility of uniform matter
Results for spin dipole frequency

\[ \omega_{spin}^2 = \frac{N}{m \int d\vec{r} z^2 n^2 \chi_M(n)} \]

Value of \( \chi \) from MC calculations of Pilati et al.

\[ k_F^0 = (24N)^{1/6} / a_{ho} \]
Spin fluctuations of a repulsive Fermi gas exhibit key features near FM transition.

Thermal fluctuations provide info about magnetic susceptibility, quantum fluctuations (small T, small N) fixed by low q behavior of spin structure factor.

Frequency of spin dipole oscillation sensitive to behavior of magnetic susceptibility. Exhibits significant quenching with respect to ideal gas value also far from the FM transition (test of Normal Fermi liquid theory)
Open questions and perspectives

- Effect of **metastability** and molecular formation on spin fluctuations.

- Identification of **quantum** fluctuations as **surface effect**. Shape dependence of subvolumes explored by laser.

- **Collisional damping** of spin oscillations (T-dependence) (connection with spin diffusion in Zwierlein talk)

- Spin fluctuations in other quantum phases (**Fermi superfluid along the BEC-BCS crossover**)

See recent MIT paper arXiv:1010.1874v1, 9 Oct 2010
A new experimental activity on ultracold atoms is starting at the CNR- BEC Centre, University of Trento

focus will be on:

pure and mixed quantum gases (Fermi - Bose, Bose – Bose)

Fermionic superfluidity

transport phenomena

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