Heat transfer in convective turbulence

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Where is Ilmenau?

Johann W. Goethe

Über allen Gipfeln
Ist Ruh'
In allen Wipfeln
Spürest Du
Kaum einen Hauch;
Die Vögelein schweigen im Walde
Warte nur, balde
Ruhest Du auch.

Karlheinz Brandenburg

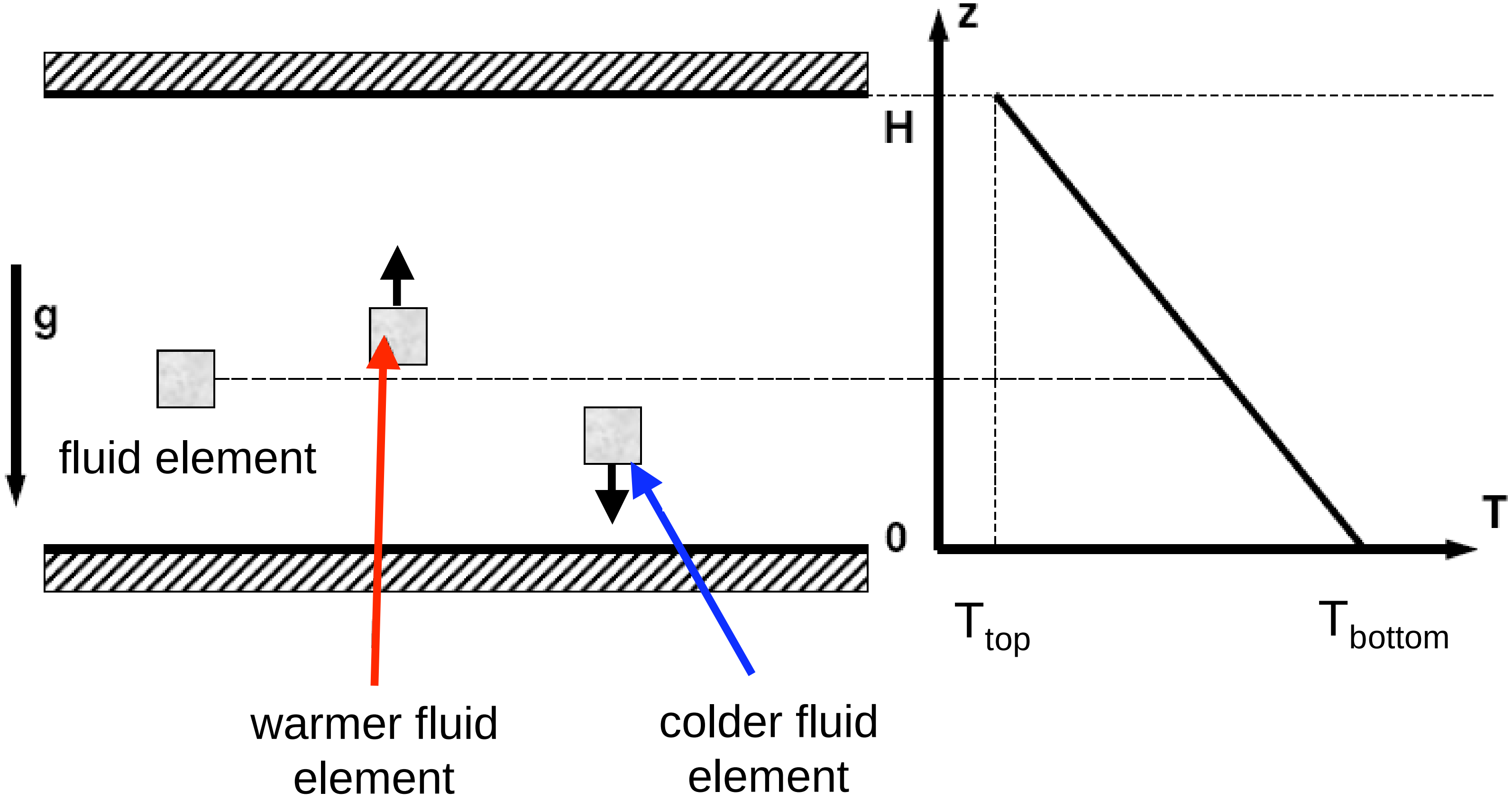
Barrel of Ilmenau

Outline

- Introduction
- Local: Boundary layer structure
- Local: Temperature and thermal dissipation
- Global: Large-scale flow patterns
- Local: Lagrangian fingerprint
- Global: Plume clusters?
- Outlook

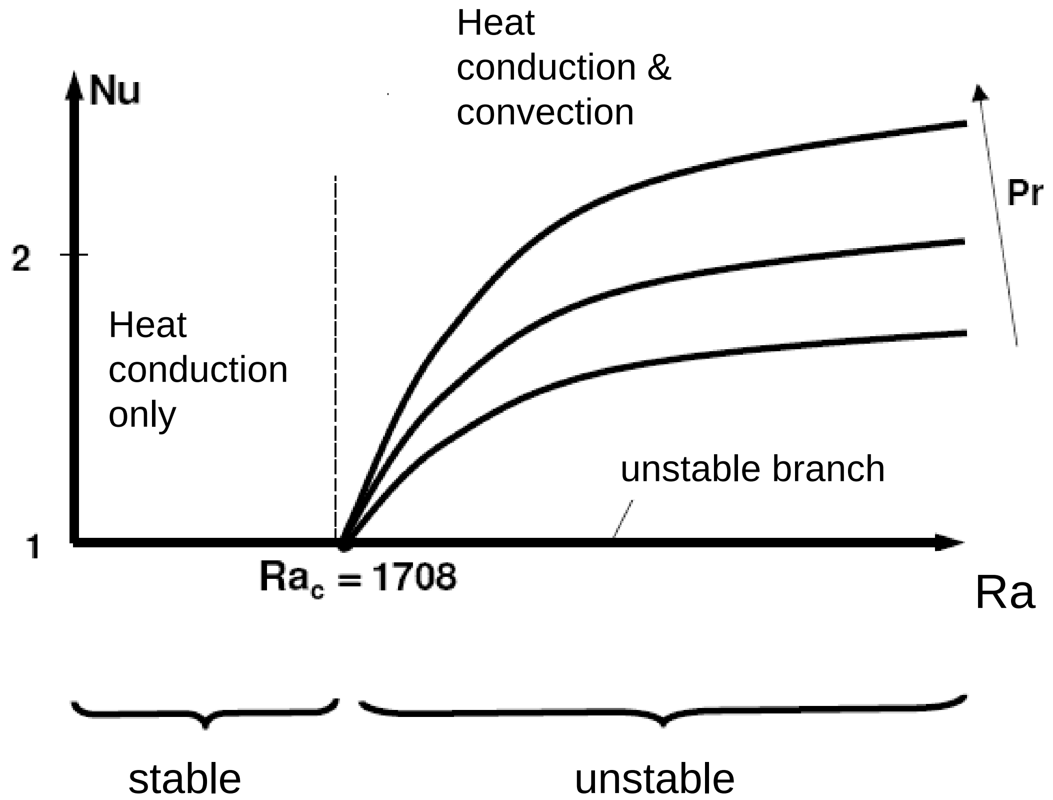
Hydrostatic equilibrium

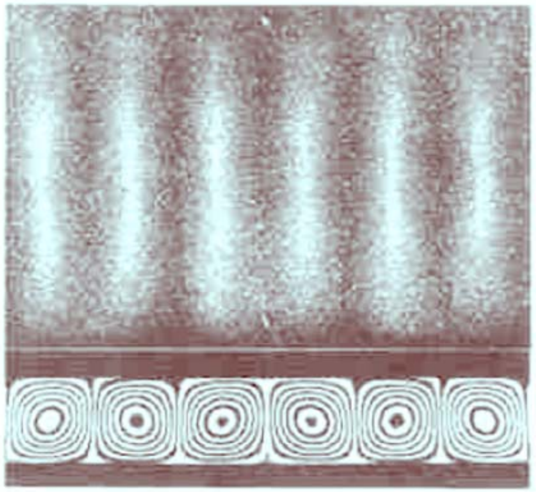
Nusselt number $$Nu = \frac{q}{q_{cond}} = 1$$



Rayleigh instability

Rayleigh, Philos. Mag. 1916; Jeffreys, Philos. Mag. 1926;
Schlüter, Lortz & Busse, J. Fluid Mech. 1965





$$Nu = \frac{q}{q_{cond}} > 1$$

Further bifurcations → Transition to convective turbulence

Model equations

Oberbeck, Ann. Phys. Chem. 1879; Boussinesq, Théorie Analytique de la Chaleur, 1903

Rotation (opt.) Buoyancy

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u}\cdot\nabla)\vec{u} = -\frac{1}{\rho_0}\nabla p - 2\Omega\vec{e}_z \times \vec{u} + \nu\nabla^2\vec{u} + \alpha g\theta\vec{e}_z$$

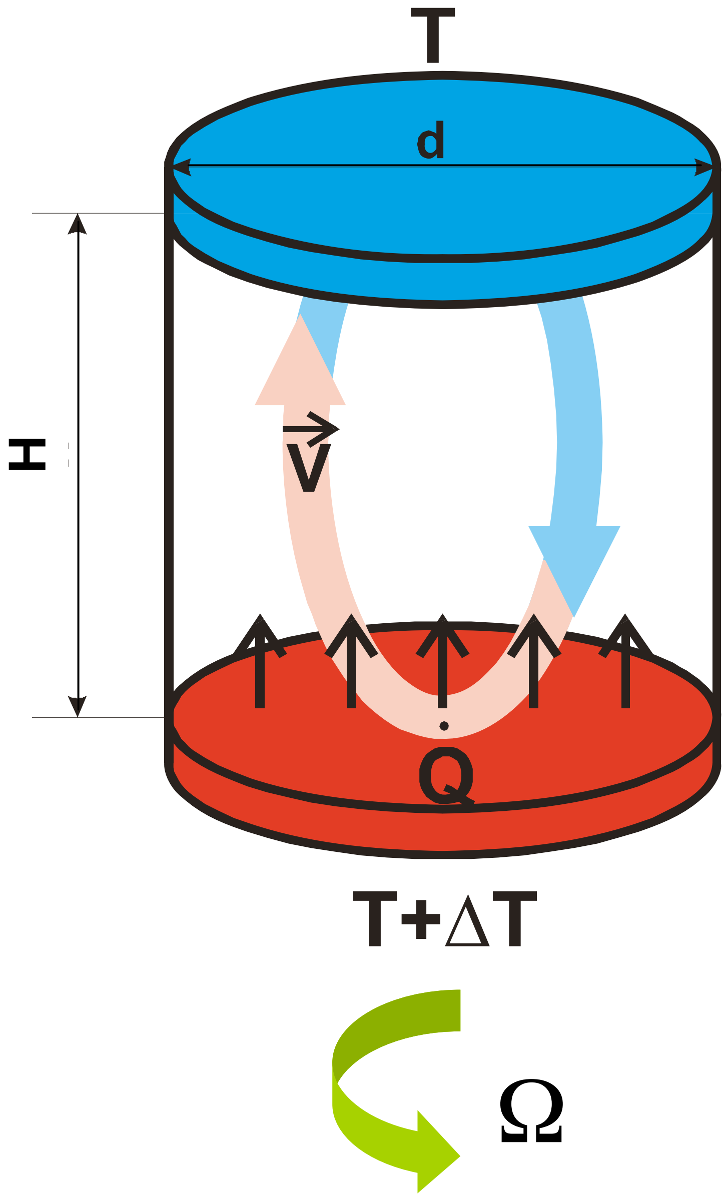
$$\frac{\partial \theta}{\partial t} + (\vec{u}\cdot\nabla)\theta = \kappa\nabla^2\theta$$

$$\nabla\cdot\vec{u} = 0$$

Approximations

- Density is linear function of temperature → Boussinesq-Approximation
- Flow is incompressible (u much smaller than speed of sound)

Rayleigh-Bénard convection



Dimensionless control parameters

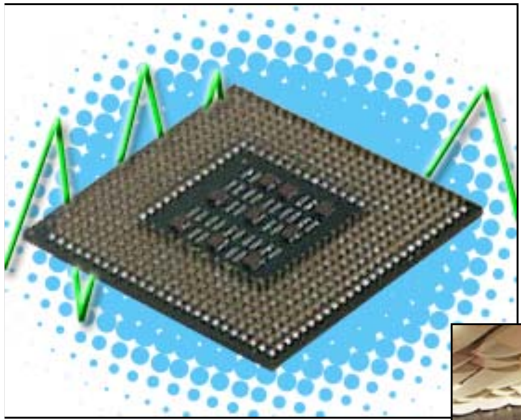
$$Ra = \frac{\alpha g \Delta T H^3}{\nu \kappa}$$

$$Pr = \frac{\nu}{\kappa}$$

$$\Gamma = \frac{d}{H}$$

$$Ta = \frac{4\Omega^2 H^4}{\nu^2}$$

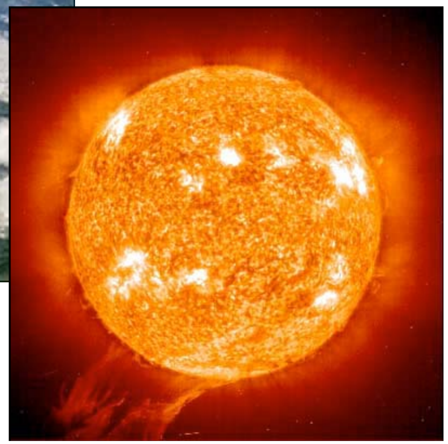
Examples



Semiconductor chips, Ra=10^6

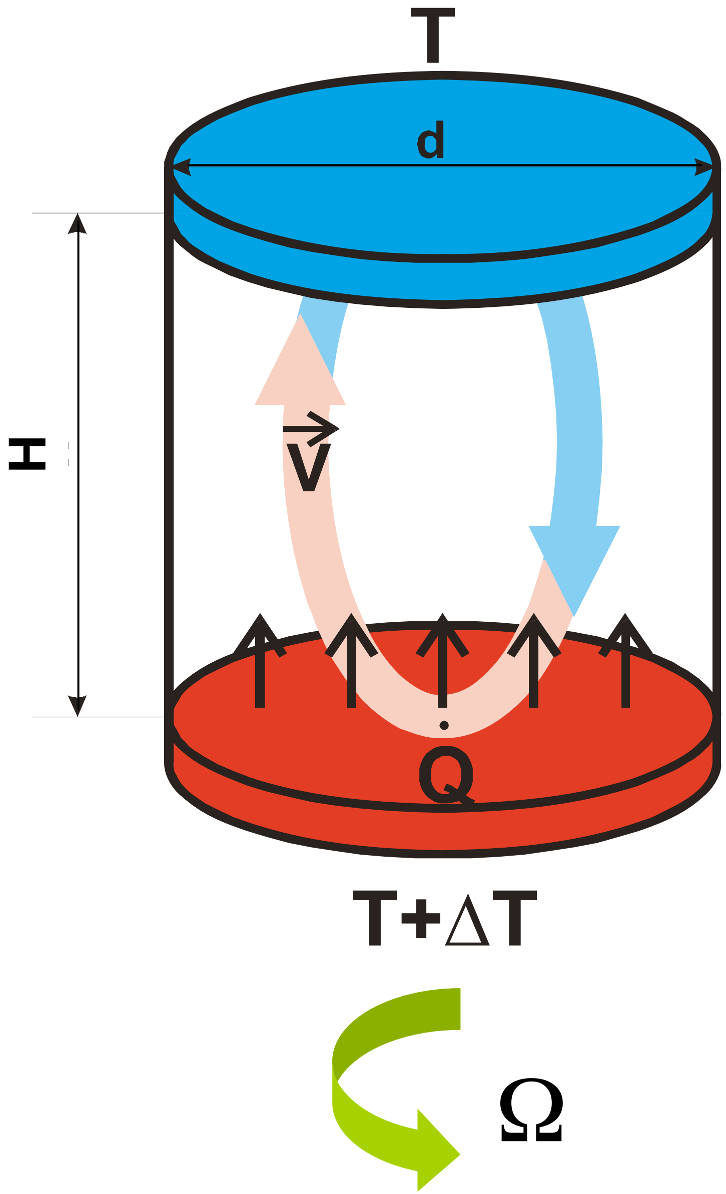
Buildings, Ra=10^{12}

Atmosphere, Ra=10^{20}



Sun, Ra=10^{23}

Rayleigh-Bénard convection



„System response“

$$Nu = \frac{\dot{Q}}{\kappa \Delta T / H} = f(Ra, Pr, Ta, \Gamma)$$

$$Re = \frac{UH}{\nu} = g(Ra, Pr, Ta, \Gamma)$$

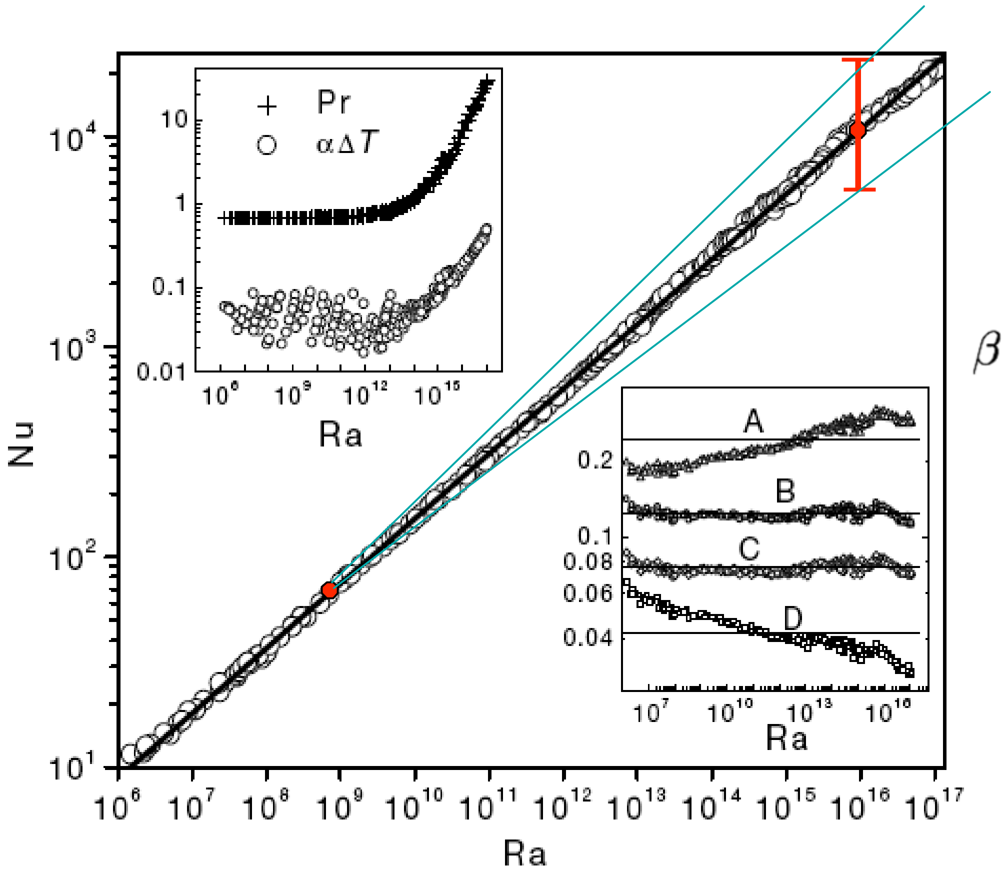
Power law for heat transfer (for fixed Pr)

$$Nu = C Ra^{\beta}$$

β=2/7 or 1/3?

Shraiman & Siggia, Phys. Rev. A 1991; Großmann & Lohse, J. Fluid Mech. 2000

Different models for the velocity boundary layer



Niemela et al., Nature 2000

$$Nu = C Ra^{\beta}$$

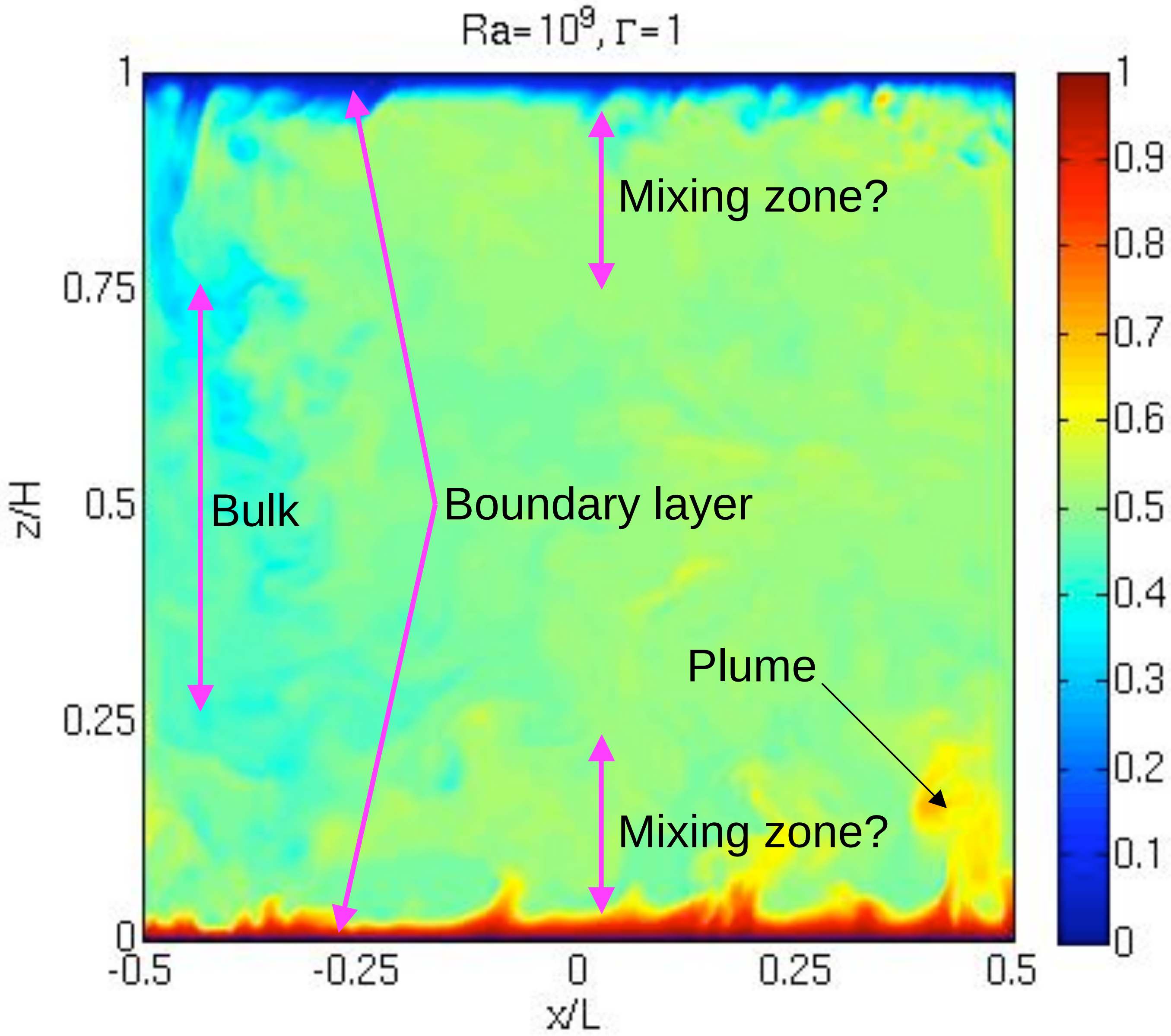
$$\beta = \frac{2}{7} \quad : \quad Nu(Ra = 10^{16}) = 4620$$

$$\beta = 0.309 \quad : \quad Nu(Ra = 10^{16}) = 10900$$

$$\beta = \frac{1}{3} \quad : \quad Nu(Ra = 10^{16}) = 26712$$

Deviation by more than 100% !!!

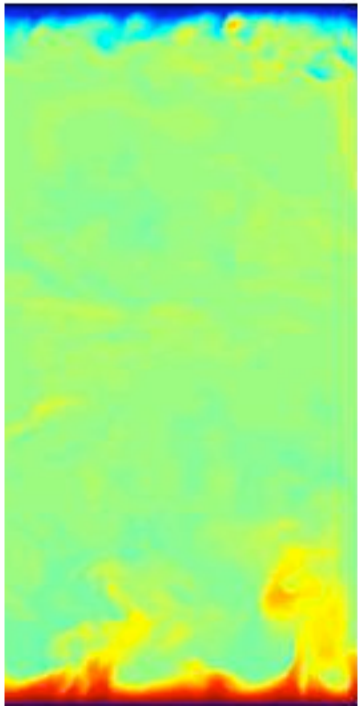
Regions in convective turbulence

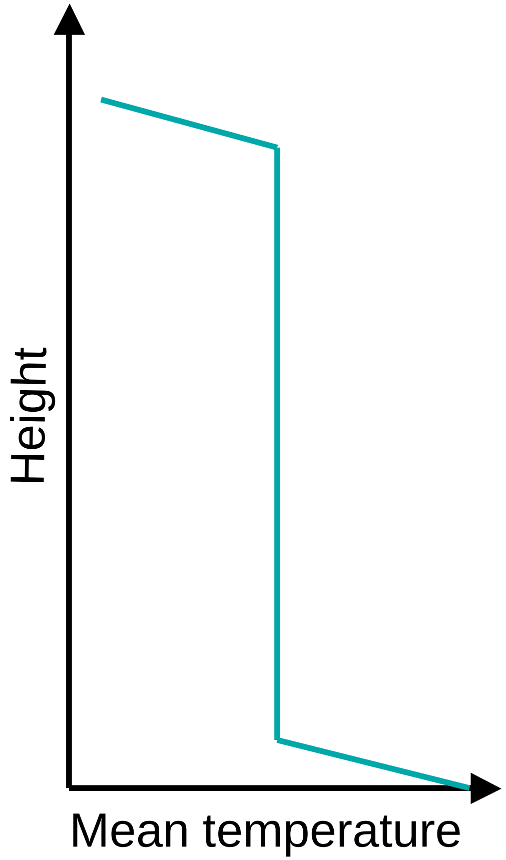


Boundary layer thickness: 0.01

Why study mean profiles?

Turbulent temperature field





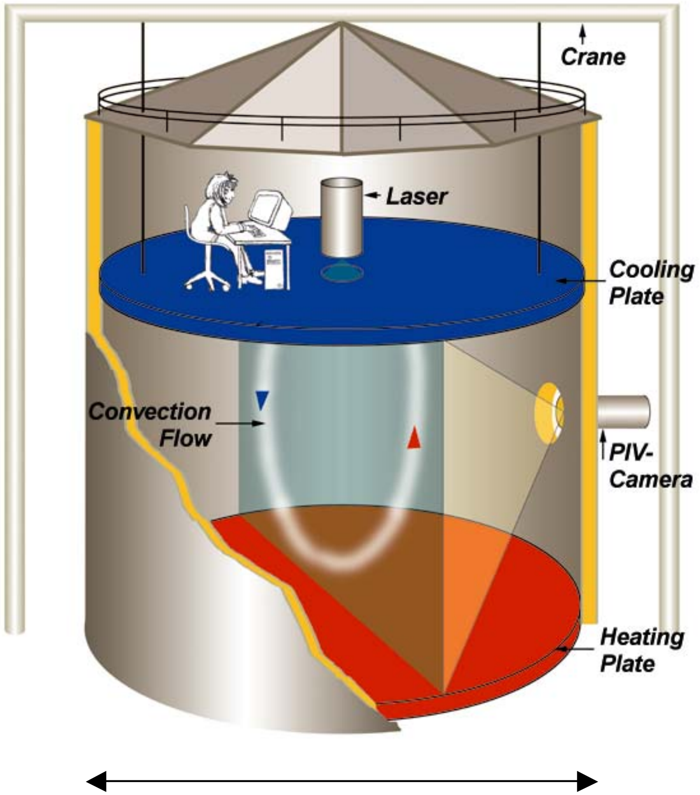
„Response“ „Perturbation“

$$Nu = C Ra^{\beta}$$

Turbulent structure ⟷ **Mean profile** ⟷ **Transport law**

Worlds biggest convection experiment

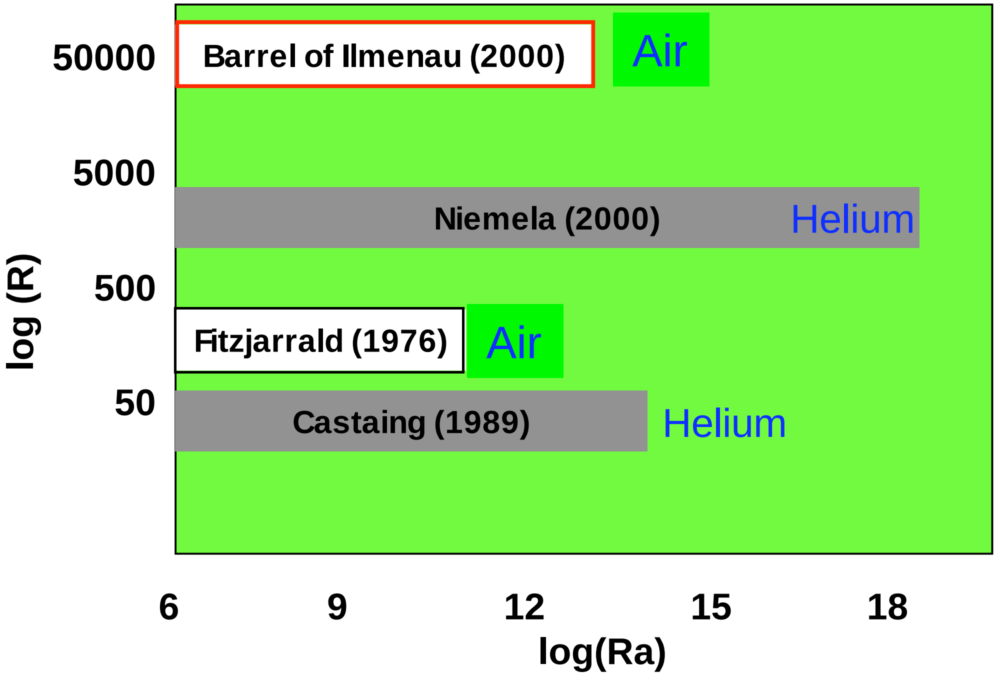
„Barrel of Ilmenau“

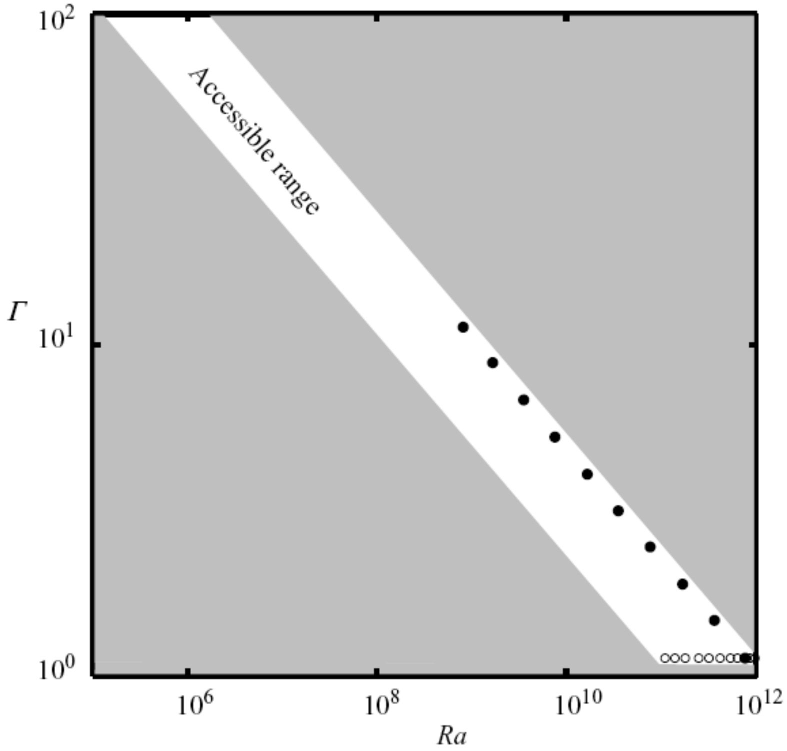


7 Meters

Accessible range in the Barrel

$$R = \frac{\text{Size of Experiment}}{\text{Size of Sensor}}$$

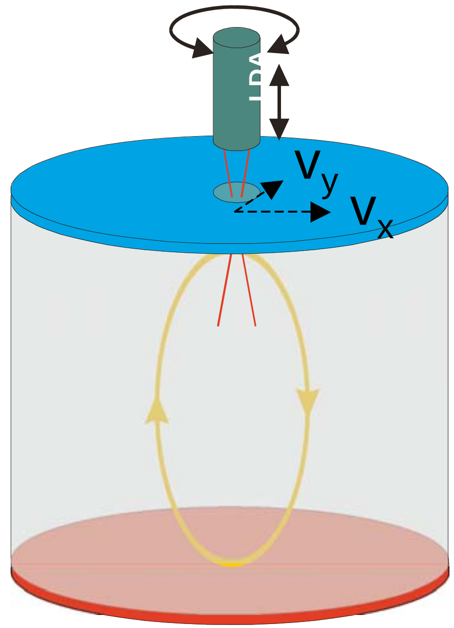


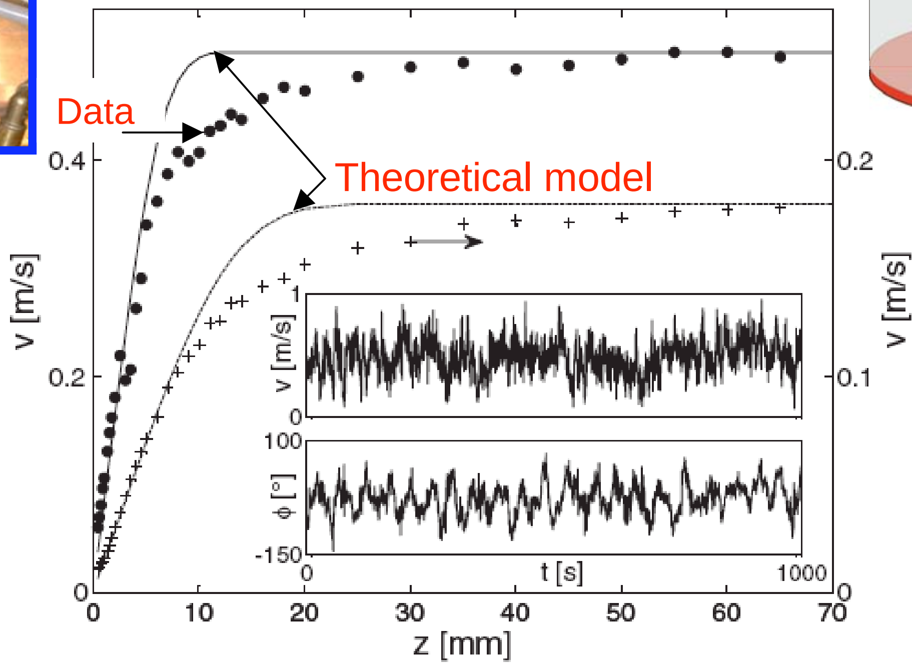


Velocity measurements

du Puits, Resagk & Thess, Phys. Rev. Lett. 2007

Laser-Doppler-Anemometry

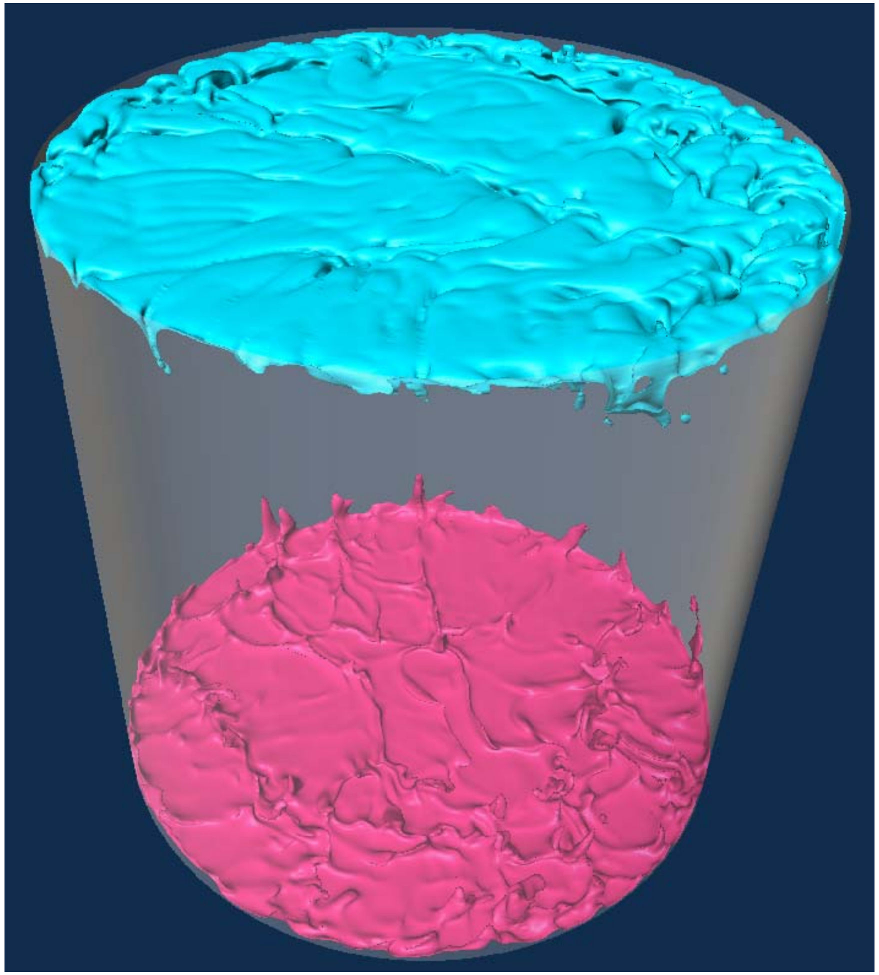




Resolve boundary layers!!

Numerical simulations in cylindrical cell

Verzicco & Orlandi, JCP 1996



Proper resolution of boundary layers limits accessible Ra

$$\delta_T = \frac{H}{2Nu}$$

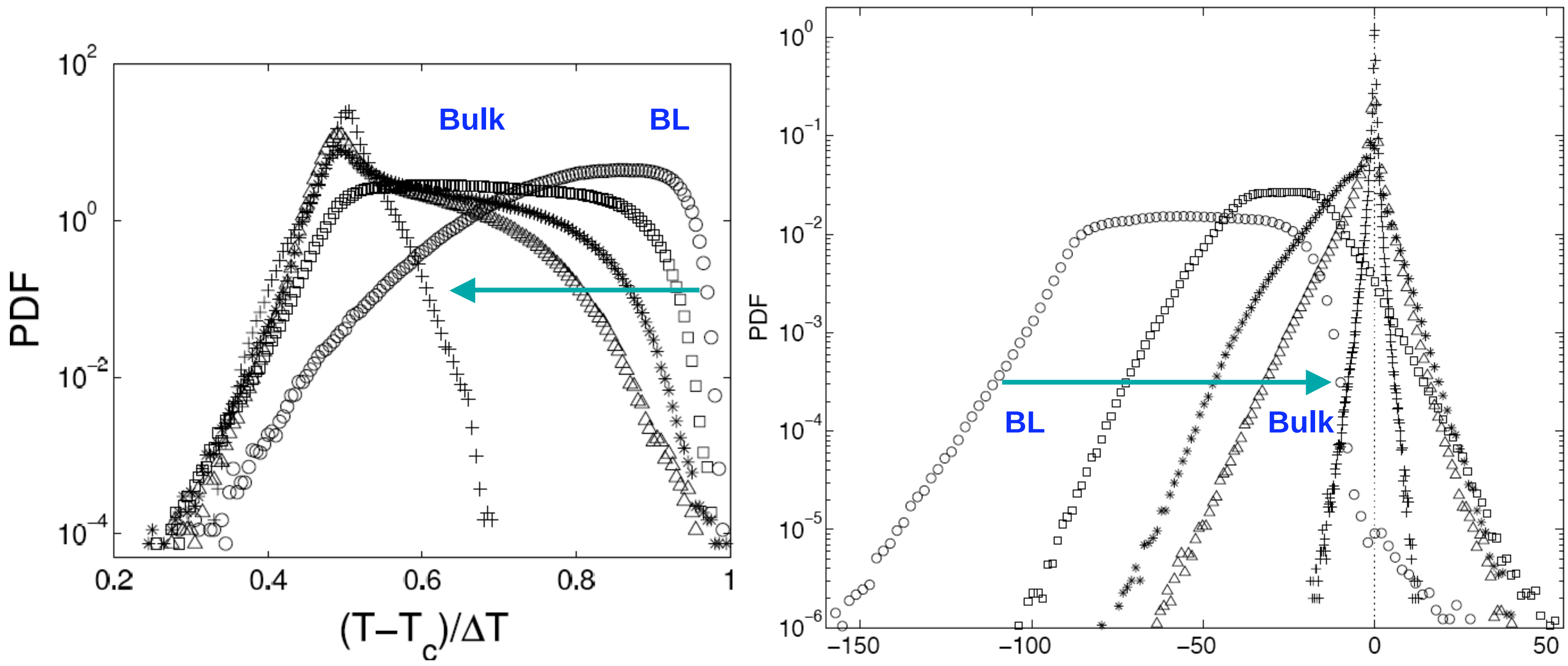
$Ra=10^9$	$\delta_T = 0.016\ H$
$Ra=10^{12}$	$\delta_T = 0.0014\ H$
$Ra=10^{17}$	$\delta_T = 0.00005\ H$

No-slip boundaries, adiabatic side walls

Second-order finite difference scheme

Temperature & temperature derivative

Emran & Schumacher, J. Fluid Mech., in revision



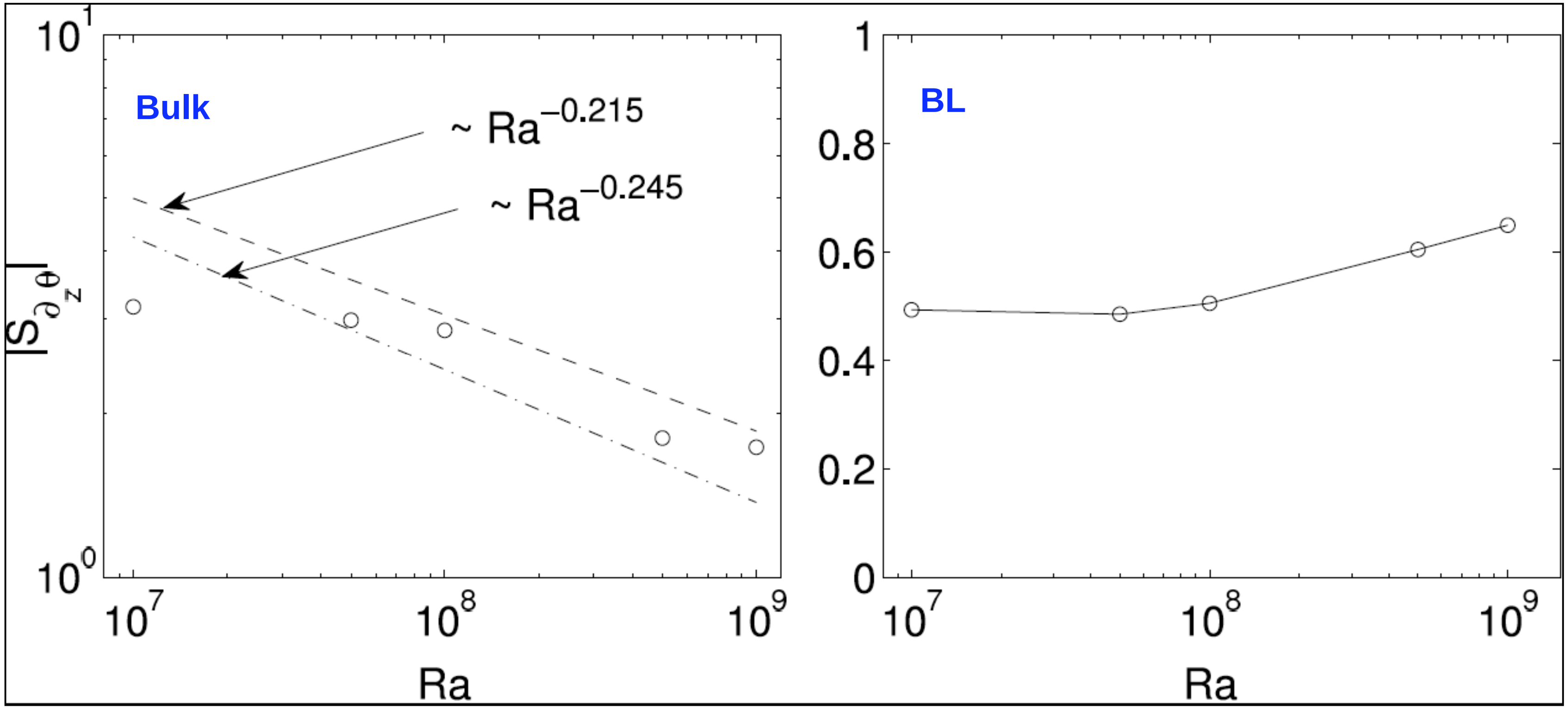
Strong vertical dependence of statistics of turbulent temperature field

Deviations from local isotropy

Schumacher & Sreenivasan, Phys. Rev. Lett. 2003

$$S_{\partial_z \theta} = \frac{\langle (\partial_z \theta)^3 \rangle}{\langle (\partial_z \theta)^2 \rangle^{3/2}}$$

$$T(\vec{x}, t) = \langle T \rangle_A(z) + \theta(\vec{x}, t)$$



Return to local isotropy (K41) for temperature fluctuations in bulk

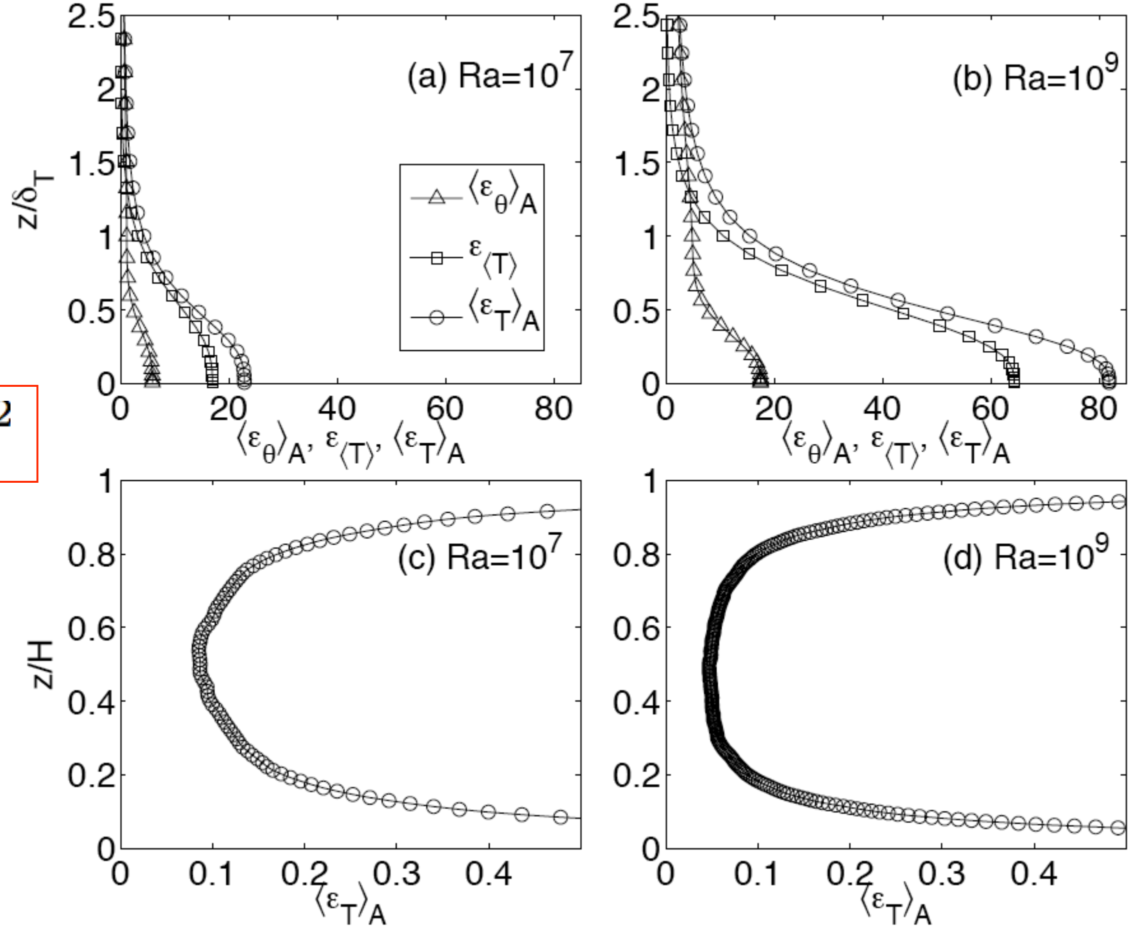
→ Formation of „superconducting core“ in high-Ra turbulence

Niemela & Sreenivasan, Phys. Rev. Lett. 2008

Thermal dissipation rate

Emran & Schumacher, J. Fluid Mech., in revision

$$\epsilon_\theta = \kappa(\nabla T)^2$$



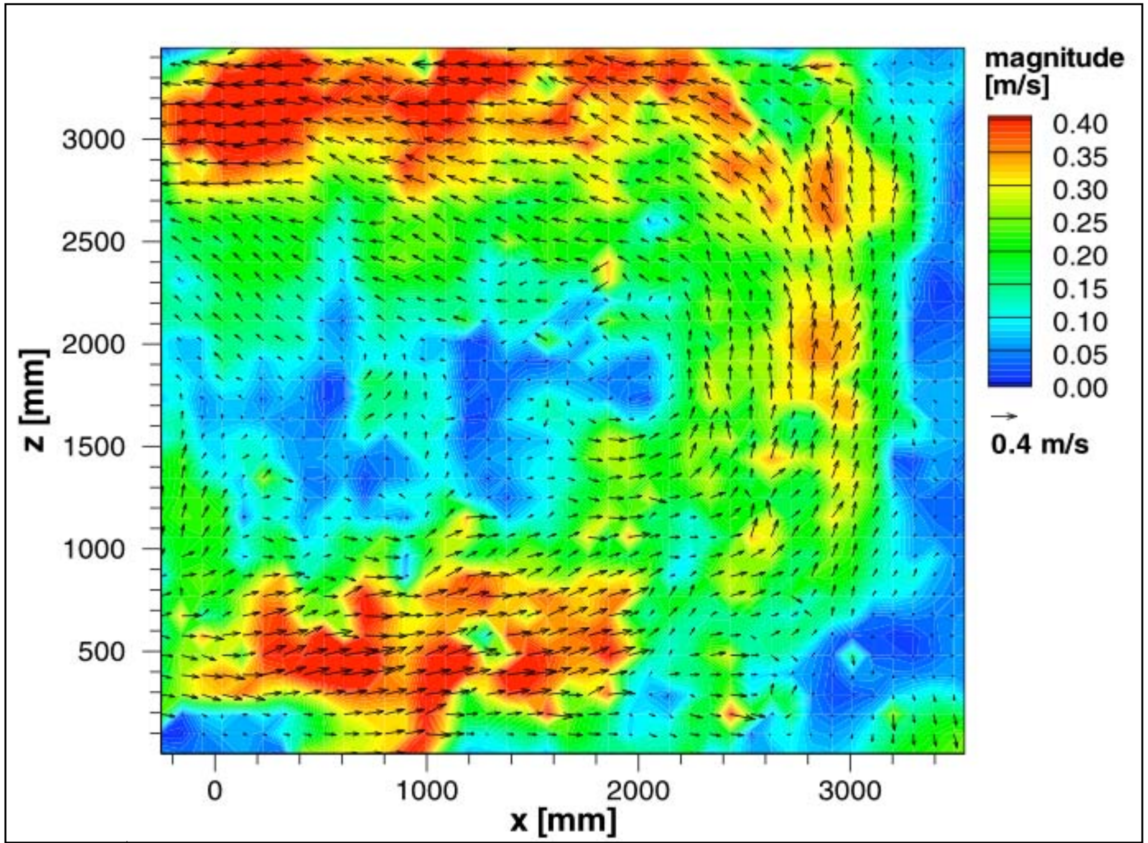
Significant contribution to dissipation due to temperature fluctuations

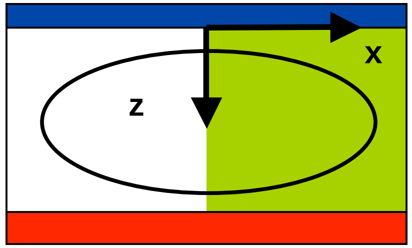
→ Does not enter the Großmann-Lohse scaling theory!

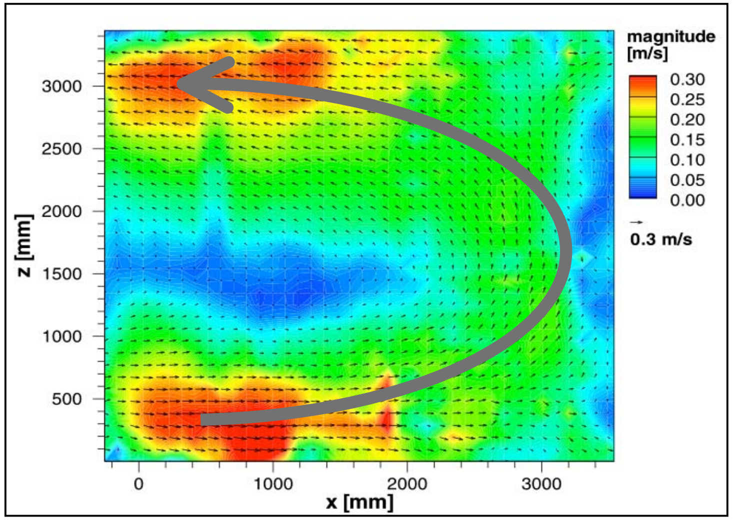
Großmann & Lohse, J. Fluid Mech. 2000

„Wind of turbulence“

Barrel: Visualisation with He bubbles



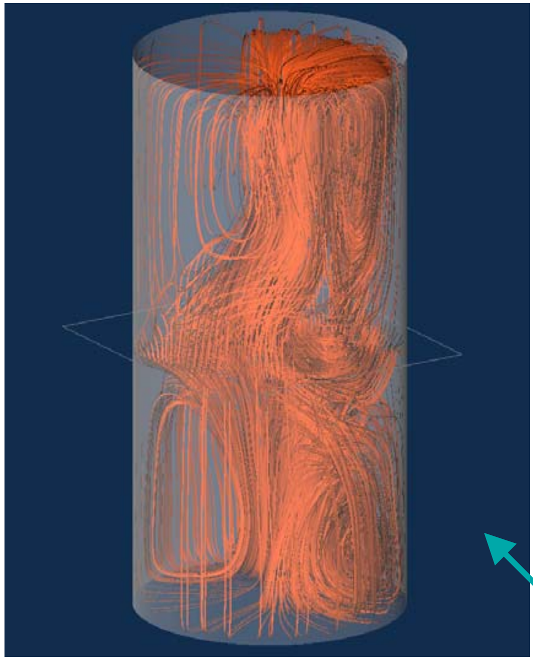




Mean (114 double frames) and snapshot

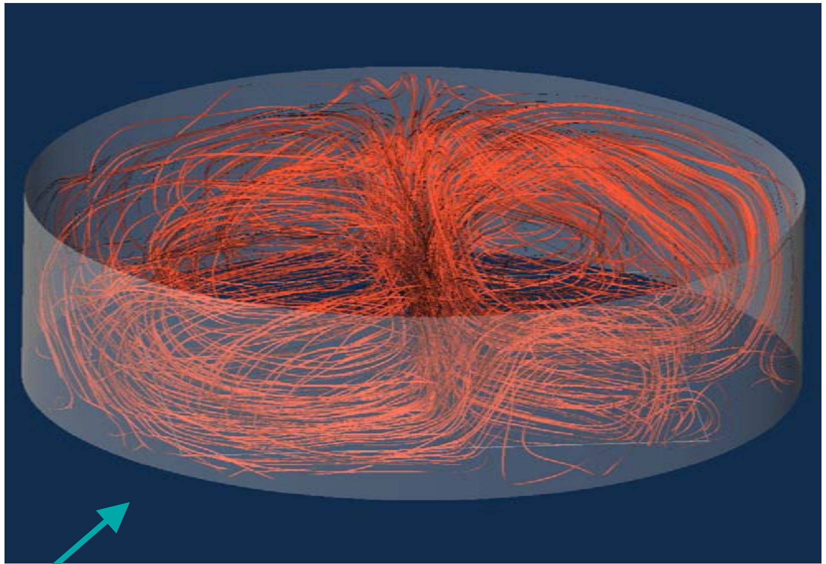
$Ra=10^{11}$, $\Gamma=2$

Geometry dependence of heat transport

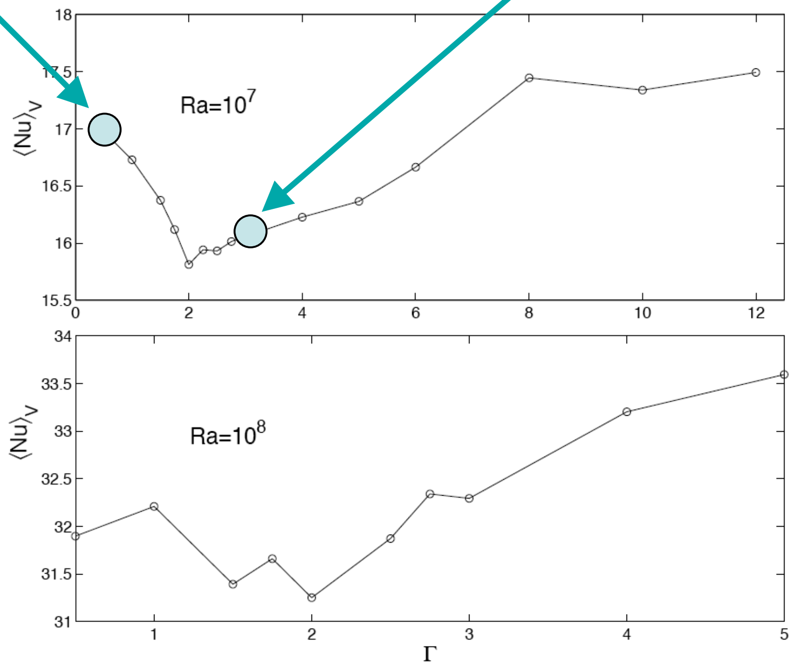


$\Gamma = \frac{1}{2}$

Time-averaged
streamlines



$\Gamma = 3$

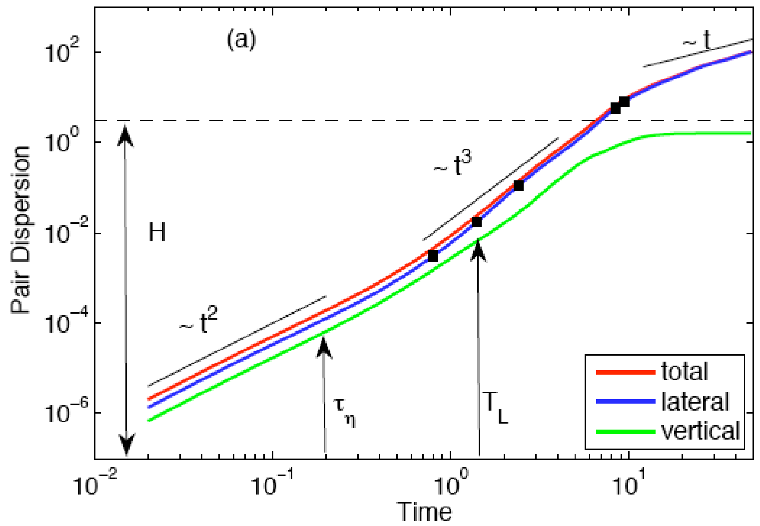


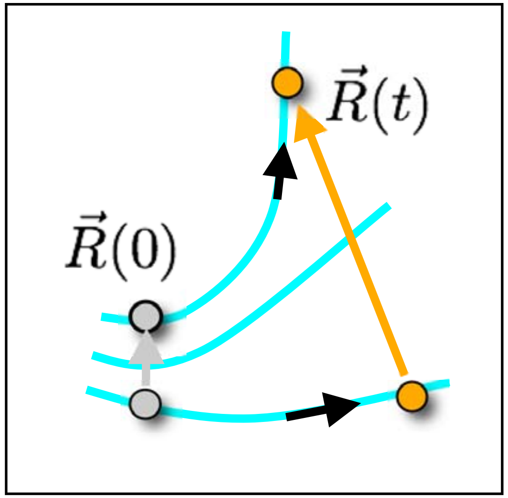
10% effect

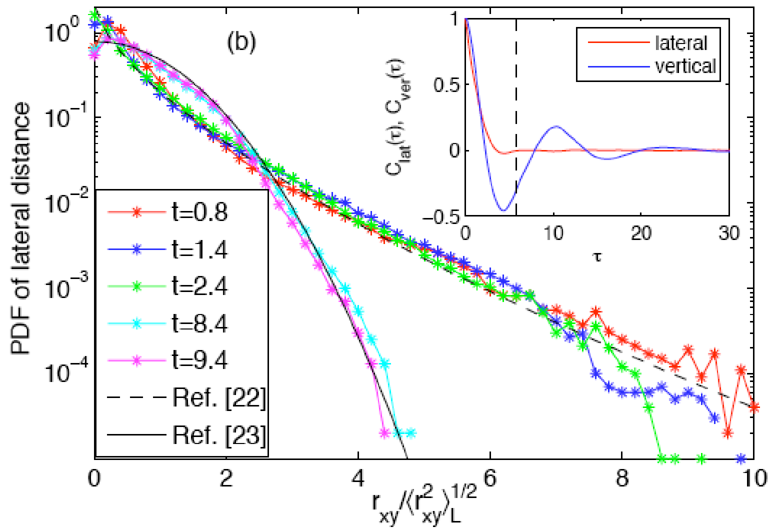
POD analysis

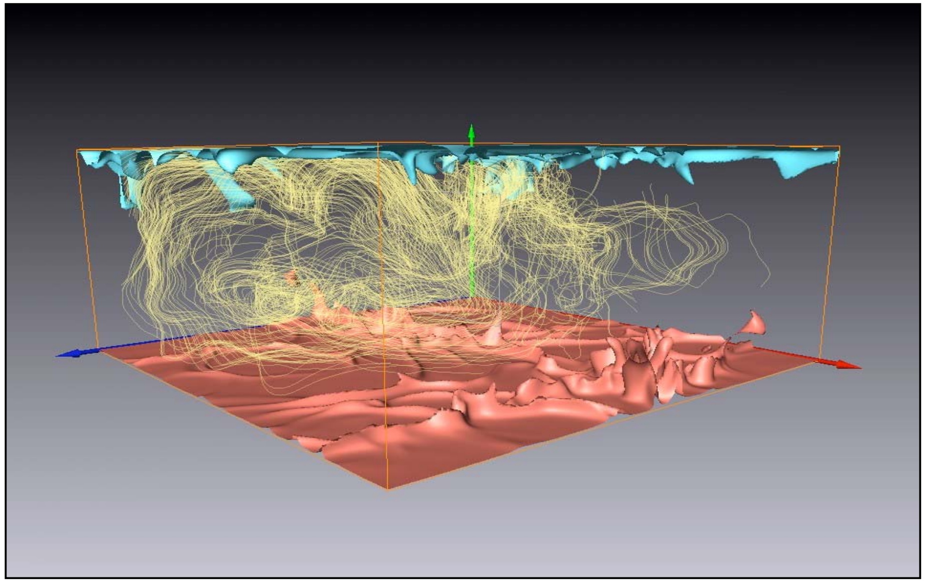
Pair dispersion in turbulent convection

Schumacher, Phys. Rev. Lett. 2008

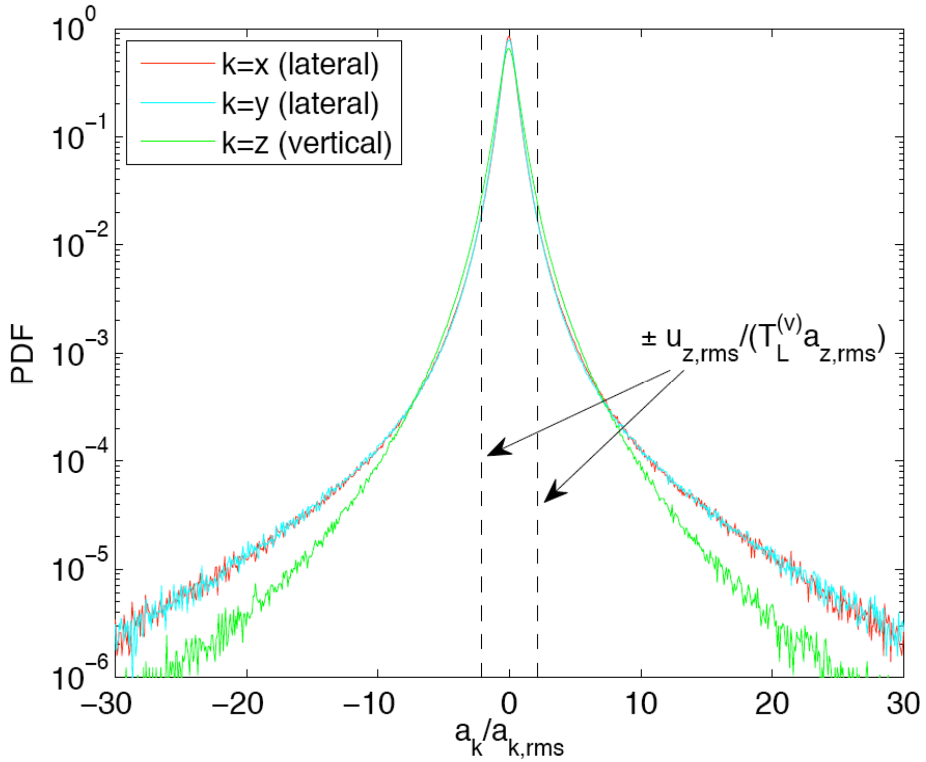


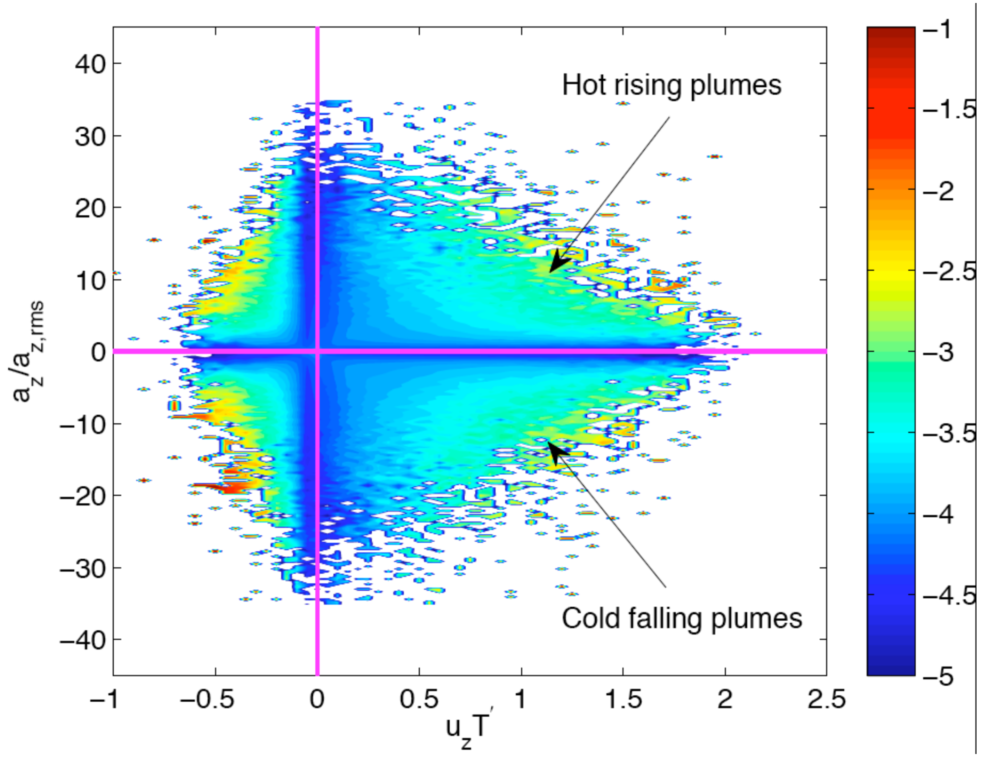






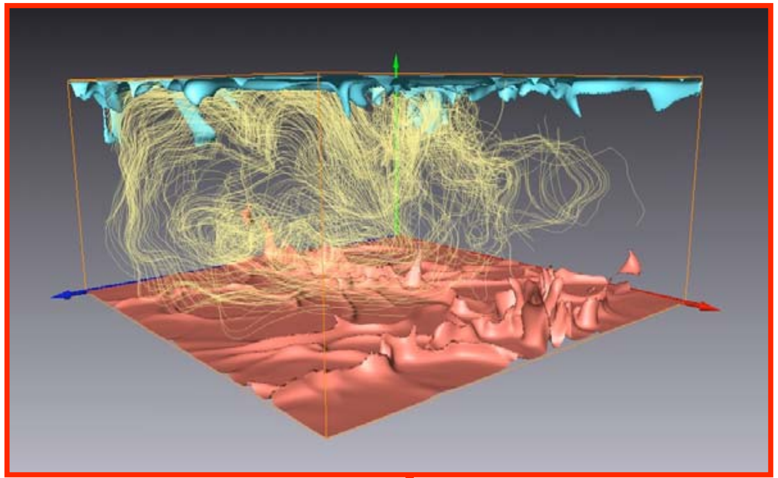
Lagrangian fingerprint of thermal plumes





$$\frac{p(u_z T', a_z)}{p(a_z)p(u_z T')}$$

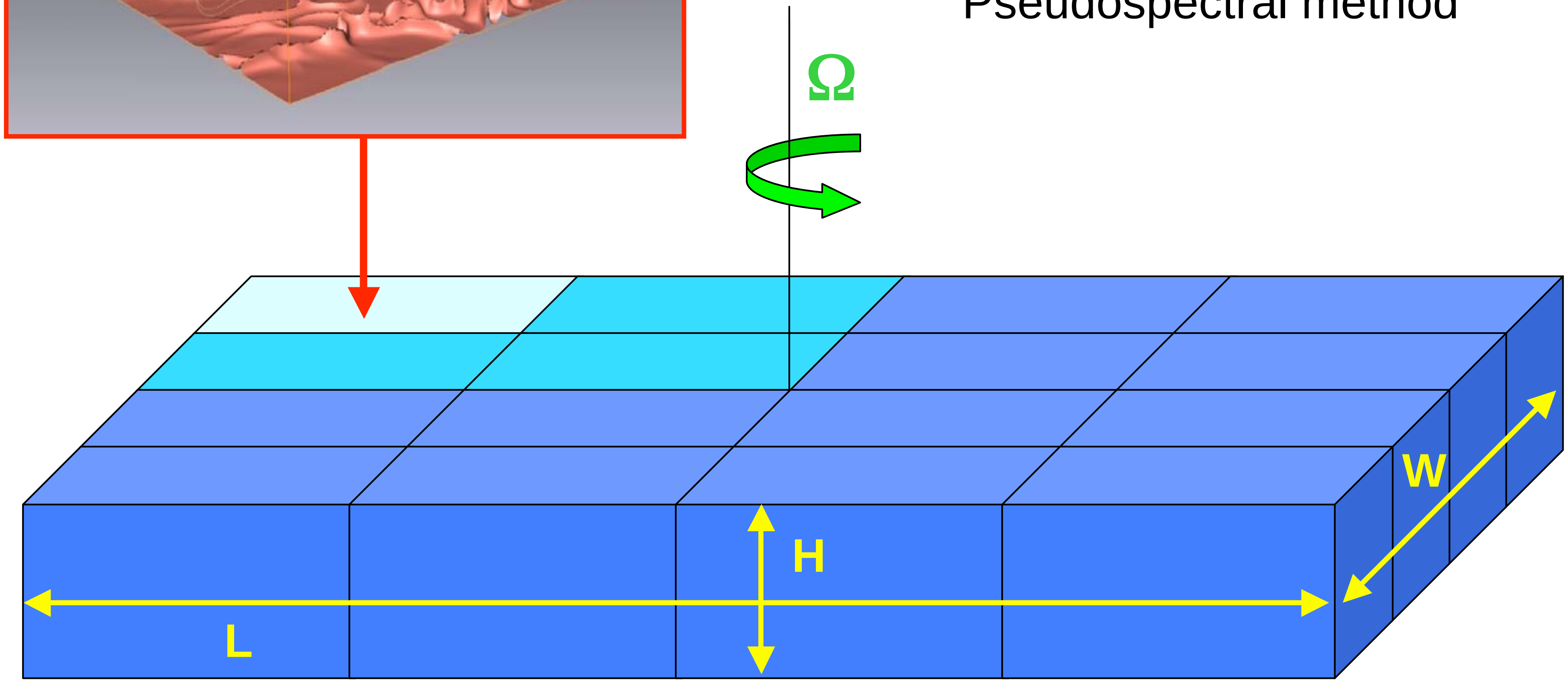
Successively flatter cells



Cartesian geometry

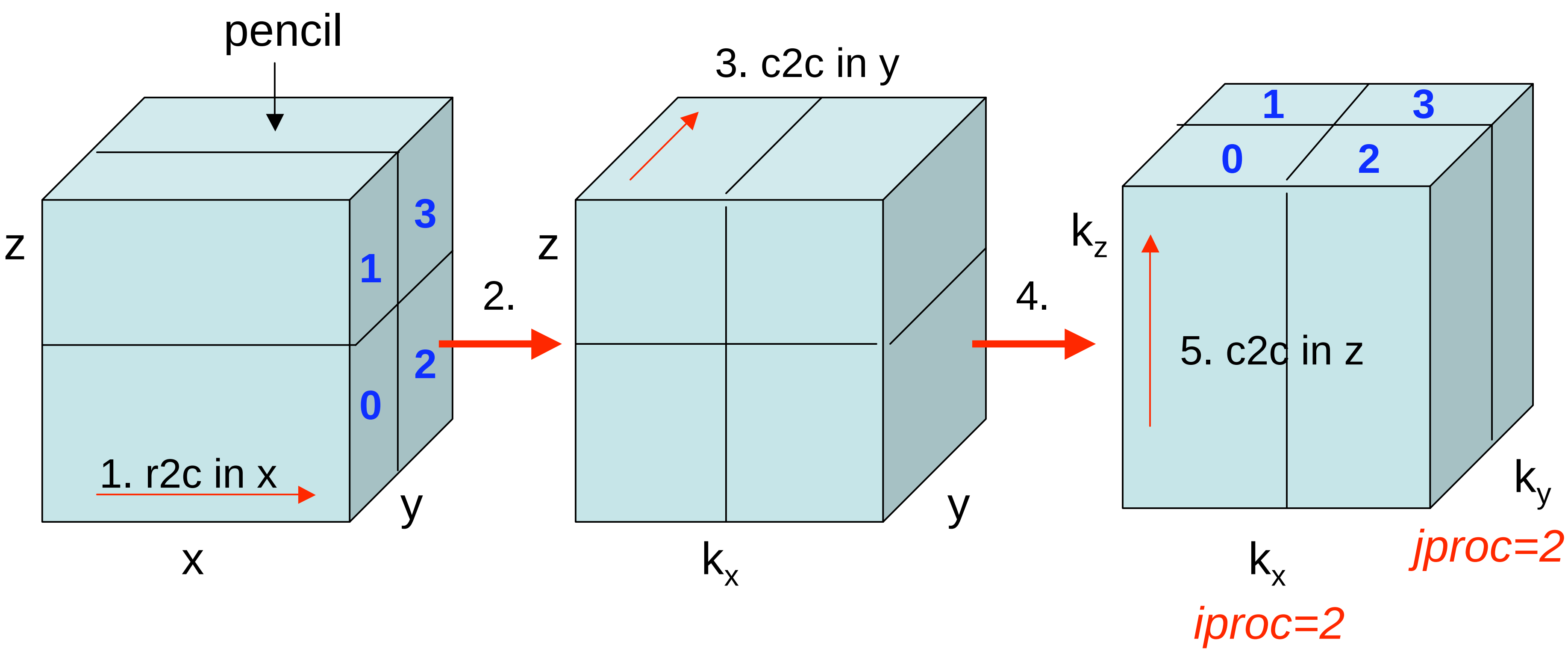
Free-slip b.c. in z
Periodic b.c. in x and y

Pseudospectral method



Volumetric 3D-FFT

D. Pekurovsky (SDSC), p3dfft package



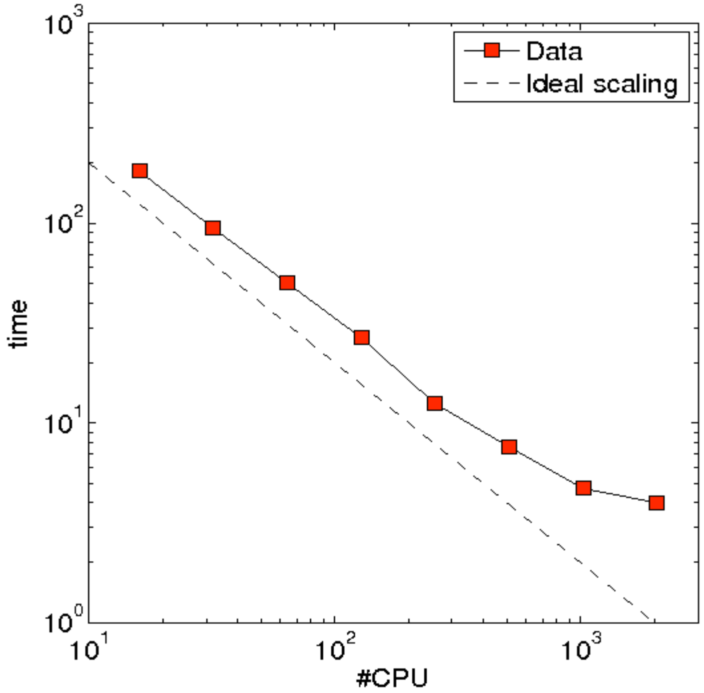
N^2 processors can be used → 2D processor grid *(iproc,jproc)*

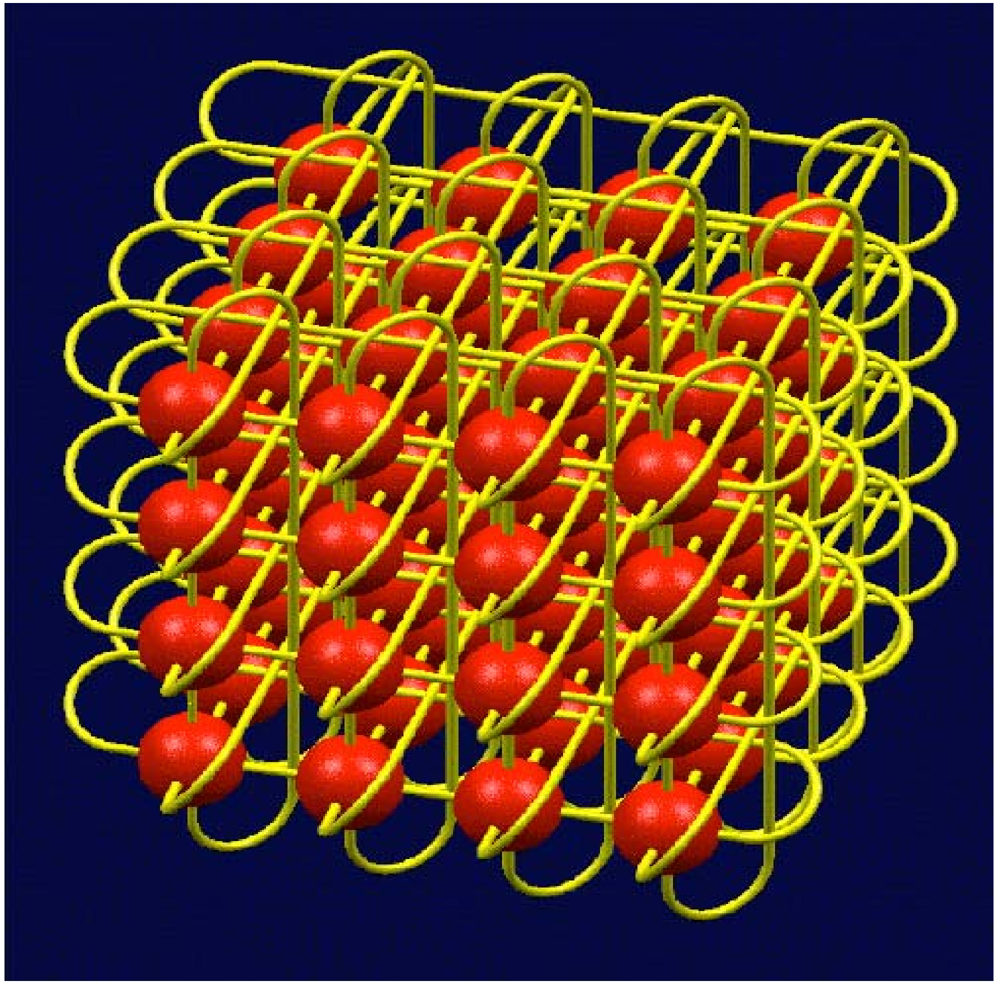
One more communication step necessary !

$N_x = N_y >> N_z$ is possible as necessary for our studies

Massively parallel supercomputing

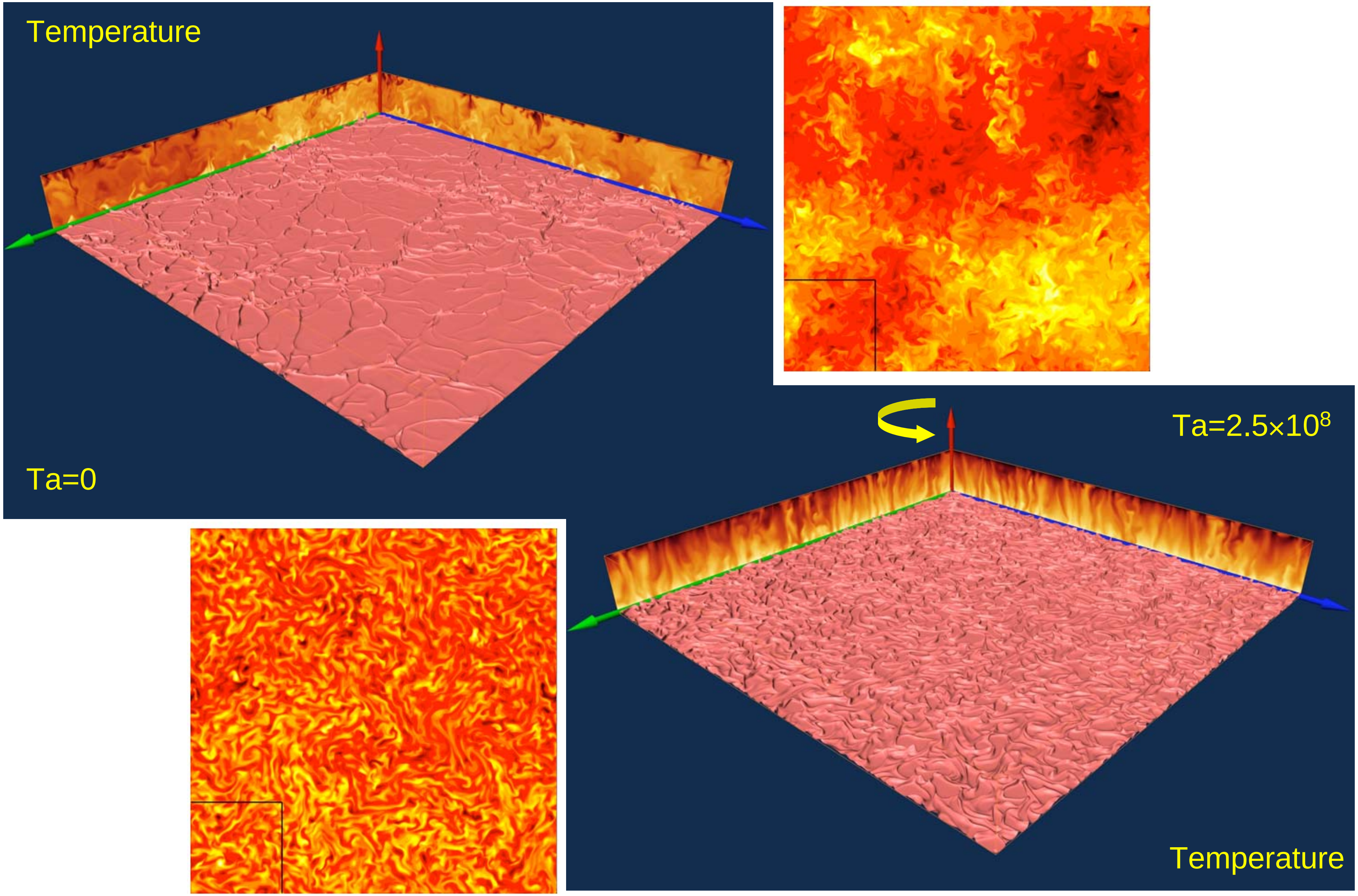
Schumacher & Pütz, Proceedings of PARCO 2007



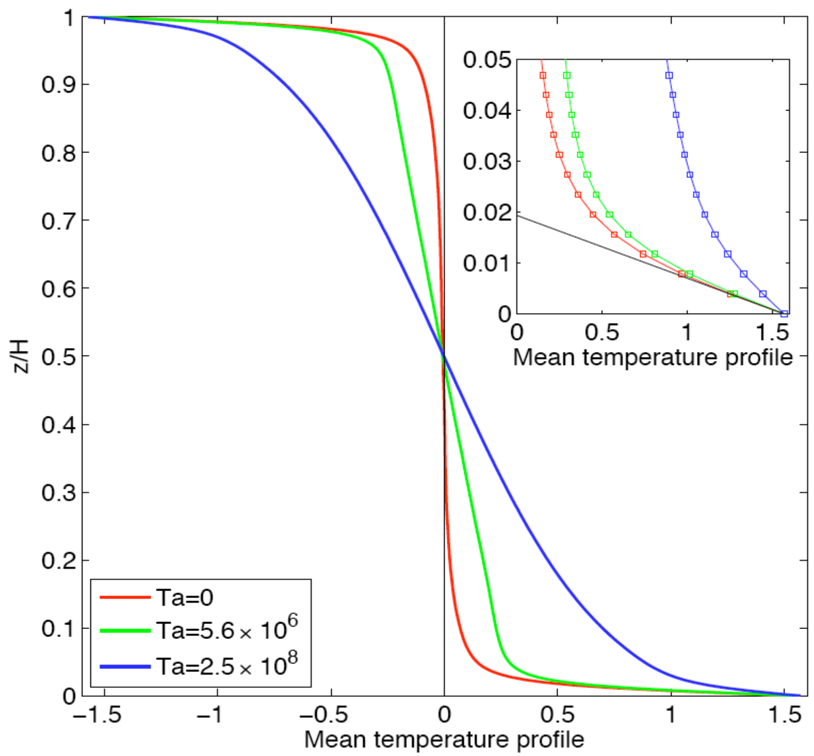


Production jobs on 4096 CPUs

Effect of rotation

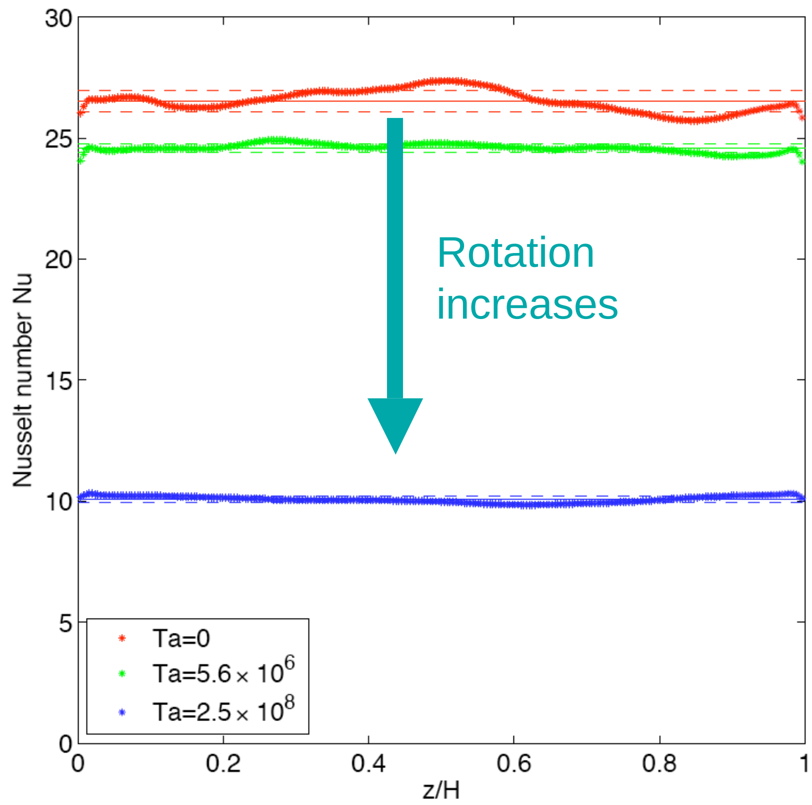


Mean temperature and heat transfer



$$Nu(z) = \frac{\langle u_z T \rangle_A - \kappa \frac{\partial \langle T \rangle_A}{\partial z}}{\frac{\kappa \Delta T}{H}}$$

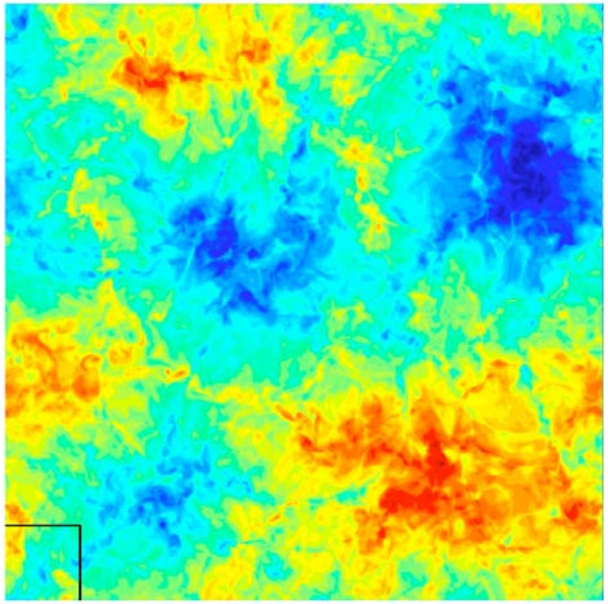
Rotation diminishes fluctuations and thus heat transport



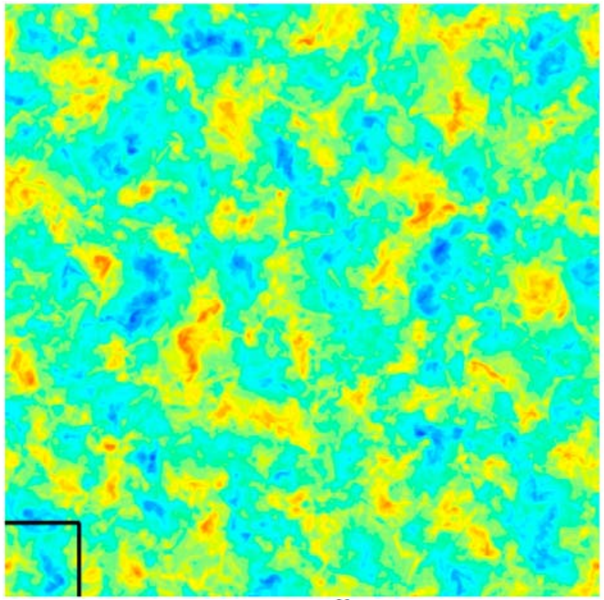
Temperature patterns

Hartlep, Tilgner & Busse, J. Fluid Mech. 2005; von Hardenberg et al., Phys. Letters A 2008

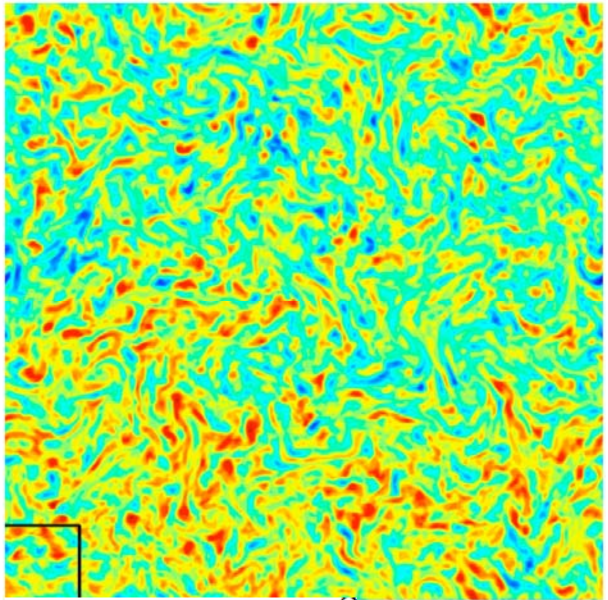
$$\overline{T}^H(x, y, t_0) = \frac{1}{H}\int_0^H T(x, y, z, t_0)\,\mathrm{d}z$$



$Ta = 0$



$Ta = 5.6 \times 10^6$

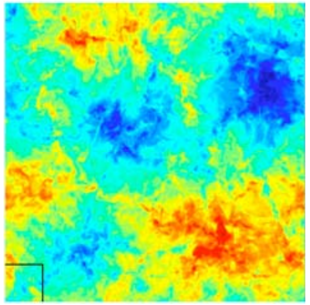


$Ta = 2.5 \times 10^8$

$$Ra = 1.1 \times 10^7 \qquad Pr = 0.7 \qquad \Gamma = 8$$

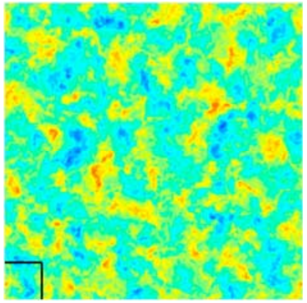
$$N_x \times N_y \times N_z = 2048 \times 2048 \times 257$$

Plume clusters?

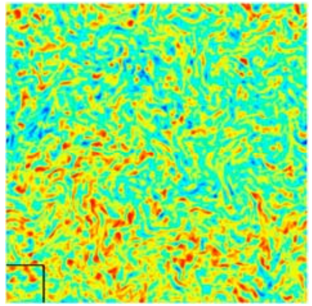


$$L_R = \frac{\sqrt{g\alpha \Delta T H}}{\Omega}$$

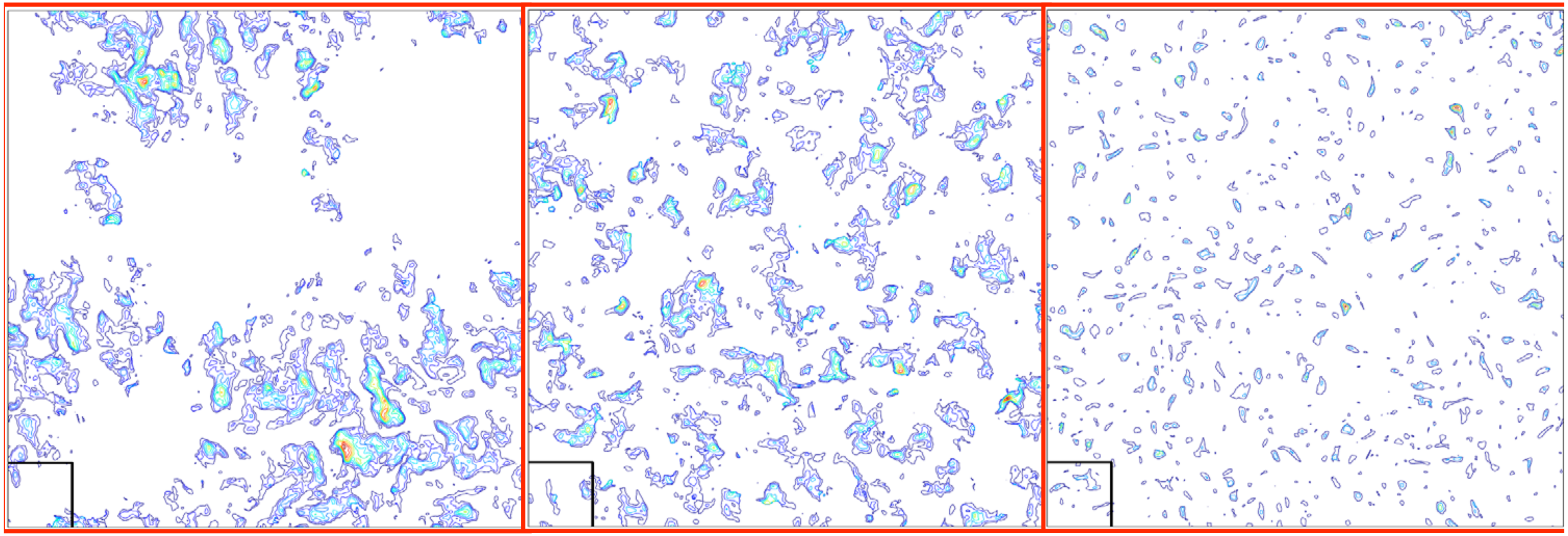
$L_R = \infty$



$L_R = H$



$L_R = H/6$



$$\overline{u_z T}^H |_{\overline{T}^H > 0} > 0.1$$

... the road ahead

- Scale analysis of plume clusters (image segmentation)
- Lagrangian analysis in large-aspect-ratio convection cells → temporal persistence of plume clusters?
- Statistical significance of thermal plumes with increasing Rayleigh number (BL thickness ~ 1/Nu)
- Together with O. Pauluis (NYU): simple extension to moist convection
  → condensation at saturation level
  → piecewise linear dependence of buoyancy on S and q
  → dry and moist lapse rates allow to tune stability

Summary

- Boundary layers (BL): neither laminar Blasius type nor turbulent Prandtl type → Scaling theories of heat transfer
- Turbulent temperature fluctuations contribute significantly to thermal dissipation in BL
- Lateral dispersion of Lagrangian tracers is Richardson-like
- Rotation prohibits formation of horizontal large-scale patterns
- Clusters of thermal plumes seem to cause large-scale temperature patterns in non-rotating case

Thanks to

- Mohammad Emran, Jorge Bailon-Cuba (TU Ilmenau)
- Ronald du Puits, Christian Resagk, Andre Thess (TU Ilmenau)
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- Roberto Verzicco (U Roma II)

- Matthias Pütz (IBM Germany)
- Jülich Supercomputing Centre (Germany)