Coarse–graining and hints of scaling in a population of 1000+ neurons

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A (partially) imagined conversation

Biologist: You physicists are so enamoured of simple models that you ignore the microscopic details. You are oversimplifying the mechanisms of life.

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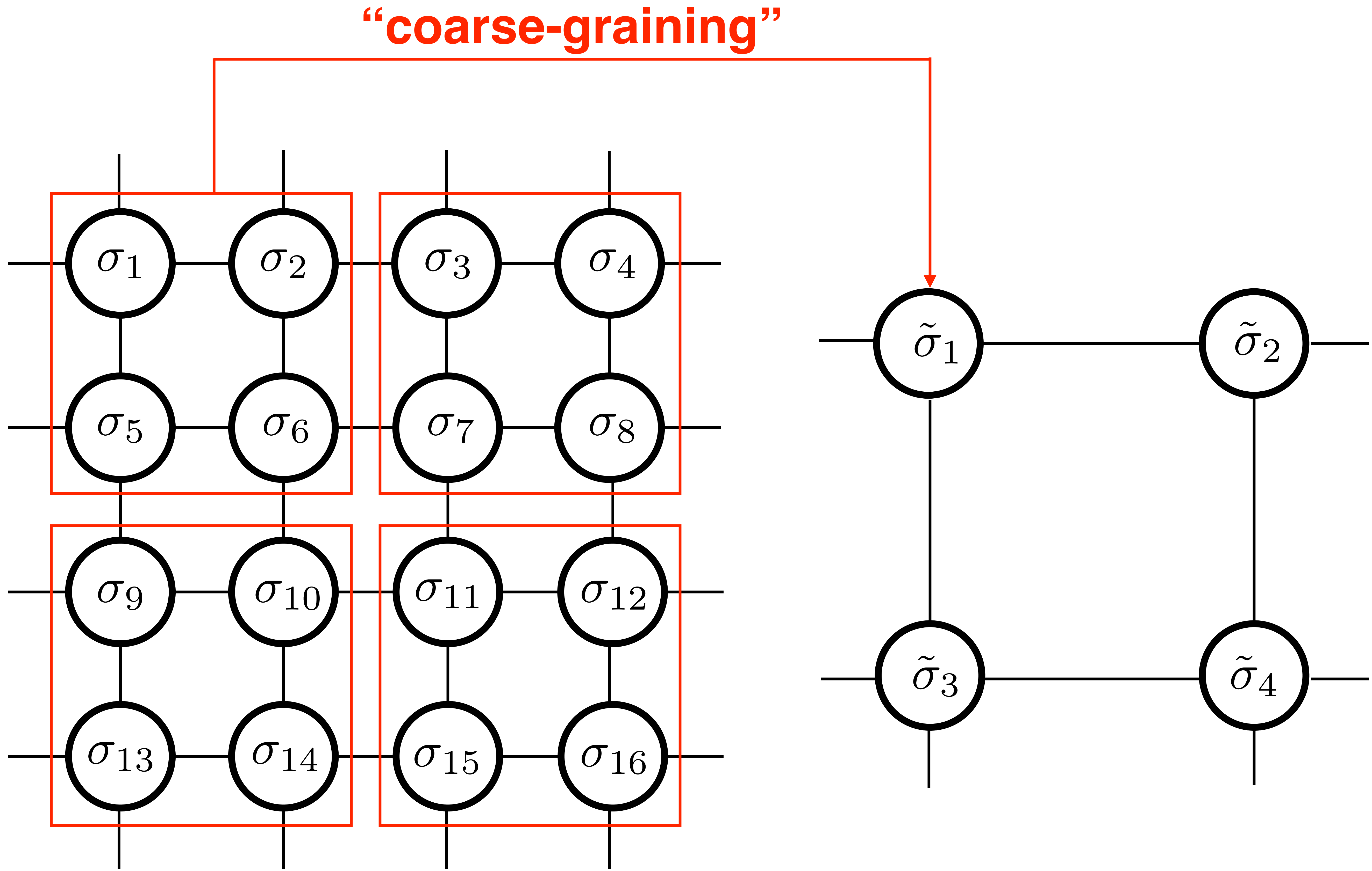
Physicist: Don’t be offended (and don’t feel special). We also oversimplify physics.

We are so used to the success of simple models that we seldom think (any more) about why they work …



$$P(\{\sigma_{\mathrm{i}}\}) \xrightarrow[\textbf{of models}]{\textbf{flow in the space}} P(\{\tilde{\sigma}_{\mathrm{i}}\})$$

What does "real space" coarse-graining mean?

microscopic variable at site j

coarse-grained variable at site i

$$\tilde{\sigma}_{\rm i} = f\left(\sum_{{\rm j}\in\mathcal{N}_{\rm i}} \sigma_{\rm j}\right)$$

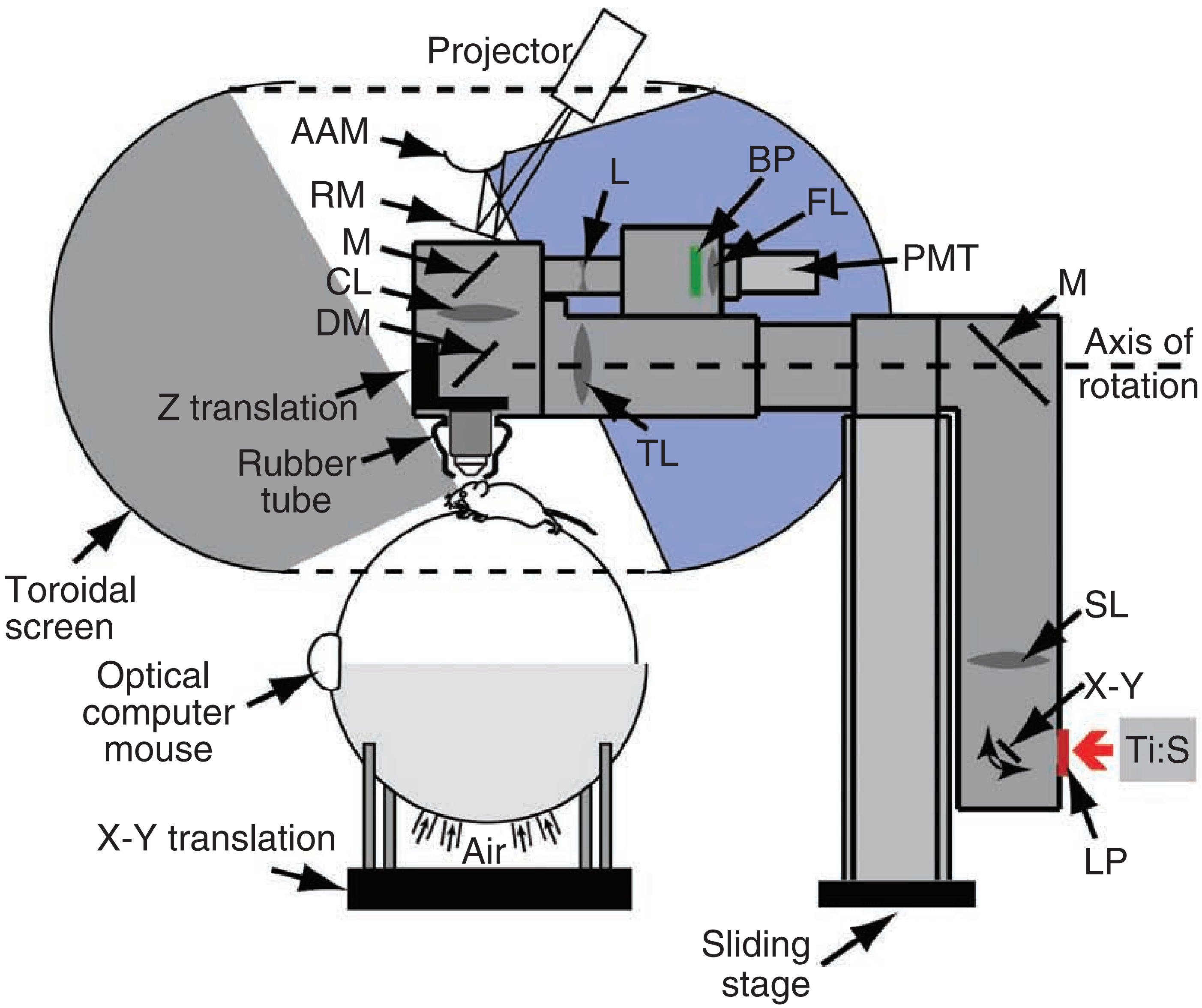
could be identity, or
restore normalization, or
majority rule to restore binary
...

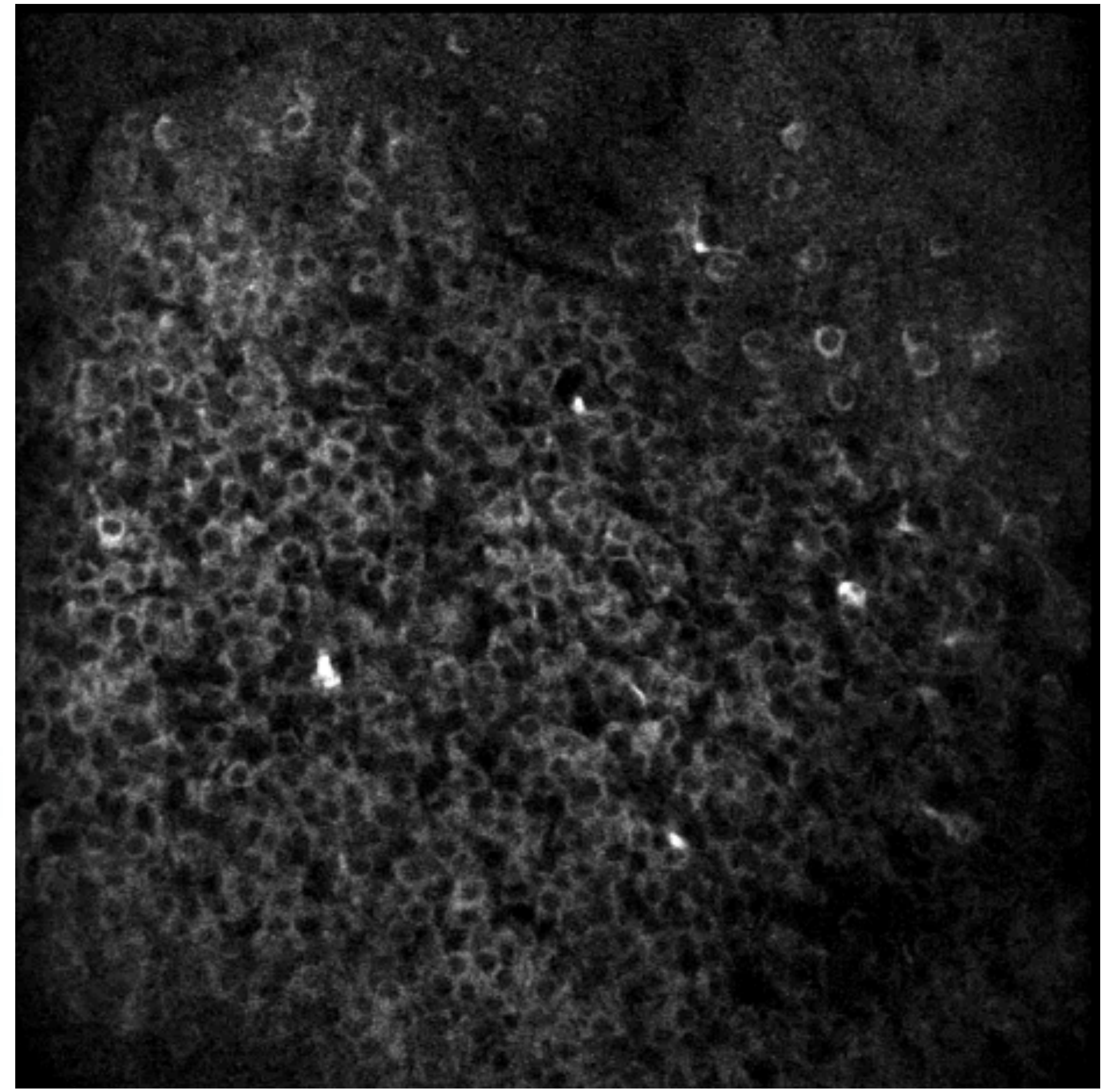
sum over neighborhood
surrounding site i

**How do we do this for neurons,
where locality isn't much of a guide?**

Try "neighbors" = maximally correlated pairs

Optical recording from hippocampal neurons as a mouse moves in a virtual environment





DA Dombeck, CD Harvey, L Tian, LL Looger, and DW Tank, *Nat Neurosci* 13:1433 (2010).
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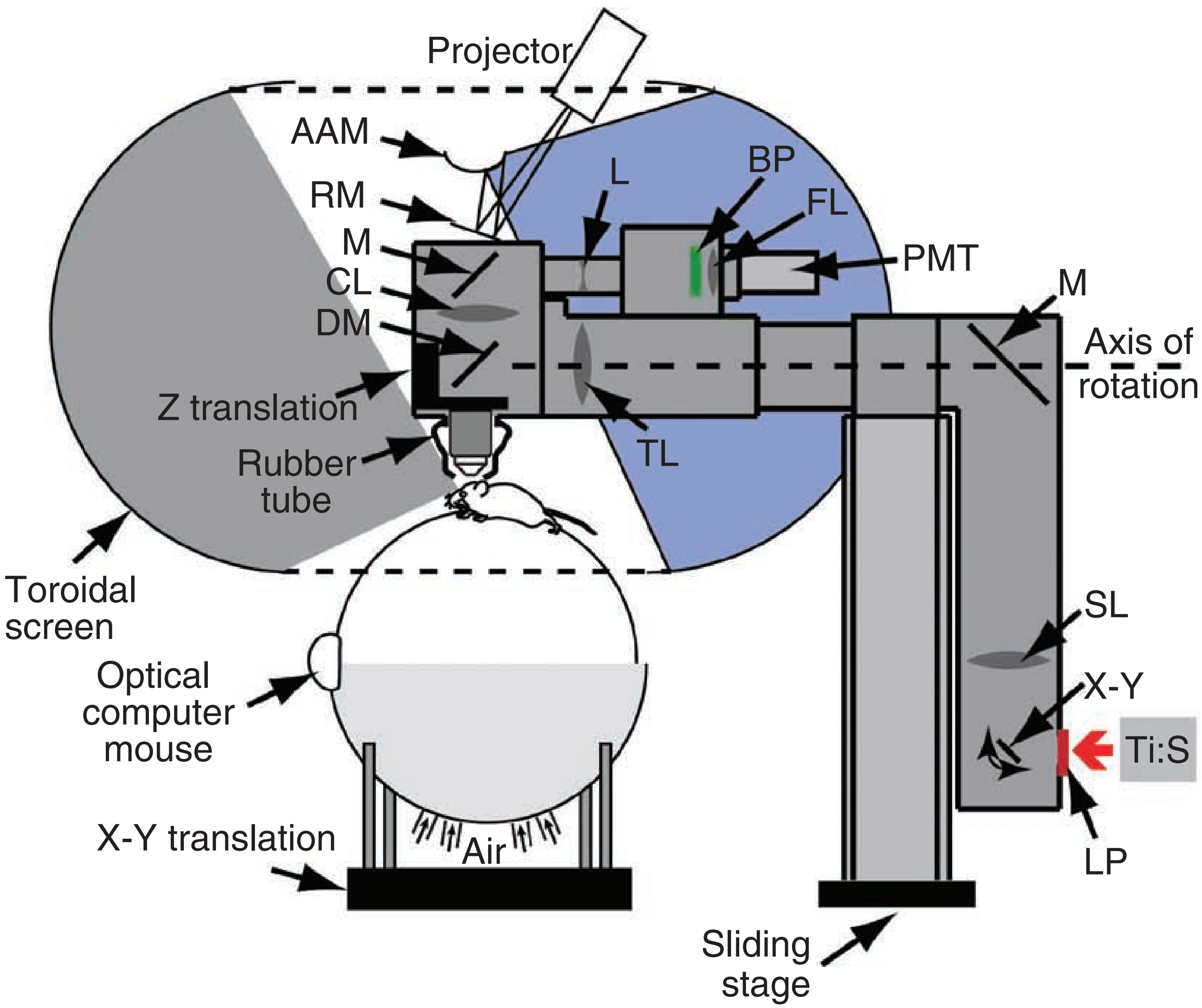

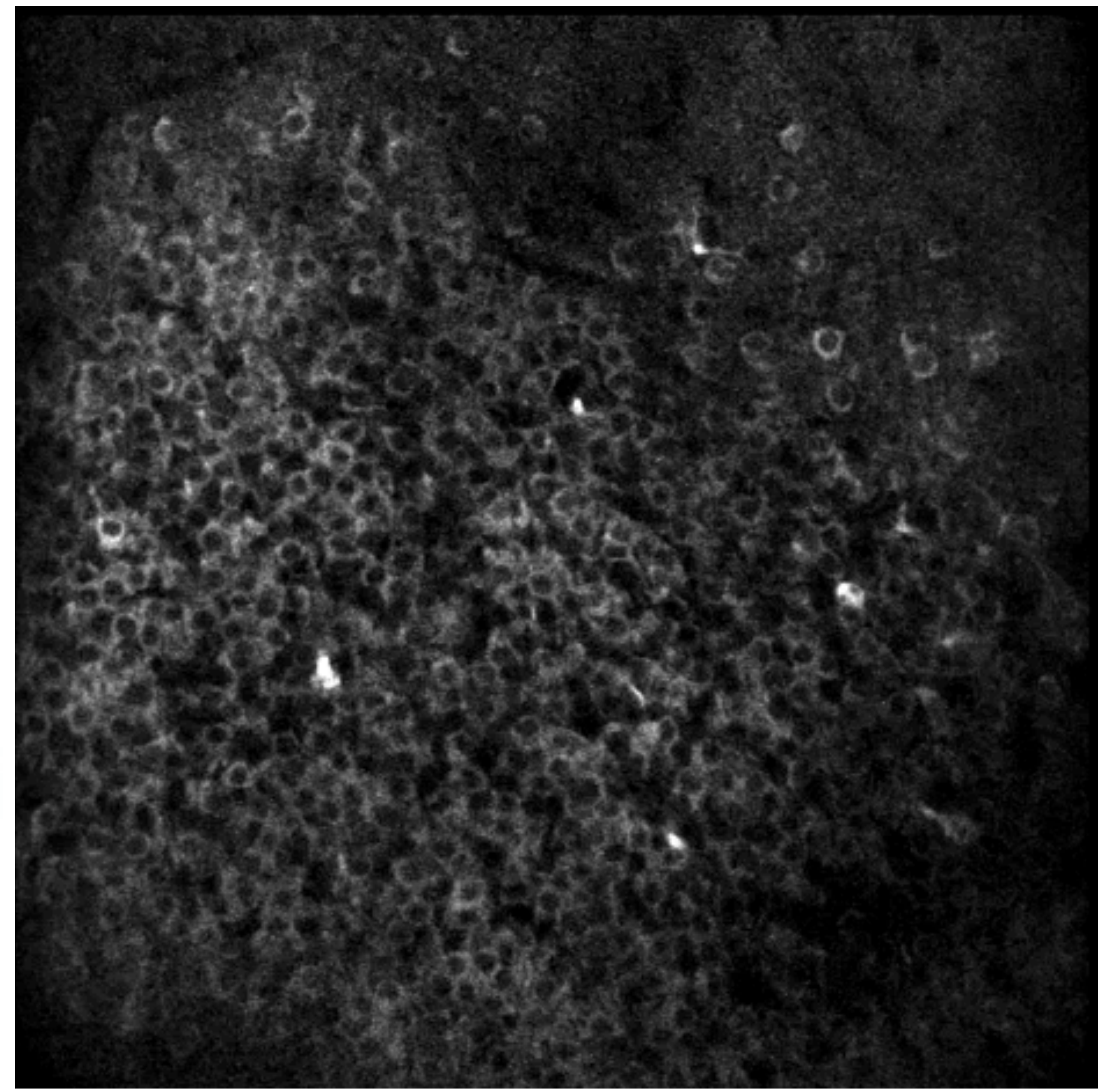

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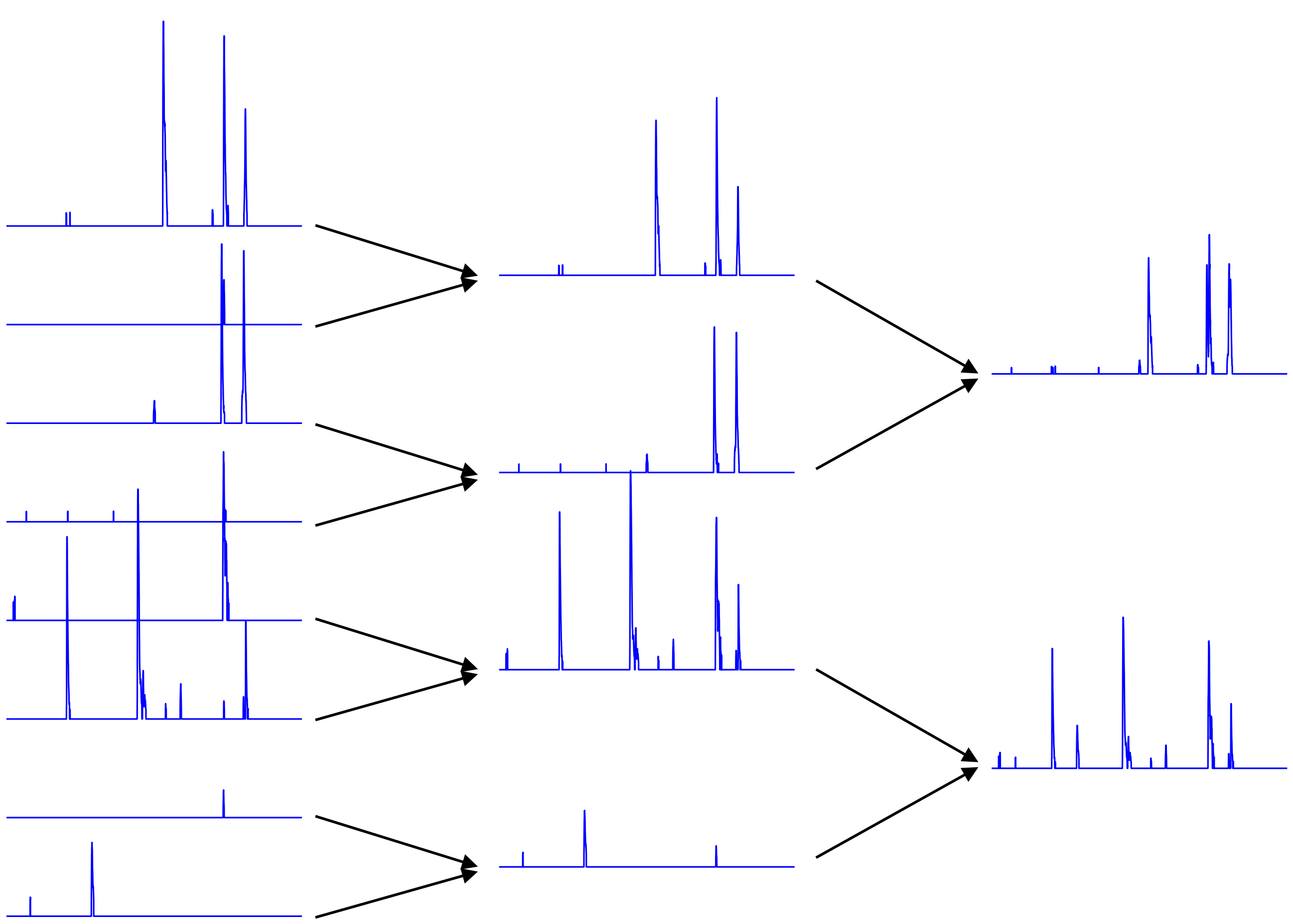
Simplest version

1. Keep continuous signals
2. Normalize so that mean nonzero signal = 1
3. Add together signals from most correlated pair, then the next …
4. Arrive at N/2 coarse-grained variables
5. Iterate

$K = 2^k$ of the original microscopic variables are grouped together after k stages of coarse-graining



Follow the distribution of individual coarse-grained variables

$$P_K(x) \equiv \frac{1}{N_K} \sum_{\mathrm{i}=1}^{N_K} \left\langle \delta\left(x - x_{\mathrm{i}}^{(K)}\right) \right\rangle$$

$$= P_0(K)\delta(x) + [1 - P_0(K)]Q_K(x)$$

normalization

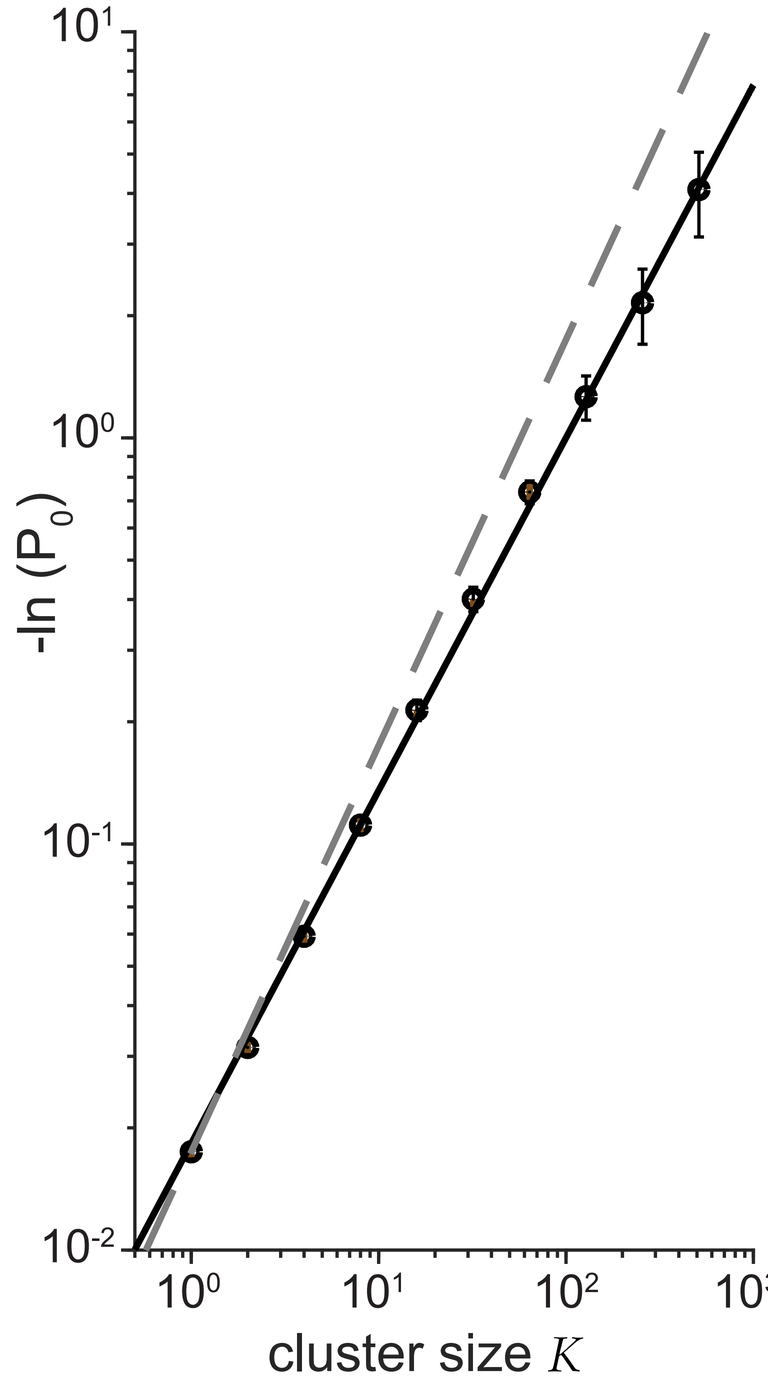
$$\int_0^\infty dx\, Q_K(x) x = 1$$

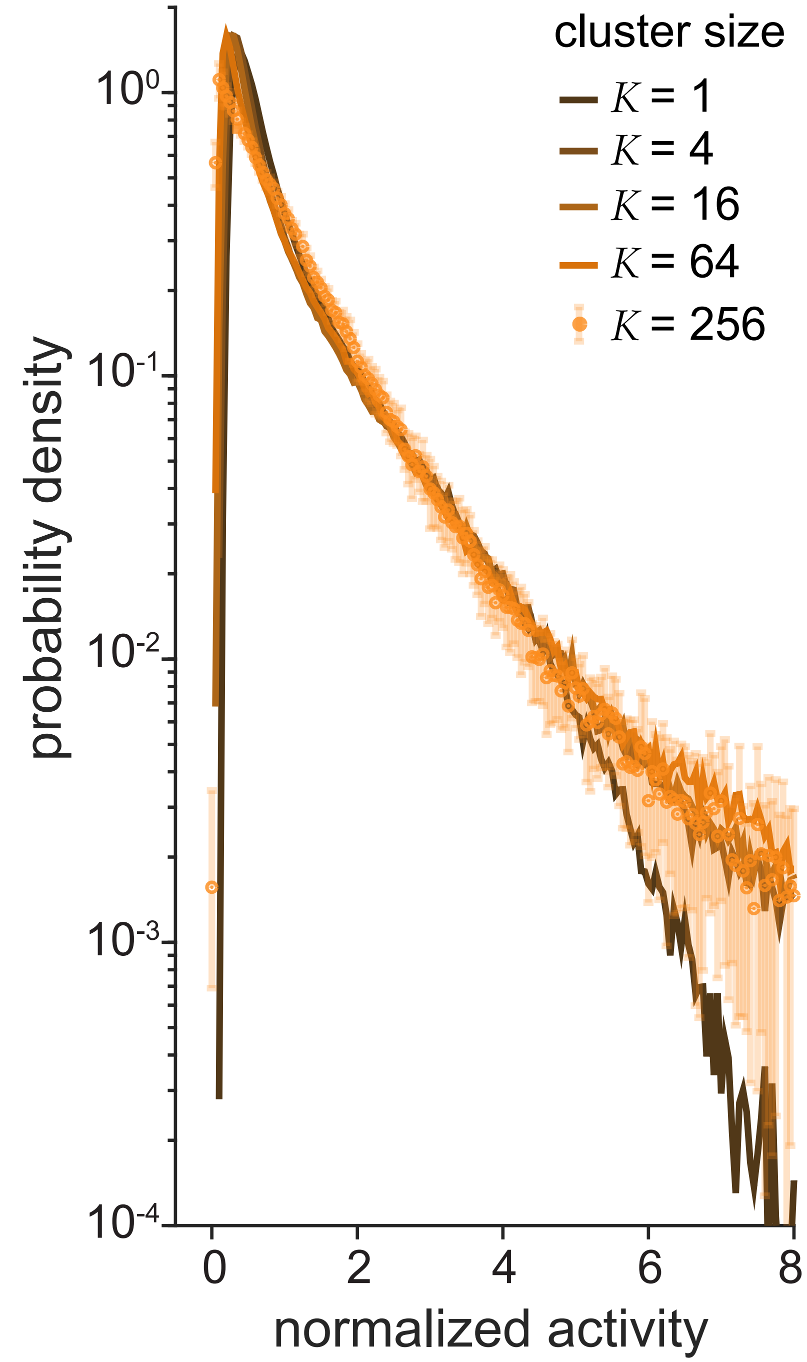
For independent neurons,

$$P_0(K) = \exp(-aK).$$

In fact,

$$P_0(K) = \exp(-aK^{\tilde{\beta}}).$$



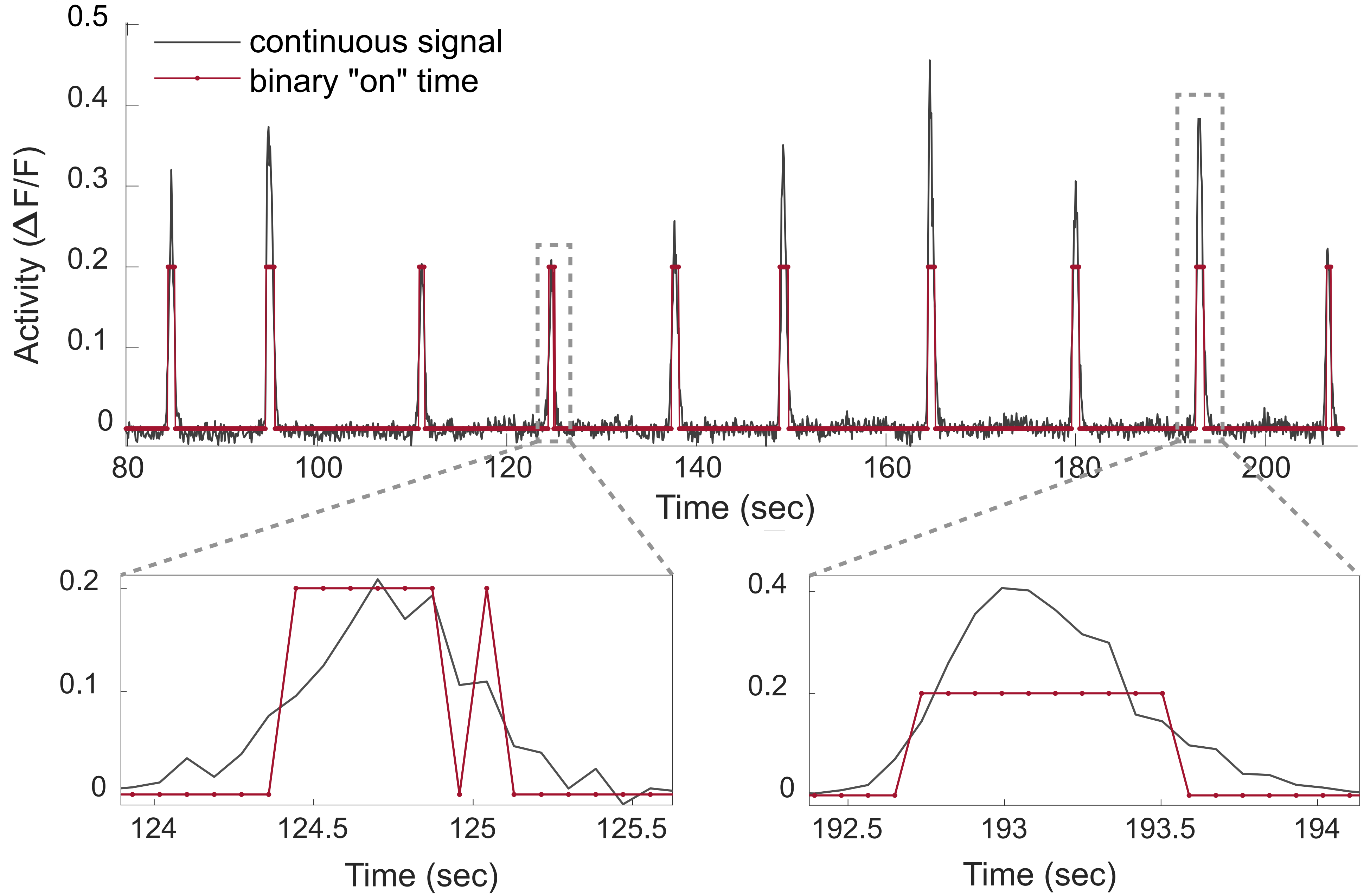


Three independent experiments, three different mice:

$$\tilde{\beta} = 0.88 \pm 0.01$$
$$= 0.89 \pm 0.01$$
$$= 0.86 \pm 0.02$$

Distribution approaches a fixed, non-Gaussian form

Can also discretize to a binary (Ising) activity variable for each neuron



$$\sigma_{\mathrm{i}}(t) = \begin{cases} \textbf{1 (active)} \\ \textbf{0 (silent)} \end{cases}$$

State of the network $\{\sigma_{\mathrm{i}}\}$

Correlations inside the clusters

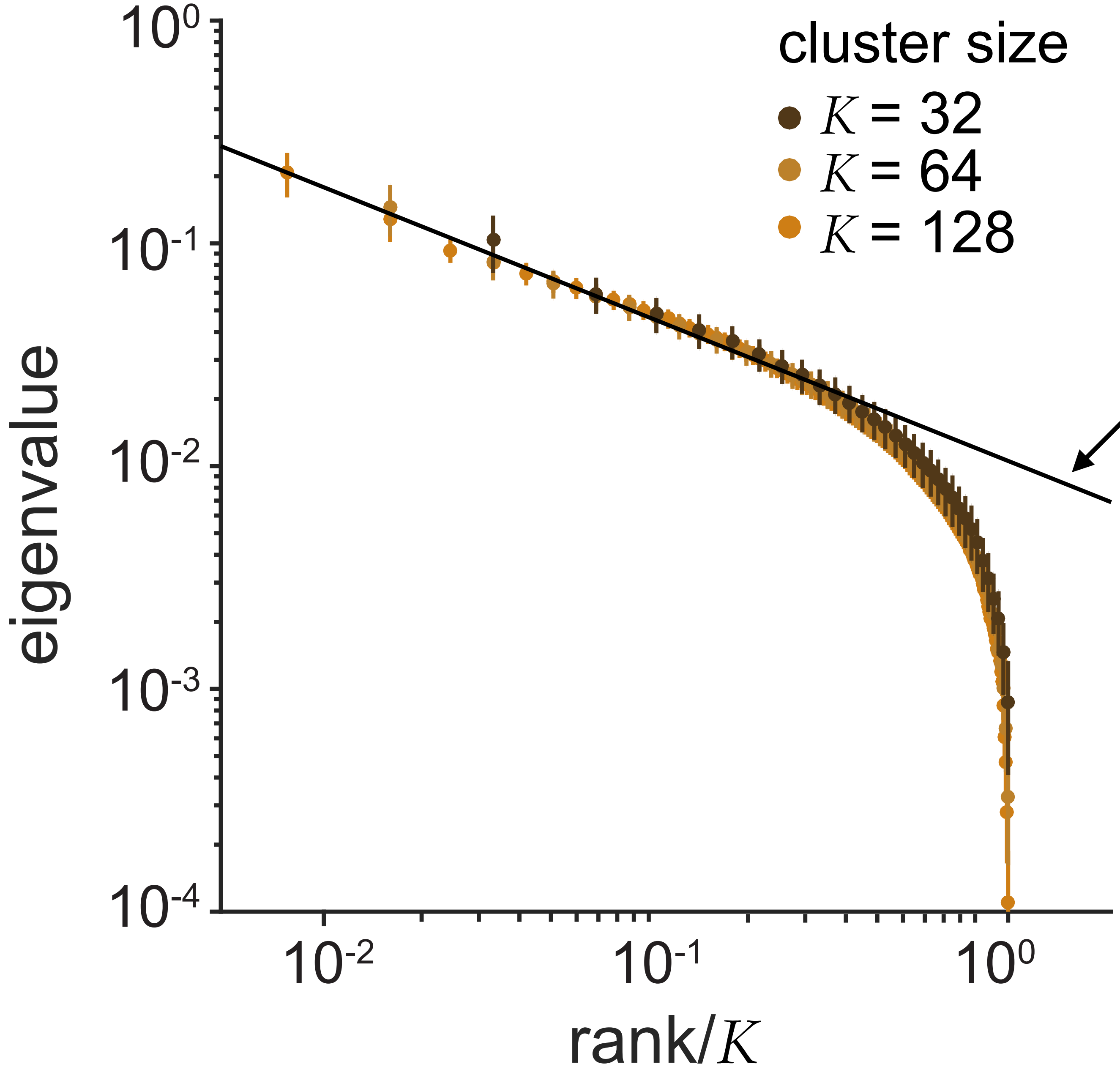
(small excursion to blackboard)

Correlations inside the clusters

$$C_{\mathrm{ij}} = \langle \sigma_{\mathrm{i}} \sigma_{\mathrm{j}} \rangle - \langle \sigma_{\mathrm{i}} \rangle \langle \sigma_{\mathrm{j}} \rangle$$

Find the eigenvalues in clusters of different sizes

(be careful about sampling problems!)



$$\lambda = A \left(\frac{K}{\mathrm{rank}} \right)^{\mu}$$

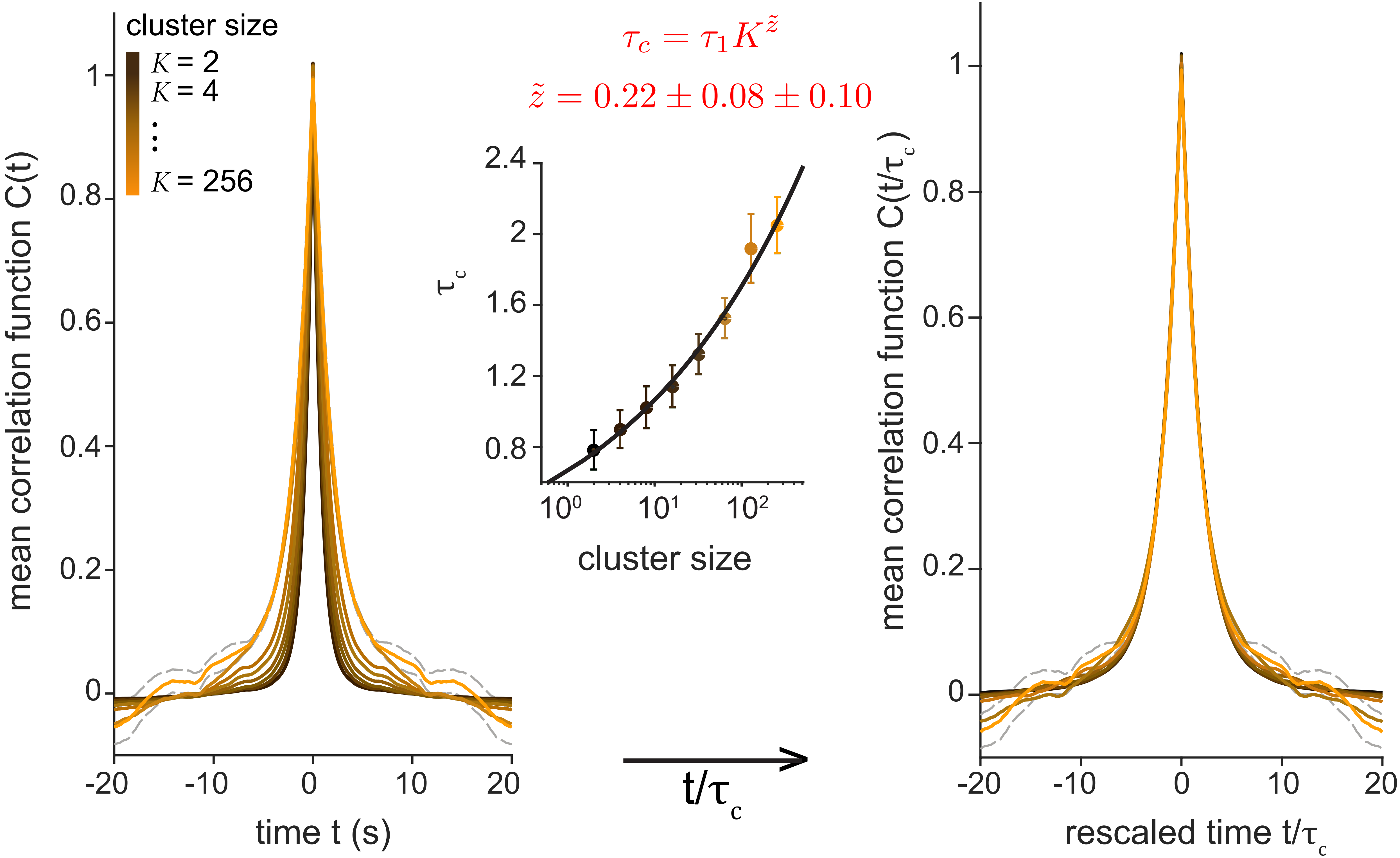
estimation error from a single experiment

$$\mu = 0.76 \pm 0.05 \pm 0.06$$

standard deviation across three experiments/mice

Larger clusters have slower dynamics ...

but these dynamics scale.



Thing to discuss, or worry about

We can do it all again in “momentum” space
Finite sample effects on eigenvalue spectra
Connection to place fields, biological function
New analysis, new artifacts

Some things suggested by the data

Self-similarity of correlation structures
Distributions of coarse-grained quantities approach fixed form
Connection to earlier discussions of criticality
Dynamic scaling - network accesses a wide range of time scales

We can do this all again in "momentum" space ...

$$C_{\mathrm{ij}} \equiv \langle \sigma_{\mathrm{i}} \sigma_{\mathrm{j}} \rangle - \langle \sigma_{\mathrm{i}} \rangle \langle \sigma_{\mathrm{j}} \rangle$$

$$\sum_{\mathrm{j}} C_{\mathrm{ij}} u_{\mathrm{j}}^{(\mathrm{r})} = \lambda_{\mathrm{r}} u_{\mathrm{i}}^{(\mathrm{r})}$$

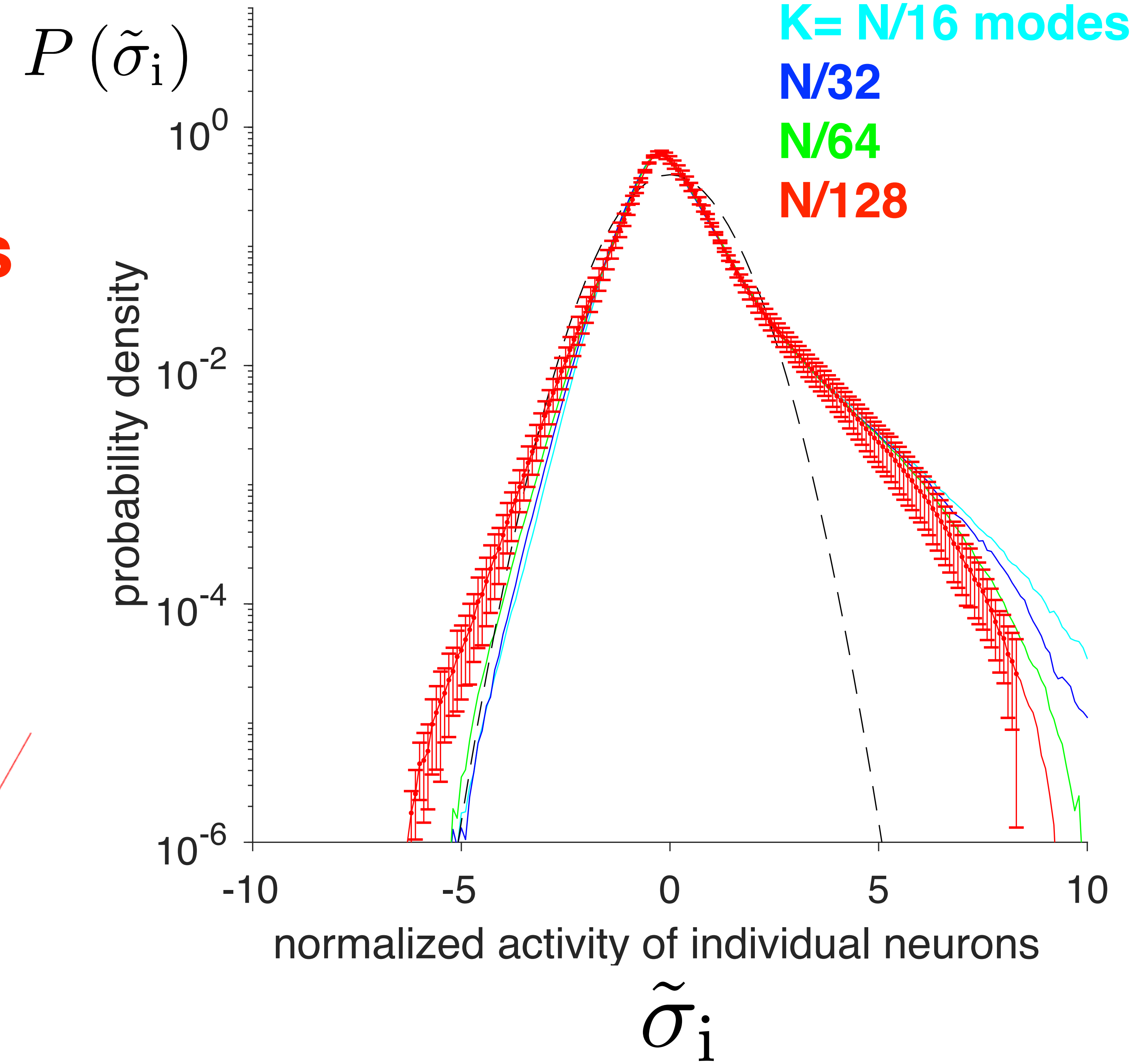
(pause for a reminder about momenta as eigenvectors of C)

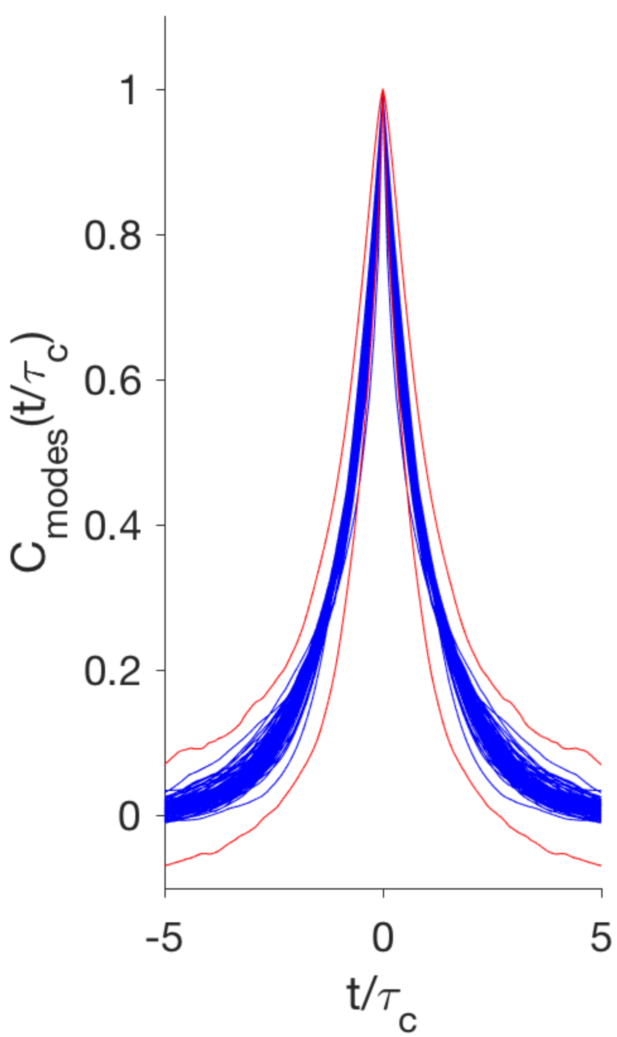
$$\lambda_1 > \lambda_2 > \cdots > \lambda_N$$

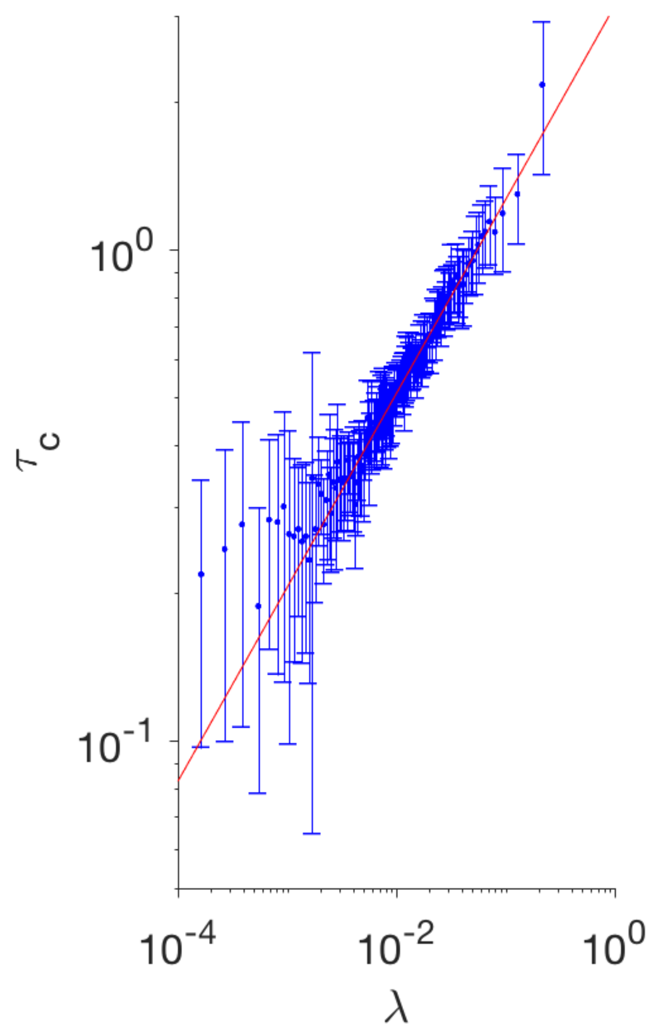
$$\hat{P}_{\mathrm{ij}}(K) = \sum_{\mathrm{r}=1}^{K} u_{\mathrm{ir}} u_{\mathrm{jr}}$$

$$\tilde{\sigma}_{\mathrm{i}} = z_{\mathrm{i}}(K) \sum_{\mathrm{j}} \hat{P}_{\mathrm{ij}}(K) \left[\sigma_{\mathrm{i}} - \langle \sigma_{\mathrm{i}} \rangle \right]$$

Distribution of coarse-grained variables approaches a fixed, non-Gaussian form







Dynamic scaling