Gravitational waves from NS-NS/BH-NS binaries

Numerical-relativity simulation

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Why NS-NS/BH-NS are important ?

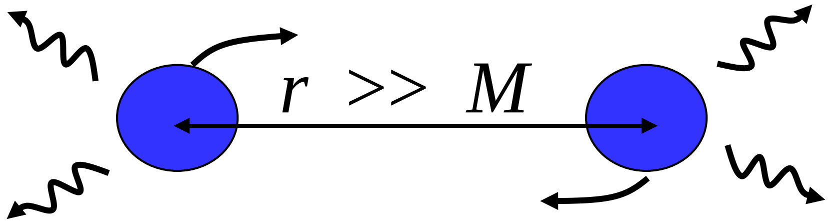
1. The **most promising sources of gravitational waves**
2. Invaluable **laboratory for studying high-density nuclear matter**
3. Possible origins of **short-hard GRBs**
4. Sources of strong **EM emission**

Numerical relativity plays a crucial role for all four issues.

NS-NS

Evolution of NS-NS ($1.35M_{sun}$-$1.35M_{sun}$)

Evolve by GW emission



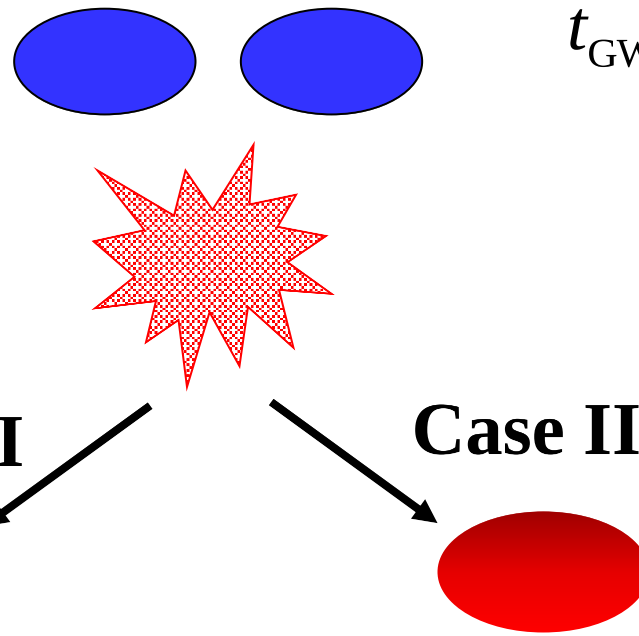
Adiabatic evolution

Tidal deformation at $r \sim$ 40-50 km

$t_{GW} \sim t_{orb}$

Dynamical evolution

Merger sets in at $f_{GW} \sim 1$ kHz



Soft EOS

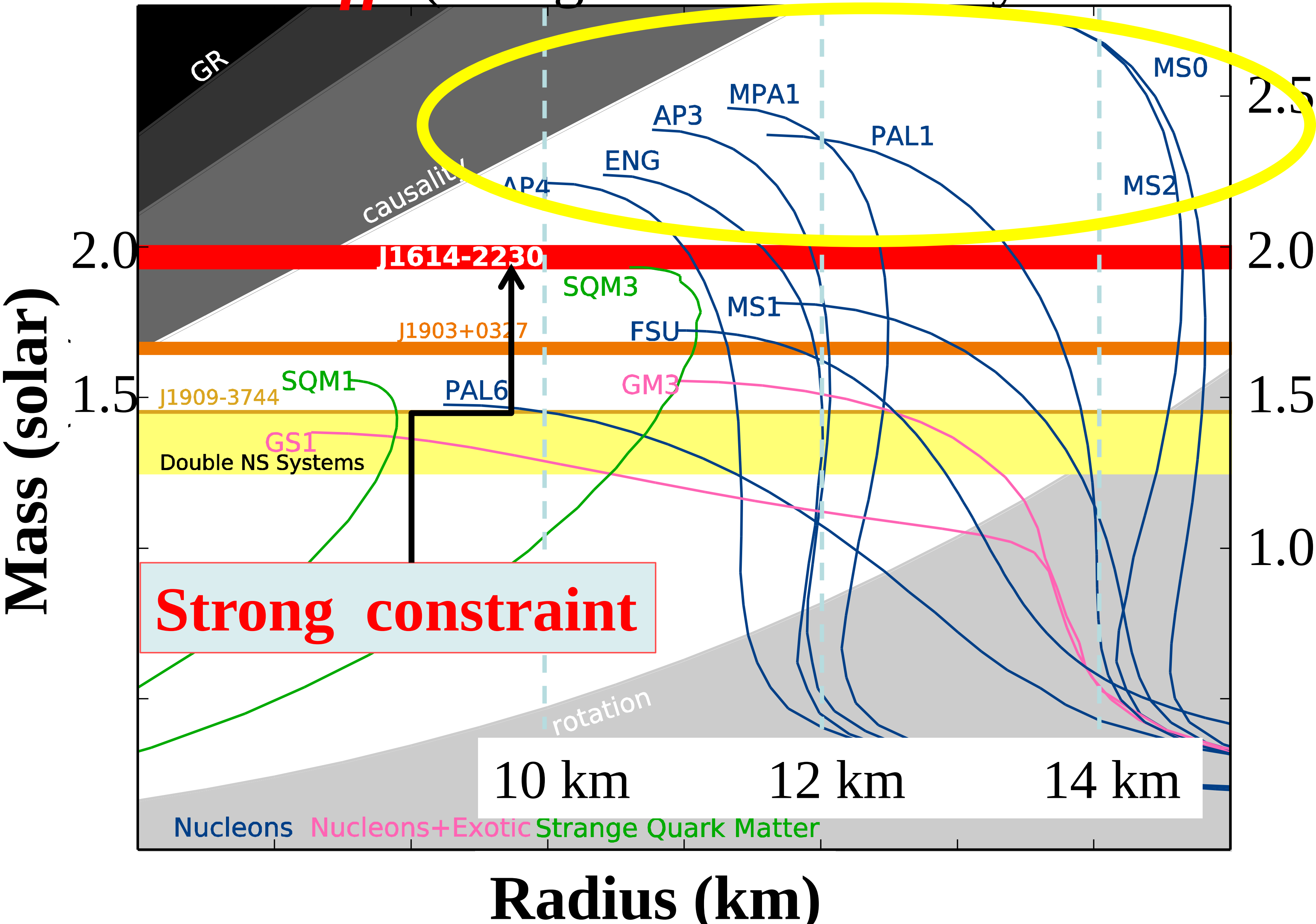
Black hole is formed

Stiff EOS

"Hypermassive NS"

Large EOS-dependence

EOS is *stiff* (though still too many candidates)



NS-NS merger with finite-temperature EOS + neutrino leakage

Example:

- EOS = Shen's EOS

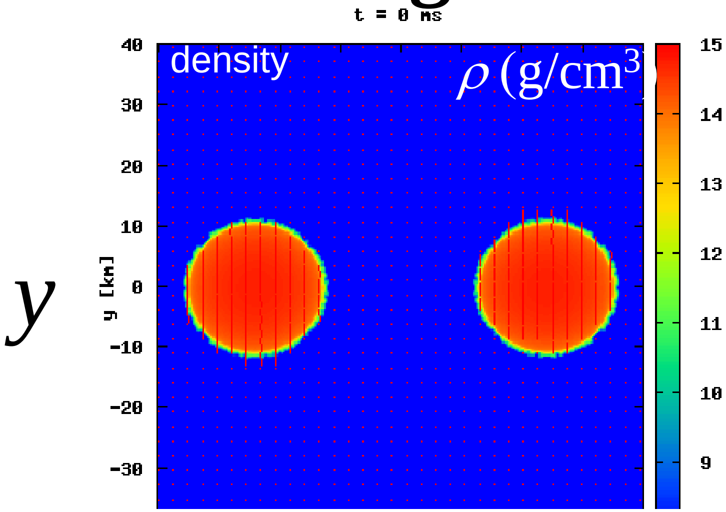
➢ Maximum mass of spherical star
  $M_{max}=2.2M_{sun}$ (T=0: zero temperature)

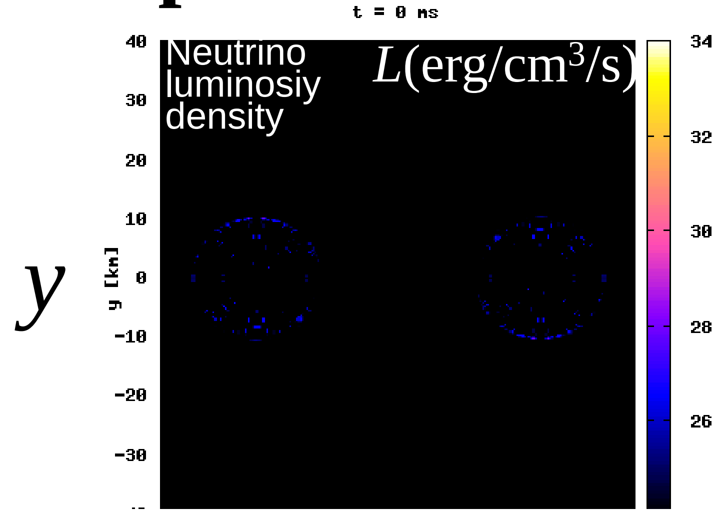
➢ R (1.4M_{sun}) ~ 14.5km → Stiff

Mass of NS-NS for simulation

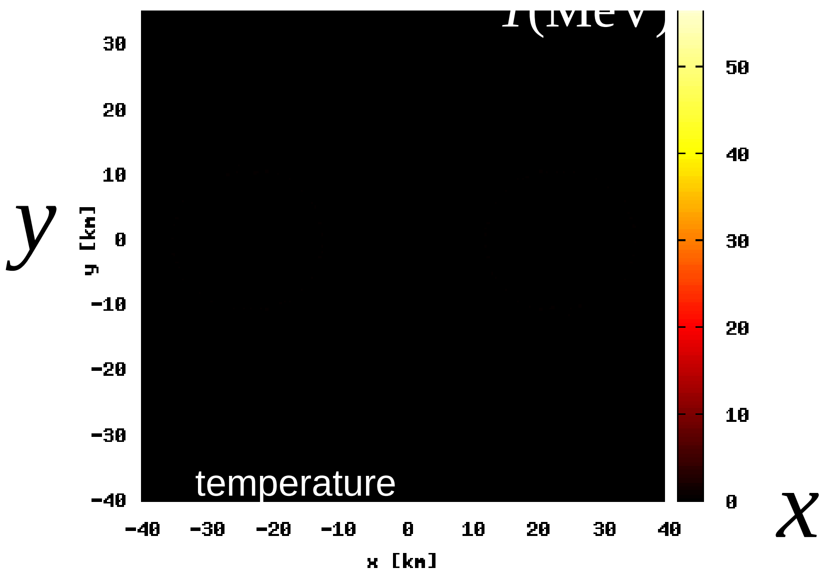
1.5—1.5 M_{sun}

NS-NS merger with finite-temp EOS + neutrino





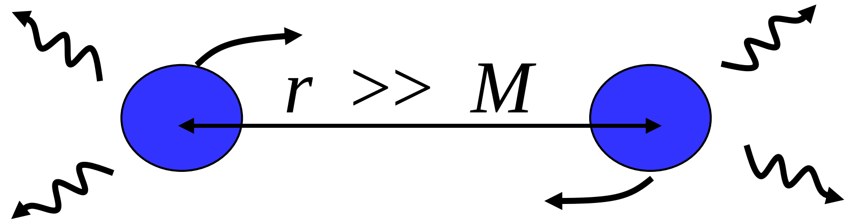
- **Long-lived hot HMNS is the outcome:**
  Supported by thermal pressure & centrifugal force



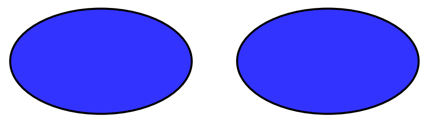
Sekiguchi, Kiuchi,
Kyutoku, Shibata
PRL107, 2011

Evolution of NS-NS ($1.35M_{sun}$-$1.35M_{sun}$)

Evolve by
GW emission

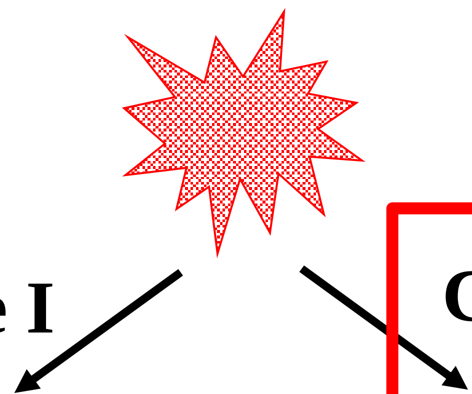


Tidal deformation



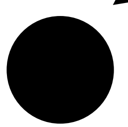
This is likely for canonical mass case

Merger sets in
at $f_{GW} \sim 1$ kHz



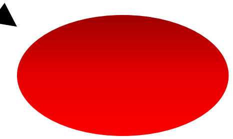
Case I

Soft EOS



Black hole is formed

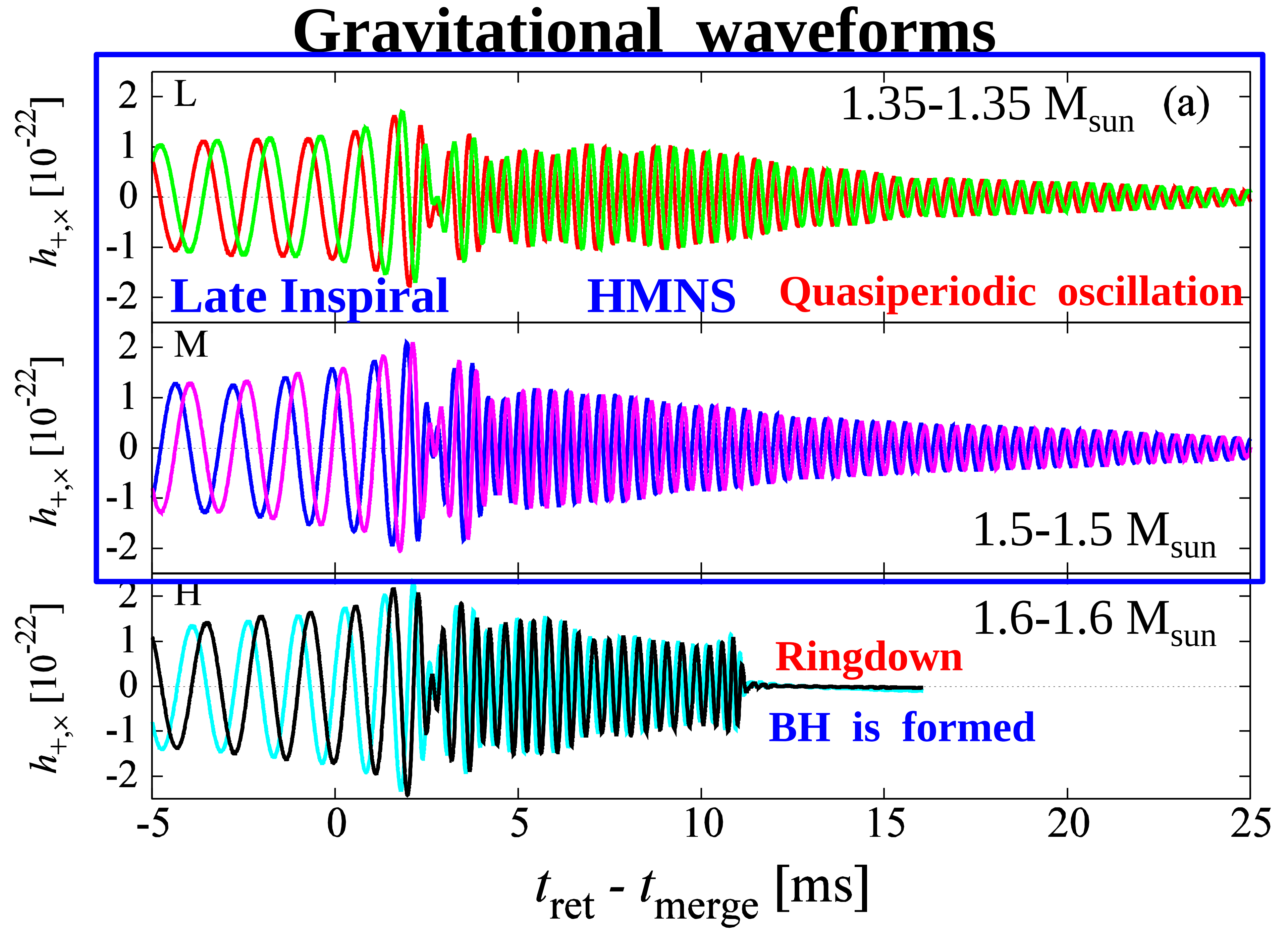
Case II



Stiff EOS

"Hypermassive NS"

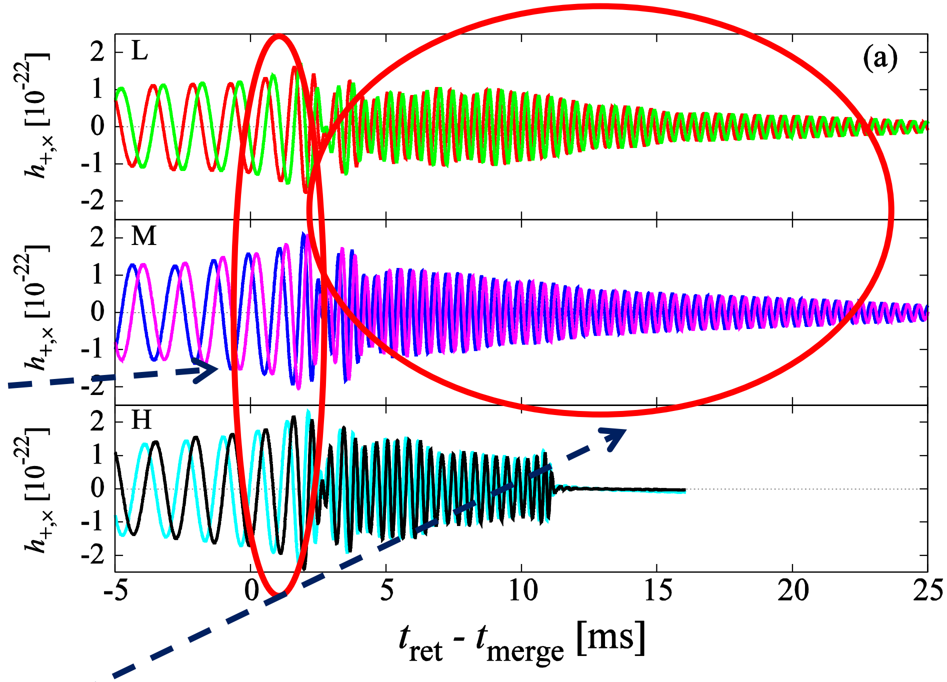
Large EOS-dependence



Gravitational waveforms
L
1.35-1.35 M_sun (a)
Late Inspiral
HMNS
Quasiperiodic oscillation
M
1.5-1.5 M_sun
H
1.6-1.6 M_sun
Ringdown
BH is formed
h_{+,×} [10^{-22}]
t_ret - t_merge [ms]
-5
0
5
10
15
20
25
2
1
0
-1
-2

Two interesting phases

1. ***Late Inspiral*** (Lai+, Damour+, Baiotti+, ….) **:**
   Effects of *tidal deformation*
   ***f* ~ 500 – 1k Hz**



2. ***Merger → HMNS*** (Janka+, Hotokezaka+)
   GW from *HMNS*
   ***f* ~ 2k – 4k Hz**

Both waveforms play an important role for constraining EOS of neutron stars

Brief introduction of numerical relativity

$$G_{\mu\nu} = 8\pi \frac{G}{c^4} T_{\mu\nu}$$

- General relativistic gravity; including GW radiation reaction

$$\left\{ \begin{array}{l} \nabla_\mu T^\mu_\nu = 0 \\ \nabla_\mu \left(\rho u^\mu \right) = 0 \\ +\text{EOS} \end{array} \right\}$$

- Hydodynamics/MHD
- Equations of state for nuclear matter

$$\left(\begin{array}{l} \nabla_\mu F^{\mu\nu} = -4\pi j^\nu \\ \nabla_{[\mu} F_{\nu\lambda]} = 0 \\ \text{Radiation } ..... \end{array} \right)$$

- Magnetic fields
- Neutrino emission
- More ..: accessorizes

Current status in the community

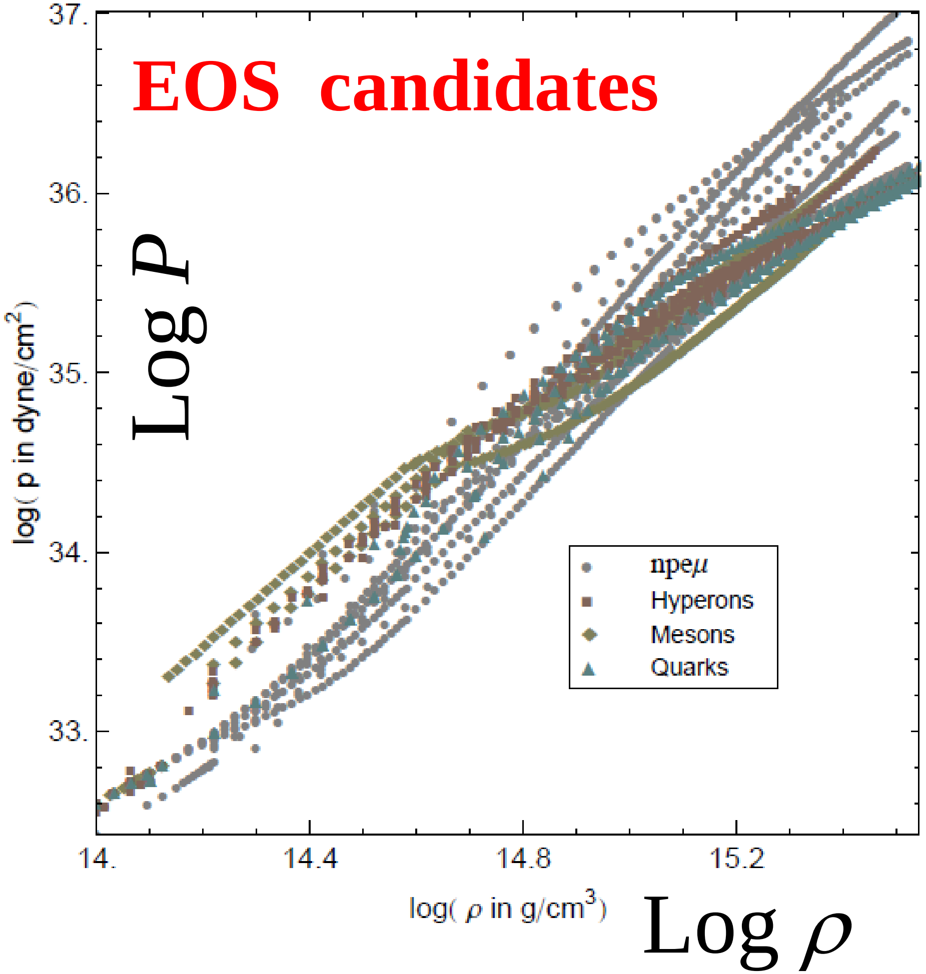
- ✓ Einstein's equation can be solved accurately (e.g., BSSN, puncture gauge, 4^{th}-order accuracy)
- ✓ Hydrodynamics equations, incorporating physical tabulated EOS, can be solved accurately (Ott)
- ✓ Magnetohydrodynamics equations in GR can be also solved accurately (Kiuchi, Etienne)
- ➢ Neutrino physics can be taken into account approximately (in progress → radiation transfer)

↓

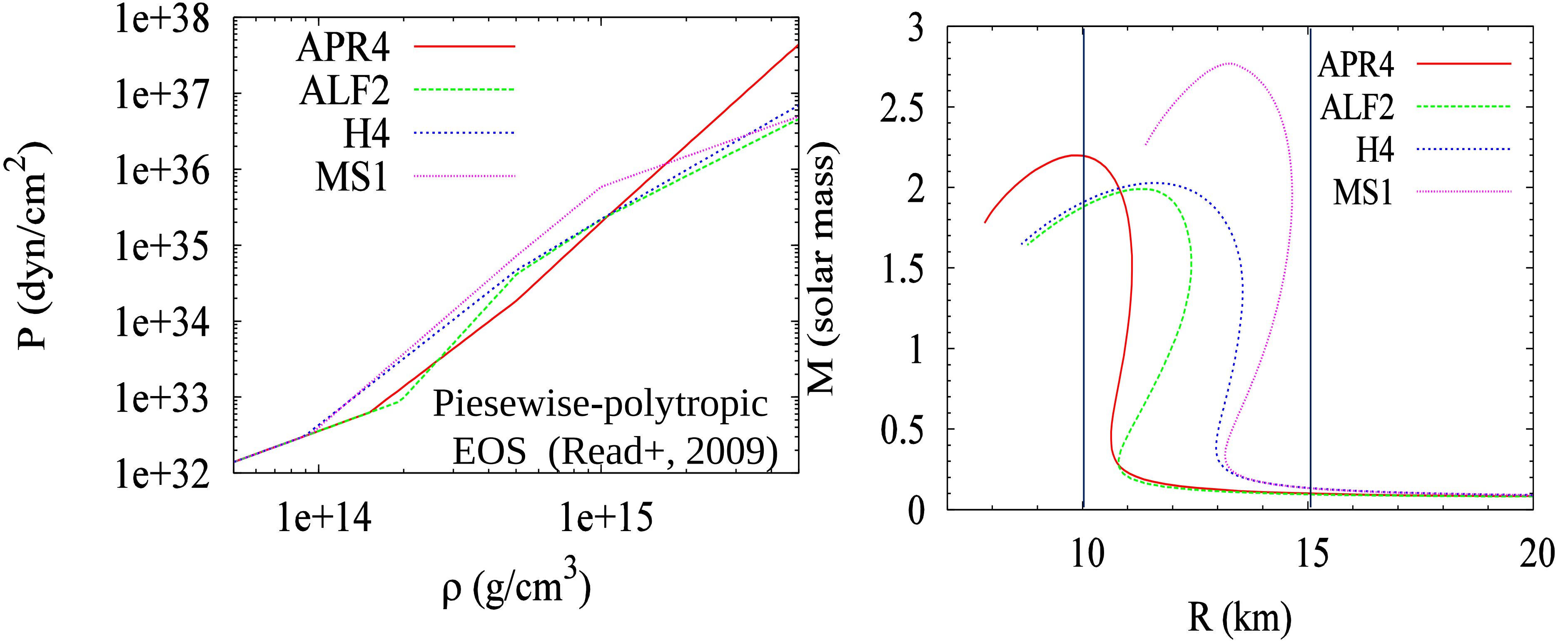
Quantitative theoretical study is now feasible !

Subtlety: NS's EOS is still unknown

- **Too many candidate EOSs by nuclear theory**
- **We have to employ many EOSs**
  → Many, systematic simulations are necessary



Four EOSs referred to in this talk



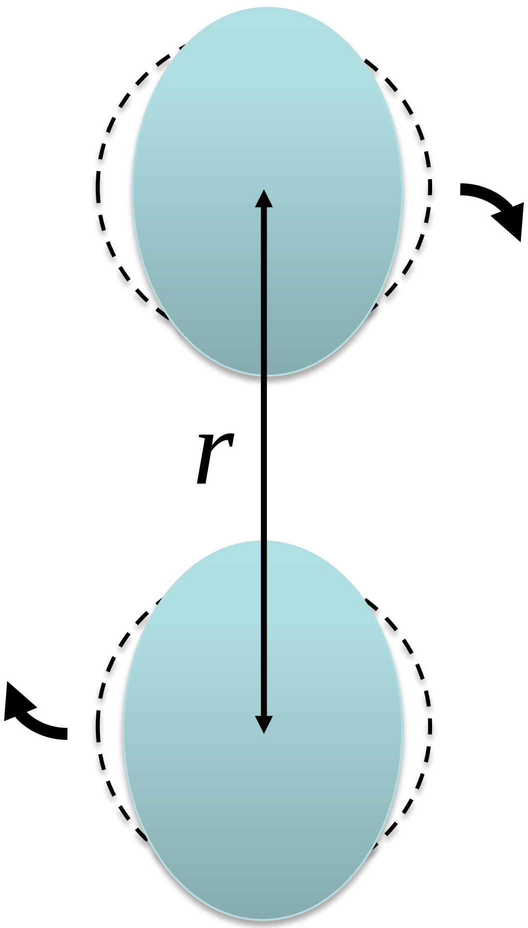
Small radius APR4 ALF2 H4 MS1 **large radius**

"Soft" ⟷ "Stiff"

1 Gravitational waves from late inspiral (Hotokezaka +)

Tidal effects in a binary inspiral

(originally pointed out by Lai+ 1992)



Close Binary System

➡ Tidal deformation;
Quadrupole is induced

$$\phi \sim -\frac{GM}{r} - \frac{C}{r^6}$$

5PN correction:

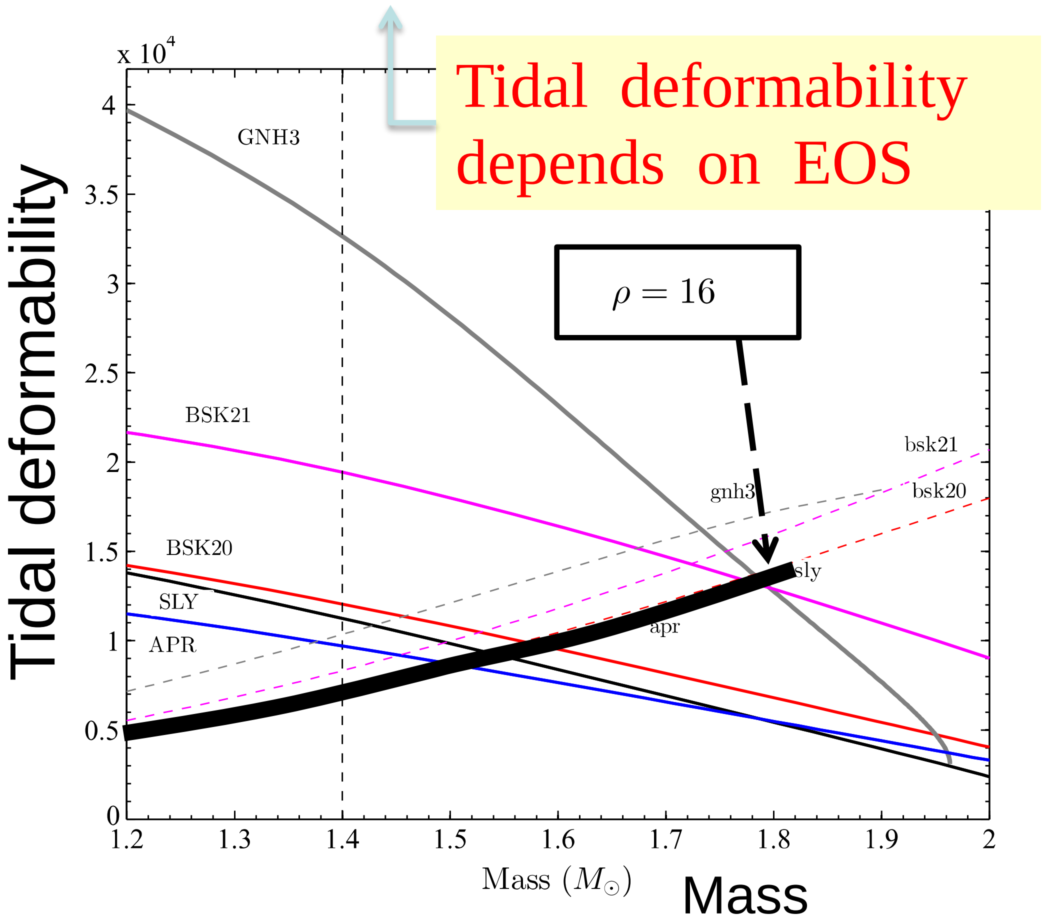
But $C \sim MR^5$, $R \sim 5$—$8\ M$

For $r \sim 2R$, it could play a role.

$$h = h(t, M_1, M_2, C_1, C_2)$$

Latest EOB study (Damour, Nagar + 2012)

A static tidal approximation:
Quadrupole moment = –λ (tidal field); linear approx.



Advanced detectors could measure tidal deformability

Calibration is necessary

Early part of the late inspiral (f < ~500 Hz)

- ✓ Tidal effects are small (linear approx. OK)
- ✓ Quasi-stationary approximation is OK
- ✓ PN and EOB including tidal effects are robust

Final part of the late inspiral (f > ~500 Hz)

- ✓ Tidal effects become stronger, could be nonlinear
- ✓ Not quasi-stationary, rather, dynamical
- ✓ Validity of analytic approaches is not trivial

→ ***Need Numerical Relativity simulations***

(See, also, Baiotti+ 2011, Bernuzzi+ 2012,
but with unrealistic EOS)

Numerical experiments

Three piecewise polytropic EOS (Read + 2009)

EOS	$\log P_1\,(\mathrm{dyne/cm^2})$	Γ_1	Γ_2	Γ_3	$C(1.35)$	$k_2(1.35)$	Tidal parameter
APR4	34.269	2.830	3.445	3.348	0.179	0.091	62.3
H4	34.669	2.909	2.246	2.144	0.146	0.115	215
MS1	34.858	3.224	3.033	1.325	0.138	0.133	332

Initial : $M\omega =$ 0.019 (track 8 – 10 orbits)

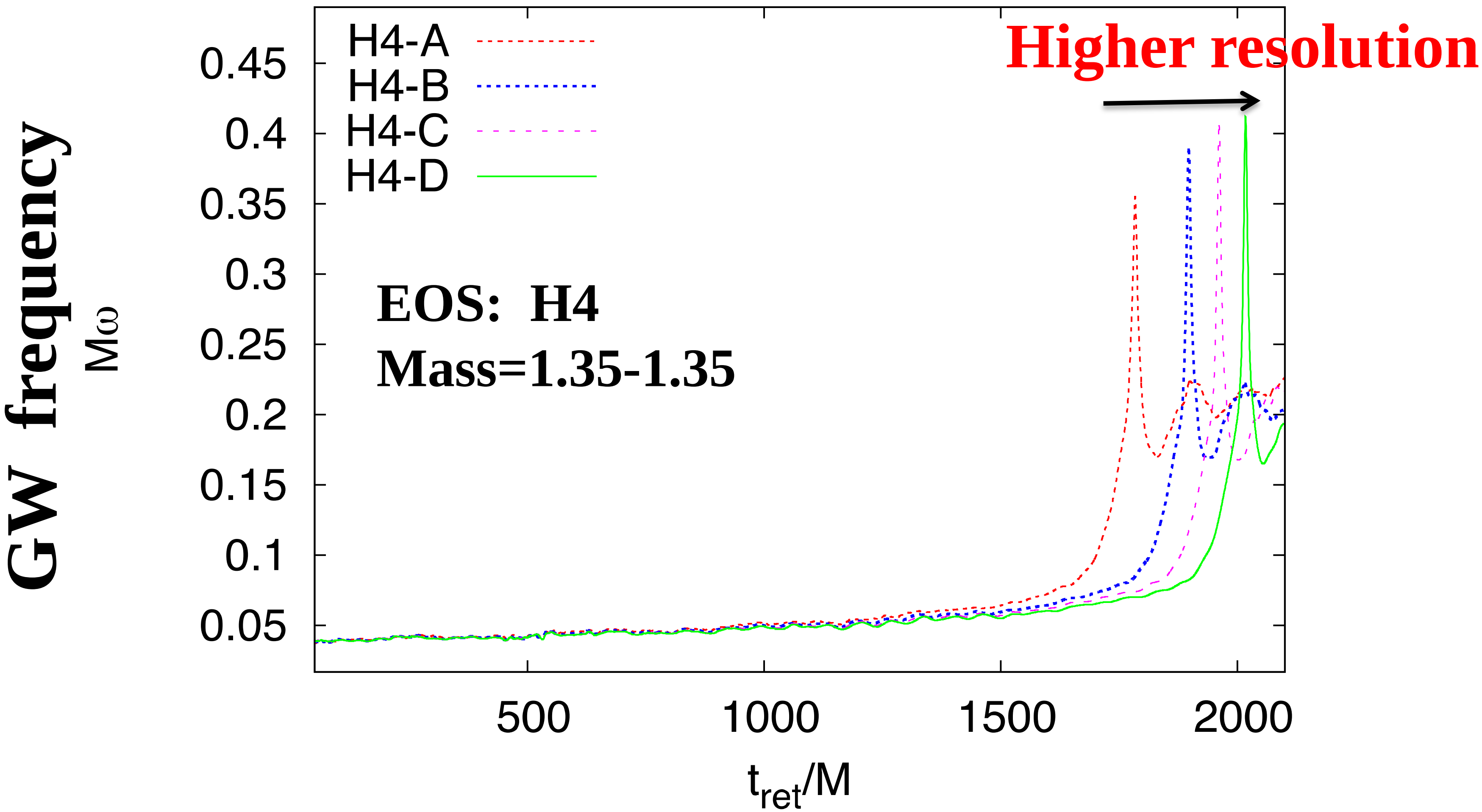
Resolution study is crucial:

Four different resolutions : N=42, 48, 54, 60

--- NS diameter is covered by 70, 80, 90, 100

→ **Then, extrapolation is taken.**

Dependence on Numerical Resolution



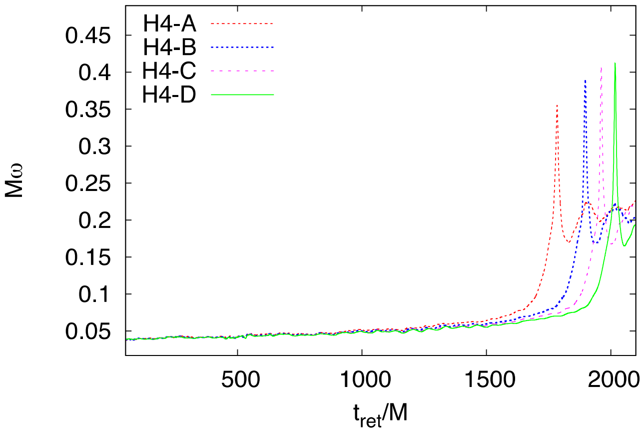
Lower resolution, larger dissipation, shorter timescale:
However, it is systematic

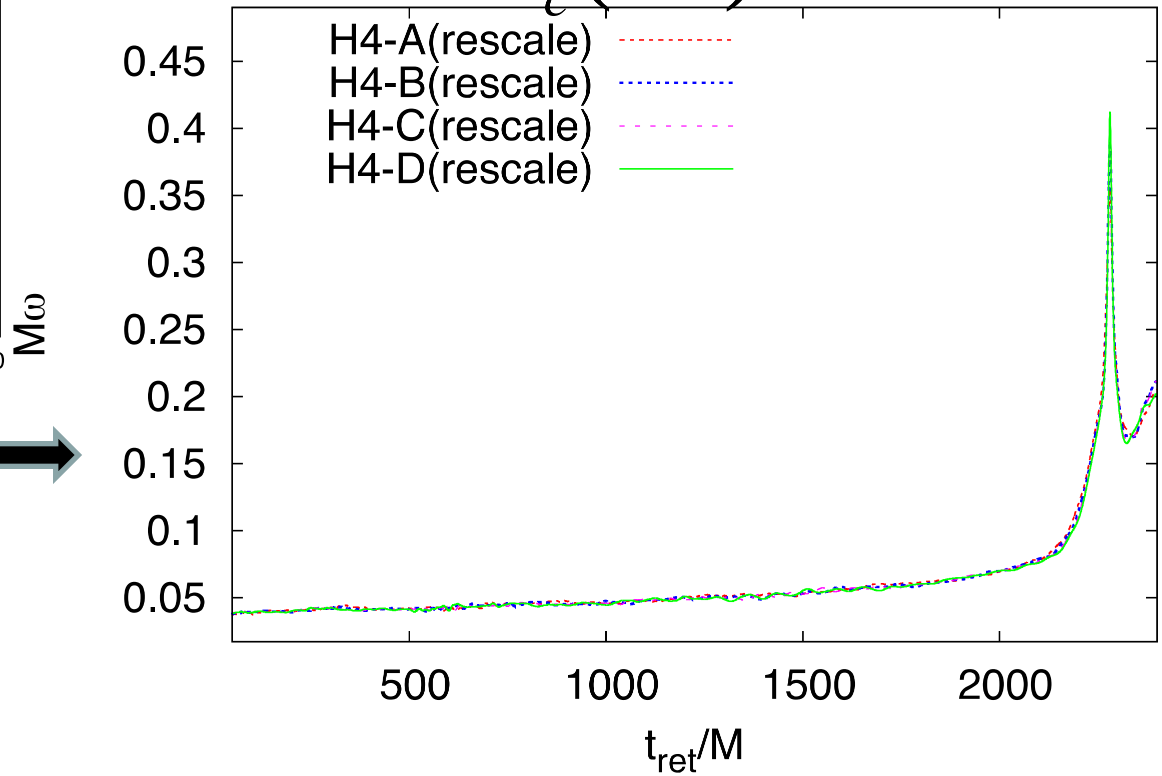
$$t_c(\Delta x) \approx t_c(0) - A(\Delta x)^{1.8}$$

Rescale the GW frequency

Rescale the time

$$t \to \tilde{t} = \frac{t_c(0)}{t_c(\Delta x)}\, t$$

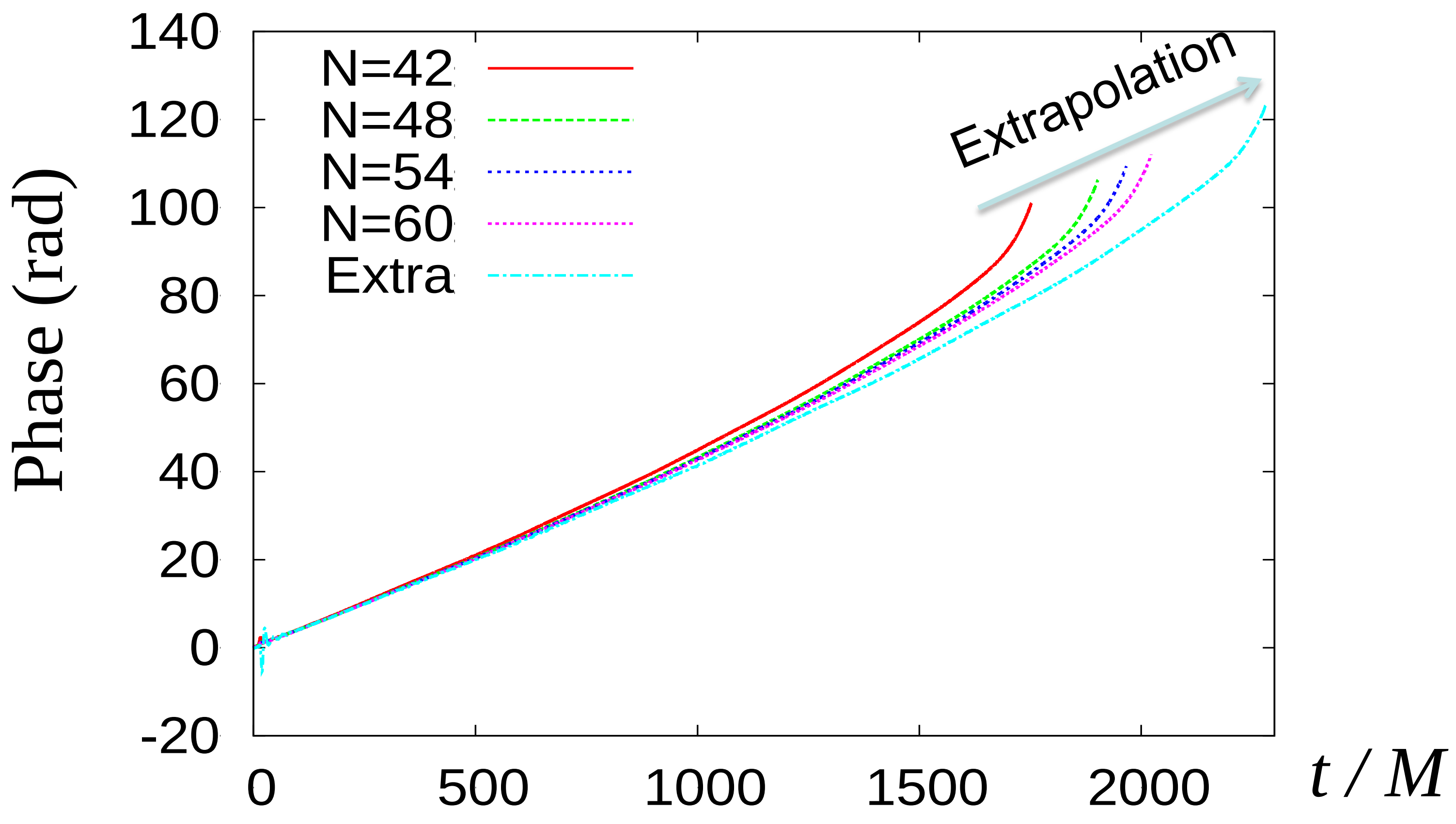




Rescaled GW frequency agrees with each other

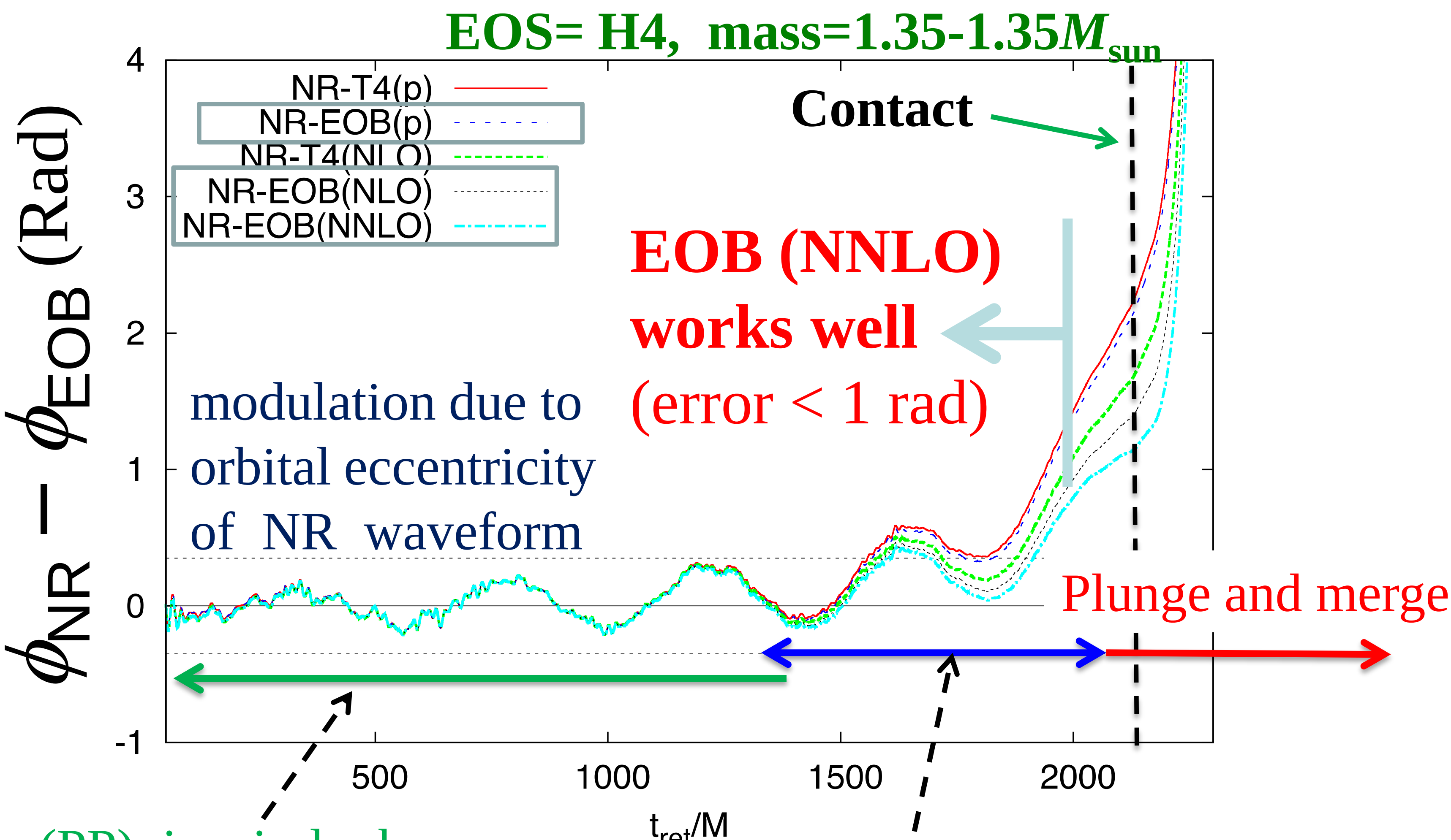
Extrapolate GW phase

$$h(t) = A(t)e^{-if(t)}$$



Phase difference between N=60 and the extrapolated one ~ 5 rad

Results: Compare NR with PN and EOB

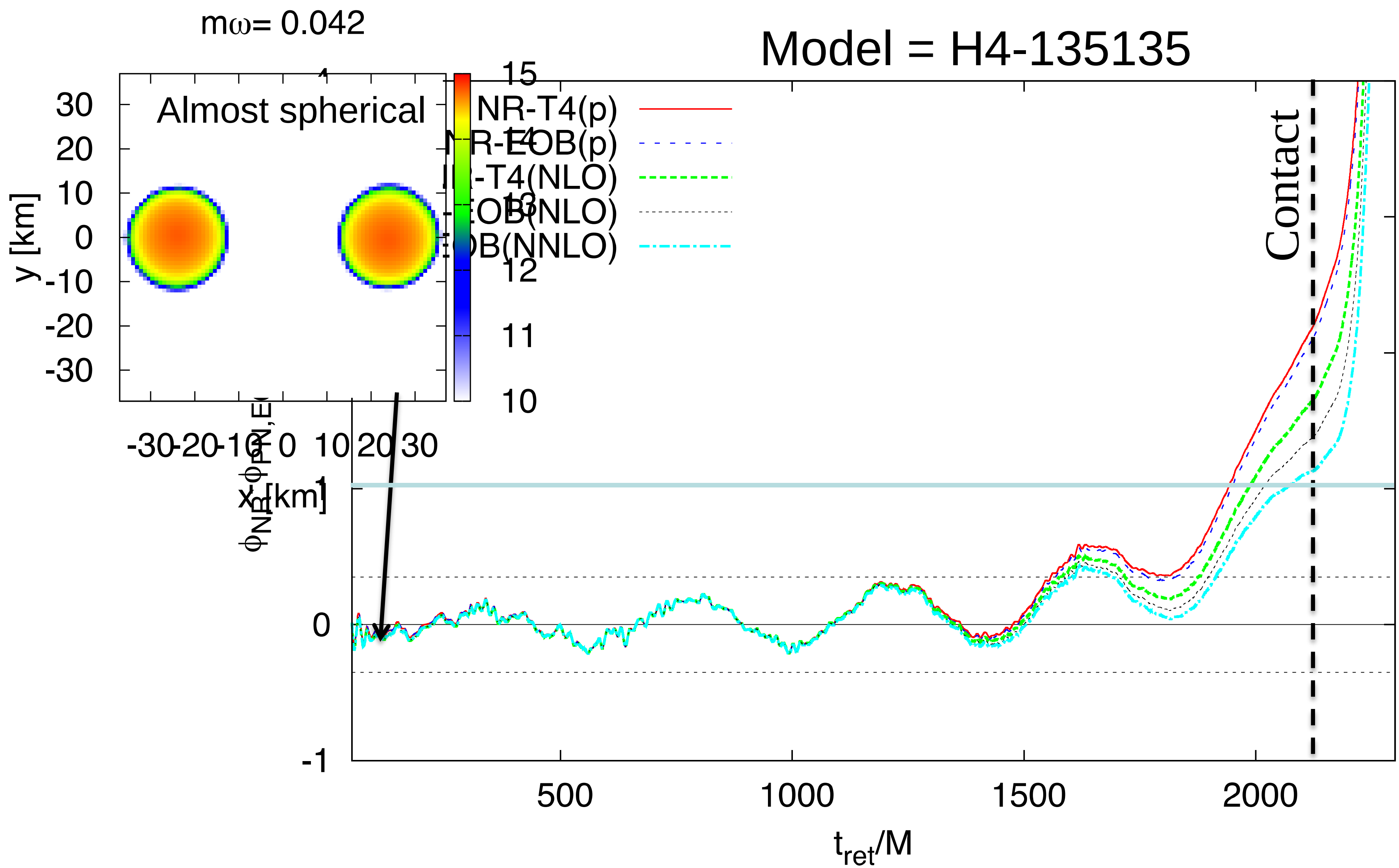


Early (PP) inspiral phase consistent with PN/EOB(p)

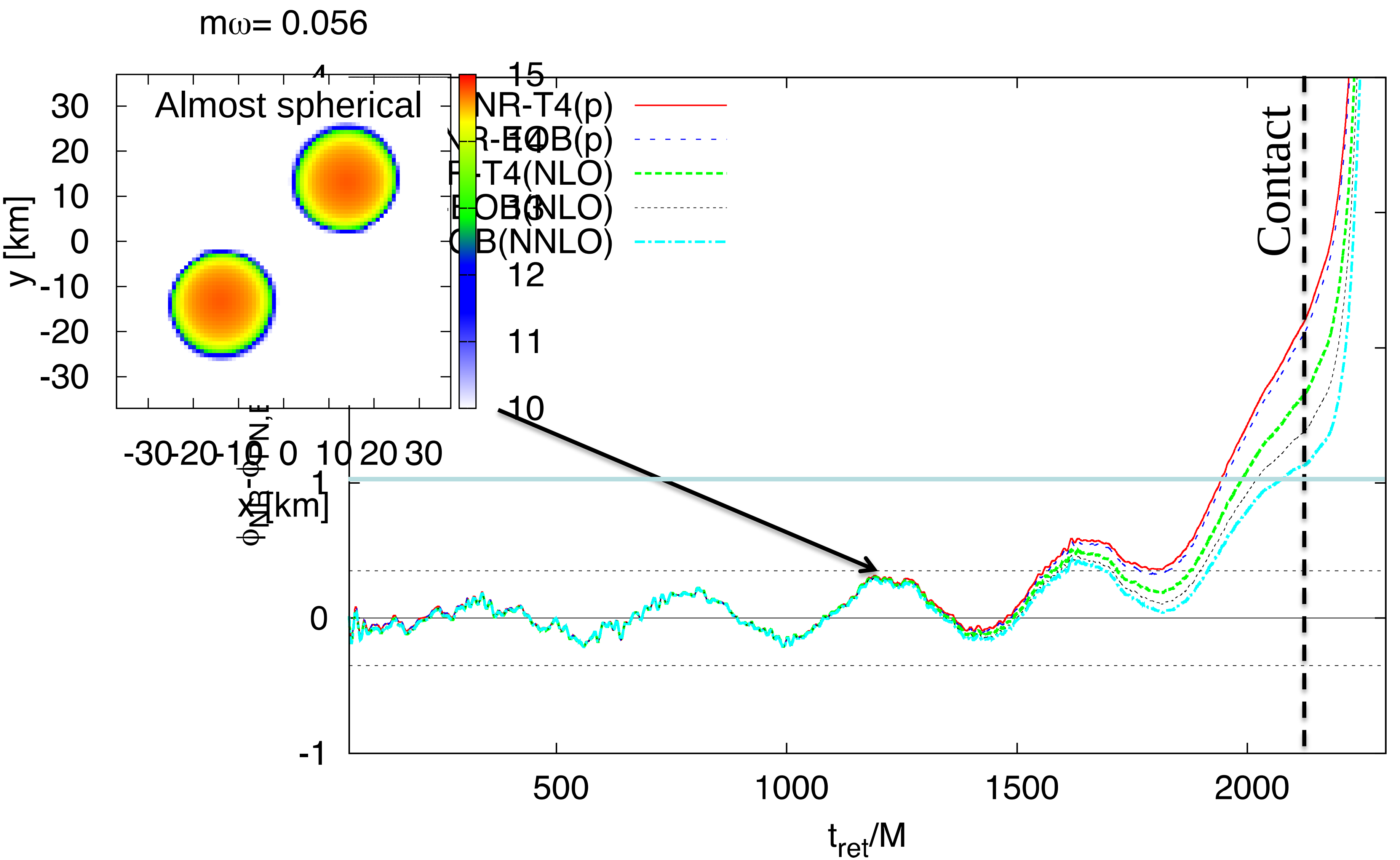
Late inspiral phase Tidal effects become strong

Refs) T4(NLO) : Vines et al ('11), EOB(NLO) : Nagar and Damour ('11), EOB(NNLO) : Bini+ ('12)

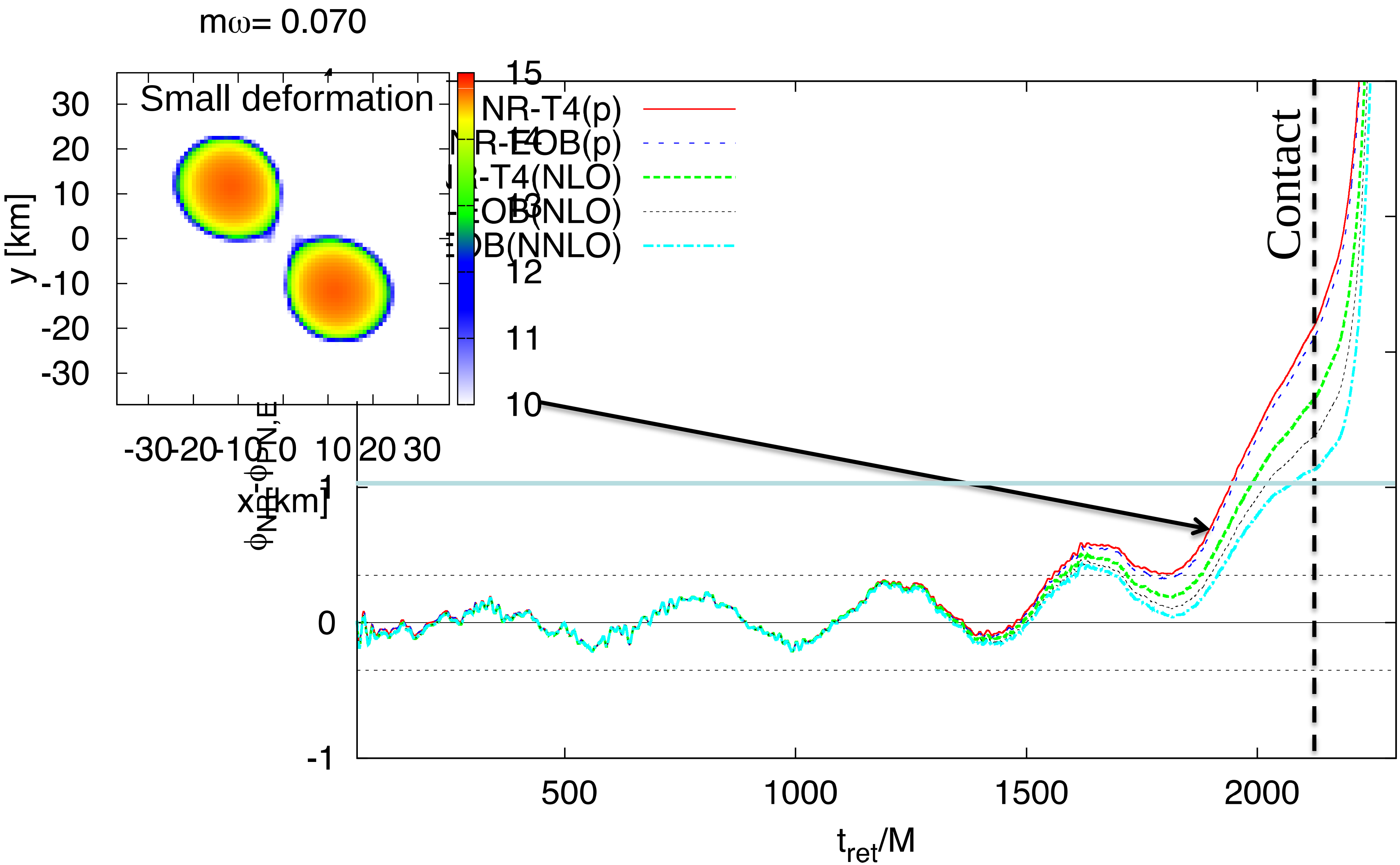
Snapshot : Early ➡ Late inspiral (f ~ 500Hz)



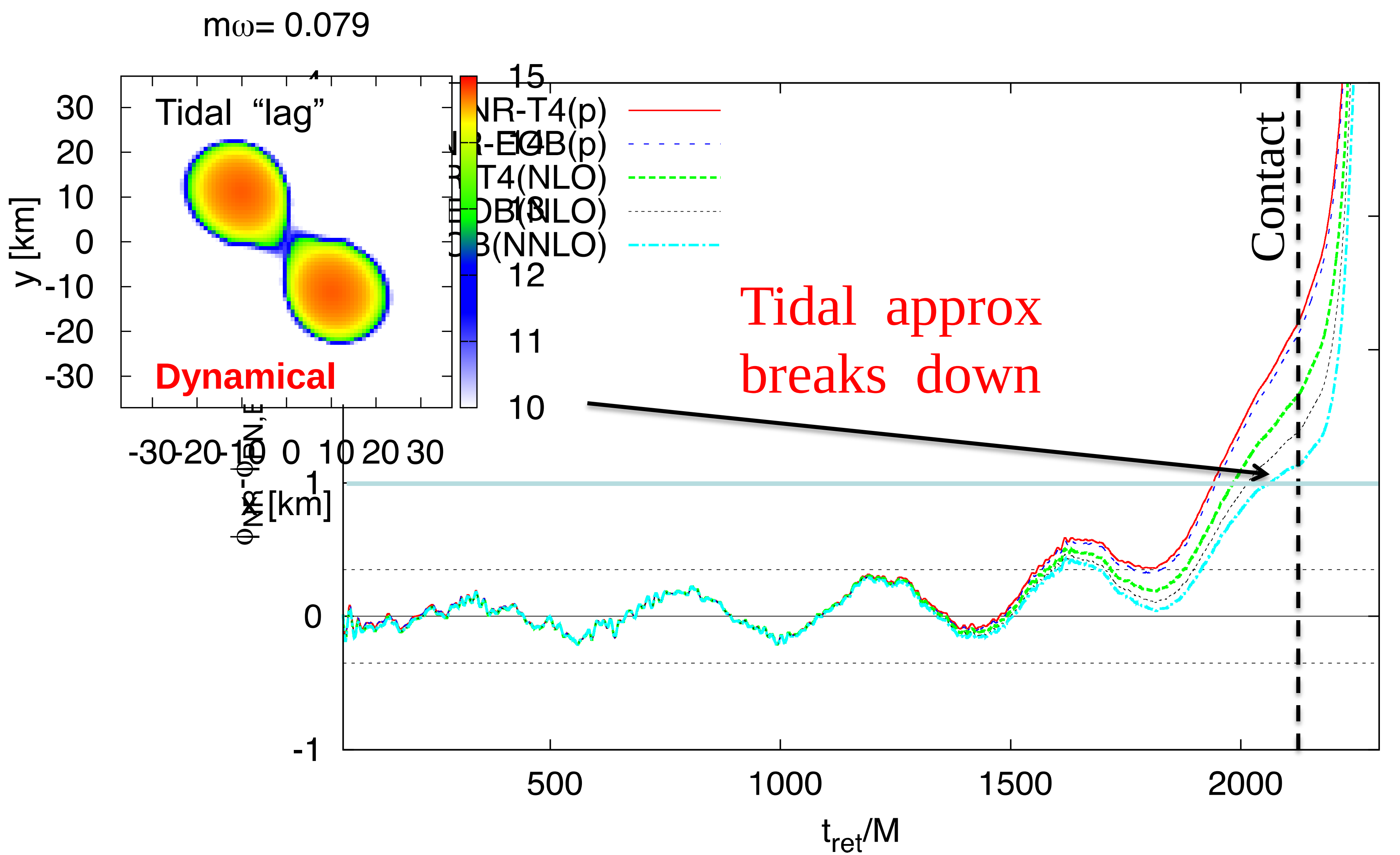
Snapshot : Late inspiral ($f \sim$ 700Hz)



Snapshot: Inspiral ➡ Plunge (f ~ 830Hz)



Snapshot : Plunge ➡ Merge (f ~ 950Hz)



Accumulated phase difference ~ 1 rad for EOS with tidal NNLO

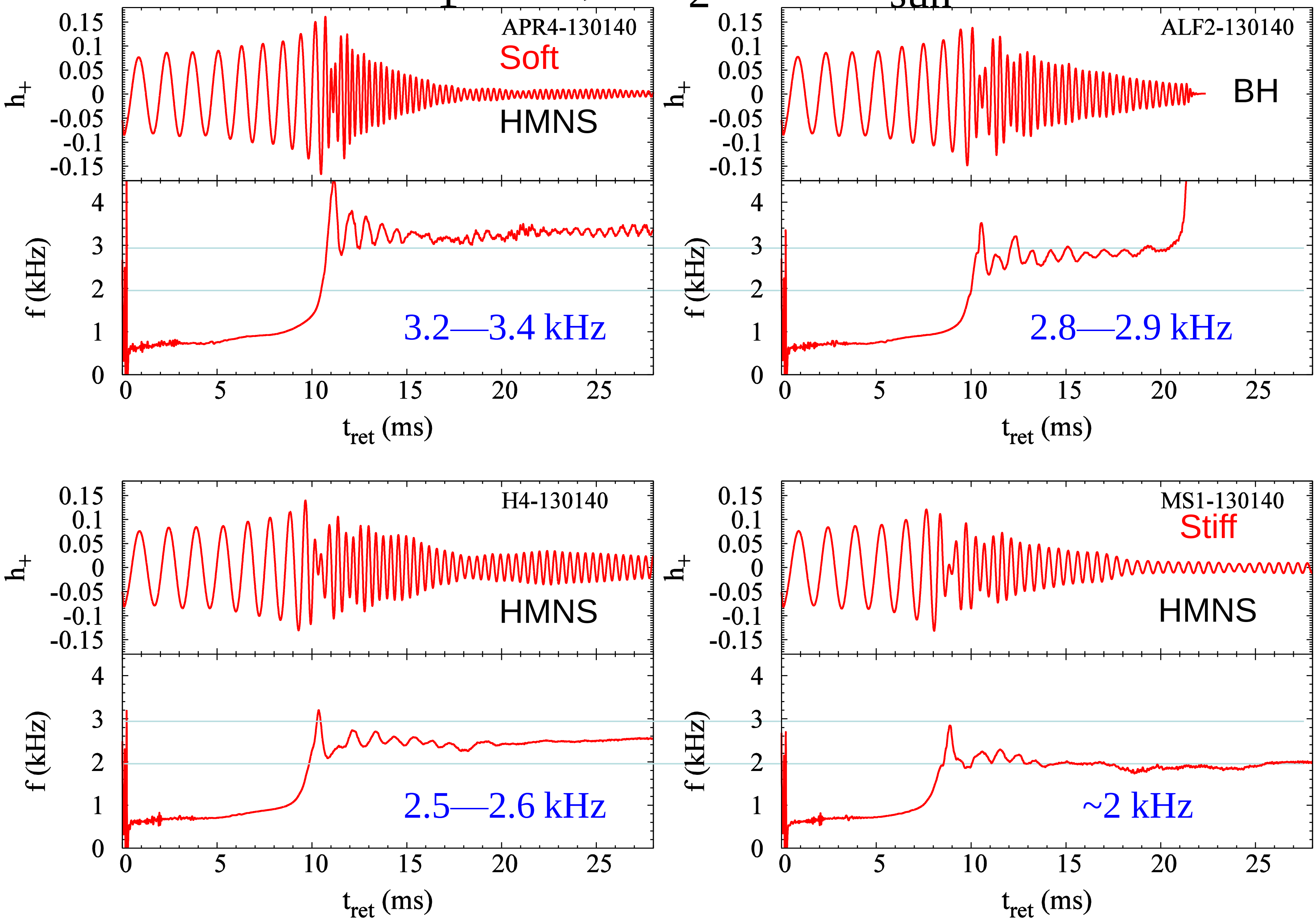
Summary for this GW

- EOB incorporating 2PN tidal effects provides a good model up to an orbit in which the tidal deformation is perturbative
- In the final inspiral phase, where tidal deformation is not written in the static manner (e.g., in the presence of lag angle), the phase error ~ 1 rad may be present

✓ Issue: Further good modeling in highly (nonlinearly) tidal-deformed phase

2 Gravitational waves from hypermassive NS

M_1=1.3, M_2=1.4M_{sun}

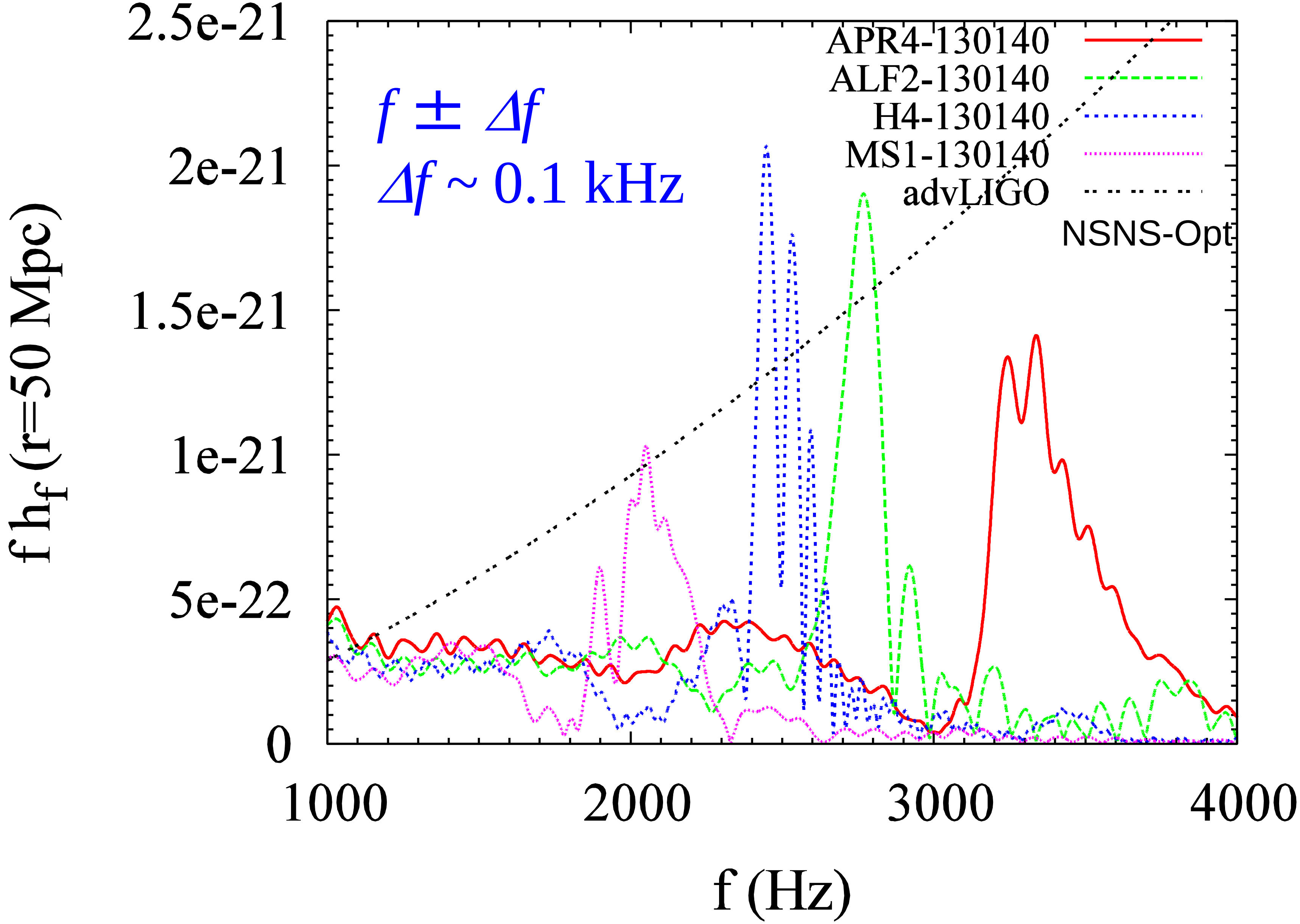


Properties of GW from HMNS

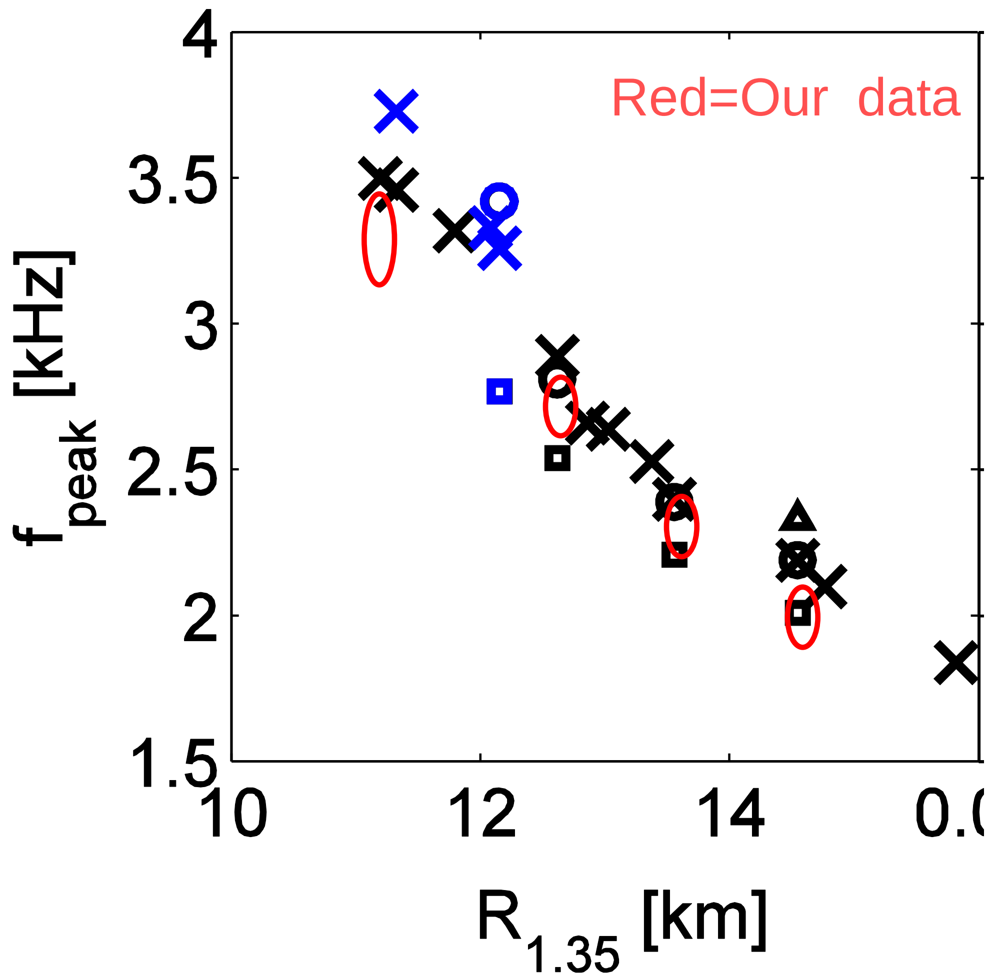
- **Gravitational-wave frequency from HMNS depends strongly on EOS**
- **The frequency has correlation with stiffness** (Janka+, 11)
- Gravitational-wave frequency appears to be approximately constant; but not exactly constant (“quasiperiodic”) due to GW reaction → Gravitational waves make a broad peak in the Fourier spectrum

↓

Fourier spectrum



Consistent with Bauswein & Janka, 2011 PRL



A correlation is present
→If f_{peak} is determined, radius is constrained

But the peak frequency would not be clear: Measurement error of R is ~ 1 km

Summary & issues for HMNS

- GW from HMNS will carry information of EOS of NS
- The NS radius may be constrained with ~ 1 km error

➢ Issues:

✓ Is it really possible to detect such GW by advLIGO/VIRGO/KAGRA ?

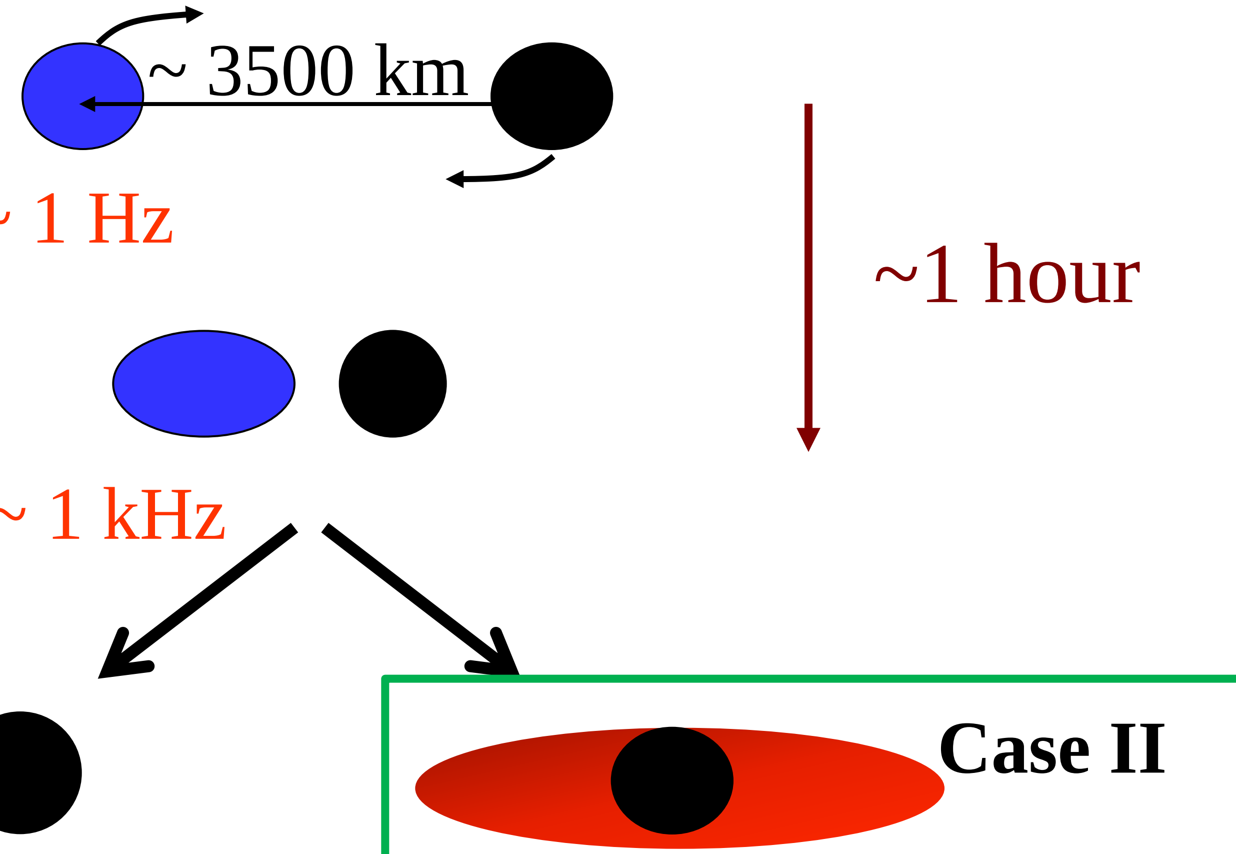
✓ If the detection is possible, how accurately the frequency is determined ?

BH-NS

Evolution of BH-NS ($4.05 M_{sun}$-$1.35 M_{sun}$)

Evolve by
GW emission
Last 1 hour ; $f_{GW} \sim 1$ Hz

Merger sets in
at $r \sim 40$ km; $f_{GW} \sim 1$ kHz



NS is disrupted

NS is swallowed by BH
for small R_{NS} or $M_{BH} >> M_{NS}$

Large EOS-dependence

Condition for tidal disruption

- BH tidal force > NS self-gravity

$$Q := \left(\frac{M_{\rm BH}}{M_{\rm NS}}\right) \le 2.0\zeta^{3/2}\left(\frac{\alpha}{1.5}\right)^{3/2}\left(\frac{c^2 R}{6GM_{\rm NS}}\right)^{3/2}$$

$$\zeta = 1\text{–}6 \quad \text{for} \quad a/M_{\rm BH} = 0\text{–}1$$

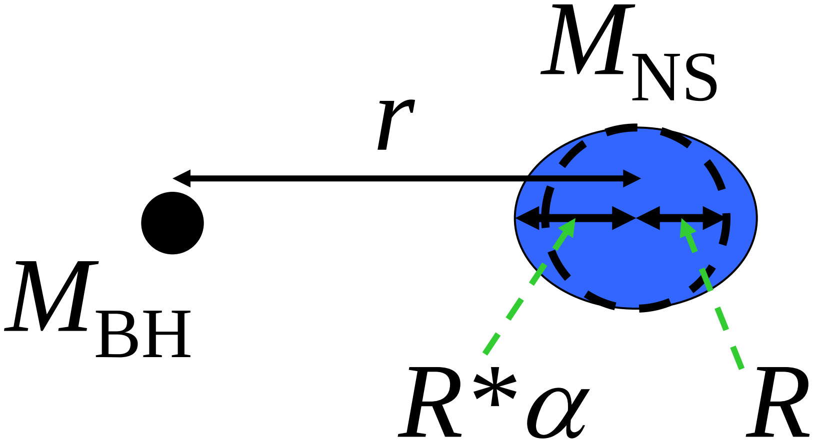
$$c = G = 1$$

✓**Low-mass BH** or

✓**Large NS radius** or

✓**Large BH spin**

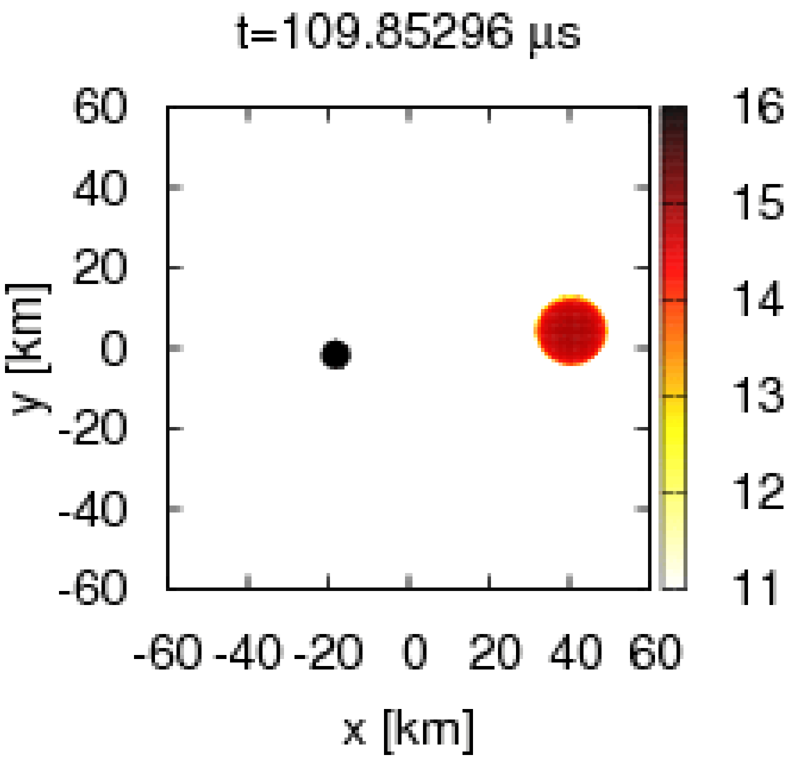
is necessary

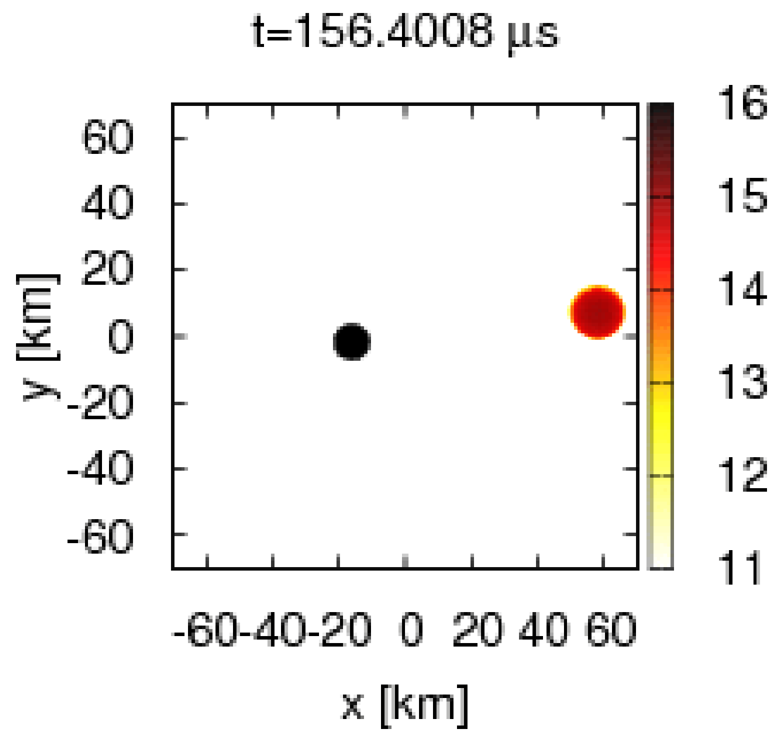


BH(a=0)-NS with piecewise polytrope

M_{BH}=2.7M_{sun}
M_{NS}=1.35M_{sun}
R=11.6 km, Q=2

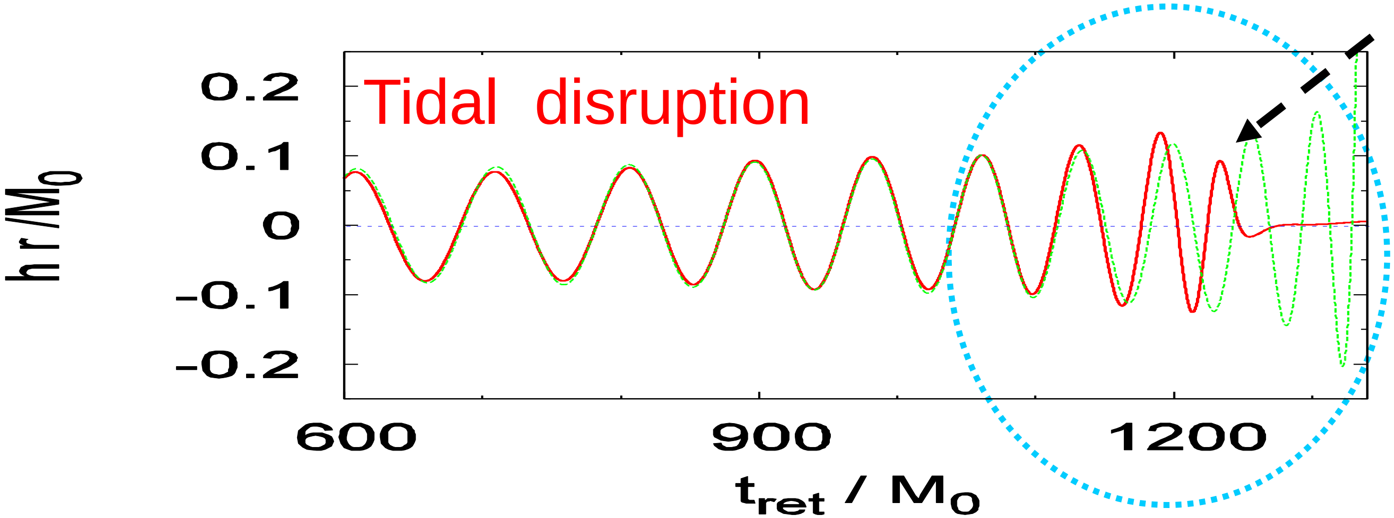
M_{BH}=4.05M_{sun}
M_{NS}=1.35M_{sun}
R=11.0 km, Q=3





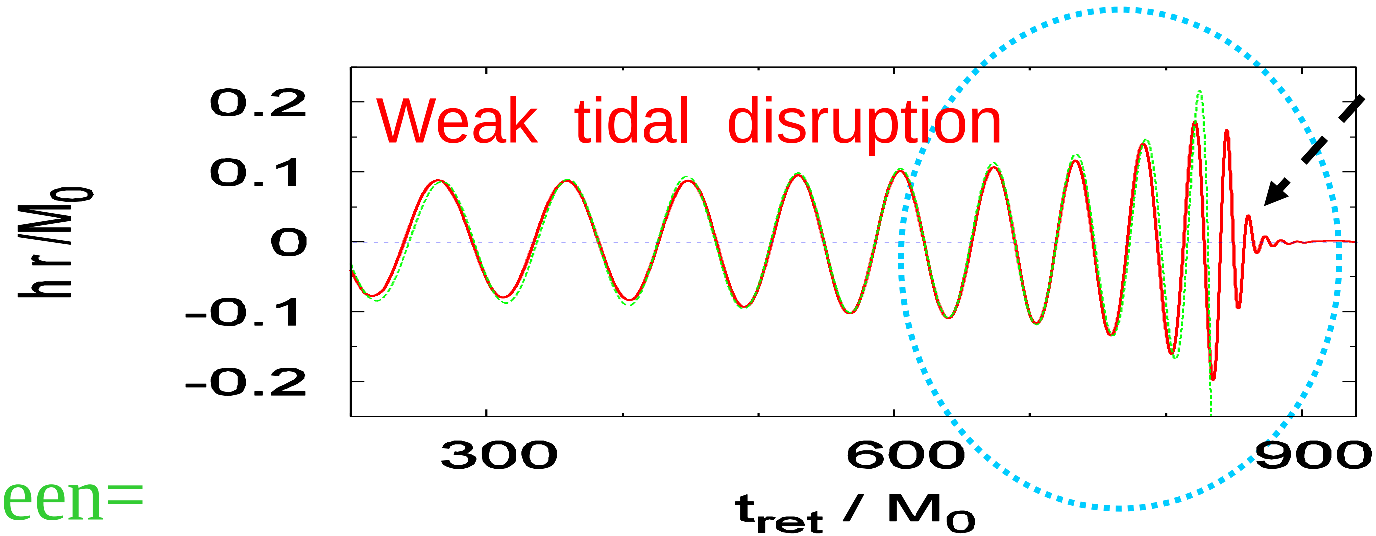
Kyutoku + PRD 2011

$M_{BH}=2.7M_{sun}$, $a=0$, $M_{NS}=1.35M_{sun}$



sudden shutdown

R=15.2 km



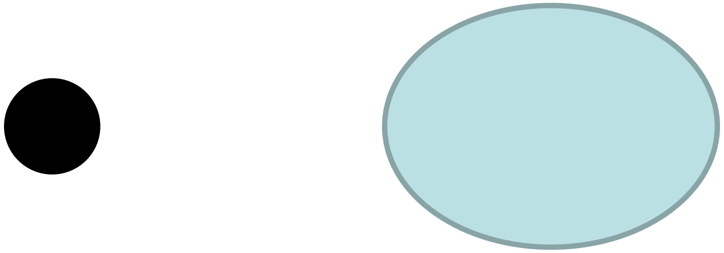
BH ringdown

R=11.6 km

Green= Tayloy T4

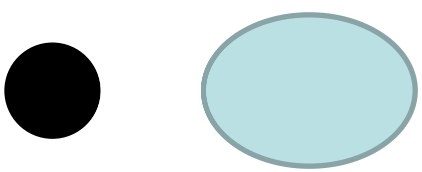
Imprint of EOS in tidal disruption

- Large NS Radius →
  tidal disruption at a
  distant orbit, *i.e.*,
  at a ***low frequency***



Assume the same mass

- Small NS Radius →
  tidal disruption
  at a ***high frequency***

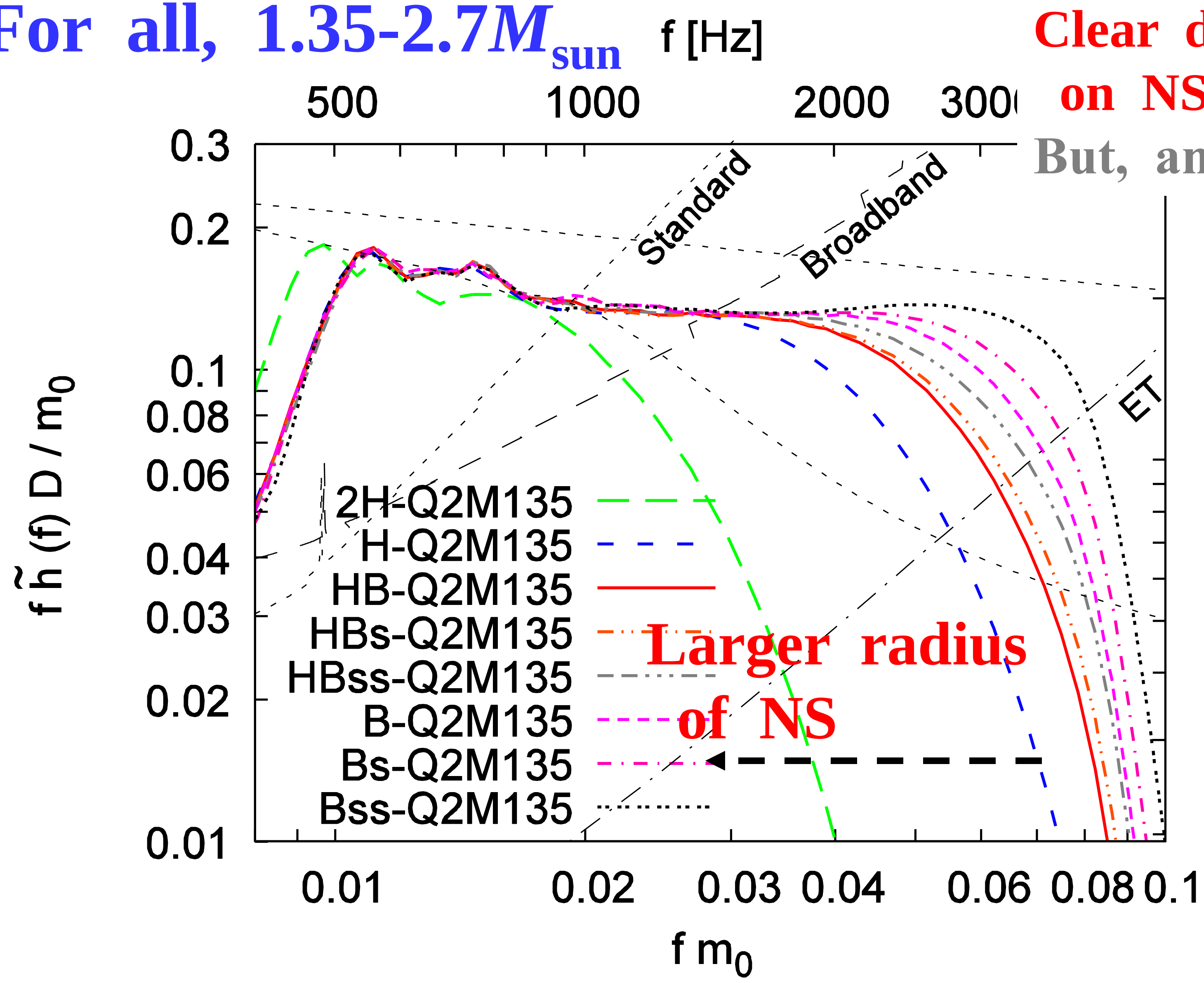


BH-NS with piecewise polytrope (*a*=0)

For all, 1.35-2.7M_{sun}

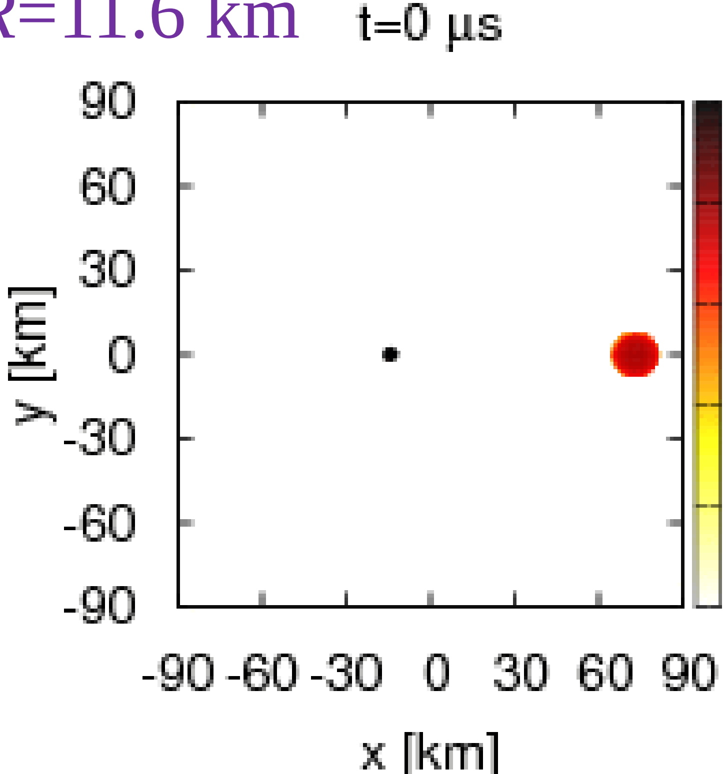
Clear dependence on NS radius

But, amp is low…

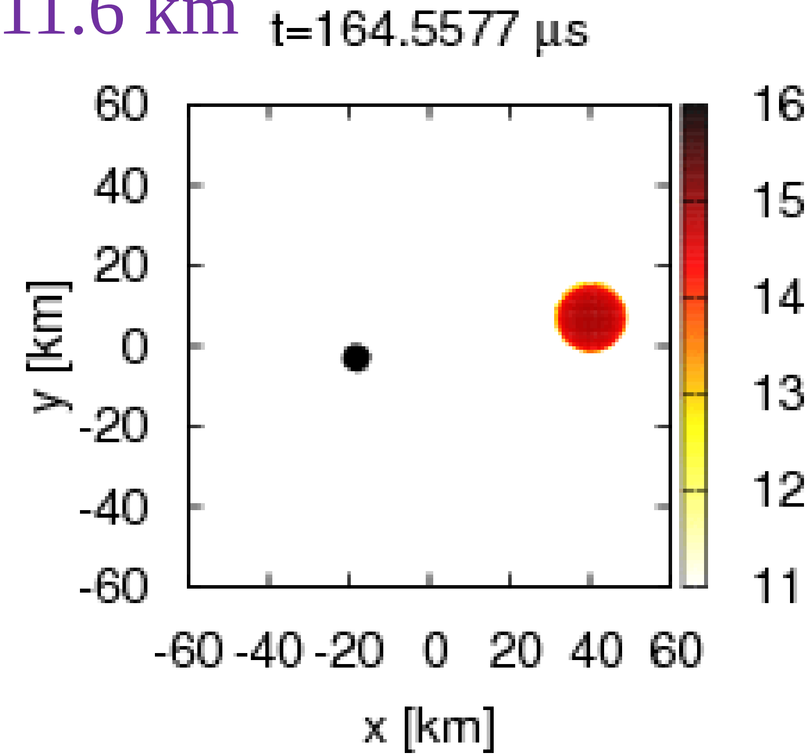


Spinning BH-NS; more promising

M_{BH}=5.4M_{sun}
a=0.75, Q=4
M_{NS}=1.35M_{sun}
R=11.6 km

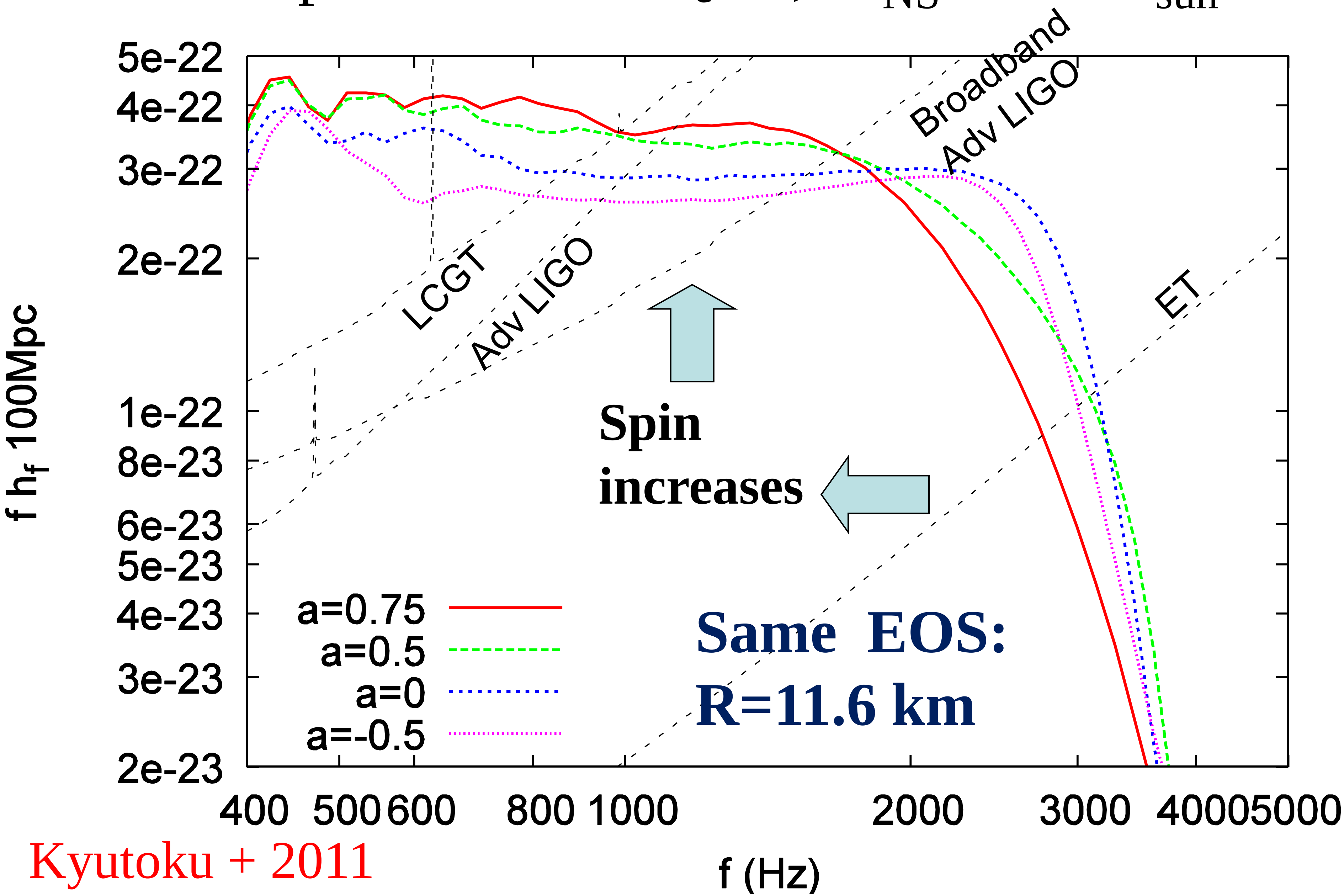


M_{BH}=2.7M_{sun}
a=–0.5, Q=2
M_{NS}=1.35M_{sun}
R=11.6 km



Kyutoku et al. 2011

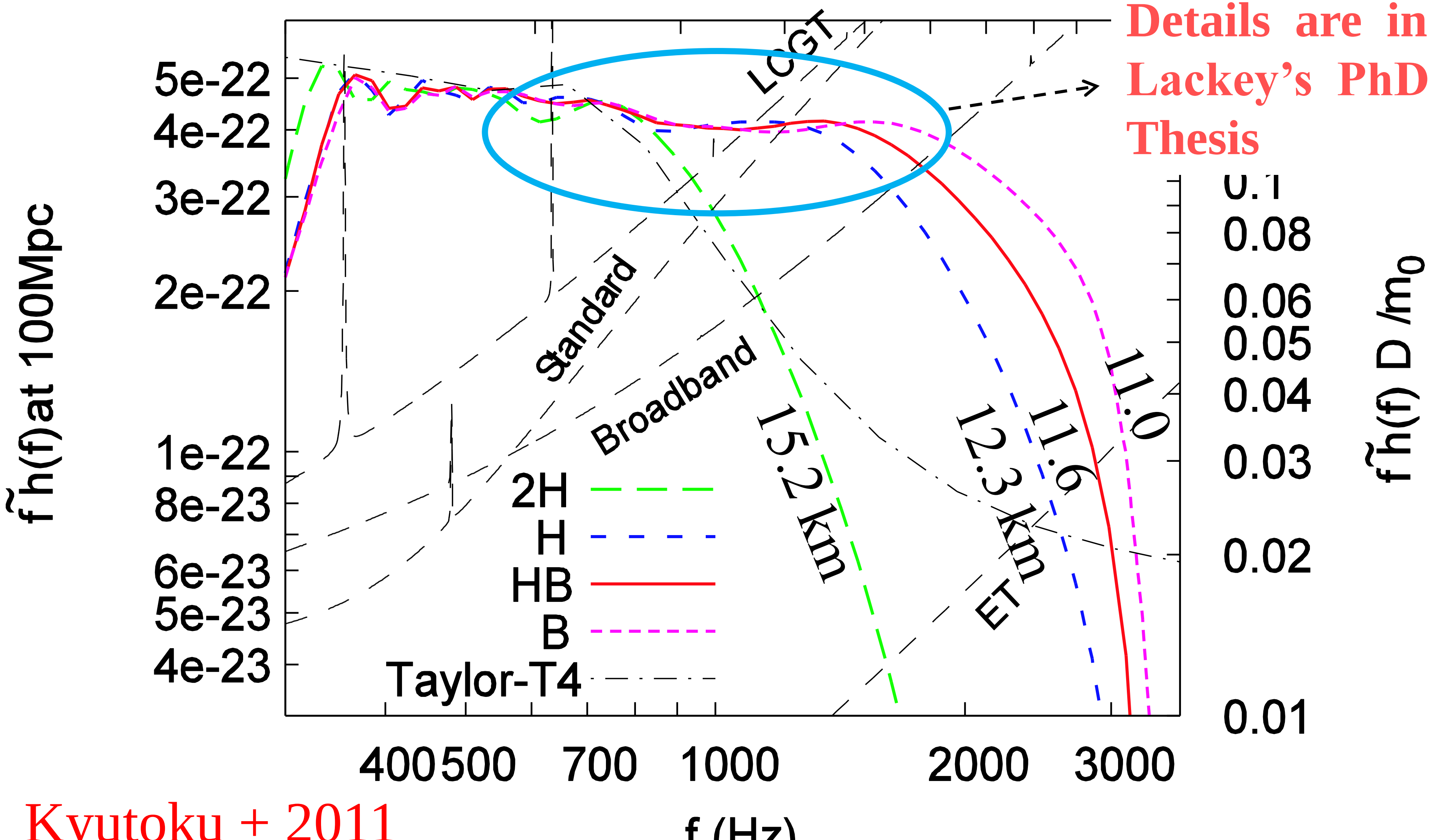
GW spectrum for Q=3, M_{NS}=1.35M_{sun}



Kyutoku + 2011

With BH spin & high-mass BH

For all, *a*=0.75 1.35-5.4M_{sun}



Details are in Lackey's PhD Thesis

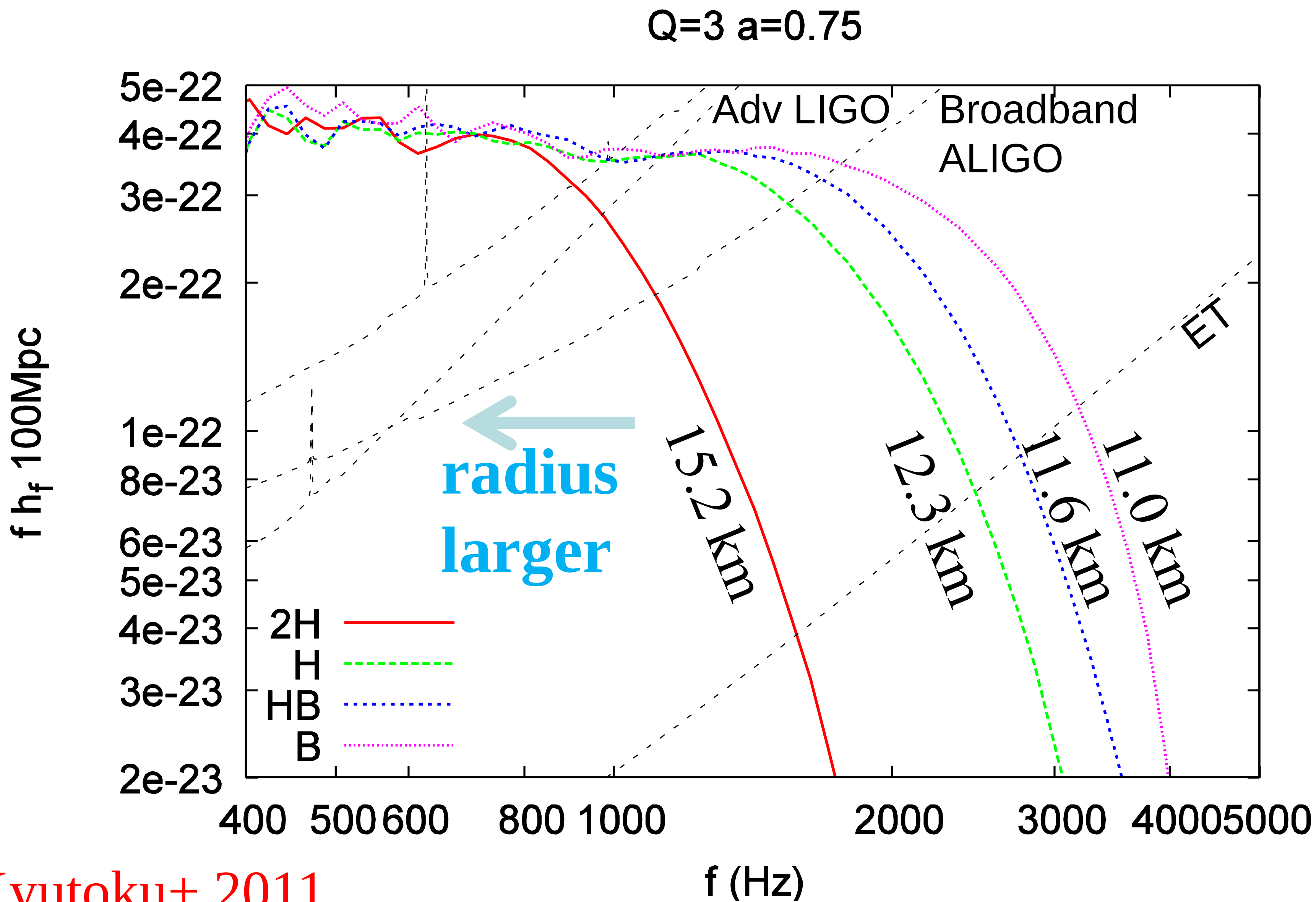
Kyutoku + 2011

Summary

- **Late-inspiral waveforms of NSNS reflect NS EOS (although it is a small effect) and EOB approach is likely to work well (currently, except for the last ~ 1 orbit)**
- **GWs from HMNS reflect NS radius; Radius may be constrained with ~1 km error for small-distance events**
- **GWs at tidal disruption reflect NS radius; high-spin BH events could constrain EOS even by advLIGO/VIRGO/KAGRA**

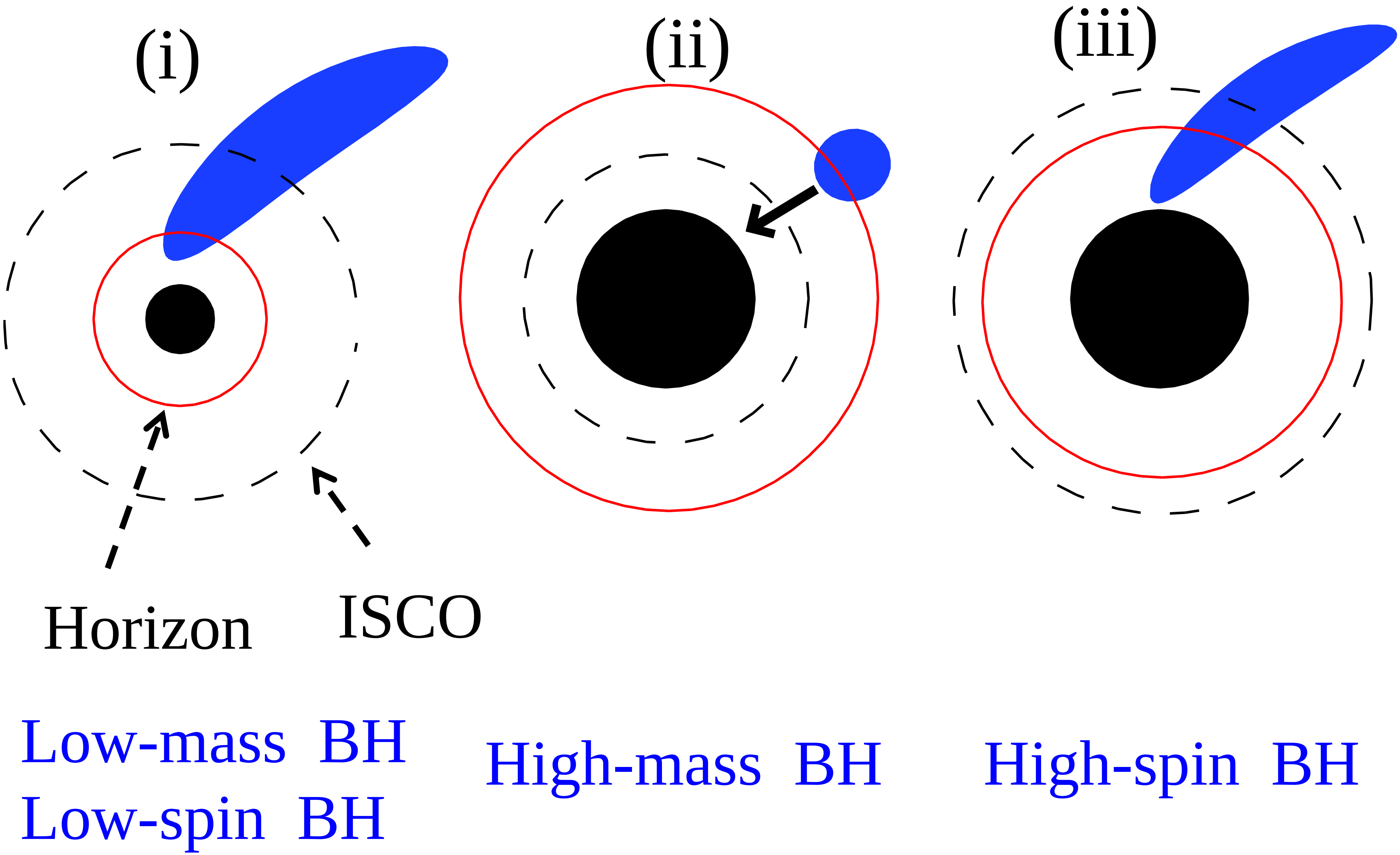
Thanks

GW spectrum for Q=3, a=0.75

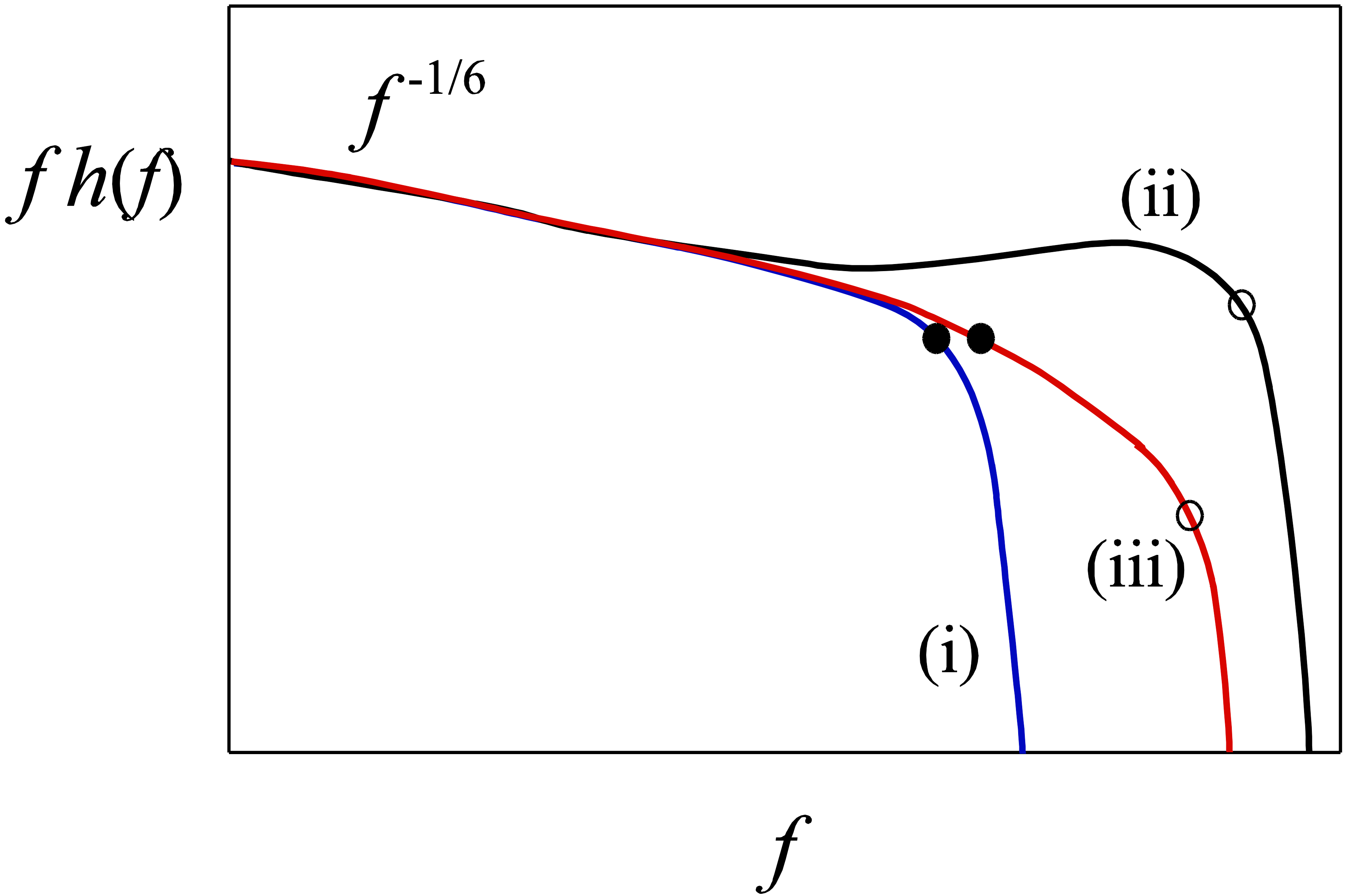


Kyutoku+ 2011

Three patterns of BH-NS merger



Spectrum type



MR relation for Shen's EOSs