Nonequilibrium Polariton Condensation: Introduction to Microcavity Polaritons

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F. M. Marchetti, M. H. Szymańska
Theory of Condensed Matter, Cambridge

April 9th 2007

Overview

- Microcavity polaritons: review of experiments.

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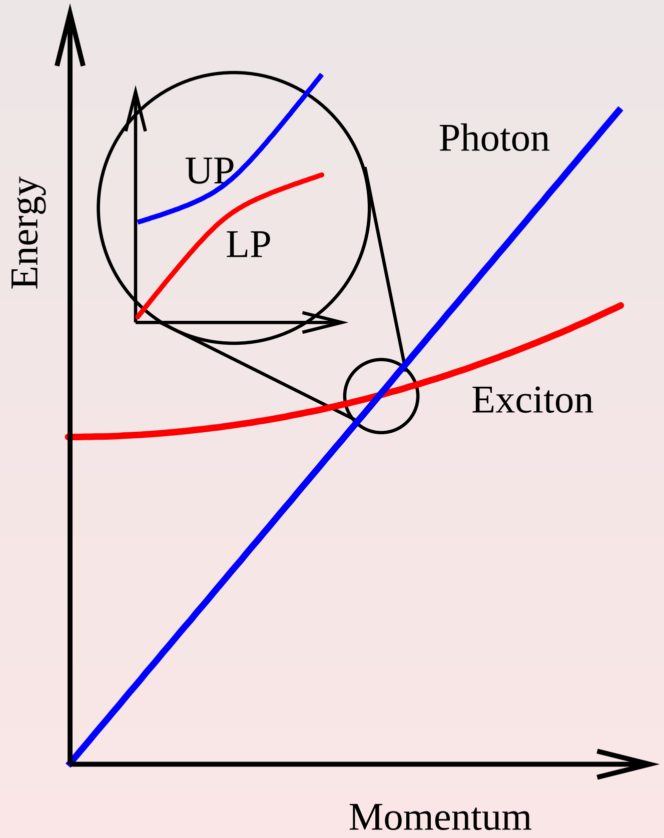
- Nonequilbrium quantum condensation

Polaritons

- Strong coupling of photons to excitons



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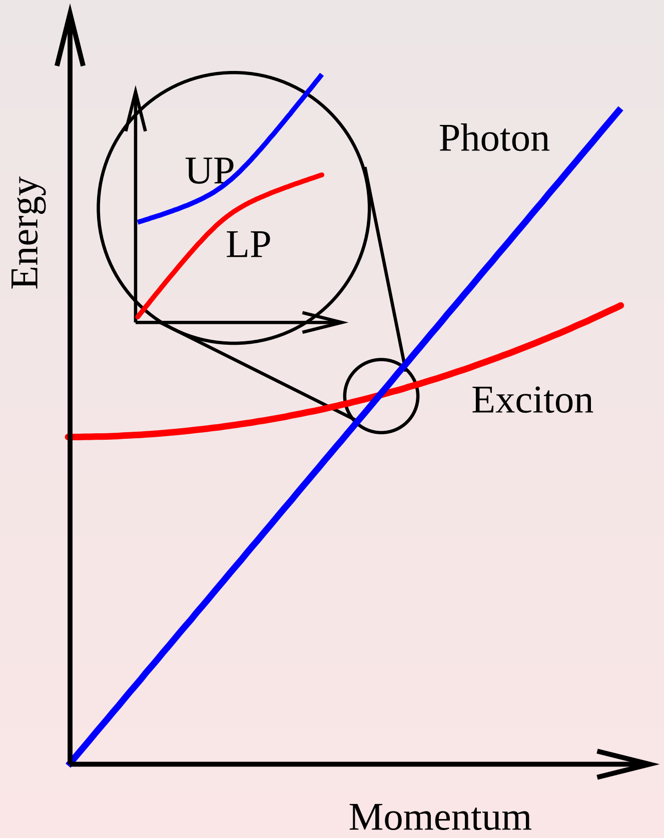

Polaritons

- Strong coupling of photons to excitons
- Anti-crossing – form two new modes
- No condensation – can relax to photon mode.

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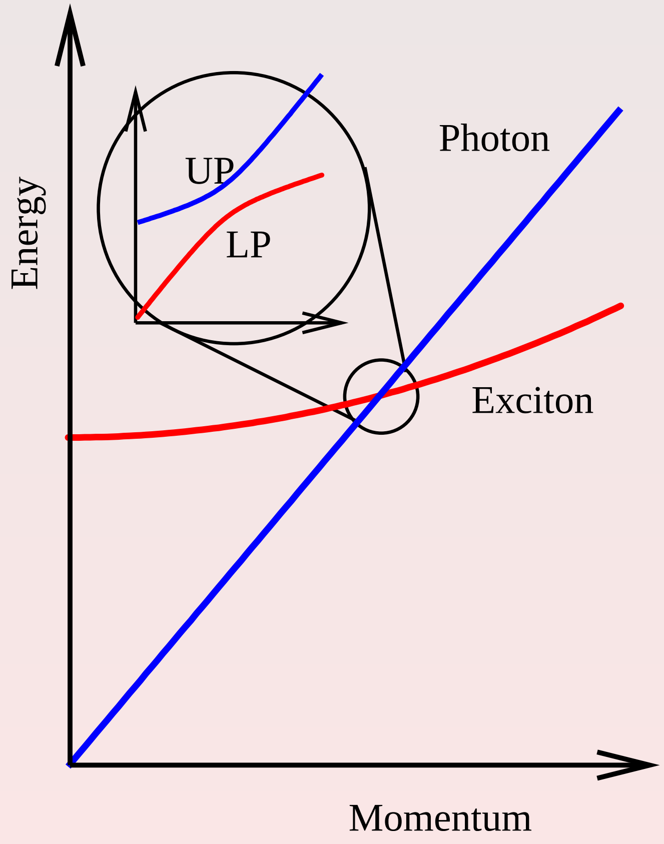

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Quantum well excitons coupled to photons confined in a microcavity.

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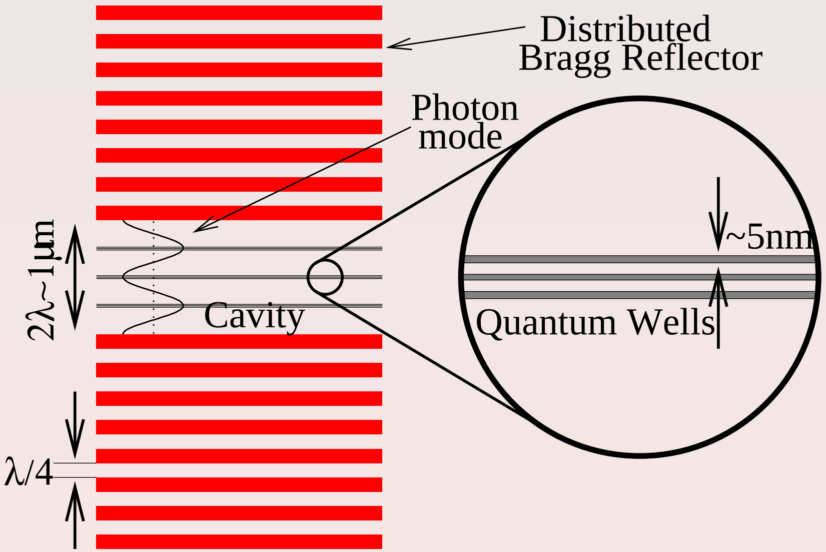

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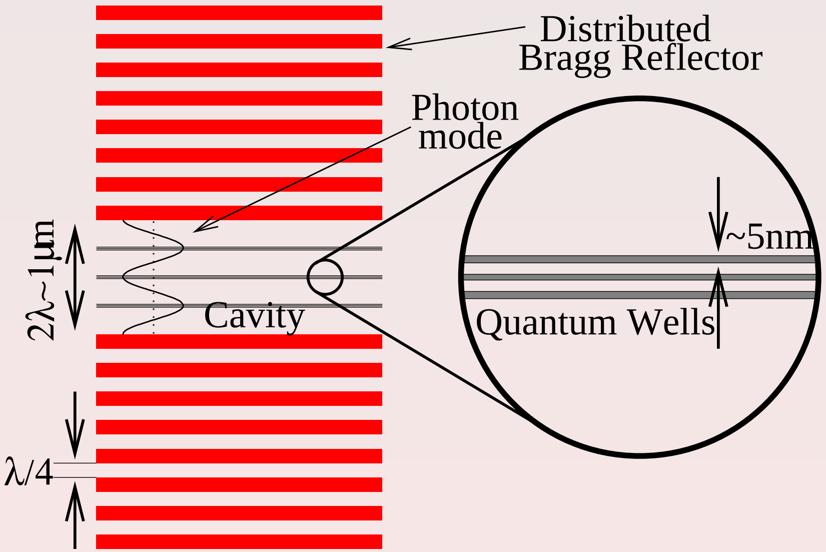

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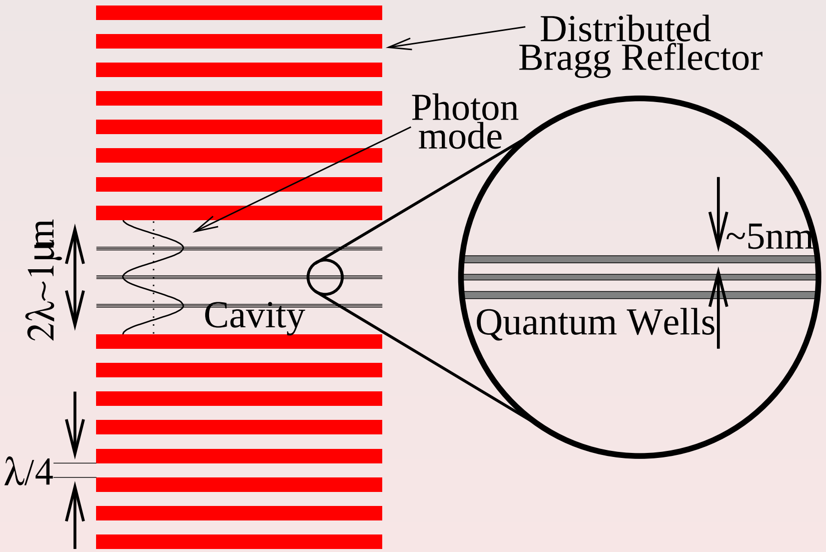

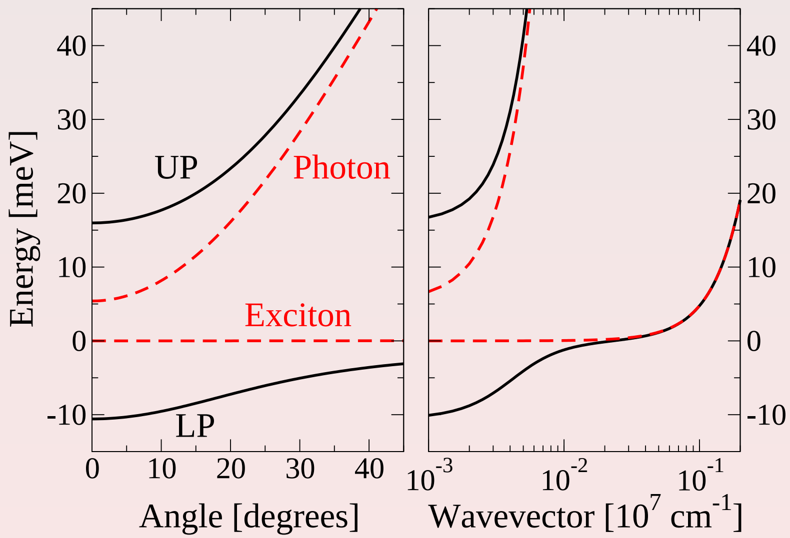

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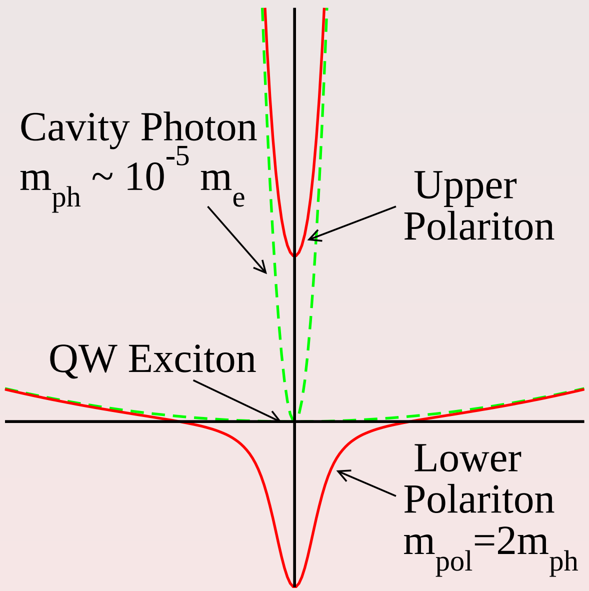

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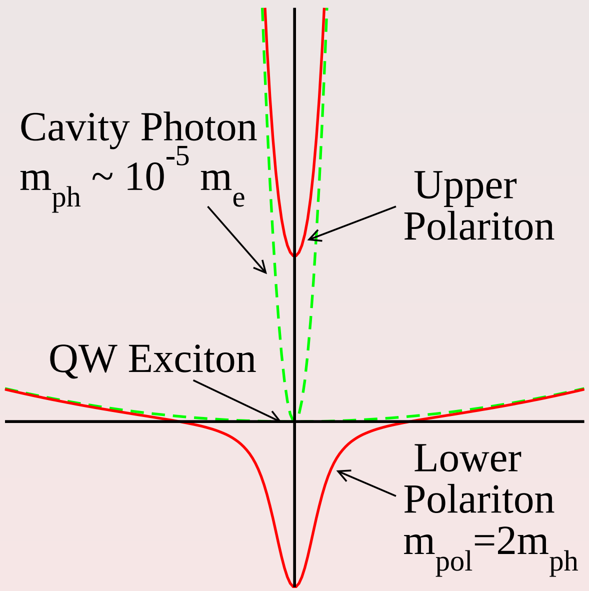
- Polariton mass $10^{-4} m_{\text{electron}}$, high T_c.



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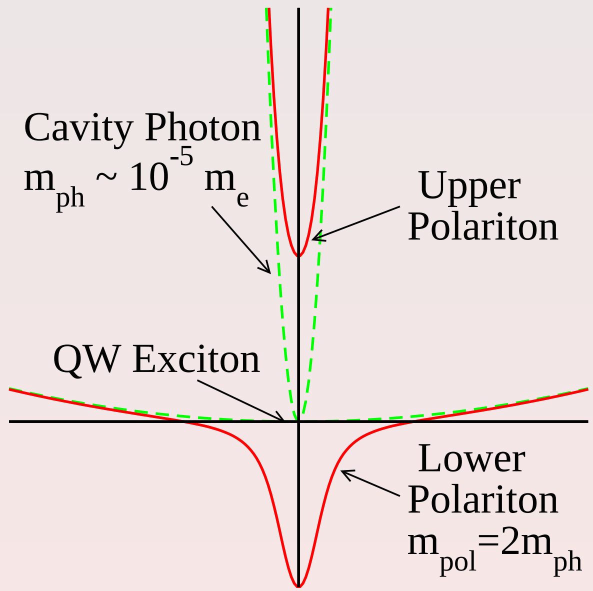

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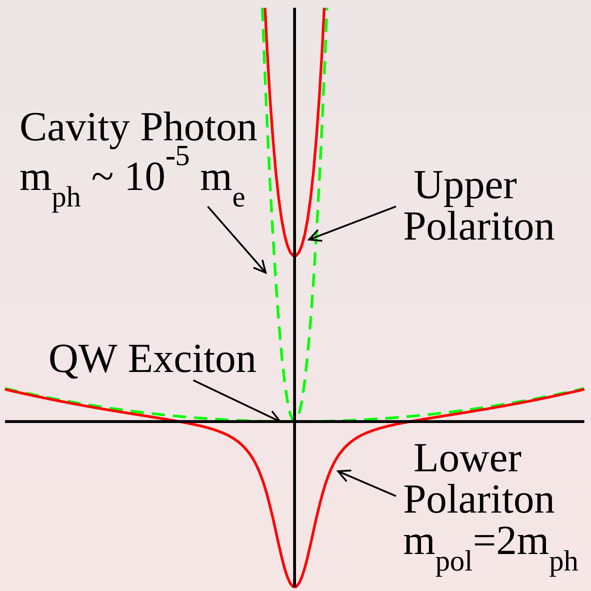

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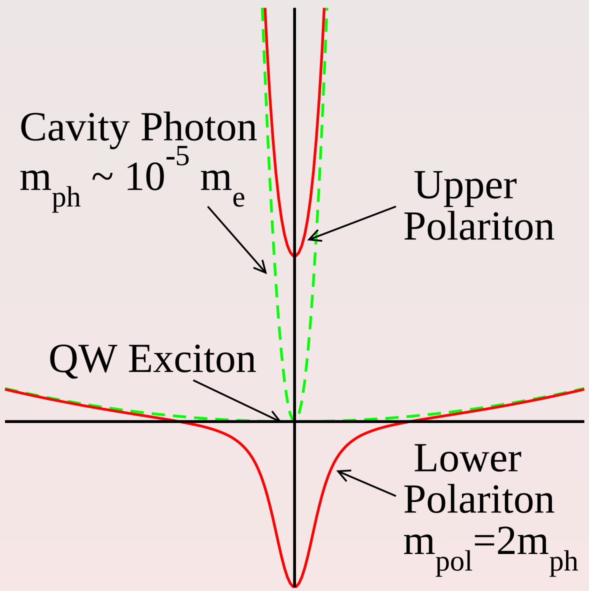
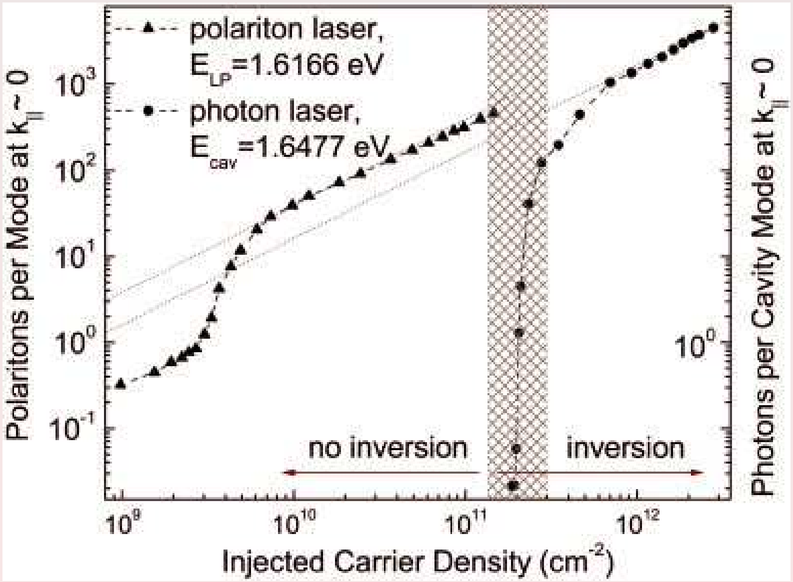


Problems?

- Cavity lifetime is short (ps), hard to thermalise.

Polariton Experiments: Photoluminescence

Non-linear Photoluminescence.



[*Deng et al.* PNAS **100** 15318]

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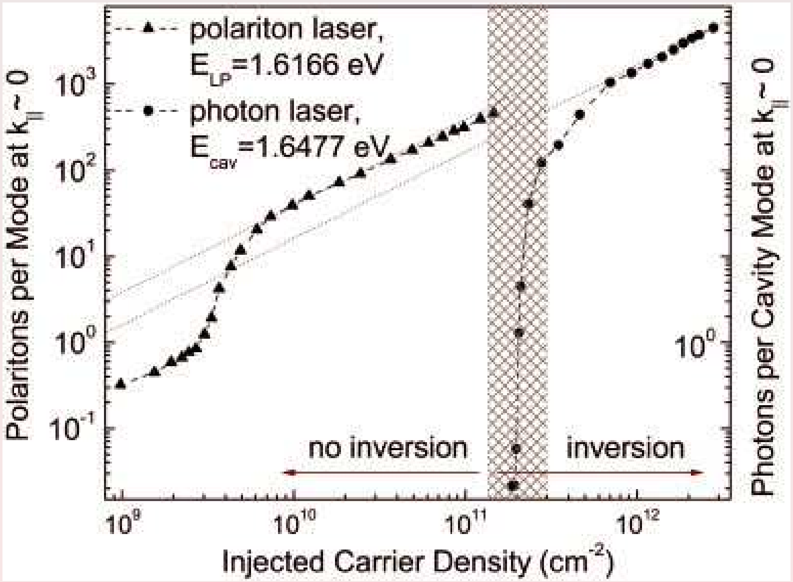

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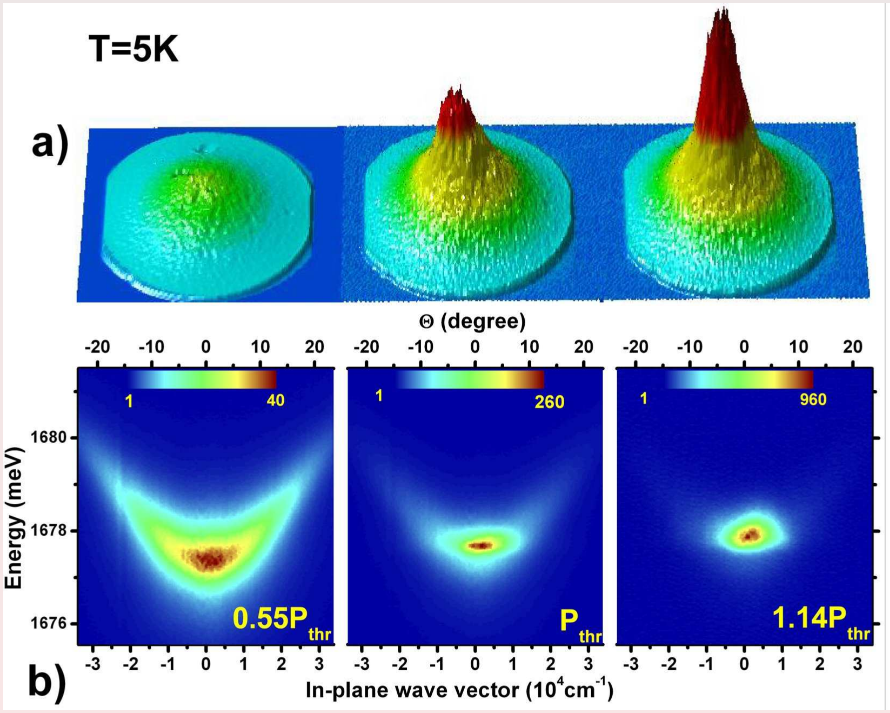
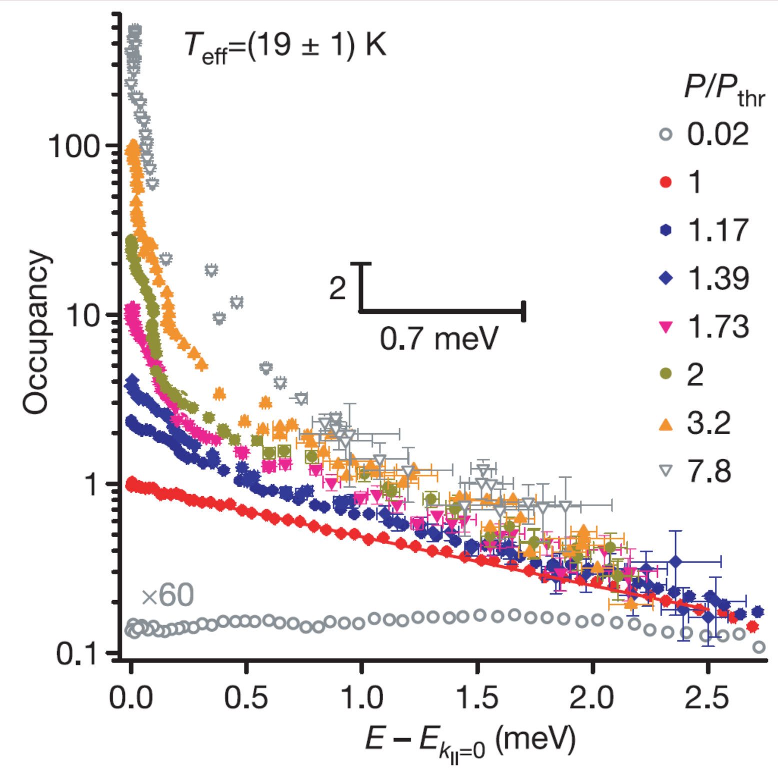


[*Kasprzak et al.* Nature **443** 409]

Polariton Experiments: Thermal distribution



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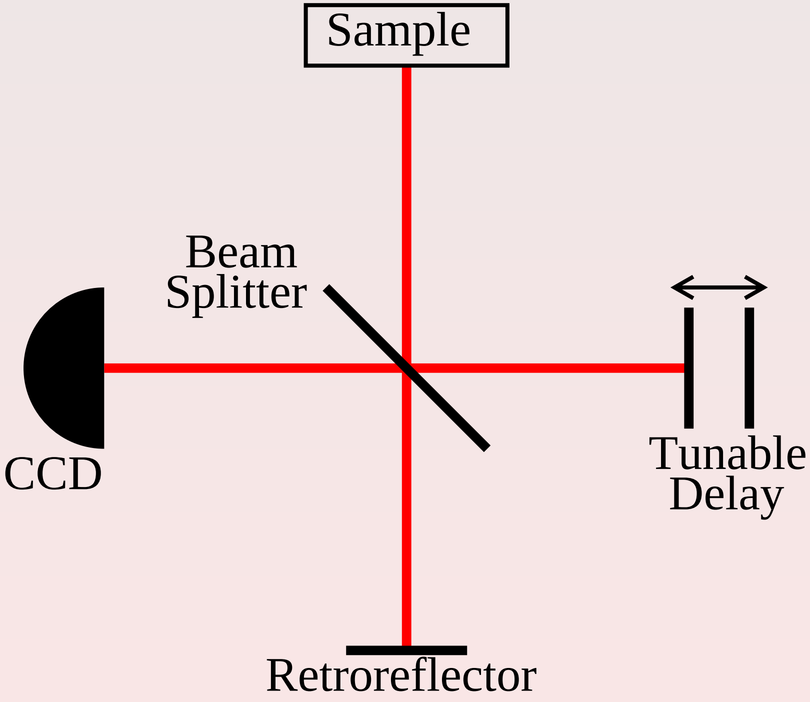
Polariton Experiments: Interference setup

Experimental configuration:



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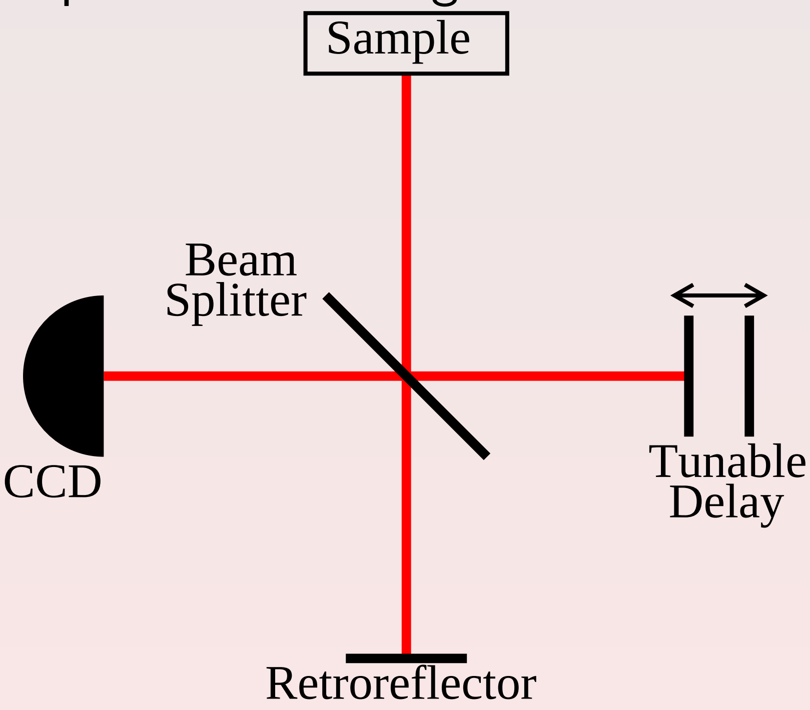

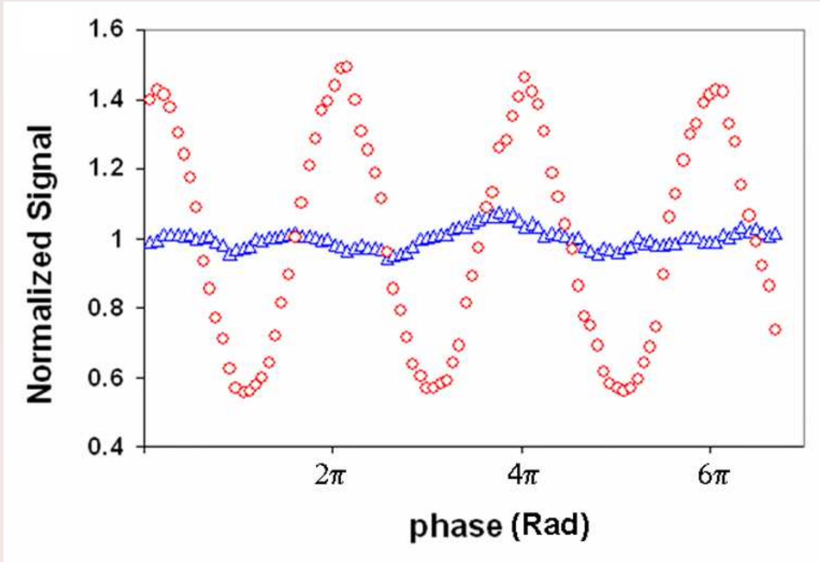

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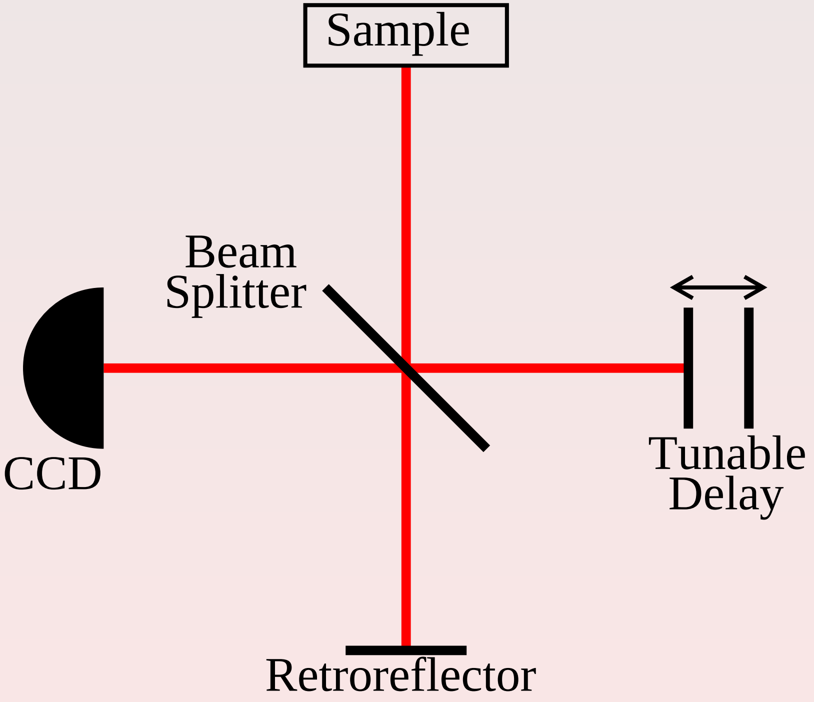

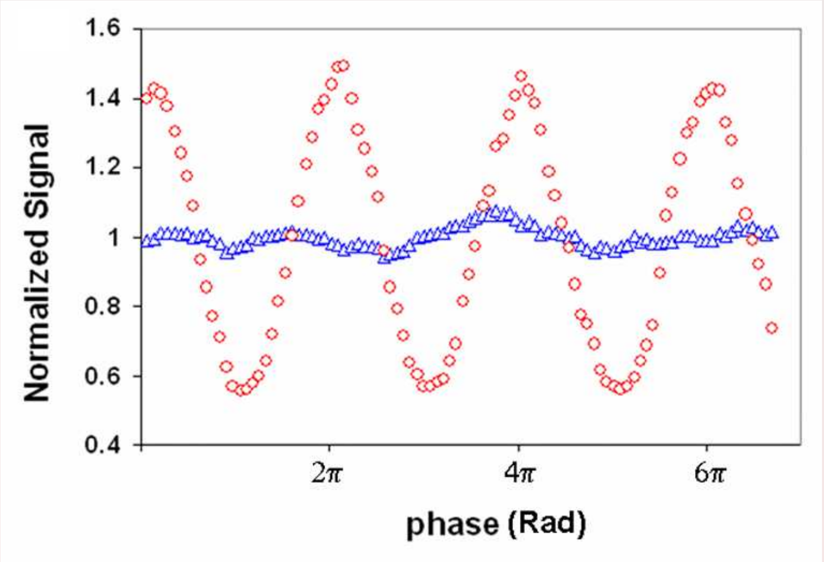

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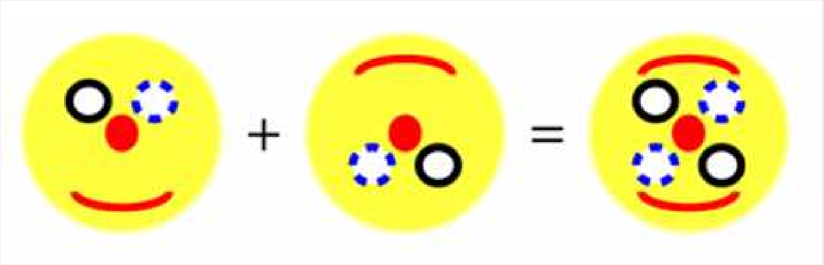
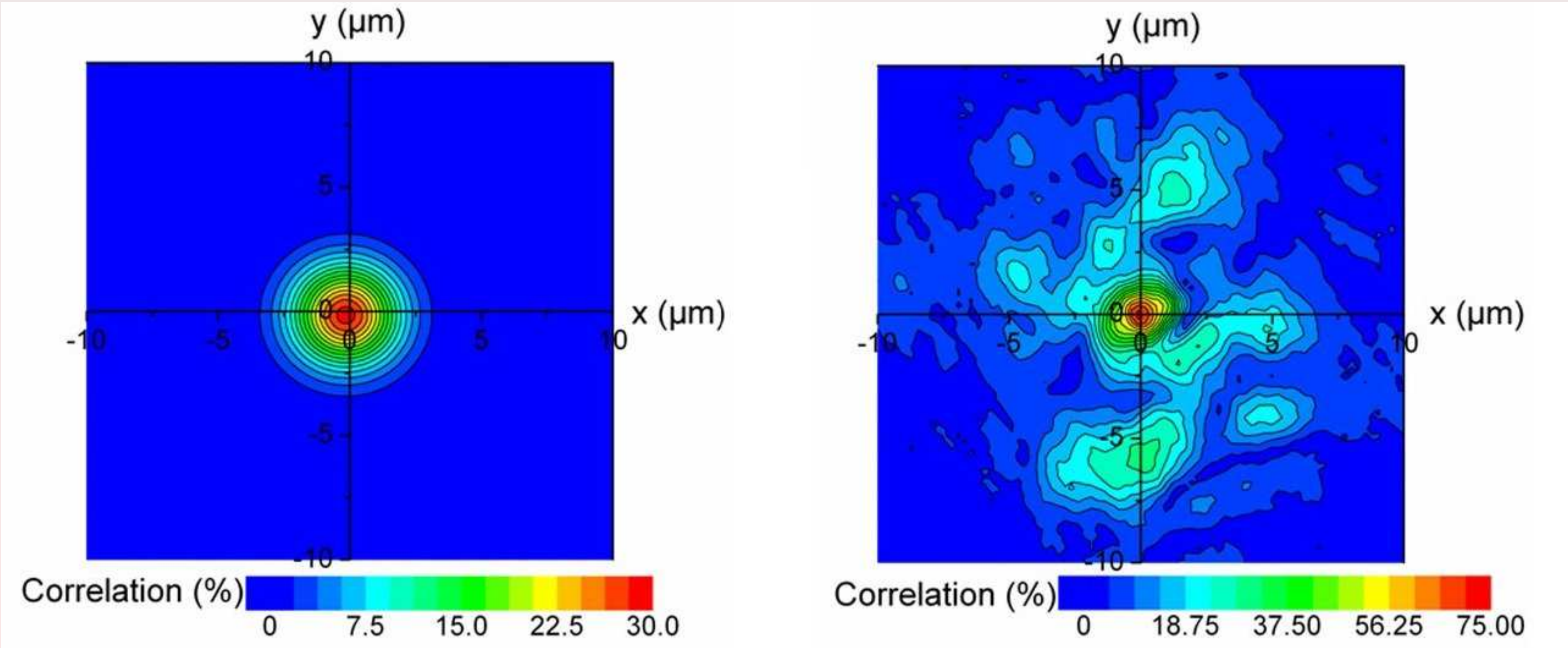


Polariton Experiments: Interference resuls

Below threshold

Above threshold



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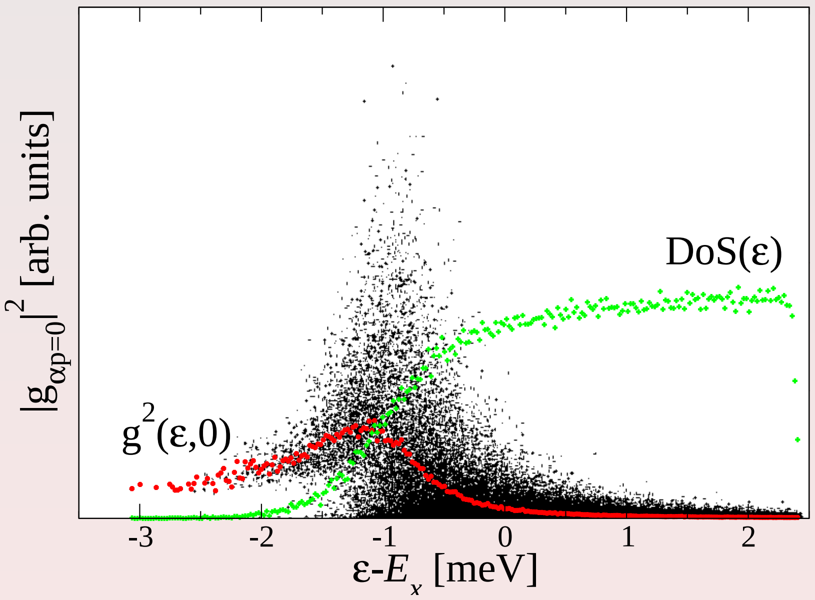
Localised two level systems

Coupling to light:



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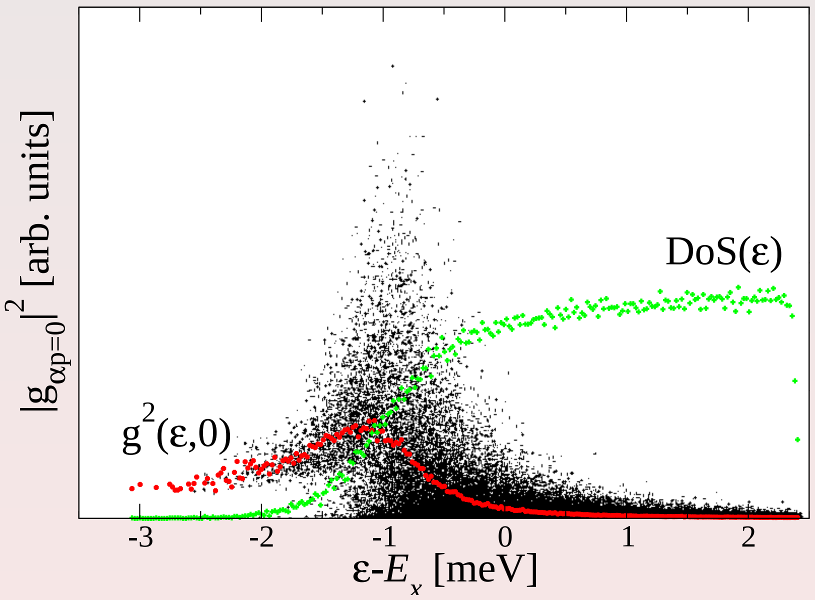

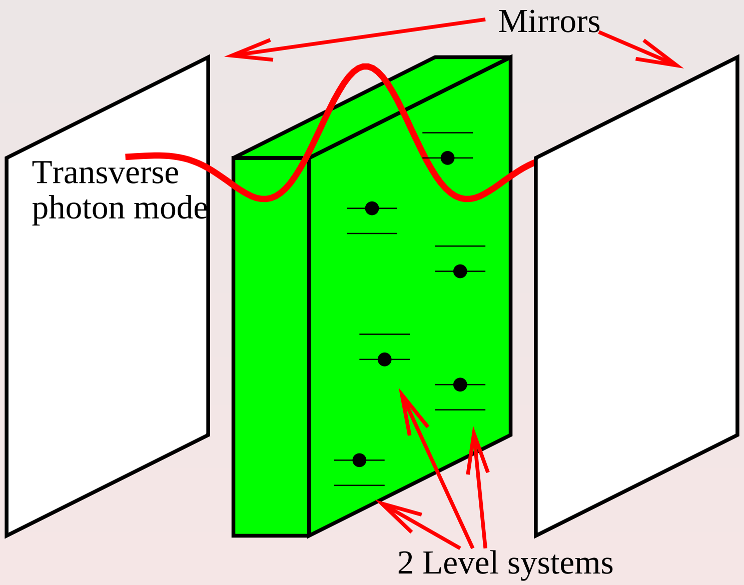

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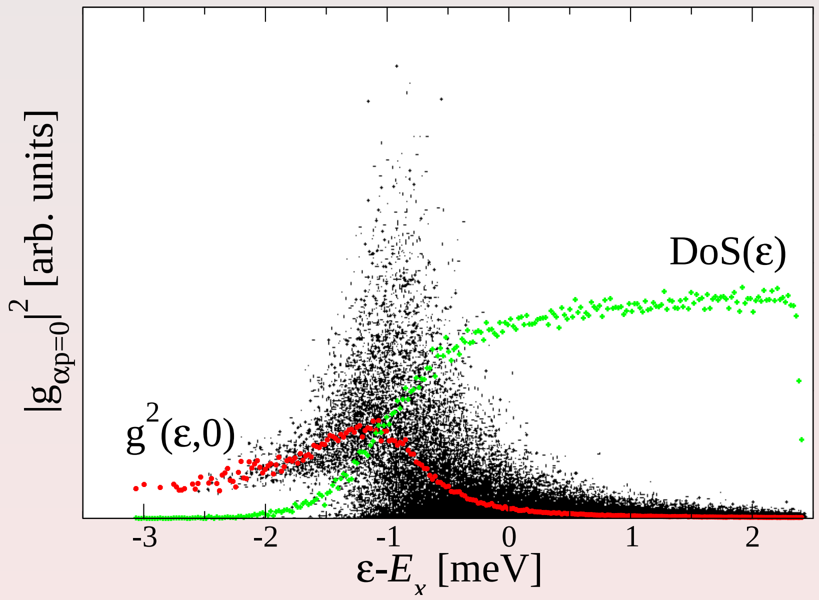

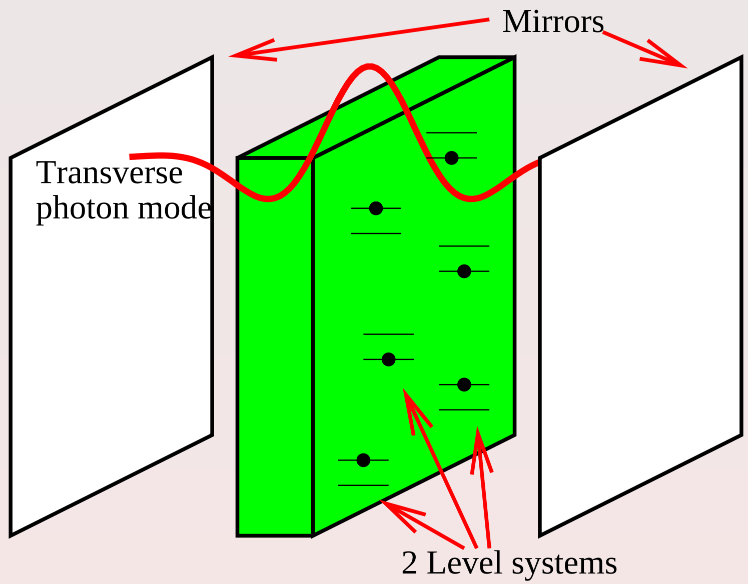

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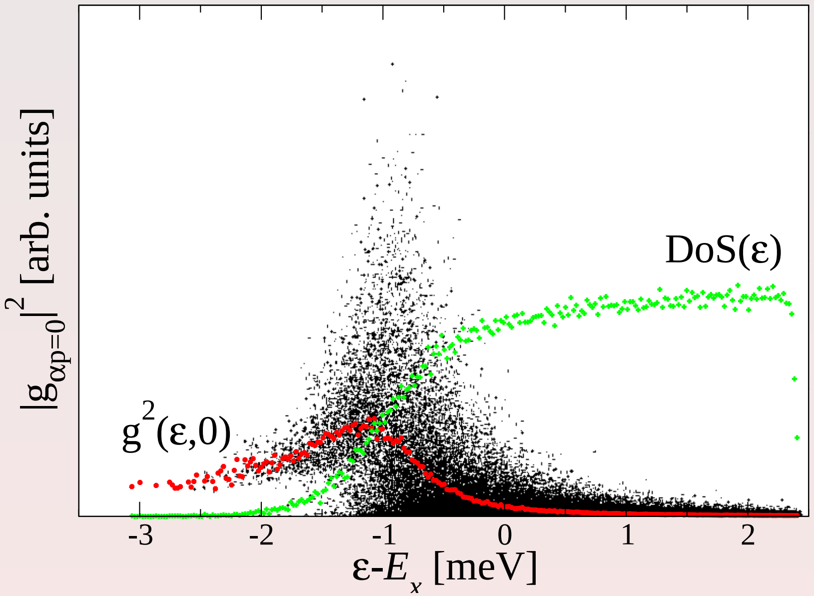

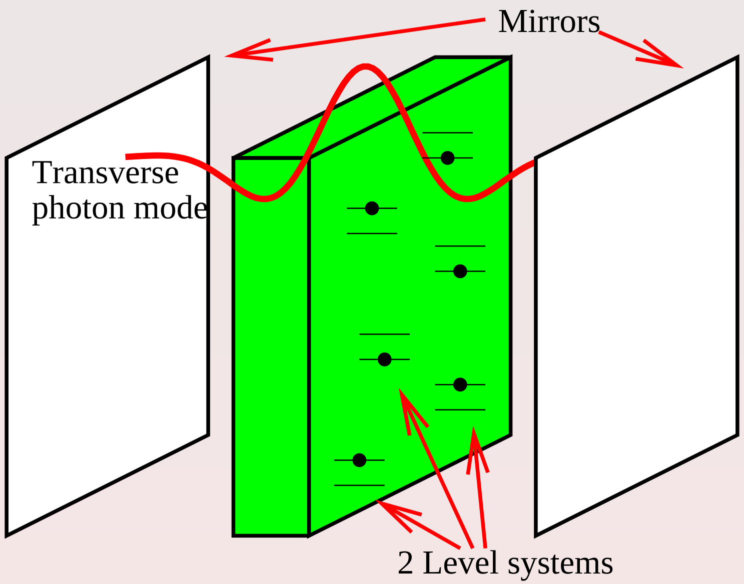

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- Energy difference between levels represents energy of bound exciton state.

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N = \sum_{\alpha=1}^{\alpha=nA} \frac{1}{2} \left(b_\alpha^\dagger b_\alpha - a_\alpha^\dagger a_\alpha + 1\right) + \sum_{k=l/\sqrt{A}} \psi_k^\dagger \psi_k.
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Comparison of physical systems:

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Mean field theory

At zero temperature, BCS-like ansatz is exact minimum

$$|\Psi\rangle = e^{\lambda(\psi_0^\dagger + \sum_\alpha X_\alpha b_\alpha^\dagger a_\alpha)} \prod_\alpha a_\alpha^\dagger |0\rangle$$

[*Eastham & Littlewood. Phys. Rev. B* **64** 235101].

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General form

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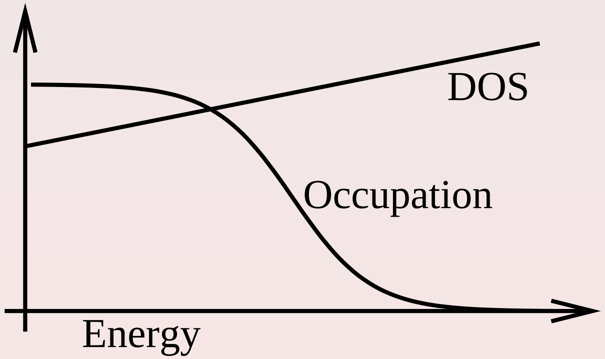

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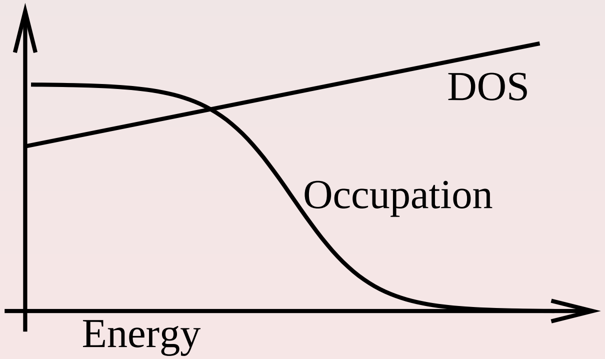

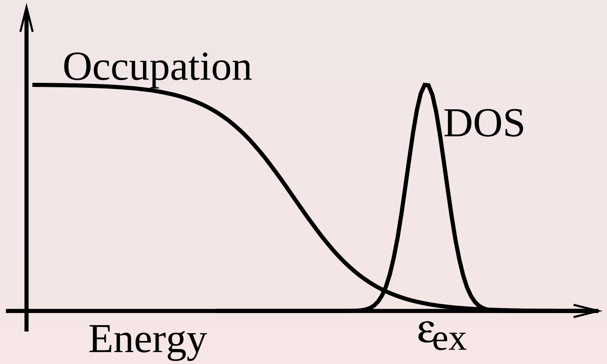

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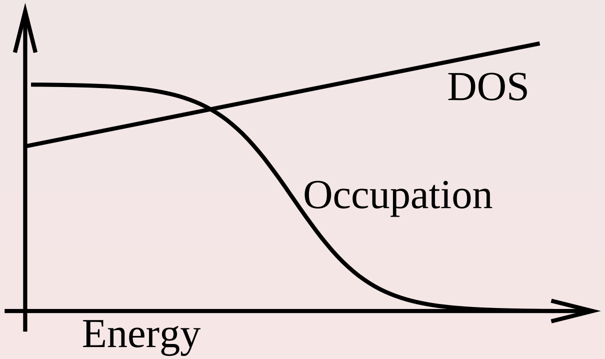

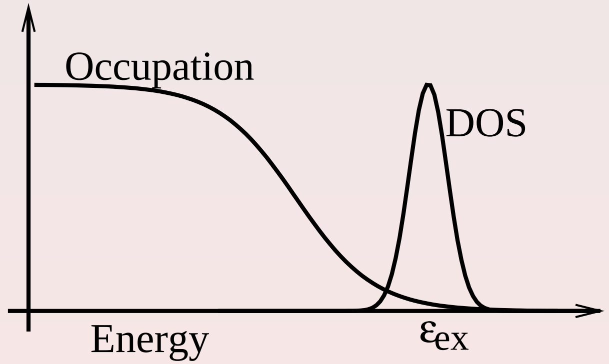

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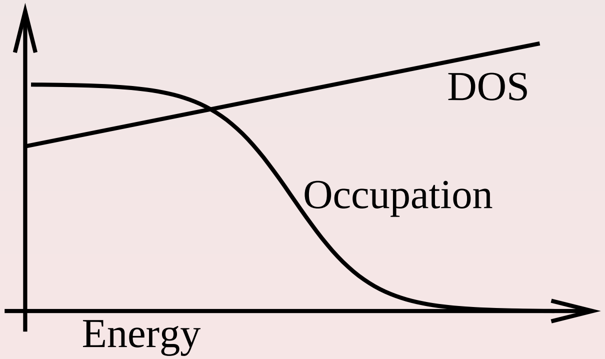

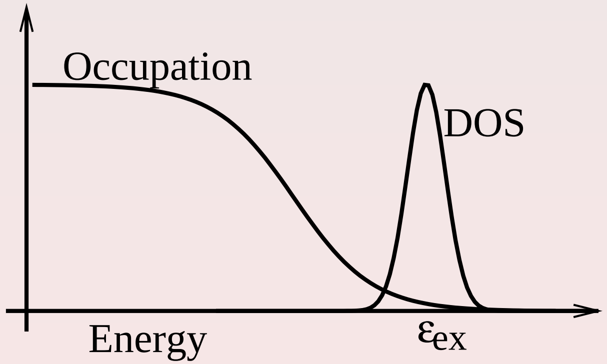

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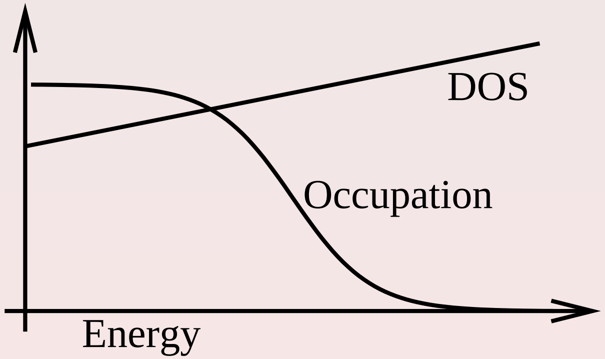

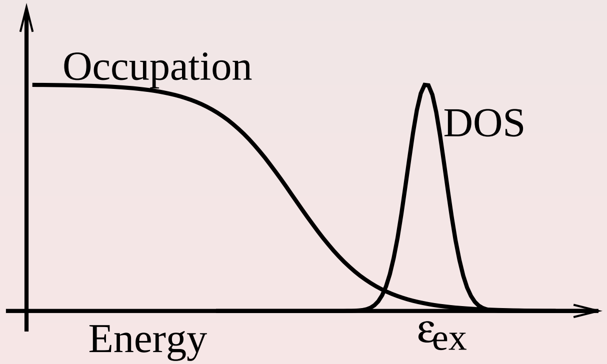

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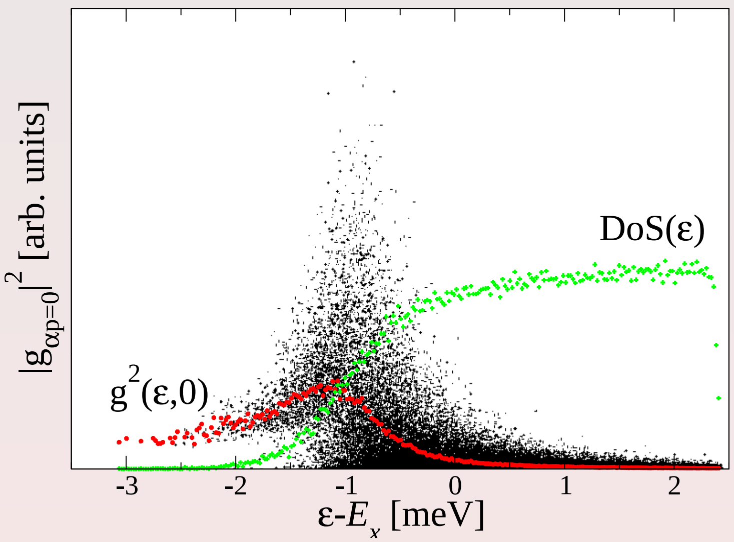
Supplementary slides

Localised two level systems



[*Marchetti et al.* PRL **96**, 066405 (2006);`cond-mat/0608096`].

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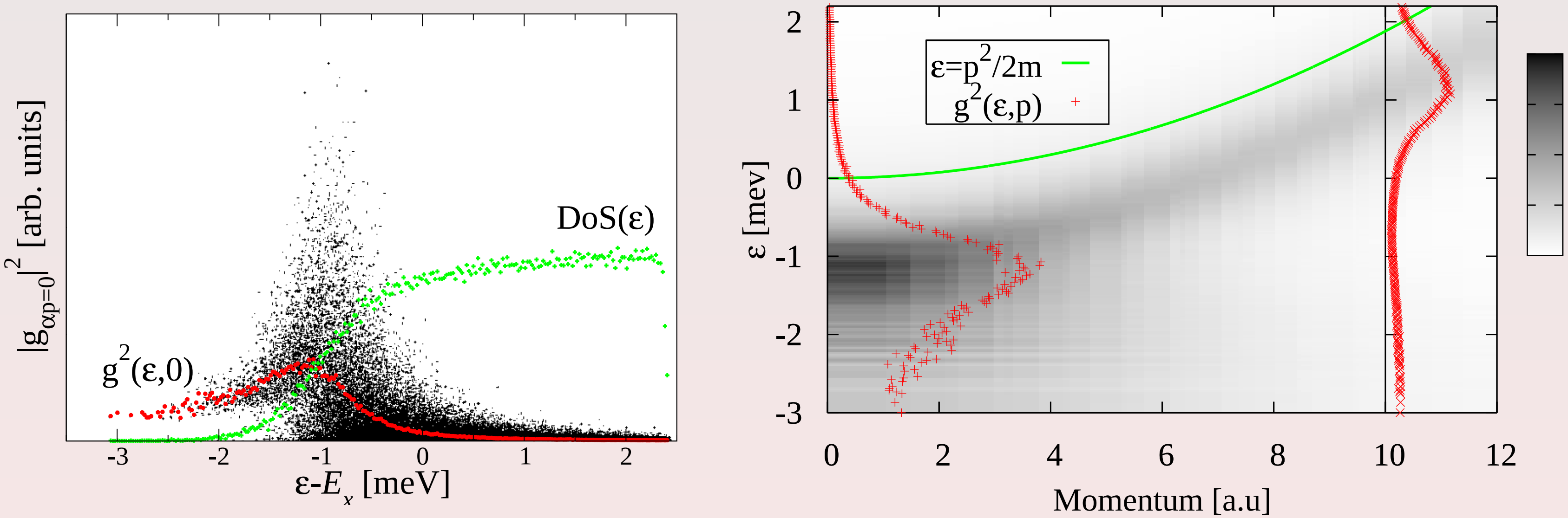

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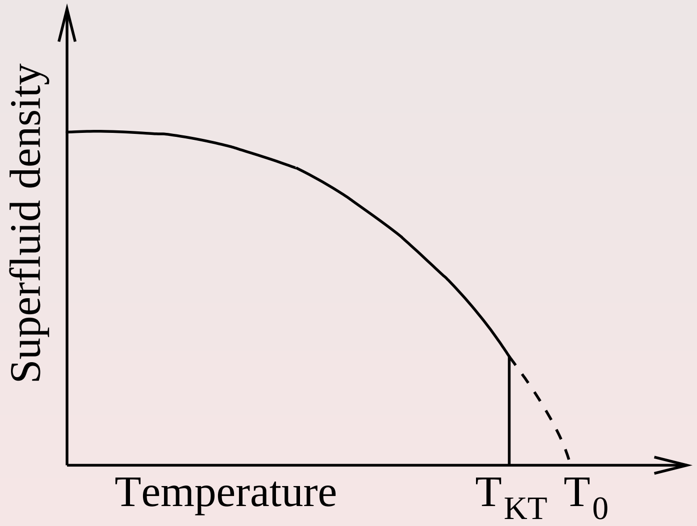
  - Boson field dynamic, with chemical potential — similar to Holland-Timmermans model, e.g. [*Ohashi & Griffin, PRA.* **67** 063612 (2003)]

Fluctuations in 2d



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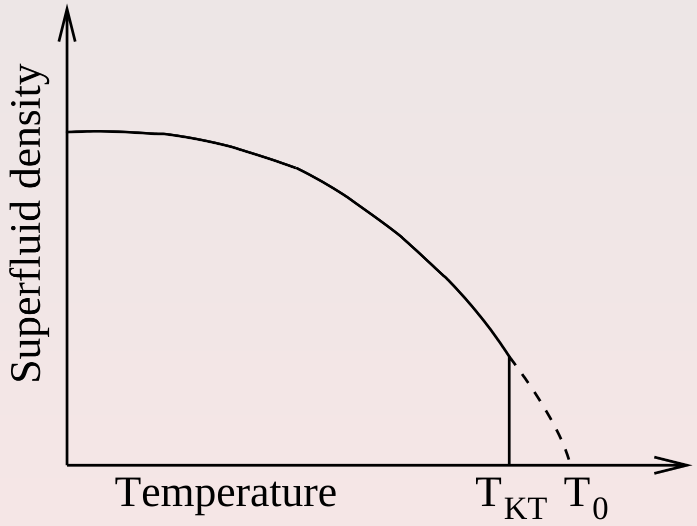

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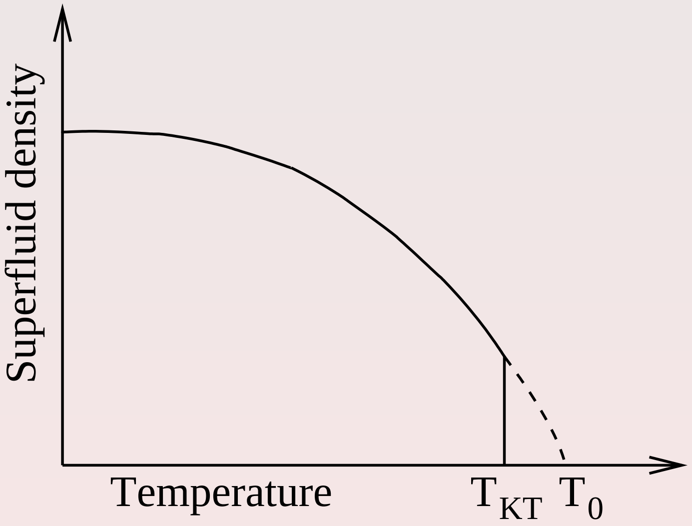

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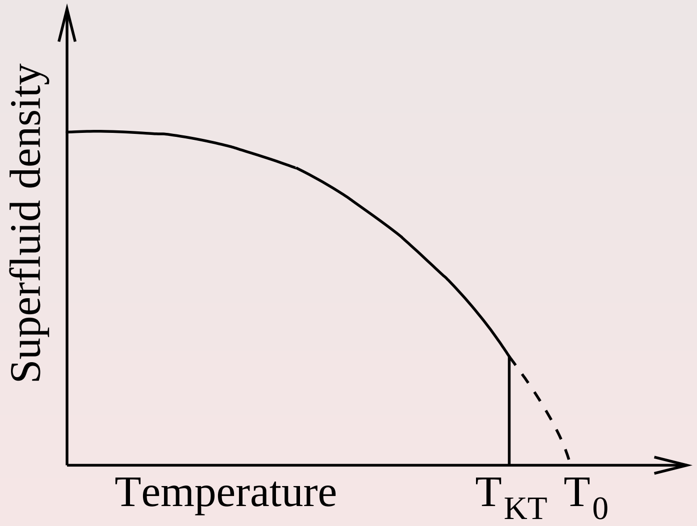

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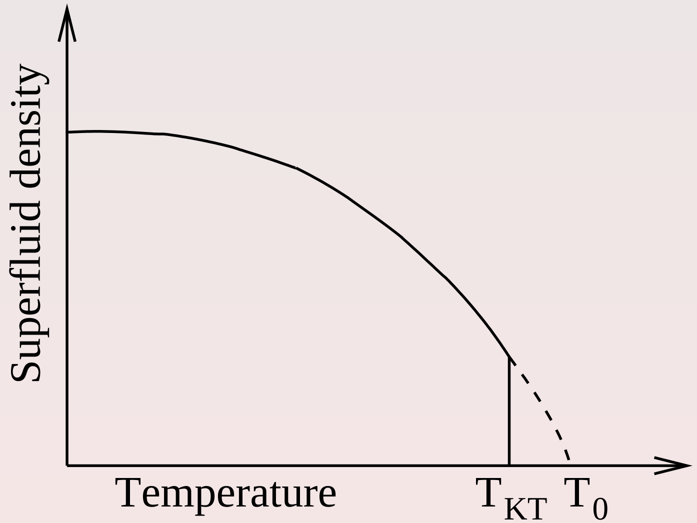

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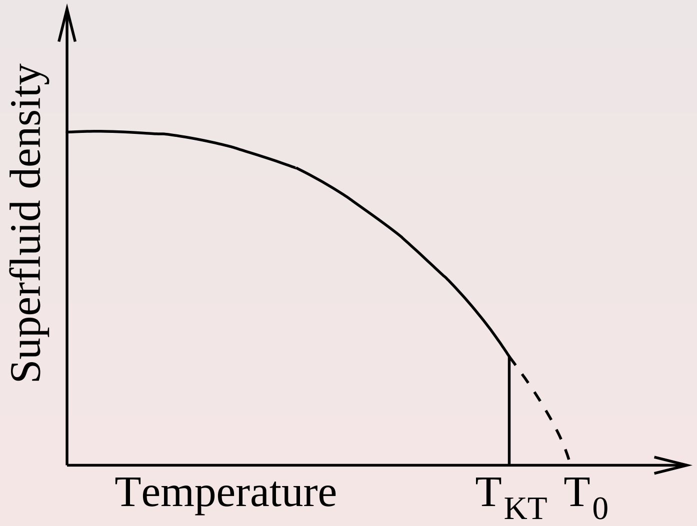

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Thus, need to find: ρ_{total} in presence of condensate.

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Condensate depletion changes critical chemical potential.

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$$H - \mu N = \sum_k (\epsilon_k - \mu) a_k^\dagger a_k + \frac{g}{2} \sum_{k,k',q} a_{k+q}^\dagger a_{k'-q}^\dagger a_k a_{k'}.$$

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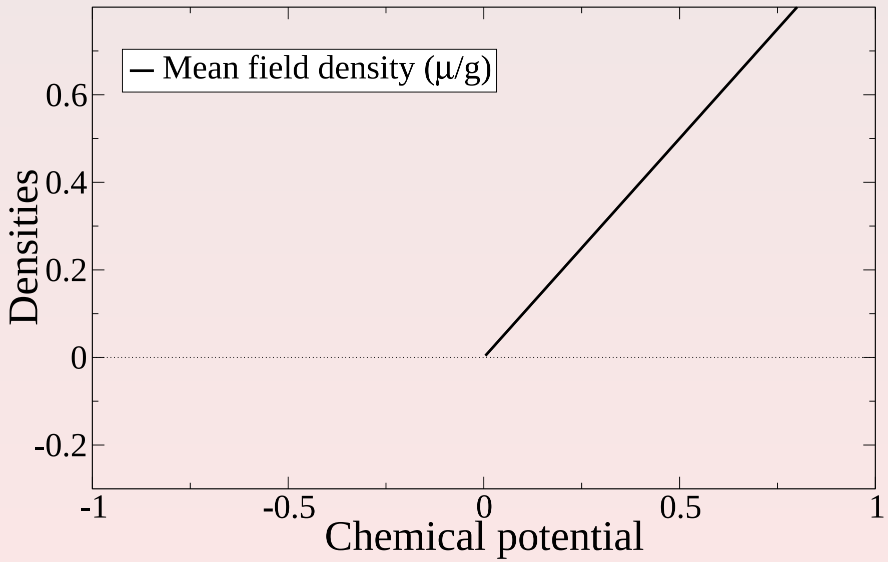

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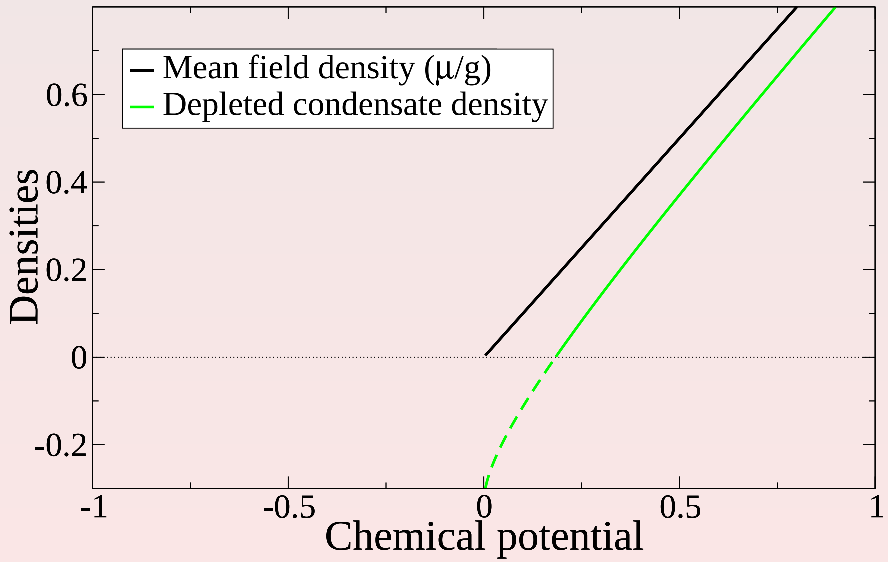

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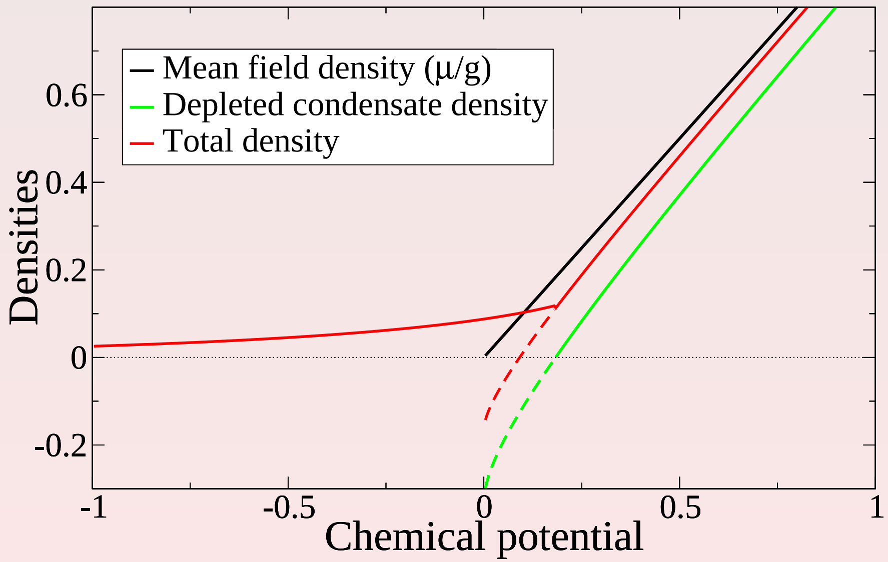
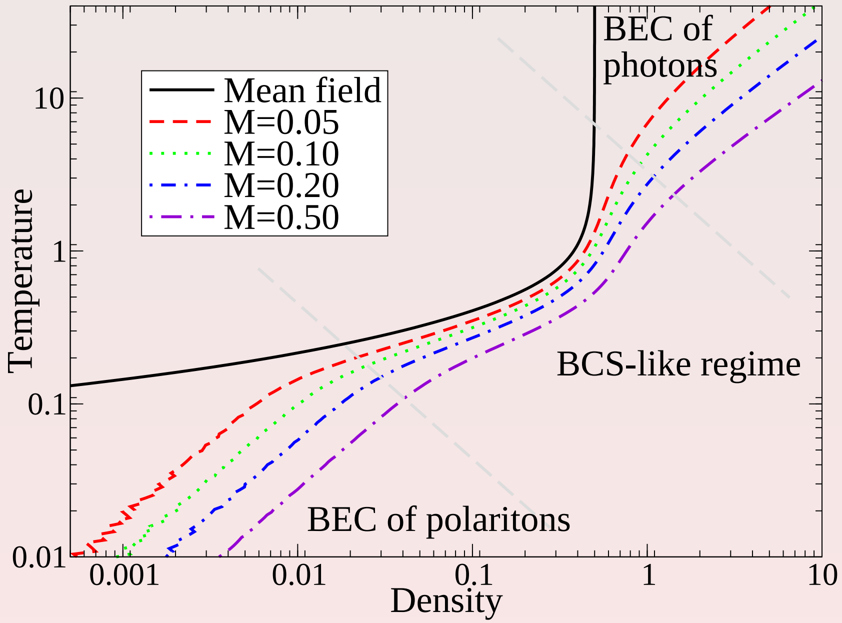


Normal state exists for $\mu < 0$: Need self energy.

The phase diagram

Calculate density where $\rho_{\text{superfluid}} = 0$.



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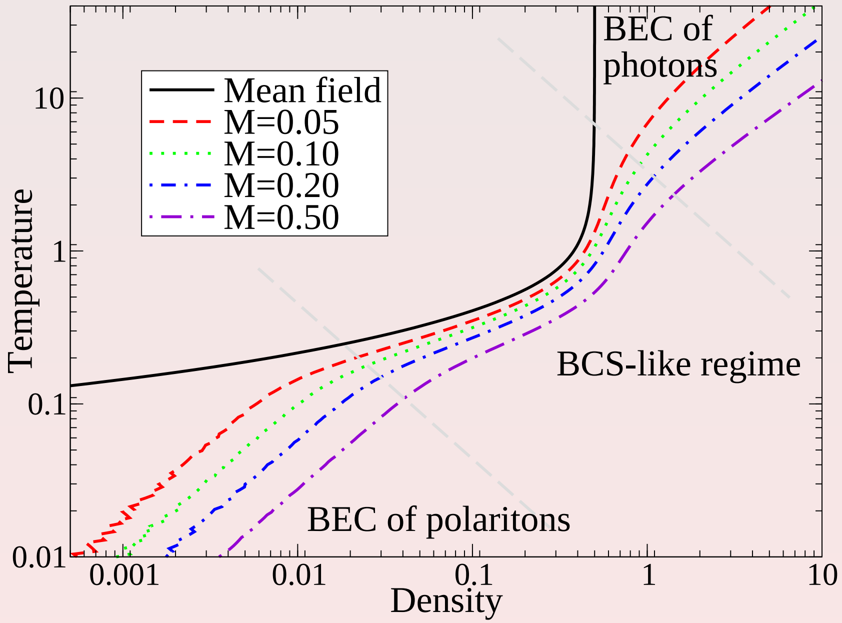

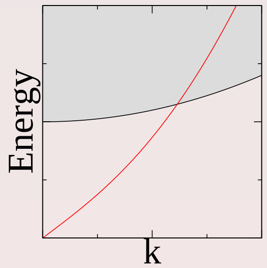

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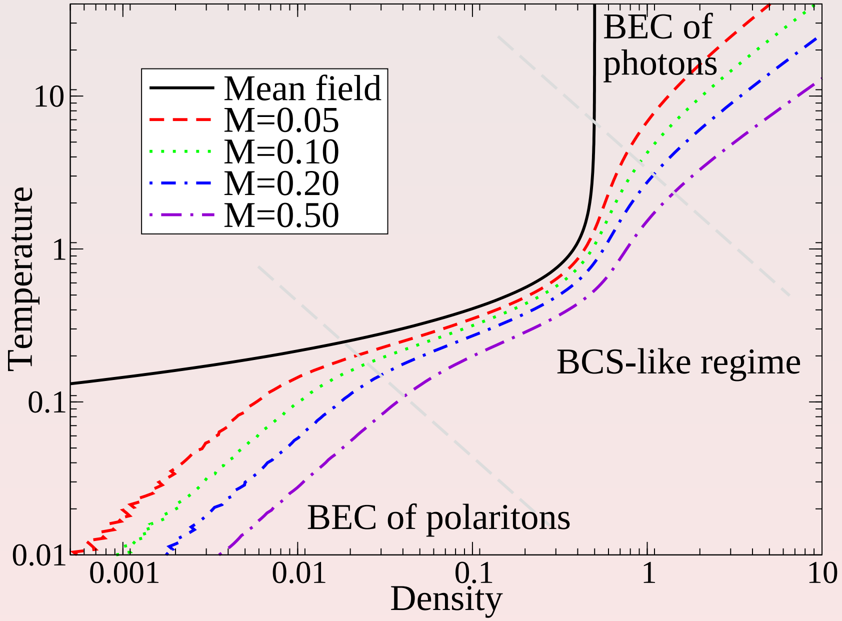

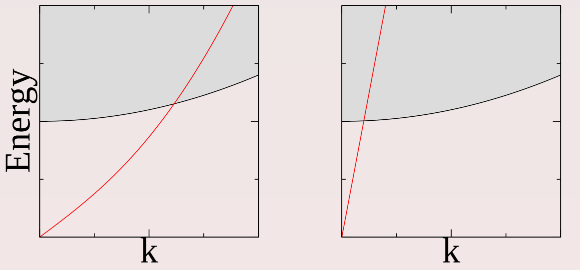

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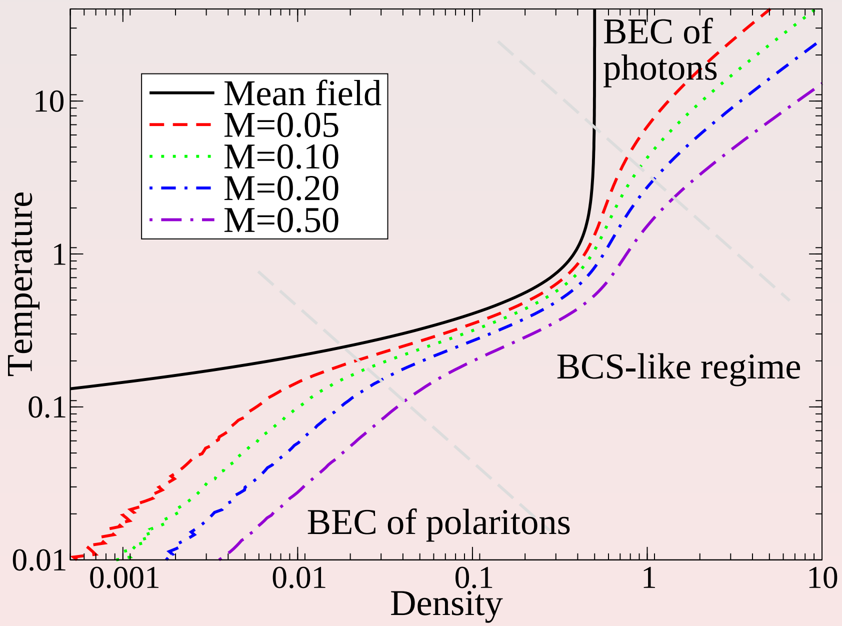

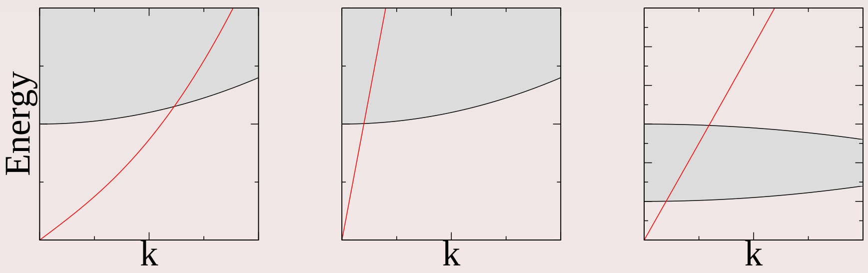

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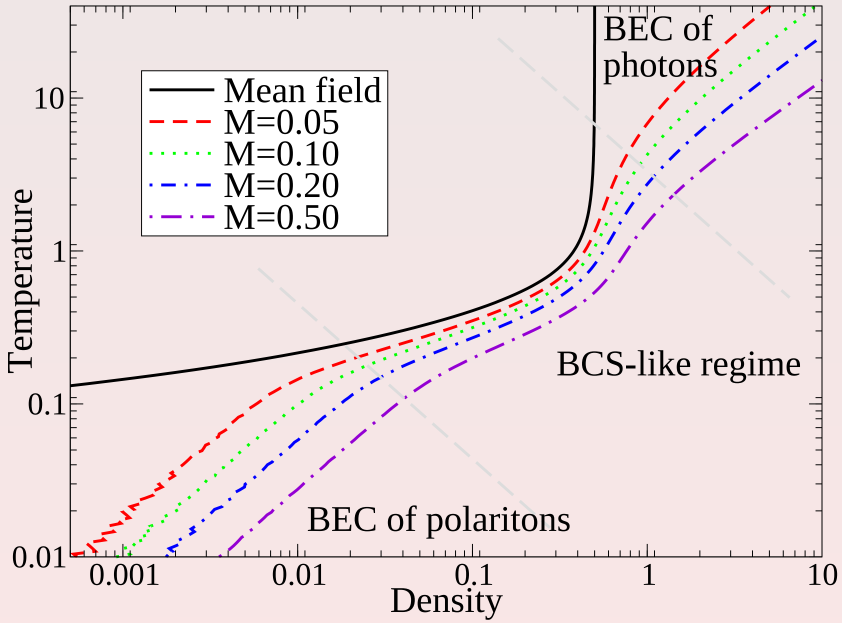

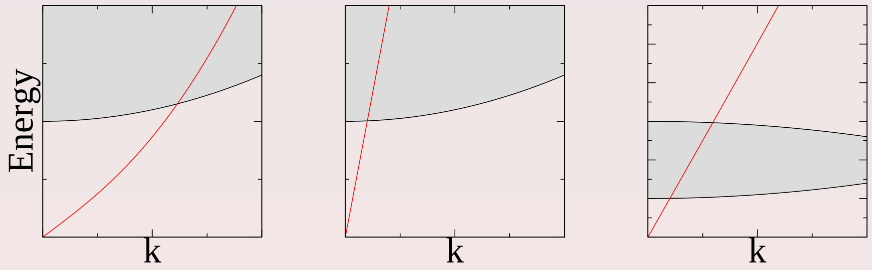

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$$T_{deg} = \frac{\rho}{m}$$

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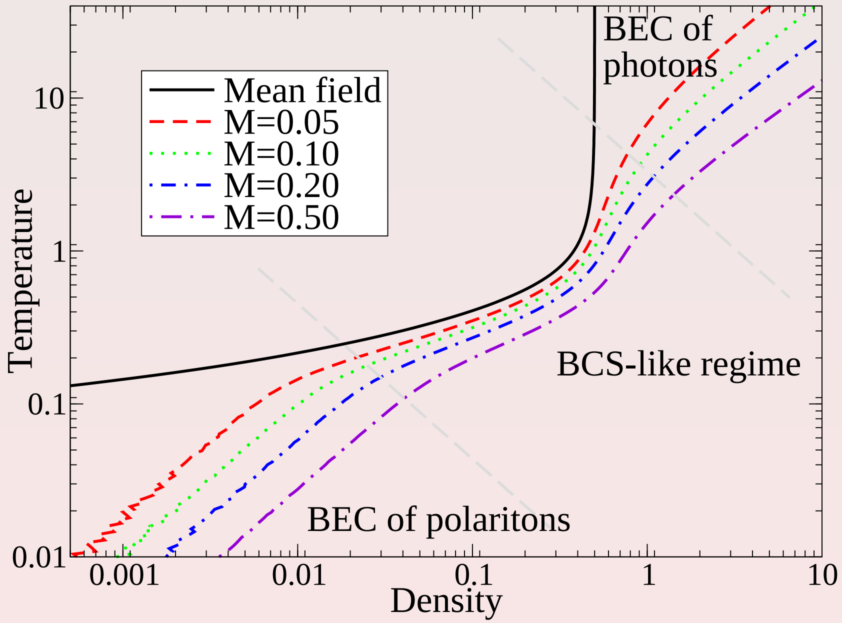

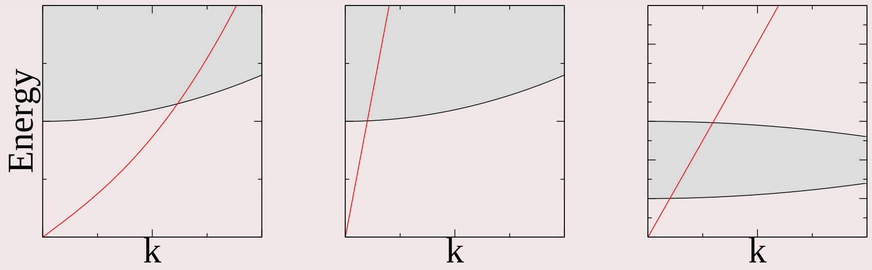

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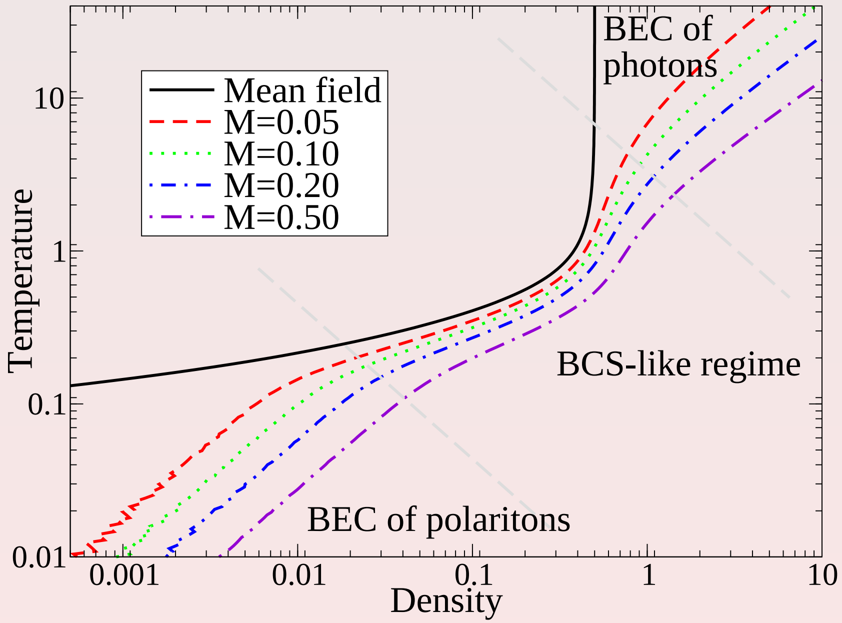

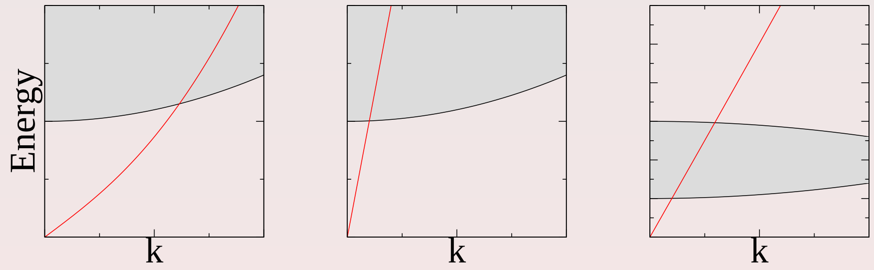

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Current experiments in BCS-like regime: $\rho_{\text{crossover}}/n \approx mg/\sqrt{n} \approx 10^{-3}$, experiments around $\rho/n \approx 0.01$.