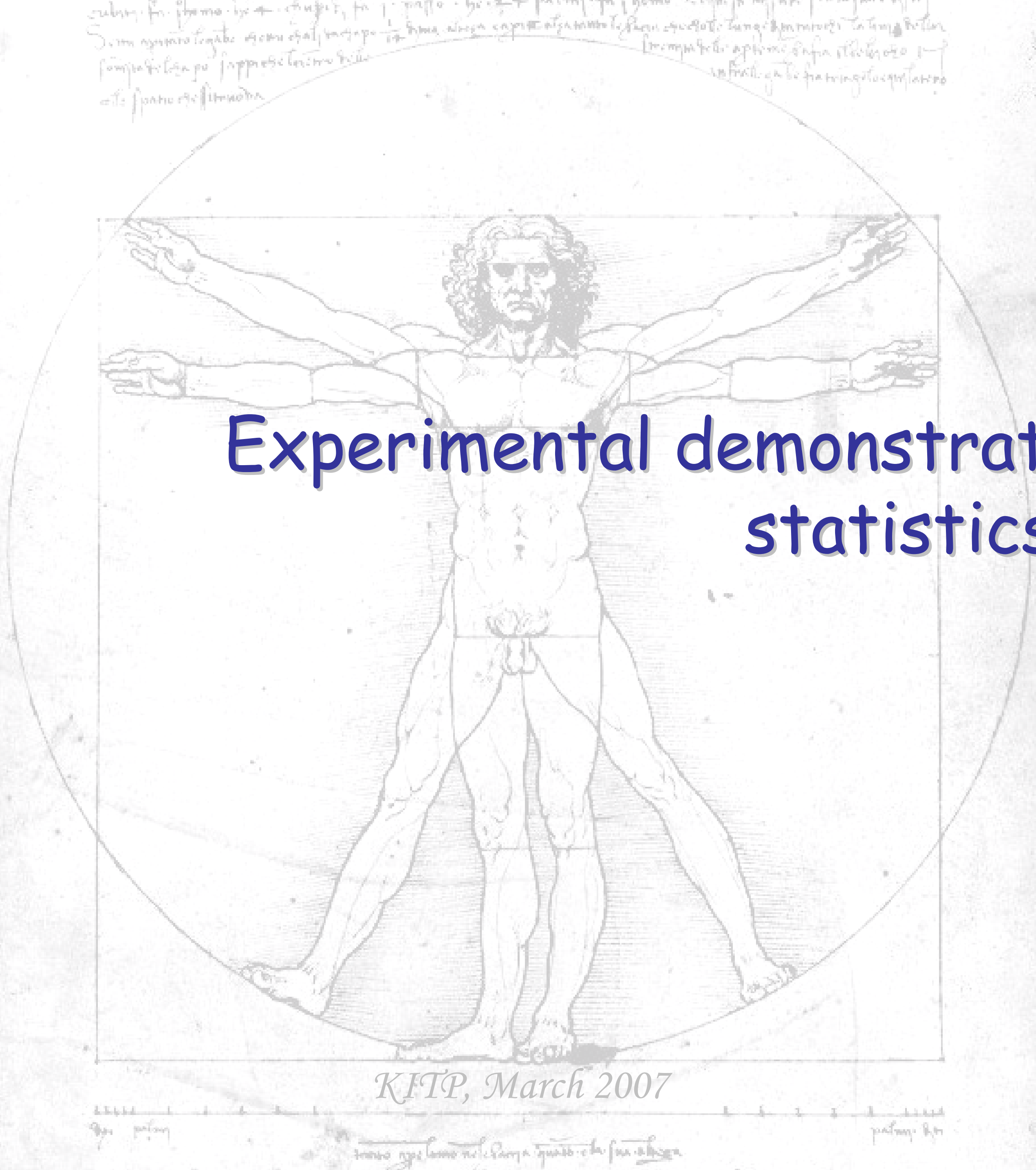

Experimental demonstration of anyonic statistics with photons

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THE ROYAL SOCIETY

KITP, March 2007

Overview

- In condensed matter **anyons** appear in ground or excited states of two dimensional systems:
  - **Superconducting** electrons in a strong magnetic field (Fractional Quantum Hall Effect)
  - **Lattice** systems (Kitaev's toric code/hexagonal lattice, Wen's models, Ioffe's model, Freedman-Nayak-Shtengel model…)
- **Energy gap** protects anyons:

  - if I get anyonic statistics I do not need gap.
- Relatively **large systems**:

  - employ largest implementable system.
- Close the gap between theory and experiment.

Overview

- Anyonic statistics is a property of a (highly entangled) wavefunction.
  - Engineer states rather than cool – same effect.

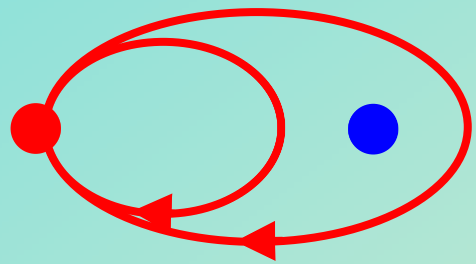
- Employ the **toric code** model.
  - One plaquette: one **anyon** and **path** of another.
  - No Hamiltonian: is like **algorithmic encoding**.

- How to *generate, manipulate, measure* anyons?
- The **control manipulations** are **exactly the same** with Hamiltonian or larger system.
- *Future work:* $H \neq 0, L >> 1$

Anyons

Anyons have non-trivial statistics.

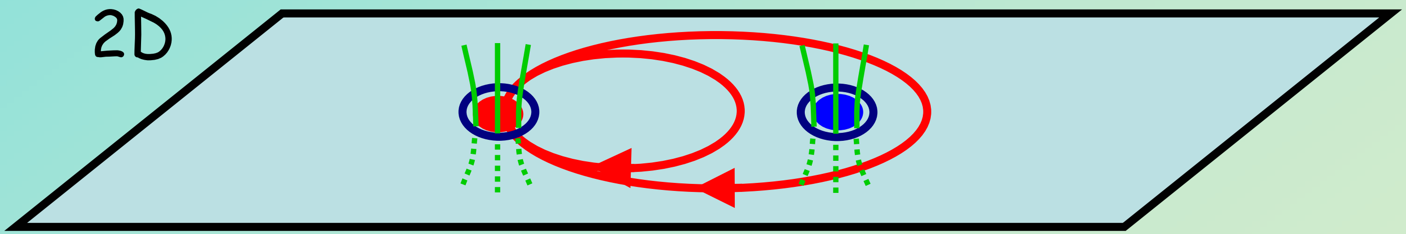
3D



Bosons $|\Psi\rangle \rightarrow |\Psi\rangle$

Fermions $|\Psi\rangle \rightarrow e^{i2\pi}|\Psi\rangle$

2D



$$|\Psi\rangle \rightarrow e^{i2\varphi}|\Psi\rangle$$

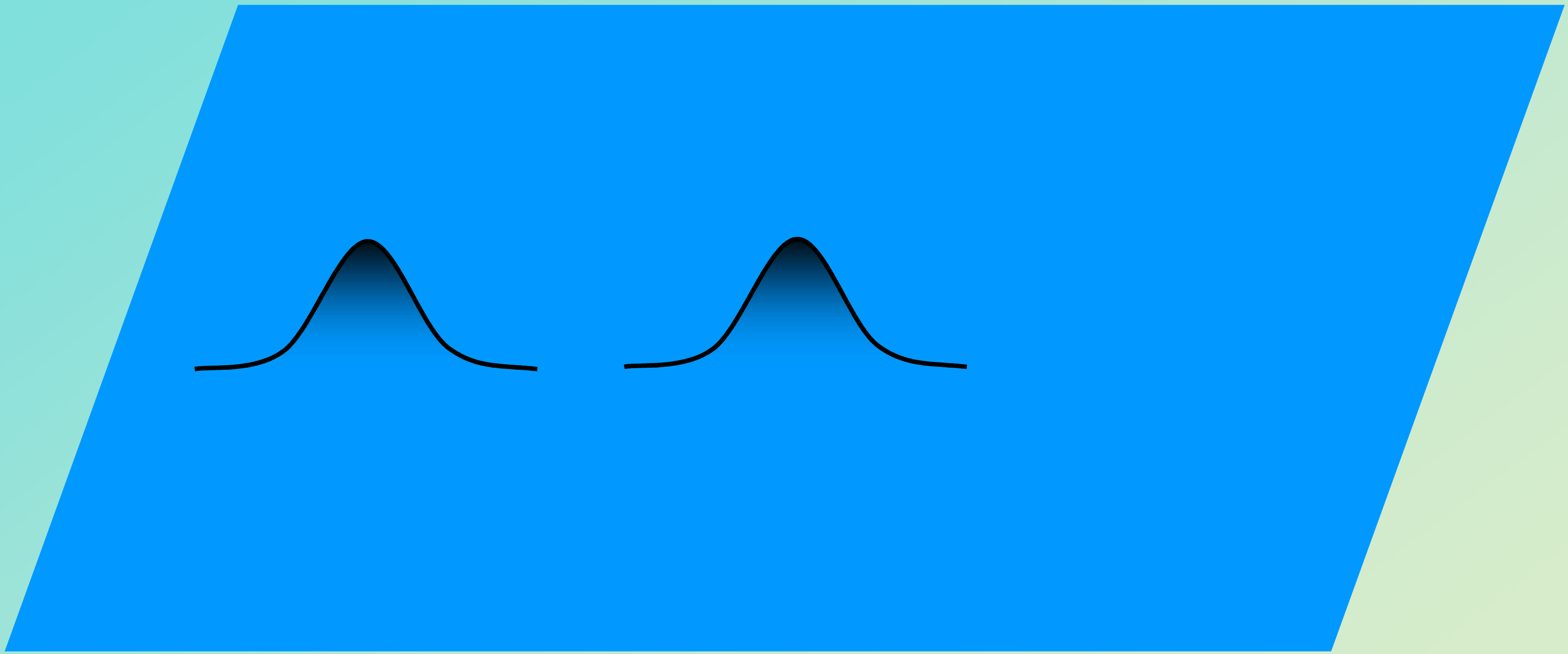
$$|\Psi\rangle \rightarrow U|\Psi\rangle$$

Anyons

Consider as composite particles of fluxes and charges.

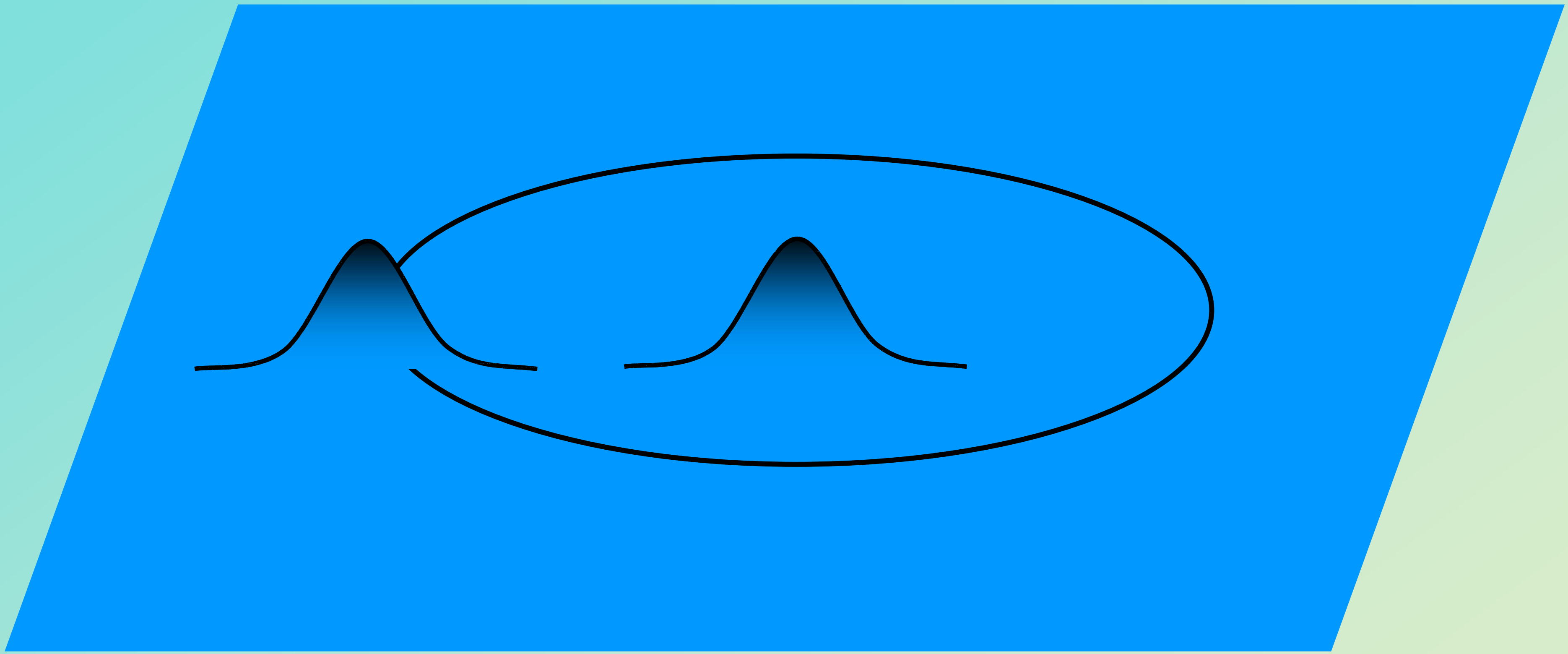
Then phase is like the Aharonov-Bohm effect.

Anyons: do they live among us?



Create two localized "things" with **effective charge** and **magnetic field.**

Anyons: do they live among us?



Create two localized “things” with **effective charge** and **magnetic field.**

Braid them -> **PHASE FACTOR:** Effective gauge theory!

Toric Code (also ECC)

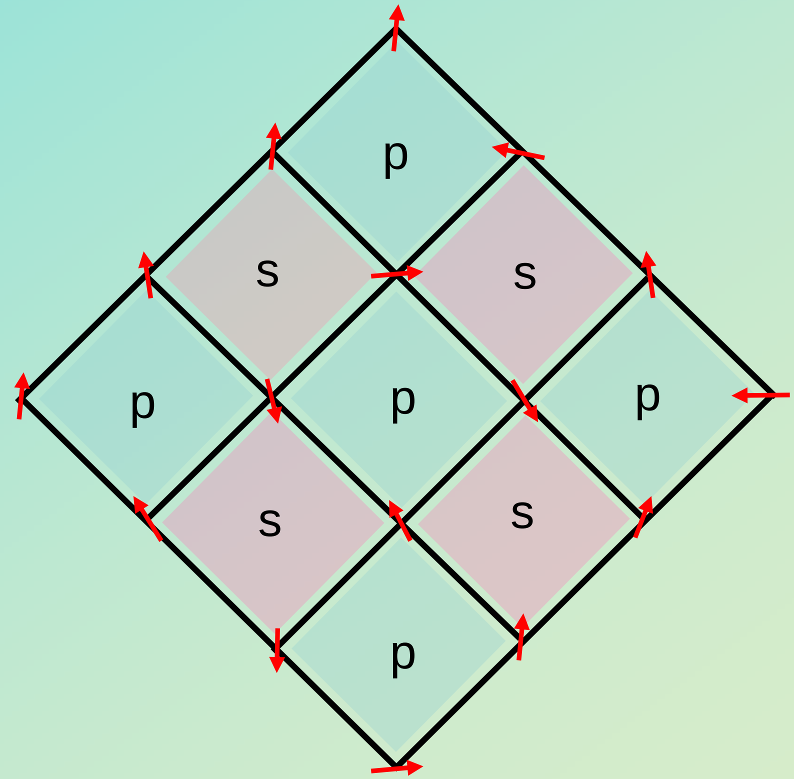
Consider the lattice Hamiltonian

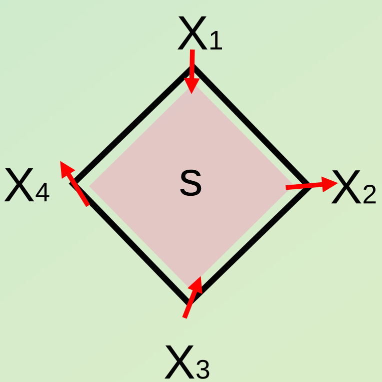
$$H = -\sum_{p} Z_1 Z_2 Z_3 Z_4 - \sum_{s} X_1 X_2 X_3 X_4$$

Spins live on the vertices.

There are two different types of plaquettes, p and s, which support ZZZZ or XXXX interactions respectively.

The four spin interactions involve spins at the same plaquette.





Toric Code

Consider the lattice Hamiltonian

$$H = -\sum_p Z_1 Z_2 Z_3 Z_4 - \sum_s X_1 X_2 X_3 X_4$$

It is easy to find the ground state of this Hamiltonian.

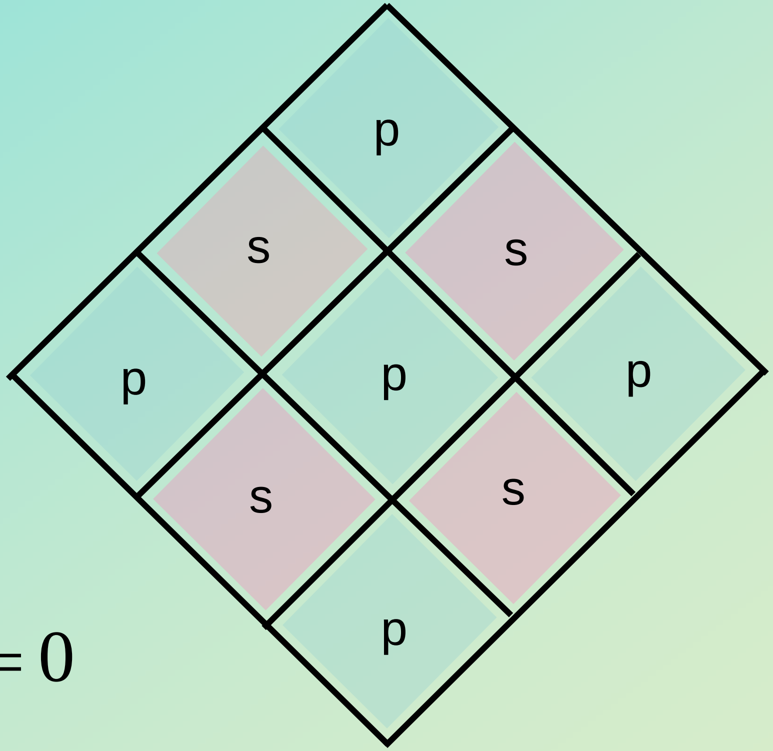
First observe that

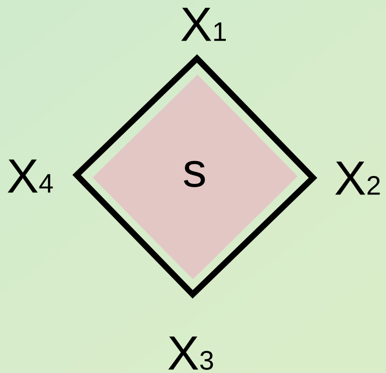
$$[H, Z_1 Z_2 Z_3 Z_4] = 0, [H, X_1 X_2 X_3 X_4] = 0$$

$$[X_1 X_2 X_3 X_4, Z_1 Z_2 Z_3 Z_4] = 0$$

$$(X_1 X_2 X_3 X_4)^2 = (Z_1 Z_2 Z_3 Z_4)^2 = 1$$

Eigenvalues of XXXX and ZZZZ terms are 1 and -1





Toric Code

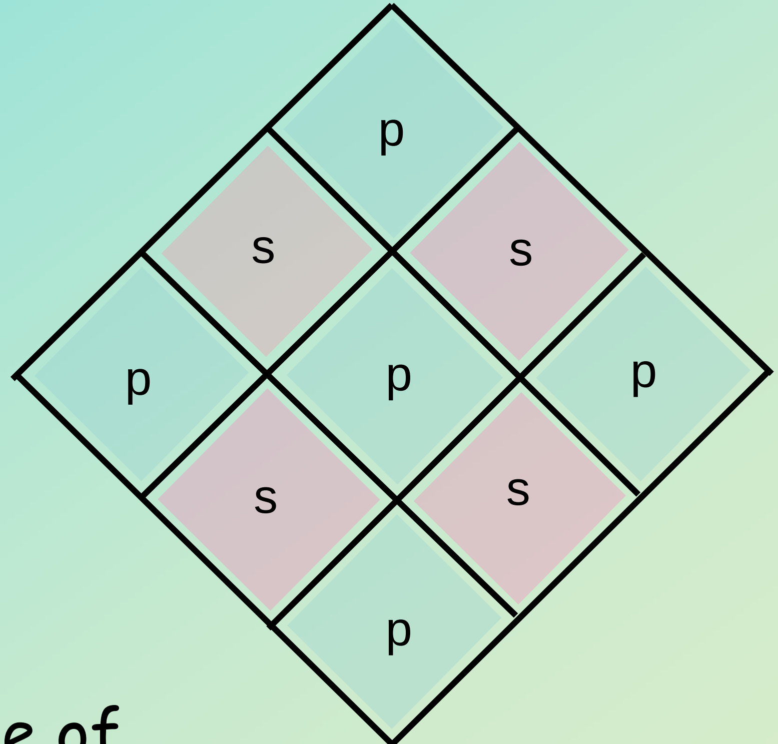
Consider the lattice Hamiltonian

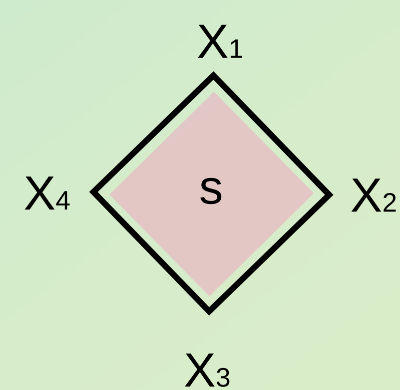
$$H = -\sum_p Z_1 Z_2 Z_3 Z_4 - \sum_s X_1 X_2 X_3 X_4$$

Hence, the ground state is:

$$\left|\xi\right\rangle = \prod_s (\mathrm{I} + X_1 X_2 X_3 X_4)_p \left|00\ldots0\right\rangle$$

The |00…0> state is a ground state of the ZZZZ terms and the (I+XXXX) term projects that state to the ground state of the XXXX term.





[F. Verstraete, et al. PRL, 96, 220601 (2006)]

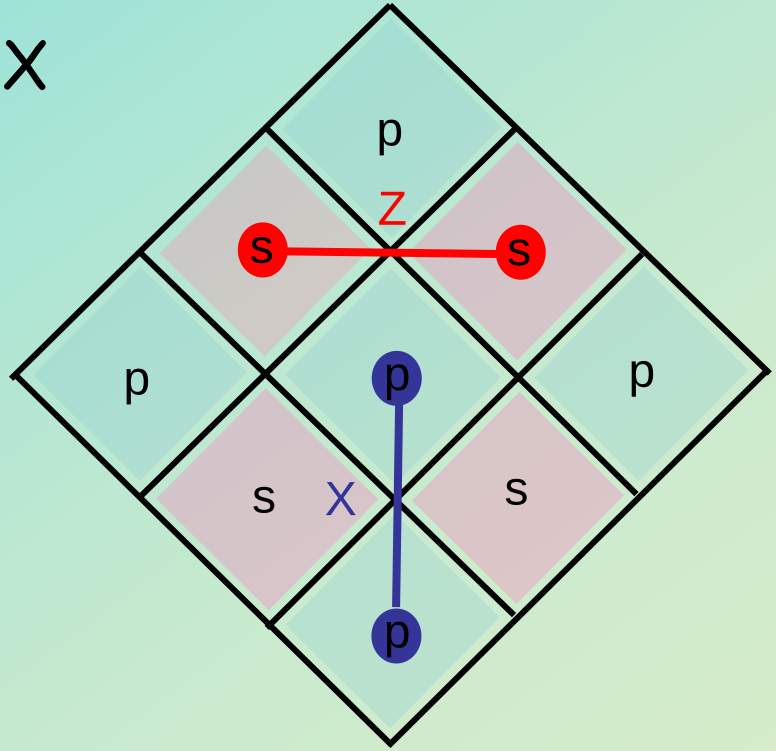
Toric Code

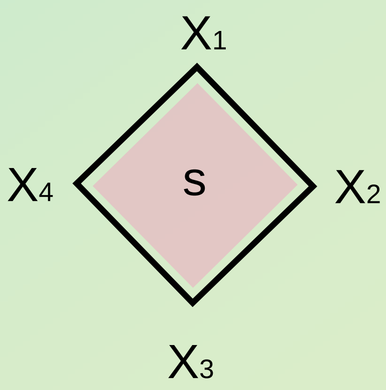
- Excitations are produced by Z or X rotations of one spin.

- These rotations anticommute with the X or Z part of the Hamiltonian, respectively.

- Z excitations on *s* plaquettes.
- X excitations on *p* plaquettes.

- X and Z excitations behave as anyons with respect to each other.

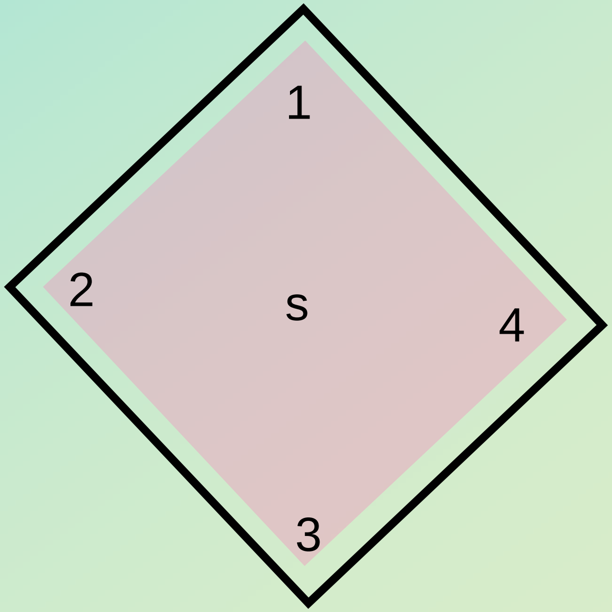




One plaquette

It is possible to demonstrate the anyonic properties with one *s* plaquette only. Then the Hamiltonian takes the form

$$H = -X_1X_2X_3X_4$$
$$-Z_1Z_2 - Z_2Z_3 - Z_3Z_4 - Z_4Z_1$$



The following state is the ground state

$$|\xi\rangle = \frac{1}{\sqrt{2}}(\mathrm{I} + X_1X_2X_3X_4)|0_10_20_30_4\rangle$$
$$= \frac{1}{\sqrt{2}}(|0_10_20_30_4\rangle + |1_11_21_31_4\rangle)$$

GHZ state!

One plaquette

One can demonstrate the anyonic statistics with only this plaquette. First create excitation with Z rotation at one spin:

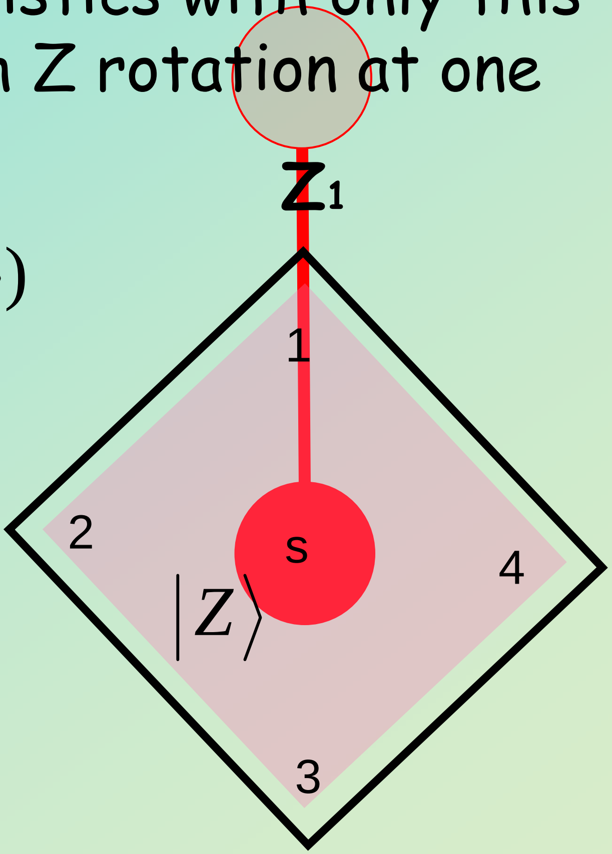
$$\left|Z\right\rangle = Z_1\left|\xi\right\rangle = \frac{1}{\sqrt{2}}\left(\left|0_1 0_2 0_3 0_4\right\rangle - \left|1_1 1_2 1_3 1_4\right\rangle\right)$$

Energy of ground state

$$H\left|\xi\right\rangle = -5\left|\xi\right\rangle$$

Energy of excited state

$$H\left|Z\right\rangle = -3\left|Z\right\rangle$$



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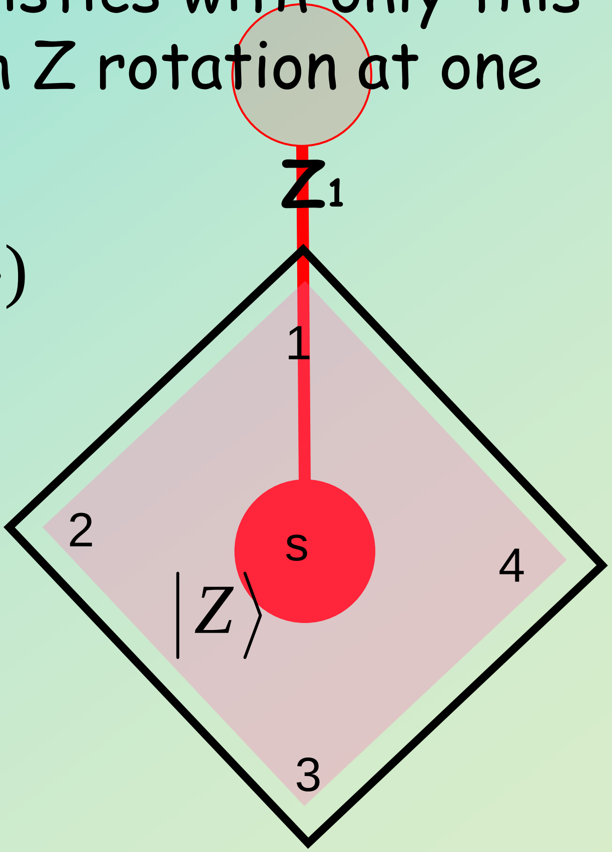
$$\left|Z\right\rangle = Z_1\left|\xi\right\rangle = \frac{1}{\sqrt{2}}\left(\left|0_1 0_2 0_3 0_4\right\rangle - \left|1_1 1_2 1_3 1_4\right\rangle\right)$$



Now we want to move an X anyon around the Z one. What we really want is the path that it traces and this can be spanned on the spins 1,2,3,4.

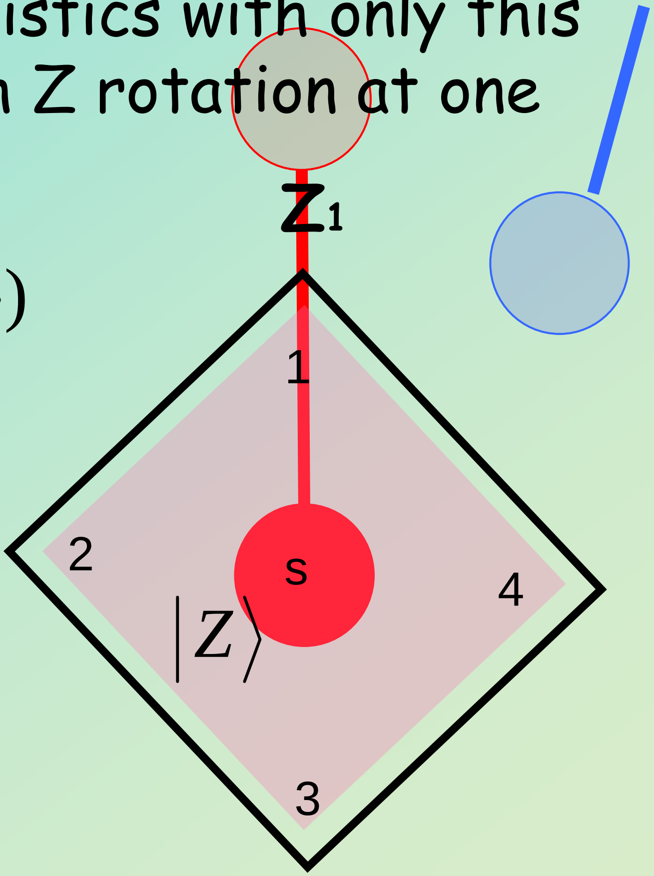
Note that the second anyon from the Z rotation is outside the system.

One plaquette

One can demonstrate the anyonic statistics with only this plaquette. First create excitation with Z rotation at one spin:

$$\left|Z\right\rangle = Z_1\left|\xi\right\rangle = \frac{1}{\sqrt{2}}\left(\left|0_1 0_2 0_3 0_4\right\rangle - \left|1_1 1_2 1_3 1_4\right\rangle\right)$$

Assume there is an X anyon outside the system. With successive X rotations it can be transported around the plaquette.



One plaquette

One can demonstrate the anyonic statistics with only this plaquette. First create excitation with Z rotation at one spin:

$$\left|Z\right\rangle = Z_1\left|\xi\right\rangle = \frac{1}{\sqrt{2}}\left(\left|0_1 0_2 0_3 0_4\right\rangle - \left|1_1 1_2 1_3 1_4\right\rangle\right)$$

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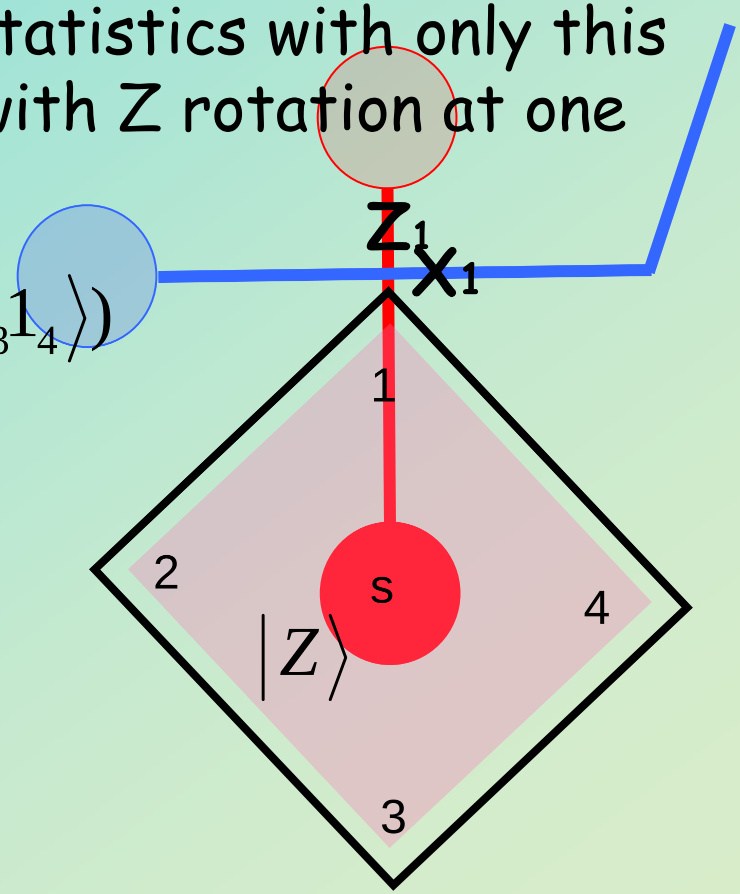
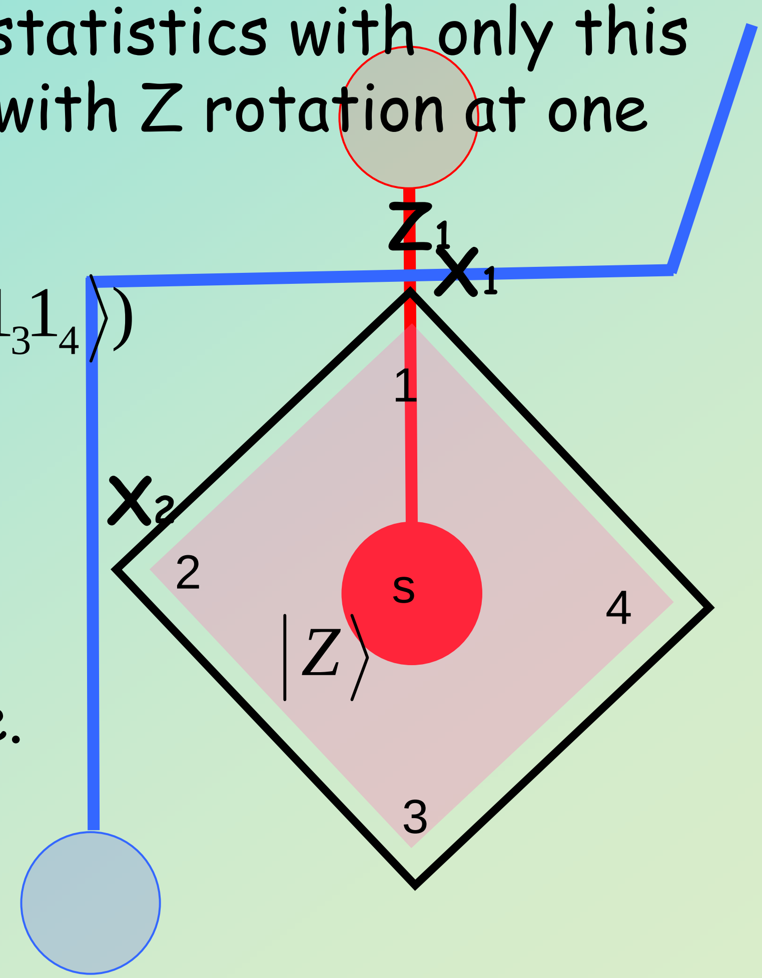


One plaquette

One can demonstrate the anyonic statistics with only this plaquette. First create excitation with Z rotation at one spin:

$$\left|Z\right\rangle = Z_1\left|\xi\right\rangle = \frac{1}{\sqrt{2}}\left(\left|0_1 0_2 0_3 0_4\right\rangle - \left|1_1 1_2 1_3 1_4\right\rangle\right)$$



Assume there is an X anyon outside the system. With successive X rotations it can be transported around the plaquette.

One plaquette

One can demonstrate the anyonic statistics with only this plaquette. First create excitation with Z rotation at one spin:

$$\left|Z\right\rangle = Z_1\left|\xi\right\rangle = \frac{1}{\sqrt{2}}\left(\left|0_1 0_2 0_3 0_4\right\rangle - \left|1_1 1_2 1_3 1_4\right\rangle\right)$$

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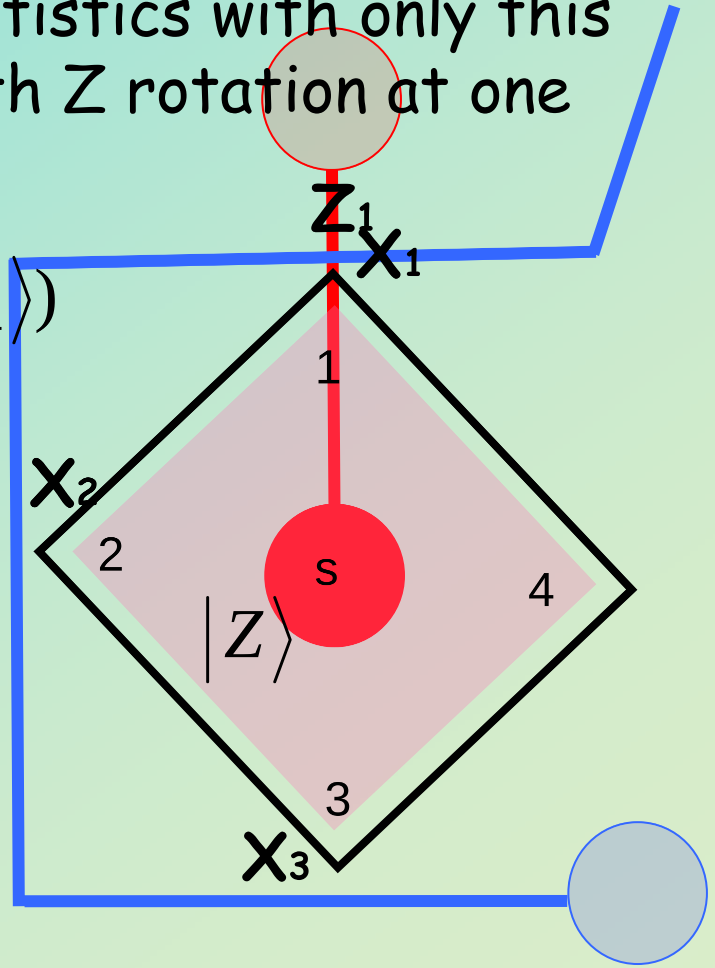
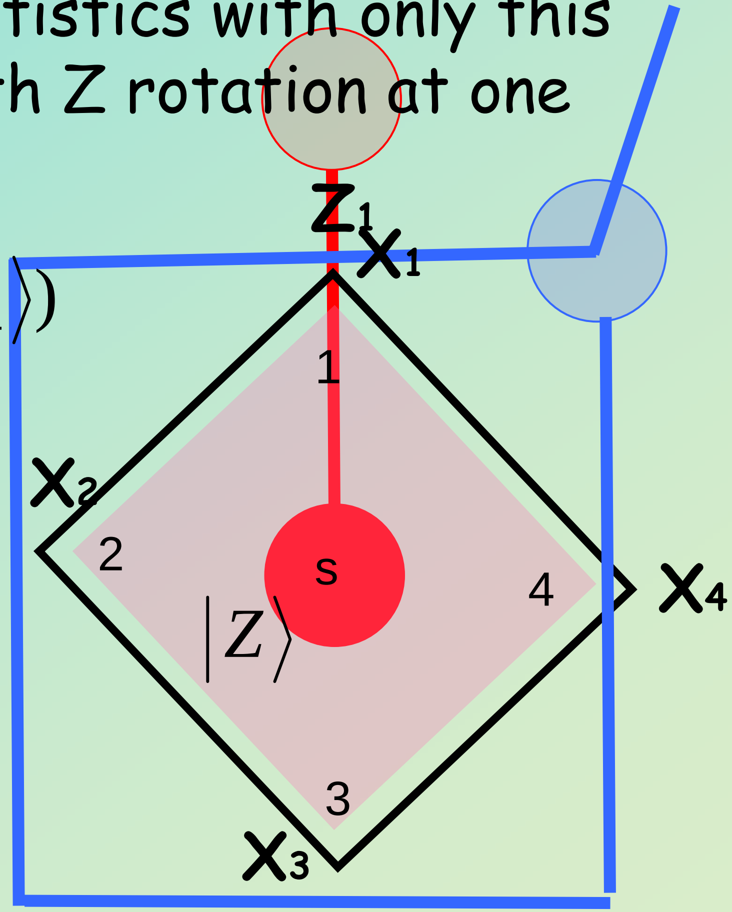


One plaquette

One can demonstrate the anyonic statistics with only this plaquette. First create excitation with Z rotation at one spin:

$$|Z\rangle = Z_1|\xi\rangle = \frac{1}{\sqrt{2}}(|0_1 0_2 0_3 0_4\rangle - |1_1 1_2 1_3 1_4\rangle)$$



Assume there is an X anyon outside the system. With successive X rotations it can be transported around the plaquette.

The final state is given by:

$$|Final\rangle = X_1 X_2 X_3 X_4 |Z\rangle =$$

$$-\frac{1}{\sqrt{2}}(|0_1 0_2 0_3 0_4\rangle - |1_1 1_2 1_3 1_4\rangle) = -|Initial\rangle$$

One plaquette

$$\left|Final\right\rangle = X_1 X_2 X_3 X_4 \left|Z\right\rangle =$$

$$-\frac{1}{\sqrt{2}}(\left|0_1 0_2 0_3 0_4\right\rangle - \left|1_1 1_2 1_3 1_4\right\rangle) = -\left|Initial\right\rangle$$

After a complete rotation of an X anyon around a Z anyon (two successive exchanges) the resulting state gets a phase π (a minus sign): hence **ANYONS**!

A property we used is that $X_1 X_2 X_3 X_4 \left|\xi\right\rangle = \left|\xi\right\rangle$ which is true.

A crucial point is that these properties can be demonstrated without the Hamiltonian!!!

An interference experiment can reveal the presence of the phase factor.

Interference Experiment

Create state

$$|\xi\rangle = \frac{1}{\sqrt{2}}(|0_1 0_2 0_3 0_4\rangle + |1_1 1_2 1_3 1_4\rangle)$$

With half of an Z rotation on spin 1, $Z_1^{1/2}$, one can create the superposition between an Z anyon and the vacuum:

$$e^{-i\varphi} Z_1^{1/2} |\xi\rangle = (|\xi\rangle + i|Z\rangle)/\sqrt{2}$$

for $\varphi = 3\pi/4$. Then the X anyon is rotated around it:

$$X_1 X_2 X_3 X_4 (|\xi\rangle + i|Z\rangle)/\sqrt{2} = (|\xi\rangle - i|Z\rangle)/\sqrt{2}$$

Then we make the inverse rotation

$$e^{i\varphi} Z_1^{-1/2} (|\xi\rangle - i|Z\rangle)/\sqrt{2} = |Z\rangle$$

Interference Experiment

That we obtained the $|Z\rangle$ state is due to the minus sign produced from the anyonic statistics.

If it wasn't there then we would have returned to the vacuum state $|\xi\rangle$.

Distinguish between $|\xi\rangle$ *and* $|Z\rangle$ states:

$H^{\otimes 4}|\xi\rangle$ has even number of 1's.

$H^{\otimes 4}|Z\rangle$ has odd number of 1's.

$$H^{\otimes 4}|\xi\rangle \propto |0000\rangle + |0011\rangle + |0101\rangle + |0110\rangle + |1001\rangle + |1010\rangle + |1100\rangle + |1111\rangle$$

$$H^{\otimes 4}|Z\rangle \propto |0001\rangle + |0010\rangle + |0100\rangle + |1000\rangle + |0111\rangle + |1011\rangle + |1101\rangle + |1110\rangle$$

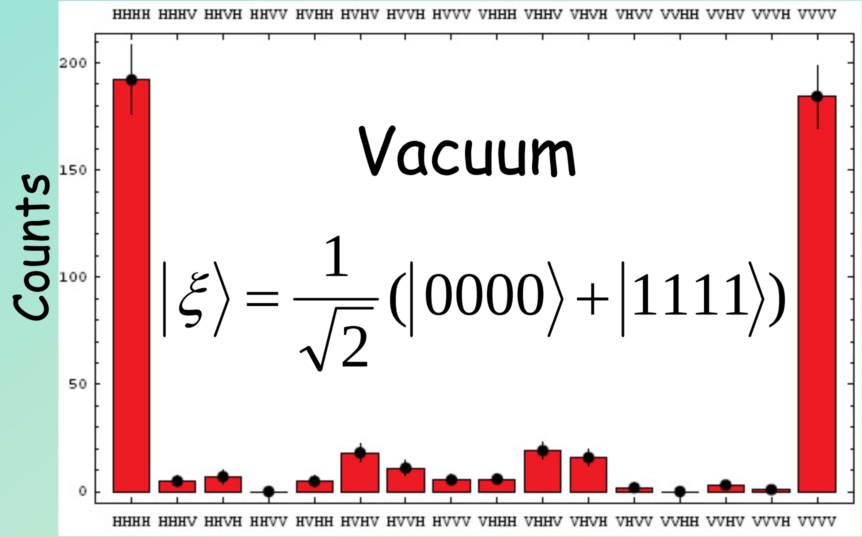
Experimental process (preliminary)

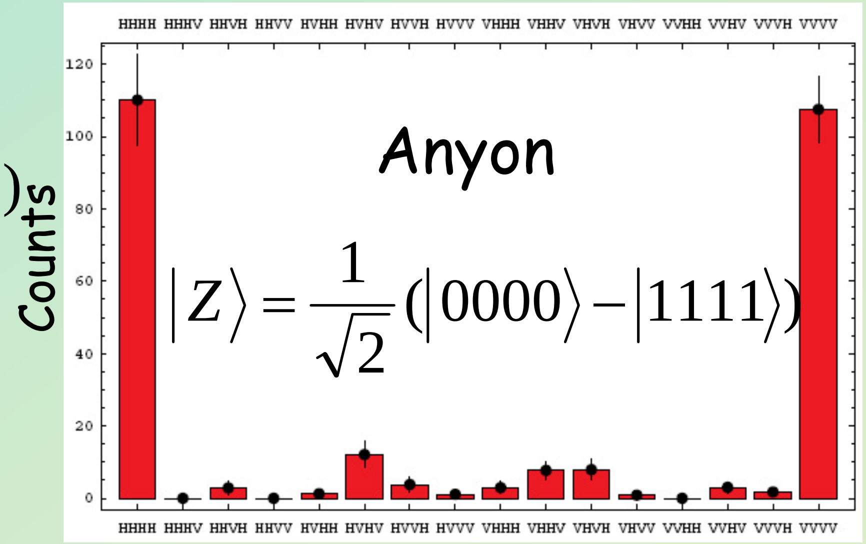
Qubit states 0 and 1 are encoded in the polarization, V and H, of four photonic modes.

The states that come from this setup are of the form:

$$
\begin{aligned}
|\Psi\rangle &= a|GHZ\rangle + b|EPR\rangle \otimes |EPR\rangle = \\
&a(|HHHH\rangle + |VVVV\rangle) + \\
&b(|VHVH\rangle + |HVVH\rangle + |VHHV\rangle + |HVHV\rangle)
\end{aligned}
$$

Measurements and manipulations are repeated over all modes.





Experimental process (preliminary)

Consider correlations:

$$tr\left[(\cos\gamma\sigma^x + \sin\gamma\sigma^y)^{\otimes 4}|\xi\rangle\langle\xi|\right] = +\cos(4\gamma)$$

$$tr\left[(\cos\gamma\sigma^x + \sin\gamma\sigma^y)^{\otimes 4}|Z\rangle\langle Z|\right] = -\cos(4\gamma)$$

Visibility > 64%

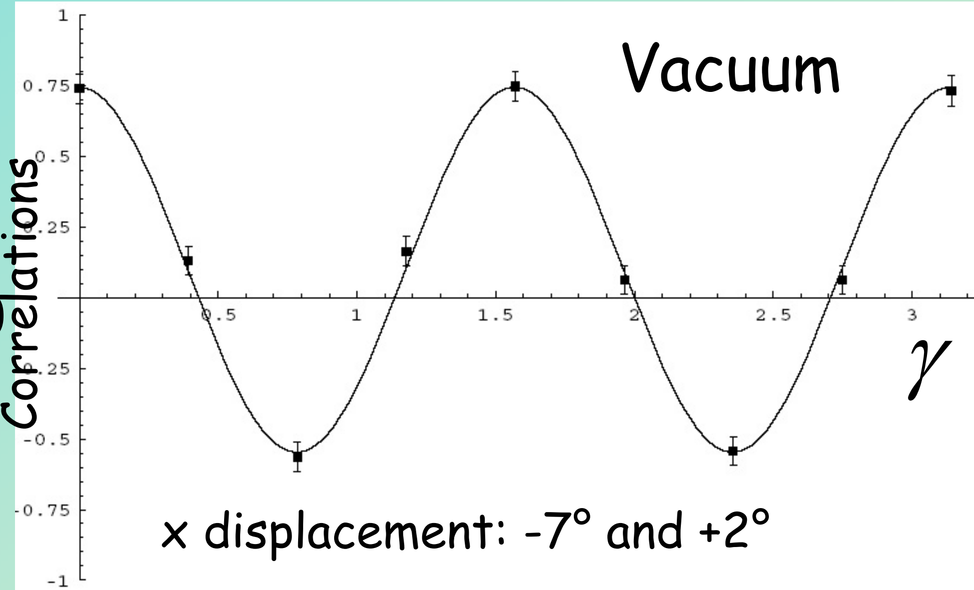
Fidelity:

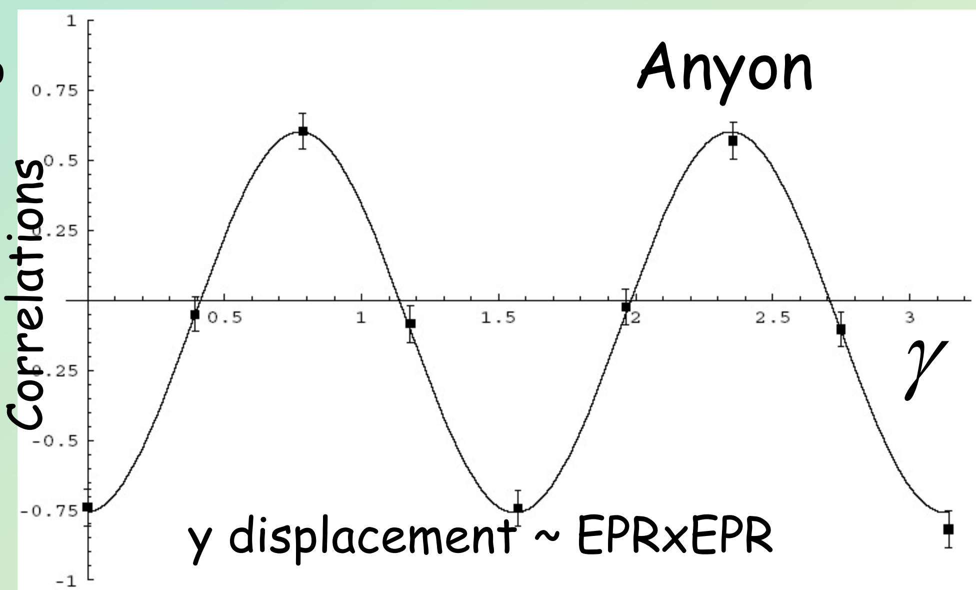
$$F = |a_1|^2 + |a_{16}|^2 + a_1^* a_{16} + a_1 a_{16}^* > 70\%$$

Witness for genuine 4-qubit entanglement:

$$W_{GHZ_4} = \frac{1}{2}\mathbf{1} - |GHZ_4\rangle\langle GHZ_4|$$

$$\Rightarrow tr(W_{GHZ_4}\rho) < 0$$





Conclusions

- Invariance of vacuum w.r.t. to closed paths:

$$\left.\begin{array}{l} XXXX|\xi\rangle = |\xi\rangle \\ Z_i Z_j|\xi\rangle = |\xi\rangle \end{array}\right\} \text{4 qubit GHZ stabilizers}$$

$$|Z\rangle = Z_i|\xi\rangle$$

- Fusion rules:

$$\left.\begin{array}{l} Z_i Z_j|\xi\rangle = |\xi\rangle \\ Z_i Z_j|Z\rangle = |Z\rangle \end{array}\right\} \quad \begin{array}{l} e \times e = 1 \\ 1 \times e = e \end{array}$$

Useful for:

- **quantum anonymous broadcasting**,
- **quantum error correction**,
- **topological quantum memory (?)** …

Non-abelian statistics can be detected similarly.

Implement Hamiltonian and larger systems.

Annals of Physics, IJQI