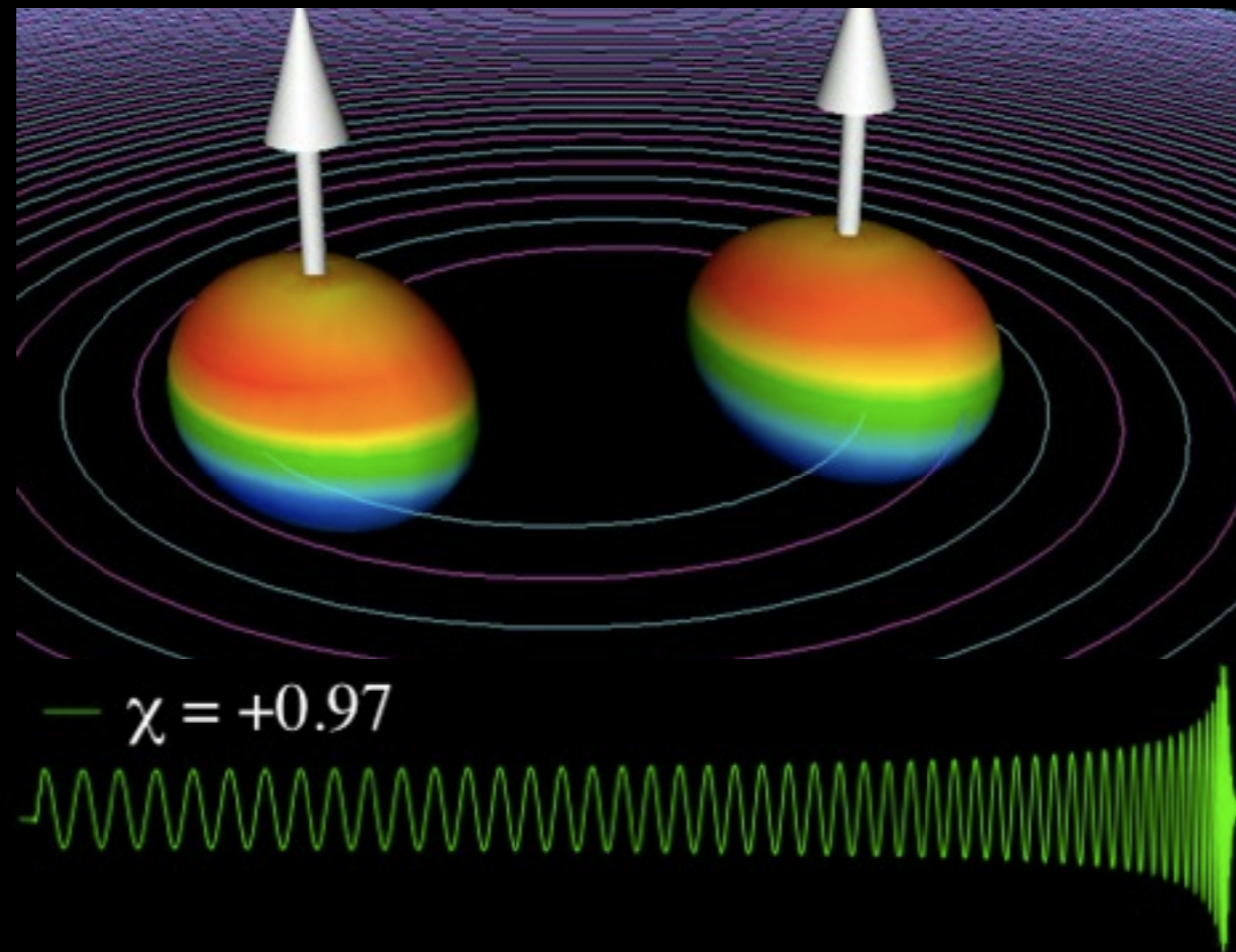


# Numerical simulations of binary black holes in the presence of spins



Geoffrey Lovelace  
Gravitational Wave Physics and Astronomy Center  
California State University, Fullerton  
July 31, 2012

# Outline

- Background & motivation
  - Spinning black holes
  - Gravitational waves (GW) from binary black holes (BBHs)
  - Modeling spinning BBHs using numerical simulations
- Results so far: some highlights
- Challenge: apply to GW data analysis
  - For best possible model waveforms, simulations must be
    - Extreme: capable of very high black-hole spin
    - Generic: handle misaligned spins & precession, unequal masses, unequal spins, extreme spins
    - Efficient: sufficiently long, accurate, & inexpensive

# Spinning black holes

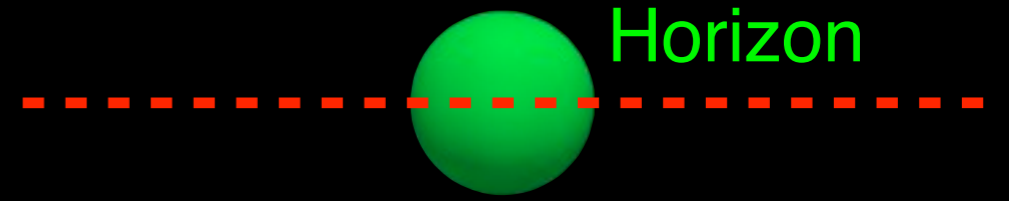
# Spinning black holes



- Black holes
  - Region of warped spacetime
  - **Horizon**: surface of black hole

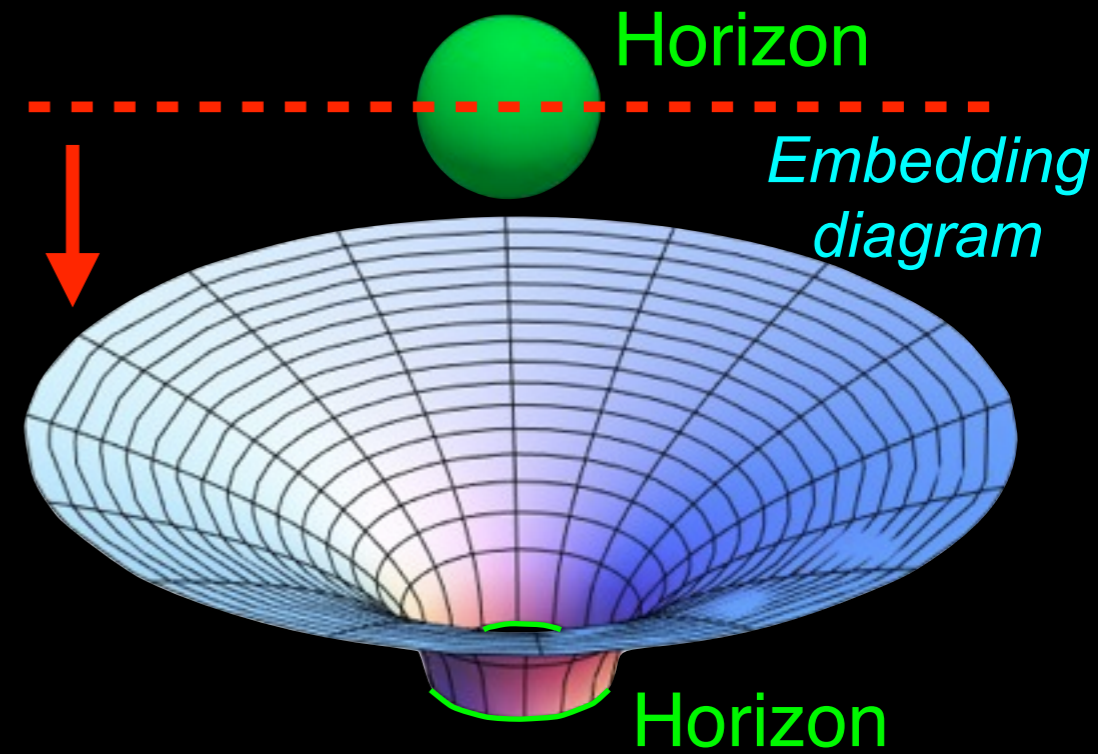
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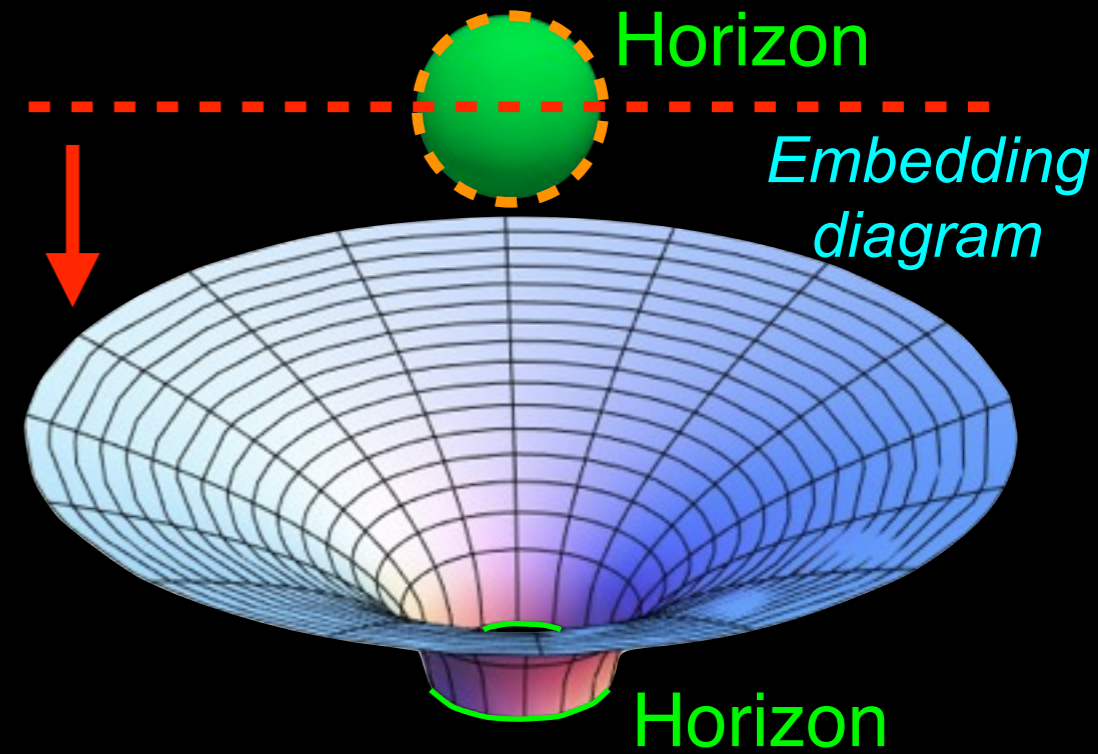
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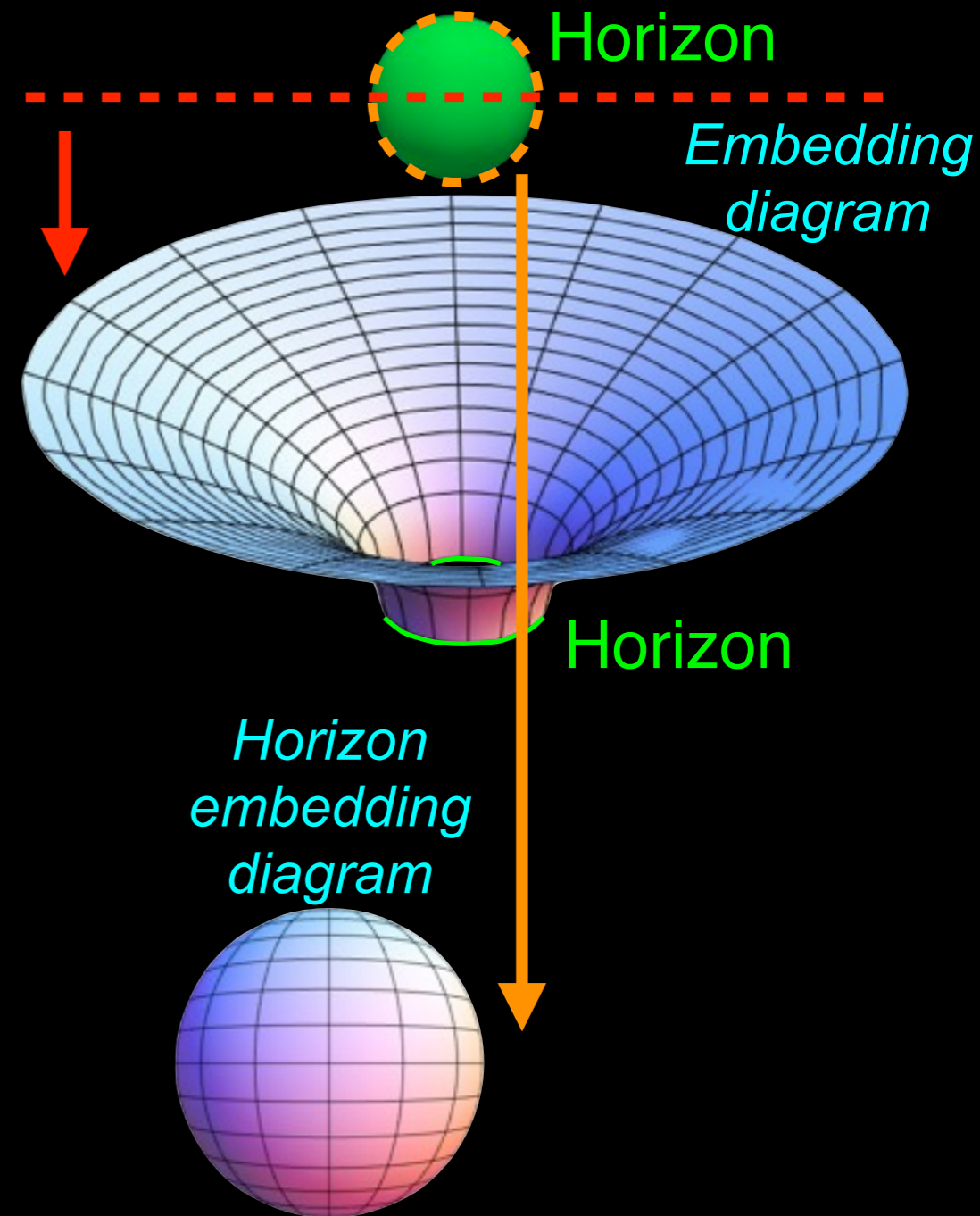
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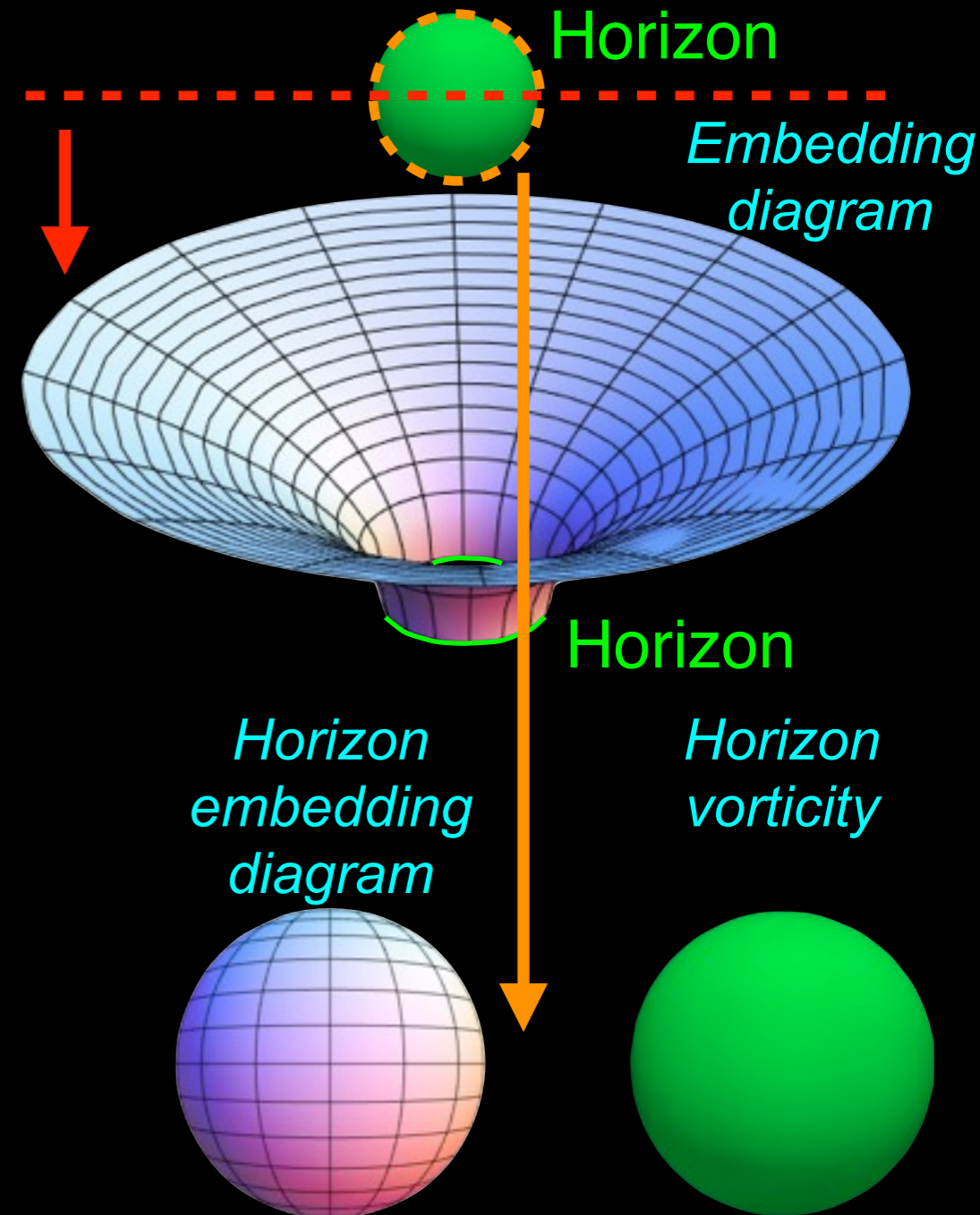
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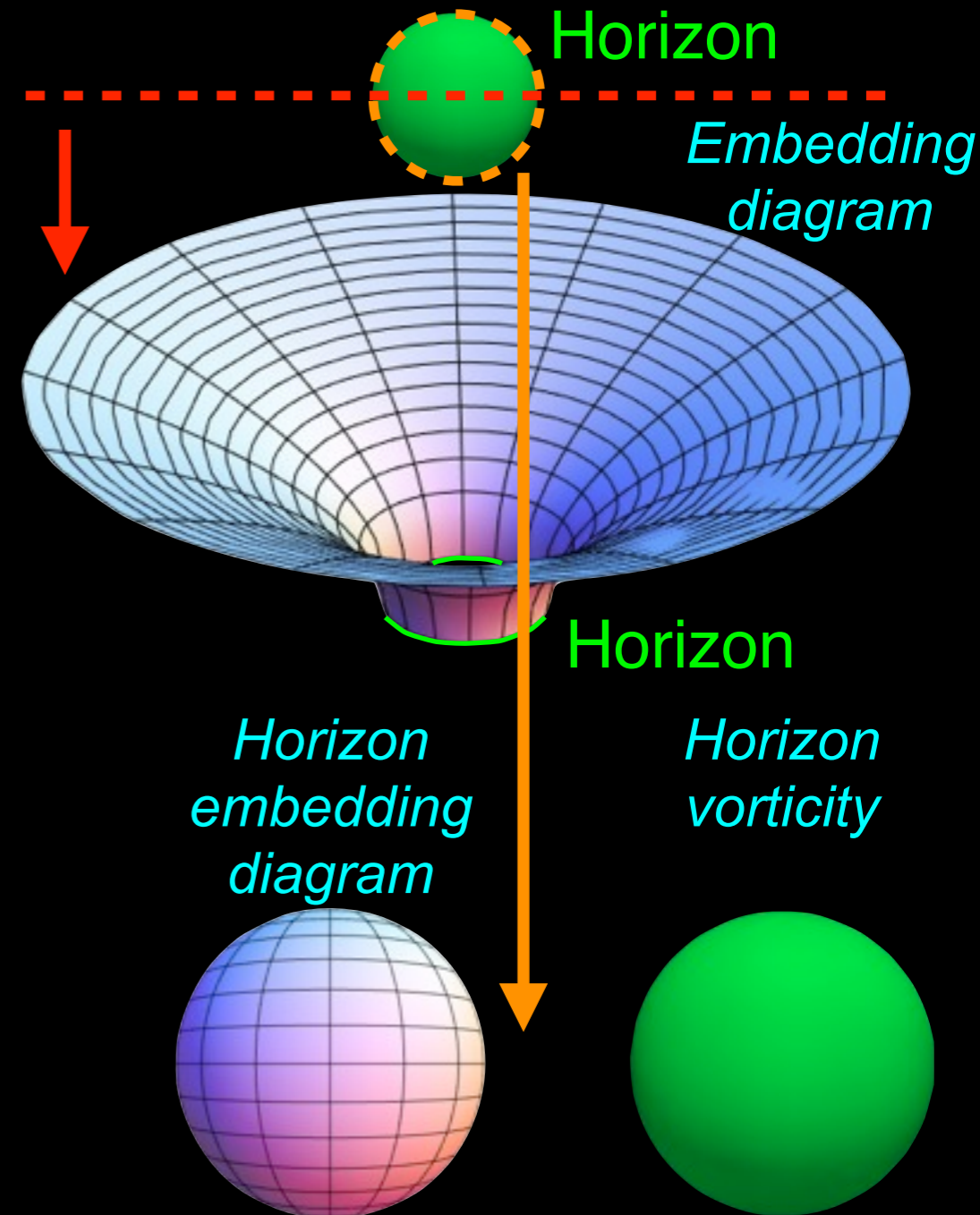
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  - Mass  $M$
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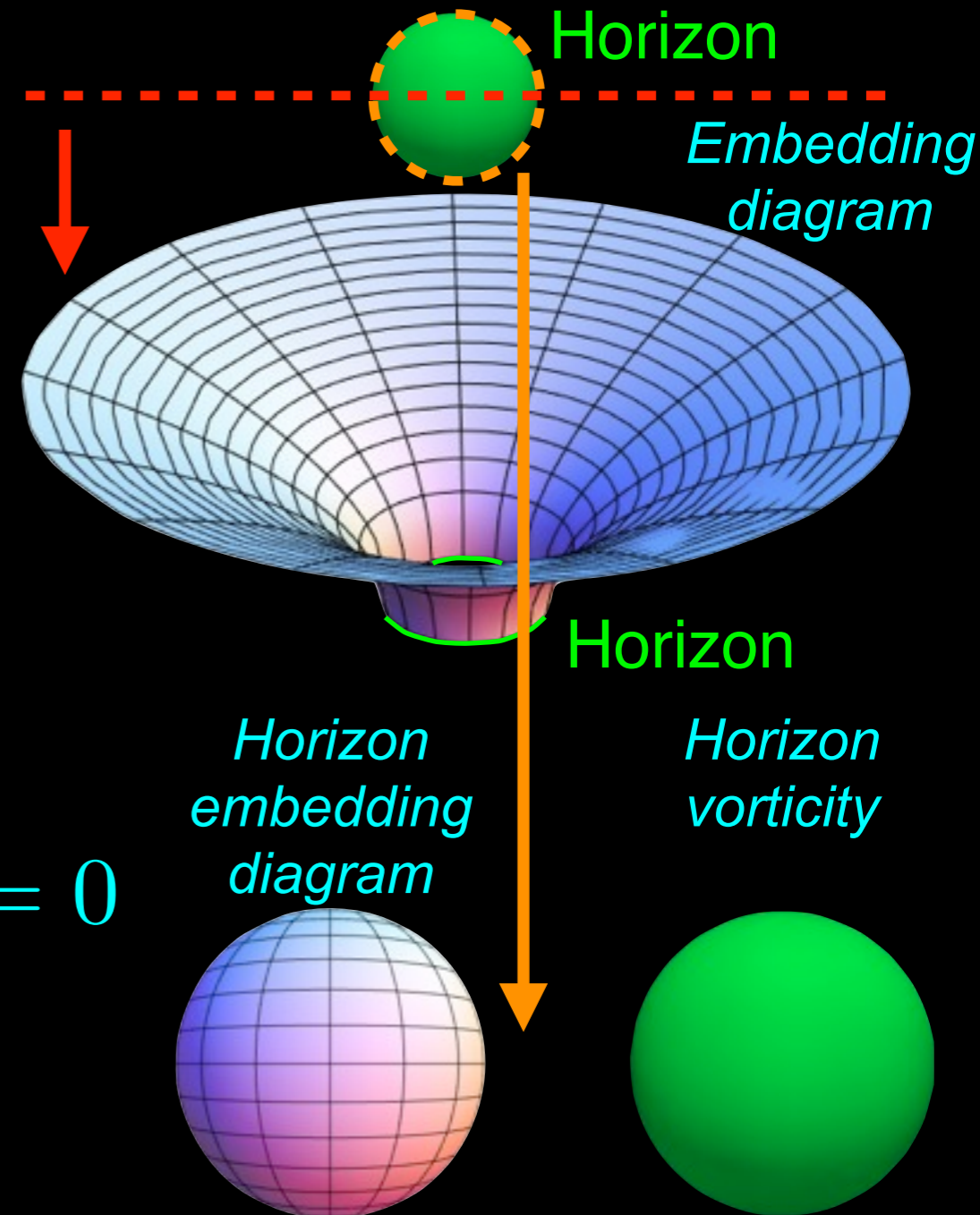
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  - Dimensionless spin

$$\chi = \frac{S}{M^2} \quad (G = c = 1)$$

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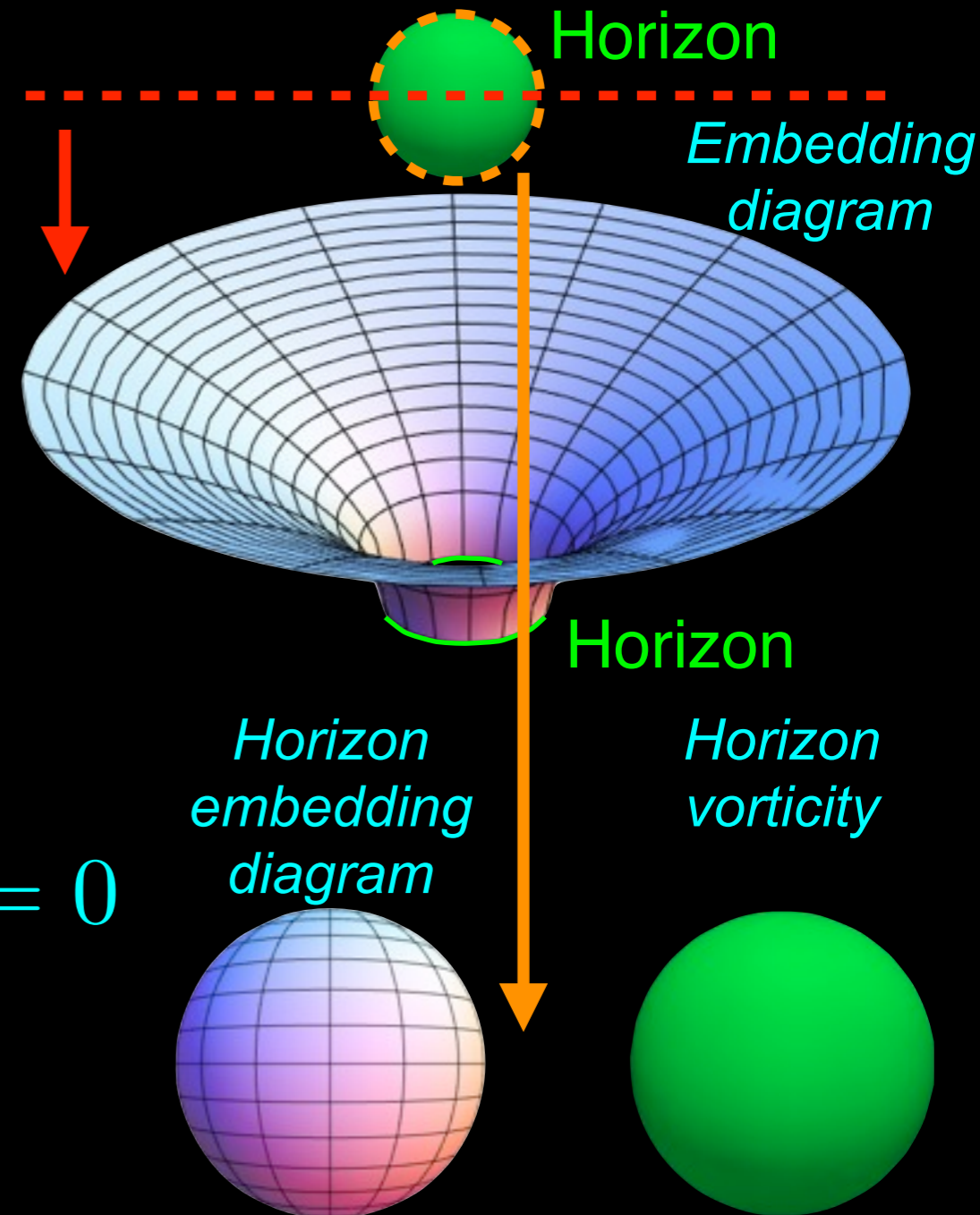
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or centrifugal forces  
destroy horizon  
(impossible)

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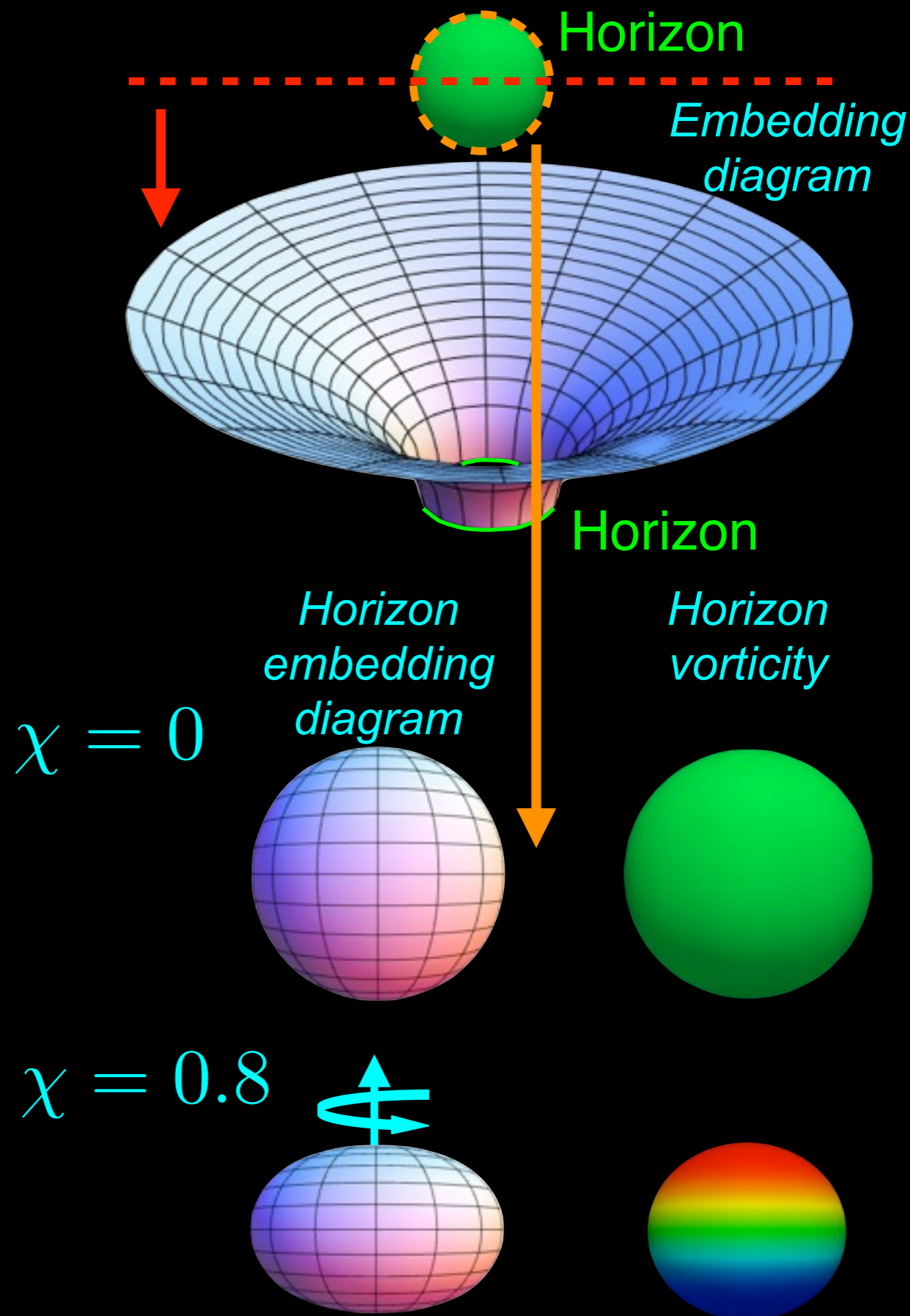
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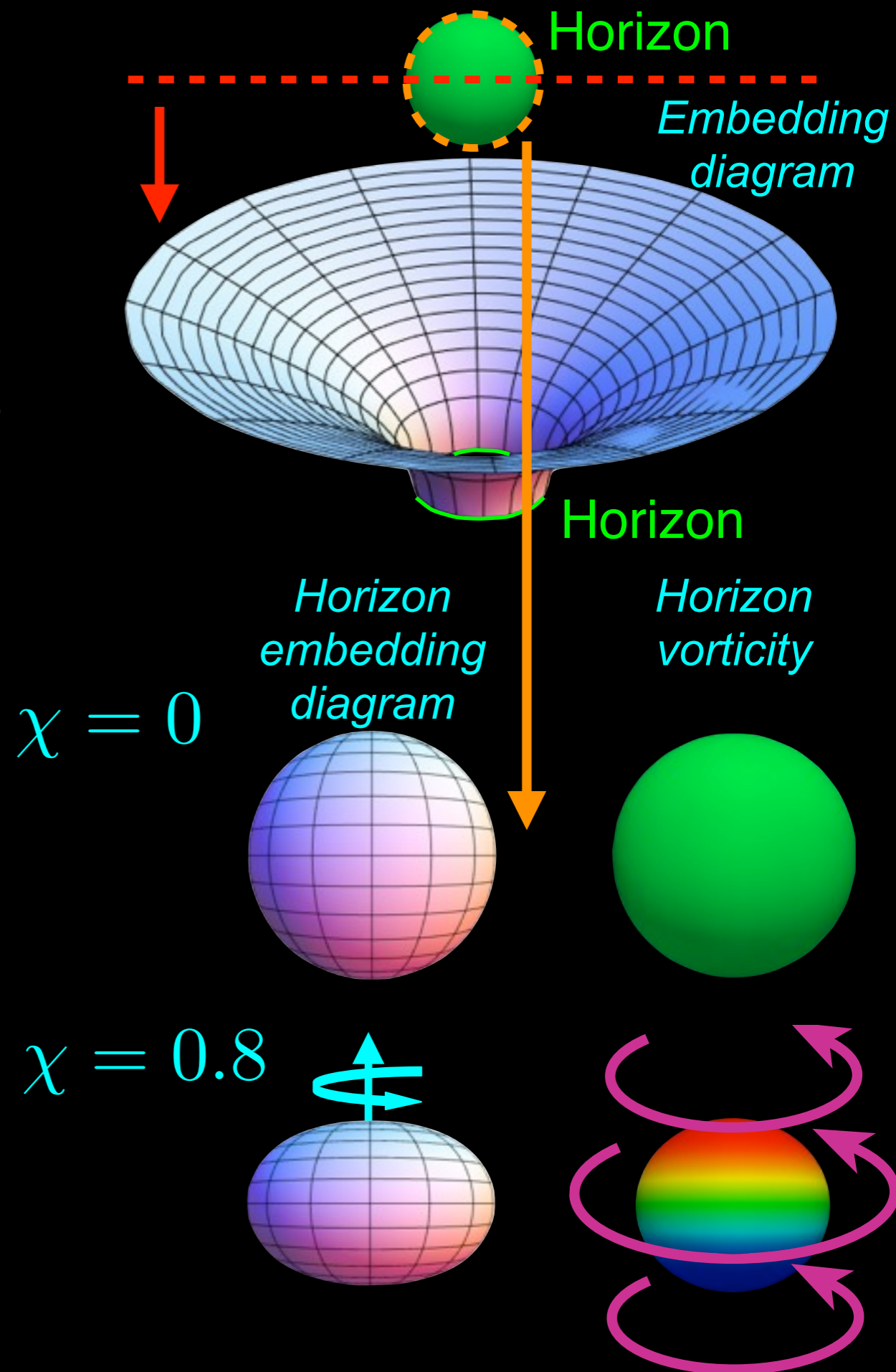
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- Frame dragging



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First candidate black hole discovered

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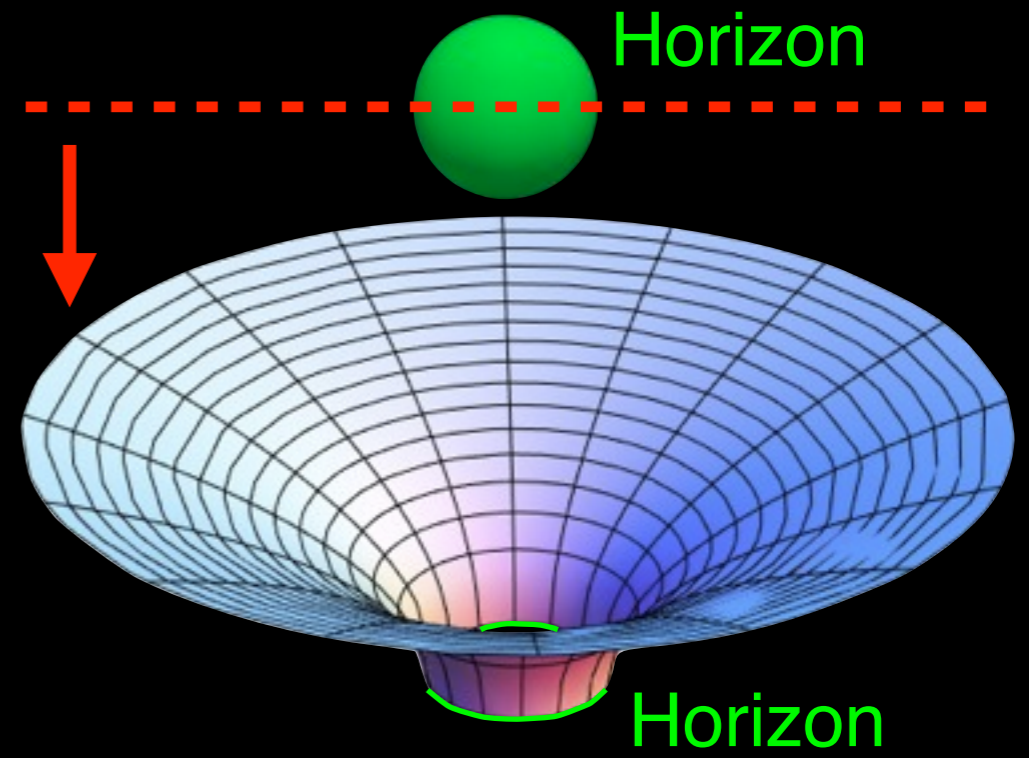
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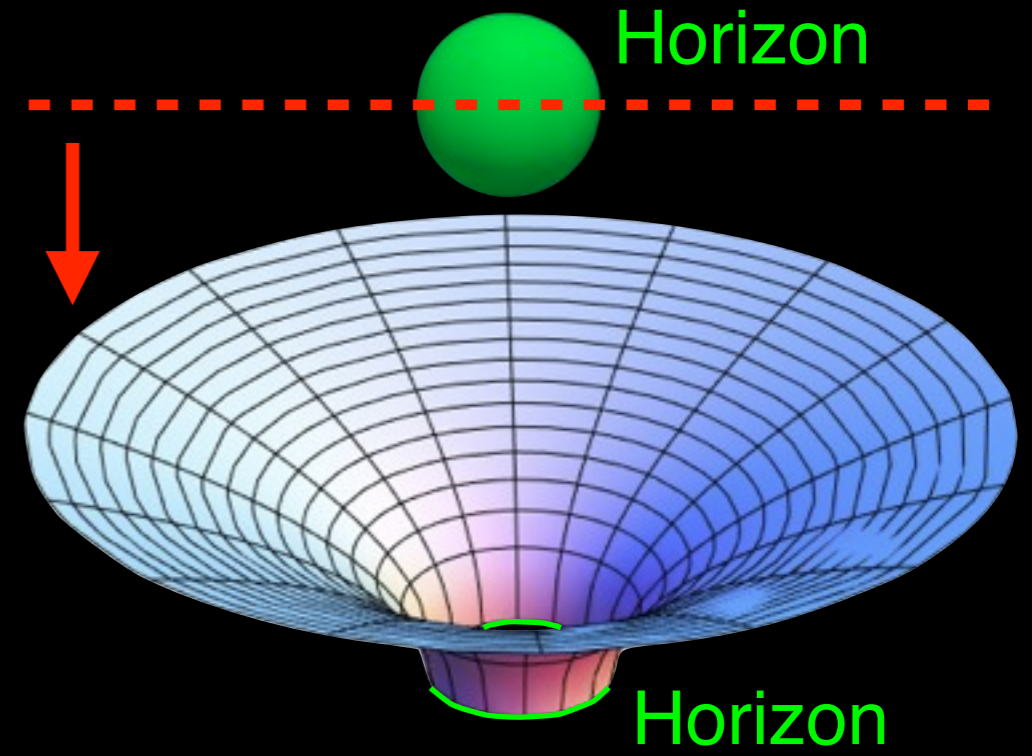
– Directly: *waves of gravity* from colliding black holes

# Black holes & grav. waves



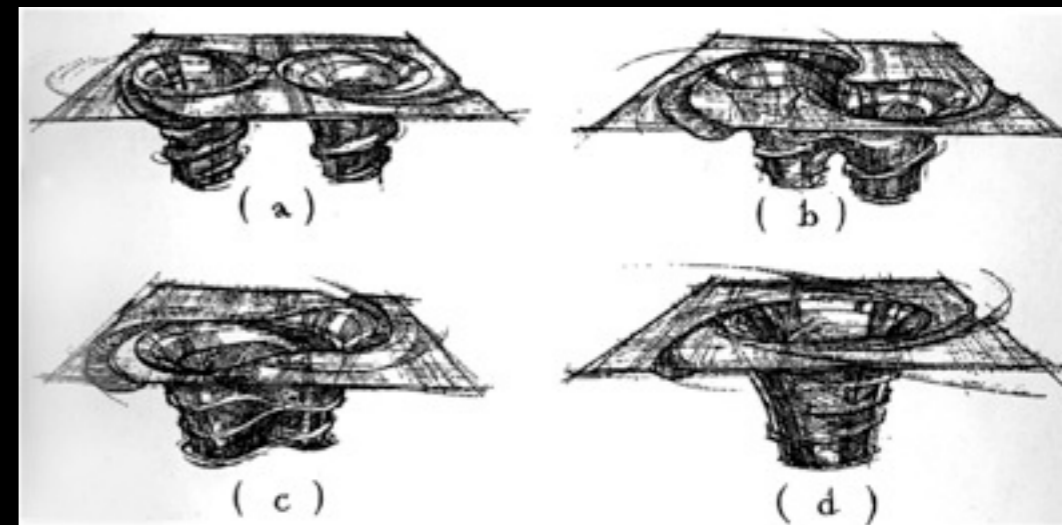
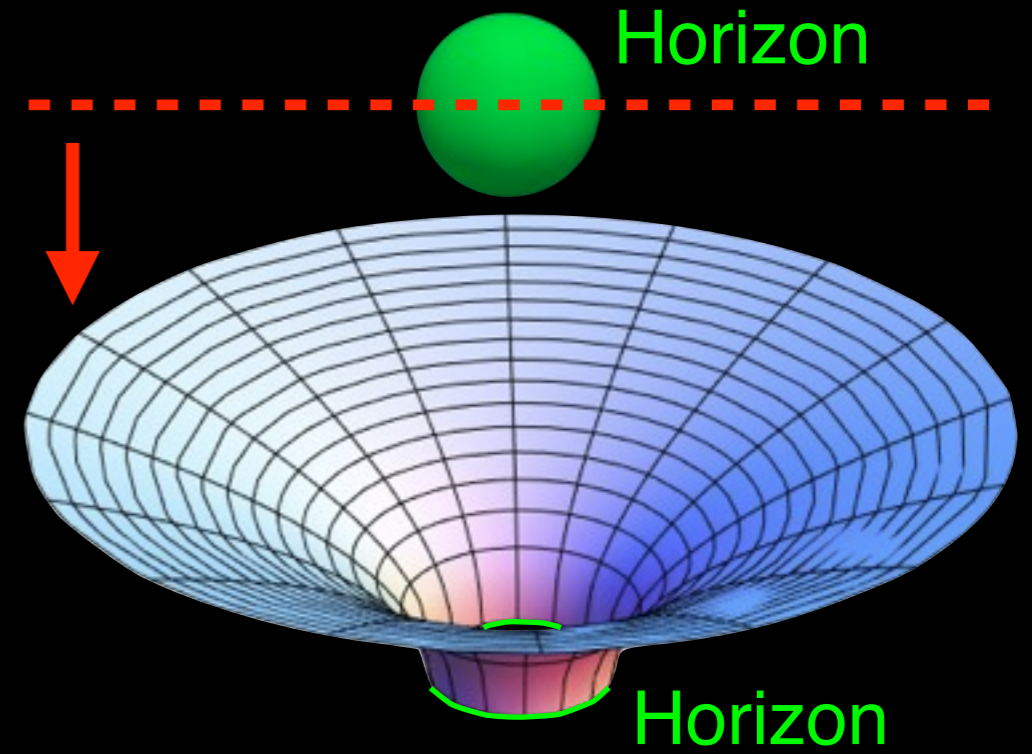
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- Coalescing black holes



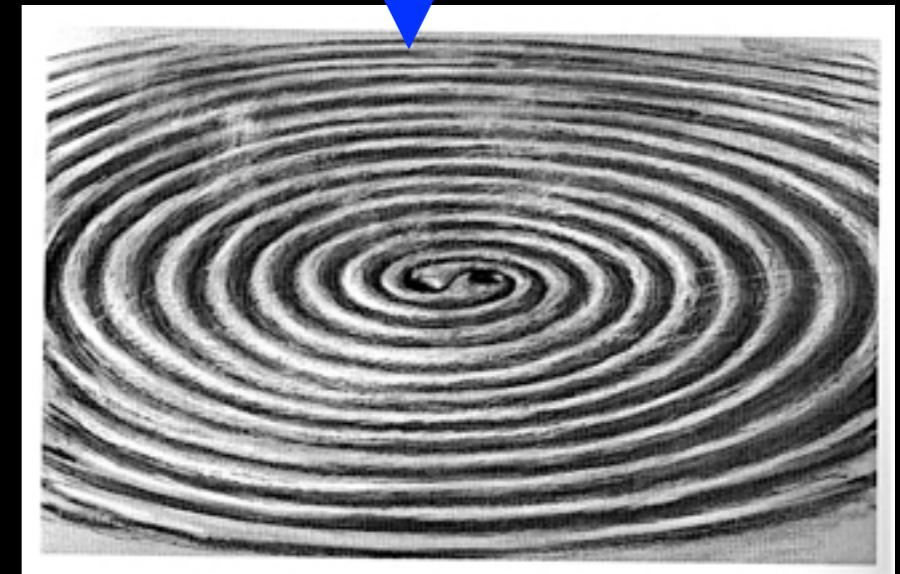
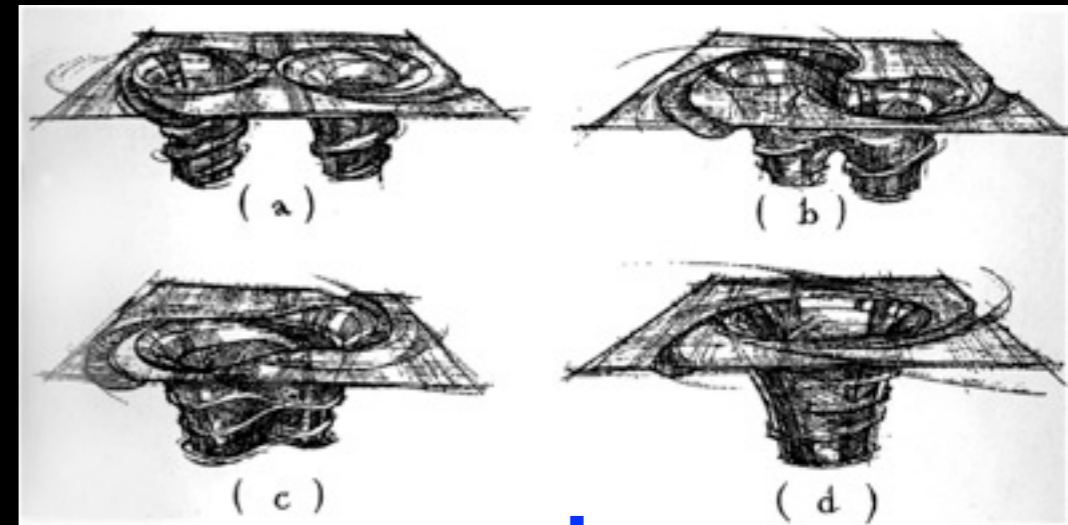
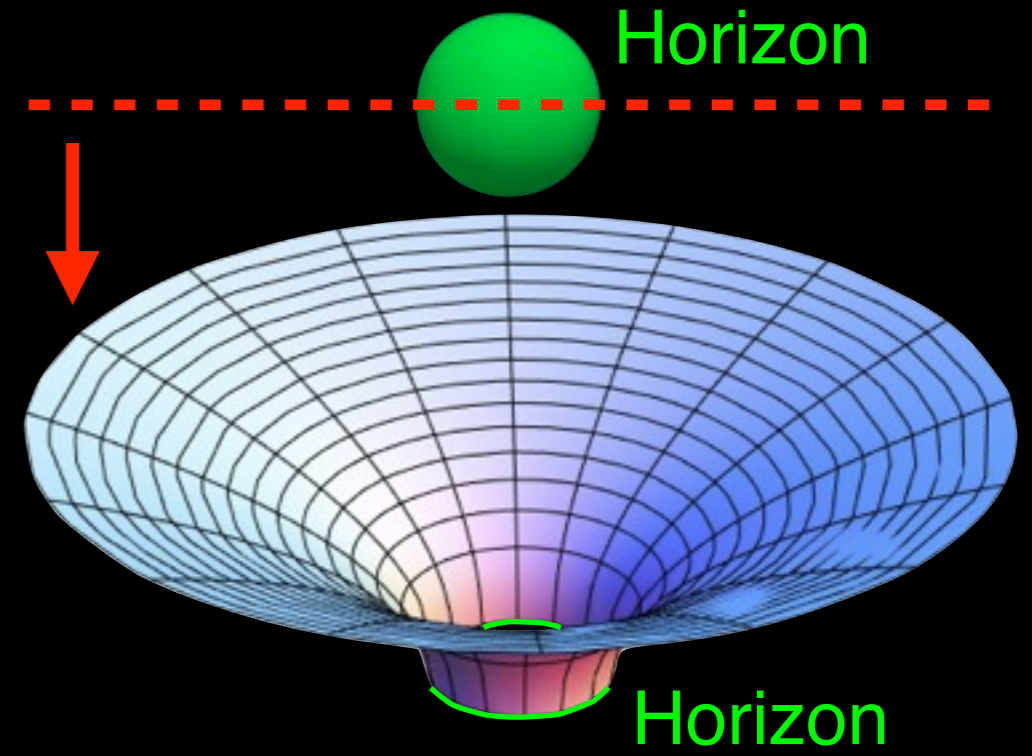
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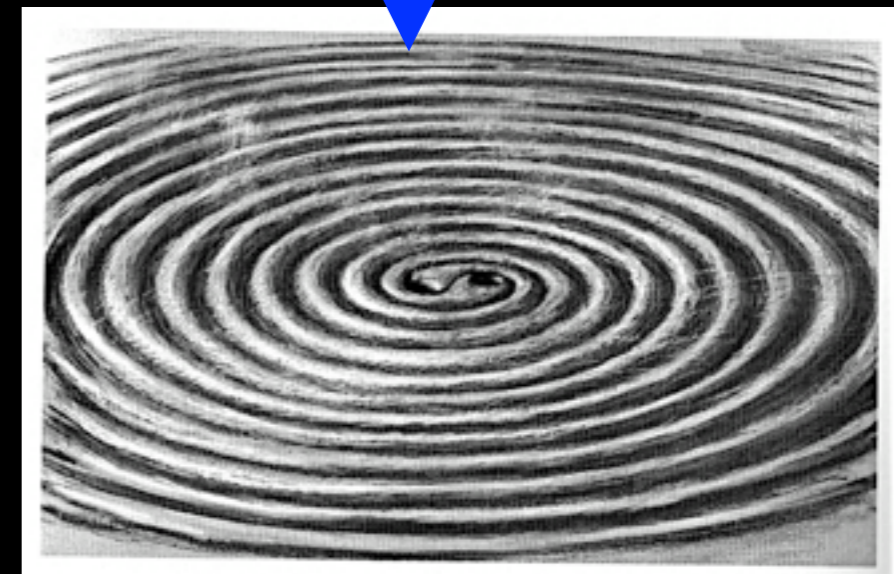
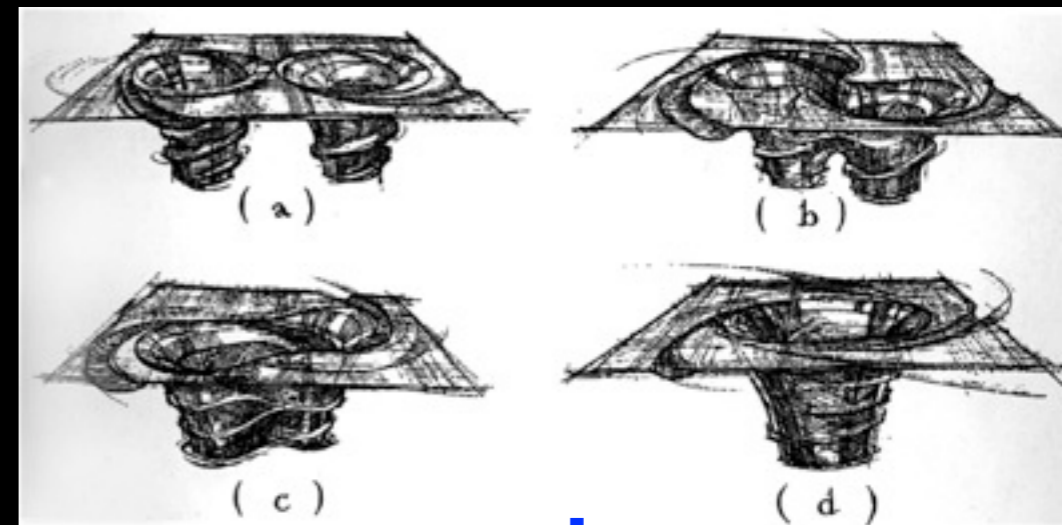
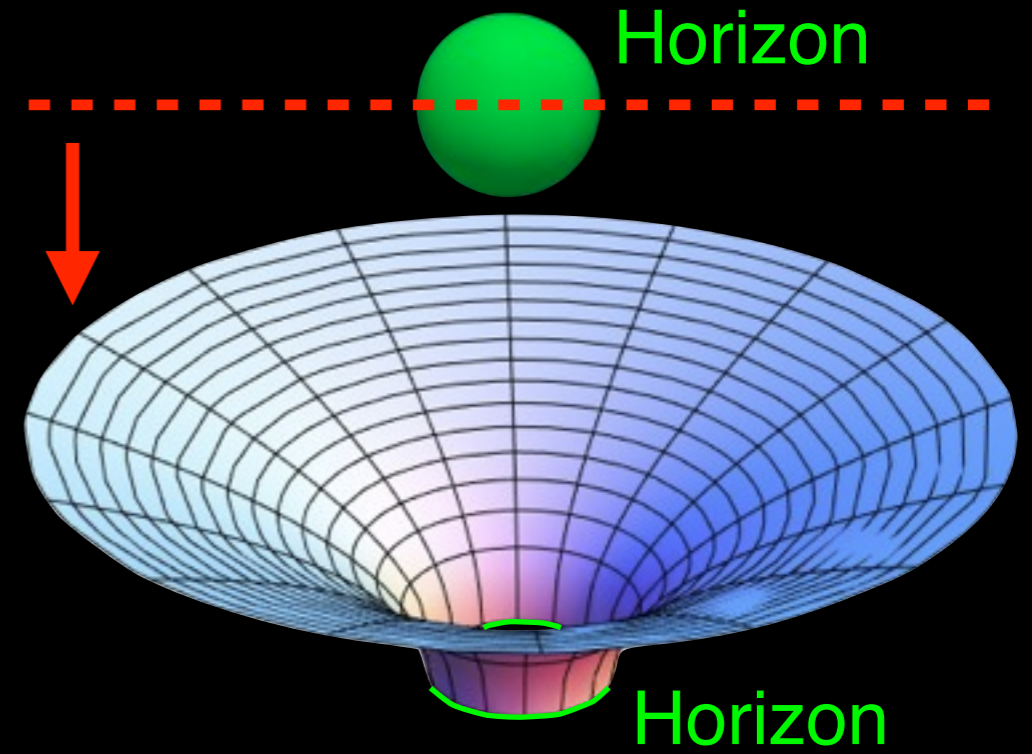
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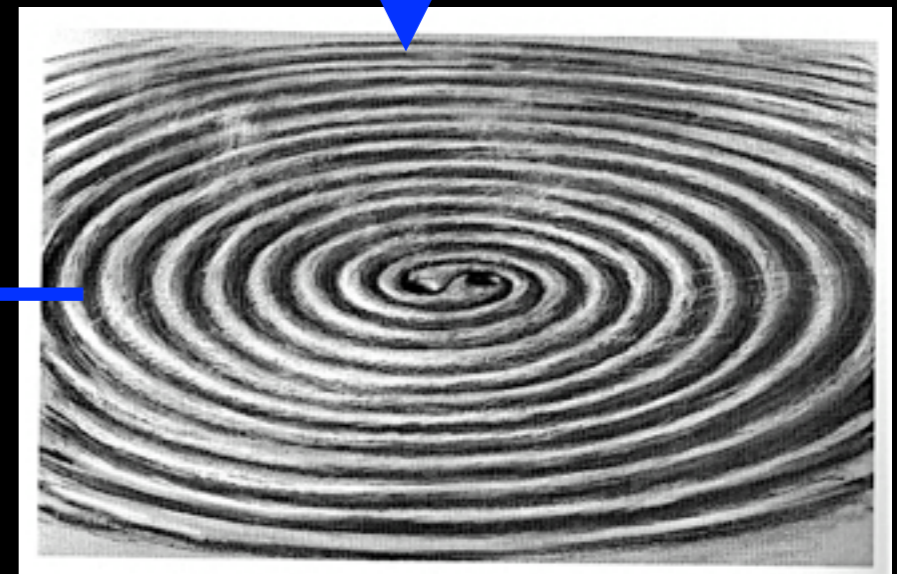
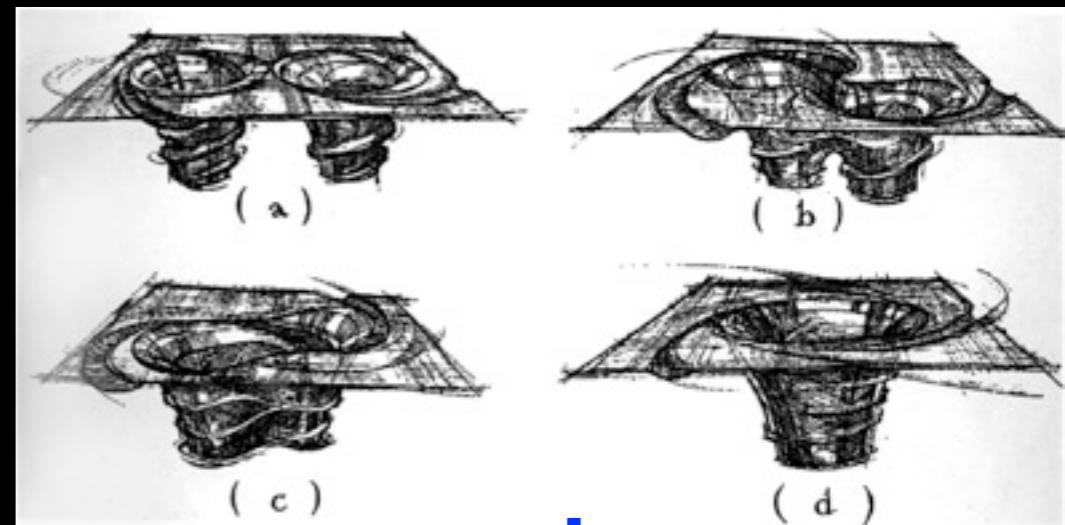
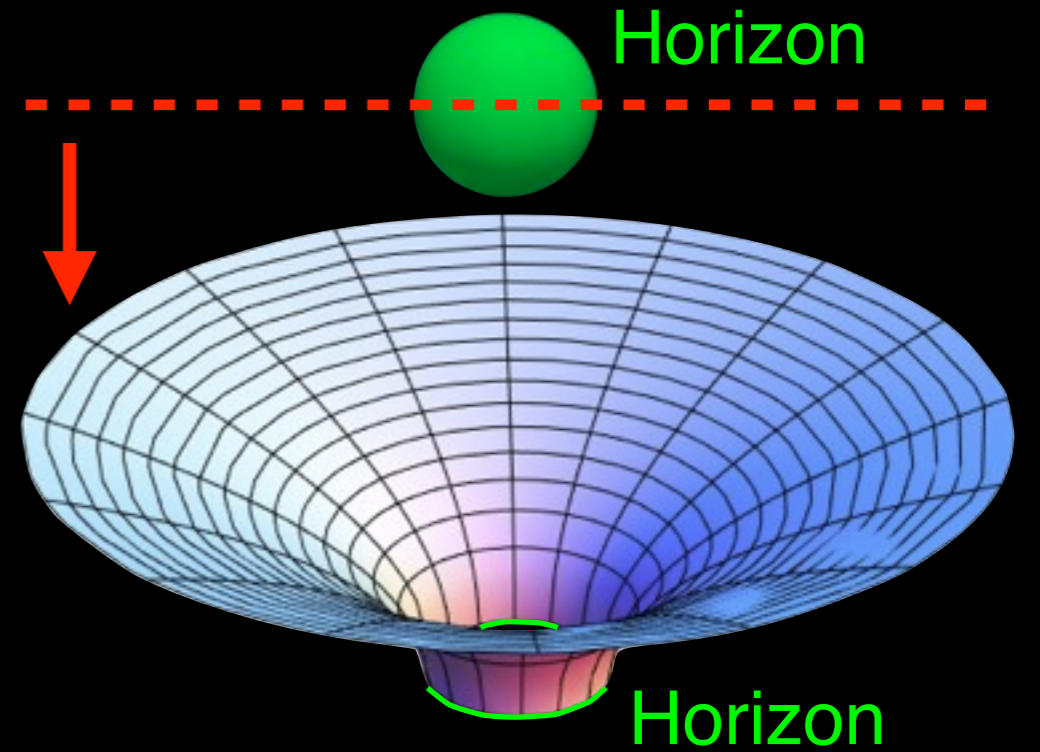
# Black holes & grav. waves

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    - Radiate  $\sim 10\%$  of mass (vs.  $\sim 0.5\%$  nuclear fusion) as *gravitational waves (GW)*



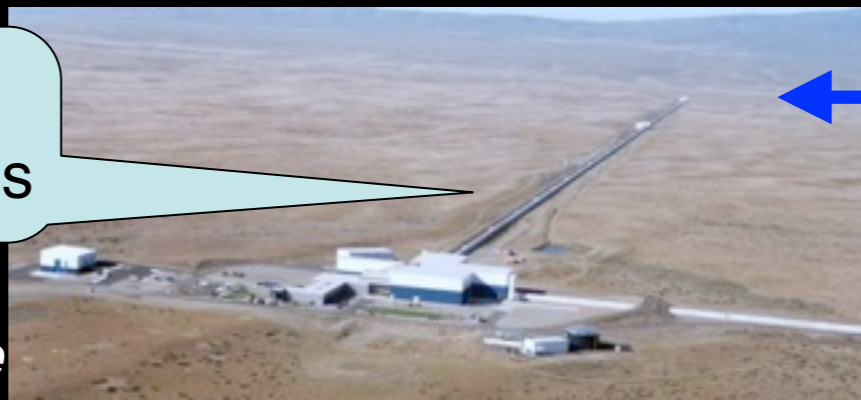
# Black holes & grav. waves

- Coalescing black holes
  - Most violent events in universe
    - Radiate  $\sim 10\%$  of mass (vs.  $\sim 0.5\%$  nuclear fusion) as *gravitational waves (GW)*
  - Waves weak at earth
    - Need accurate GW predictions
      - To find GW in noisy detector data
      - To estimate source properties



GW vary 4km arm length by  $10^{-18}$  m = 0.1% atomic nucleus

Images courtesy  
LIGO, Kip Thorne



# Colliding black holes

# Colliding black holes

- Warped spacetime
  - Obeys general relativity (GR)

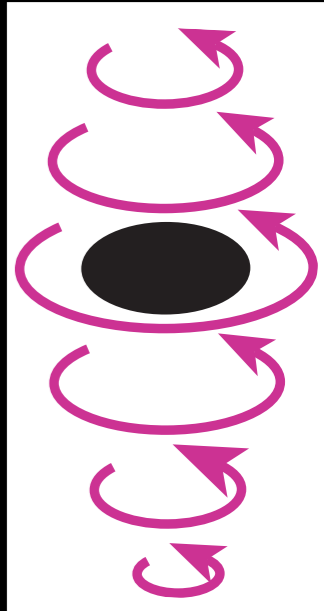
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  - History: black hole solutions
    - 1915: GR, Spherical black hole
    - 1963: Rotating black hole

Single black hole

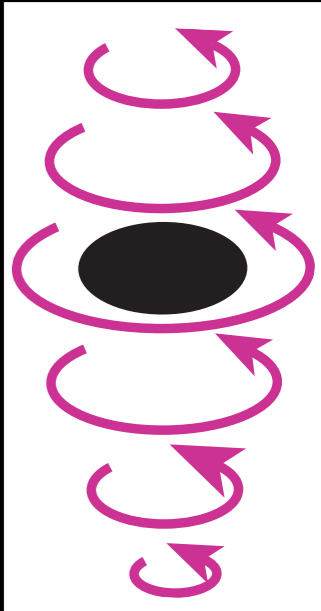


*Images courtesy Kip Thorne*

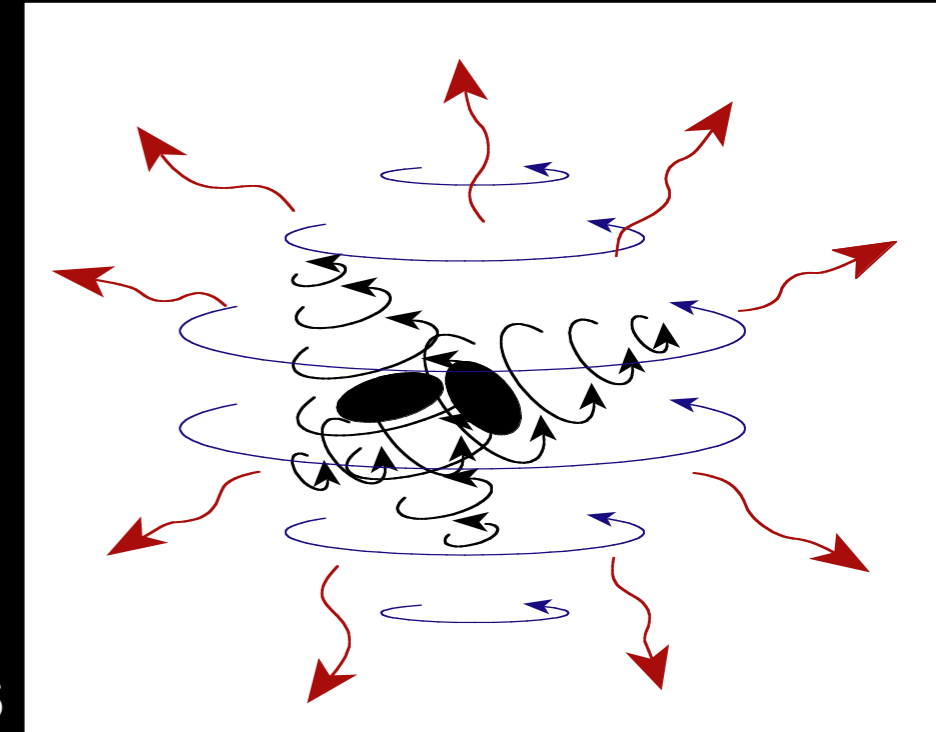
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    - 2005: First inspiral, collision of 2 black holes  
(only possible via computer simulation)  
Pretorius, PRL **95**,  
121101 (2005)

Single black hole



Images courtesy Kip Thorne



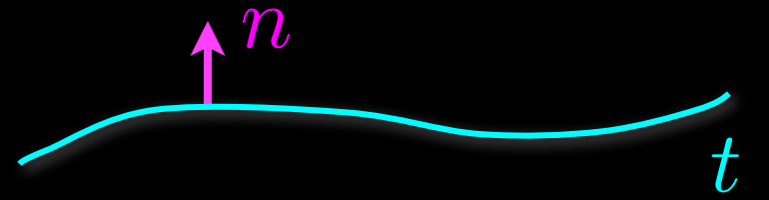
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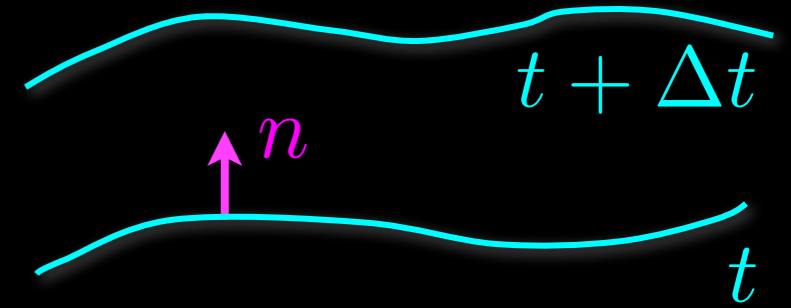
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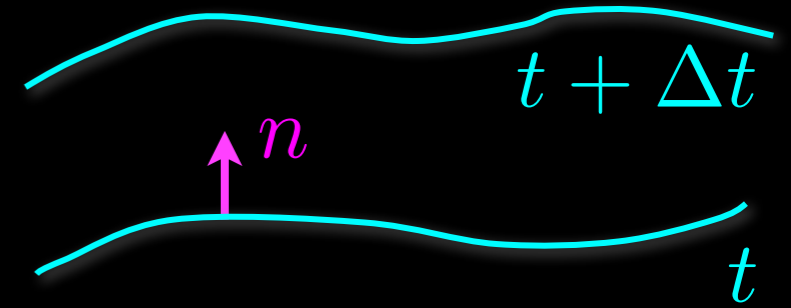


# Simulating colliding black holes



# Simulating colliding black holes

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- Initial data
  - Must satisfy constraints



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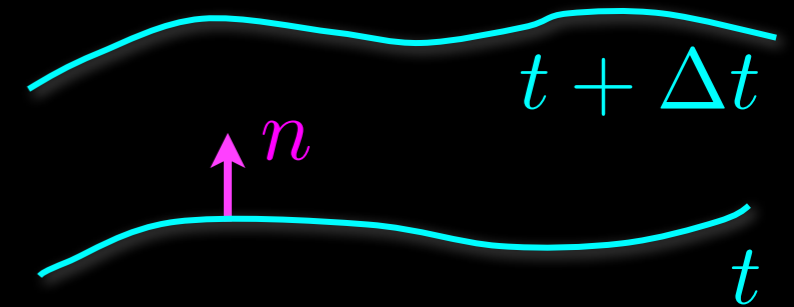
- Solve constraint equations
- Get desired physics (holes about to merge)

- Evolution

- Step from current time to next time

- Challenges

- Constraints must stay satisfied
- Coordinates must be well behaved
- Remove singularities
- Boundary conditions
- Computational cost



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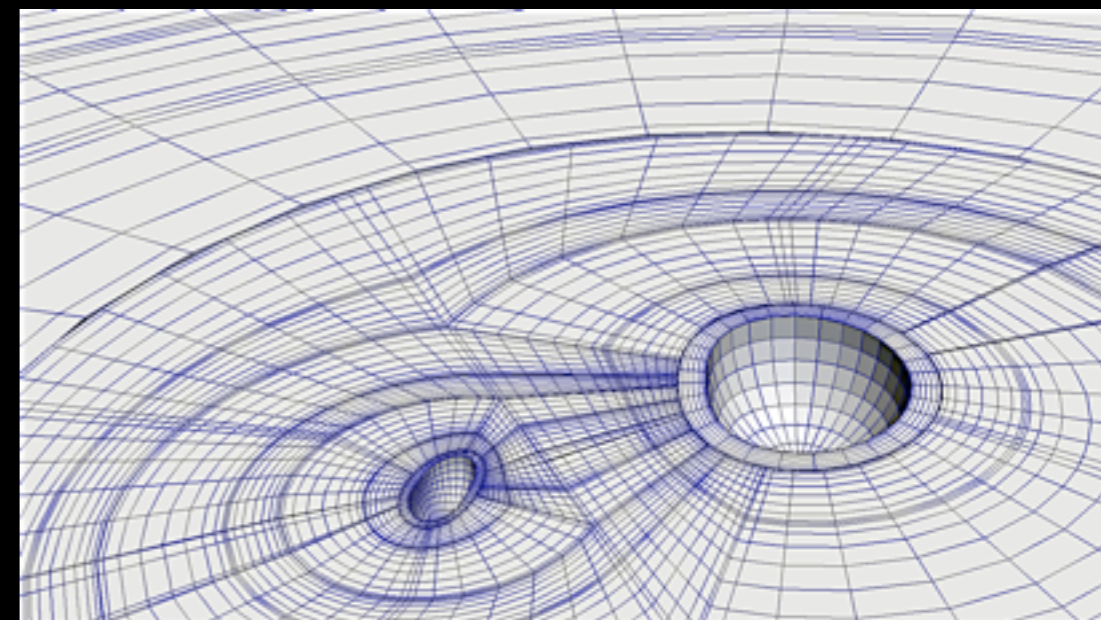
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Evolution	BSSNOK evolution eqs. High-order finite-difference Moving puncture	Generalized harmonic evolution eqs. Pseudospectral Excision
Codes	BAM, Hahndol, LazEv, Lean, Llama, MayaKranc, UIUC	SpEC

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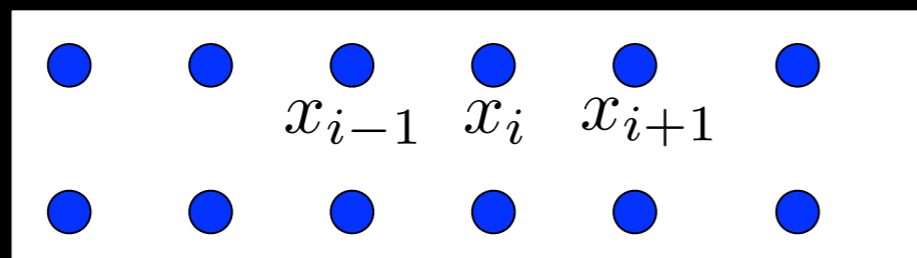
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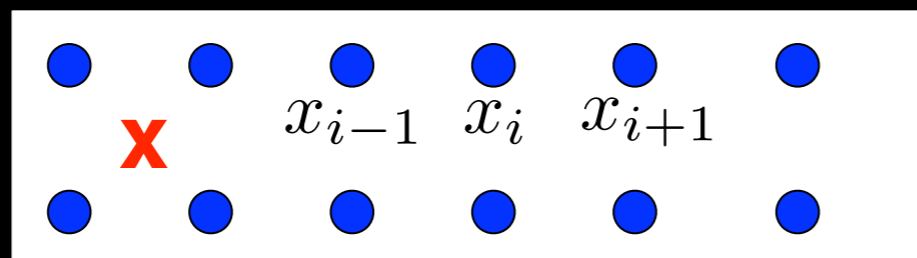
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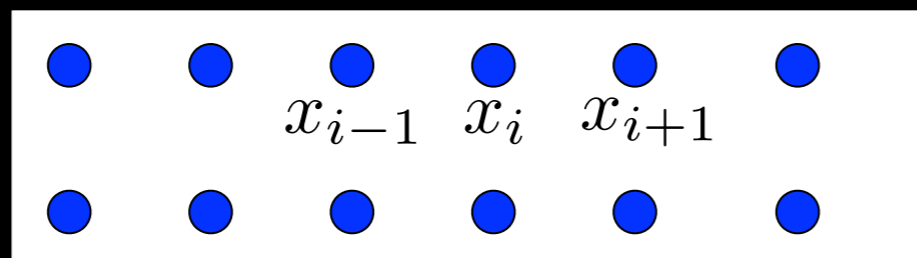
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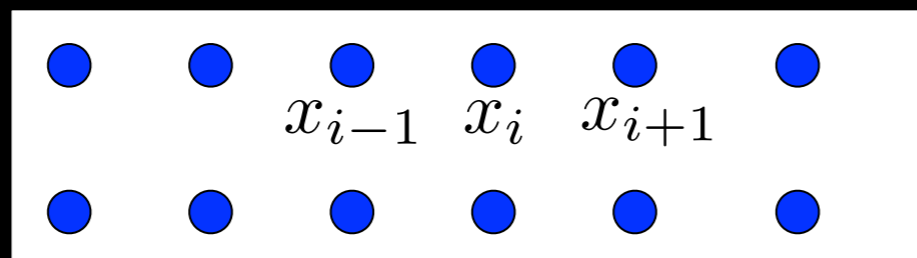
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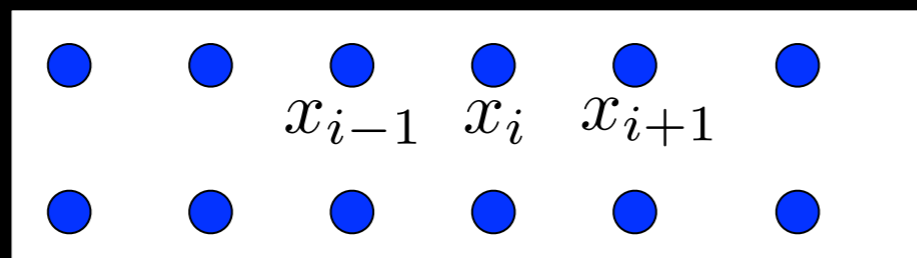
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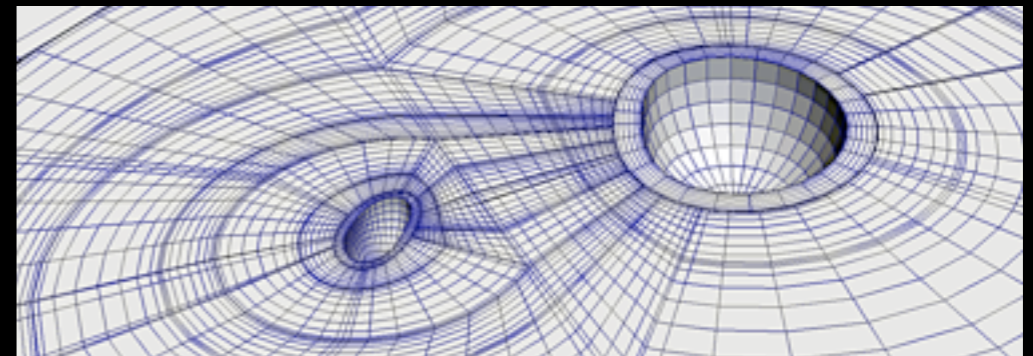
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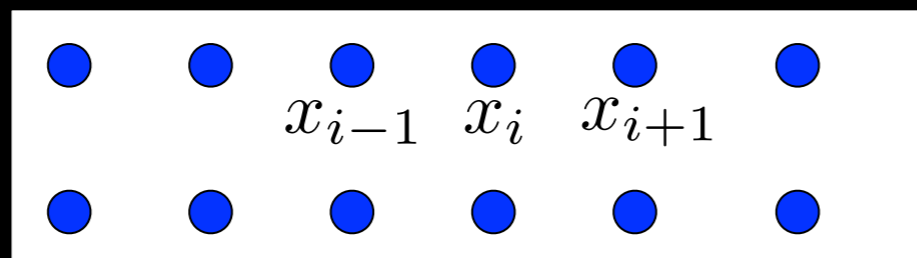
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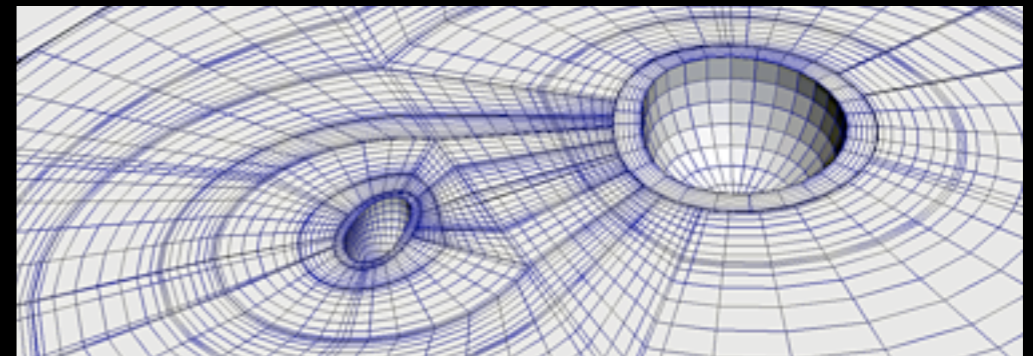
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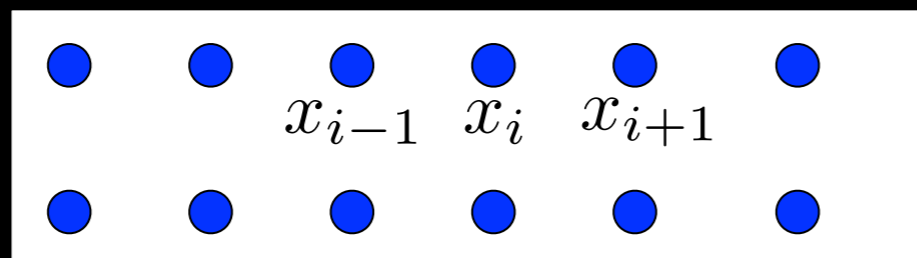
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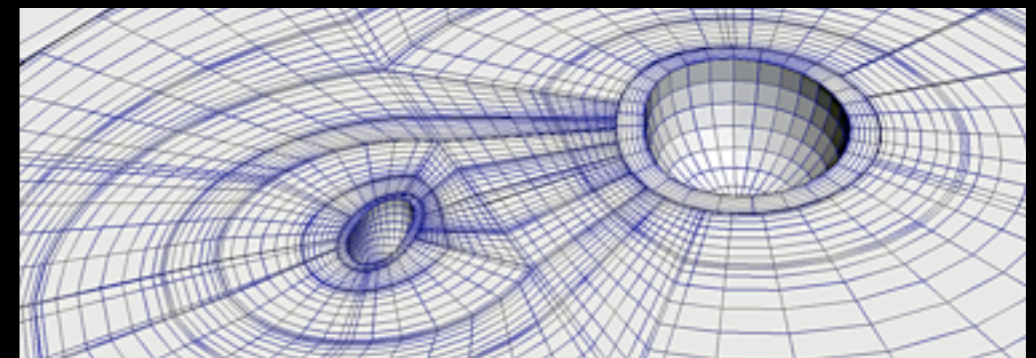
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**Advantages** Robust, open-source frameworks, many more simulations so far

High accuracy at lower cost, longer simulations, very high black-hole spins

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# Some results

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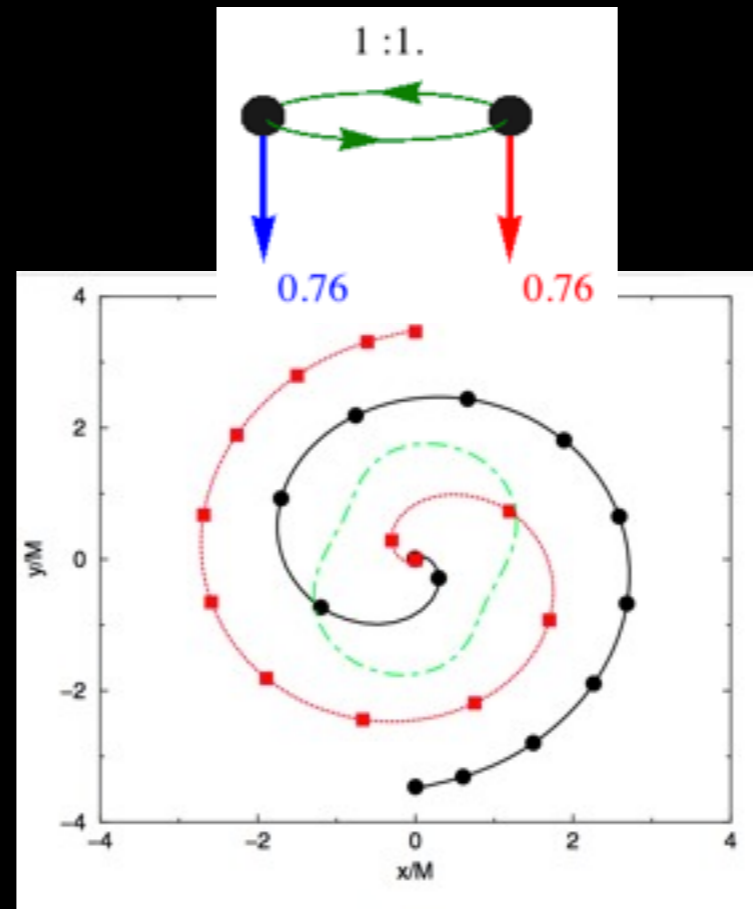
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# Some results

- Orbital hangup
  - First simulation of BBH merger with spin  
Campanelli, Lousto,  
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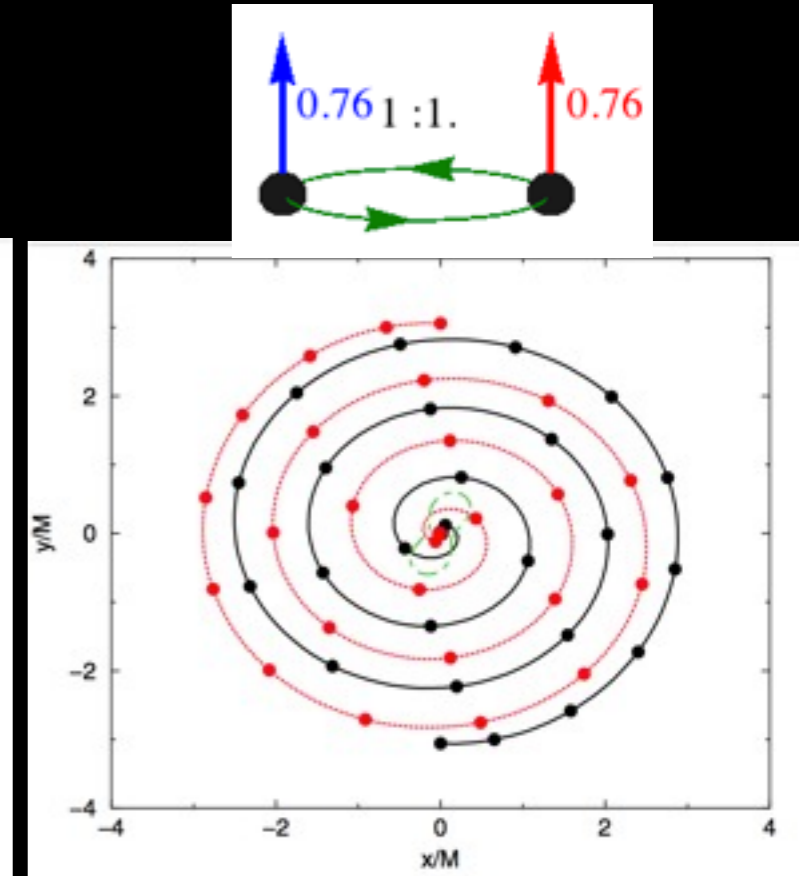
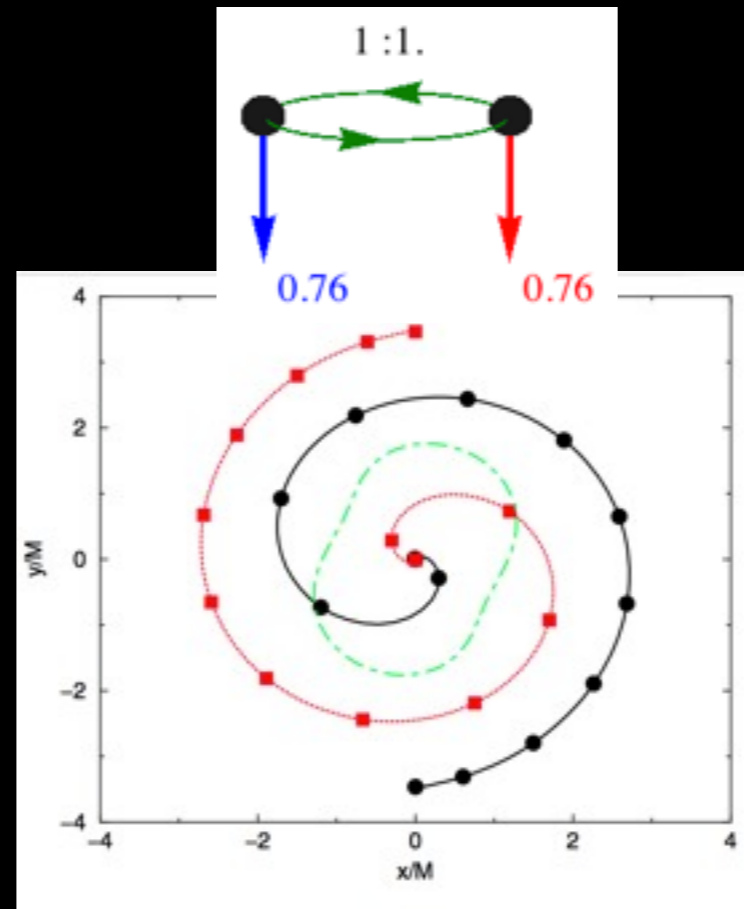
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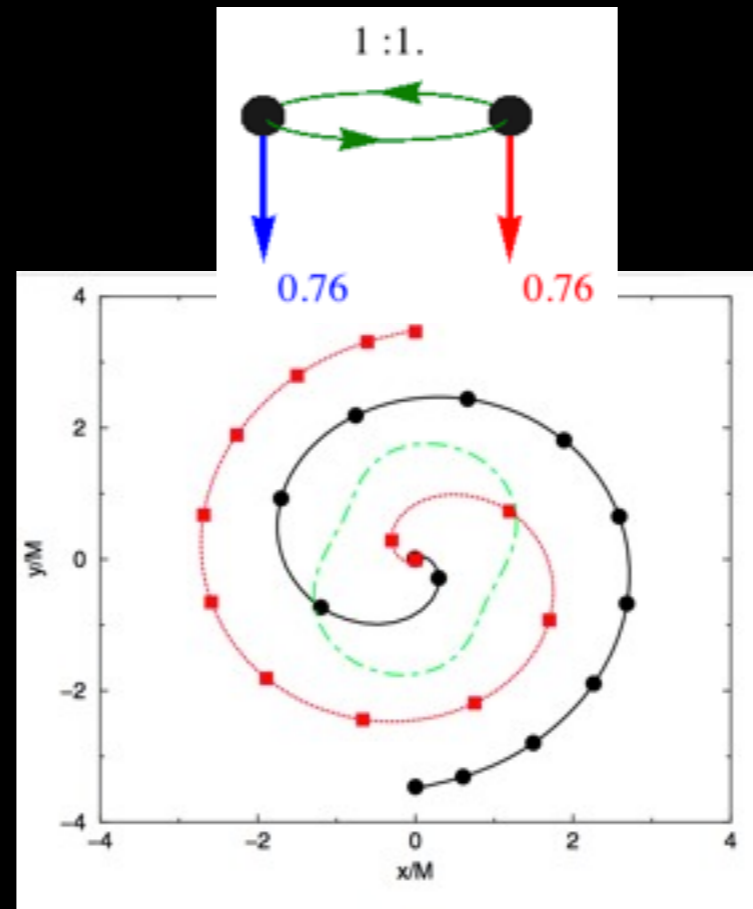
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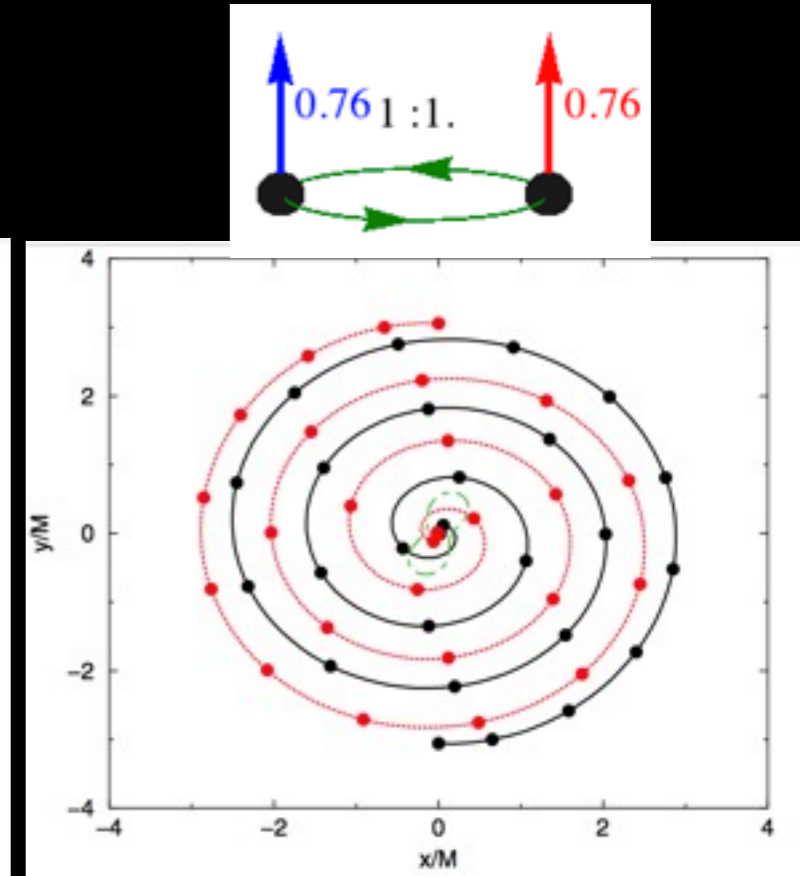


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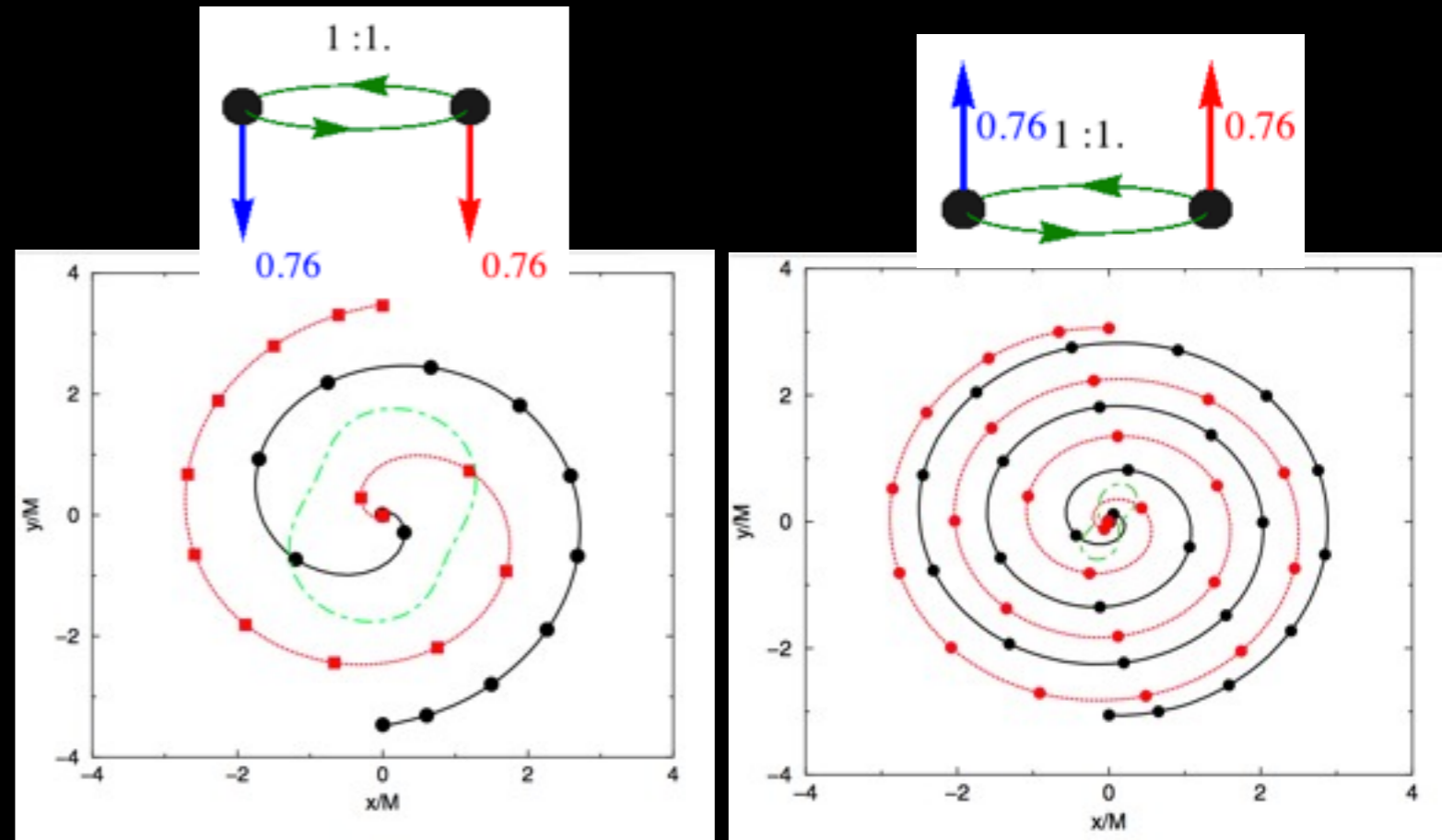
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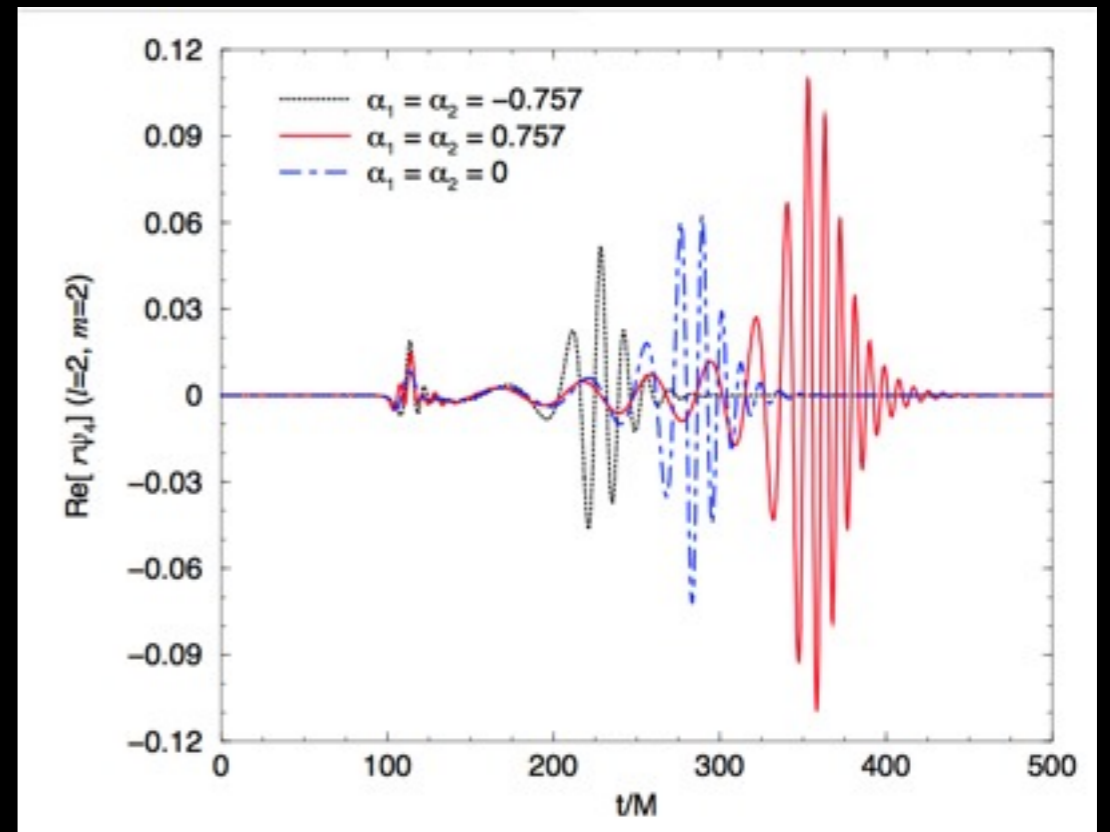


Figure by Campanelli appearing in  
Centrella et al,  
RMP 82, 3069 (2010).



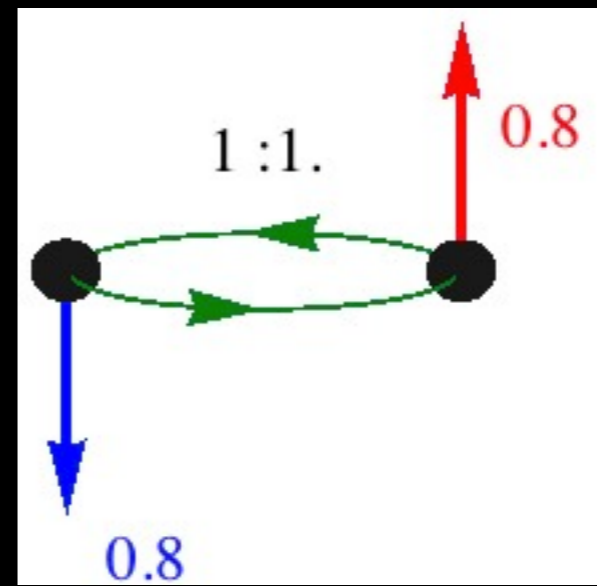
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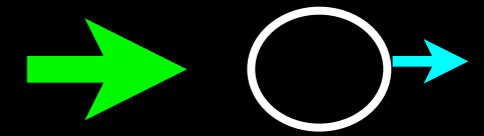
- Recoil

# Some results

- Recoil
  - First spinning recoil



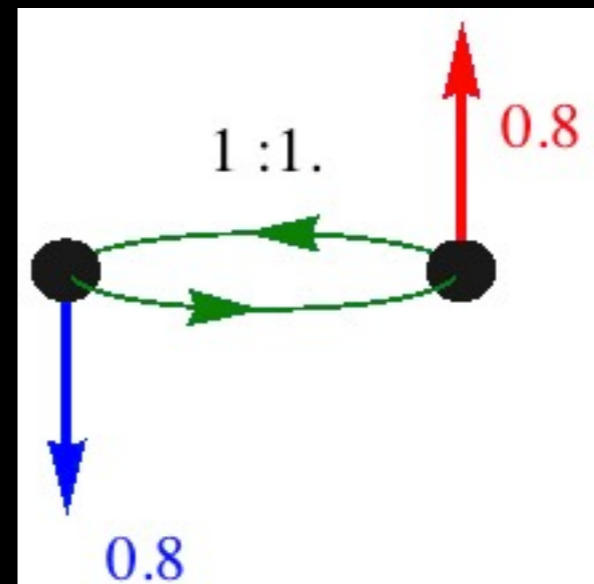
$$v \approx 400 \text{ km/s}$$



Herrmann et al, APJ 661, 430 (2007)

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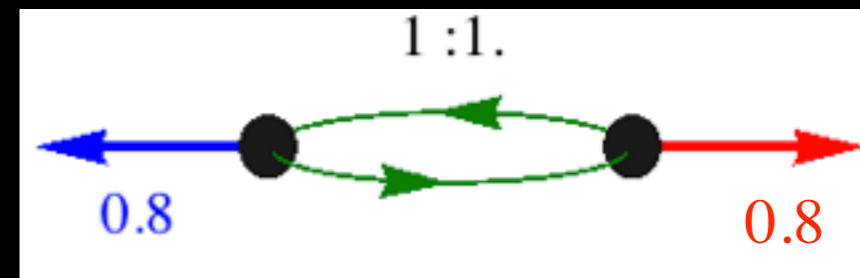
- Recoil
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  - Superkick
    - Maximize over orbital phase
    - Can exceed galactic escape velocity ( $\sim 2000$  km/s)



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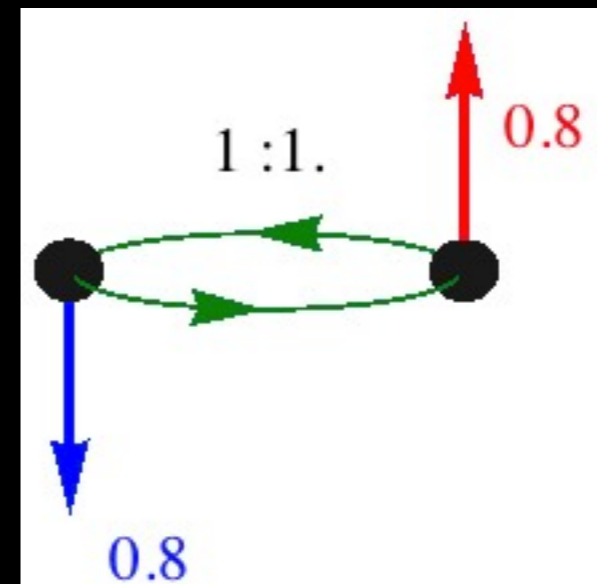
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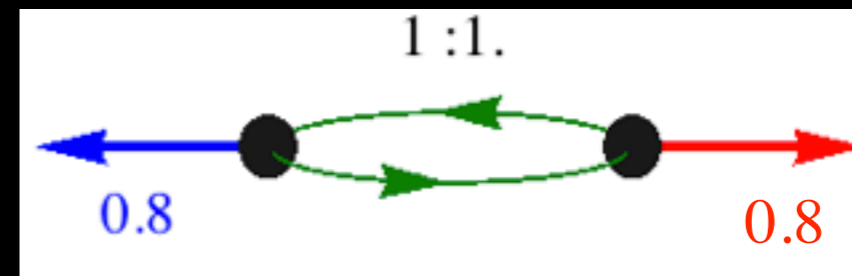
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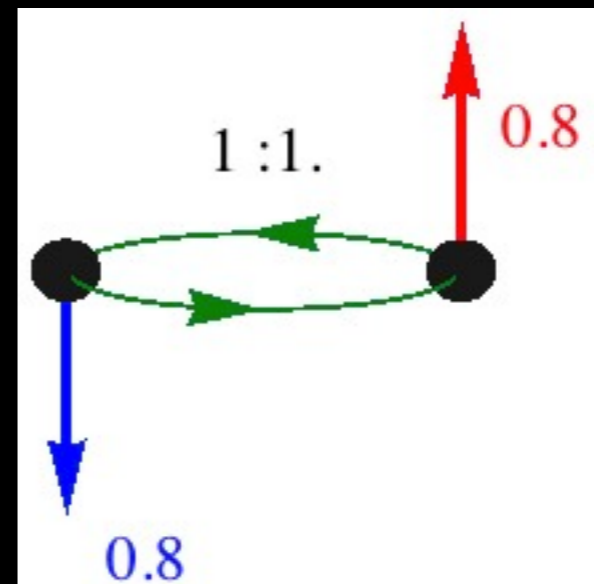
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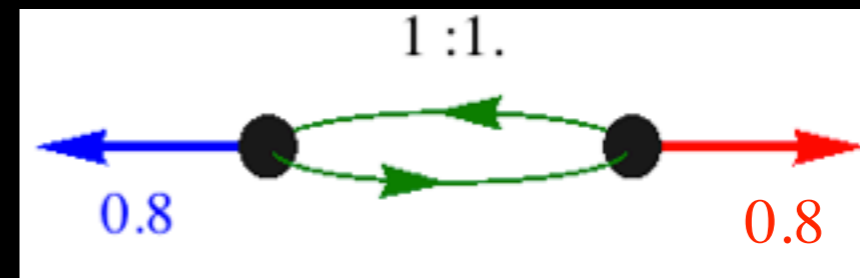
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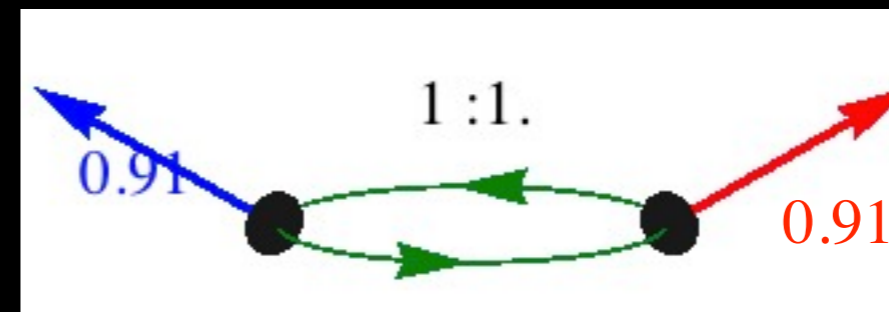
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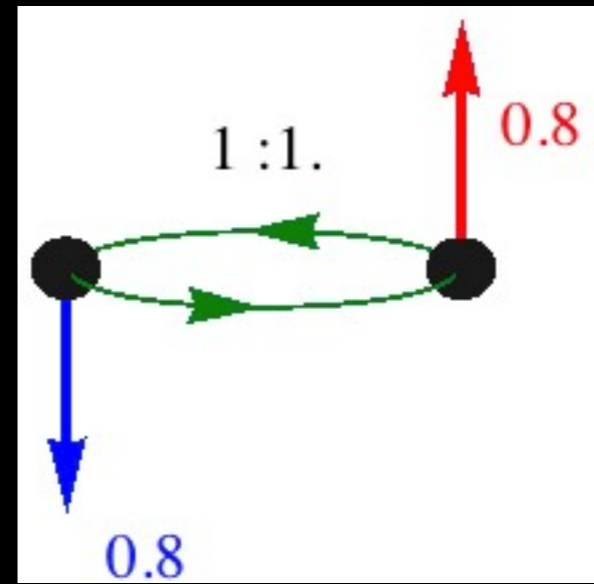


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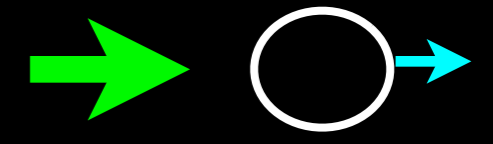
Lousto & Zlochower,  
PRL 107, 231102 (2011)

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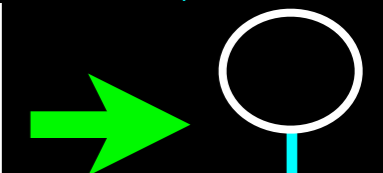
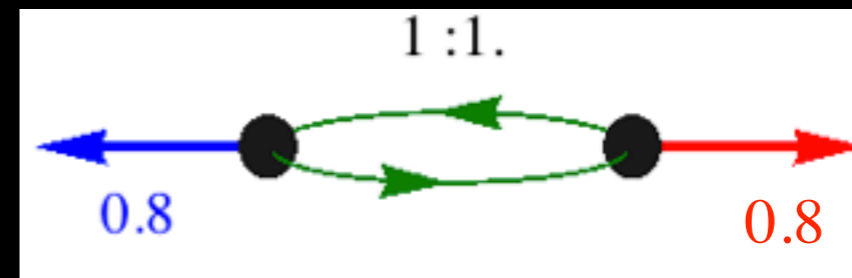


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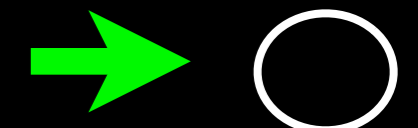
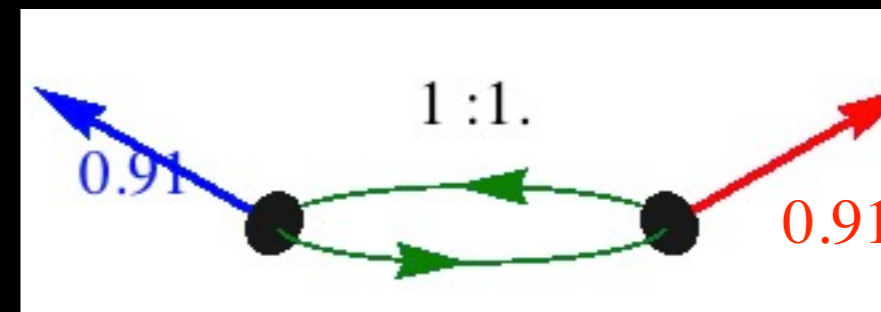


Herrmann et al, APJ **661**, 430 (2007)

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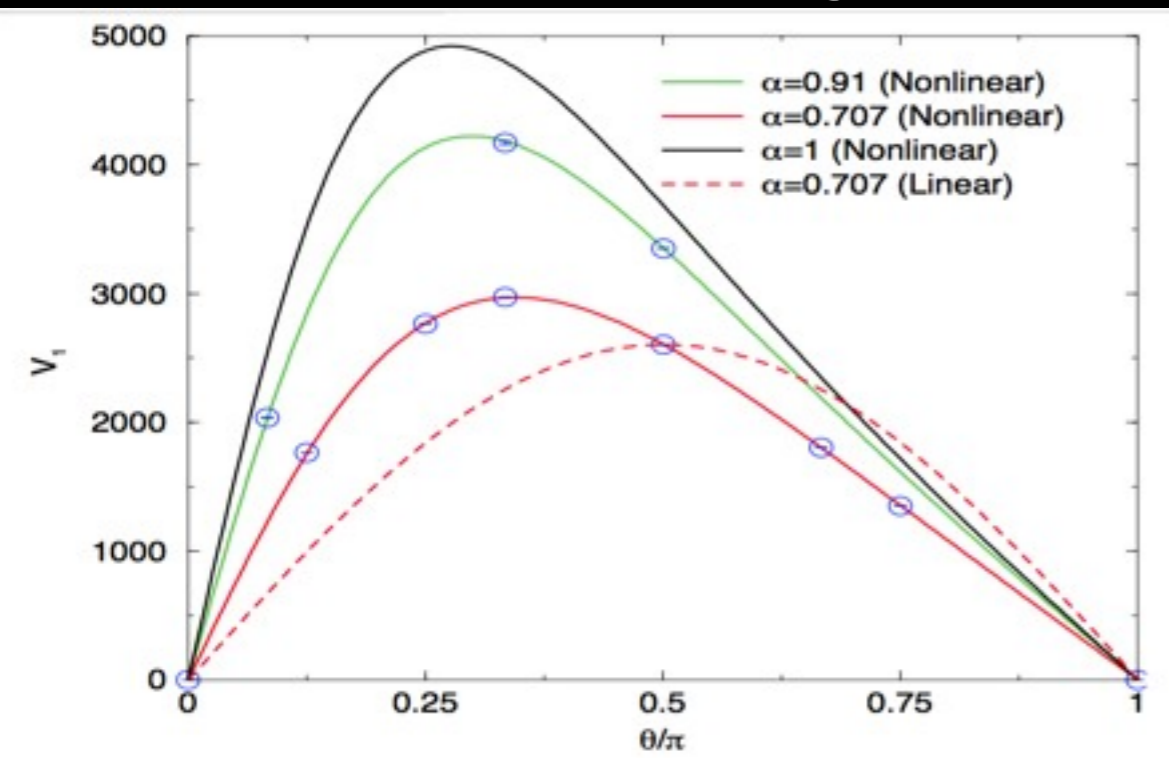


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# BH-BH mergers with nearly extremal spin



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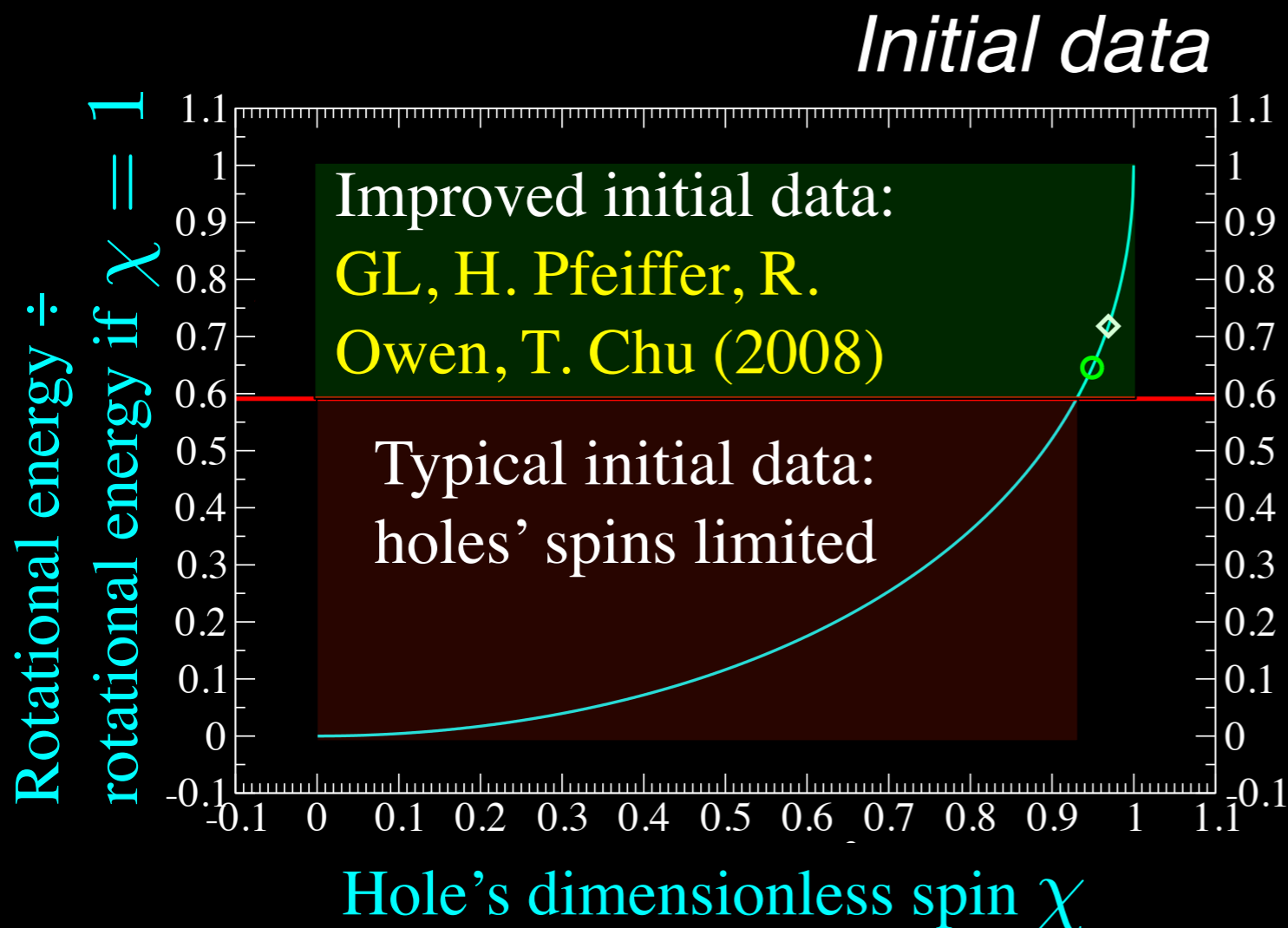
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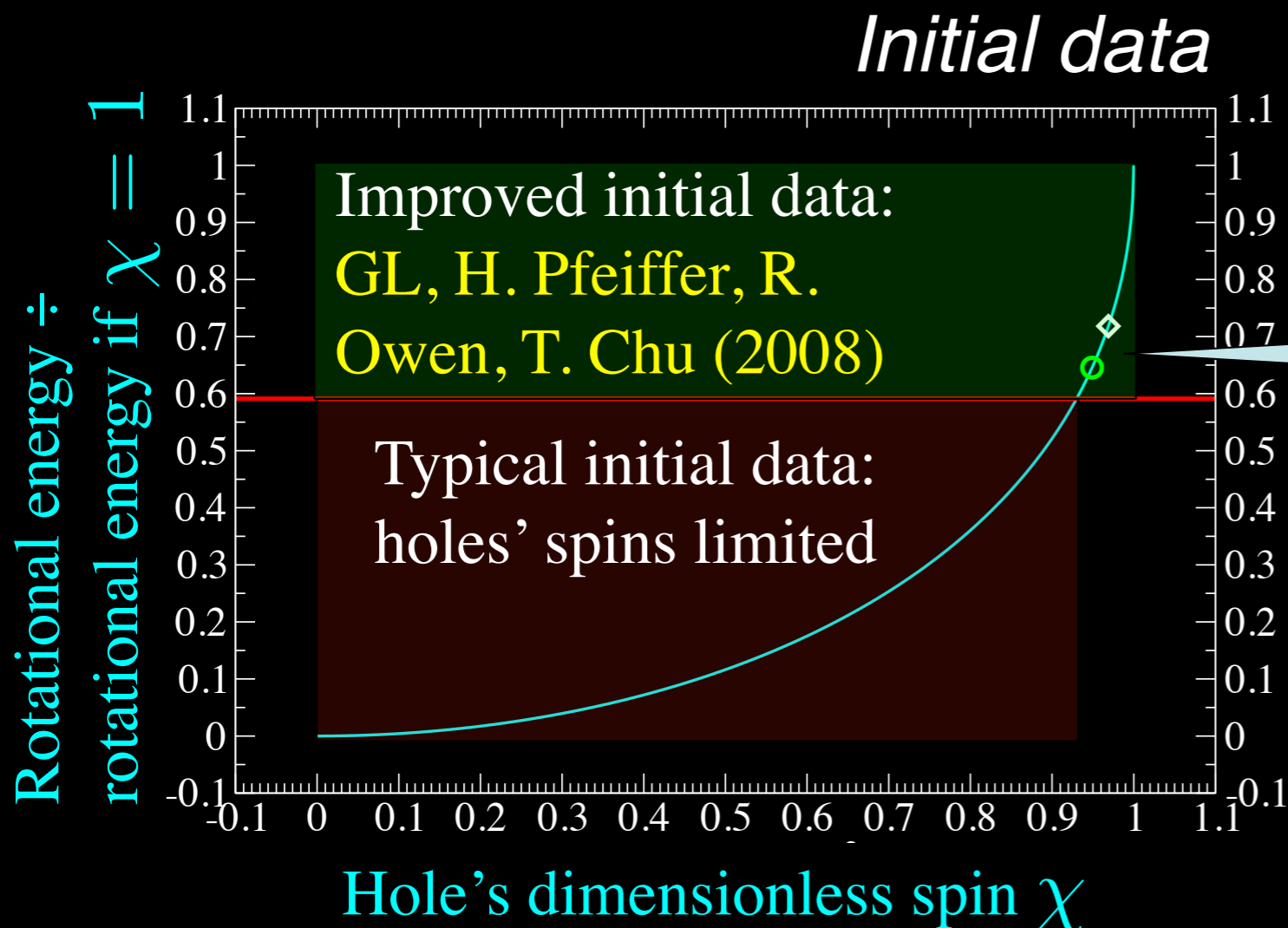


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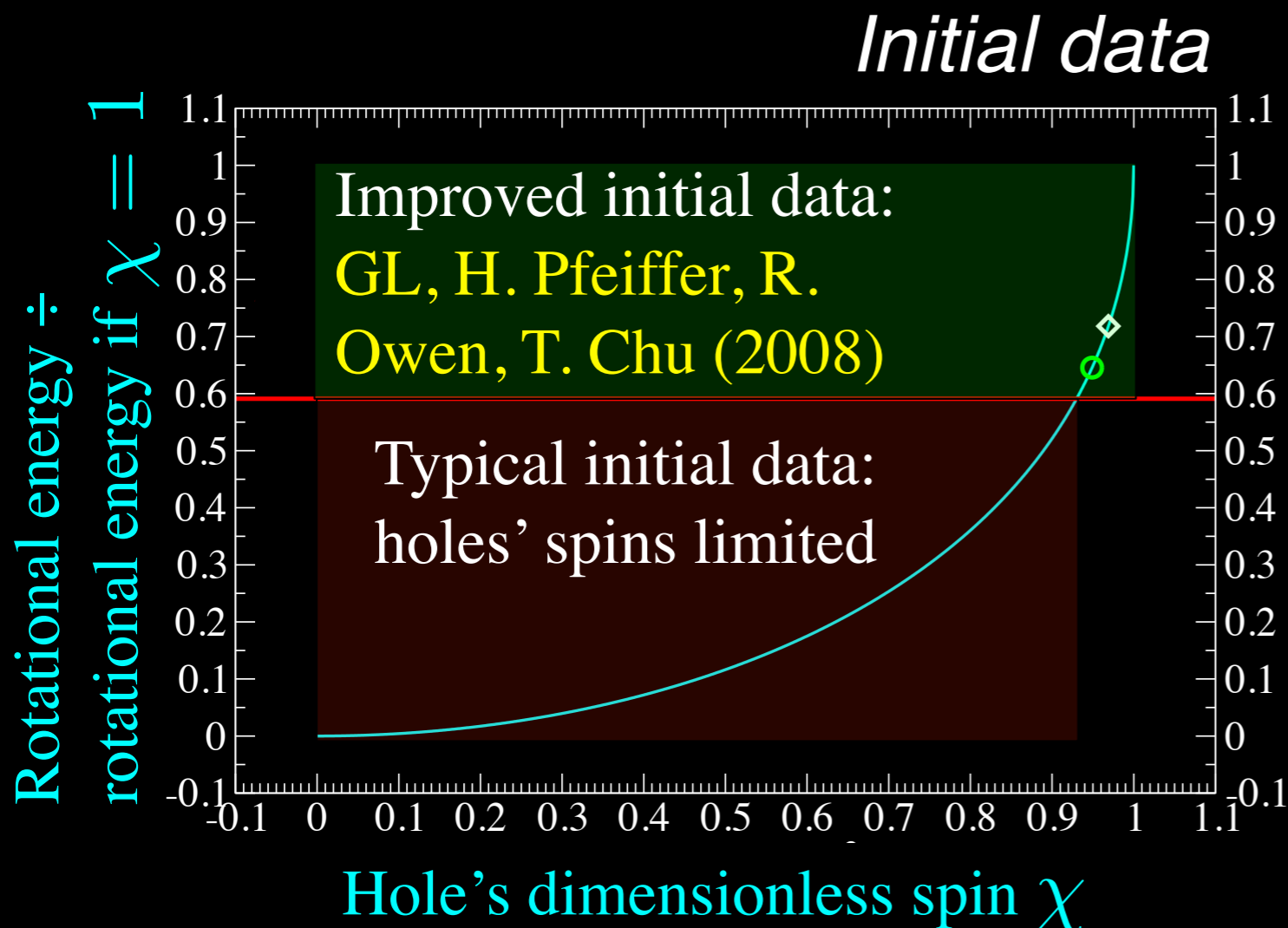
Highest black-hole spins simulated to date (only to exceed the “typical” (“Bowen-York”) initial-data limit

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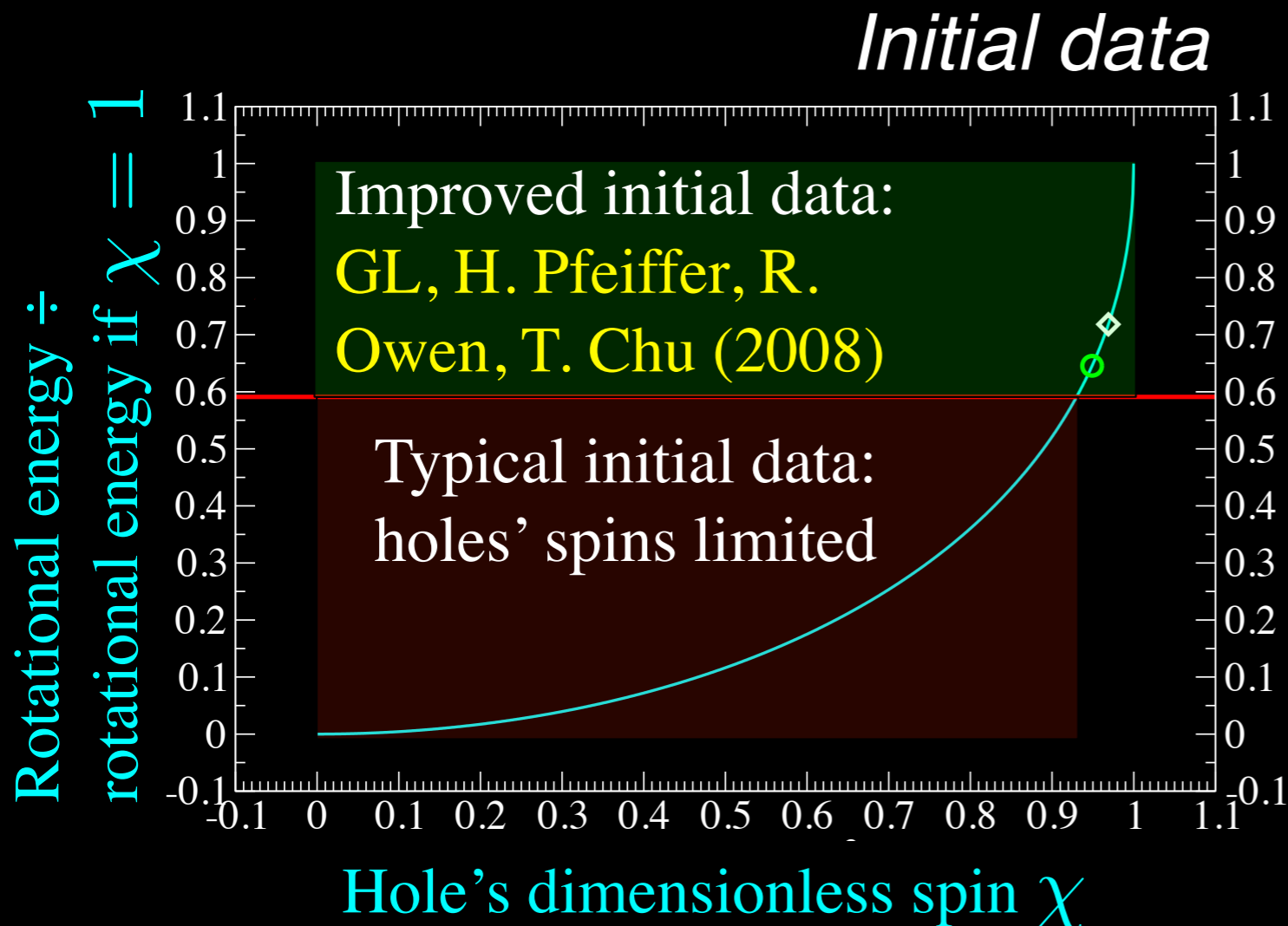
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## *Evolution*

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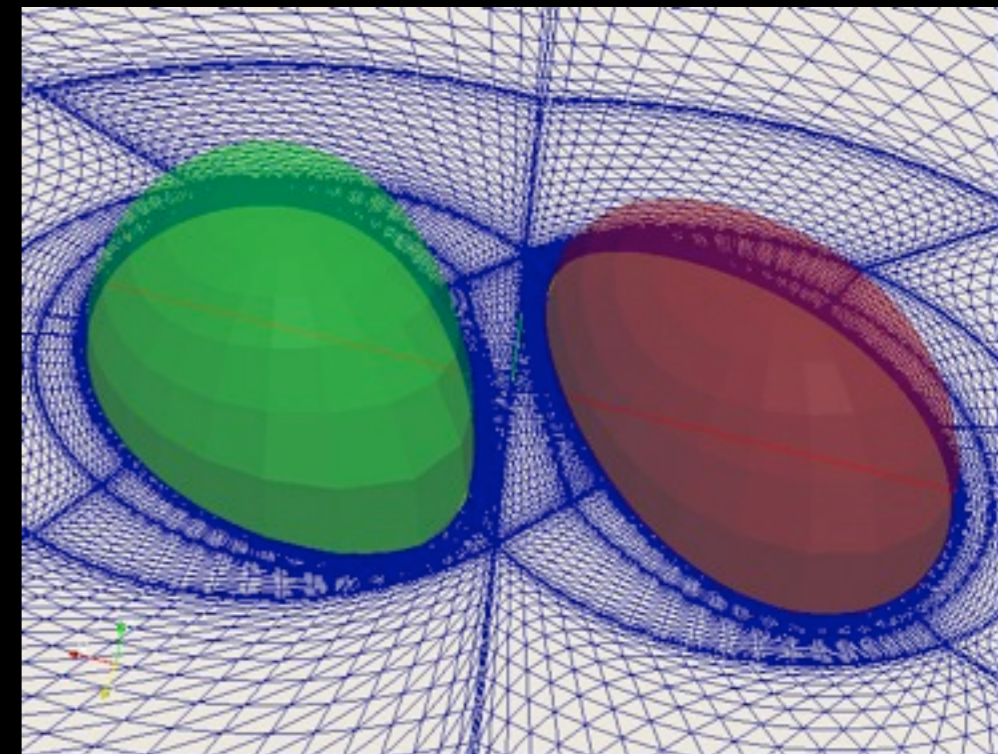
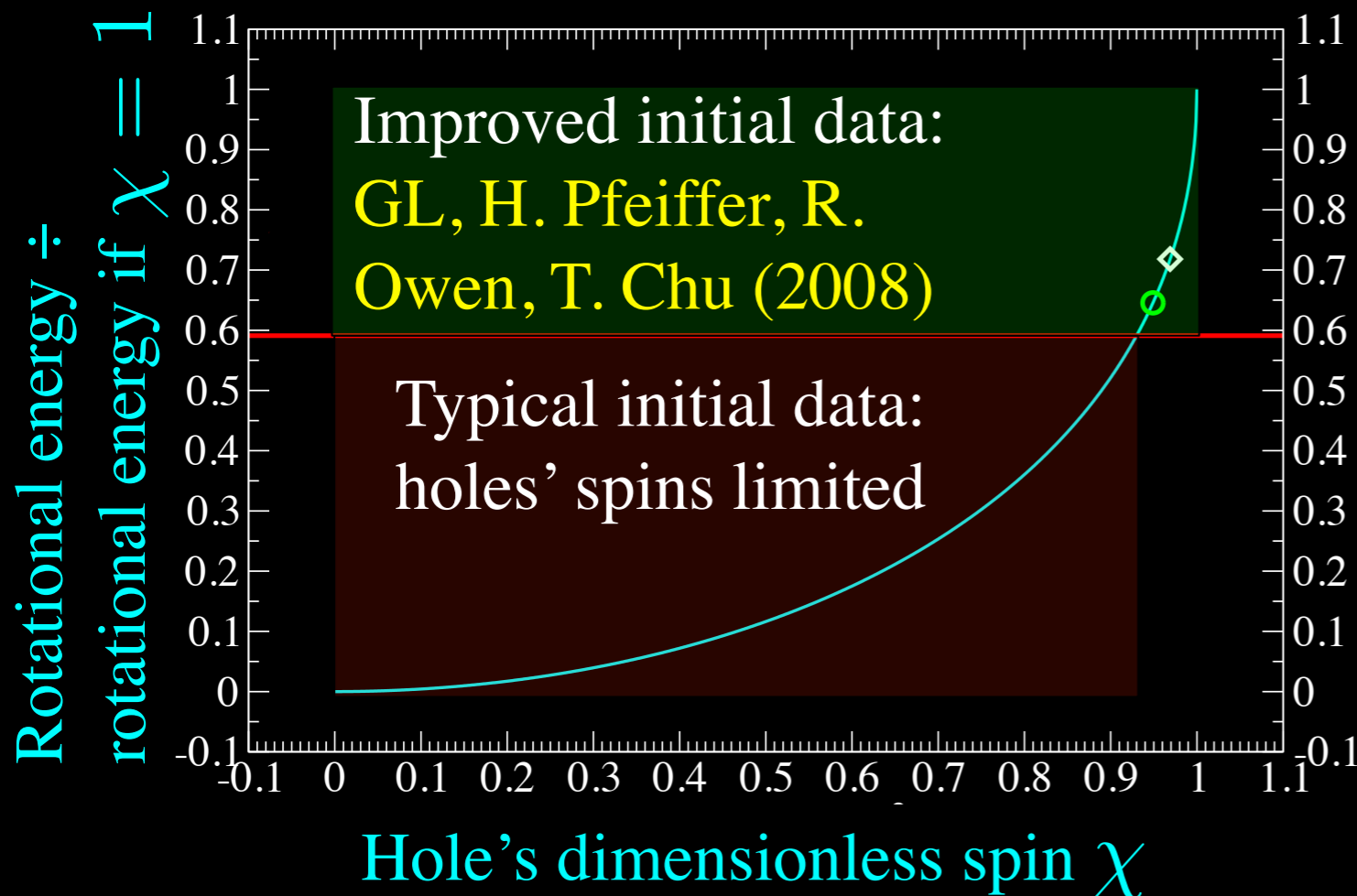
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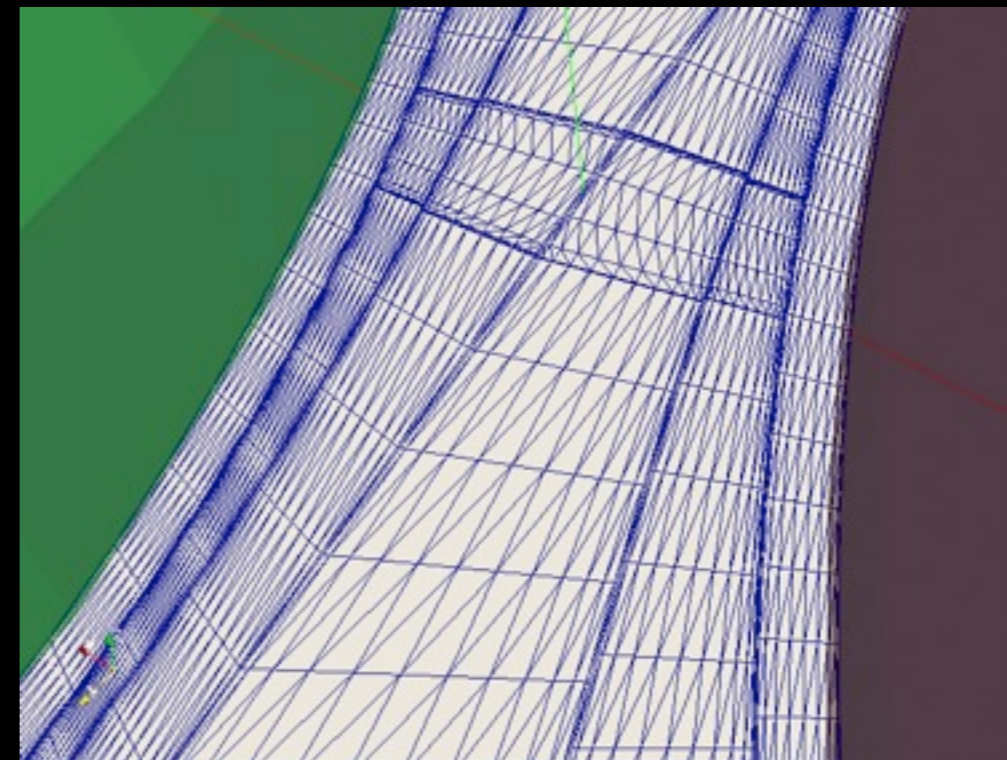
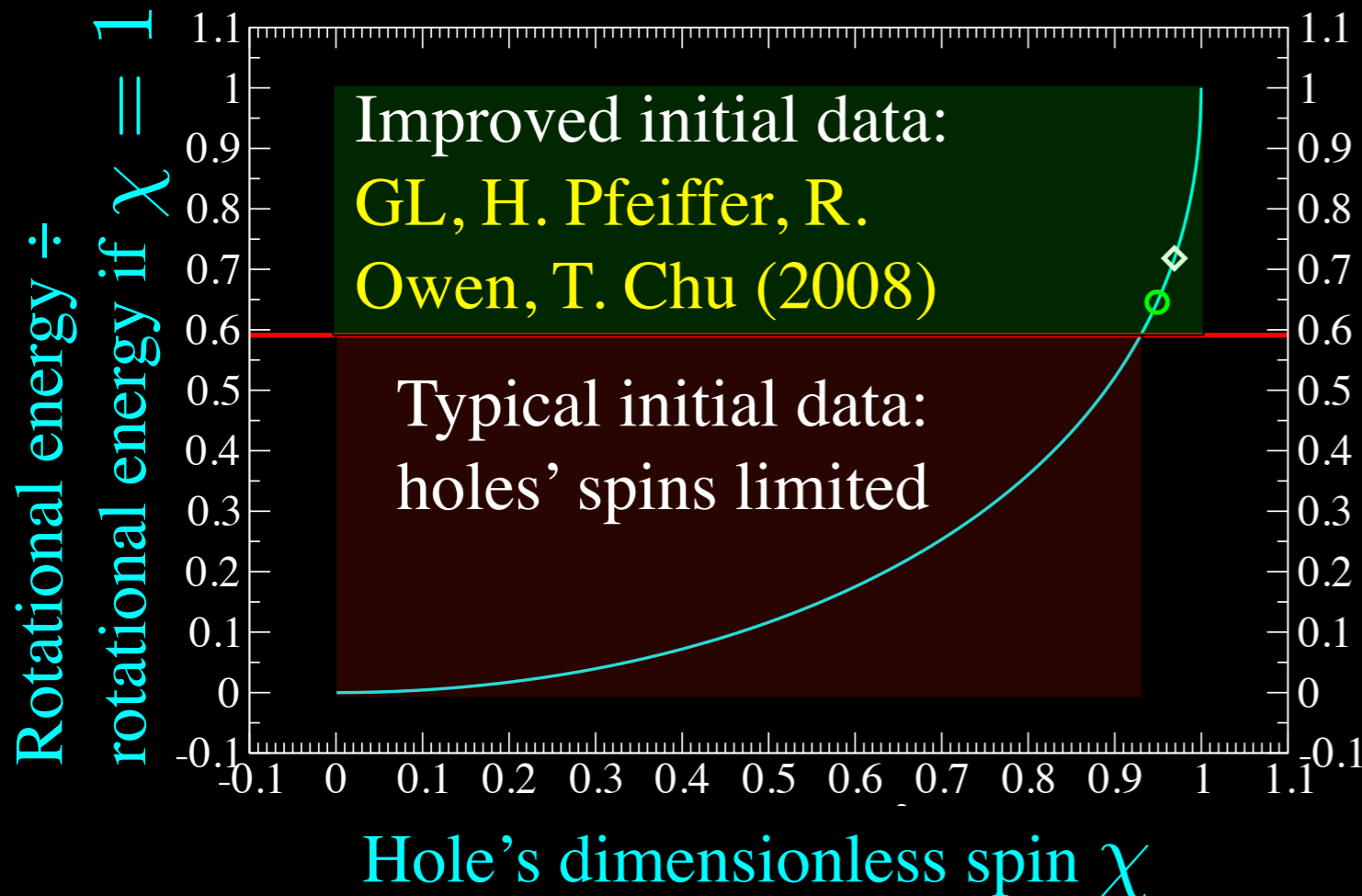
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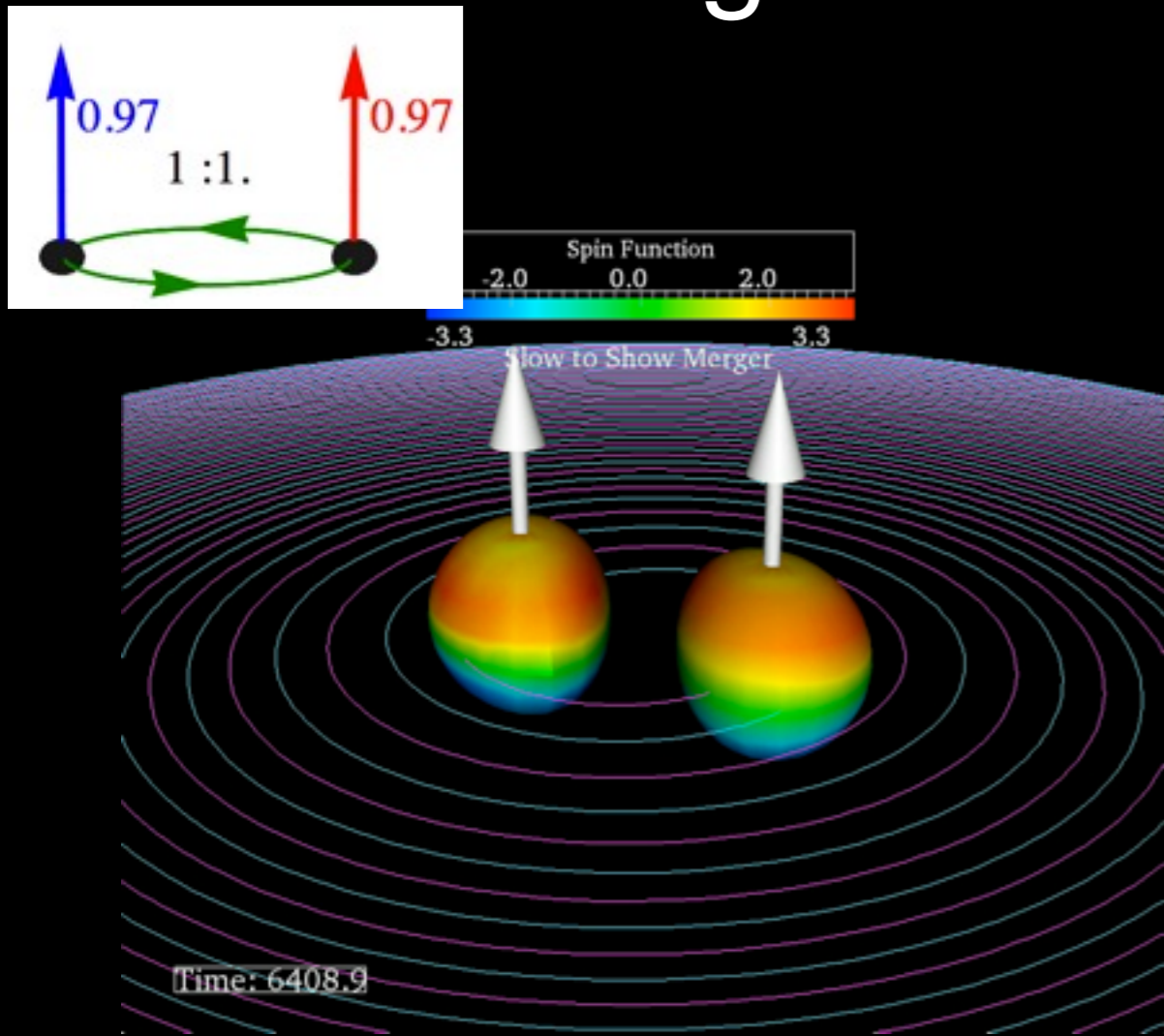
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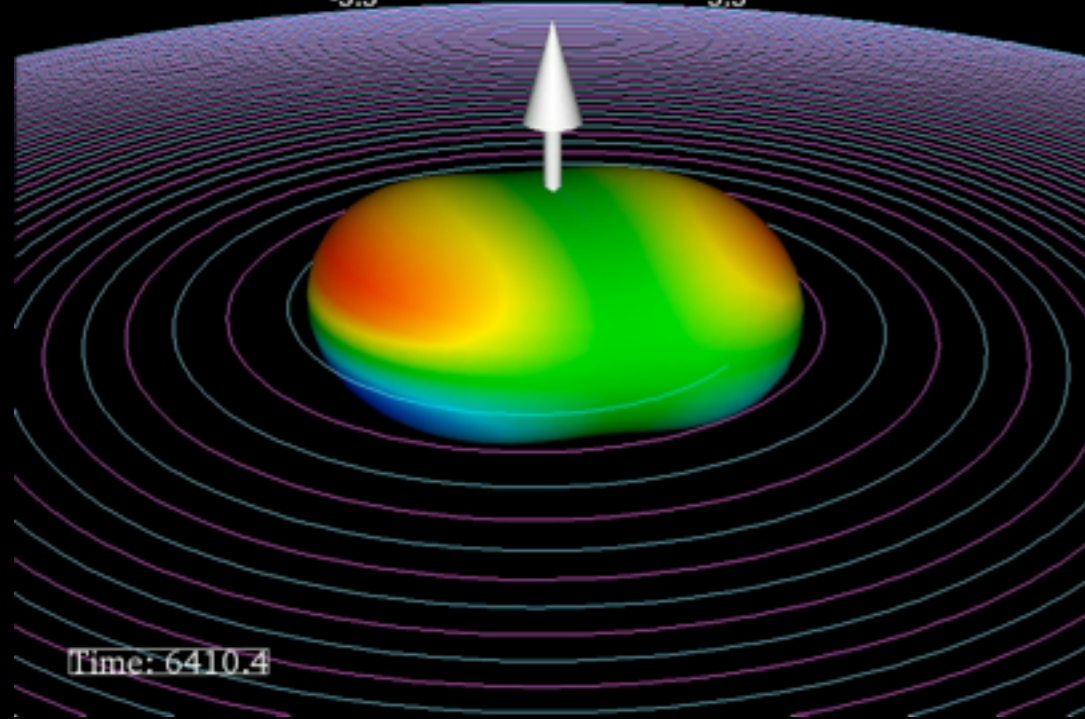
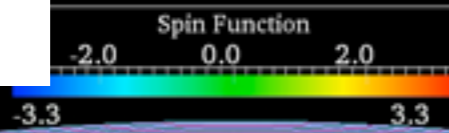
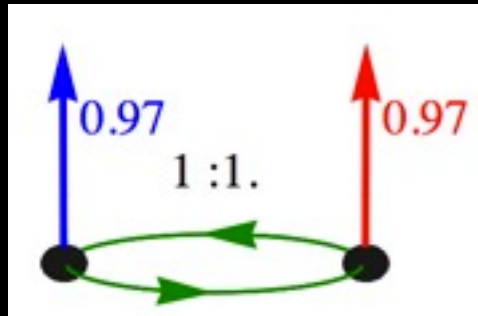
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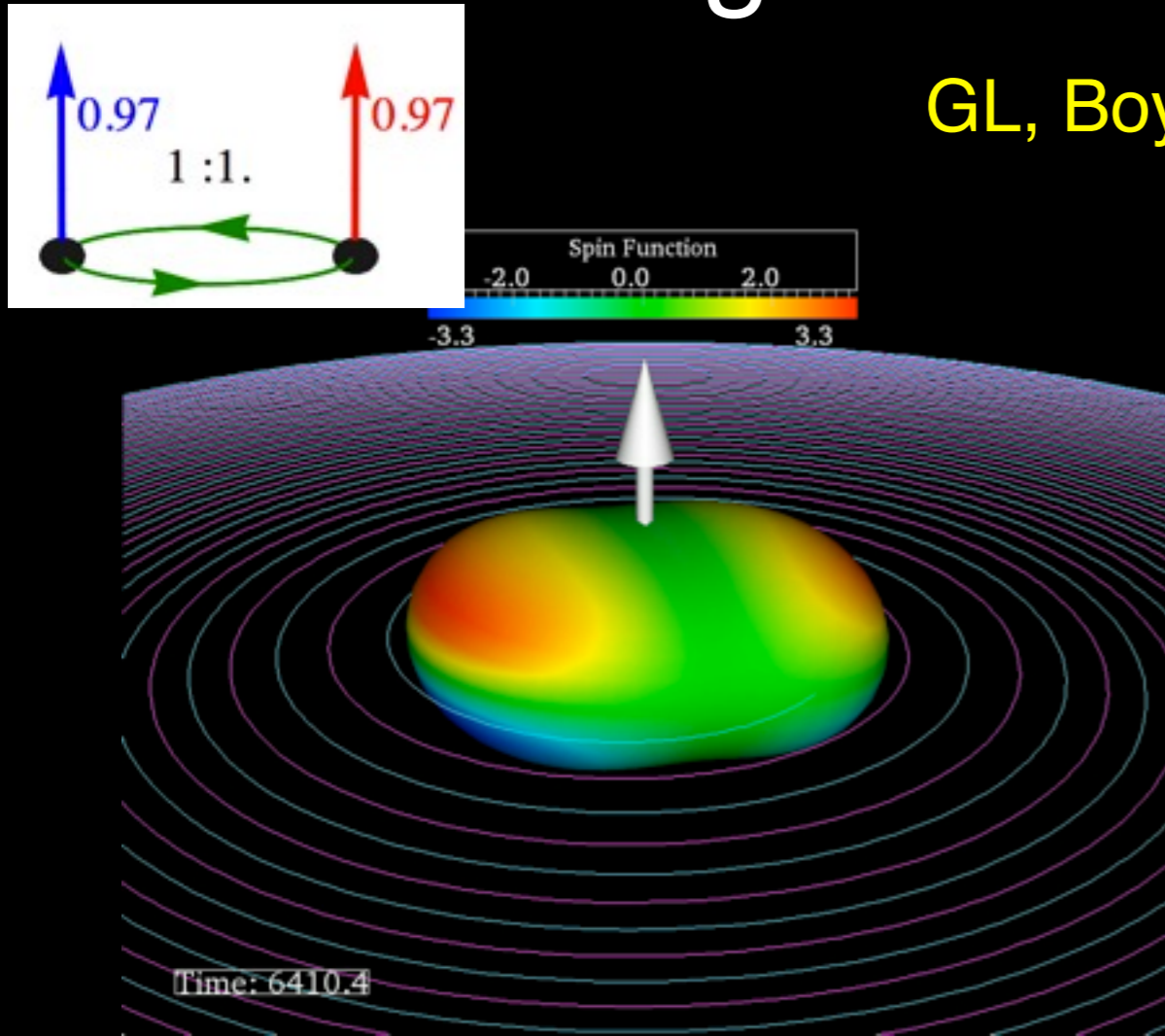
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GL, Boyle, Scheel, Szilágyi, CQG **29**, 045003 (2012)



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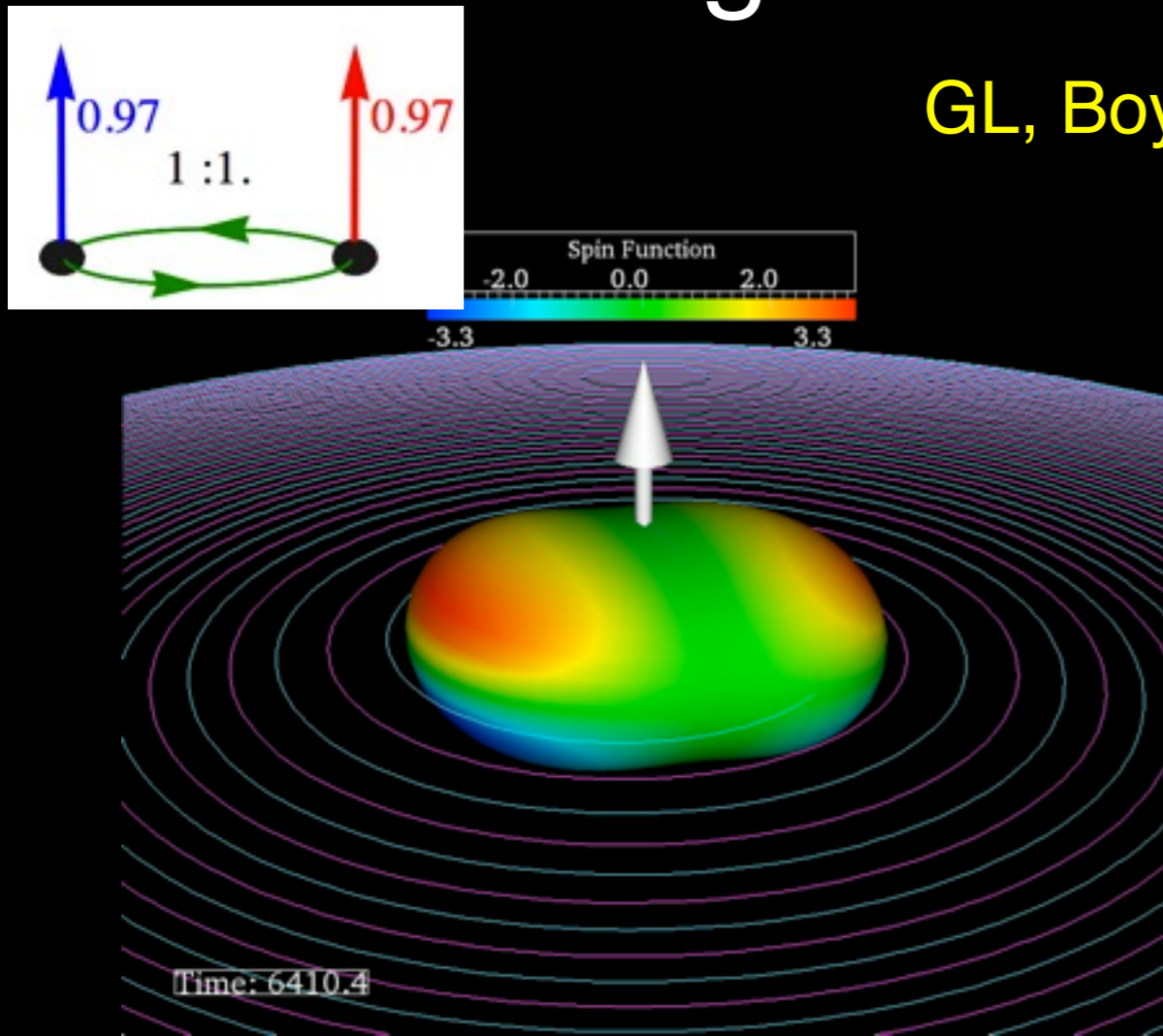
25.5 orbits

11% of initial mass radiated

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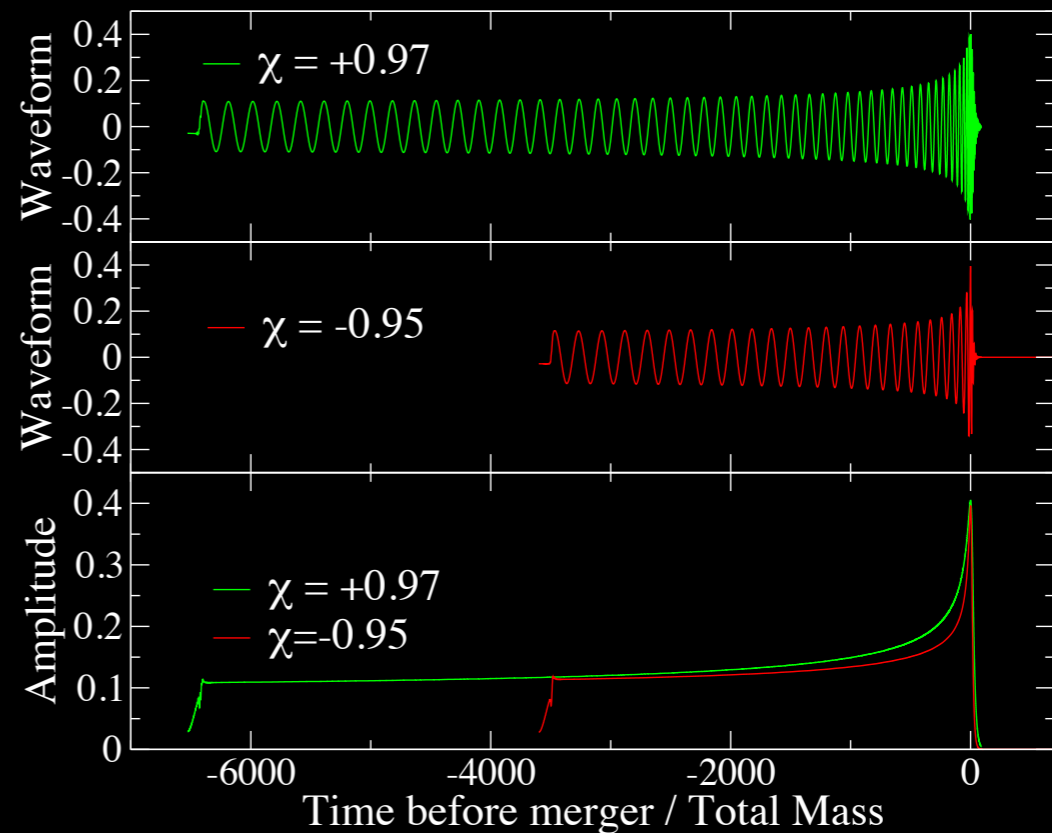
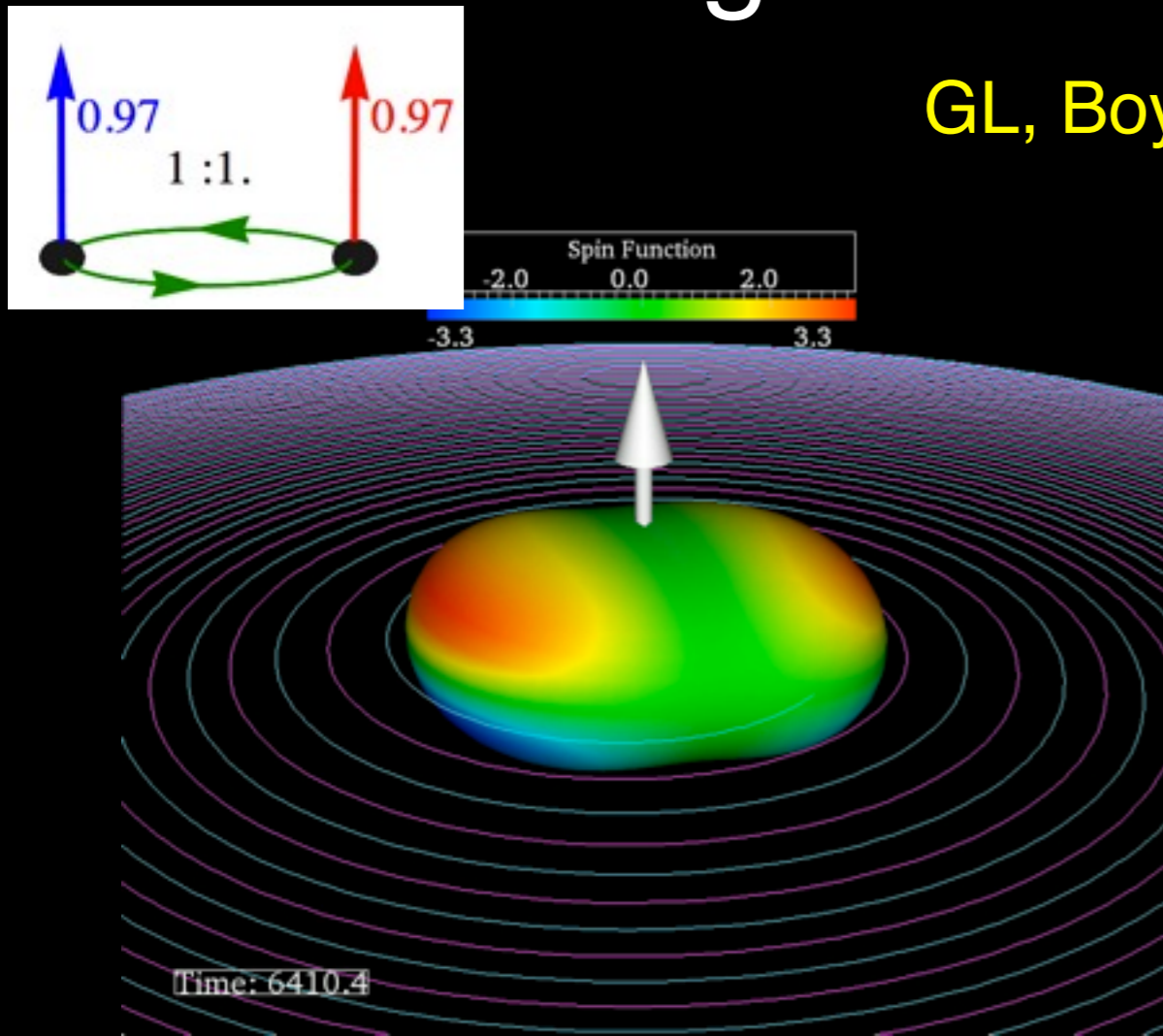
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110k cpu-hours

$\approx 120$  days wallclock time

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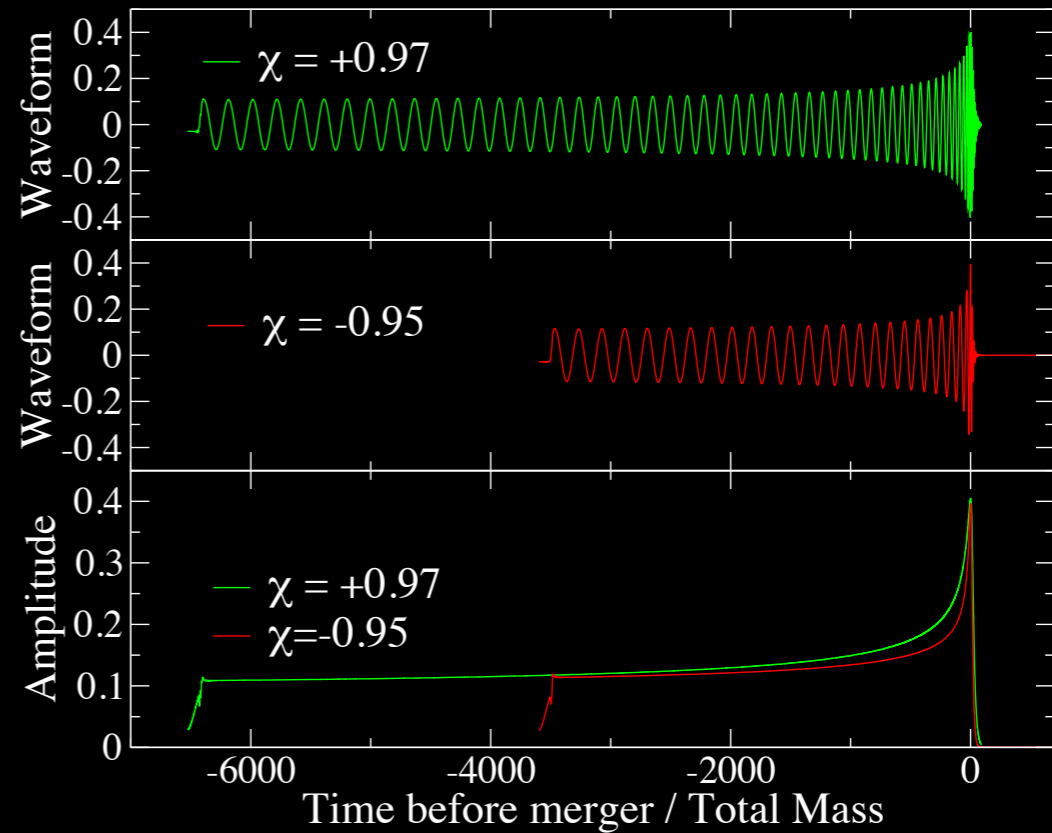
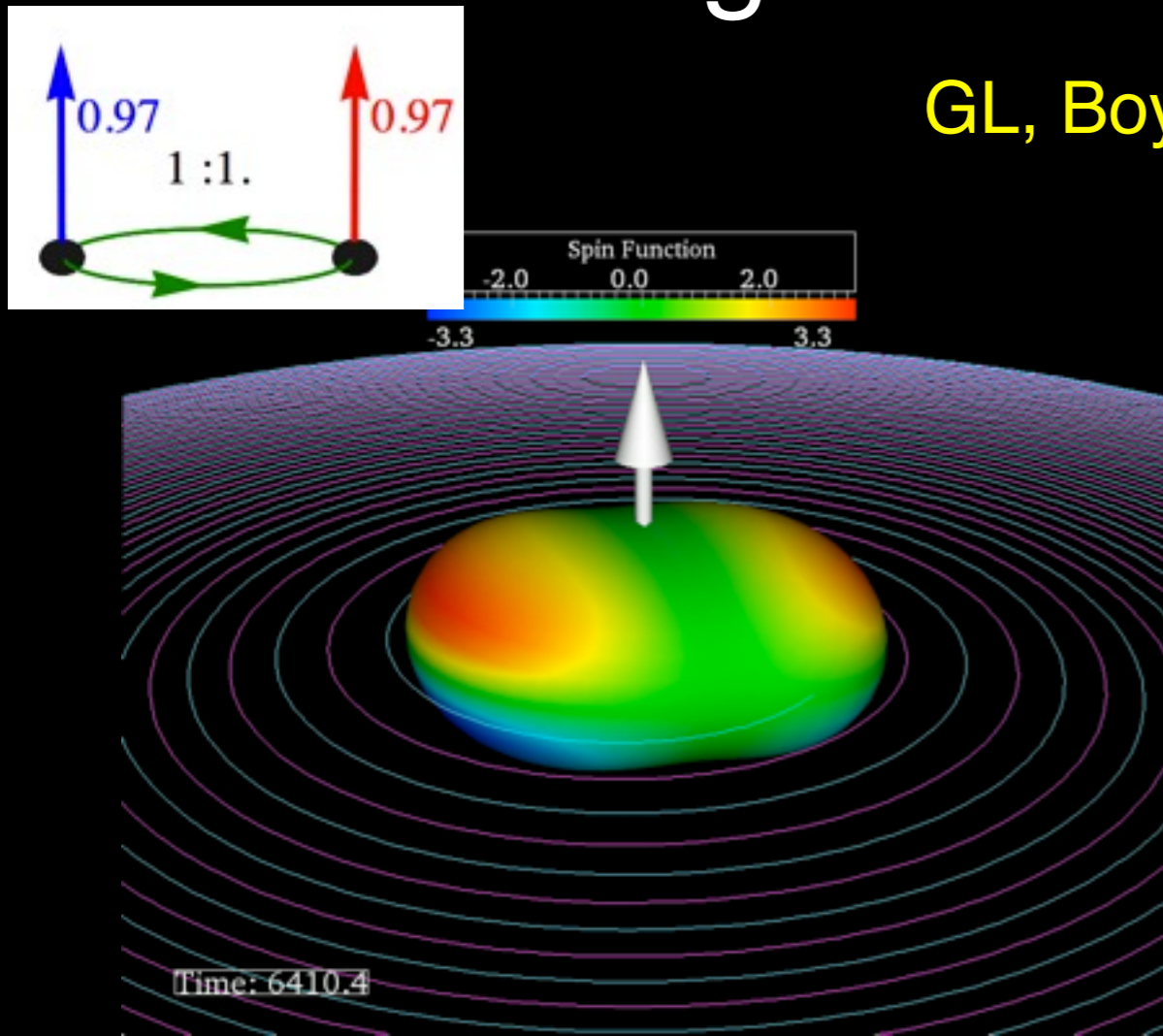
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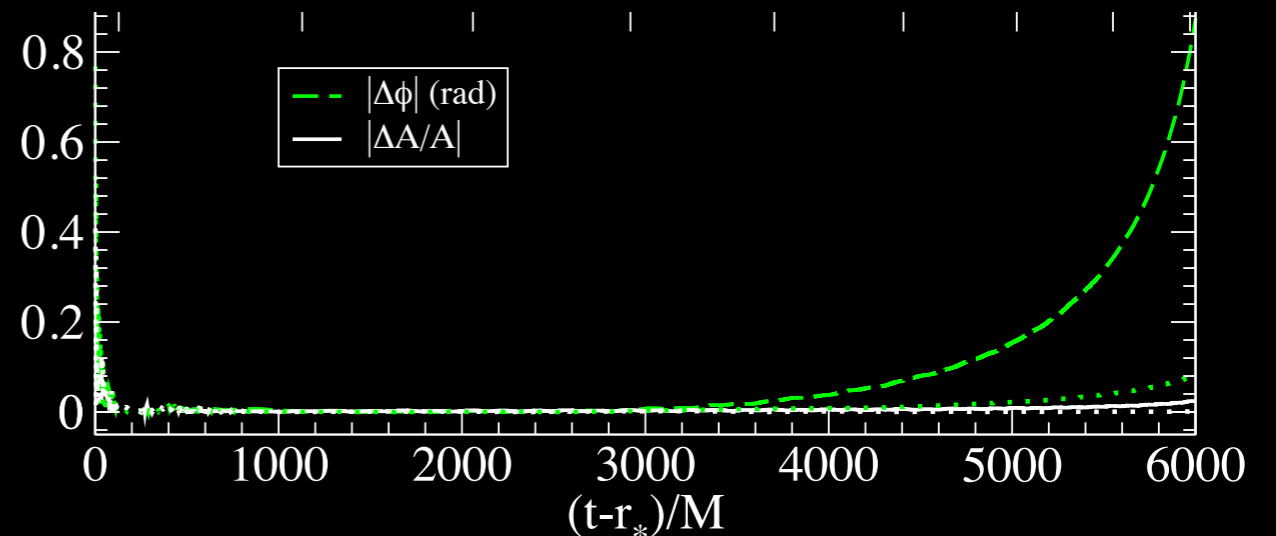
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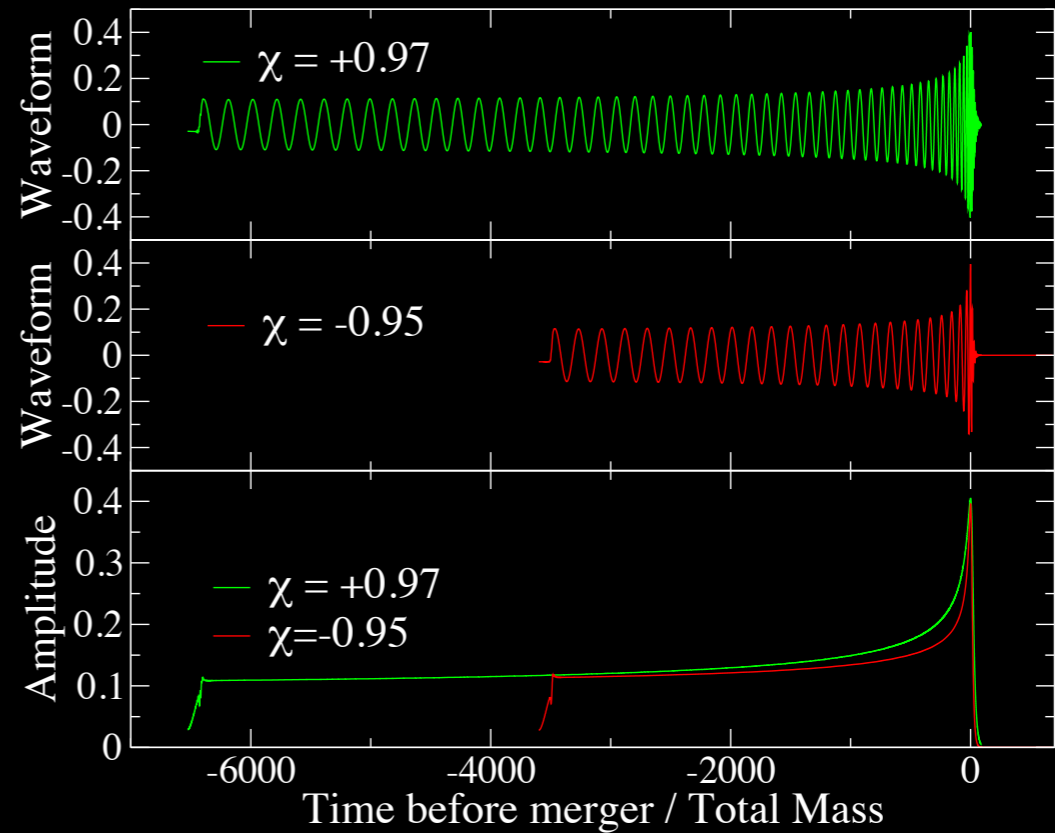
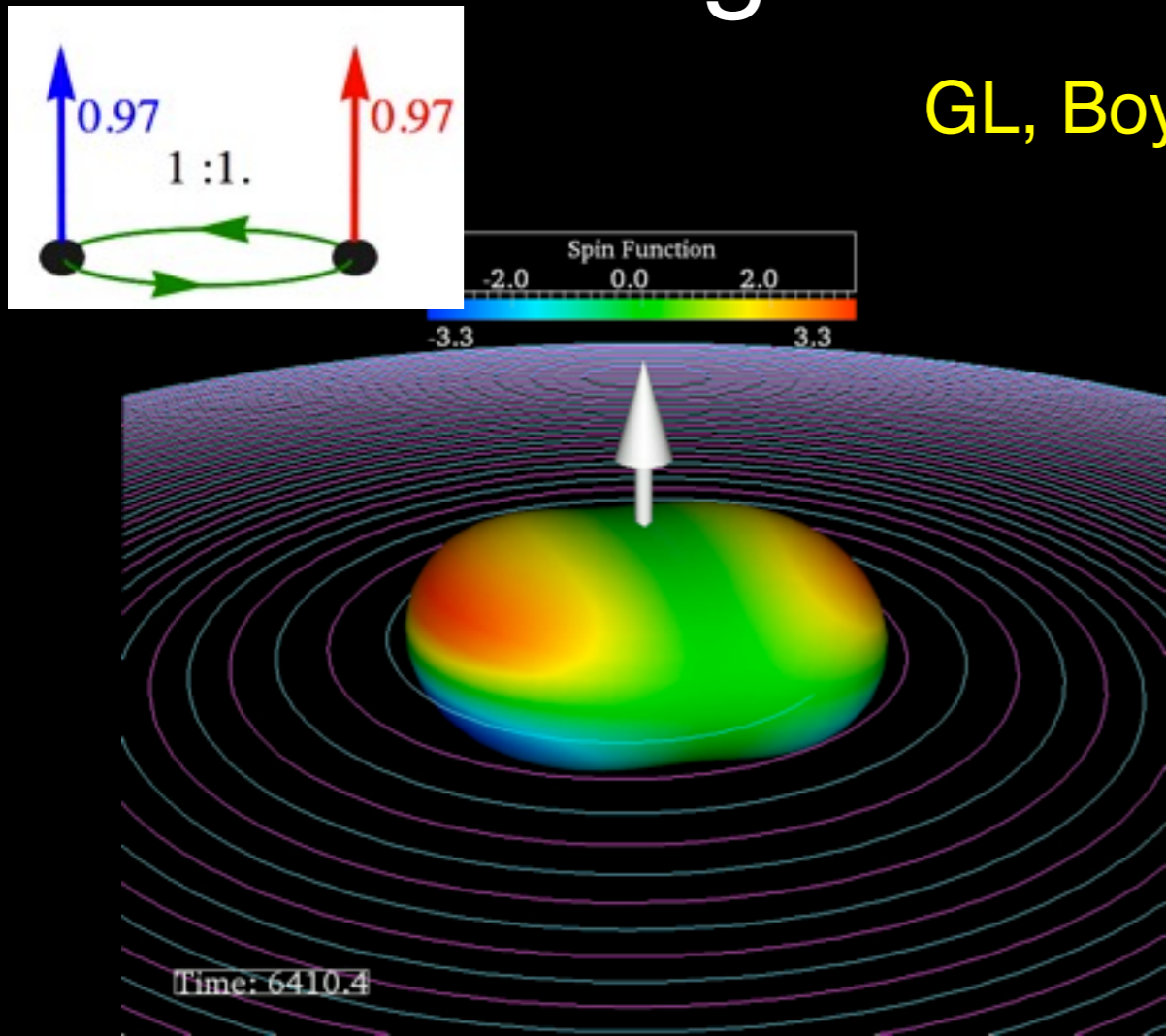
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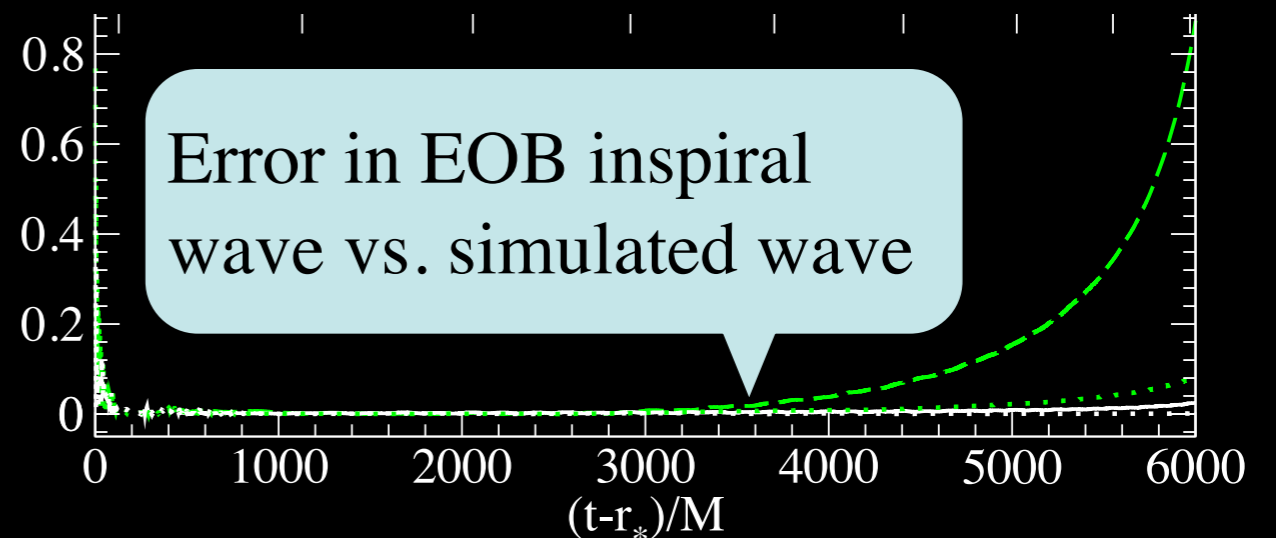
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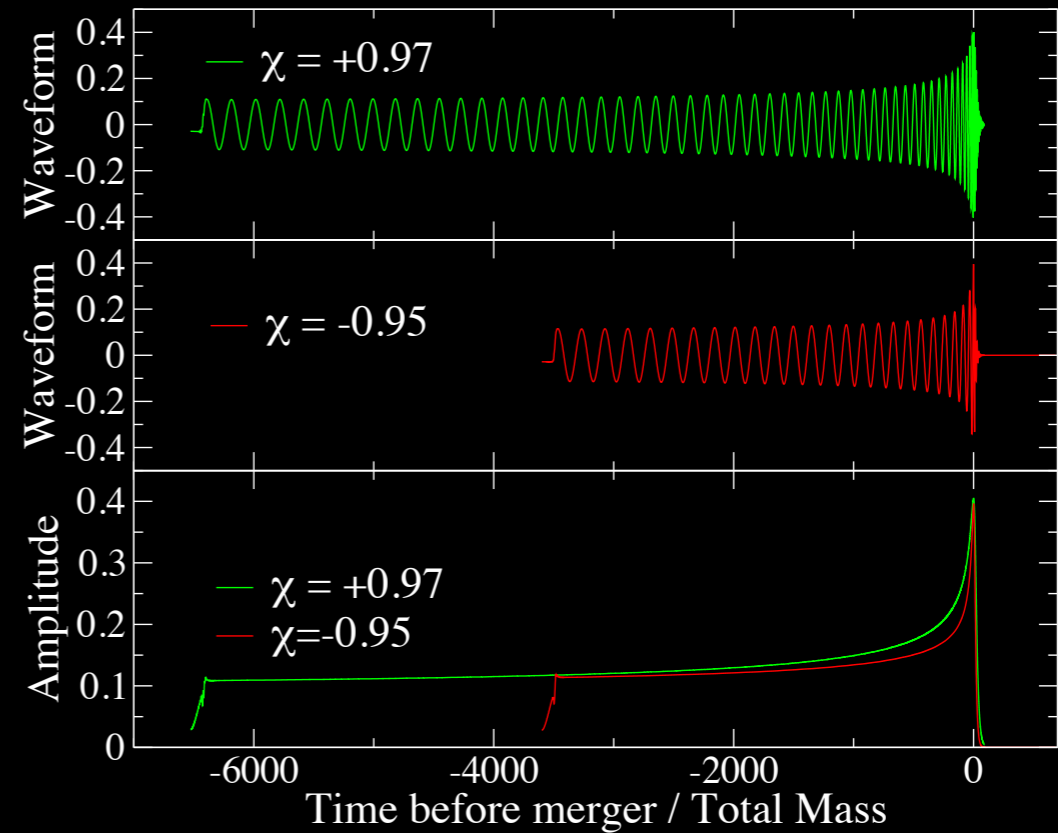
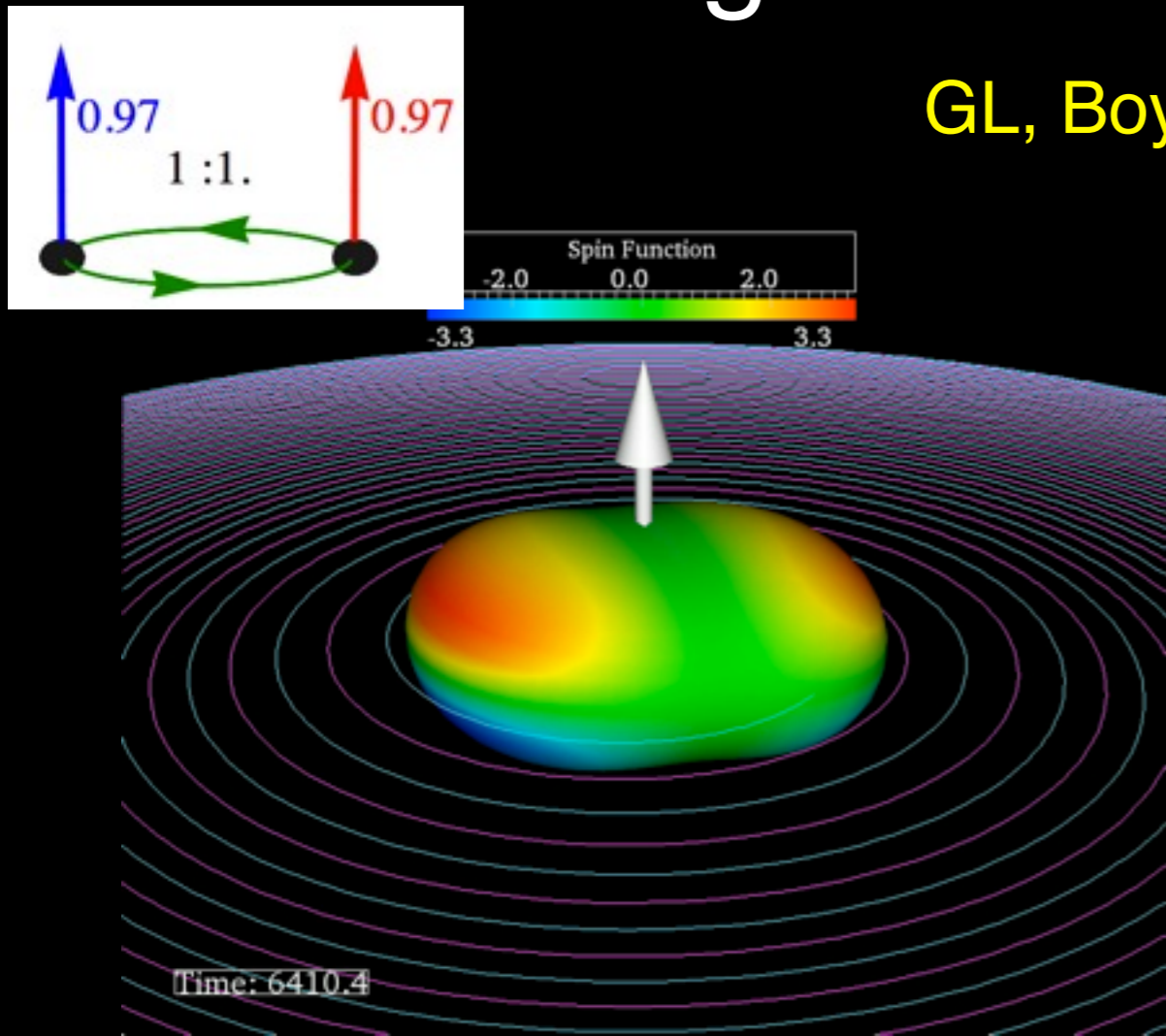
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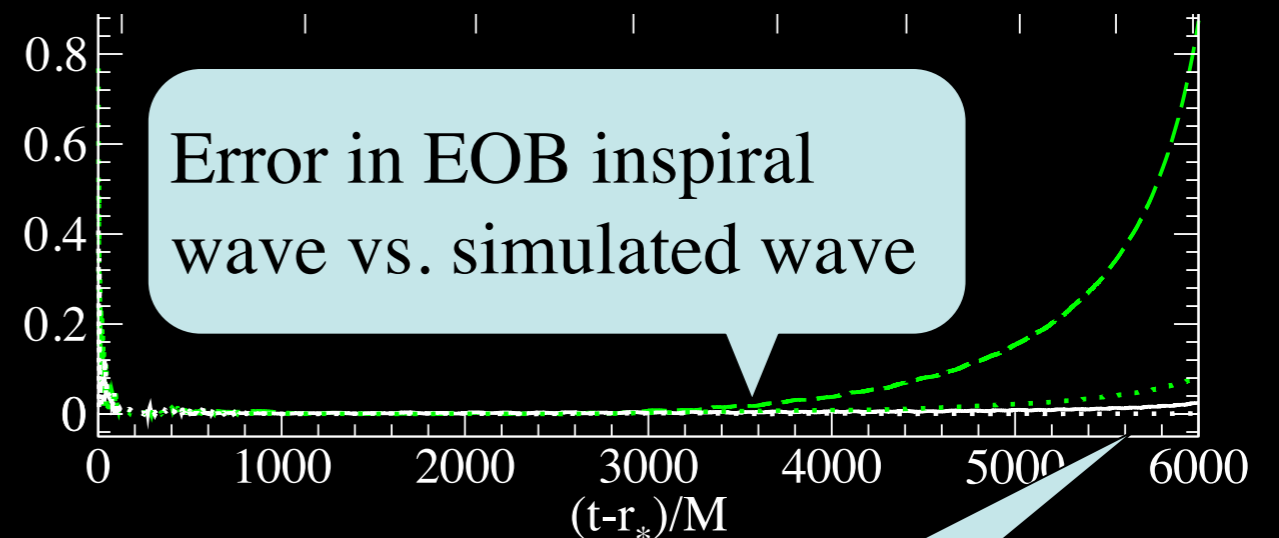
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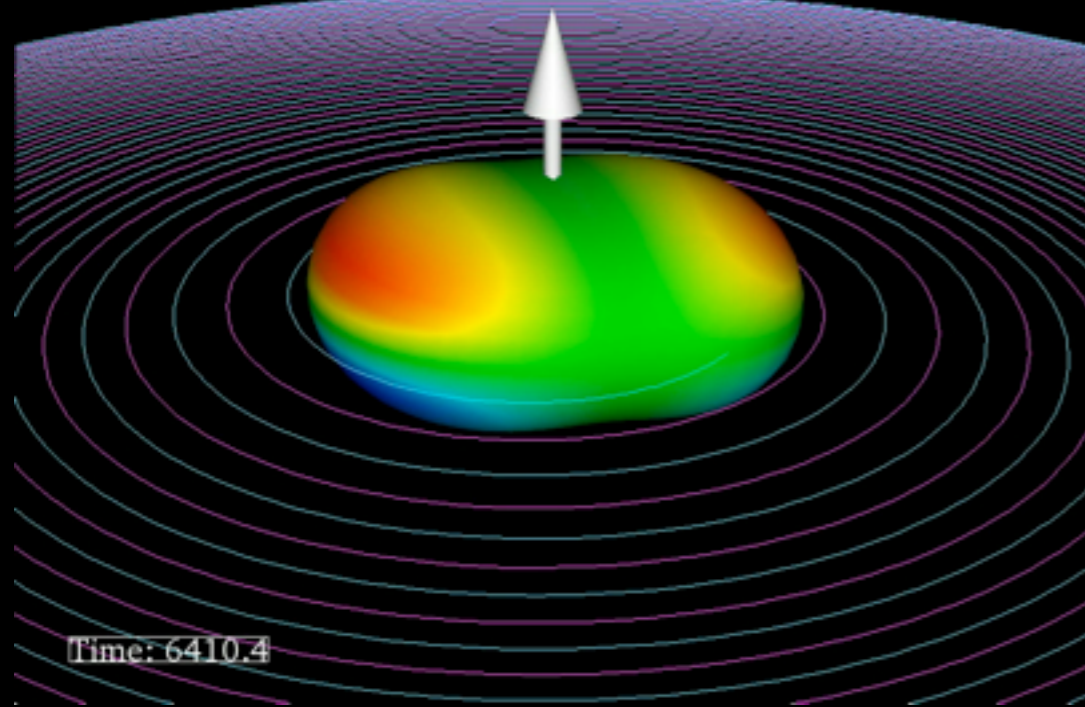
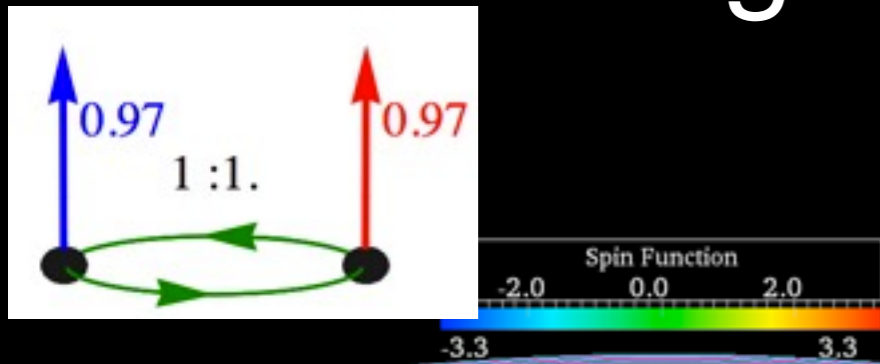
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Dotted lines: numerical error

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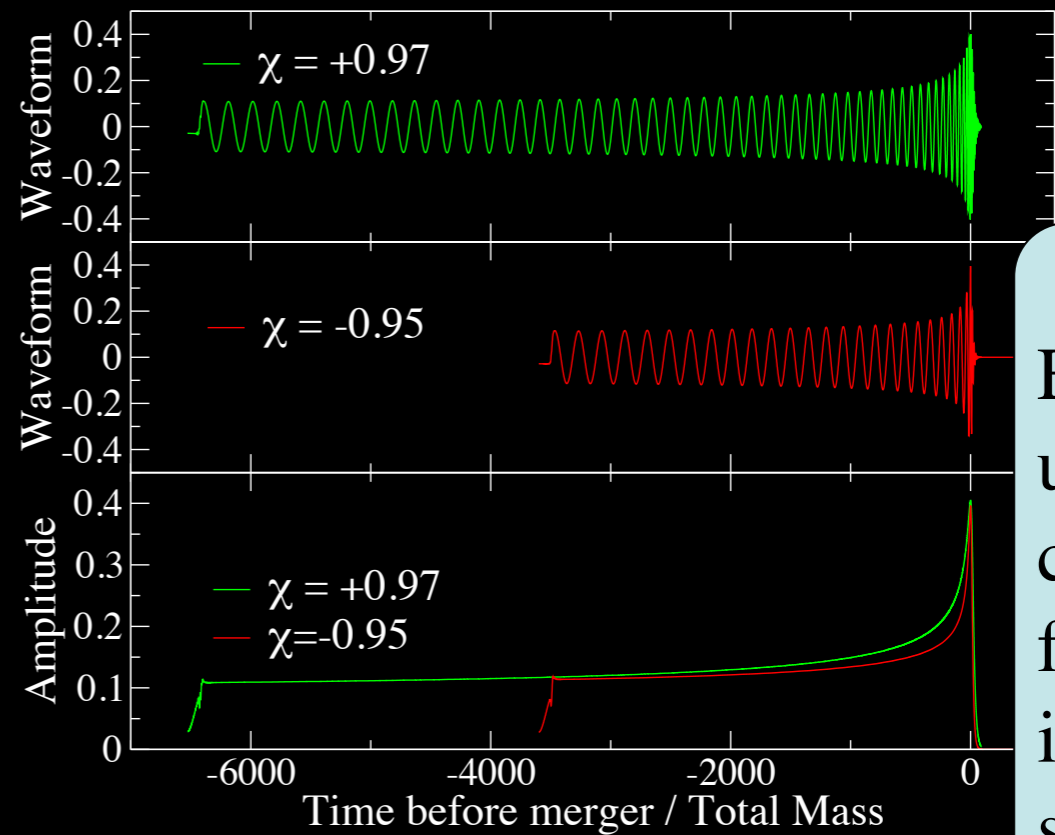
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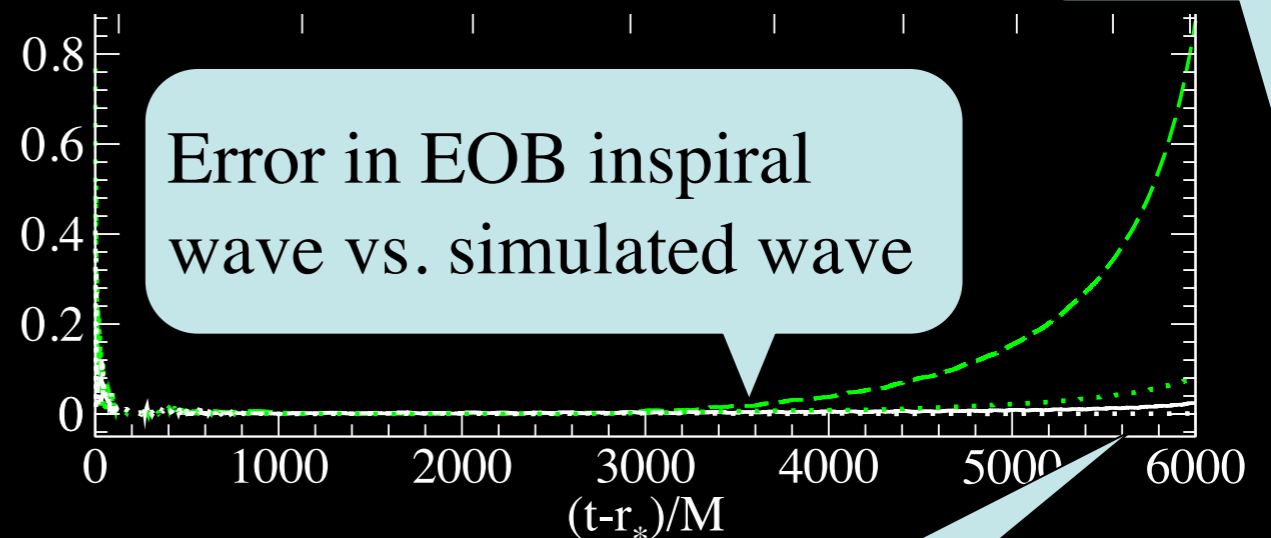
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EOB  
unreliable  
currently  
for merger  
if aligned  
spin  
 $\chi \gtrsim 0.7$



Error in EOB inspiral  
wave vs. simulated wave

Dotted lines: numerical error



# Precession

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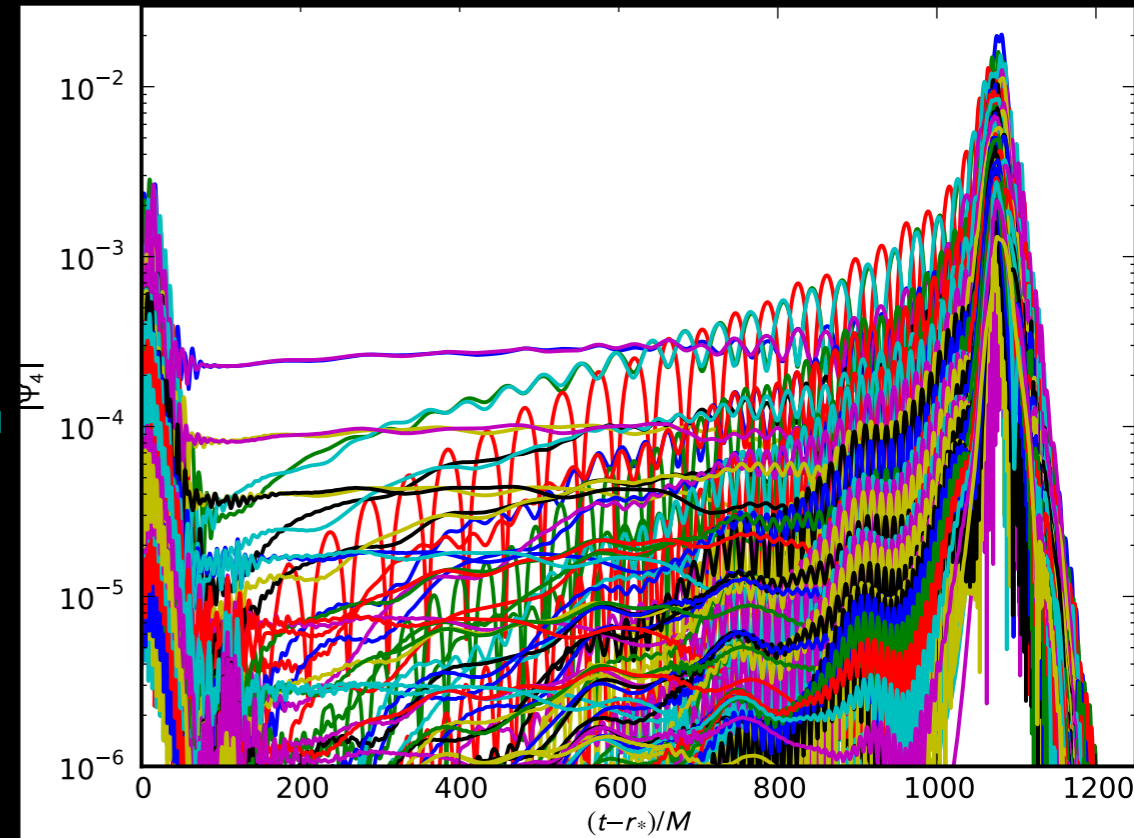
- 7D parameter space:  
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Wave amplitude  $Y_{lm}$



Time

Figure courtesy M. Boyle & L. Kidder

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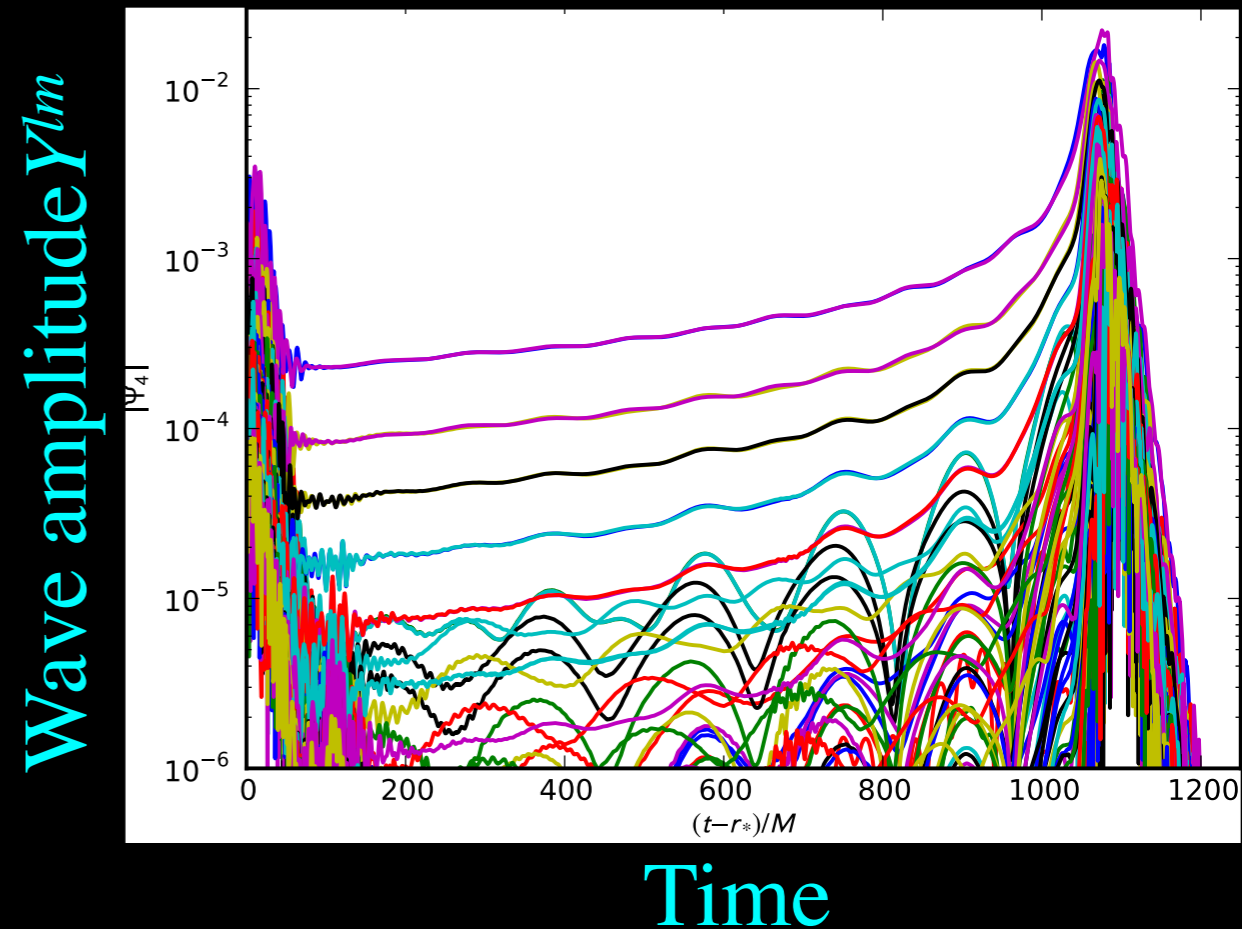
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Radiation-aligned frame

Schmidt et al, PRD **84**, 024046 (2011)

O'Shaughnessy et al,  
PRD **84**, 124002 (2011)

Figure courtesy M. Boyle & L. Kidder



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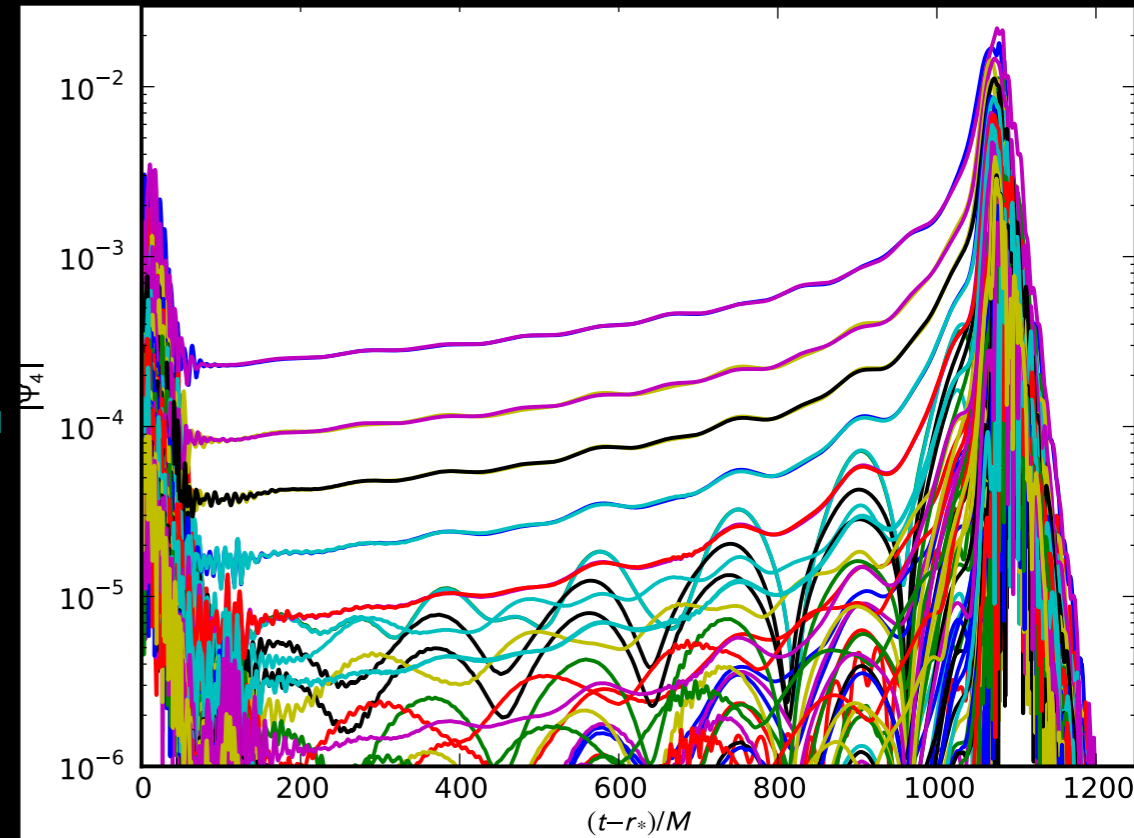
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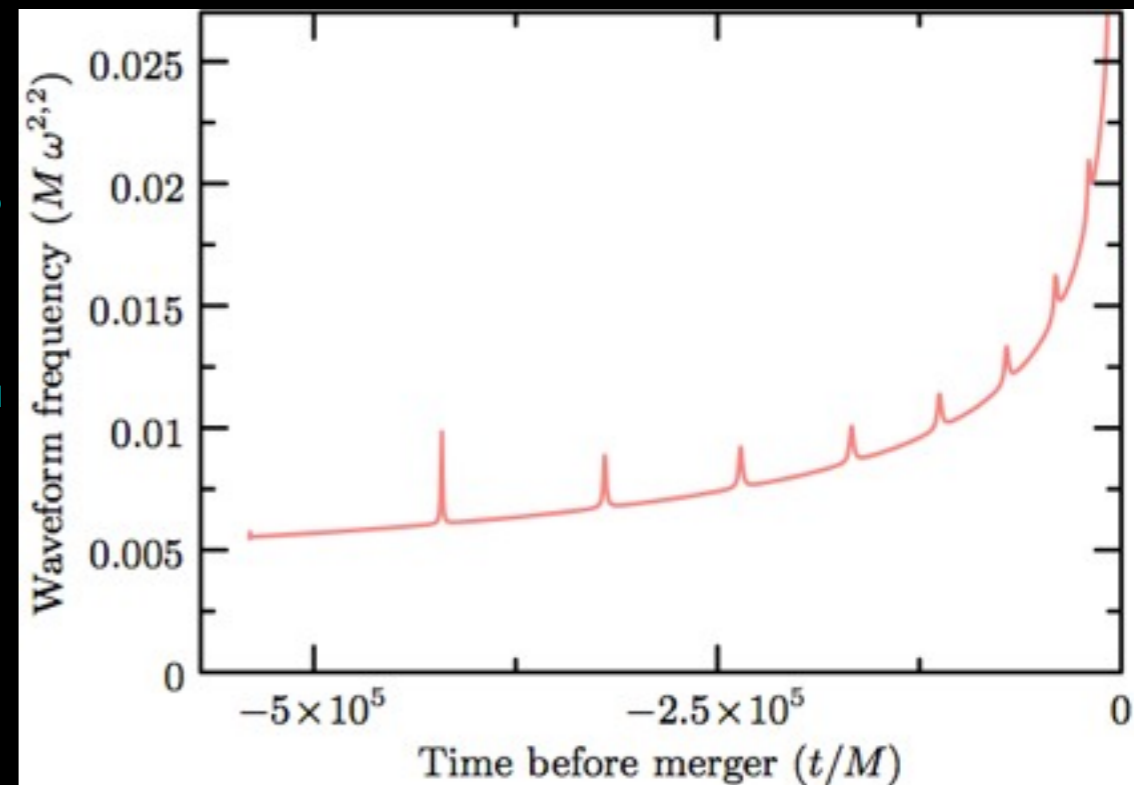
Figure courtesy M. Boyle & L. Kidder

Wave amplitude  $Y_{lm}$



Time

Wave frequency  $Y_{22}$



Time before merger

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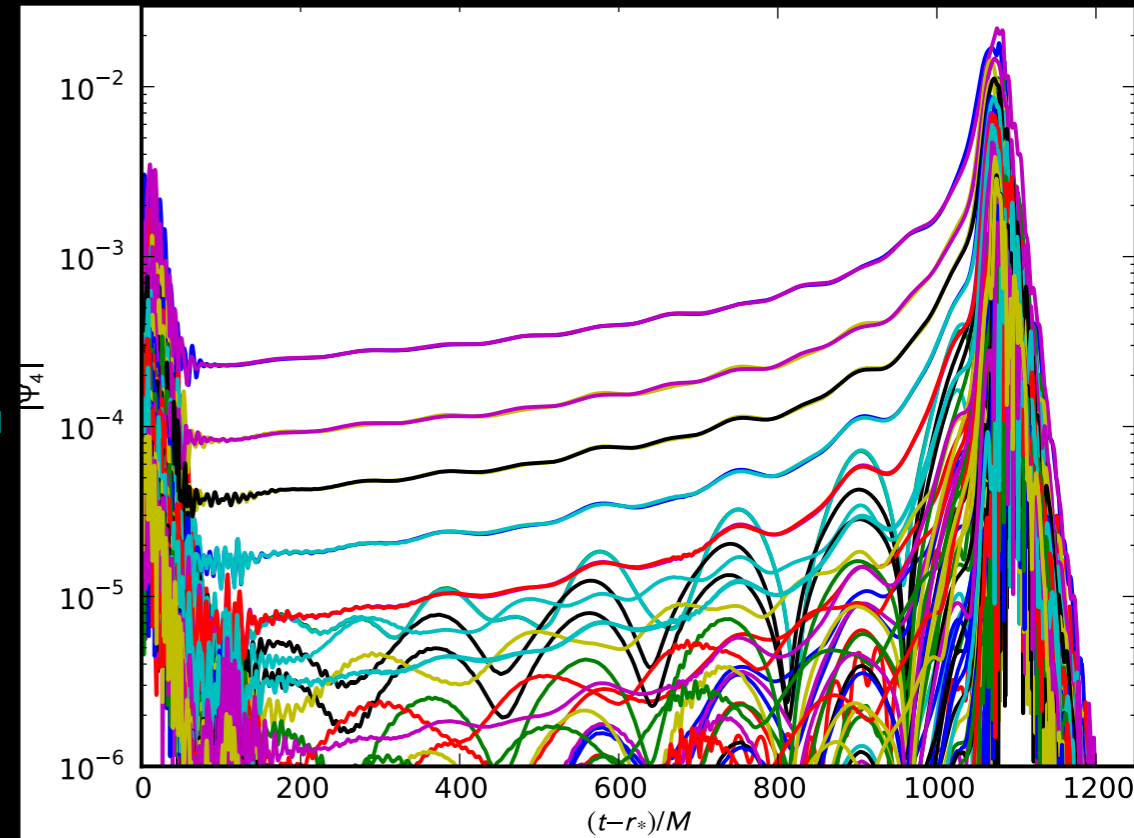
O'Shaughnessy et al,  
PRD **84**, 124002 (2011)

Minimal-rotation frame

Boyle et al, PRD **84**, 124011 (2011)

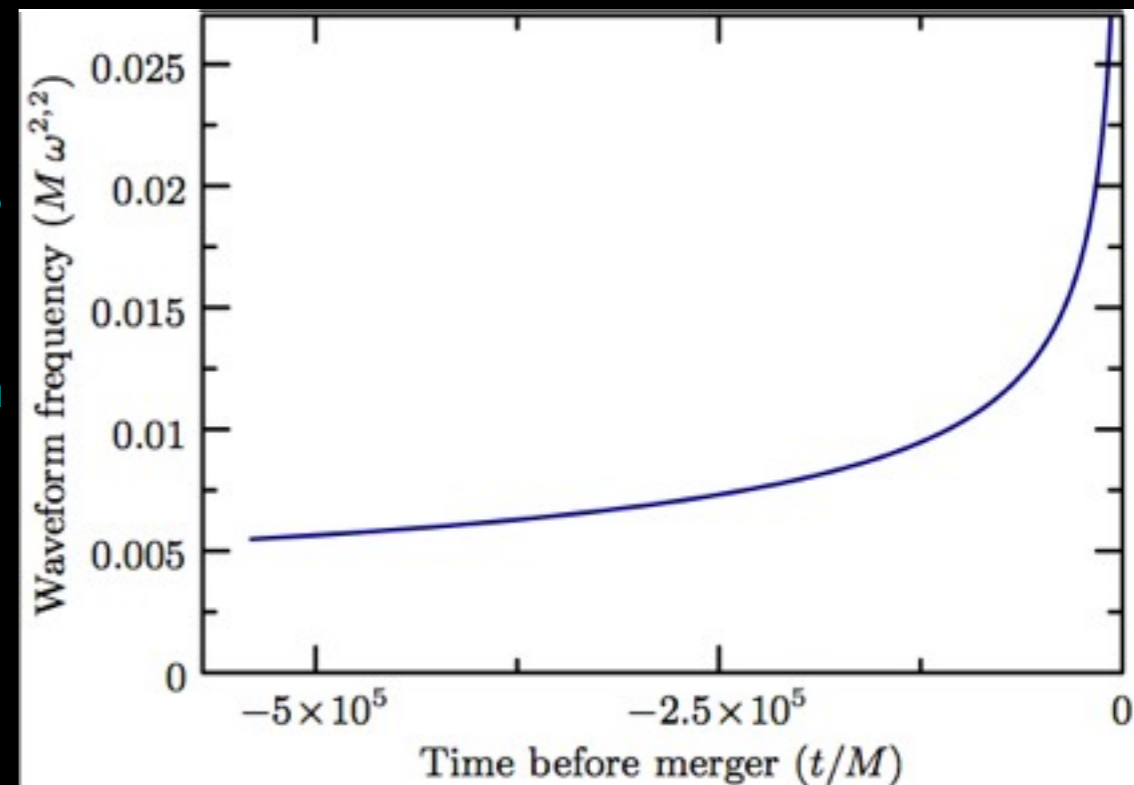
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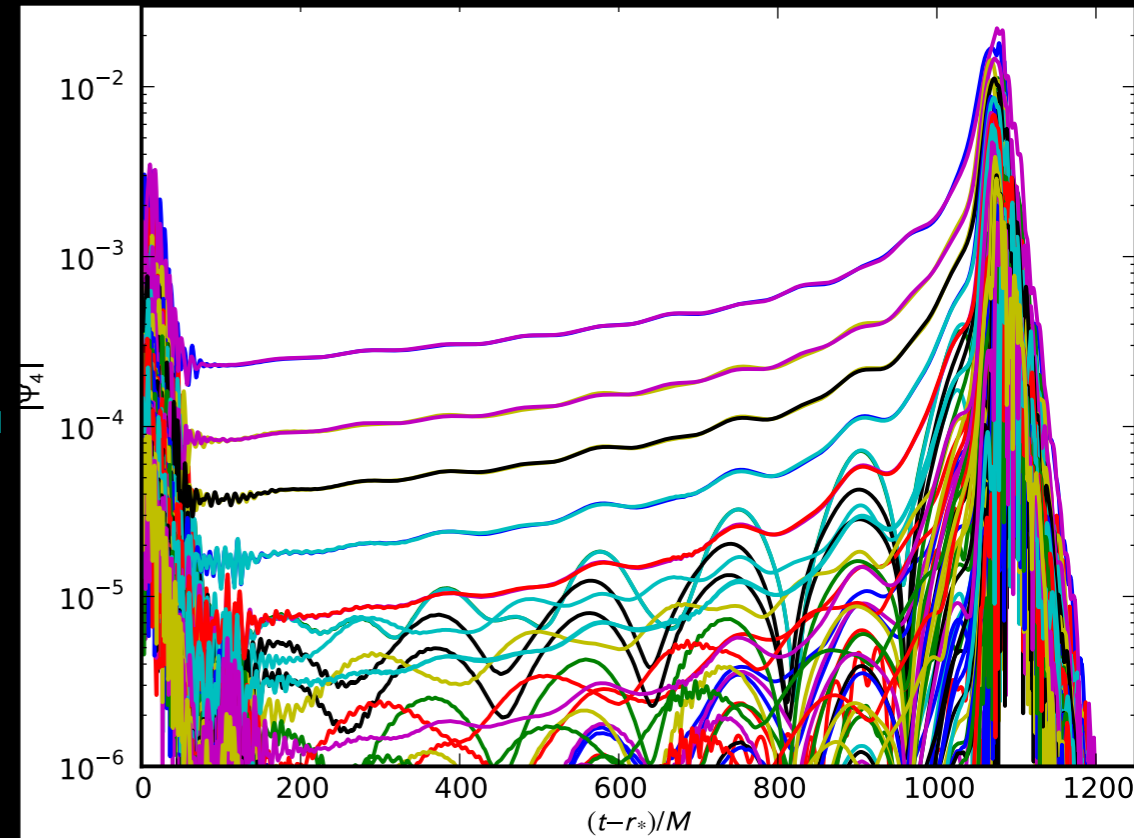
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Approximate mapping of  
precessing inspiral waveforms  
to non-precessing

Schmidt et al, (2012), arXiv:1207.3088

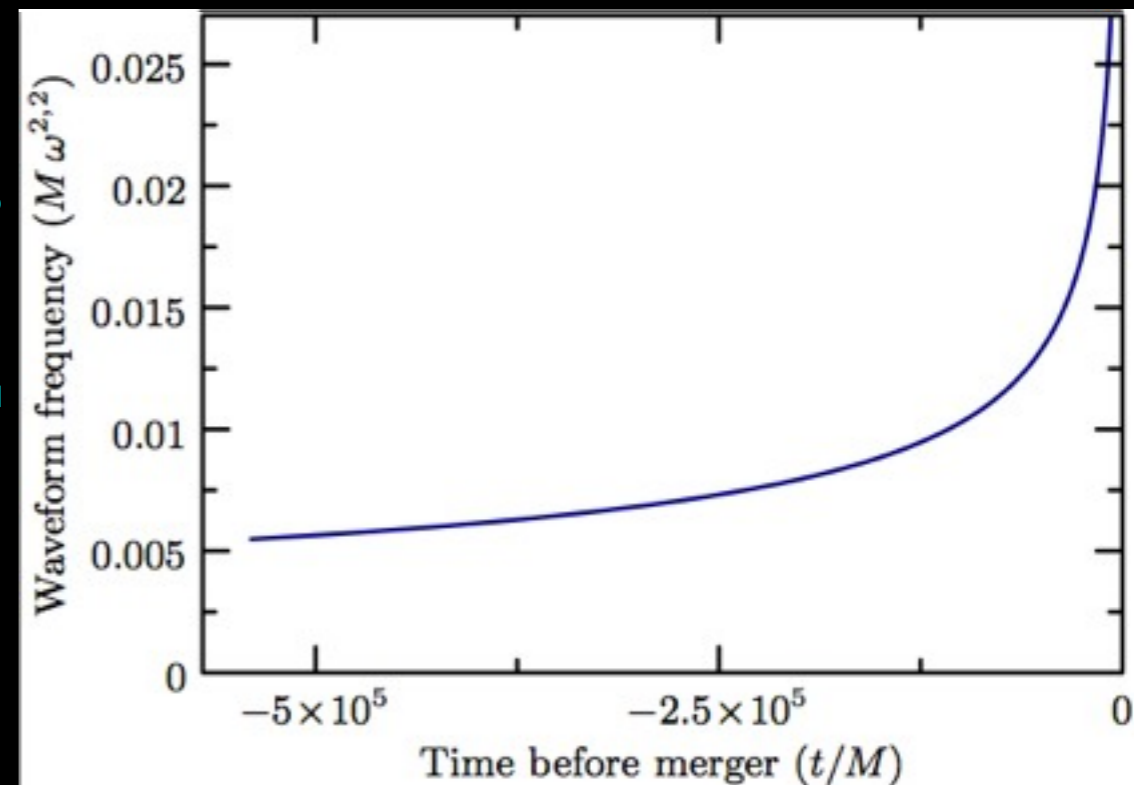
Figure courtesy M. Boyle & L. Kidder

Wave amplitude  $Y_{lm}$



Time

Wave frequency  $Y_{22}$



Time before merger

# Precession



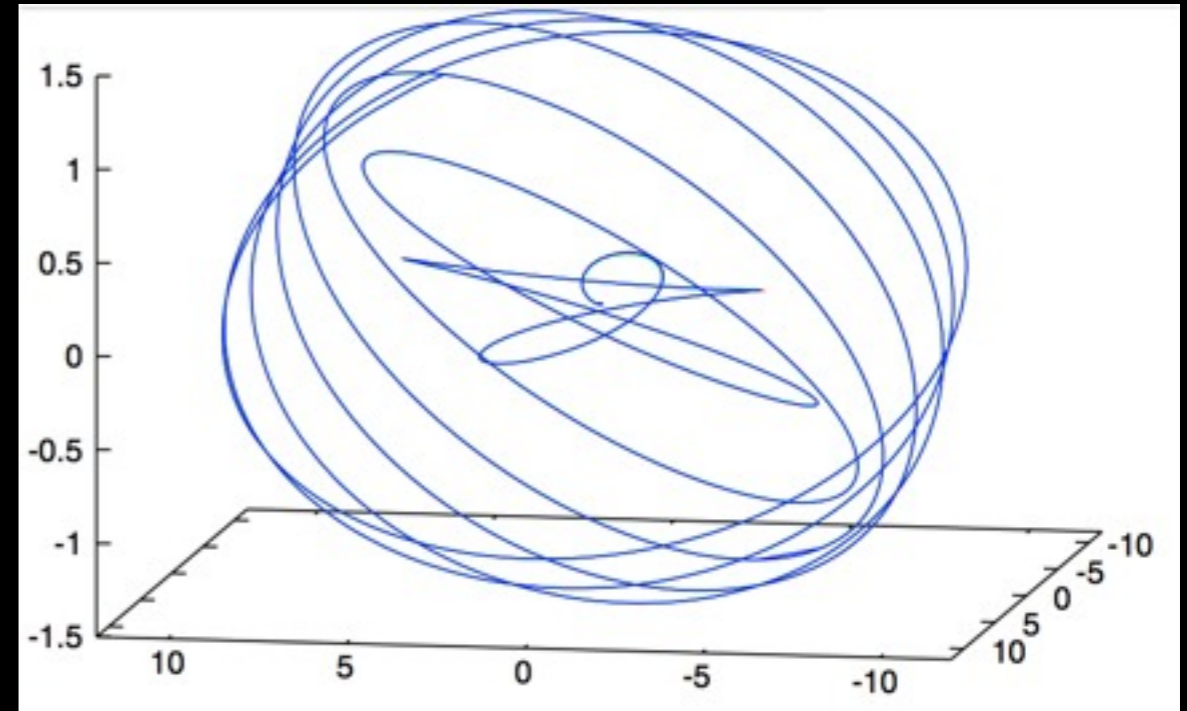
# Precession

- Current results

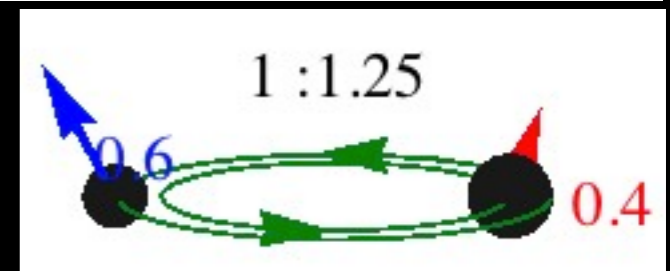
# Precession

- Current results
  - Some short simulations, but few  $\sim 10$ -orbit or longer waveforms

Position difference  $\vec{x}_1 - \vec{x}_2$



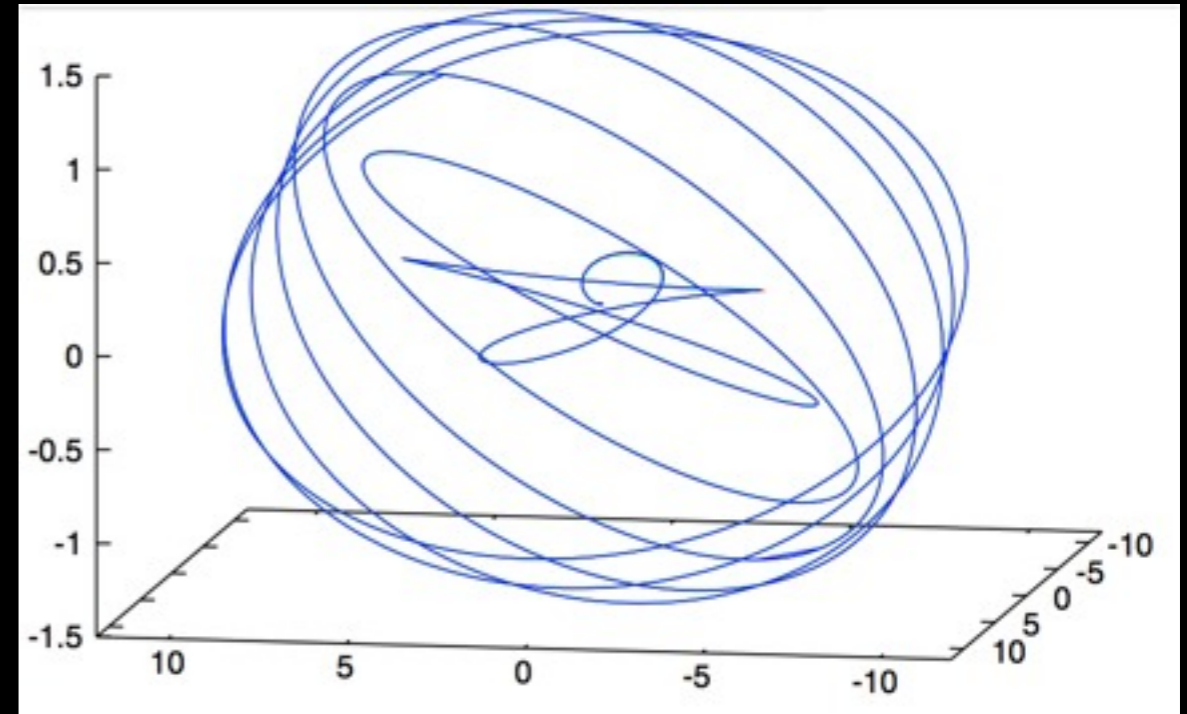
Campanelli et al,  
PRD 79, 084010  
(2009)



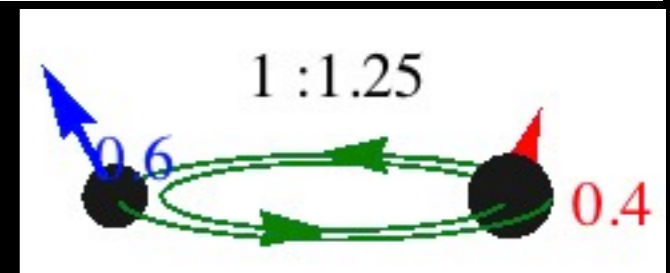
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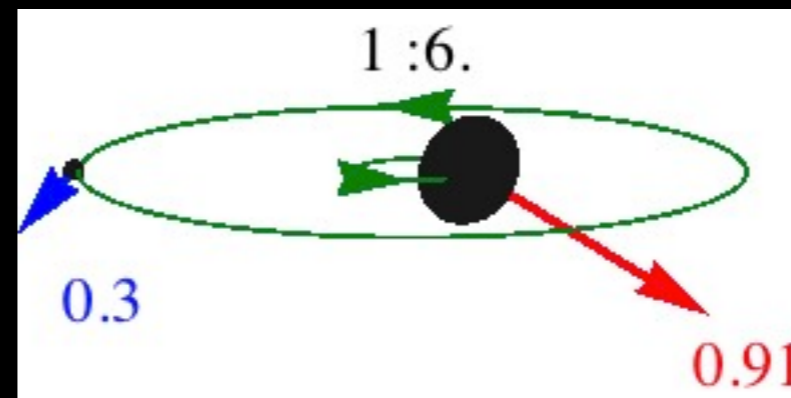
Position difference  $\vec{x}_1 - \vec{x}_2$



Campanelli et al,  
PRD 79, 084010  
(2009)



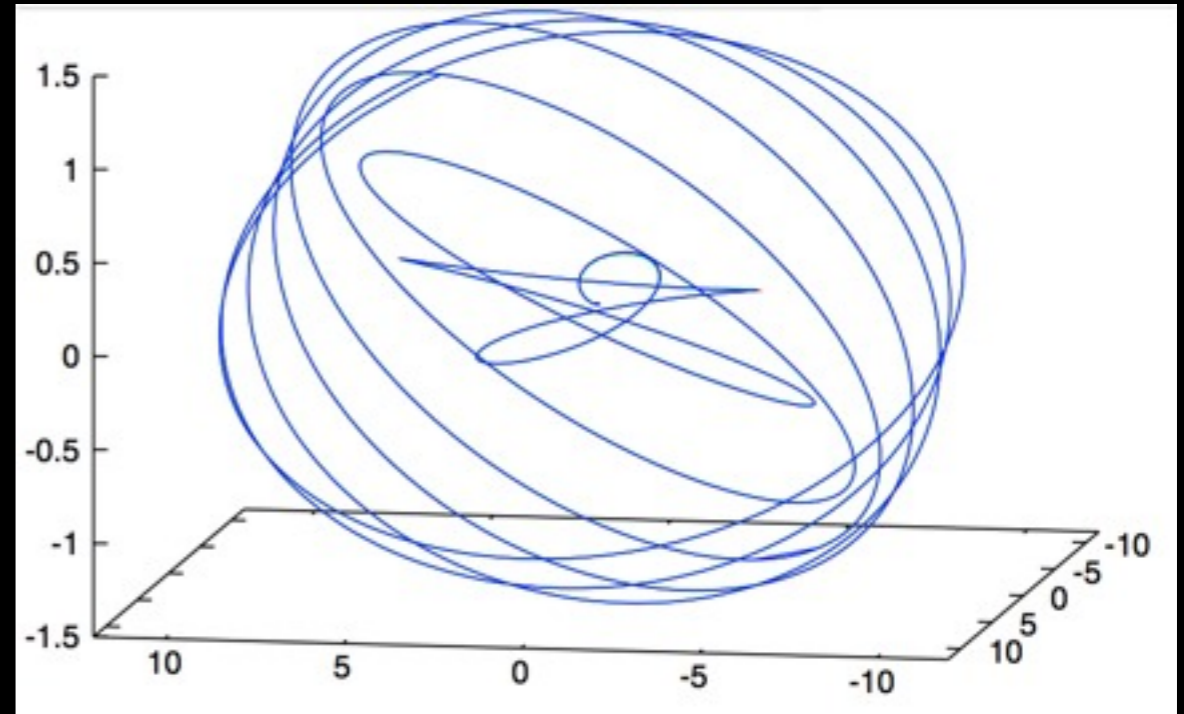
SXS (black-holes.org)  
collaboration in preparation



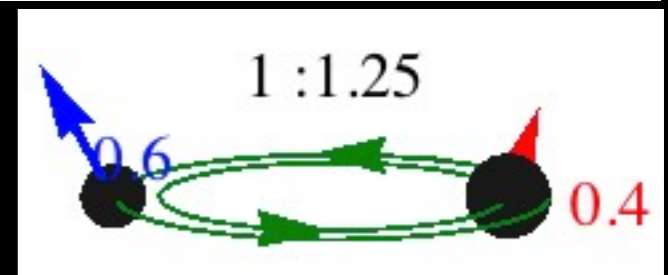
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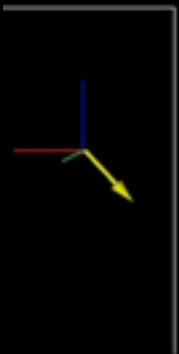
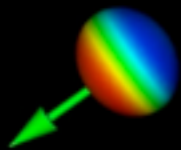
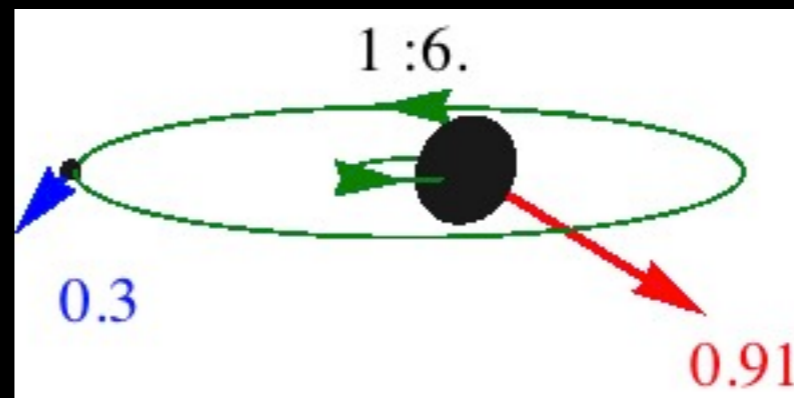
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# Applying simulations to GW data analysis

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- Template waveforms

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- Template waveforms
  - Goal: predict waveforms for LIGO
    - 7-dimension parameter space
      - Mass ratio  $q$
      - black-hole spins  $\vec{\chi}_1, \vec{\chi}_2$

# Applying simulations to GW data analysis

- Template waveforms

- Goal: predict waveforms for LIGO

- 7-dimension parameter space

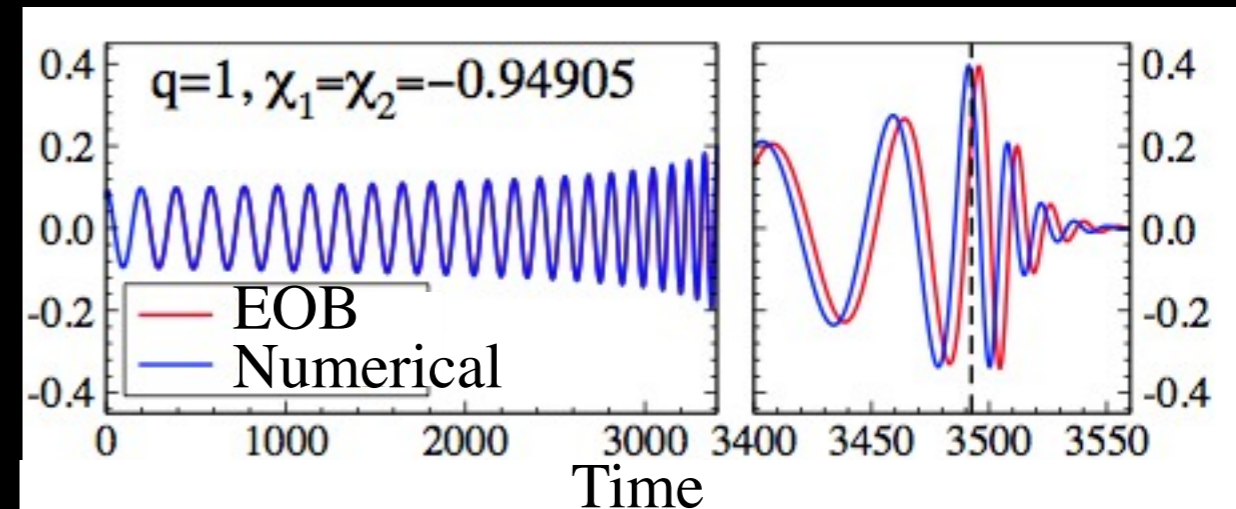
- Mass ratio  $q$

- black-hole spins  $\vec{\chi}_1, \vec{\chi}_2$

- Strategies

- Simulations to calibrate & validate approx., phenomenological waveforms

- e.g. “EOB”: Taracchini et al, PRD **86**, 024011 (2012)





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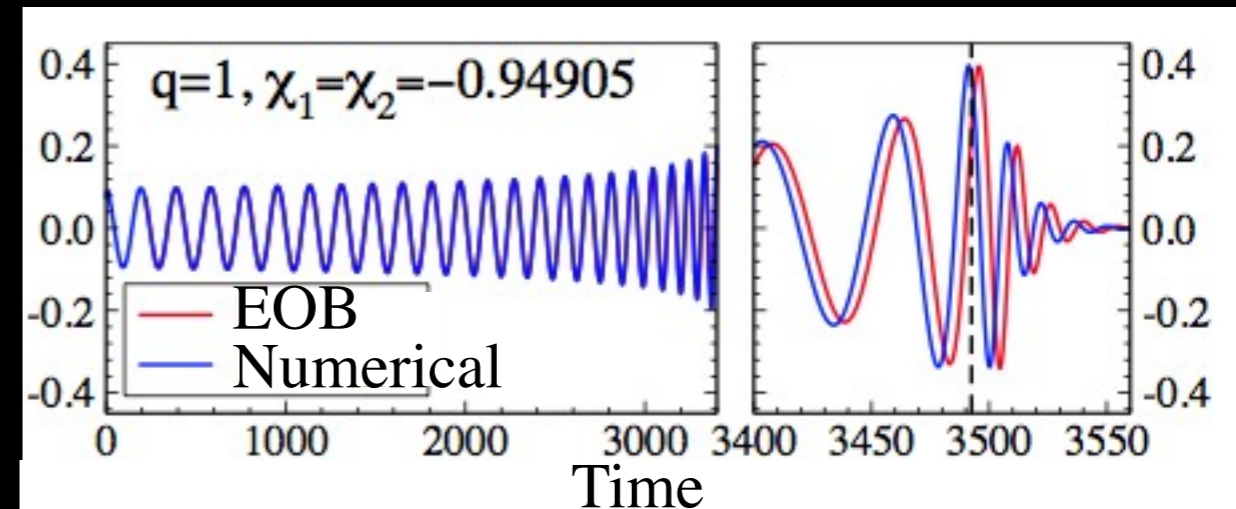
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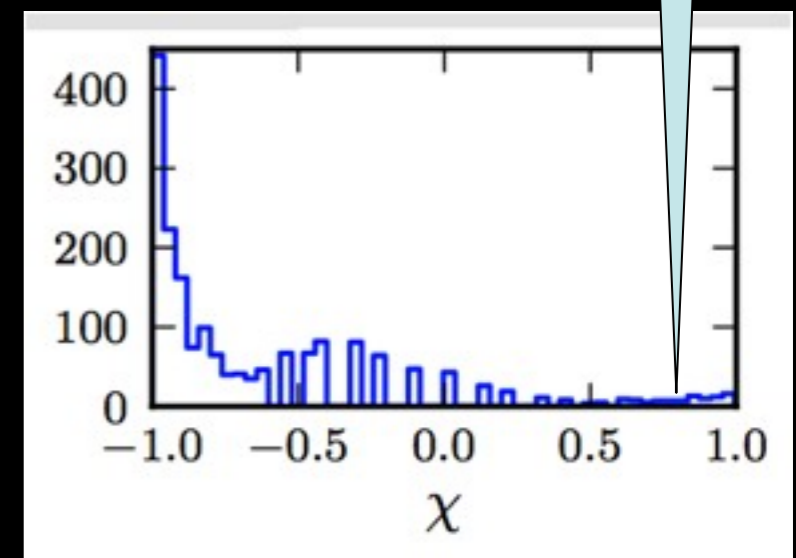
- e.g. “EOB”: Taracchini et al, PRD 86, 024011 (2012)

- Reduced basis (R.B.) Hermann et al (2012), arXiv:1205.6009

- choose “most important” waveforms, interpolate between them



# elements chosen by R.B. for equal-, aligned-spin post-Newtonian waves



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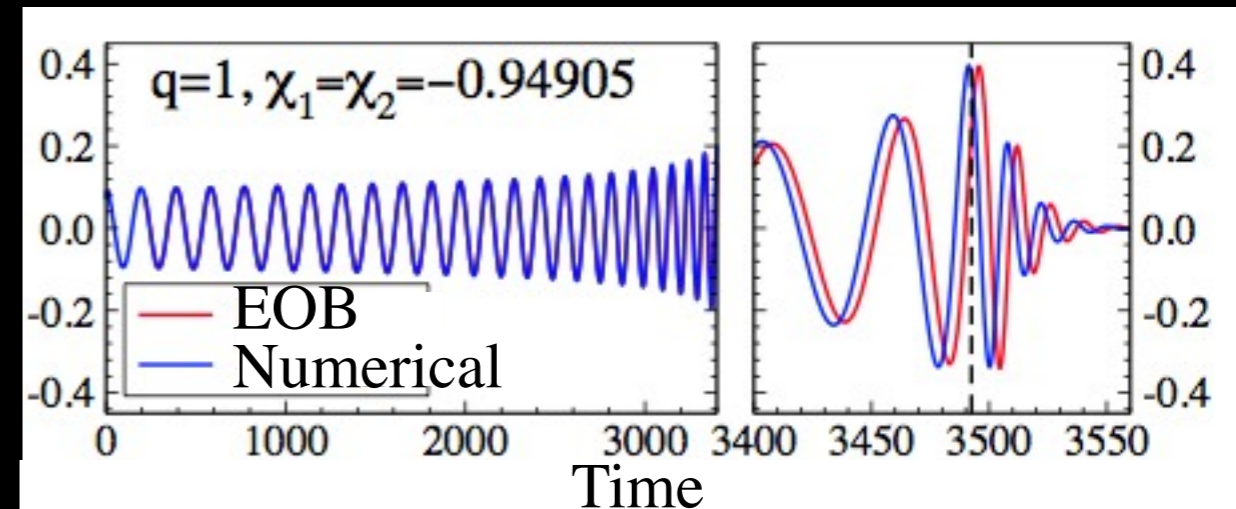
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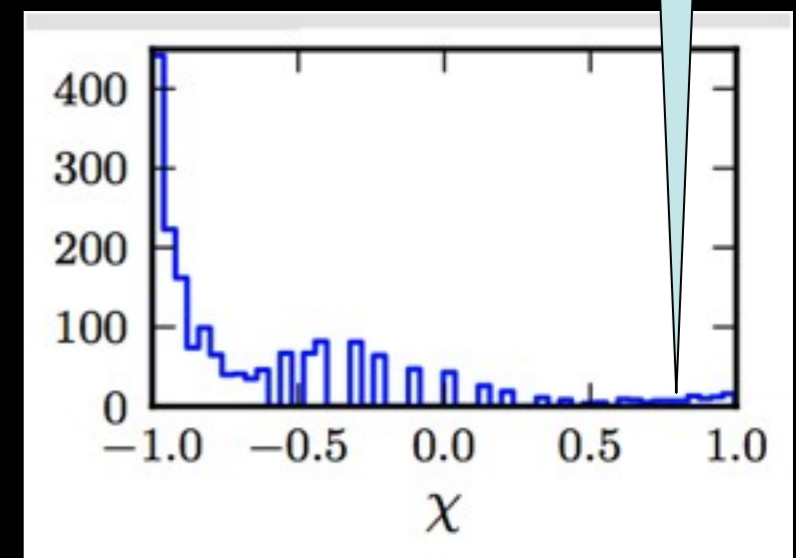
- choose “most important” waveforms, interpolate between them

- How many waveforms?

- Uncertain, but guess  $O(10^3)$

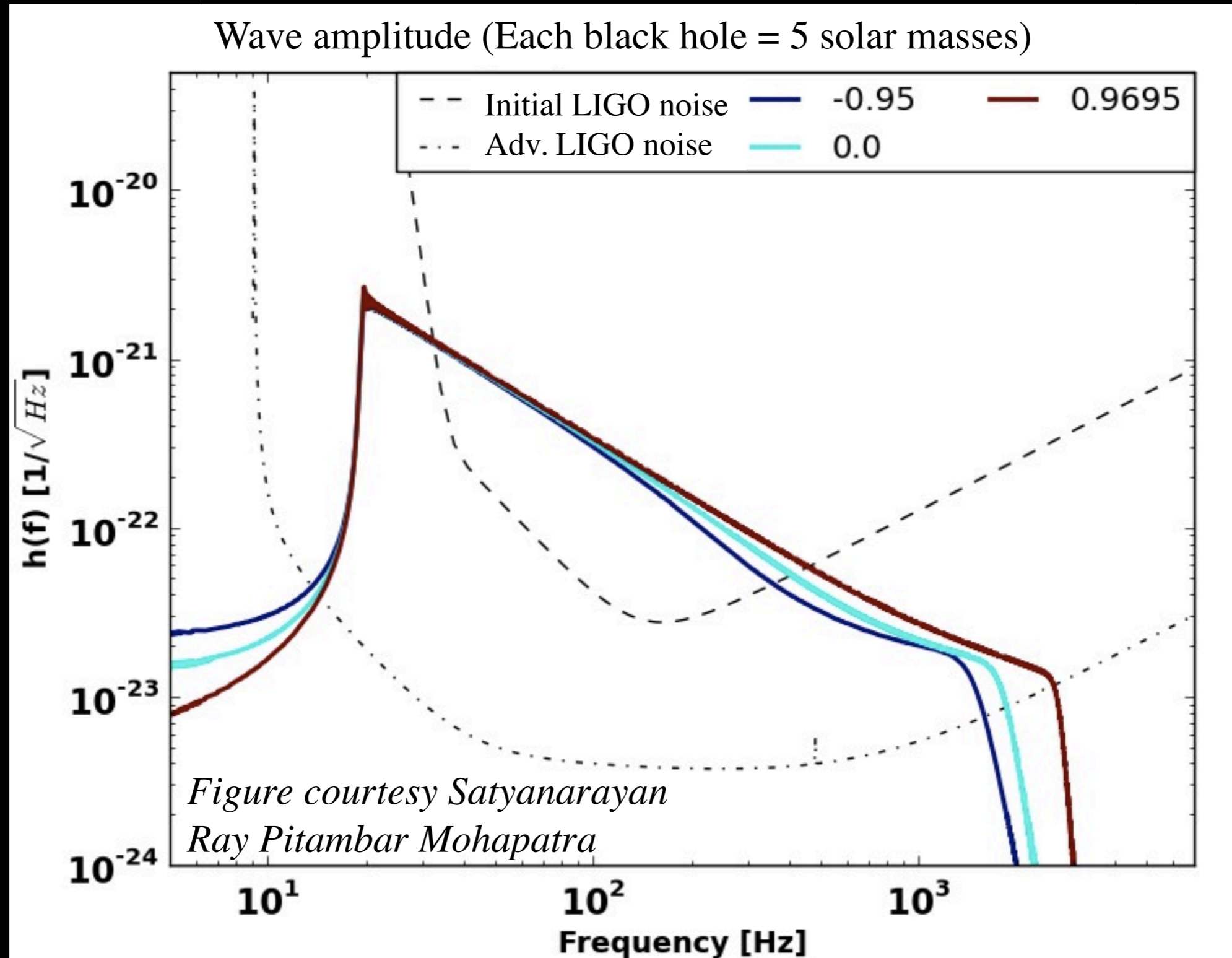


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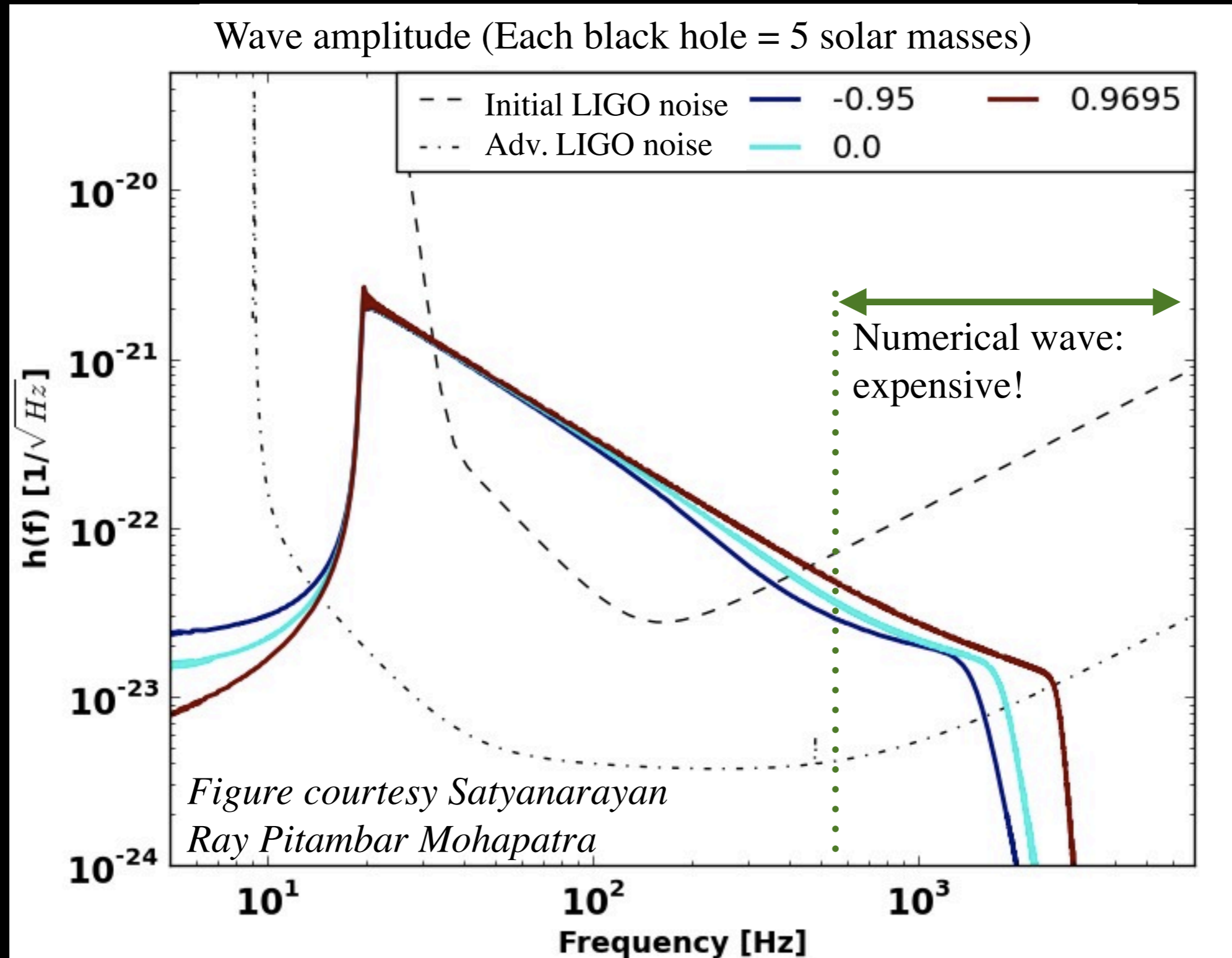
# Applying simulations to GW data analysis

- Hybrid waveforms



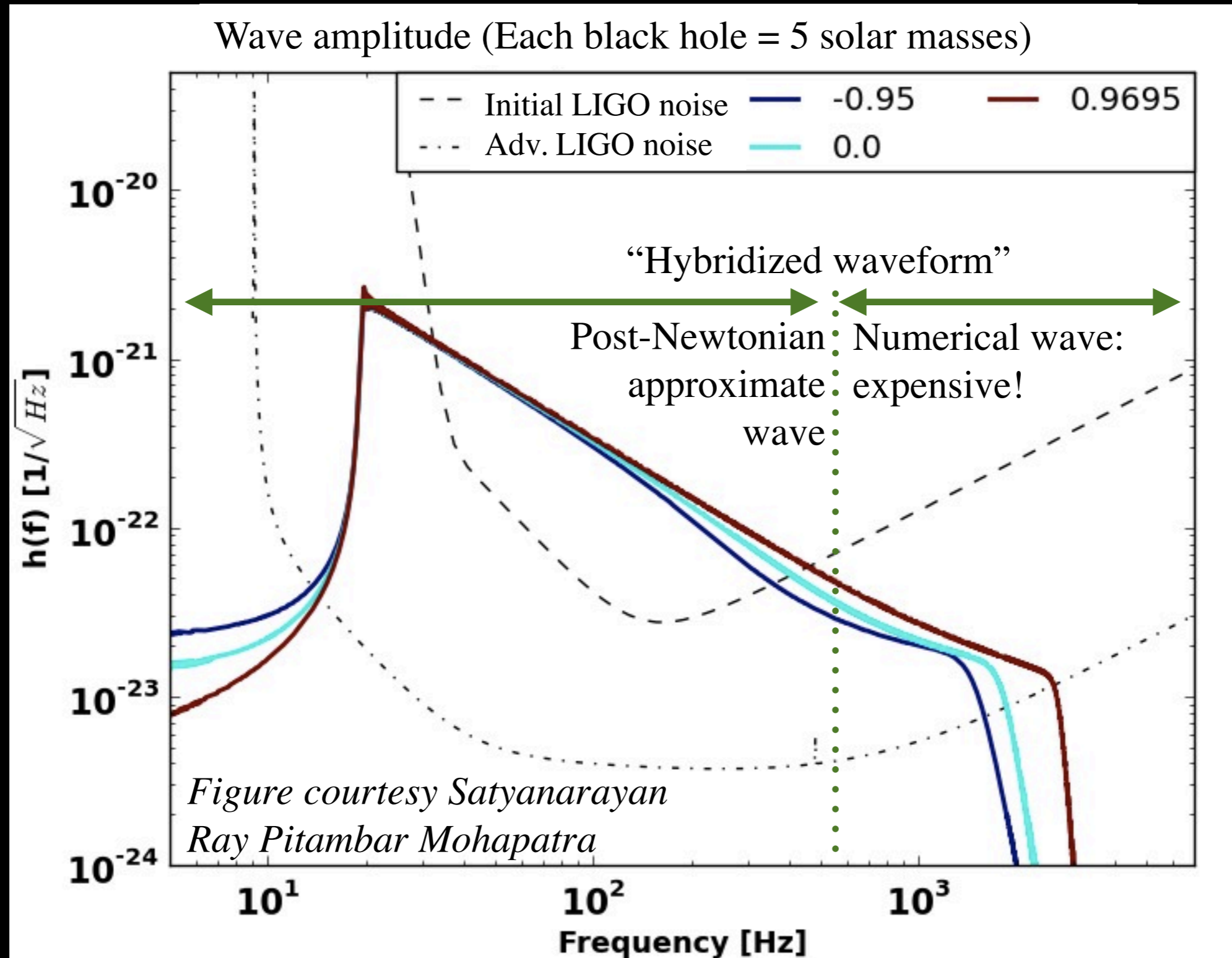
# Applying simulations to GW data analysis

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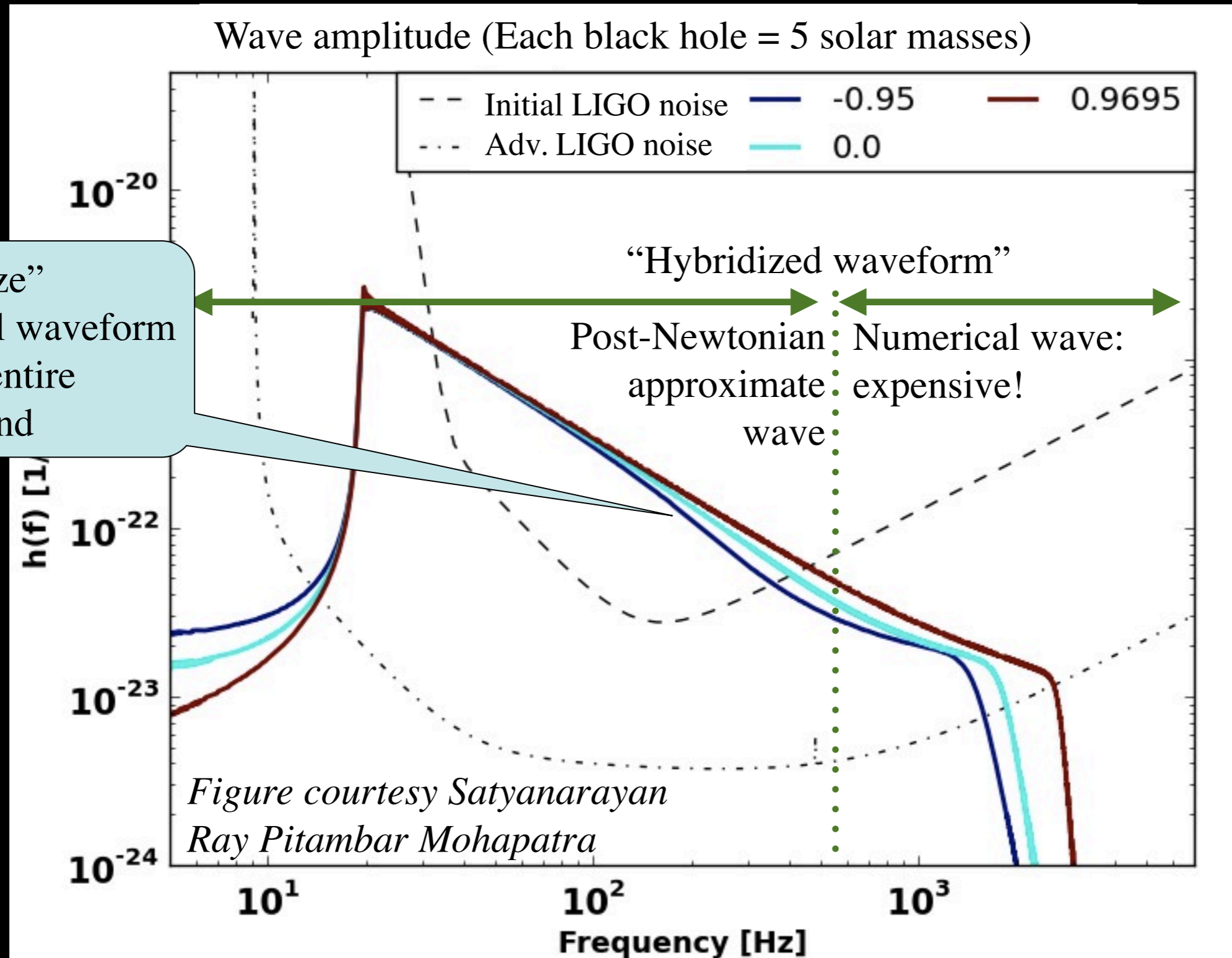
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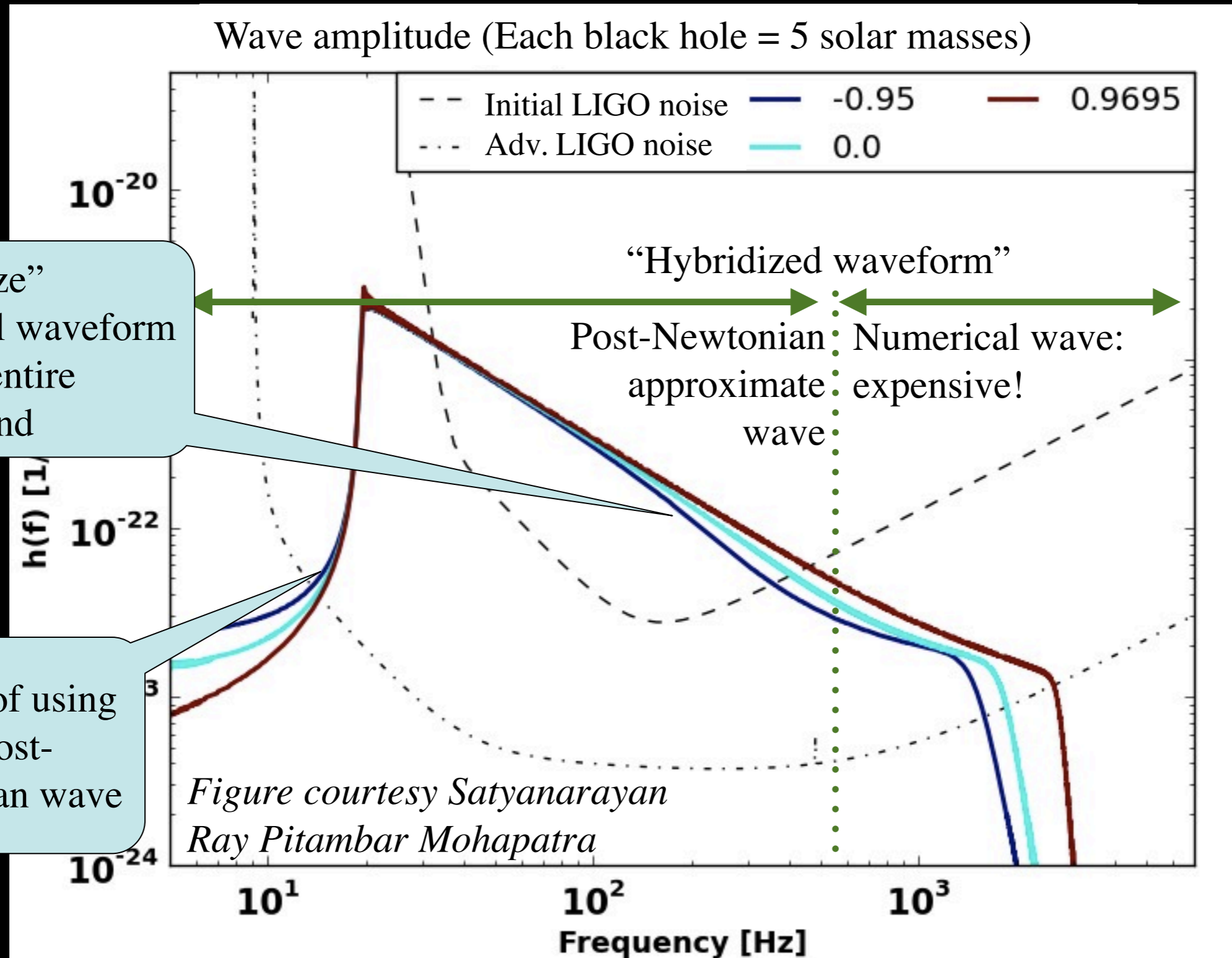
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# Applying simulations to GW data analysis

- Hybrid waveforms



# Numerical INjection Analysis (NINJA)

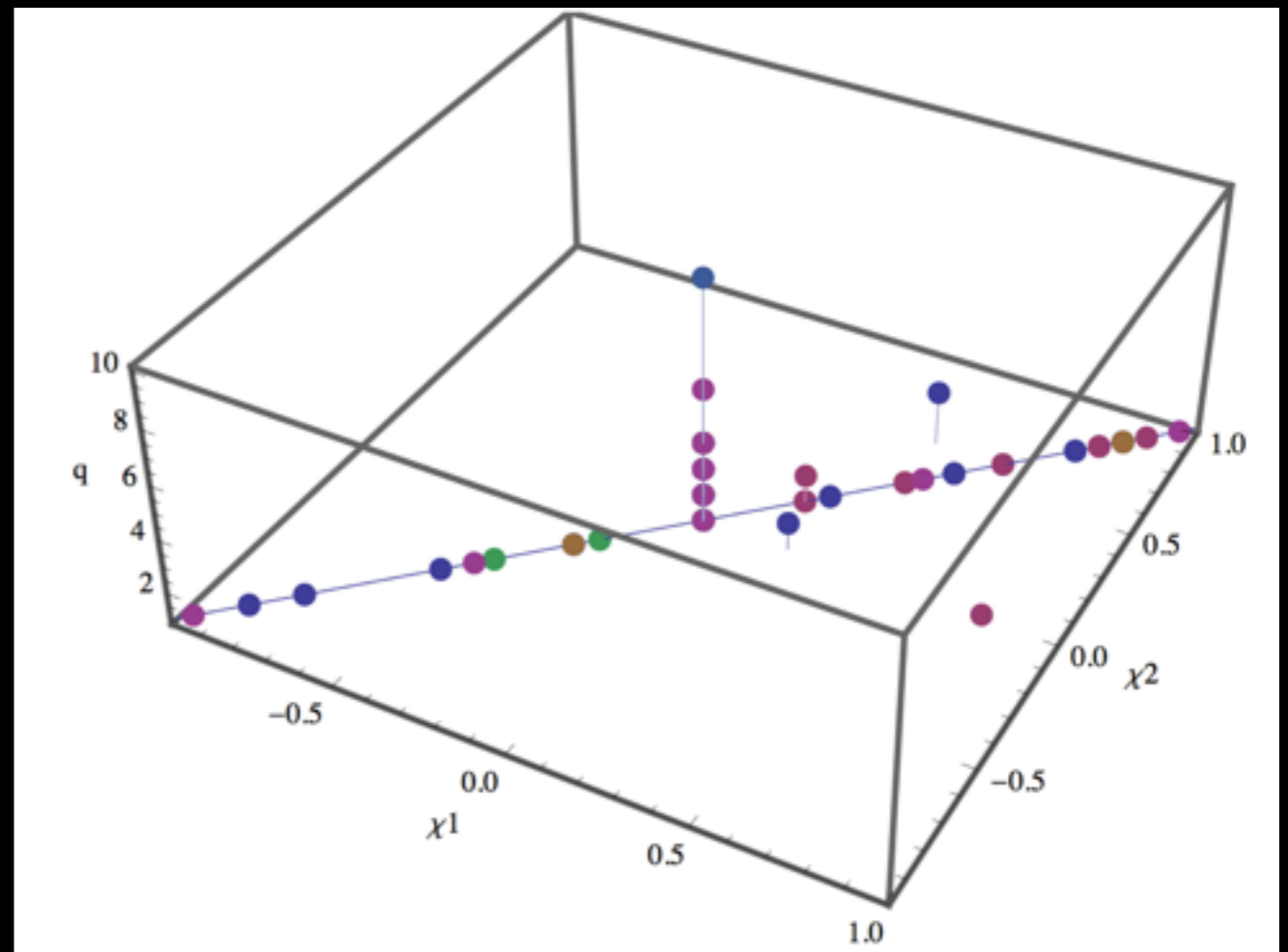


# Numerical INjection Analysis (NINJA)

- NINJA-2 project goals
  - Inject contributed numerical waveforms into LIGO data
  - Search for injected waveforms

# Numerical INjection Analysis (NINJA)

- NINJA-2 project goals
  - Inject contributed numerical waveforms into LIGO data
  - Search for injected waveforms
- Current status
  - Catalog contains 63 hybrid waveforms
    - Mostly unequal mass
    - No precession (i.e. spin aligned with orbital ang. momentum)
  - Catalog paper on arXiv (public release coming soon)
  - Now injecting waveforms & searching for them



Ajith *et al* (2012), arXiv:1201.5319  
see also poster by Thilina Dayanga

# Numerical Relativity / Analytical Relativity (NRAR)

precessing  
not precessing

# Numerical Relativity / Analytical Relativity (NRAR)

- NRAR project goals

- Simulate new waveforms, including **precession**
- Use numerical waveforms to calibrate approximate, analytic waveform templates

**precession**  
not **precession**

# Numerical Relativity / Analytical Relativity (NRAR)

- NRAR project goals

- Simulate new waveforms, including **precession**
- Use numerical waveforms to calibrate approximate, analytic waveform templates

- Current status

- 35 waveforms
- Moderate mass ratios & spins, some precession
- Catalog not yet complete

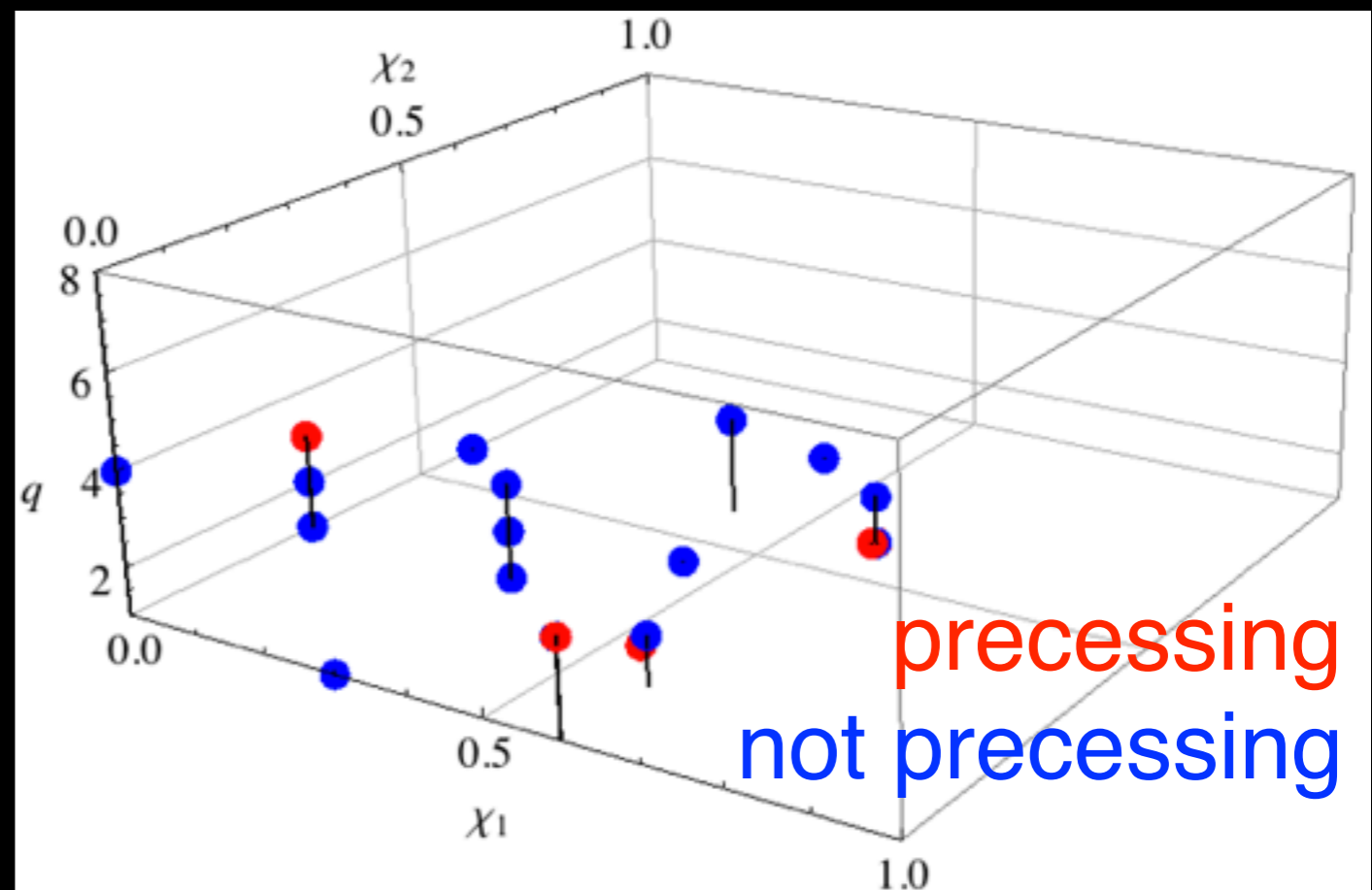


Figure courtesy NRAR project  
see also talk by Yi Pan Thursday, 11:00AM

# Efficiency

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- Cost

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- Cost

- Typical cost per simulation

Puncture codes

SpEC code

CPU hours

$O(100k \text{ cpu hours})$

$O(20k \text{ cpu hours})$

Wallclock time

$O(\text{months})$

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# Efficiency

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		CPU hours	Wallclock time
– Typical cost per simulation	Puncture codes	$O(100k \text{ cpu hours})$	$O(\text{months})$
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- Goals

- Many ( $\approx 1000$ ) simulations to calibrate templates
- Few rapid [ $O(\text{days})$ ] simulations: interface with data analysis
- New methods needed

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CPU hours

Wallclock time

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|----------------|-------------------|-----------|
| Puncture codes | O(100k cpu hours) | O(months) |
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- Few rapid [ $O(\text{days})$ ] simulations: interface with data analysis

- New methods needed

Hannam et al, PRD **82**, 124052, (2010)

Damour et al, PRD **83**, 024006 (2011)

Ohme et al, PRD **84**, 064029 (2011)

MacDonald et al, CQG **28**, 134002 (2011)

Boyle, PRD **84**, 064013 (2011)

- Accuracy & length

- Parameter estimation

- Much longer waveforms than common today
- Better post-Newtonian for better hybrid waveforms

See also talk by Sascha Husa (Thursday, 12:00PM)

# Summary & questions

- Summary

- Spinning BBH important GW sources for LIGO
- Considerable progress so far
- Challenges remain
  - Goal: for GW data analysis, need large number of generic waveforms & extreme waveforms
    - Also need improved approximate, analytic models (post-Newtonian, EOB)
  - Goal: greatly reduce computational cost

- Some questions

- Priorities: what simulations most useful for data analysis?
- Production: biggest obstacles to large GW catalogs, including precessing waveforms?

