Polarons in Immersed Optical Lattices

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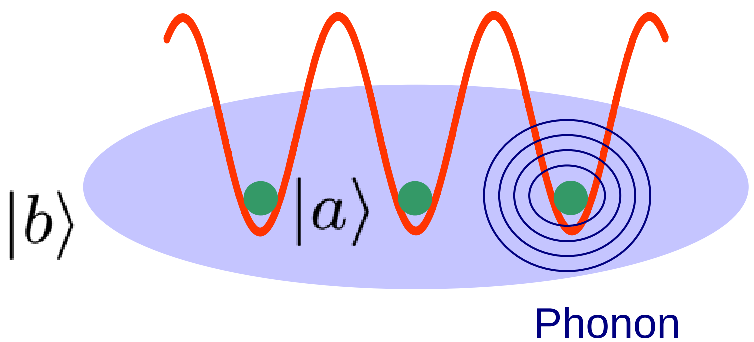
Immersed optical lattices

M. Bruderer, A. Klein, S.R. Clark and DJ, preprint

M. Bruderer and DJ, New J. Phys. **8**, 87 (2006)

Immersed Lattice

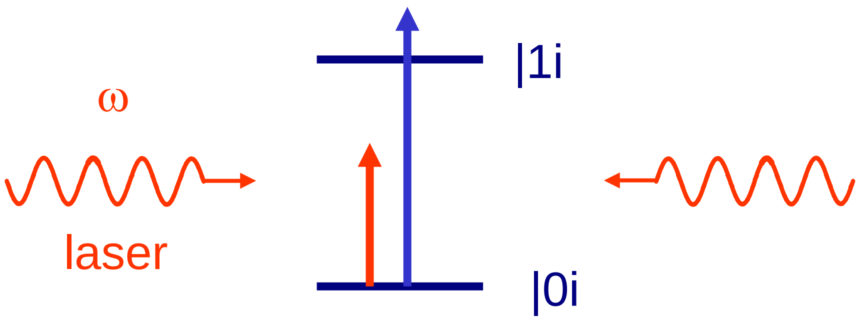
- We consider a mixture of two degenerate atomic species
  - ⇨ Species |ai is trapped by an optical lattice
  - ⇨ The second species |bi does not see the lattice but is magnetically trapped

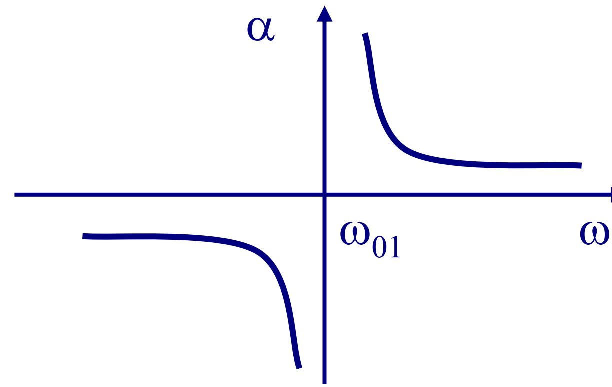


- We consider the limit where the background gas contains many more particles than the optical lattice
  - ⇨ The background acts as a bath
  - ⇨ The interaction between bath and lattice atoms creates phonons
  - ⇨ We study the effects of these phonons on the dynamics of the lattice atoms

Optical Potentials

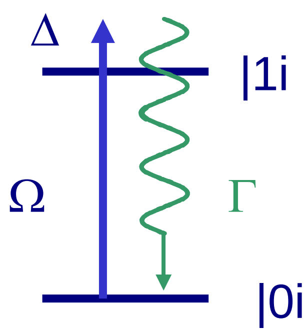
- AC – Stark shift





$$E_{AC} = \alpha I \text{ where } I \propto |\mathcal{E}|^2$$

- Spontaneous emission



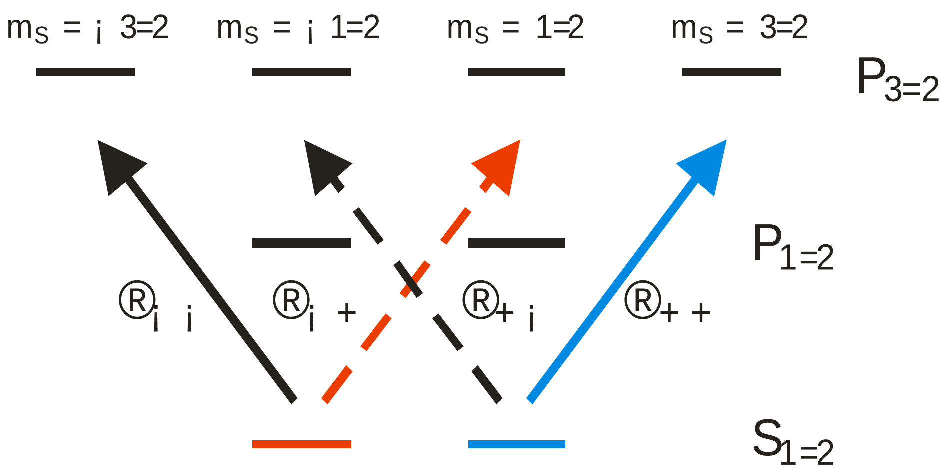
shift: $\nu = \dfrac{\Omega^2}{4\Delta + 2i\Gamma}$

$$\frac{\text{AC –Stark shift } <\{\nu\}}{\text{Spontaneous emission } I\{\nu\}} ¼ \frac{\Delta}{\Gamma} À 1$$

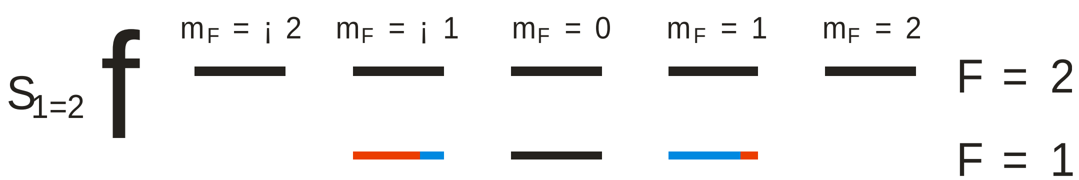
→ Spontaneous emission rates of less than $1s^{-1}$

State dependent potentials

- Use two species with different transition frequencies
- ‘Magic’ frequencies for hyperfine states in Alkali atoms



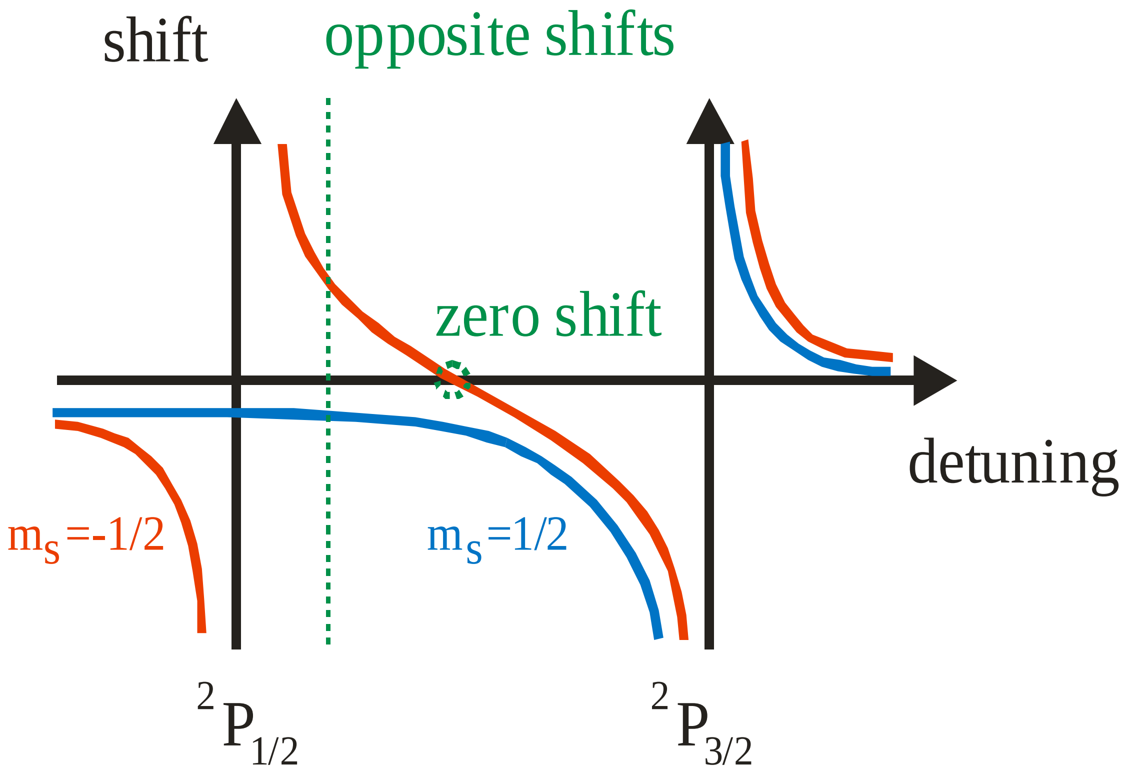
Fine structure of ^{87}Rb, ^{23}Na



Hyperfine structure of ^{87}Rb, ^{23}Na

State dependent optical lattice

- AC Stark shift due to σ^+ laser light

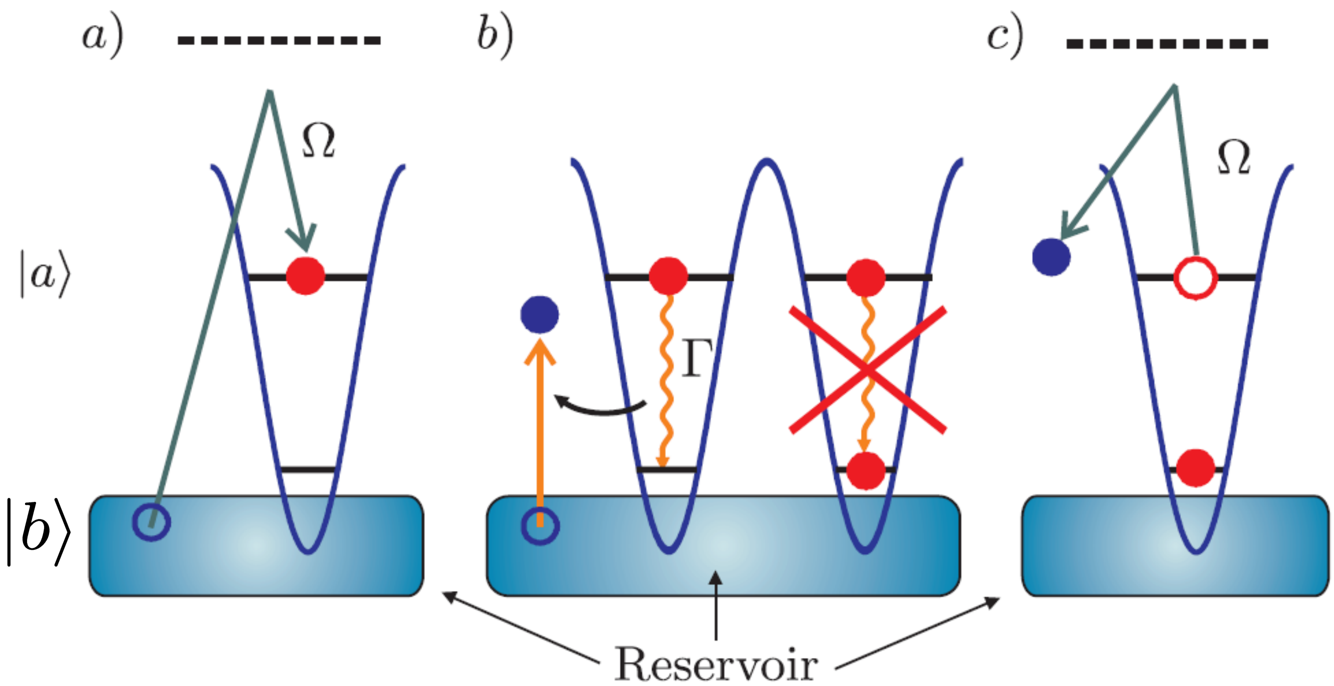


- DJ et al., PRL 1999
- O. Mandel et al., Nature 2003

Initialization of a fermionic register

A. Griessner *et al.*, Phys. Rev. A **72**, 032332 (2005).

- Start with an empty optical lattice immersed in an ultracold Fermi gas



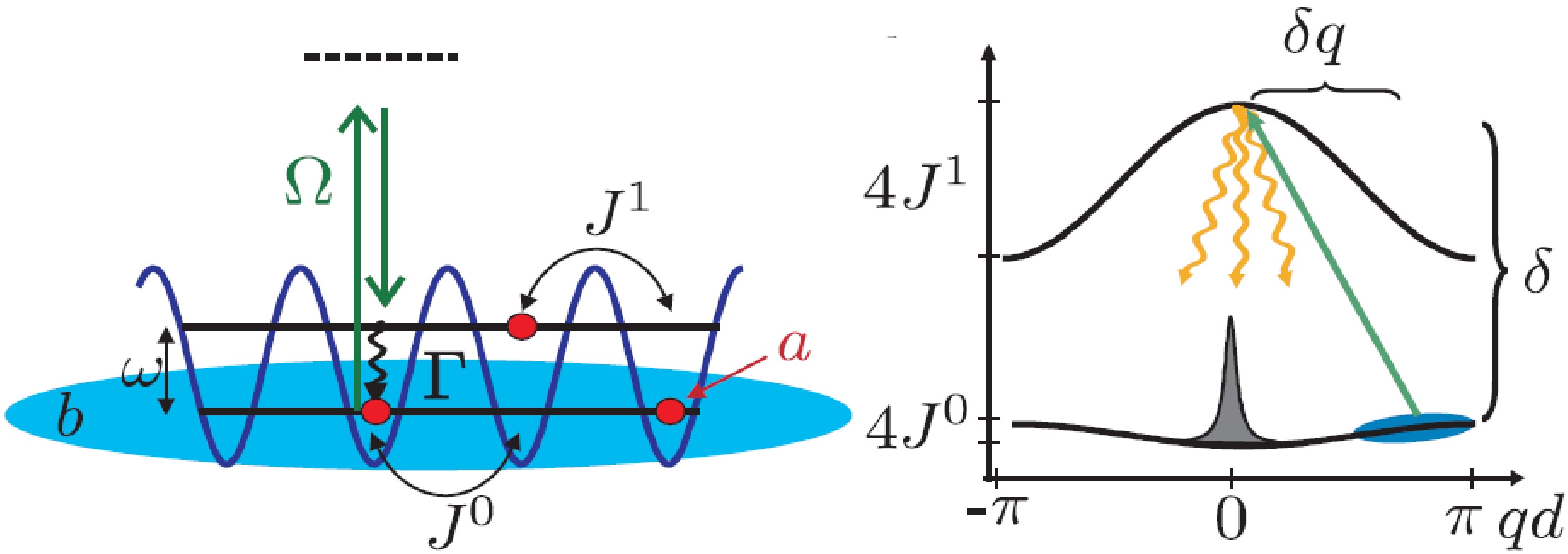
⇨ a) load atoms into the first band

⇨ b) incoherently emit phonons into the reservoir

⇨ c) remove remaining first band atoms

Cooling by superfluid immersion

A. Griessner *et al.*, Phys. Rev. Lett. **97**, 220403 (2006); A. Griessner *et al.*, New J. Phys. **9**, 44 (2007).



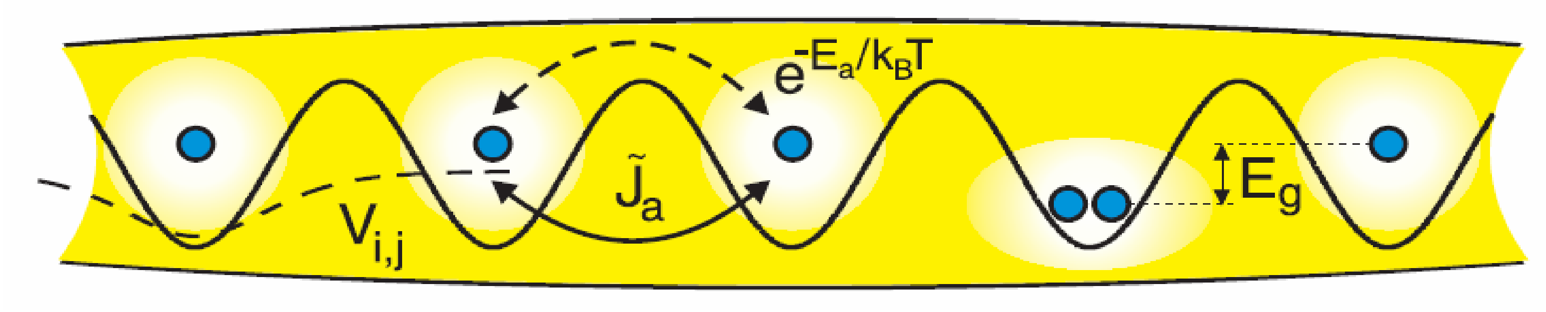
- Lattice fermions immersed in a BEC
- Atoms with higher quasi-momentum q are excited
- They decay via the emission of a phonon into the BEC
- They are collected in a dark state in the region q¼0
- Analysis of an iterative map in terms of Levy statistics

$$\mathcal{M}_j : \hat{\rho}_j \rightarrow \hat{\rho}_{j+1} \equiv \left(\hat{\mathcal{D}} \circ \hat{E}_j\right) \hat{\rho}_j$$

See KITP talk by P. Zoller

Interactions of lowest band atoms with a BEC

- We consider atoms moving in the lowest band only, BEC at finite temperature

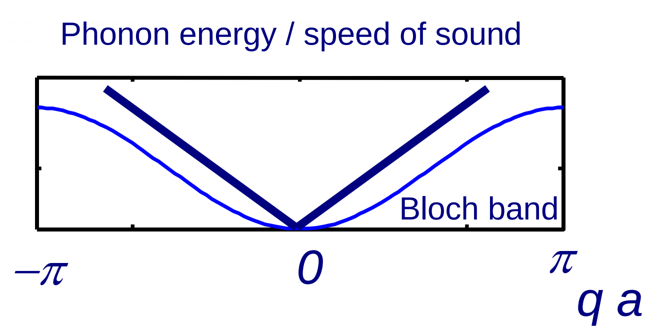


- The total Hamiltonian is $H = H_\chi + H_B + H_I$

$$\hat{H}_\mathrm{B} = \int \mathrm{d}\mathbf{r}\, \hat{\phi}^\dagger(\mathbf{r}) \left[-\frac{\hbar^2 \nabla^2}{2m_b} + V_\mathrm{ext}(\mathbf{r}) + \frac{g}{2} \hat{\phi}^\dagger(\mathbf{r}) \hat{\phi}(\mathbf{r}) \right] \hat{\phi}(\mathbf{r})$$

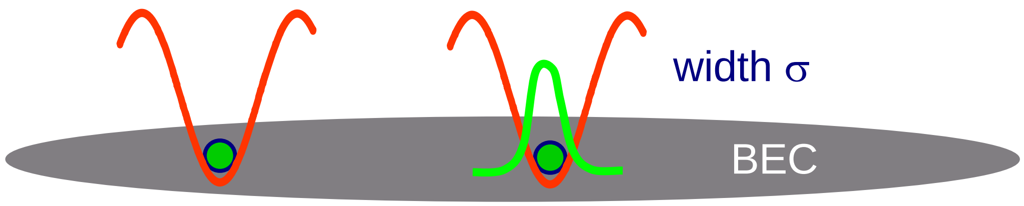
$$\hat{H}_\mathrm{I} = \kappa \int \mathrm{d}\mathbf{r}\, \hat{\chi}^\dagger(\mathbf{r}) \hat{\chi}(\mathbf{r}) \hat{\phi}^\dagger(\mathbf{r}) \hat{\phi}(\mathbf{r}),$$

and H_χ covers the dynamics of the lattice atoms



A single impurity in a BEC

- A deep optical lattice realizes atomic quantum dots (see A. Recati et al. PRL 2005)
  - ➡ Species |bi is a BEC in 1D, 2D or 3D
  - ➡ Impurities are assumed to interact independently (dÀξ)



$n_0\sigma^3 \gg 1$

- ➡ The condensate density operator

$$\hat{\phi}^\dagger(\mathbf{x})\hat{\phi}(\mathbf{x}) = n_0 + \hat{n}(\mathbf{x},t)$$

- ➡ The impurity is trapped inside the BEC

$$\psi_\sigma(\mathbf{x}-\mathbf{x}_0) \propto \frac{1}{a_\perp^2\,\sigma}\exp\left[-\left(\frac{x-x_0}{\sqrt{2}a_\perp}\right)^2-\left(\frac{y-y_0}{\sqrt{2}a_\perp}\right)^2-\left(\frac{z-z_0}{\sqrt{2}\sigma}\right)^2\right]$$

Dephasing of a quantum dot

- With H_χ=0 and

$$\hat{H}_B = \frac{1}{2}\int \mathrm{d}^D\mathbf{x}\left(\frac{n_0}{m}(\nabla\hat{\varphi})^2 + g\hat{n}^2\right)$$

⇨ The temporal correlation function of the impurity is given by

$$\langle\hat{\chi}^\dagger(0)\hat{\chi}(\tau)\rangle \propto \left\langle\ \exp\left(-i\frac{\kappa}{g}\int d\mathbf{x}|\psi_\sigma(\mathbf{x},\tau)|^2\{\hat{\varphi}(\mathbf{x},\tau)-\hat{\varphi}(\mathbf{x},0)\}\right)\right\rangle$$

- Turn the dephasing into fringe visibility by Ramsey interferometry
- Measure coarse grained phase correlations
- Spatial and temporal correlations accessible

⇨ In terms of Bogoliubov excitations the resulting Hamiltonian is an independent boson model (easy to solve)

$$H = \left(\kappa n_0 + \sum_{\mathbf{k}}\left(g_{\mathbf{k}} b^\dagger_{\mathbf{k}} + \mathrm{h.c.}\right)\right)\hat{\chi}^\dagger\hat{\chi} + \sum_{\mathbf{k}}\omega_{\mathbf{k}}\left(b^\dagger_{\mathbf{k}}b_{\mathbf{k}} + \frac{1}{2}\right) + \mathrm{const}$$

with $g_{\mathbf{k}}$ overlap matrix elements for the impurity-Bogoliubov excitations coupling

Measuring properties of the BEC

- We obtain for a D dimensional BEC and long waiting time τ ! 1

$$\langle \hat{\chi}^\dagger(0)\hat{\chi}(\tau)\rangle \propto \begin{cases} C_T \exp\left[-\left(\frac{\kappa}{g}\right)^2 \frac{mc\,k_B T}{2\hbar^2 n_0}\,\tau\right] & \text{for } D=1 \\ C'_T\left(\frac{\sigma}{c\tau}\right)^\nu & \text{for } D=2 \\ \exp\left[-\left(\frac{\kappa}{g}\right)^2 \frac{m\,k_B T}{(2\pi)^{3/2}\hbar^2 n_0 \sigma}\right] & \text{for } D=3 \end{cases}$$

$$\nu = (\kappa/g)^2 m k_B T/(2\pi\hbar^2 n_0)$$

⇨ Measure temperature, speed of sound, …

⇨ A SAT in a 1D BEC cannot be used as a qubit

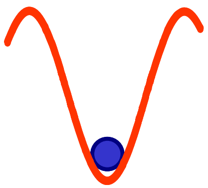
⇨ The frequencies of the Bogoliubov excitations are not changed by the presence of the impurity

Atom interferometry

A.D. Cronin, J. Schmiedmayer, D.E. Pritchard, review article, to be published

- Use atoms instead of photons for interferometric measurement
  - Apply a π/2 pulse (a Hadamard gate)

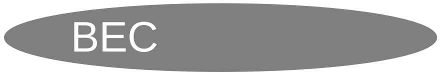
$$\phi(x)|a\rangle \rightarrow \phi(x)(|a\rangle + |c\rangle)$$



  - Split the wave function e.g. using state dependent potential

$$\rightarrow \phi(x)|a\rangle + \psi(x,t)|c\rangle$$

  - By interaction with e.g. a BEC one arm of the interferometer acquires a phase α



$$\rightarrow \phi(x)|a\rangle + \mathrm{e}^{\mathrm{i}\alpha}\psi(x,t)|c\rangle$$

  - Combine the two components yielding a kinematic phase β and apply π/2 pulse

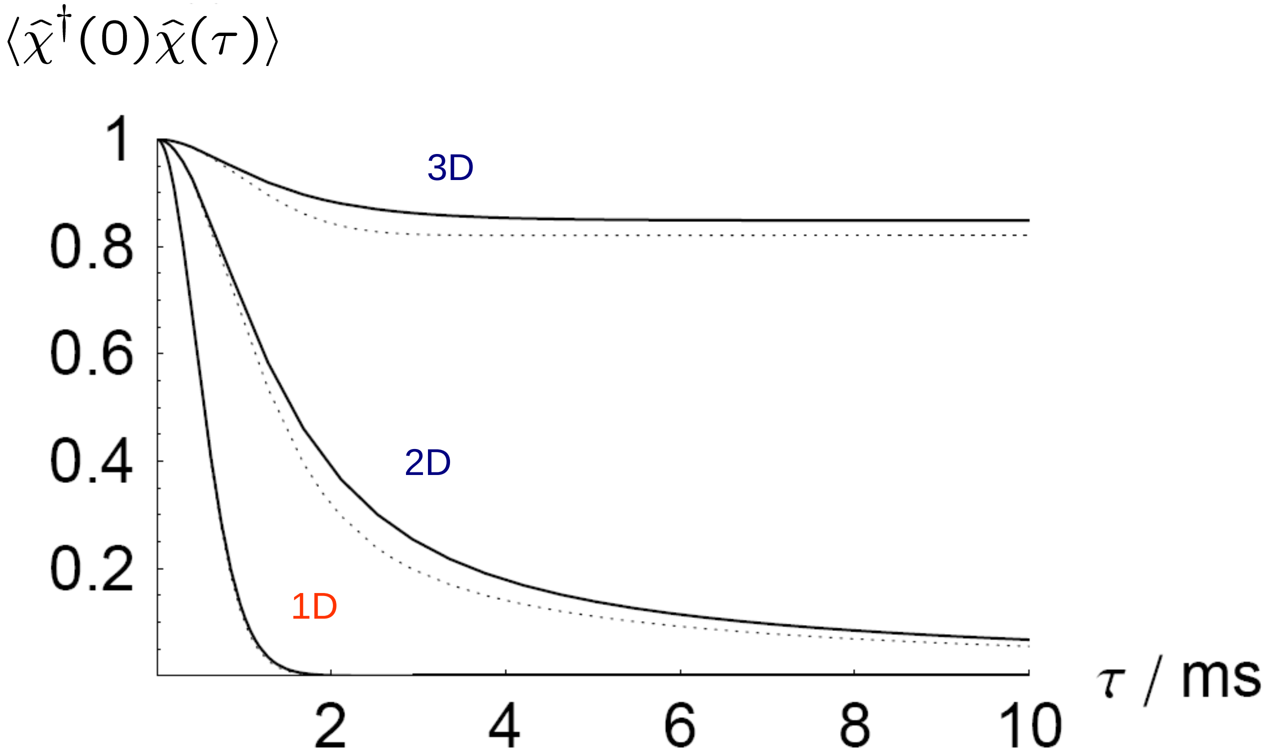
$$\rightarrow \phi(x)[(1+\mathrm{e}^{\mathrm{i}(\alpha+\beta)})|a\rangle + (1-\mathrm{e}^{\mathrm{i}(\alpha+\beta)})|c\rangle]$$

  - Measure population in states |ai and |ci to determine α

$$p_a = \cos^2(\alpha+\beta) \qquad p_c = \sin^2(\alpha+\beta)$$

move via laser parameters

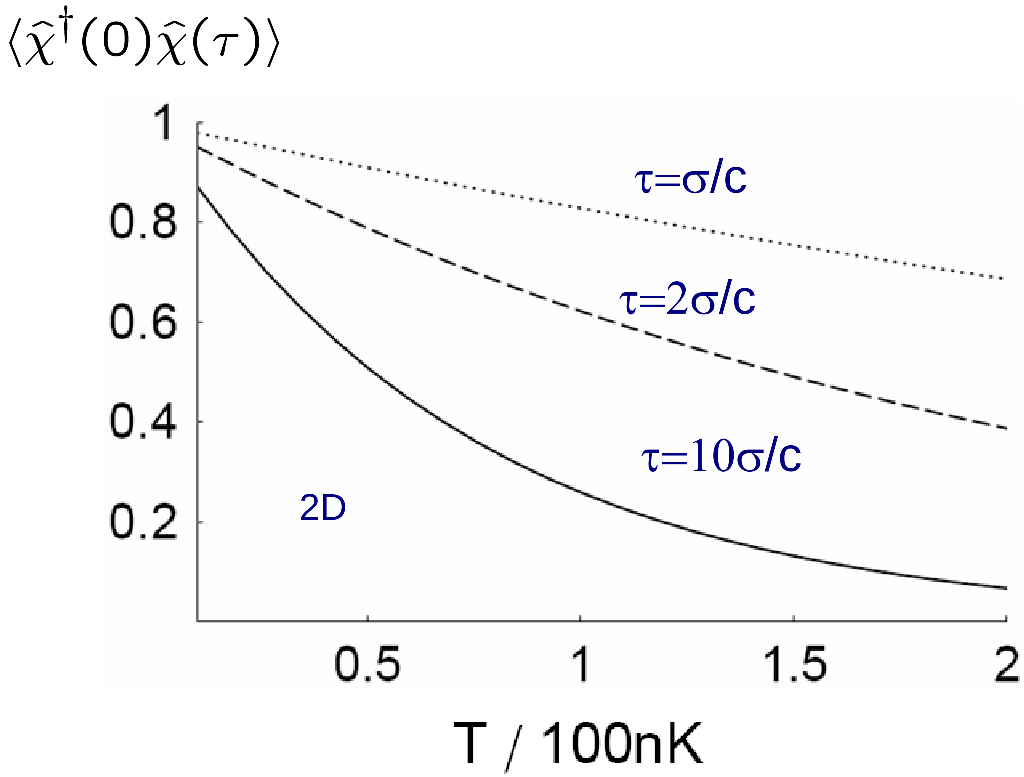
BEC properties



$$m = 10^{-25} kg,\ l_0 = 2\times 10^6 m^{-1},\ c = 10^{-3} ms^{-1}$$

$$\sigma = 10^{-6} m \qquad \kappa = g \qquad T = 2\times 10^{-7} K$$

BEC properties



$$m = 10^{-25}kg,\ l_0 = 2\times 10^6 m^{-1},\ c = 10^{-3} ms^{-1}$$

$$\sigma = 10^{-6} m \qquad \kappa = g$$

Immersed optical lattice (I)

- We do not specify H_χ yet but assume $\hat{\chi}(\mathbf{r}) = \sum_\nu \eta_\nu(\mathbf{r})\hat{a}_\nu$ and solve GPE at κ=0
- For sufficiently weak BEC-impurity coupling $|\kappa|/gn_0(\mathbf{r})\xi^D(\mathbf{r}) \ll 1$ we assume the deviation hδφ(r)i » κ
- The linear order in δφ does not vanish and we obtain

$$\widehat{H}_I = \kappa \int \mathrm{d}\mathbf{r} \widehat{\chi}^\dagger(\mathbf{r})\widehat{\chi}(\mathbf{r})\widehat{\phi}_0(\mathbf{r}) \left[\delta\widehat{\phi}^\dagger(\mathbf{r}) + \delta\widehat{\phi}(\mathbf{r})\right]$$

- Carry out a Bogoliubov transformation to diagonalize the quadratic terms in δφ and obtain a Hubbard-Holstein model

$$\widehat{H} = \widehat{H}_\chi + \sum_{\nu,\mu} \omega_\mu \left[M_{\nu,\mu}\widehat{b}_\mu + \mathrm{h.c.}\right]\widehat{n}_\nu + \sum_\mu \omega_\mu b_\mu^\dagger b_\mu$$

- Here $n_\nu = a_\nu^y a_\nu$ and $M_{\nu,\mu}$ describes the coupling of mode function η_ν to Bogoliubov excitation ˙µ

Immersed optical lattice (II)

- We apply a unitary Lang-Firsov transformation

$$\widehat{H}_{\mathsf{LF}} = \mathrm{e}^{\widehat{S}}\widehat{H}\mathrm{e}^{-\widehat{S}} \qquad \widehat{S} = -\sum_{\nu,\mu}\left(M_{\nu,\mu}\widehat{b}_{\mu} - \text{h.c.}\right)\widehat{n}_{\nu}$$

- We specialize to the case where H_χ is a BHM (parameters U_a and J_a) and find the transformed Hamiltonian

$$\begin{aligned}\widehat{H}_{\mathsf{LF}} &= -J_a\sum_{\langle i,j\rangle}(\widehat{X}_i\widehat{a}_i)^\dagger(\widehat{X}_j\widehat{a}_j) + \left(\frac{U_a}{2} - E_P\right)\sum_j \widehat{n}_j(\widehat{n}_j - 1) \\ &\quad + (\mu - E_P)\sum_j \widehat{n}_j - \sum_{i\neq j} V_{i,j}\widehat{n}_i\widehat{n}_j + \sum_{\mathbf{q}}\omega_{\mathbf{q}}\widehat{b}^\dagger_{\mathbf{q}}\widehat{b}_{\mathbf{q}}\end{aligned}$$

- X_ν is a unitary Glauber displacement operator for the phonon cloud

$$\hat{X}^\dagger_\nu = \exp\left[\sum_\mu\left(M^*_{\nu,\mu}\hat{b}^\dagger_\mu - M_{\nu,\mu}\hat{b}_\mu\right)\right]$$

- For a sufficiently deep lattice $V_{i,j} = (\kappa^2/\xi g)\,\mathrm{e}^{-2|i-j|a/\xi}$ and $E_p = V_{i,i}/2$, where *a* is the lattice spacing

Small hopping term and low BEC temperature

- For J/E_P¿1 and k_BT/E_P¿1 we treat the hopping term as a perturbation and find

$$\hat{H}^{(1)} = -\tilde{J}\sum_{\langle i,j\rangle}\hat{a}_i^{\dagger}\hat{a}_j + \tilde{\mu}\sum_j \hat{n}_j + \frac{1}{2}\tilde{U}\sum_j \hat{n}_j(\hat{n}_j - 1) - \frac{1}{2}\sum_{i\neq j} V_{i,j}\,\hat{n}_i\hat{n}_j$$

where

$$\tilde{\mu} = \mu - E_p,\ \tilde{U} = U - 2E_p \text{ and } \tilde{J} = J\,\langle\langle \hat{X}_i^{\dagger}\hat{X}_j\rangle\rangle.$$

- hh.ii denotes the average over the thermal phonon bath and gives

$$\langle\langle \hat{X}_i^{\dagger}\hat{X}_j\rangle\rangle = \exp\left\{-\sum_{\mathbf{q}\neq 0}|M_{0,\mathbf{q}}|^2[1-\cos(\mathbf{q}\cdot\mathbf{a})](2N_{\mathbf{q}}+1)\right\}$$

  - ⇨ $N_{\mathbf{q}}$ is the thermal occupation of the Bogoliubov excitation **q**
  - ⇨ The hopping bandwidth thus decreases exponentially with T and κ.

Generalized master equation

- For larger temperature T¿T$_c$ we derive a generalized master equation for the occupation probabilities P$_i$(t) and find

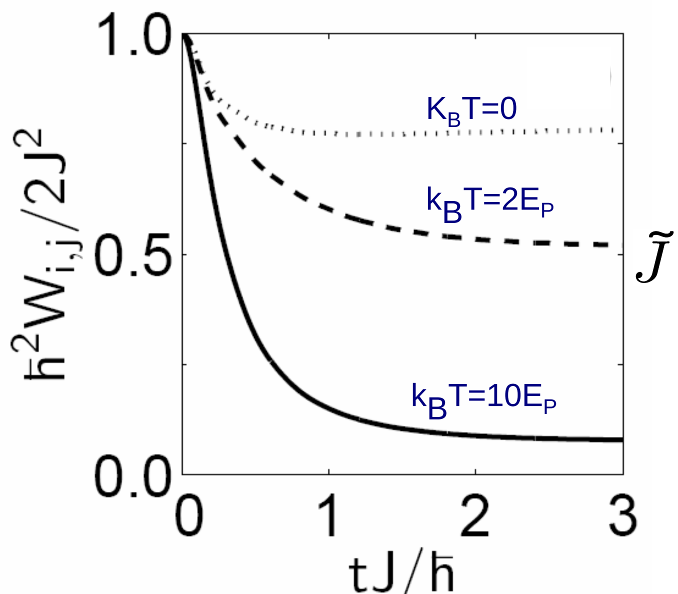
$$\frac{\partial P_i(t)}{\partial t} = \int_0^t \mathrm{d}s \sum_j \left[W_{i,j}(s)P_j(t-s) - W_{j,i}(s)P_i(t-s)\right]$$

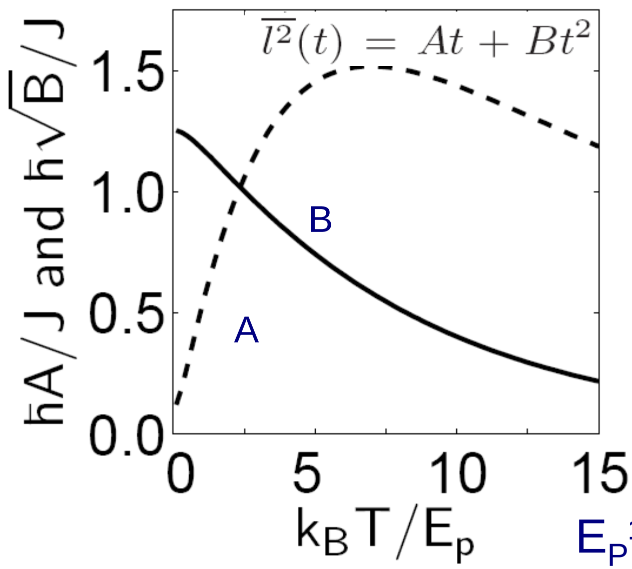
- This describes a transition from coherent to incoherent transport
  - ⇨ Small temperatures (wave)

$$W_{i,j}(s) = 2\tilde{J}^2\Theta(s)$$

  - ⇨ Large temperature (diffusion)

$$W_{i,j}(s) = 2w_{i,j}\delta(s)$$





Quantum statistics with classical particles, D. Gottesman, cond-mat/0511207.

E$_P$¼11nK

Polaron clusters

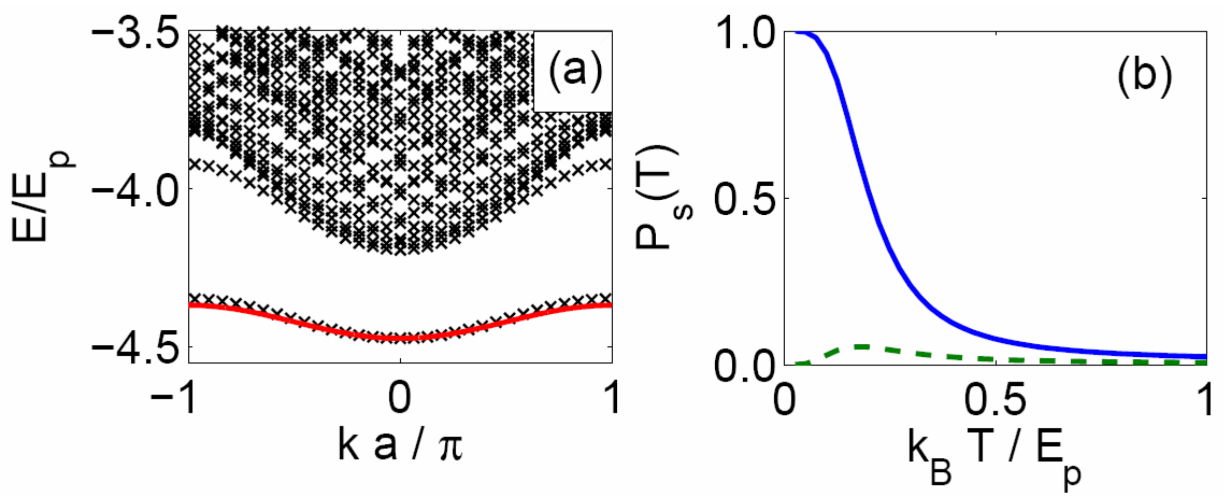
- Low temperature $k_B T \lesssim E_p$ and $\tilde{U} \gg V_{j,j+1}, \tilde{U} \gg \tilde{J}$

- The off-site interactions lead to the formation of clusters of polarons in adjacent sites with biding energy [see T. Holstein, Annals of Physics **8**, 343 (1959)]

$$E_b(s) \approx (s-1)V_{j,j+1}$$

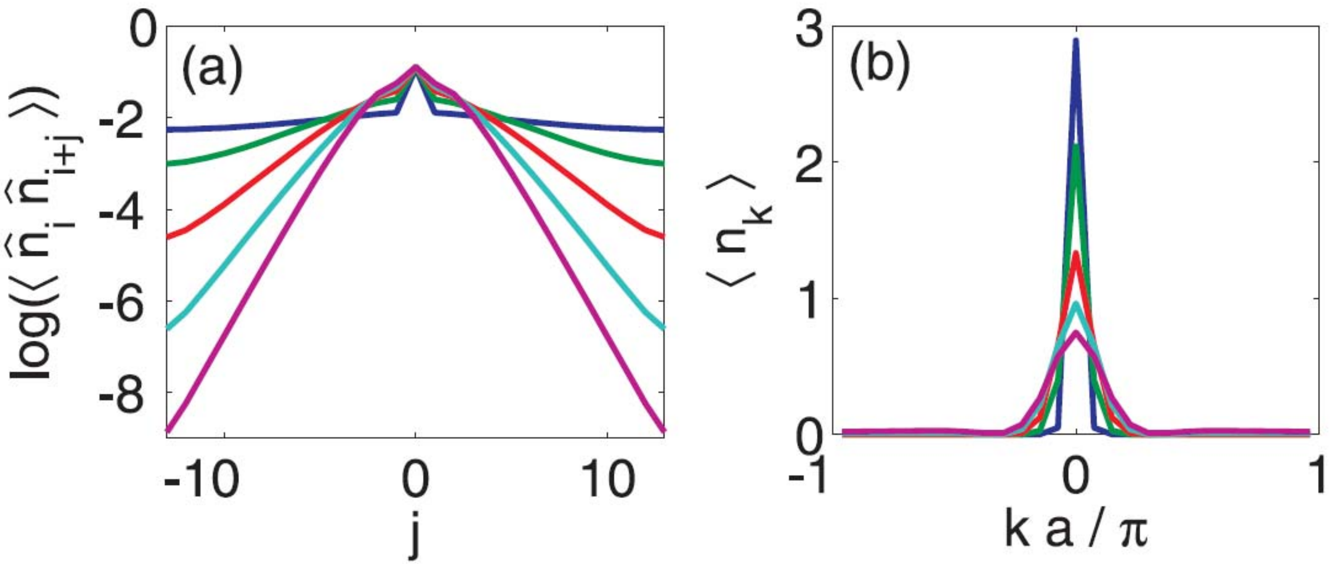
and a lowest band

$$E_k(s) \approx -E_b(s) - 2\tilde{J}^s (V_{j,j+1})^{1-s} \cos(ka)$$



Observing polaron clusters

- Measure density-density correlations and broadening in momentum distribution



- Exponential localization in real space
- Broadening in momentum space
- Increased particle number fluctuations

$$\kappa / E_R \lambda = \{4.0, 6.1, 8.1, 10.1, 12.1\} \times 10^{-2}$$

Outlook

- Anisotropic lattices and off-site interactions
  - ⇨ Hopping rates differently suppressed along different axes
  - ⇨ Off-sites interactions become anisotropic
  - ⇨ Fermions moving in the first excited band
- Probing the BEC
  - ⇨ Measurements using entangled impurities
  - ⇨ Making the impurities small compared to a_s → GPE approach breaks down
- Experiments
  - ⇨ Bose-Bose mixture setup in M. Inguscio's group
  - ⇨ State dependent lattice under construction
  - ⇨ Joint work on immersion experiments starting in May

People

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