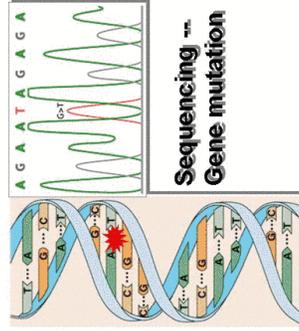


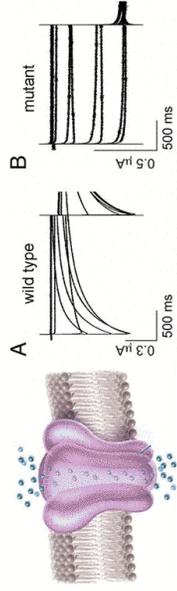


## Math instead of mice

Genetics and arrhythmia mechanisms: Insights  
from theoretical models



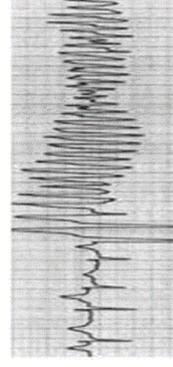
Sequencing --  
Gene mutation



Channel recording -- Abnormal kinetics



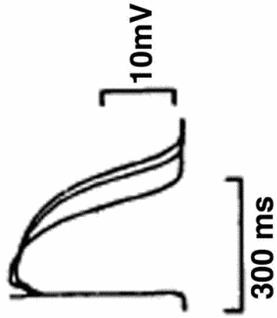
How do mutations  
propagate as  
emergent dynamics  
to disrupt protein, cell  
and tissue behavior?



Abnormal ECG

## Why math instead of mice?

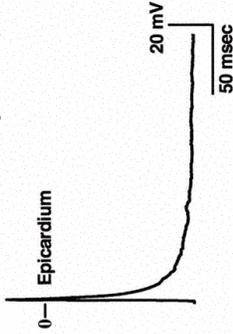
Human action potential



QuickTime™ and a TIFF (Uncompressed) decompressor are needed to see this picture.

Franz, M.R. et al. J Clin Invest. 1988; 82(3): 972-979

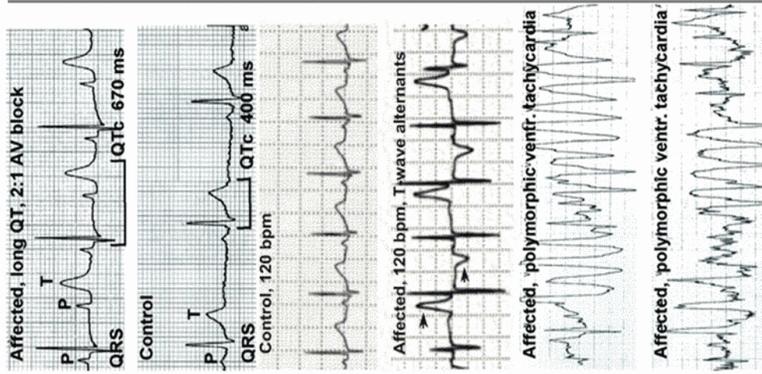
Murine action potential



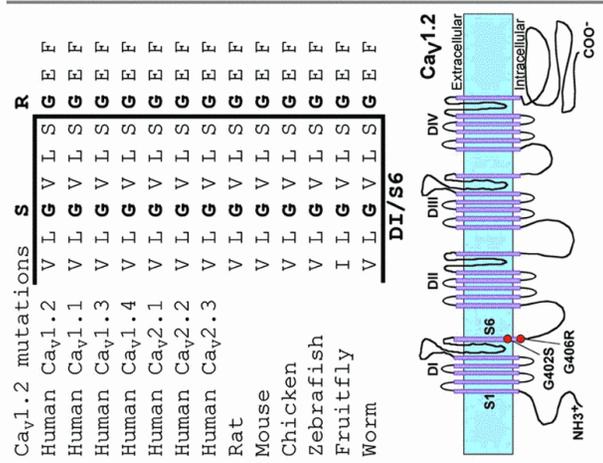
≈300 grams  
60-80 bpm

≈0.3 grams  
500 bpm

Anumonwo et al. Circ Res. 2001 Aug 17;89(4):329-35

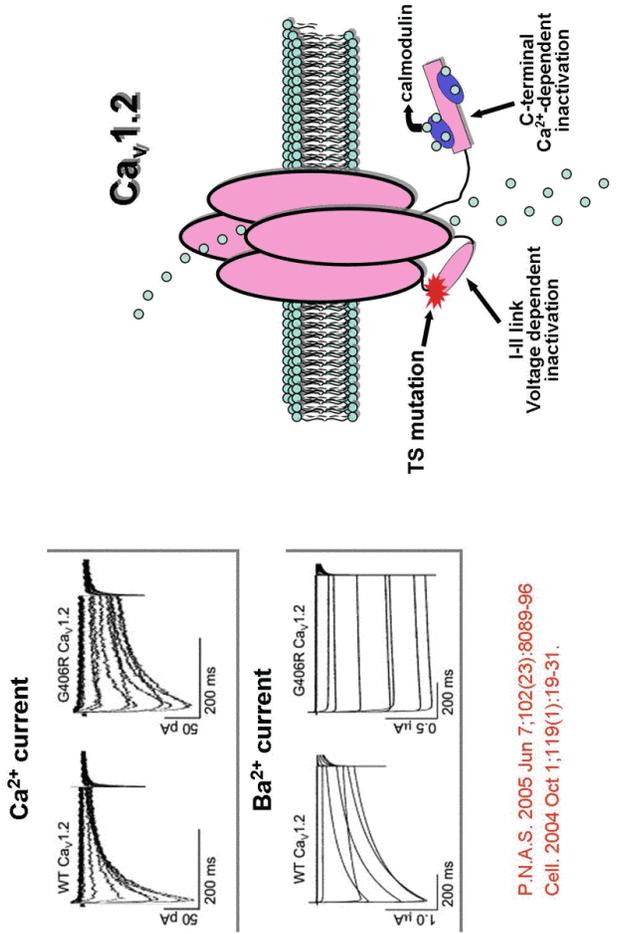


## Timothy Syndrome



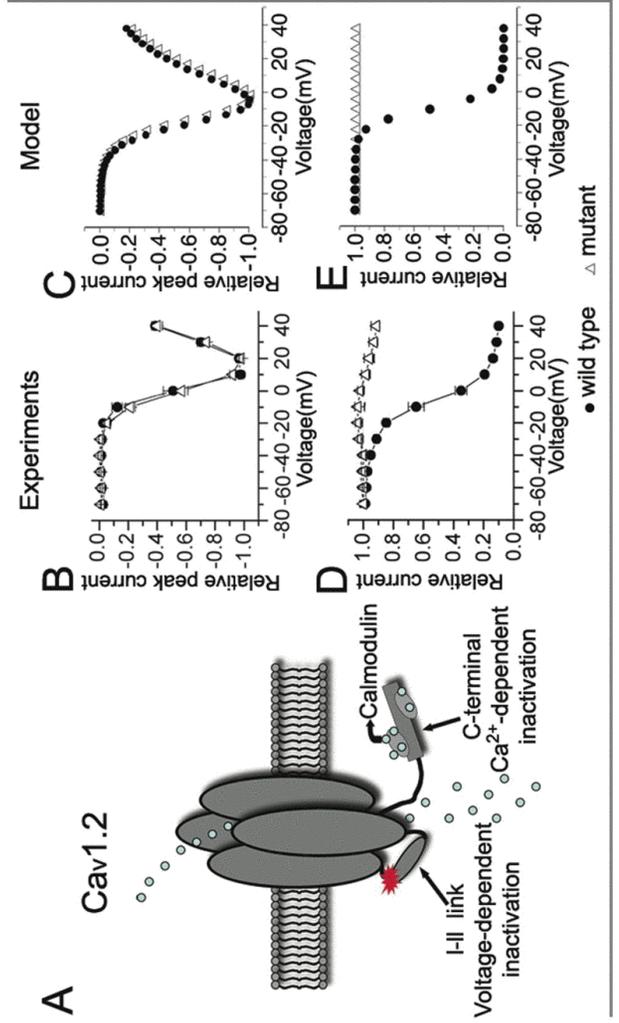
P.N.A.S. 2005 Jun 7;102(23):8069-96  
Cell. 2004 Oct 1;119(1):19-31.

### The TS mutation disrupts voltage-dependent inactivation of $I_{CaL}$

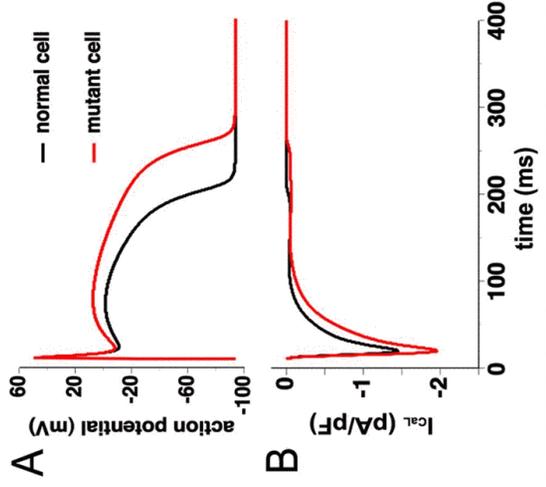


P.N.A.S. 2005 Jun 7;102(23):8089-96  
 Cell. 2004 Oct 1;119(1):19-31.

### The Timothy Syndrome mutation eliminates voltage-dependent inactivation of $Ca_v1.2$



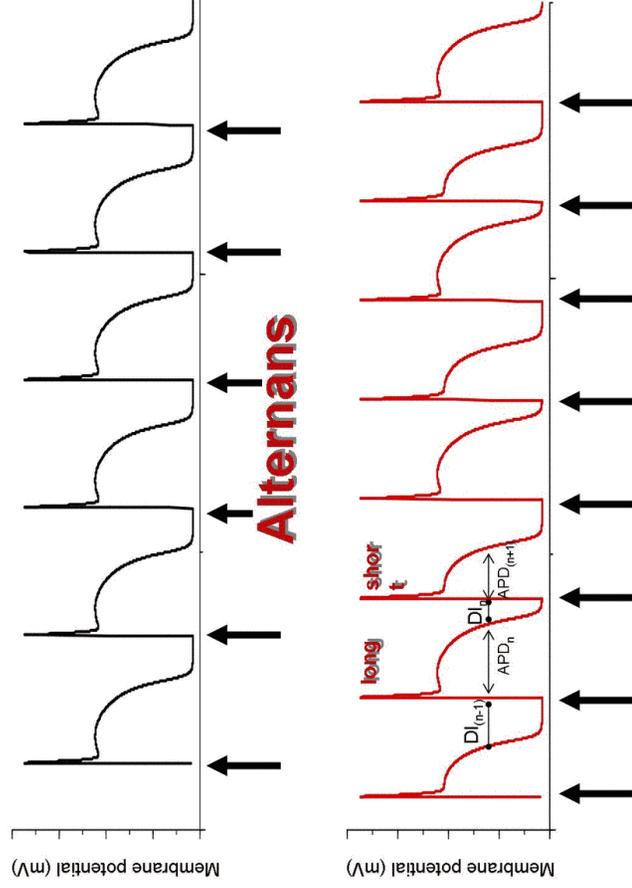
**The TS mutation prolongs action potential duration (APD)**



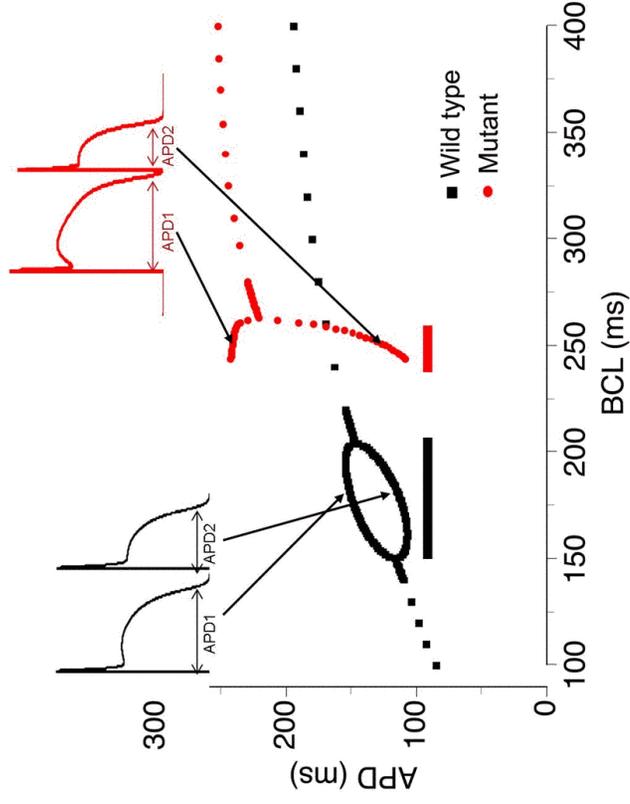
Physiological rate (BCL 500 ms)

Fox JJ, McHarg JL, Gilmour RF Jr. Am J Physiol Heart Circ Physiol. 2002 Feb;282(2):H516-30.

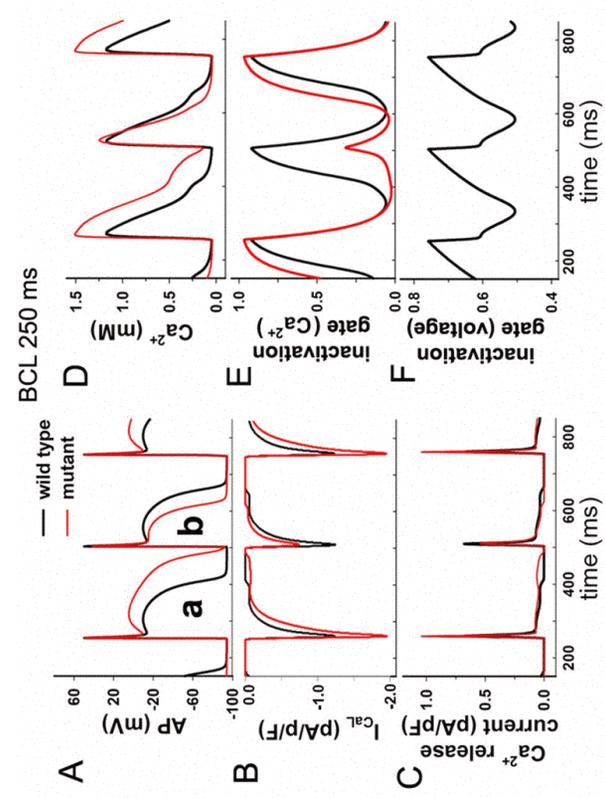
**Normal Dynamics**



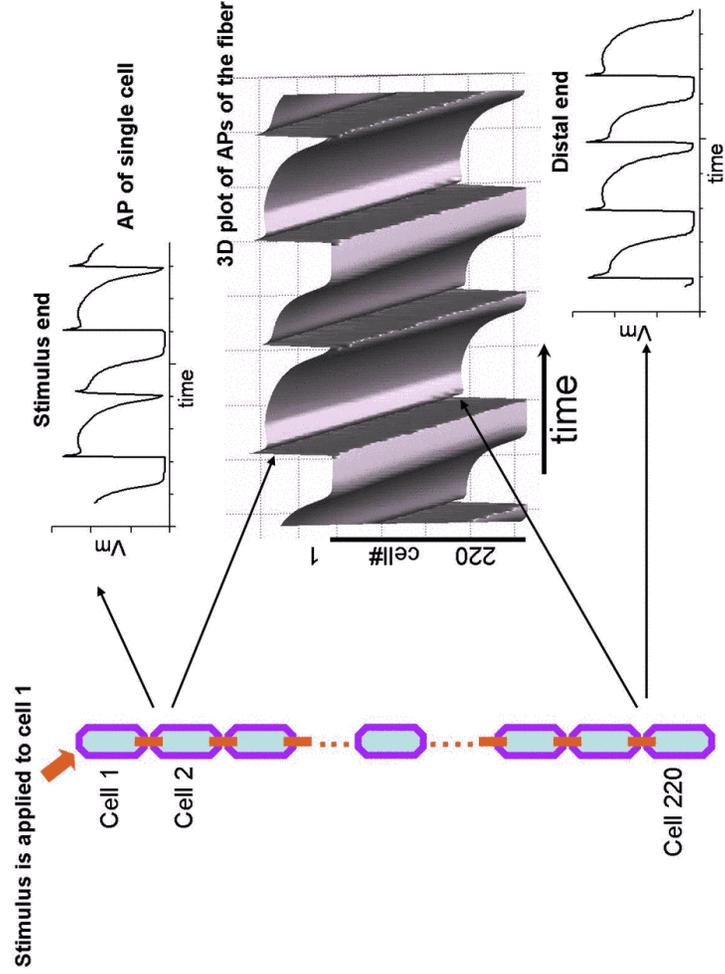
**The TS mutation alters single cell dynamics**



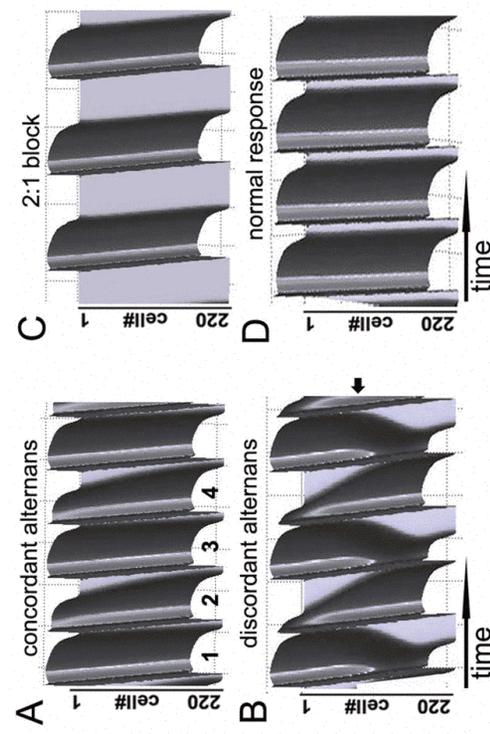
**Ca<sup>2+</sup> related events in alternans**



# 1-D Cardiac Fiber



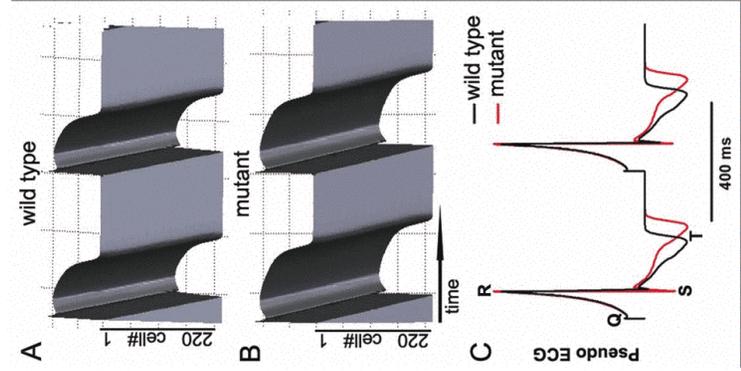
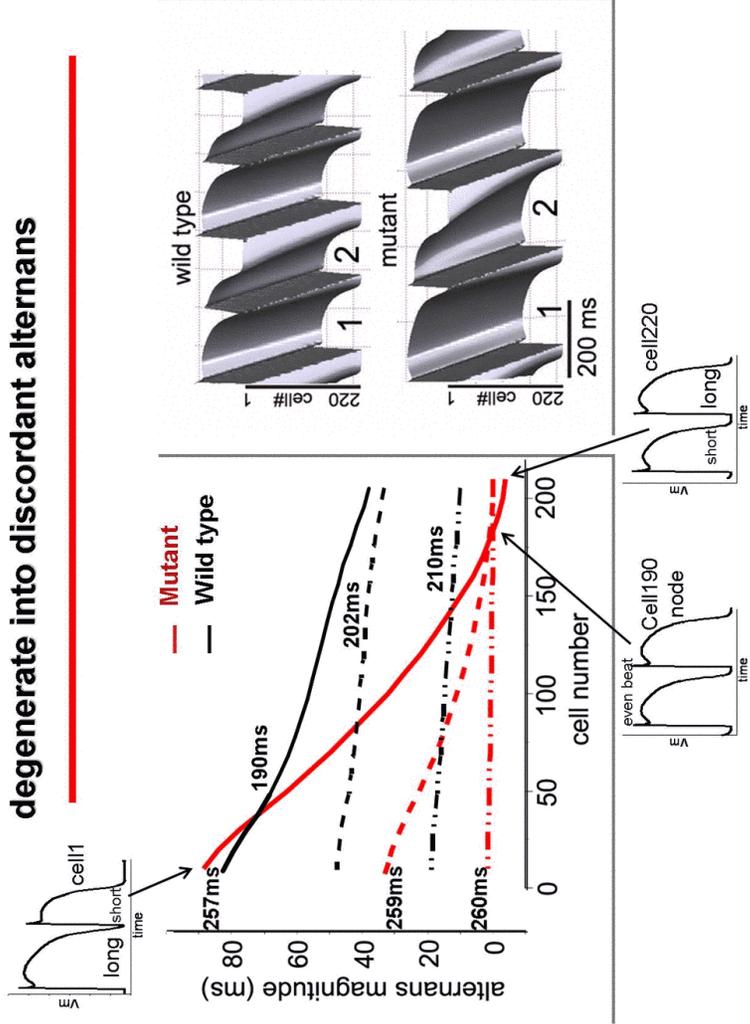
## The TS mutation shifts the window for discordant alternans ~70 ms



**Table: Threshold BCL of arrhythmic events for fibers of normal and mutant cell**

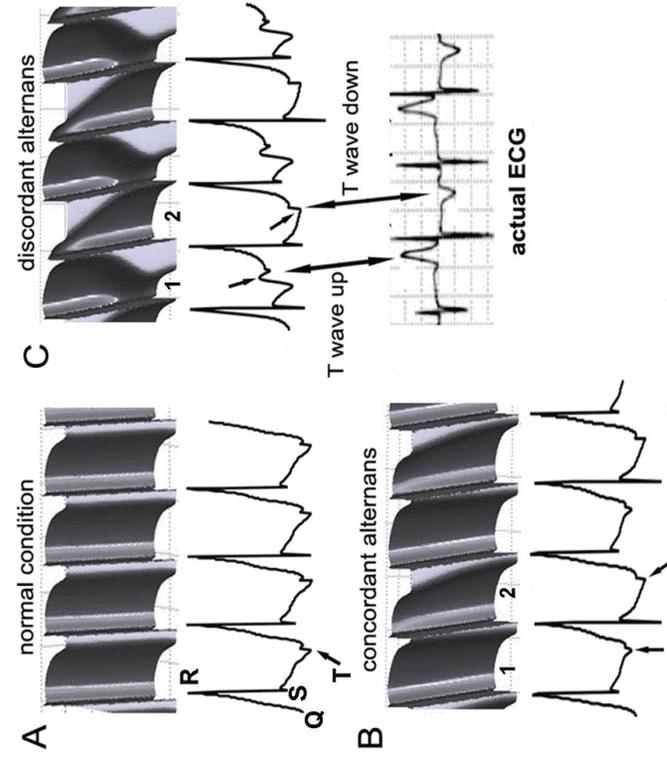
		Threshold BCL (ms)	
		Discordant Alternans	2:1 block
Wild type	Concordant Alternans	210	188
Mutant	Concordant Alternans	259	185
	Discordant Alternans	257	249

**The TS mutation causes concordant alternans to rapidly degenerate into discordant alternans**



**QT interval prolongation**

## T-wave anomalies in computed ECGs



## Summary

- 1 The TS mutation disrupts the rate-dependent dynamics in a single cardiac cell and promotes the development of alternans.
- 2 In coupled tissue concordant alternans is observed at much slower heart rates in mutant, and once initiated, rapidly degenerates into discordant alternans and conduction block.
- 3 The ECG computed from mutant tissue exhibits T-wave alternans and alternating T-wave inversion at physiologically relevant pacing rates.

## Acknowledgements

Zheng I. Zhu Ph.D.



QuickTime™ and a TIFF (Uncompressed) decompressor are needed to see this picture.

Alfred P. Sloan Foundation

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The Lab

Zheng Zhu Ph.D.

Jun Xu Ph.D.

Ronit Vaknin

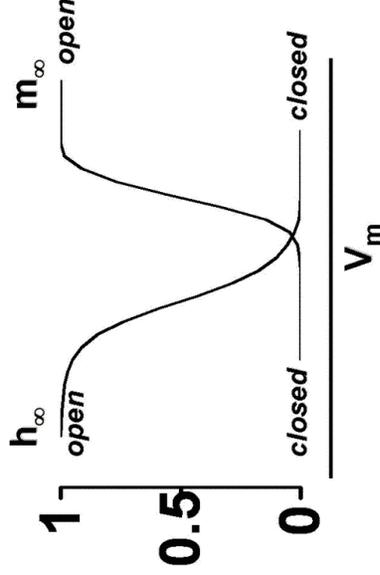
## Detailed Na<sup>+</sup> channel gating models

- Usefulness - what can we learn?
- The relationship between implicit kinetic properties of channel gating and observed phenomena at other system scales.
- Non-equilibrium considerations
- Simulation of pharmacological interventions - predictive computable models.

# The Hodgkin-Huxley framework

Variables  $h$  and  $m$  represent voltage dependent inactivation and activation gates

$$I_{Na} = m^3 h * g_{Na} * (E - E_{Na})$$



Markov models can incorporate mutations that affect discrete transitions

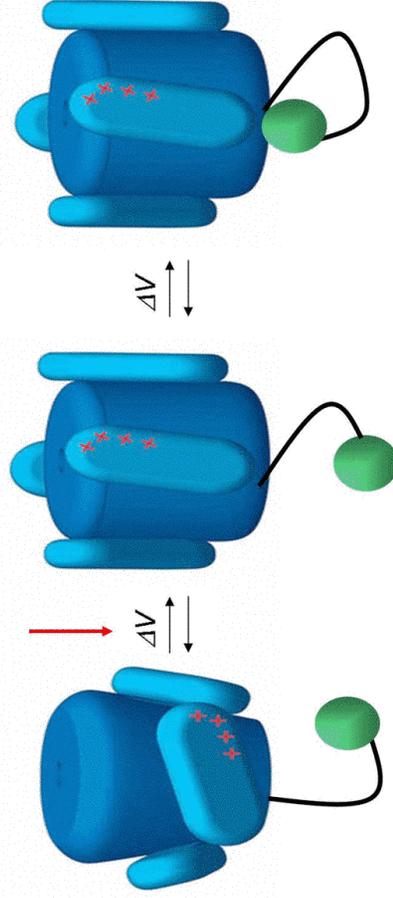
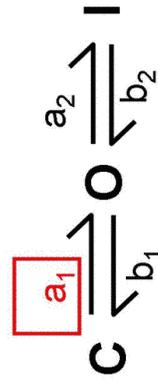


Figure adapted from Y. Jiang et al. Nature. 2003 May 1;423(6935):33-41.

## Computation of state probabilities



$$\frac{dP_j}{dt} = \sum_{j=1}^N [k_{ji} \cdot P_i(t, V_m)] - \sum_{j=1}^N [k_{ij} \cdot P_j(t, V_m)]$$

$$dC/dt = P(O) \cdot b_1 - P(C) \cdot a_1$$

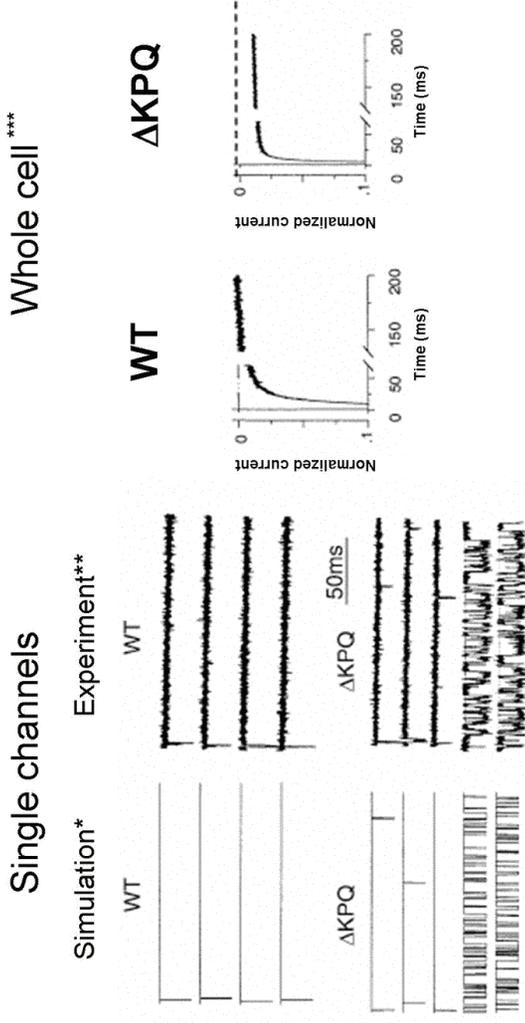
$$dO/dt = P(C) \cdot b_1 + P(I) \cdot b_2 - (P(O) \cdot (a_2 + b_1))$$

$$dI/dt = P(O) \cdot a_2 - P(I) \cdot b_2$$

How do we build a model?



**Kinetics of  $I_{Na}$**

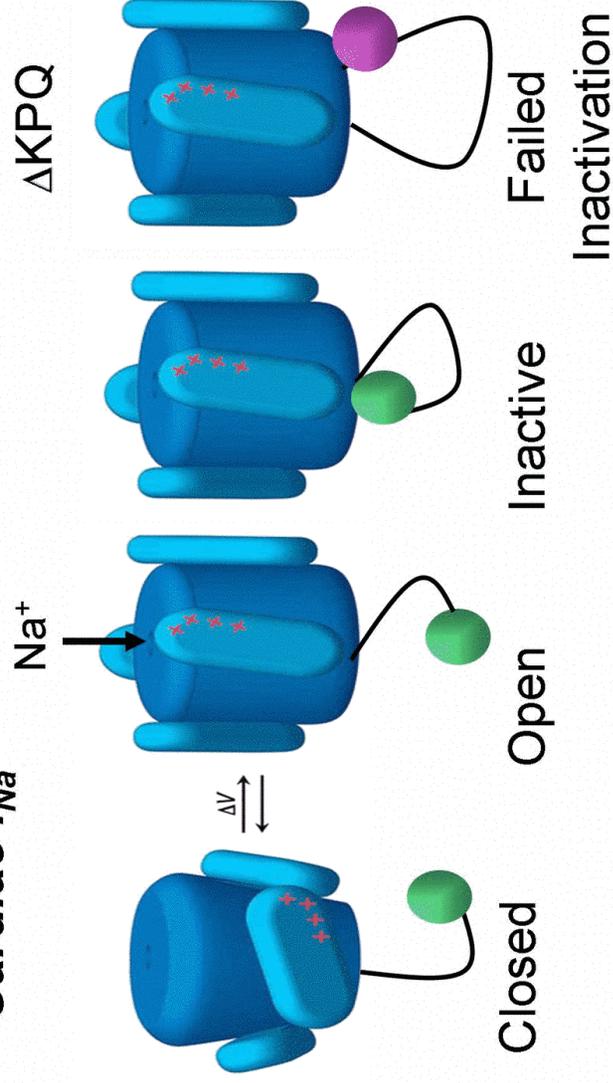


\* Clancy and Rudy. 1999. Nature. 400. 566-569.

\*\*Chandra, Starmer and Grant. 1998. AJP. H1643-H1654

\*\*\* Bennett et al. 1995. Nature. 376. 683-685.

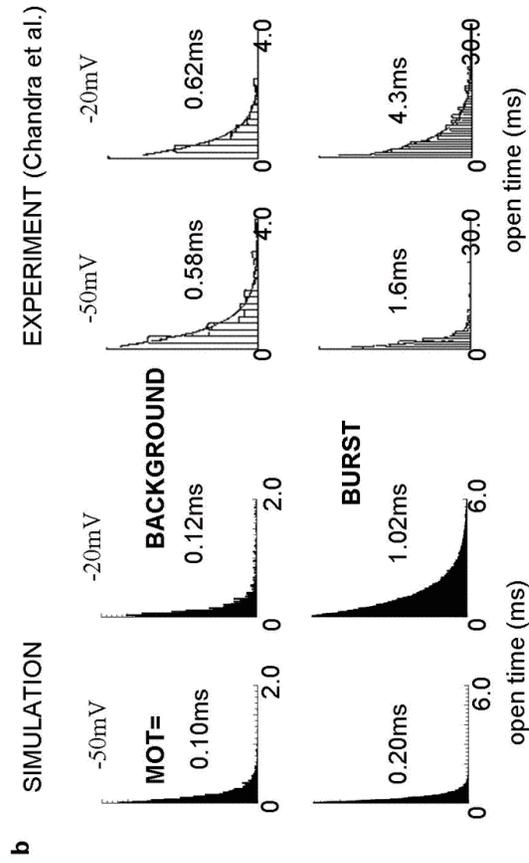
**Cardiac  $I_{Na}$**



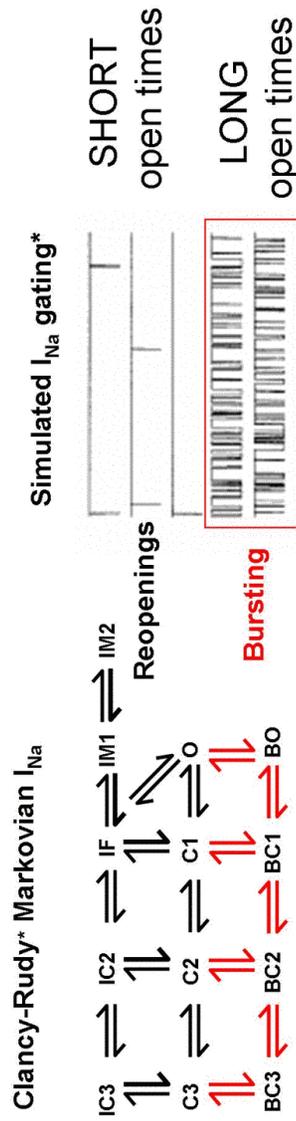
**$\Delta KPQ$  channels may fail to inactivate**

Multiple channel open times suggest “modal” gating

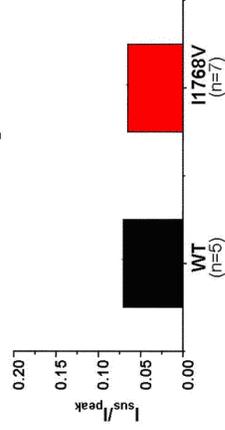
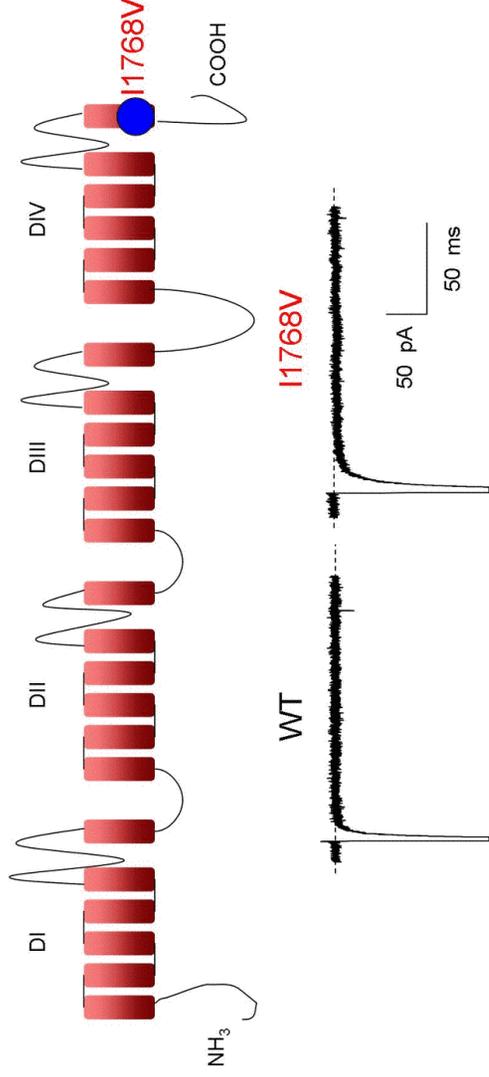
Two MOTs = two modes



Modeling kinetics of  $\Delta KPQ I_{Na}$

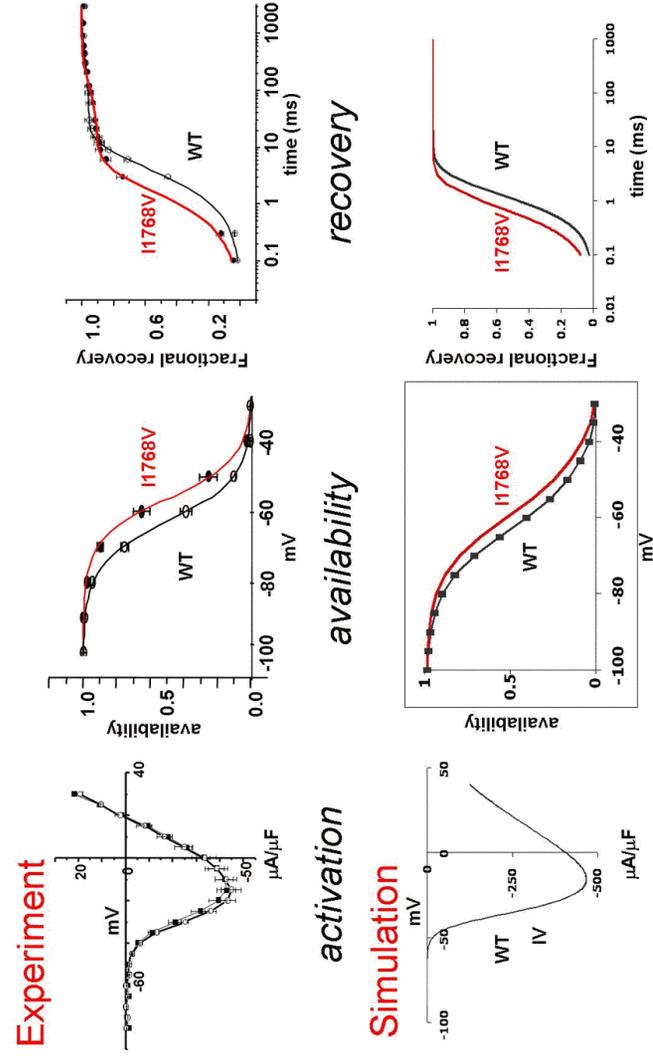


The I1768V Long-QT mutation lacks channel bursting

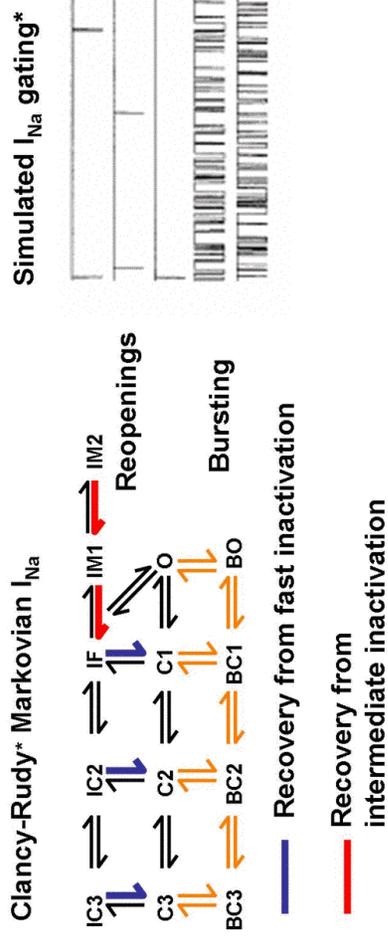


Clancy, C.E.,  
Tataeyama, M., Liu, H.,  
Wehrens, X.H.T. and  
Kass, R.S. Circulation.  
2003;107:2233-223.

Kinetic properties of WT and I1768V cardiac I<sub>Na</sub>

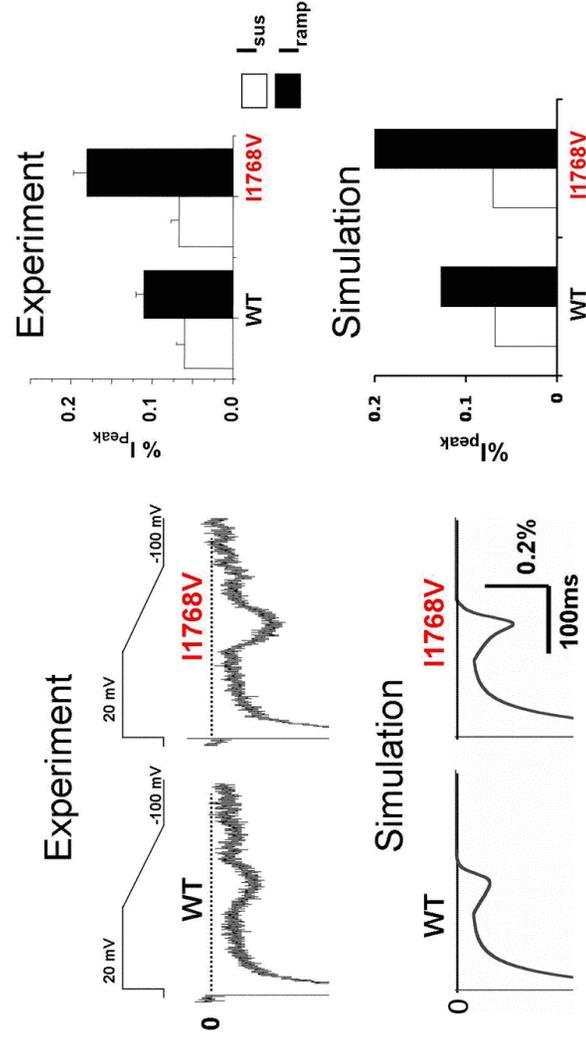


## Background



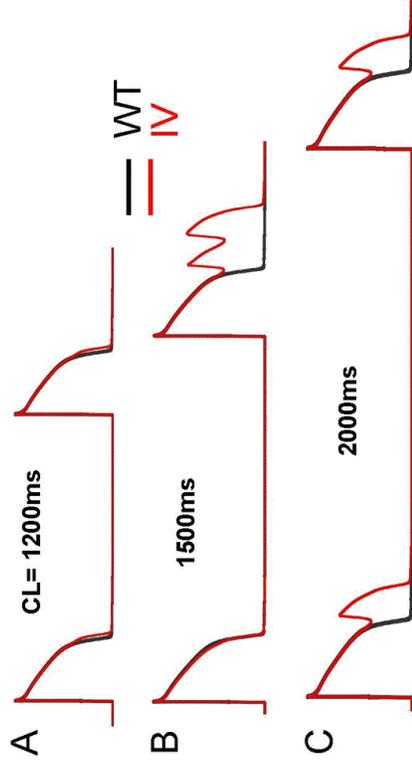
\*Clancy and Rudy. Circulation; 2002;105:1208-1213.  
 Clancy and Rudy. 1999. Nature. 400. 566-569.

## The I1768V mutation results in larger current during repolarization

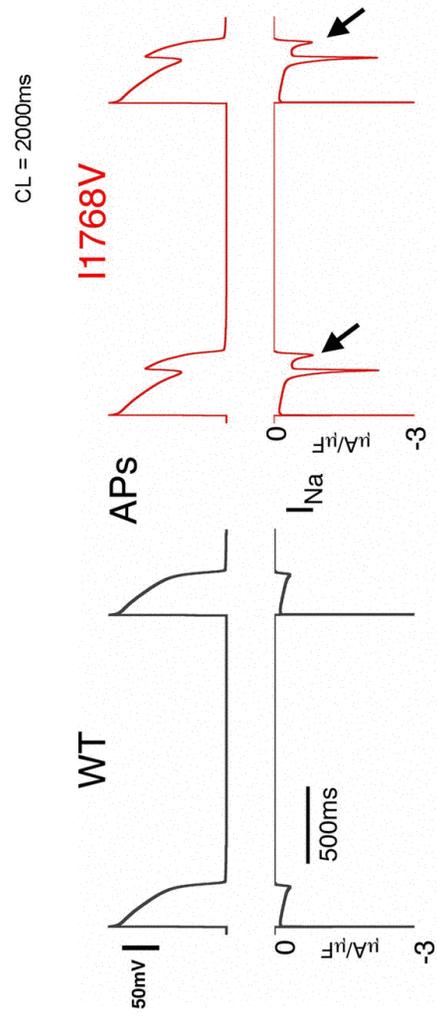


**The I1768V mutation results in faster recovery from inactivation allowing for channel reopening during repolarization.**

Channel reopenings disrupt the balance of current during repolarization and underlie arrhythmic EADs.



**The mechanism of AP prolongation by I1768V**



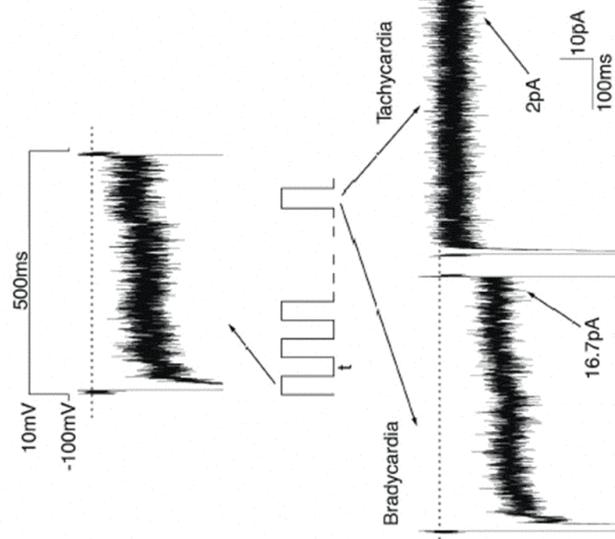
clancy et al. figure 4

## Summary

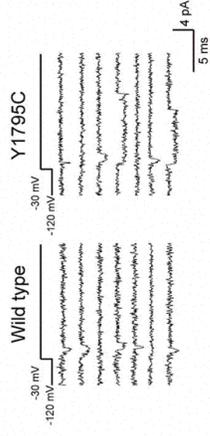
- Faster recovery from inactivation subsequent to channel opening results in reopening of a small population of channels.
- Background channel reopening results in the generation of late current during AP repolarization.
- Mutation induced alterations in channel gating during repolarization may underlie LQTS phenotypes.

Mechanisms of rate dependence of LQTS arrhythmias:

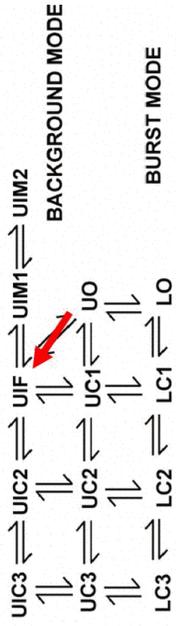
Insights from electrophysiological experiments and theoretical models.



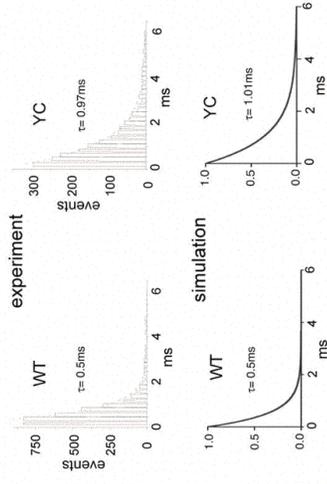
**A Background openings**



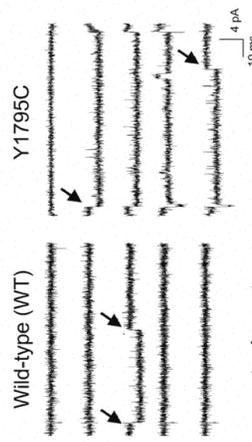
**Na<sup>+</sup> channel model**



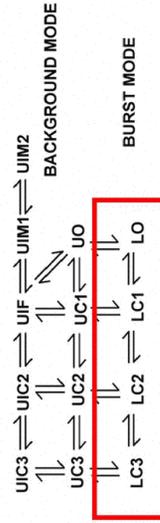
**B Mean open times**



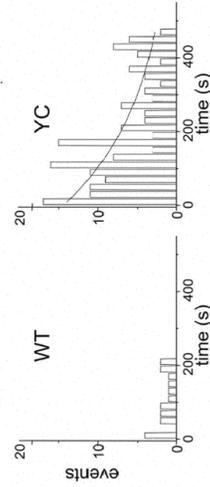
**A bursting of single channels**



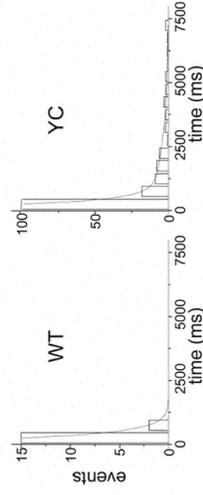
**Na<sup>+</sup> channel model**

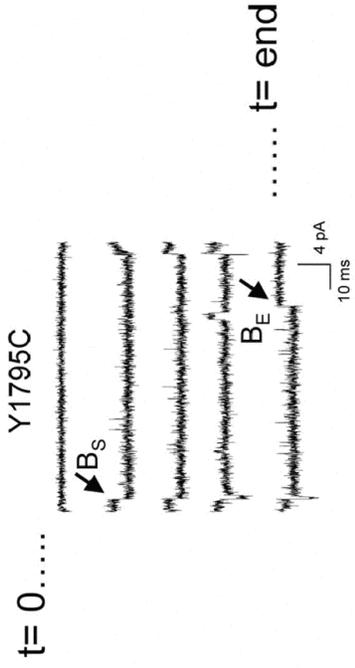


**B latency to burst**

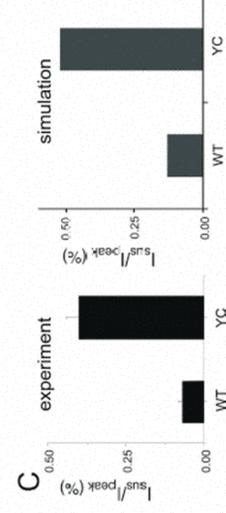
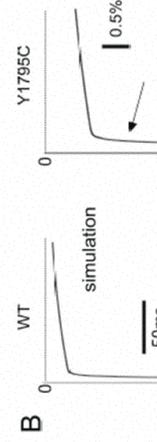
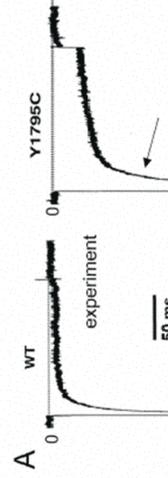
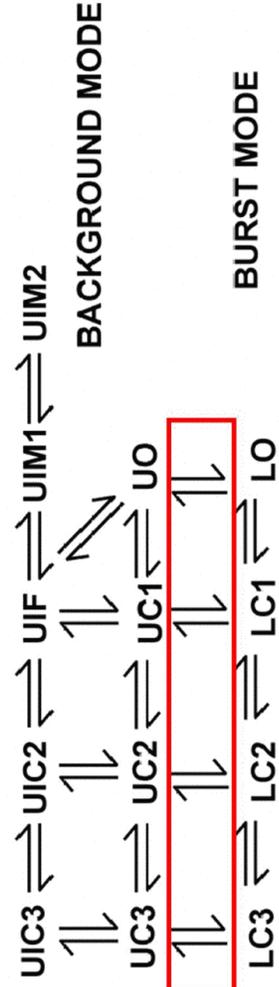


**C burst mode dwell time**

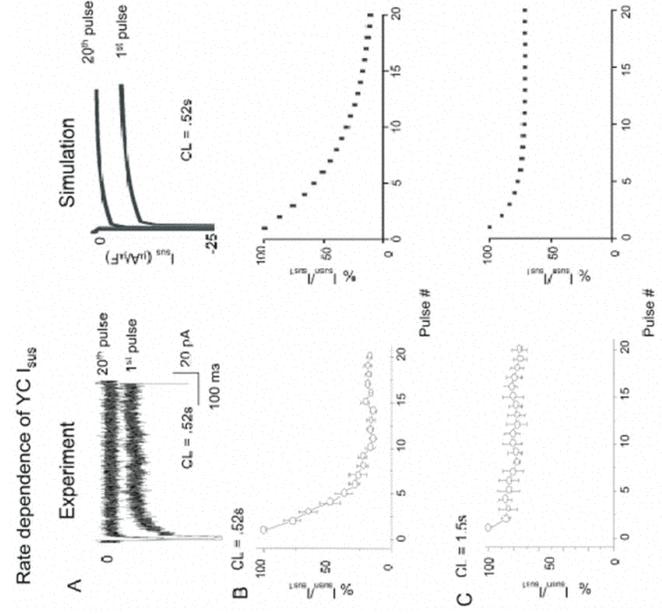
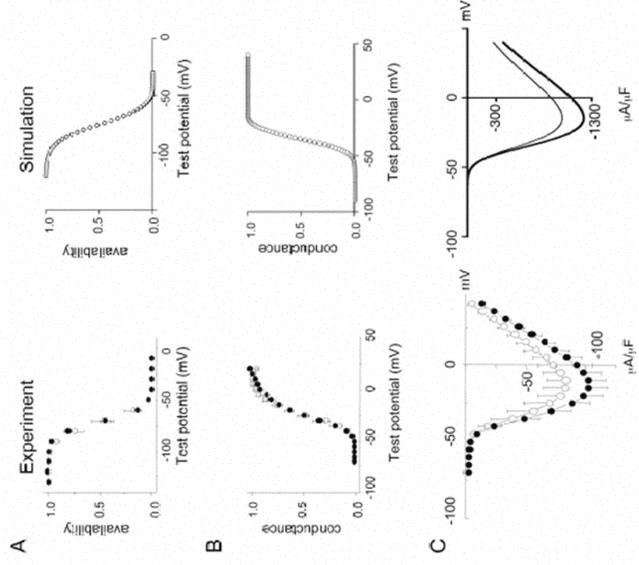




### Na<sup>+</sup> channel model



The Y1795C mutation does not affect other gating features



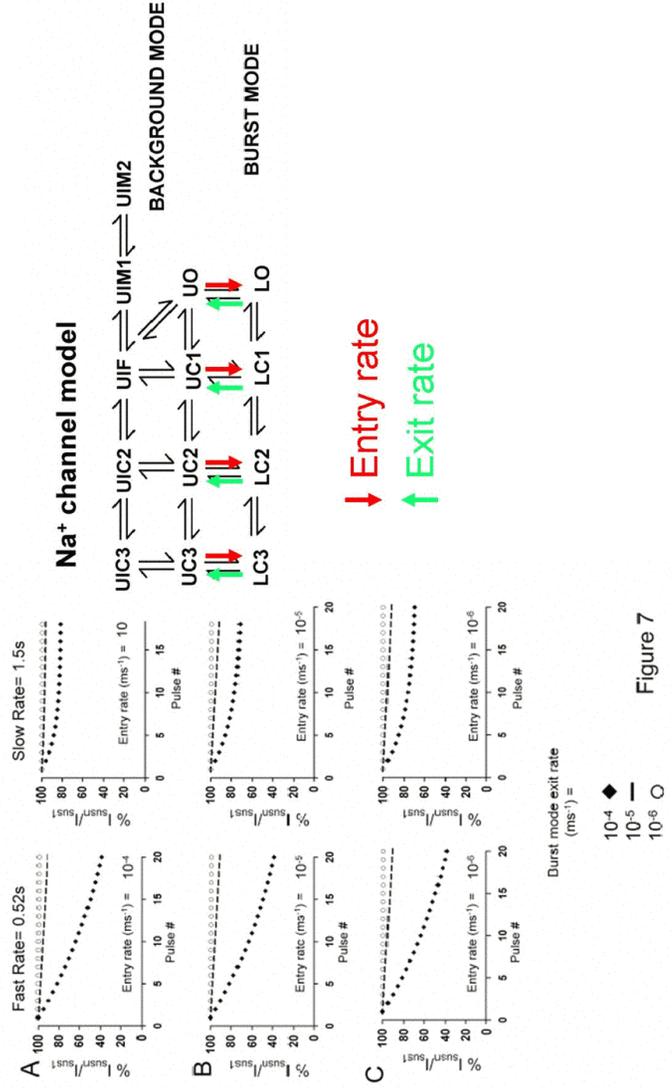


Figure 7

# Acknowledgements

Robert S. Kass (Columbia University)

Yoram Rudy (Washington University)

Michihiro Tateyama (Columbia University)