Deconfined criticality, Runaway flow to strong coupling, and Superfluid--Solid quantum phase transitions

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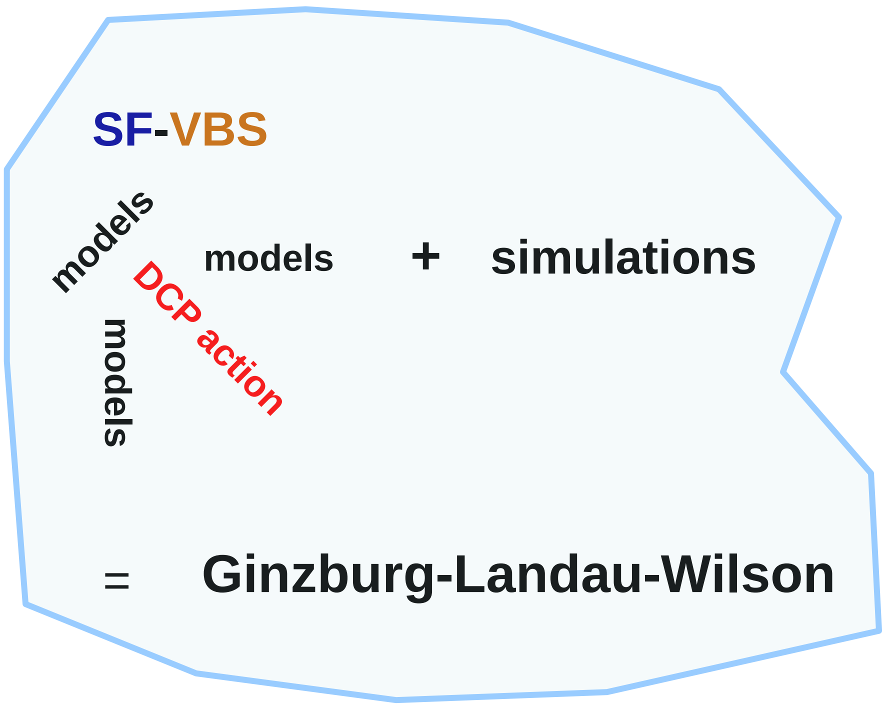

Annals of Physics 321, 1601 (2006)
Prog. Theor. Phys. Jap. 160 (2005)
PRL 93 230402 (2004)

National Science Foundation
WHERE DISCOVERIES BEGIN

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1. Results for microscopic models.

- GLW phenomenology:
  multicomponent order-parameter with finite modulus,

  or

- vortices in the valence-bond solid / deconfined criticality

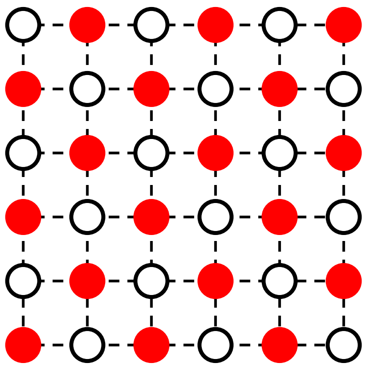
2. Phase diagram of the deconfined critical action.

- Numerical flowgram method

- Wilson RG flow and proof of the weak-first-order transition
  into the SF phase.

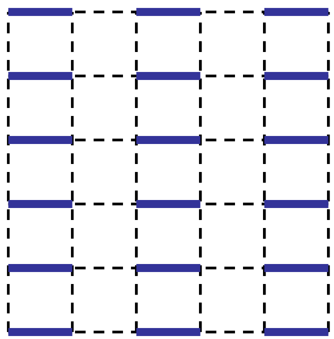
Solid = insulator with broken translational symmetry (integer filling excluded)

Checkerboard solid (anti-ferromagnet)

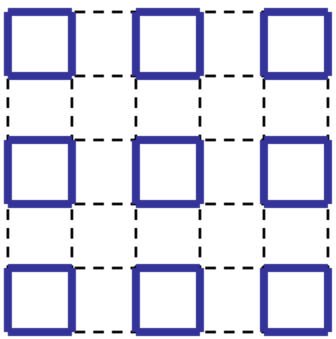


Valence-bond solids (VBS)

Columnar VBS



Plaquette VBS



Density-order

$$\langle n_i n_j \rangle$$

Current (or bond) order

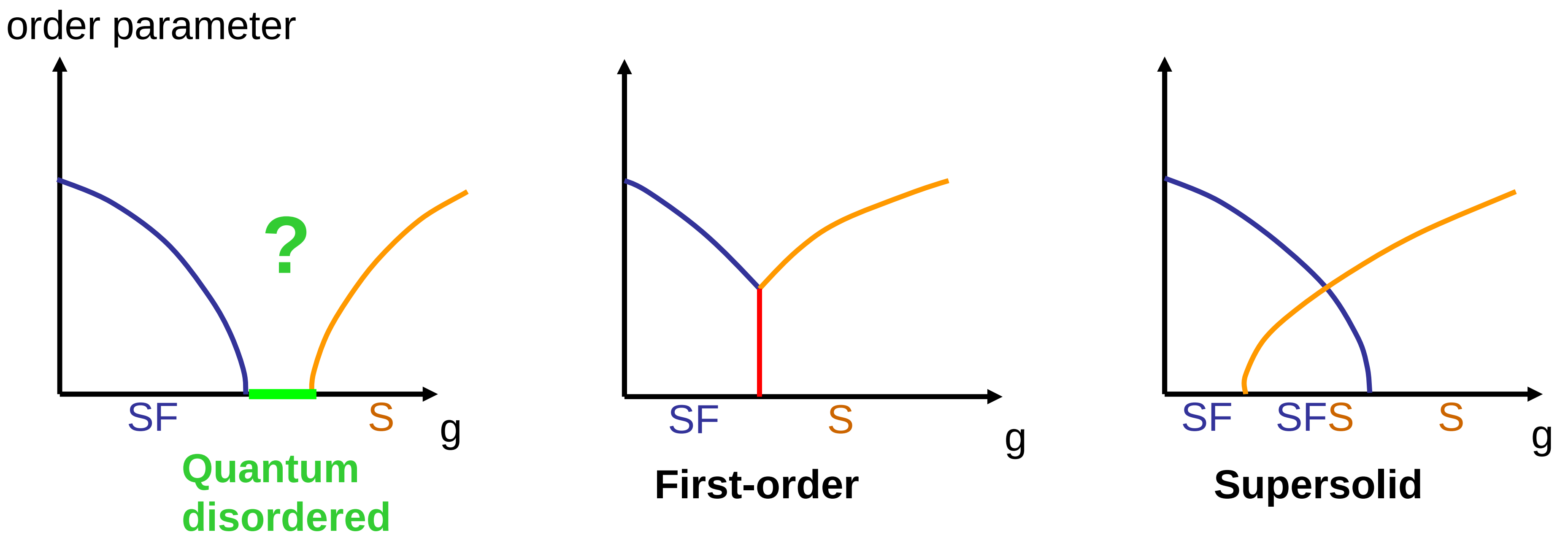
$$\langle J_i^2 J_j^2 \rangle$$

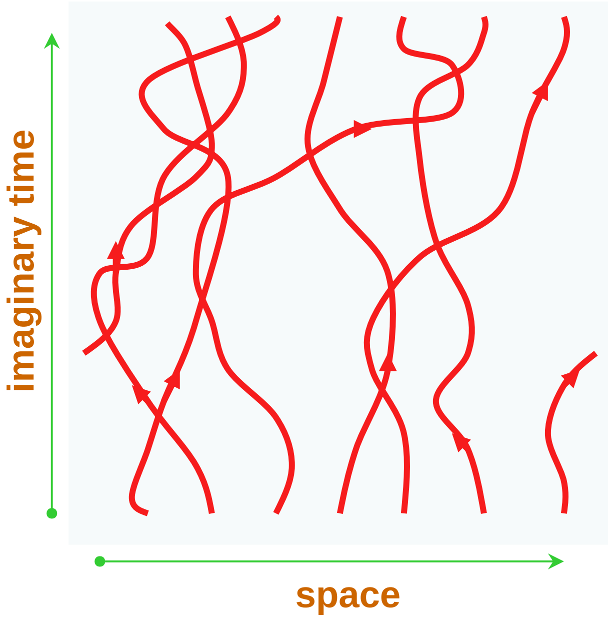
Superfluid and Solid orders and transitions:

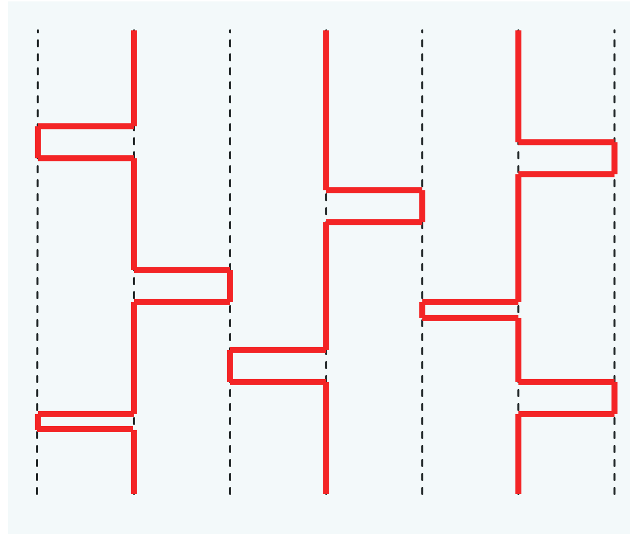
$\underbrace{\Psi \qquad M \qquad B_x, B_y \qquad D_x, D_y \qquad \ldots}$

order parameters:

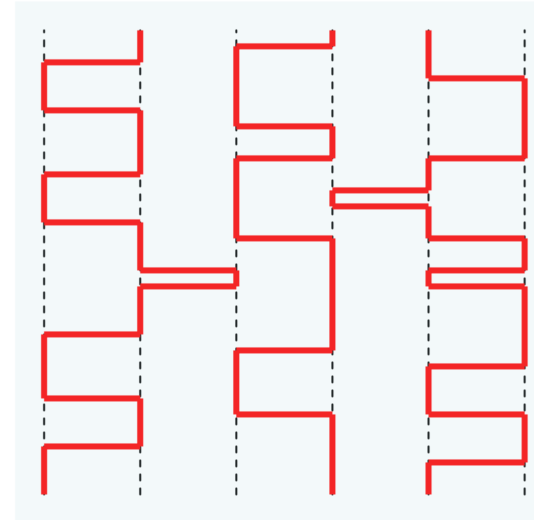
"Naïve" Ginzburg-Landau (expansion from the generic quantum disordered phase)







CB



VBS

structured worldlines
site or bond order

$\langle n_i n_j \rangle$ $\langle J_i^2 J_j^2 \rangle$

“Spagetti worldlines”
winding numbers

$\rho_S \sim \langle W^2 \rangle$ ⇒ **SF**

SF and **S** parameters are **NOT** independent

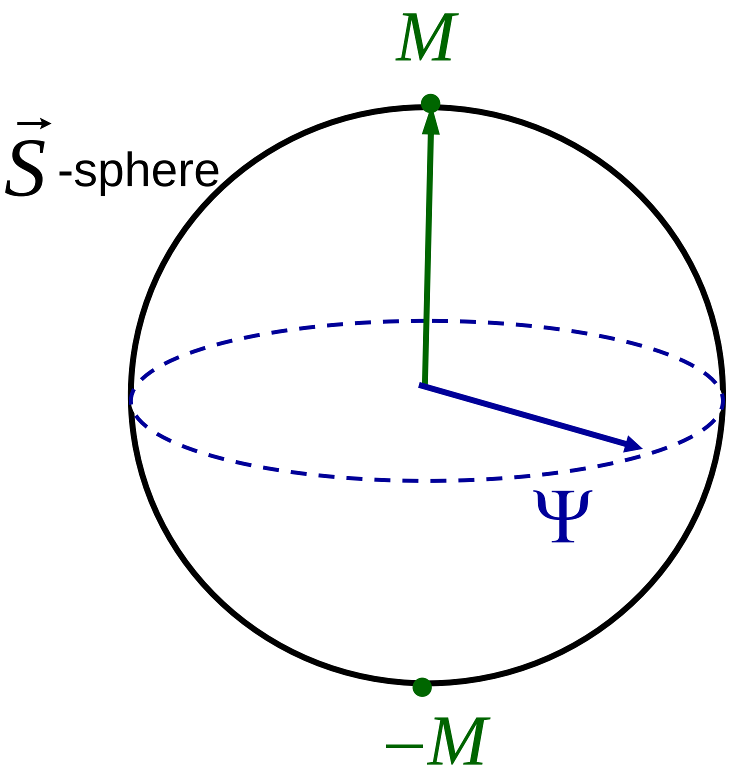
not being naïve …

$$\vec{S} = (\underbrace{S_1, S_2}_{\Psi}, \underbrace{S_3}_{M}, \underbrace{S_4, S_5}_{B_x, B_y} \ldots)$$

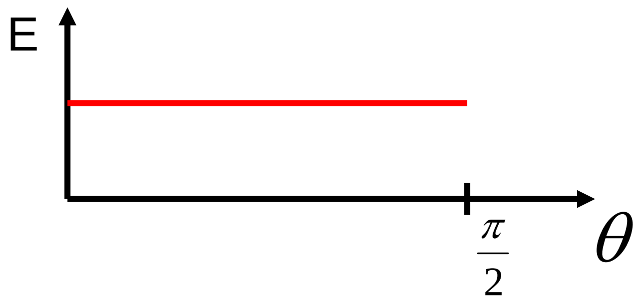
- multi-component order parameter; the symmetry of S is ALWAYS broken in the ground state

$$|\vec{S}| \neq 0$$

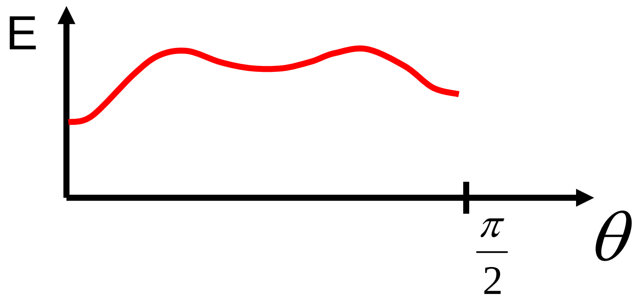
O(3)-case:



Heisenberg point = exact O(3)-symmetry



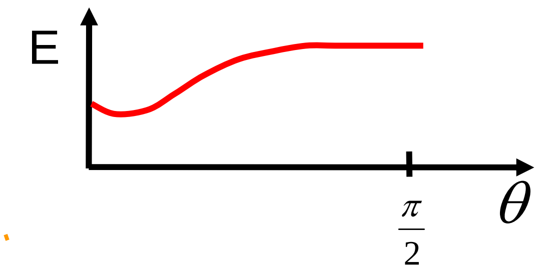
Generic case

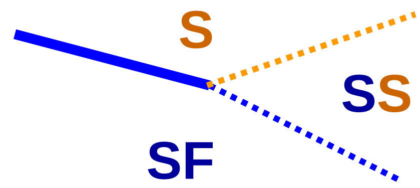


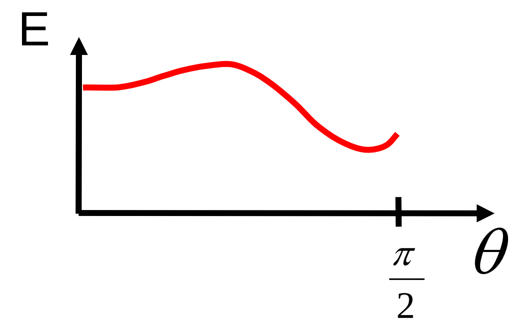
Away from the high-symmetry point

SF-Solid
SF-Supersolid } second order

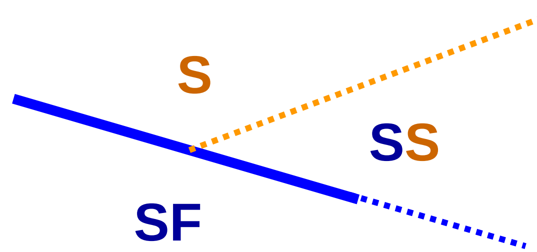
Supersolid-Solid weak first-order







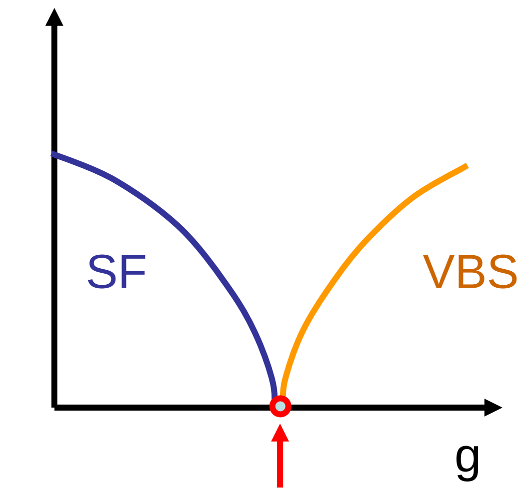
or, I-order SF-Supersolid



SF and **VBS** orders are deeply related … May be even self-dual at the critical point!

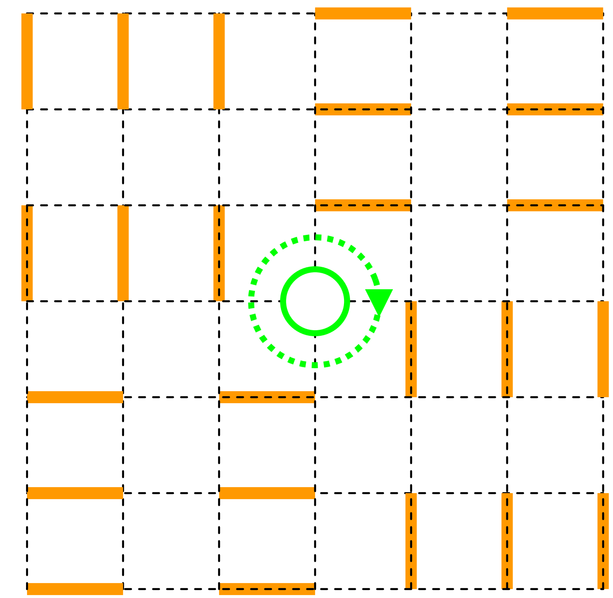
O.I. Motrunich and A. Vishwanath, Phys. Rev. 70, 075104 (2004).
T. Senthil, A. Vishwanath, L. Balents, S. Sachdev, and M.P.A. Fisher, Science, 303, 1490 (2004):

Deconfined criticality, does not fit the standard Landau-Ginsburg-Wilson paradigm.



Deconfined spinons
= vortices in the VBS order

$$B_x + iB_y$$



DCP action:

$$S = \int d^2r\, d\tau \sum_{a=1,2} \left| \left(\partial_\mu - iA_\mu \right) z_a \right|^2 + s|z|^2 + u\left(|z|^2\right)^2 + v|z_1|^2 |z_2|^2 + \kappa \left(\varepsilon_{\mu\nu\beta} \partial_\nu A_\beta \right)^2$$

Ordered SF state for matter fields =
(duality mapping) “quantum disordered” for vortex fields

Do LG expansion for vortex fields!

L. Balents, L. Batosch, A. Burkov, S.Sachdev, K. Sengupta, PRB 71, 144508 and 144509 (2005).

Filling factor 1/2 ⟶ dual magnetic flux/plaquette 1/2 ⟶ two species of vortexes + gauge field coupling

at the SF-VBS critical point

DCP action

DCP action:

$$S_\psi = \int d^2 r d\tau \sum_{a=1,2} \left|\left(\partial_\mu - iA_\mu\right)\psi_a\right|^2 + s|\psi|^2 + u\left(|\psi|^2\right)^2 + v|\psi_1|^2|\psi_2|^2 + \kappa\left(\varepsilon_{\mu\nu\beta}\partial_\nu A_\beta\right)^2$$

(Field theory)

$$S_{XY} = -J\sum_{r,\mu}\left[\cos\left(\Delta_\mu\varphi_1 - A_\mu\right) + \cos\left(\Delta_\mu\varphi_2 - A_\mu\right)\right] + \kappa\sum_{\square}(\nabla\times A)^2$$

(XY)

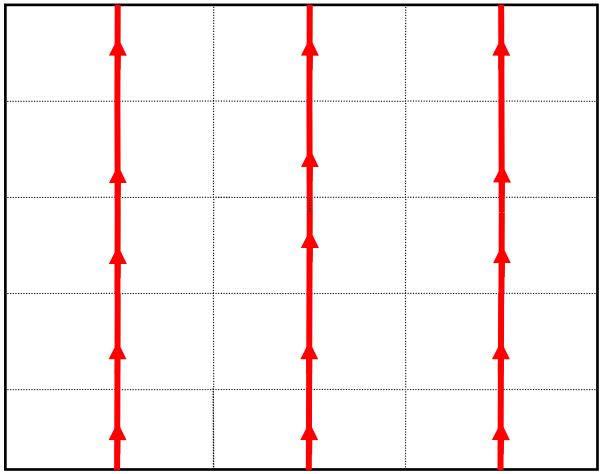
$$S_J = U\sum_{r,\mu,a} j_{a\mu}{}^2(r) + g\sum_{rr',\mu} Q(r-r')\left(j_{1\mu}(r) + j_{2\mu}(r)\right)\cdot\left(j_{1\mu}(r') + j_{2\mu}(r')\right)$$

j-current

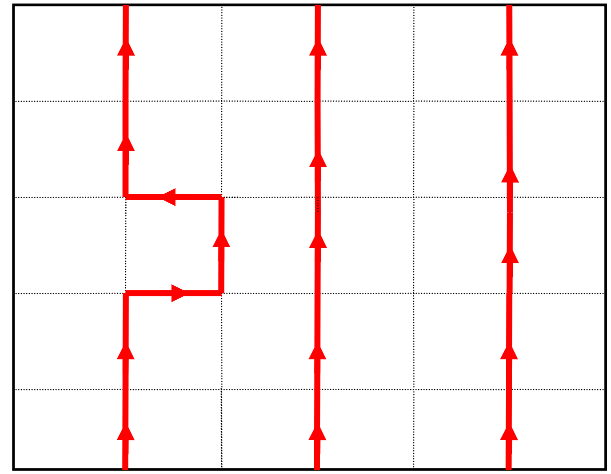
$$Q^{-1}(q) = \sum_\mu \sin^2(q_\mu/2) \;\rightarrow\; Q(r) \square 1/r$$

Mappings:

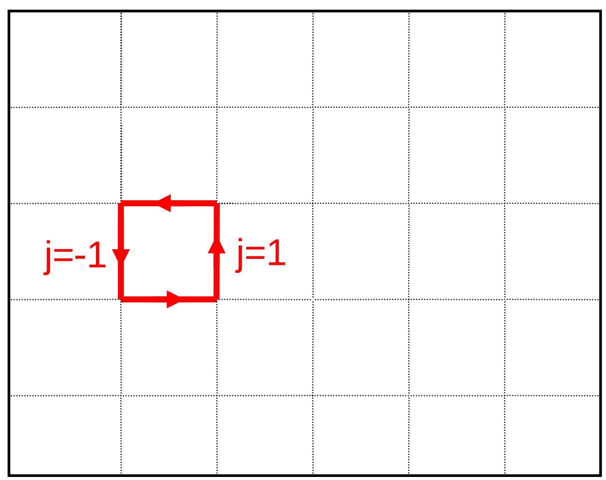
Path integral (imaginary time worldlines)



Ideal vacuum



fluctuation



fluctuation - ideal vaccum

= closed oriented loops
or j-currents

Mappings:

“High-T” expansion (expansion in kinetic energy)

$$S = -t\sum_{<rr'>}\left(\psi_r^*\psi_{r'}\, e^{iA_{<rr'>}} + c.c.\right) + \sum_r U\left|\psi_r\right|^4 - \mu\left|\psi_r\right|^2 + \kappa\sum_{\square}[\nabla\times A]^2$$

$$Z = \iint DA D\psi\, e^S$$

$$= \iint DA \prod_r \int d\psi_r\, e^{S_r} \prod_{b=<rr'>} \sum_{n_b=0}^{\infty} \frac{(-t)^{n_b}}{n_b\,!} (\psi_r^*\psi_{r'})^{n_b} \sum_{m_b=0}^{\infty} \frac{(-t)^{m_b}}{m_b\,!} (\psi_r\psi_{r'}^*)^{m_b} e^{iA_b(n_b-m_b)} e^{S_A}$$

$$= \sum_{\{j_b=n_b-m_b\}_{CP}} \exp\{-g\sum_{rr'} Q(\mathbf{r}-\mathbf{r}')\,\mathbf{j}_r\mathbf{j}_{r'}\}\; W(\{j_b\}) = \sum_{\{j_b=n_b-m_b\}_{CP}} e^{S_J}$$

closed loop configurations

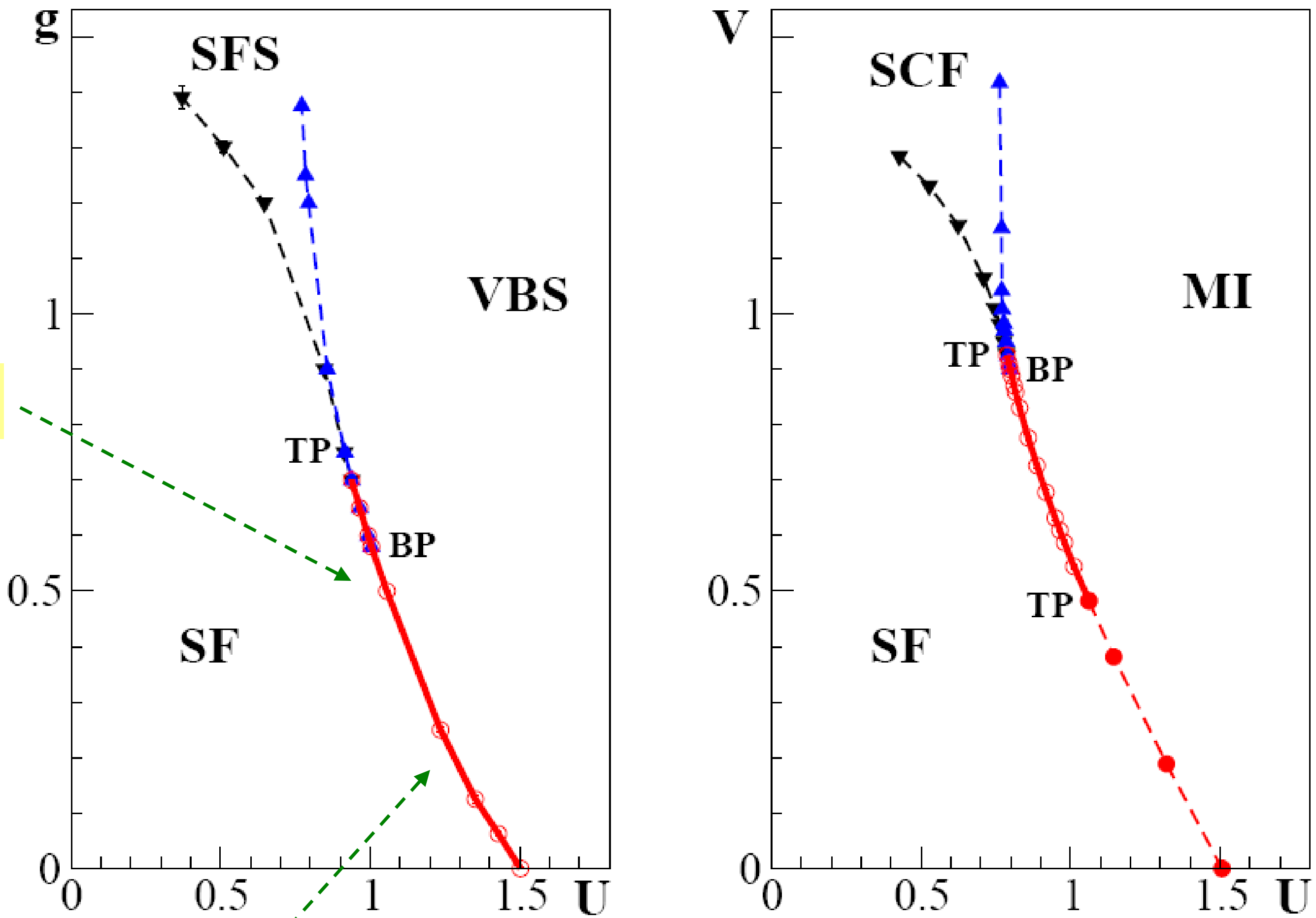
$$\mathbf{j}_r = \mathbf{j}_{1r} + \mathbf{j}_{2r}$$ (DCP)

(XY-model is the “fixed modulus” approximation of S_ψ)

Phase diagram of the DCP action

short-range version

$gQ(r-r') \to V\delta(r-r')$



g ~ 0.5 - weak I-order

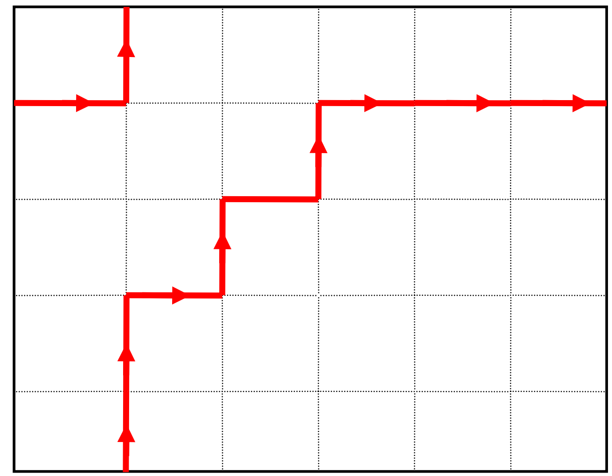
small g maps to g > 0.5

RG flow: $g_{eff}(L) \propto gL$

Winding numbers = MC "blessing"

(Pollock, Ceperley '86)

$$M_x = 1$$

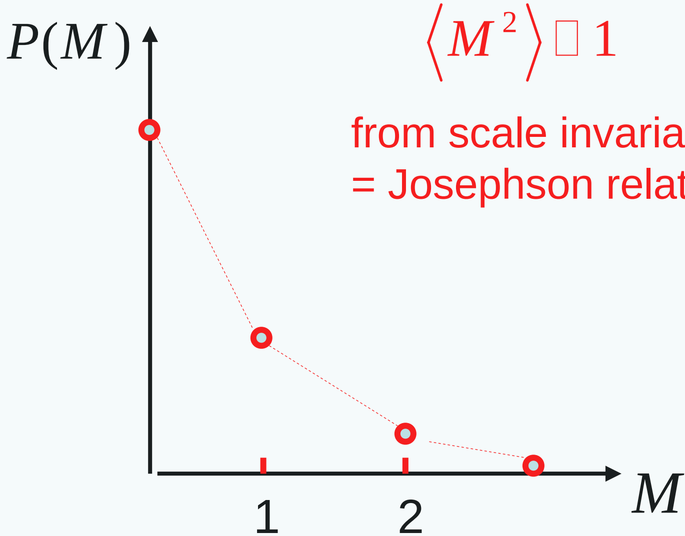


Superfluid stiffness:

$$\Lambda_s = (\rho_s / m) = TL^{d-2} \langle M^2 \rangle$$

J-current through any crossection

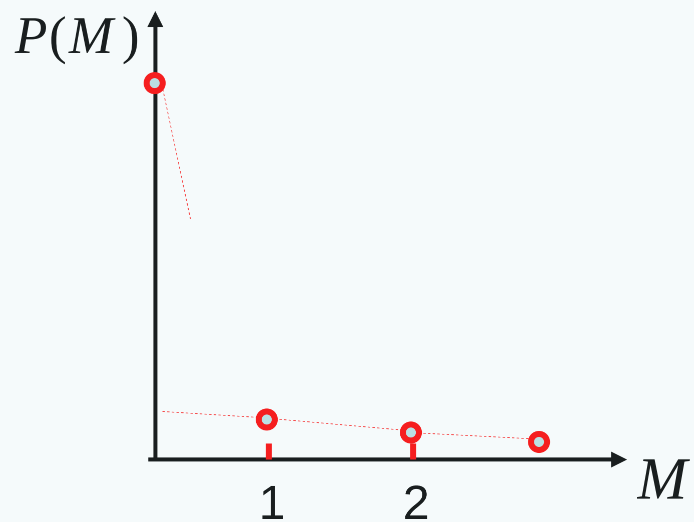
Universal distribution at the continuous critical point:



$$\langle M^2 \rangle \sim 1$$

from scale invariance
= Josephson relation

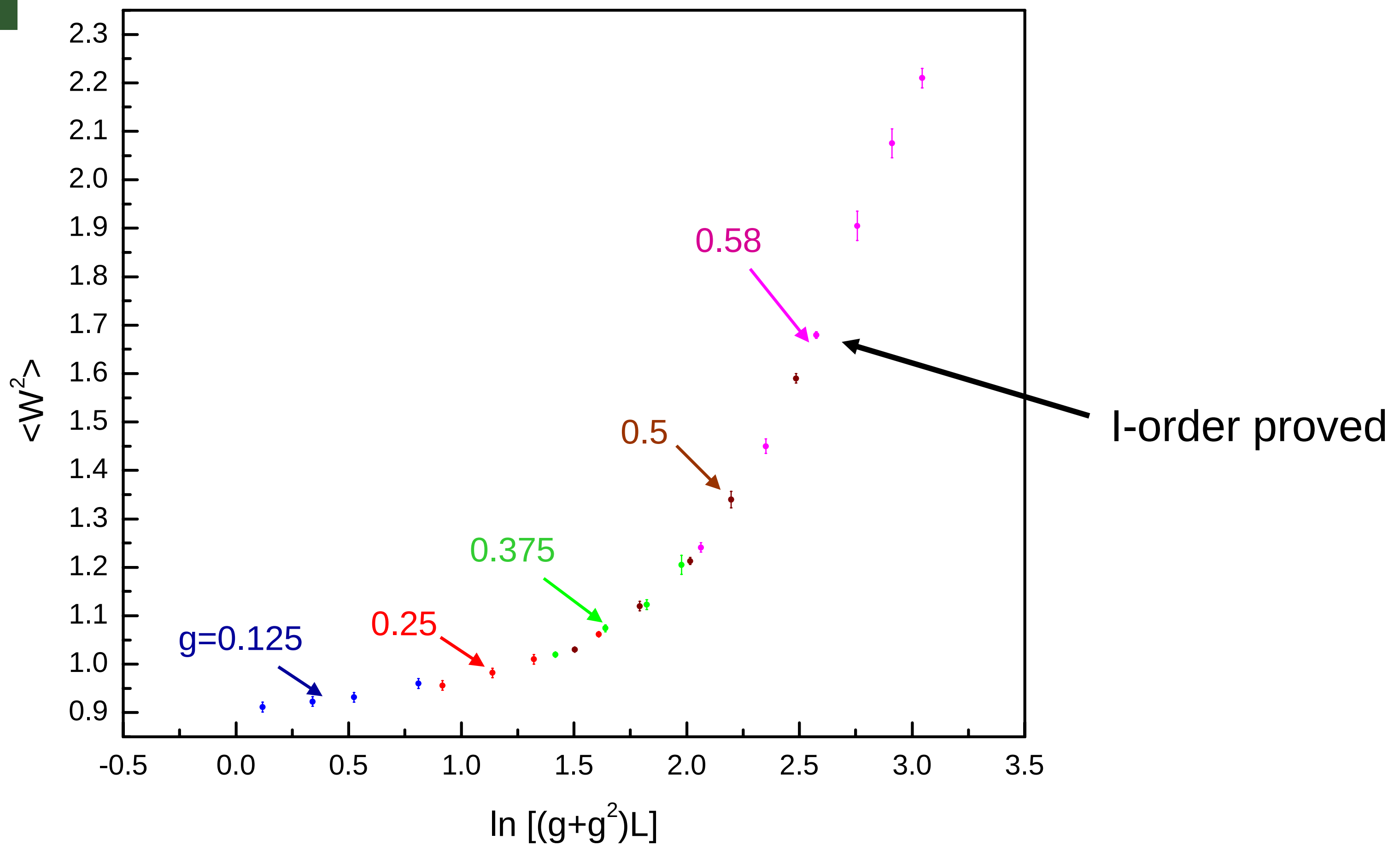
I-order SF-S transition:



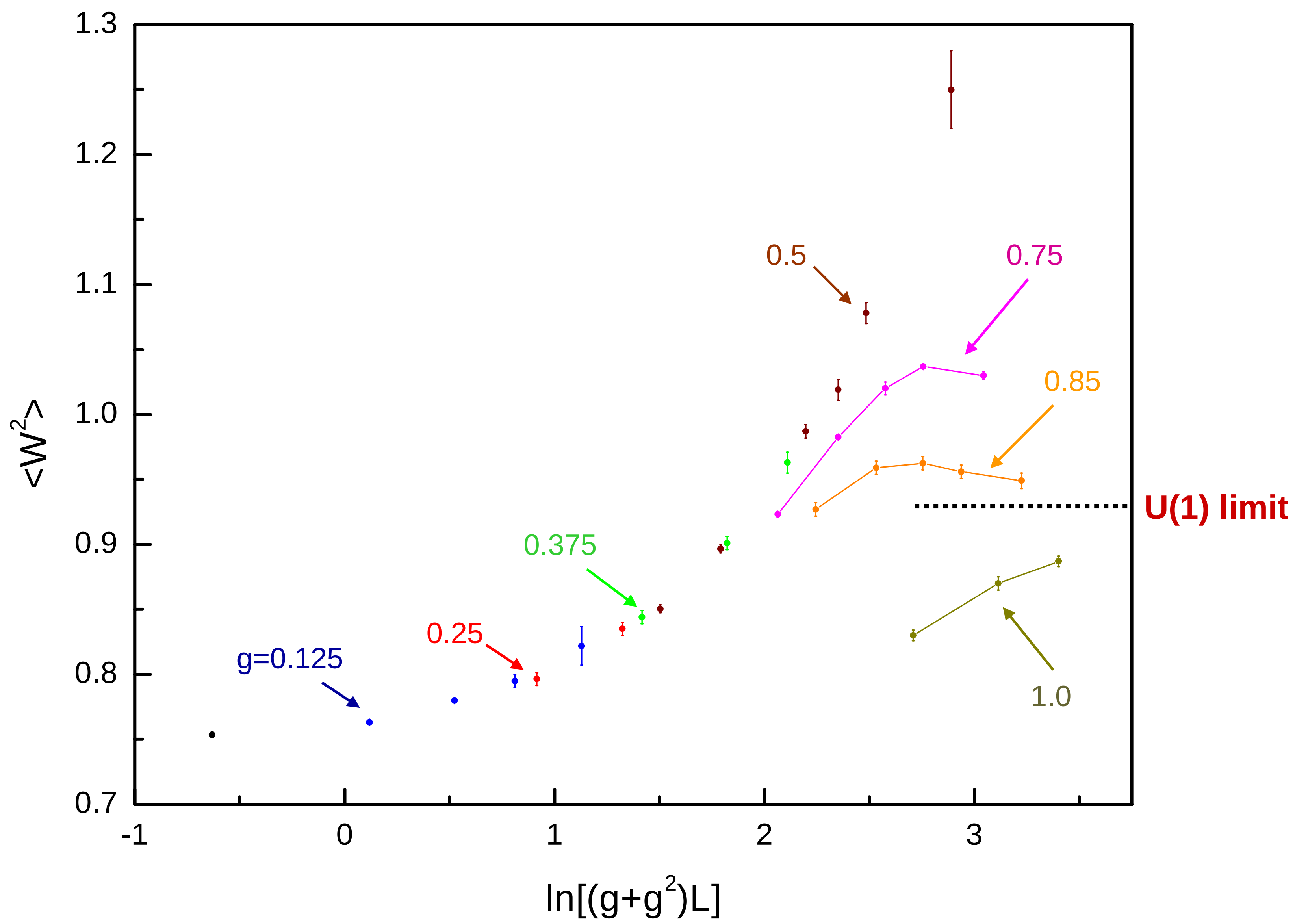
Critical point criterion: $\sum_{W \neq 0} P(W)/P(0) = 1$ Plot: $\langle W^2 \rangle = \sum_W W^2 P(W)$

For the scale-invariant II-order transition: $\langle W^2 \rangle \to const$ as $gL \to \infty$

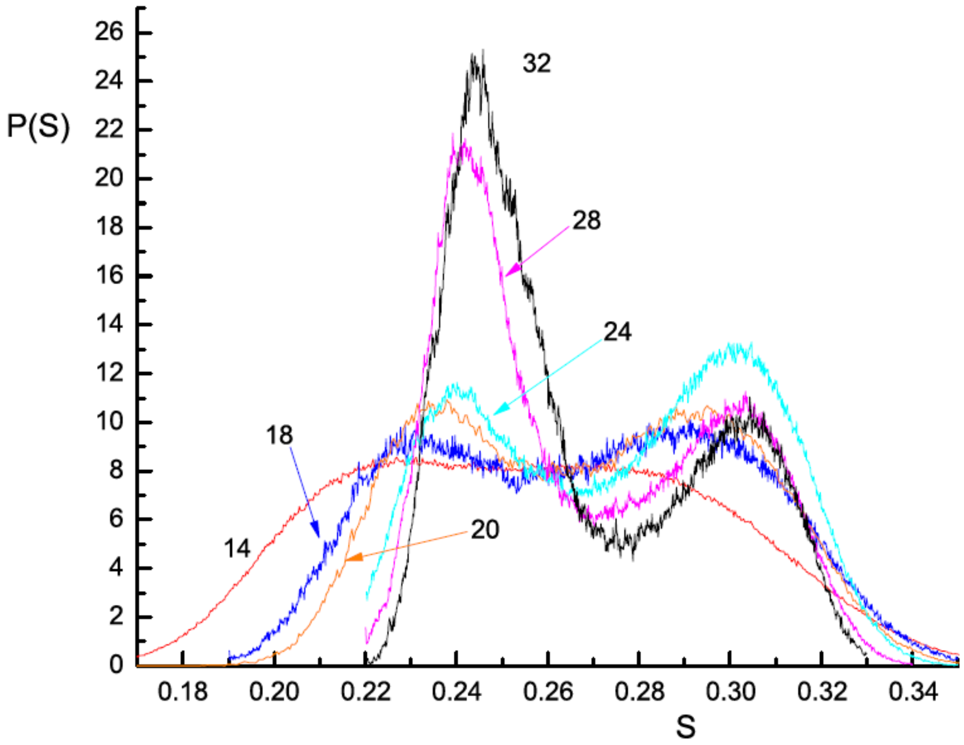
$j_1 + j_2$ channel



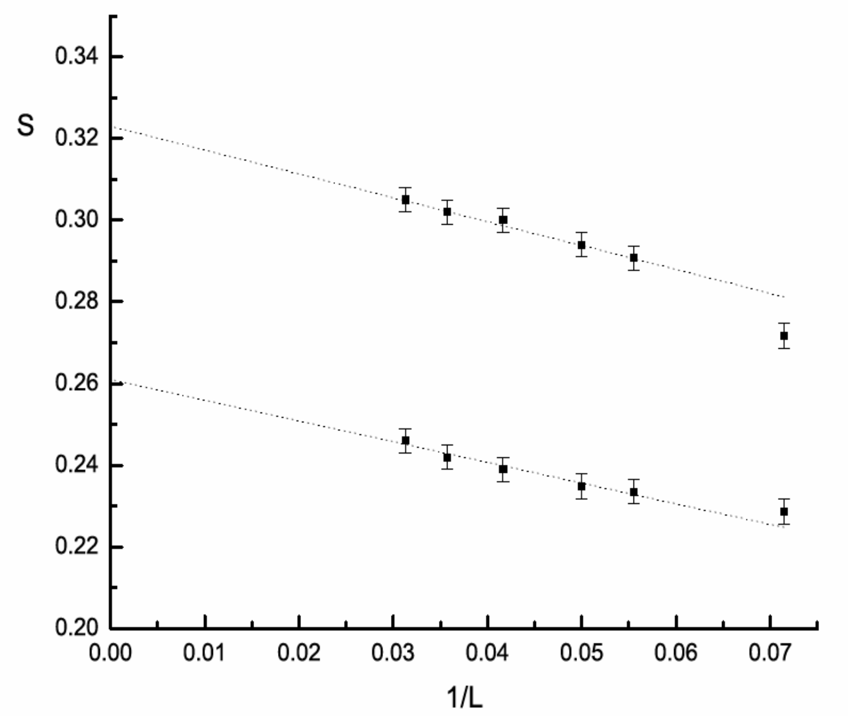
$j_1 - j_2$ channel



Proof of first-order transition at g=0.58



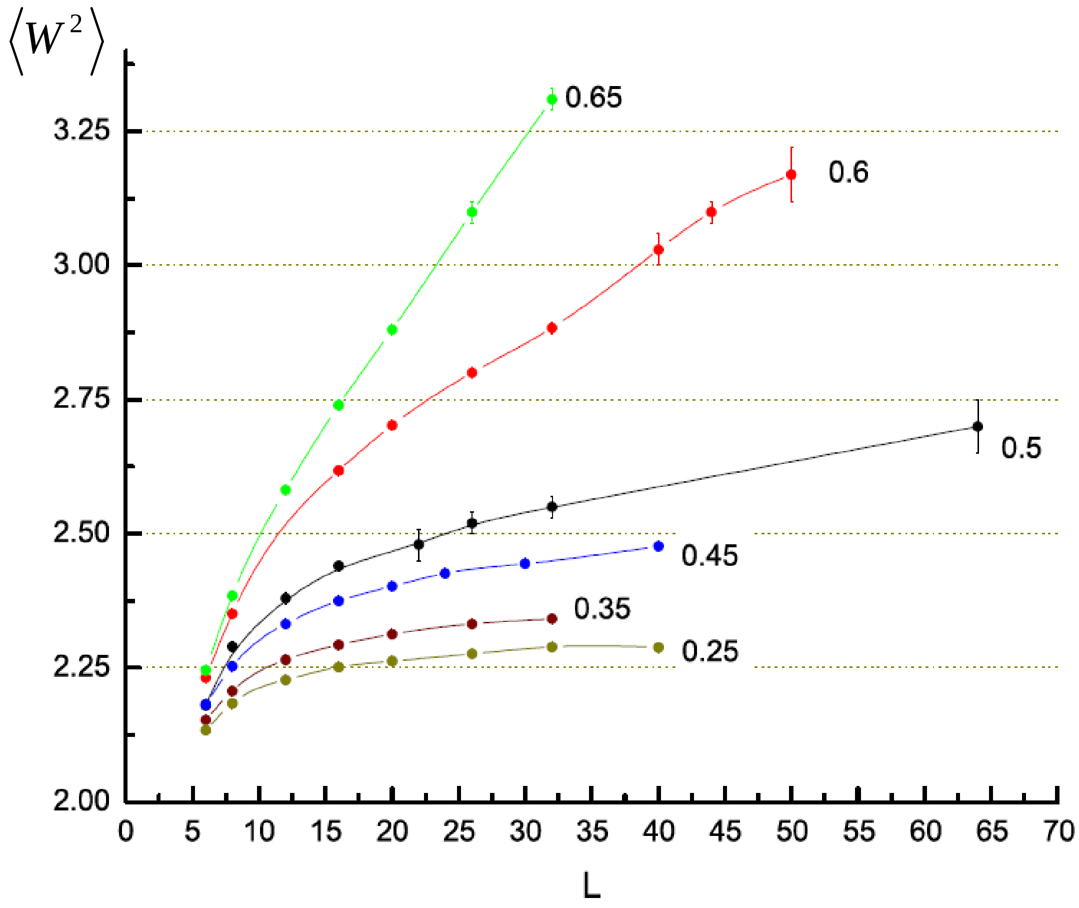
Probability distributions for action at the critical point for g=0.58



Scaling of peak positions with system size L

Compare with the short-range model across the TP

$j_1 + j_2$ channel



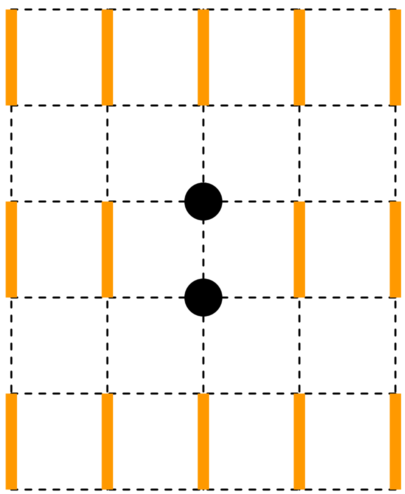
Data collapse is not possible!

A. Particle = confined state of two spinons

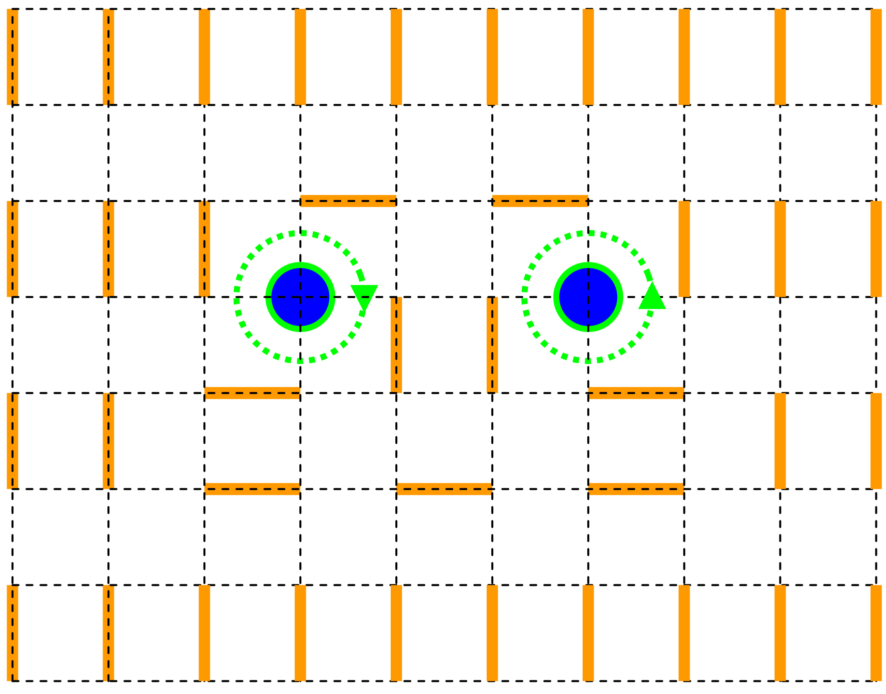
vs

B. Spinon = bound state of vortex in the **VBS** order and particle

A

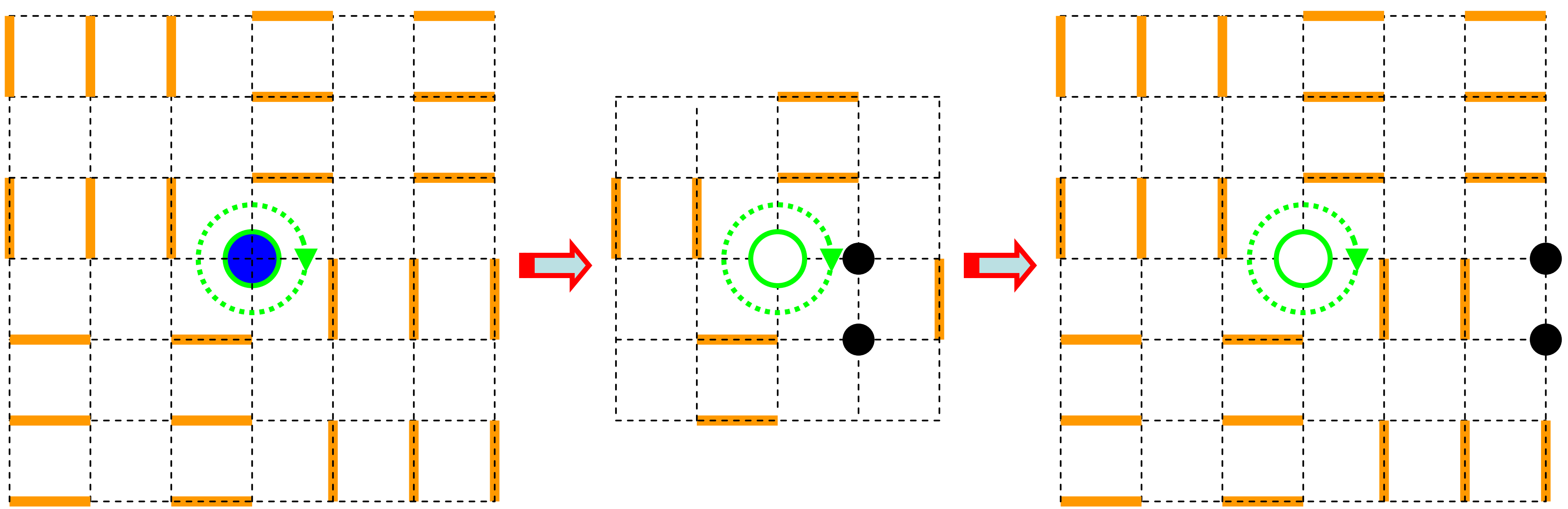


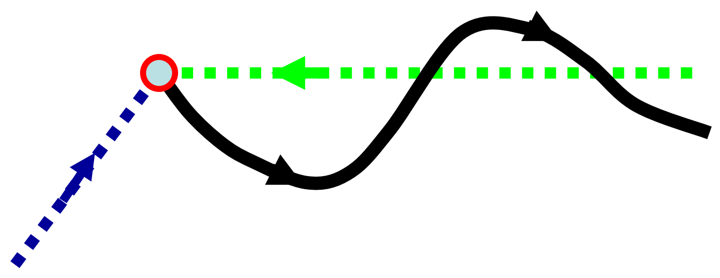
= confined spinons



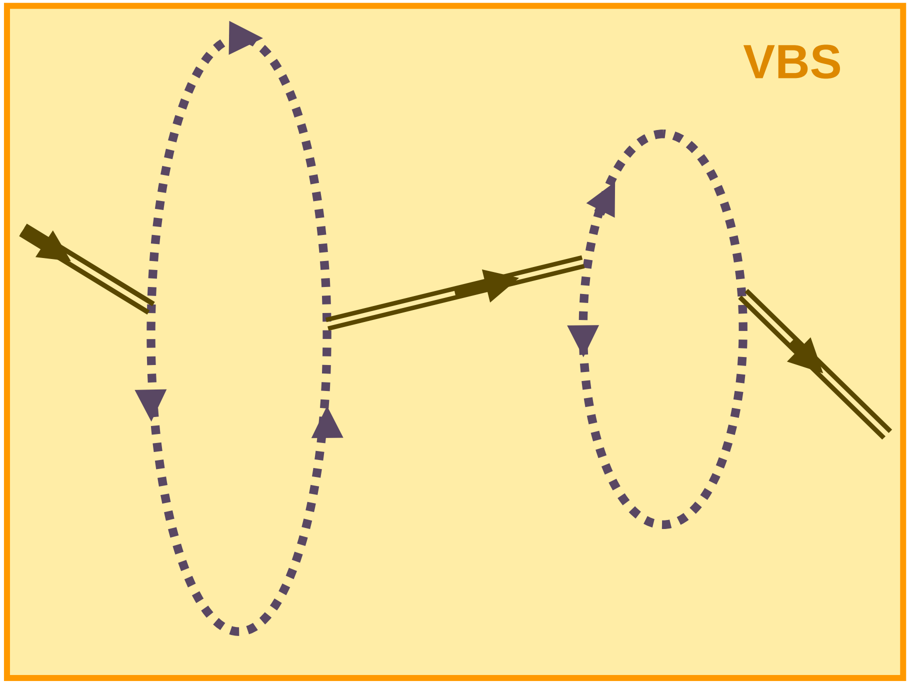
B

Treating particles as separate fileds



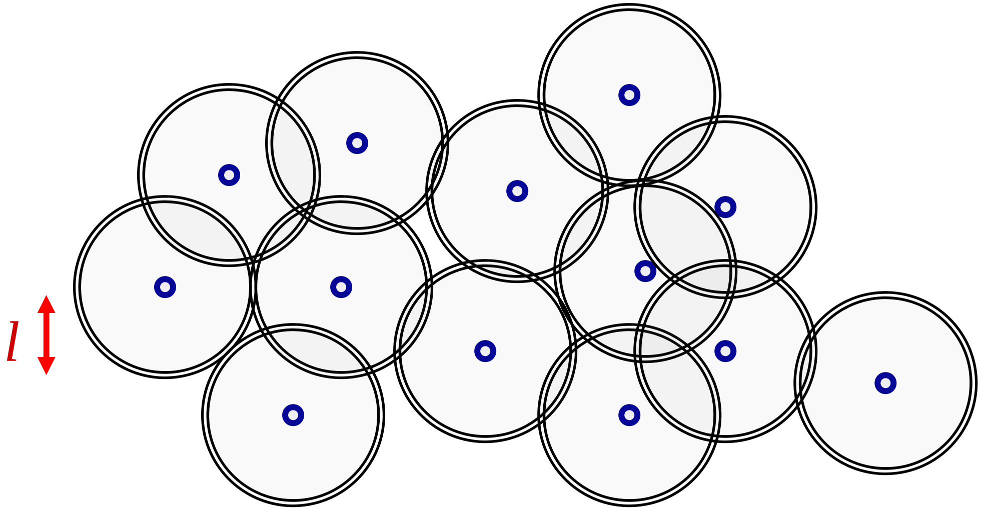


“Mott ^{2} transition”:



particles delocalize before spinon loops proliferate (or spinons deconfine) –

“Mott transition” at $nl^2 \sim 1$



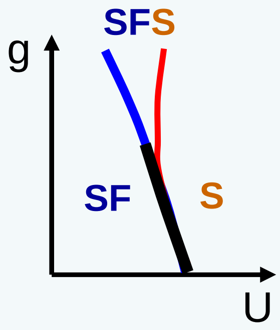
Conclusions:

1. **Numerically, all SF-S transitions observed so far were I-order,**

   in agreement with the GWL theory for

$$\left|\vec{S} = (\underbrace{S_1, S_2}_{\Psi}, \underbrace{S_3}_{S_{CB}}, \underbrace{S_4, S_5}_{B_x, B_y}\ldots)\right| \neq 0$$

2. **It appears that the DCP action is the theory of weak I-order transition into the SF phase (often through the SFS phase)**



3. **Particle delocalization on crystal defects (spinons, domain walls, etc.) may be the mechanism preventing deconfined criticality from happening**