

KITP

# Entanglement Renormalization

12<sup>th</sup> of February 2007

Guifre Vidal

University of Queensland (Brisbane, Australia)

- Simulation of Quantum Many-Body Systems

### University of Queensland

G.V.

Roman Orus (postdoc)

Glen Evenbly (PhD)

Jacob Jordan (PhD)

Sukhi Singh (PhD)

### Collaborators

Frank Verstraete (Univ. Vienna)

Ignacio Cirac (MPQ Garching)

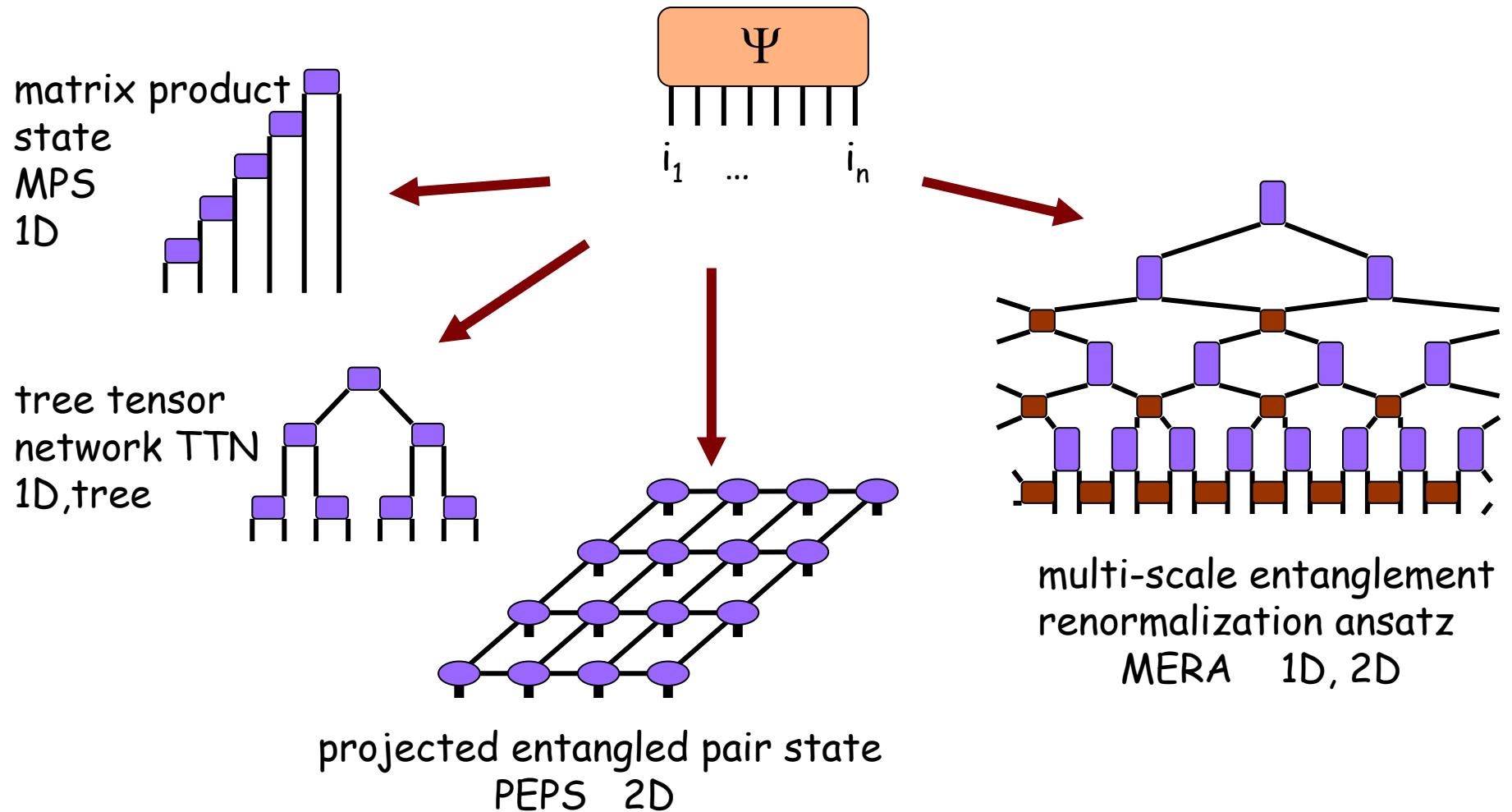
Miguel Aguado (MPQ Garching)

Lluís Masanes (Univ. Cambridge)

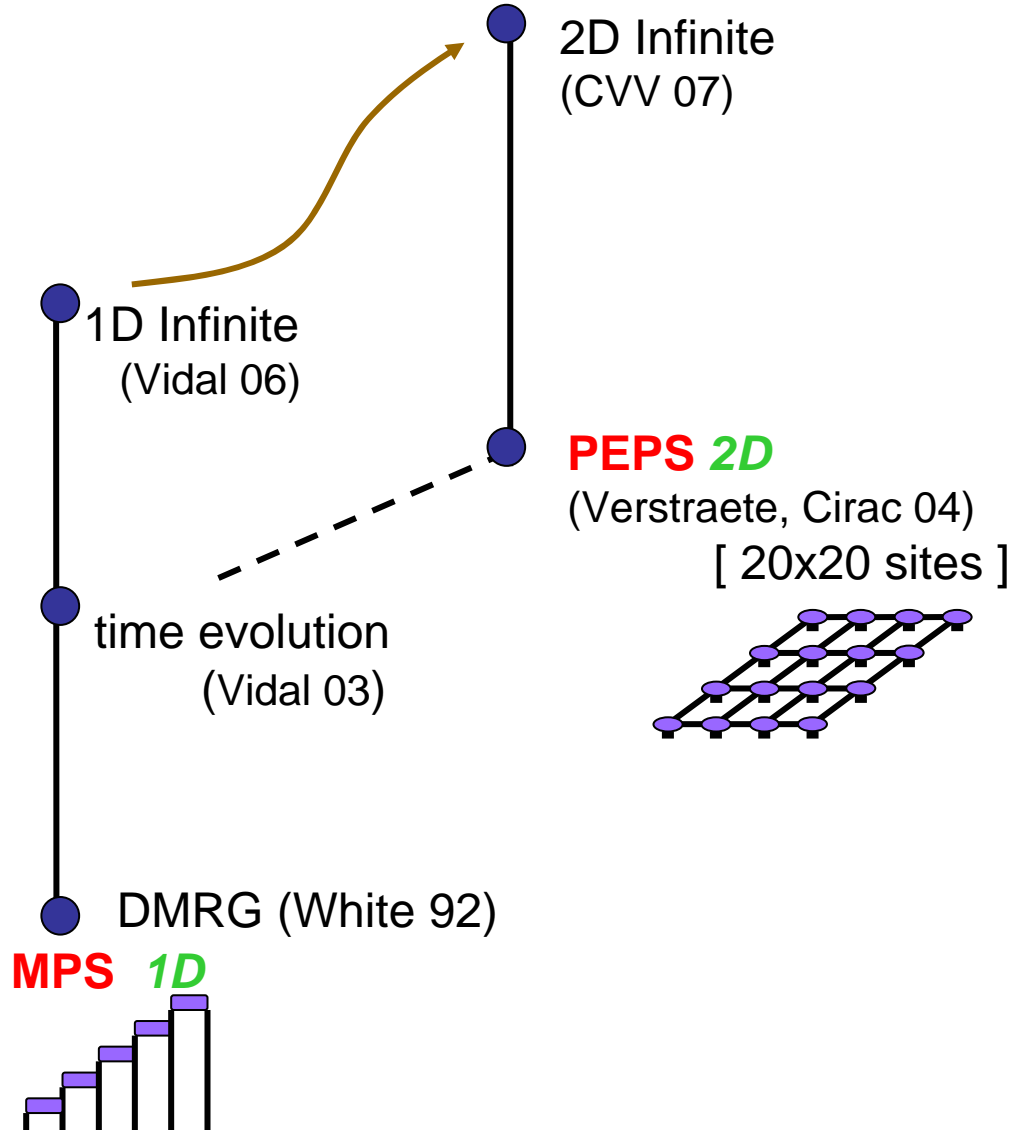
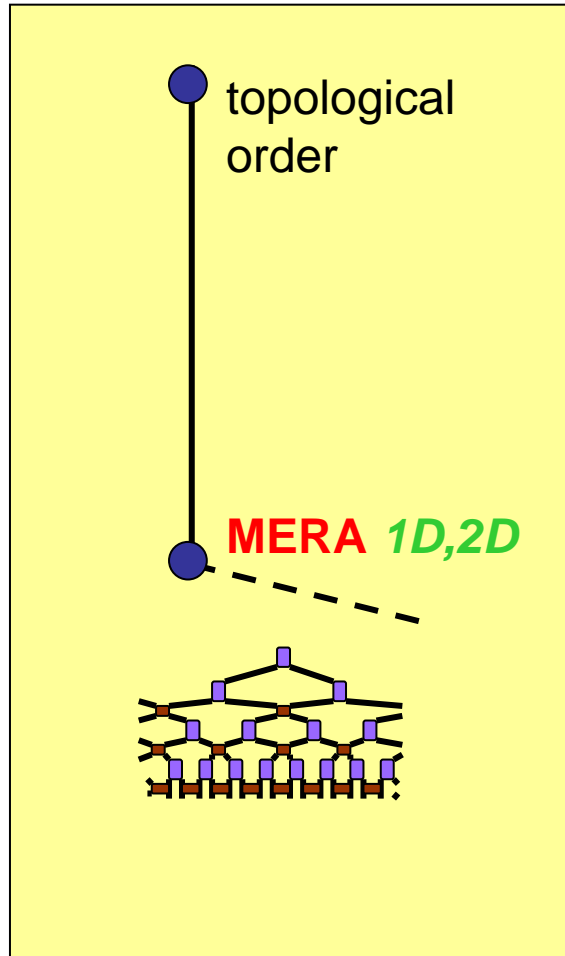
Huan-Qiang Zhou (Chongqing Univ.)

• Efficient description of n-qubit states

$$|\Psi\rangle = \sum_{i_1 \cdots i_n} \Psi_{i_1 \cdots i_n} |i_1 \cdots i_n\rangle \quad 2^n \text{ coefficients } \Psi_{i_1 \cdots i_n}$$

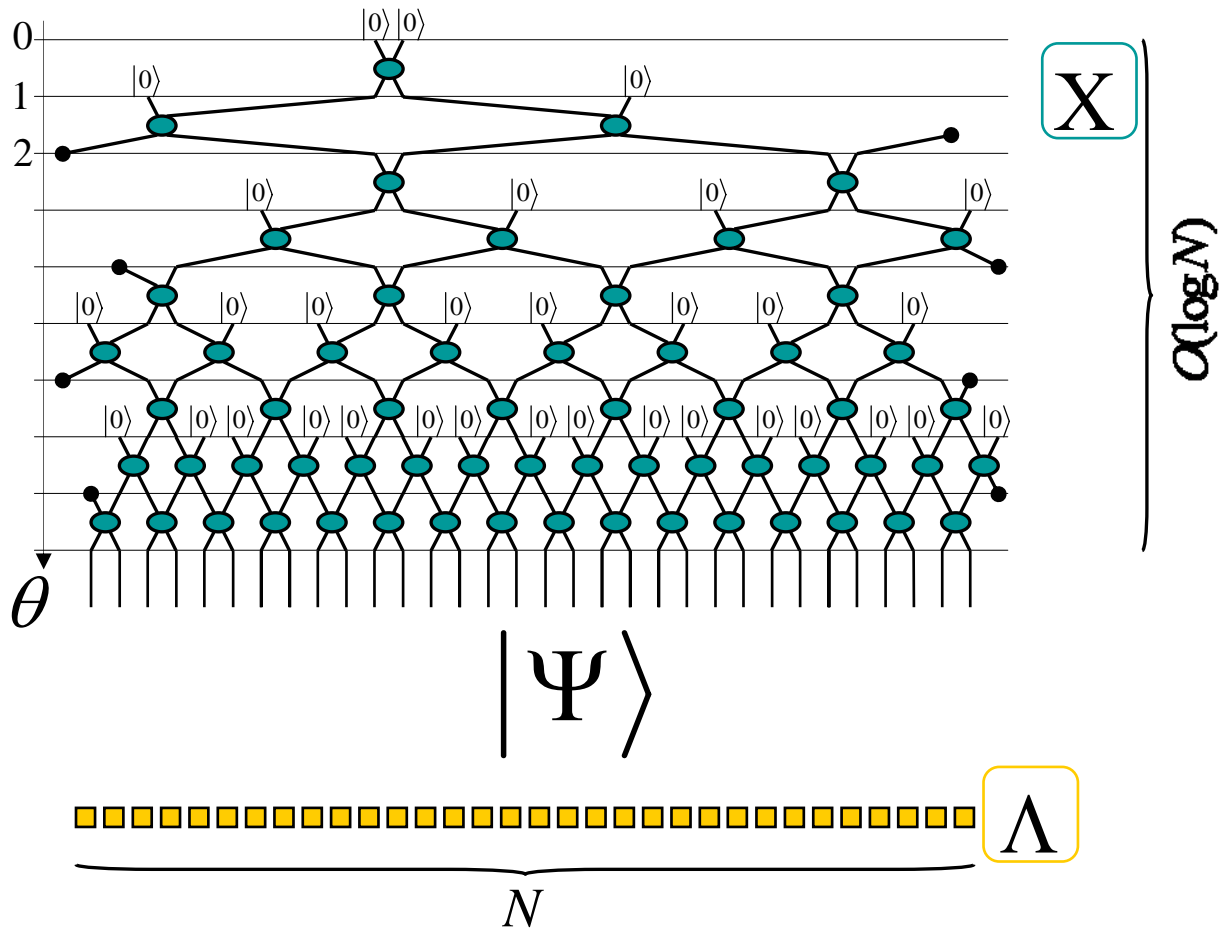


# Summary of recent developments



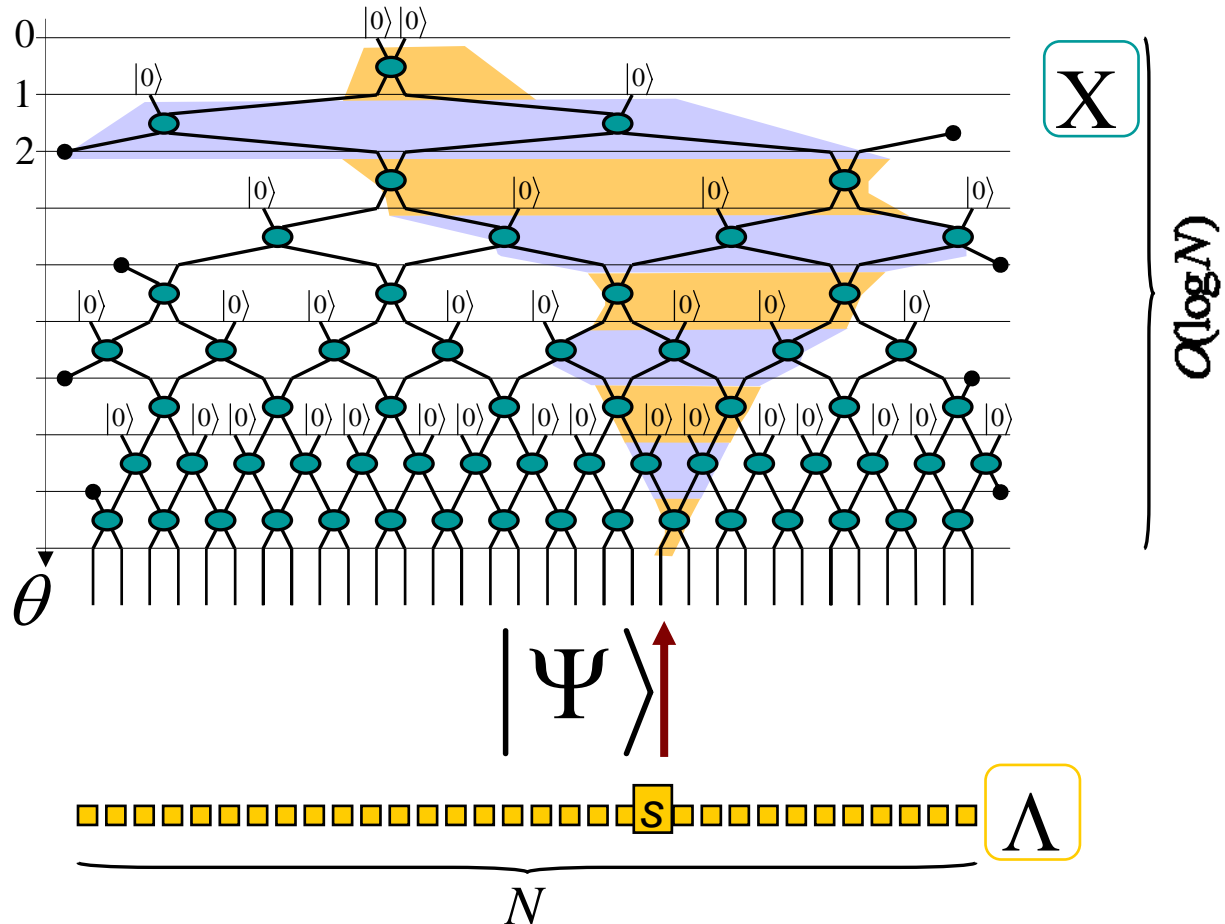
# Multi-Scale Entanglement Renormalization Ansatz (MERA)

GV, cond-mat/0512165, quant-ph/0610099



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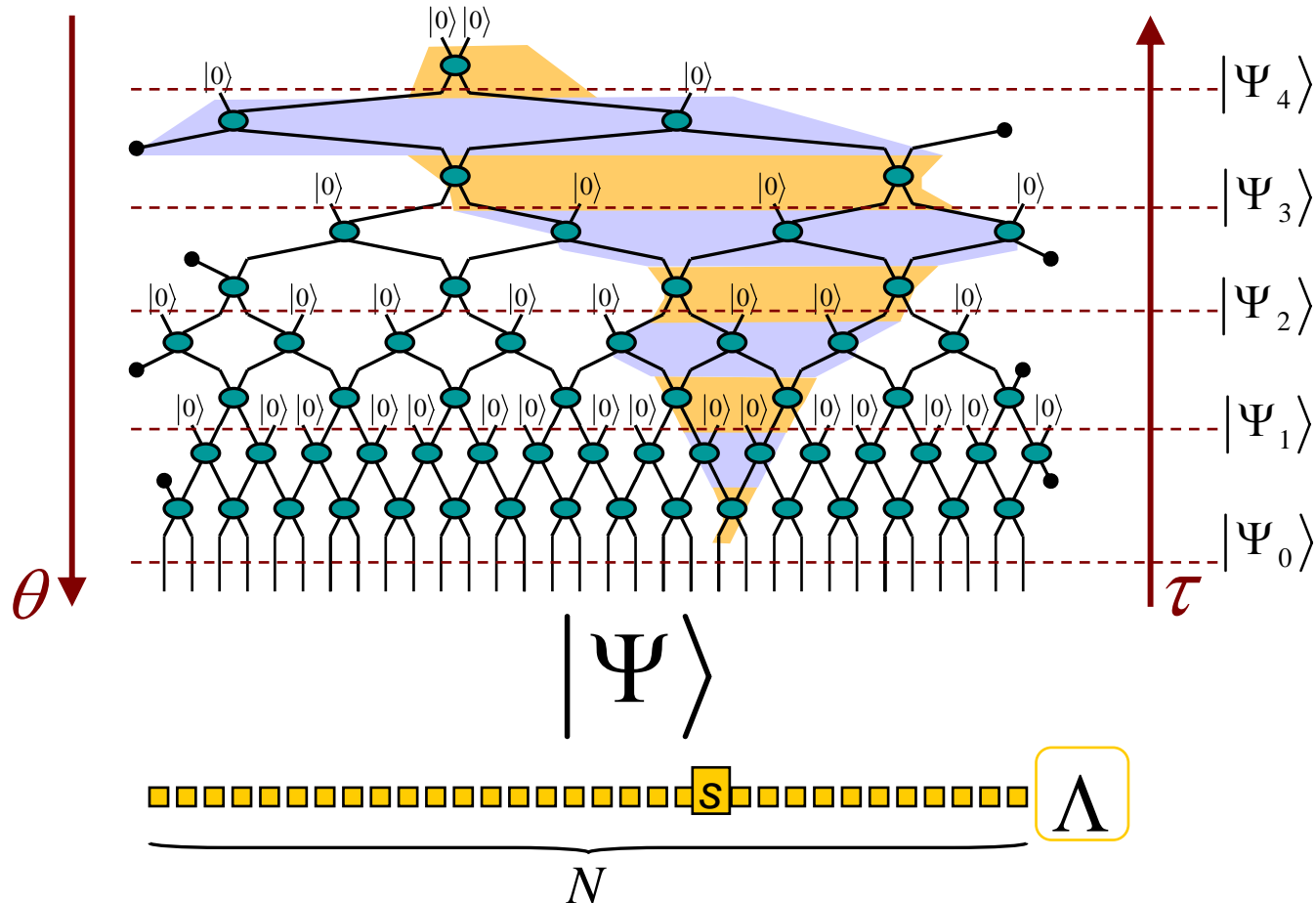


- Causal cone has bounded width !!!

➔ Local reduced density matrices can be computed efficiently

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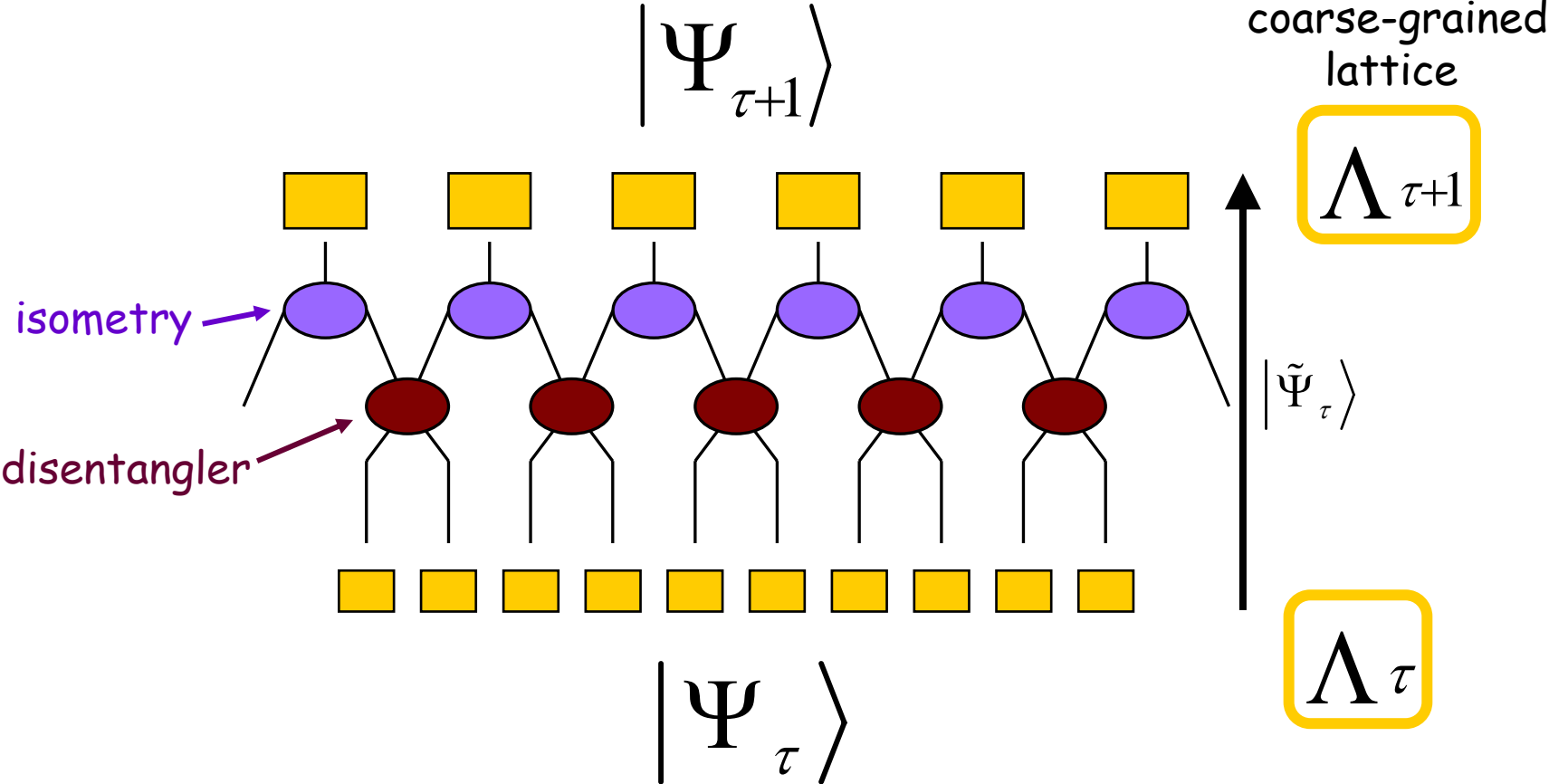


- Causal cone has bounded width !!!

- ➔ Local reduced density matrices can be computed efficiently
- ➔ Renormalization group (or coarse-graining) transformations

# Entanglement Renormalization

GV, cond-mat/0512165, quant-ph/0610099





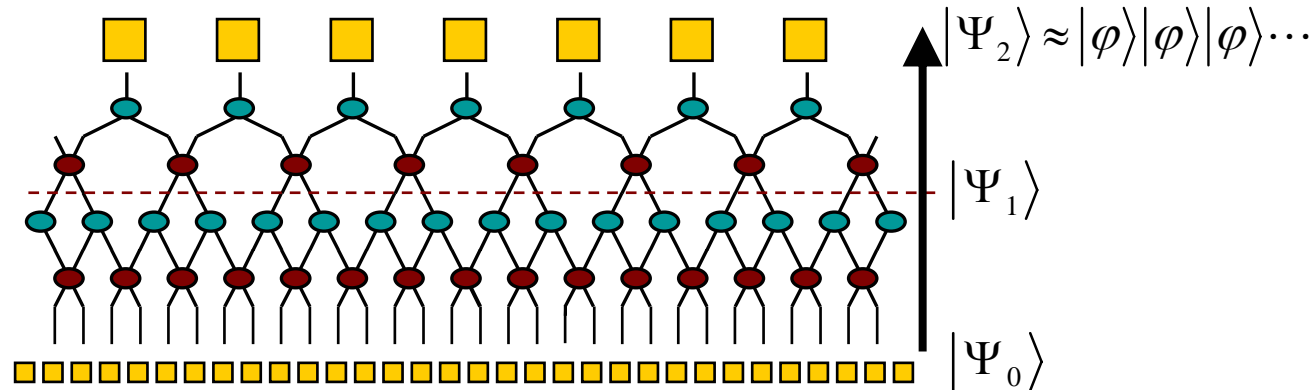
# Entanglement Renormalization

GV, cond-mat/0512165, quant-ph/0610099

- Ground state of a local Hamiltonian:

- Gapped systems  
(finite correlation length)

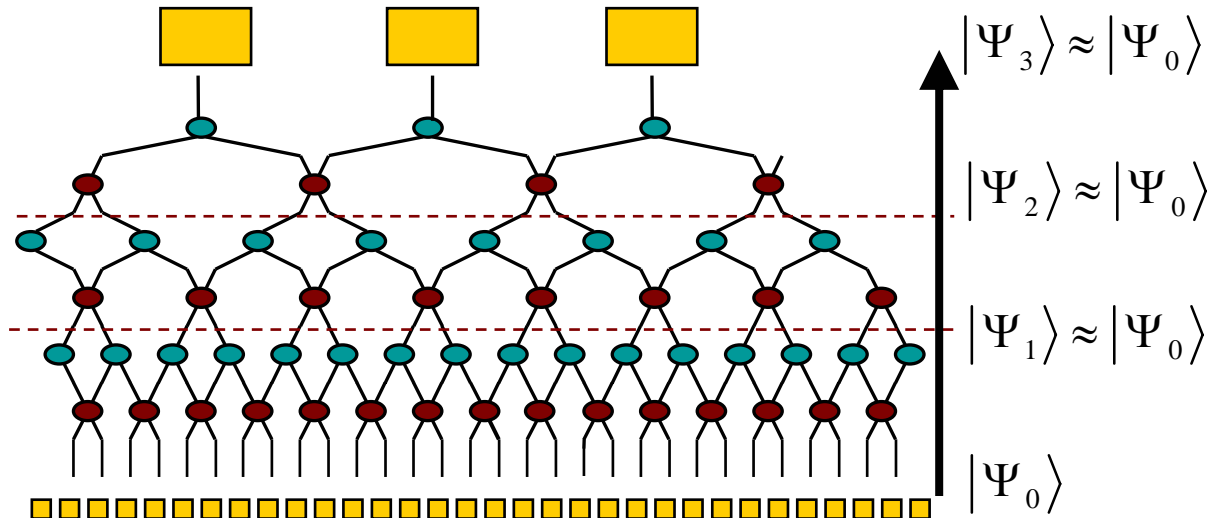
- ❖ Unentangled state after several coarse-graining transformations



- Critical systems  
(quantum phase transition)

Glen Evenbly, GV

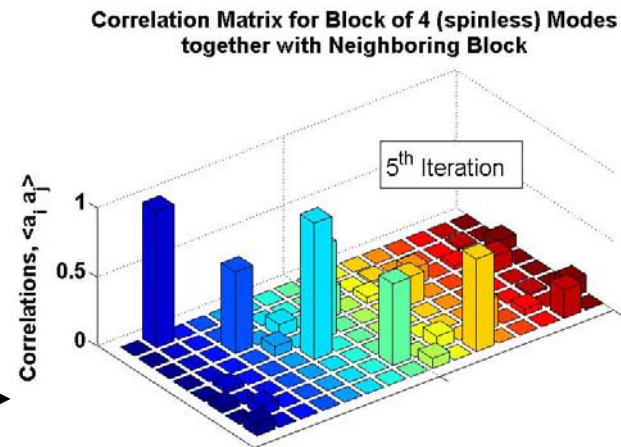
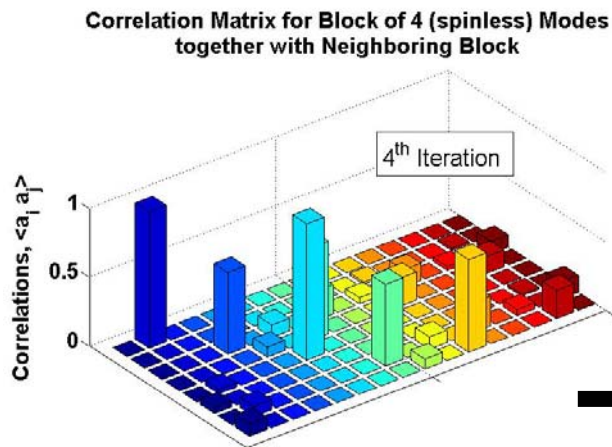
- ❖ Self-similar MERA  
(ground state always remains entangled)



# Entanglement Renormalization

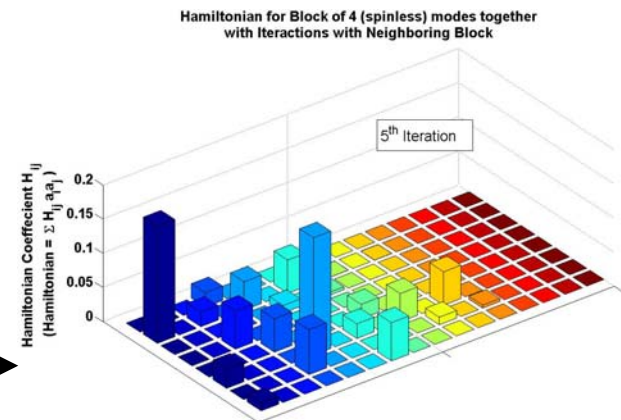
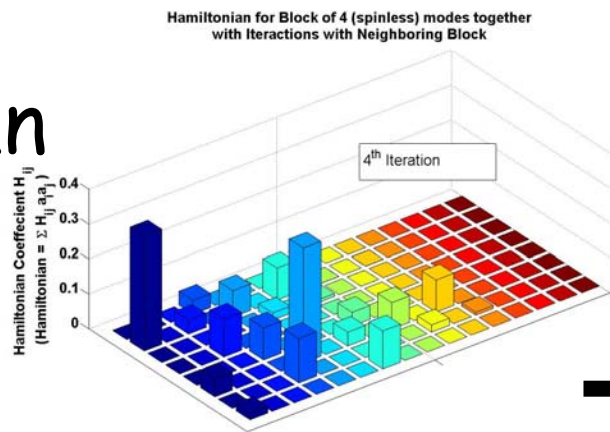
(discrete!) scale invariance in critical systems

## State



RG transformation

## Effective Hamiltonian



RG transformation

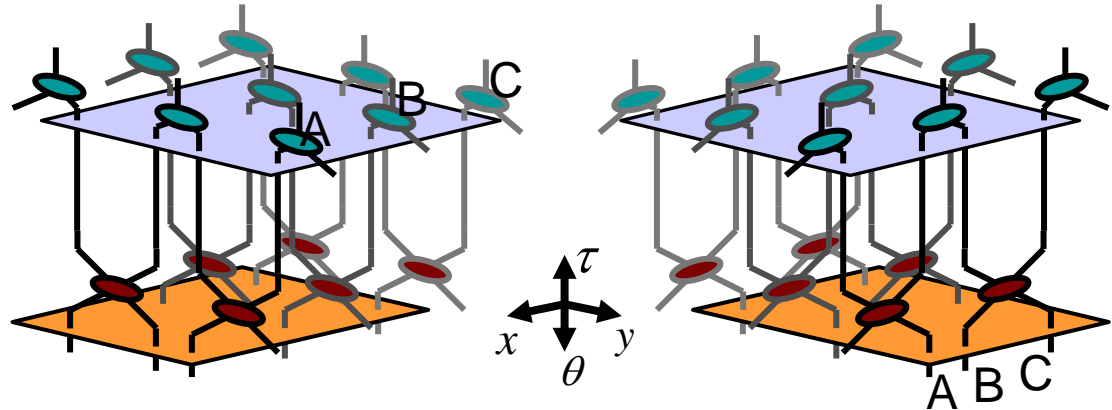
# Entanglement Renormalization

GV, cond-mat/0512165, quant-ph/0610099

- Ground states with topological order:

- 2D MERA

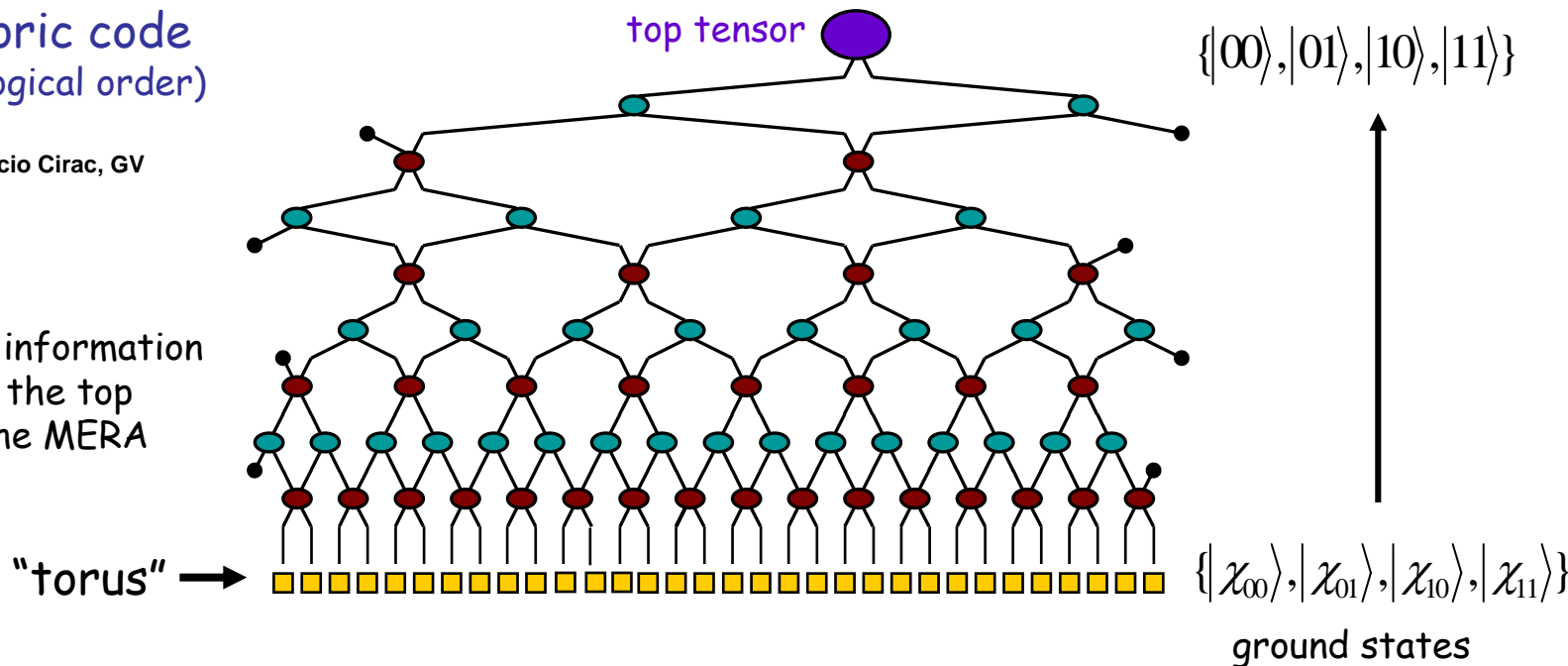
- ❖ More complicated structure (2+1) dimensional quantum circuit bounded causal cone



- Kitaev's toric code (contains topological order)

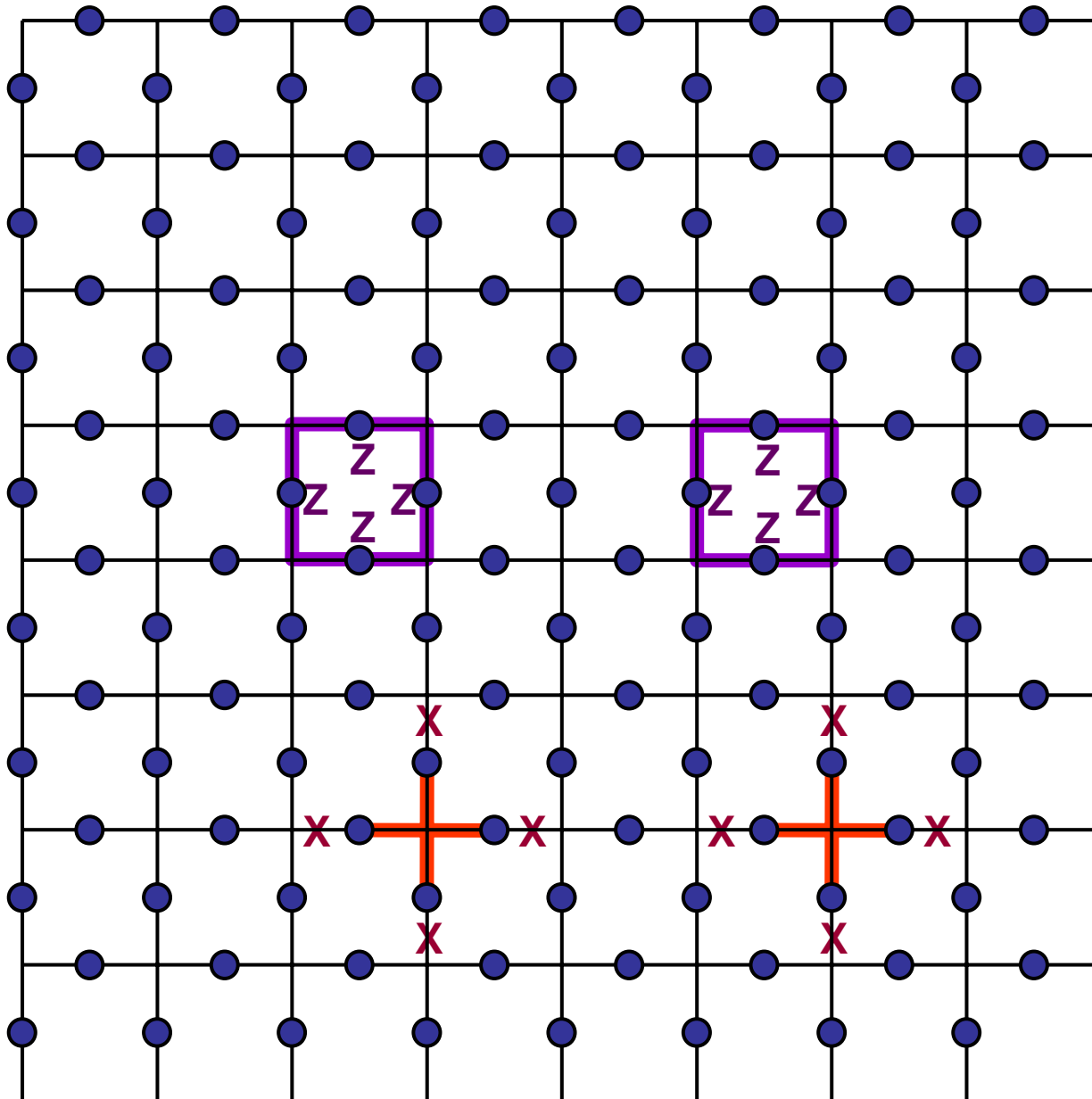
Miguel Aguado, Ignacio Cirac, GV

- ❖ Topological information is stored in the top tensor of the MERA

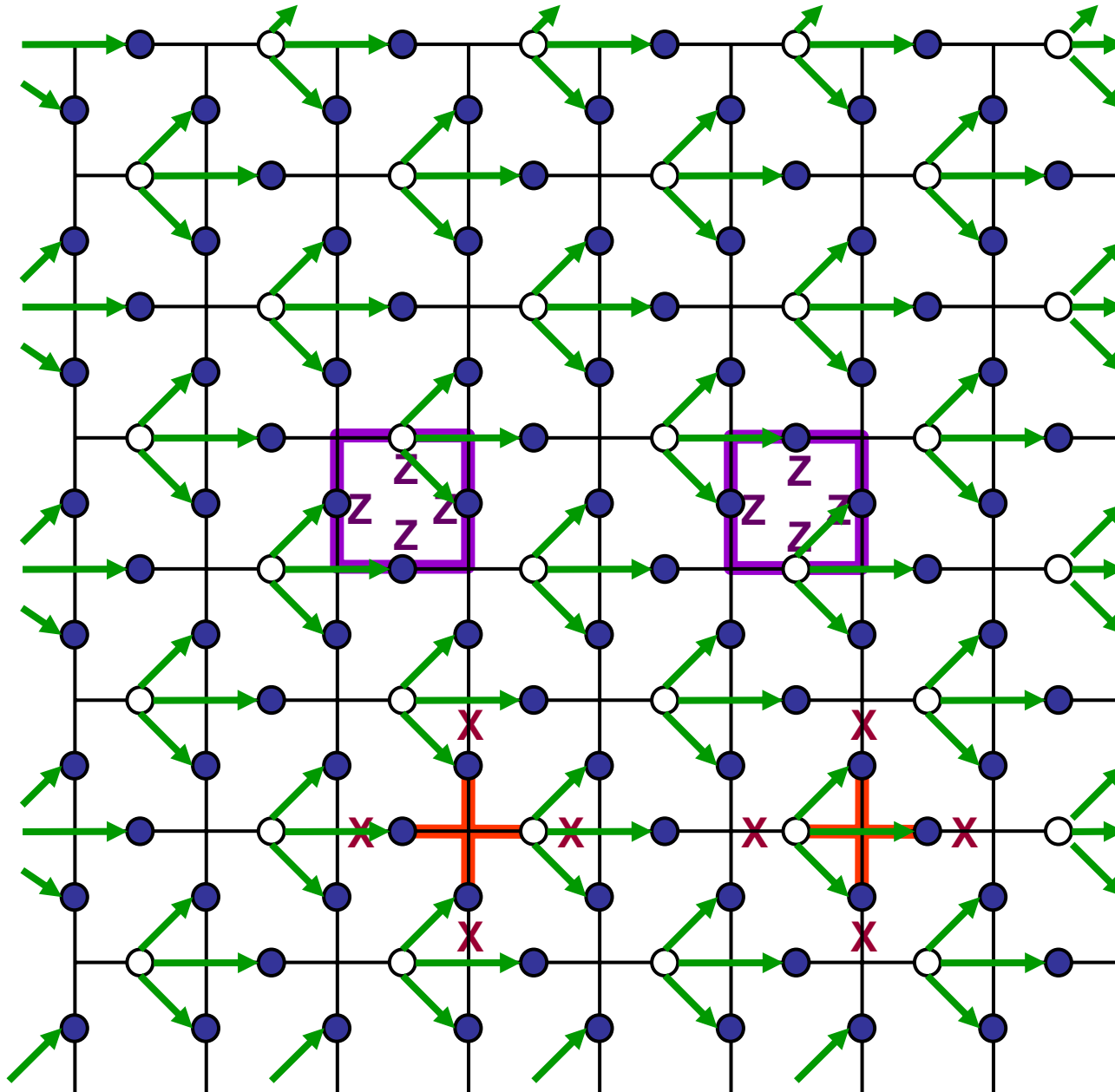


- Transformation of
  - stabilizers
  - topological charges
- Excitations
- Fixed point of RG flow?

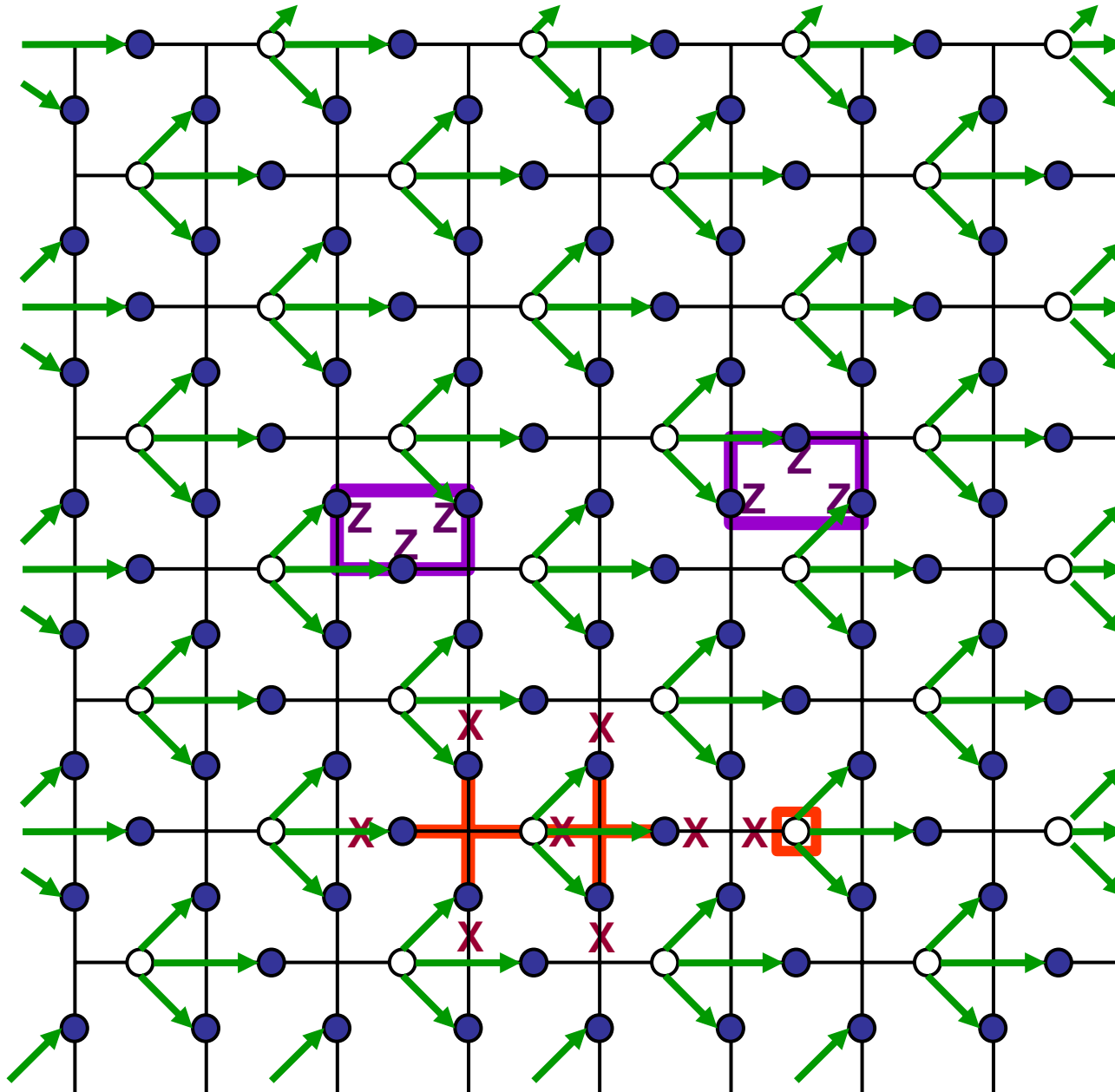
# Kitaev's code on a torus



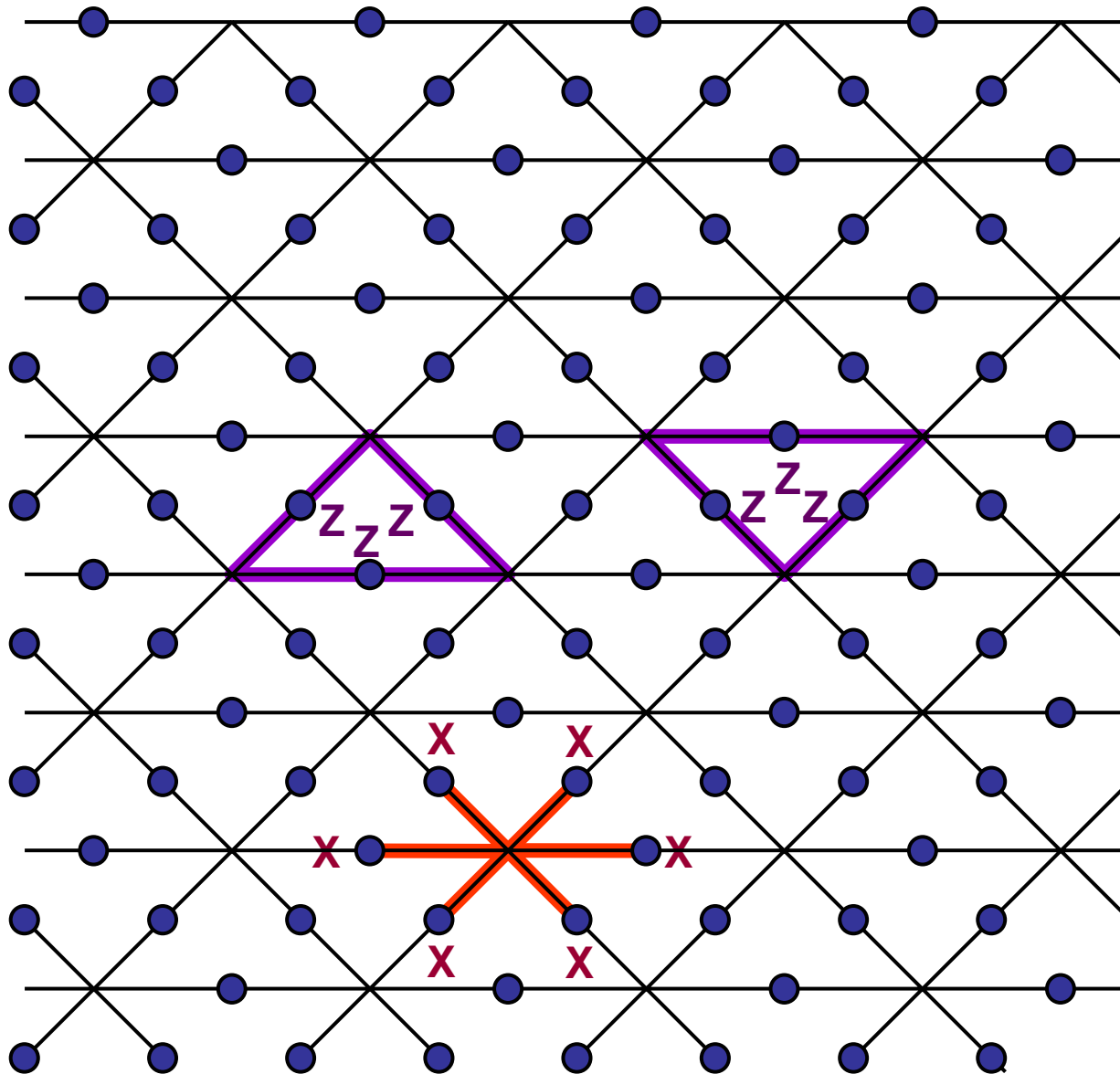
# Disentangler



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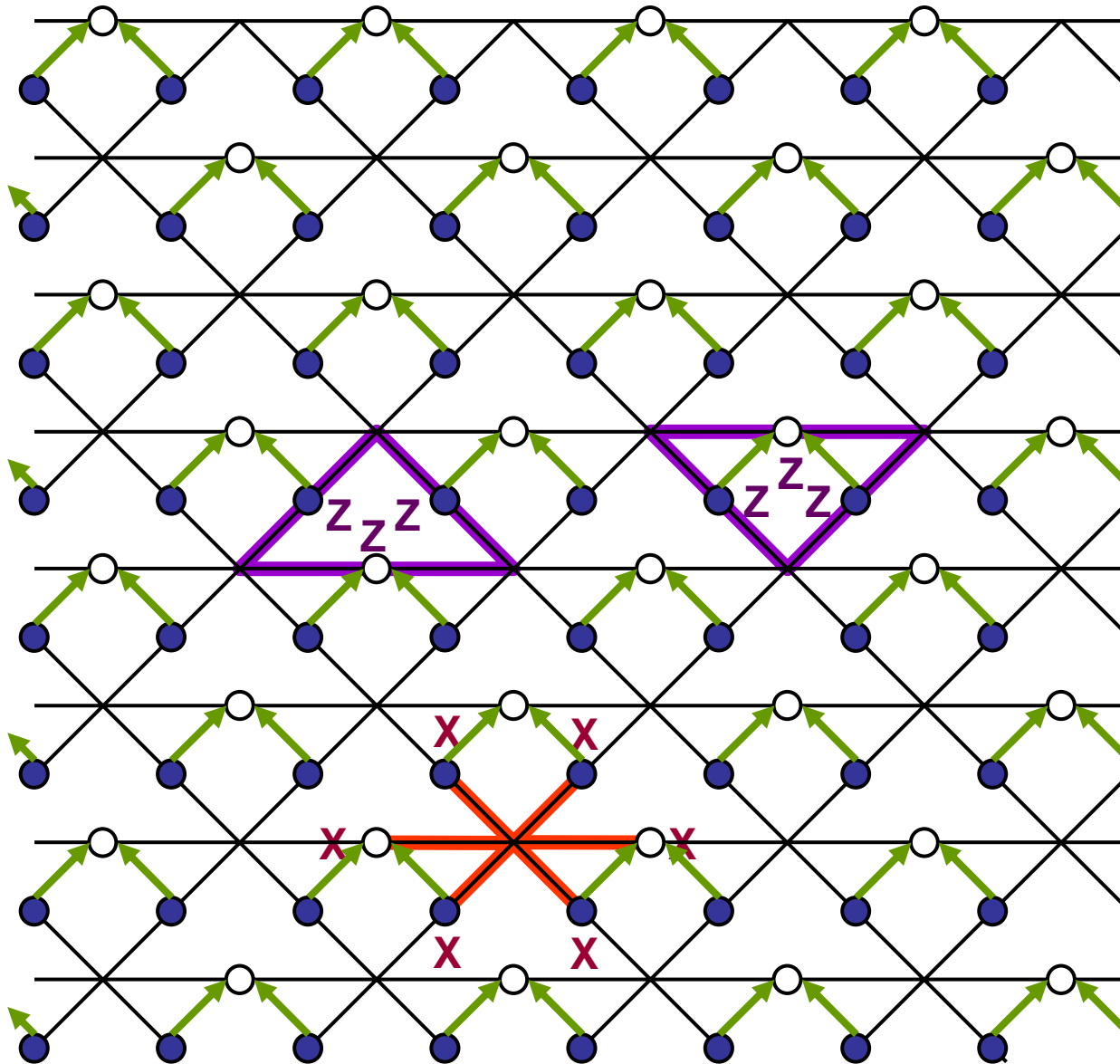


# Levin-Wen model on a torus

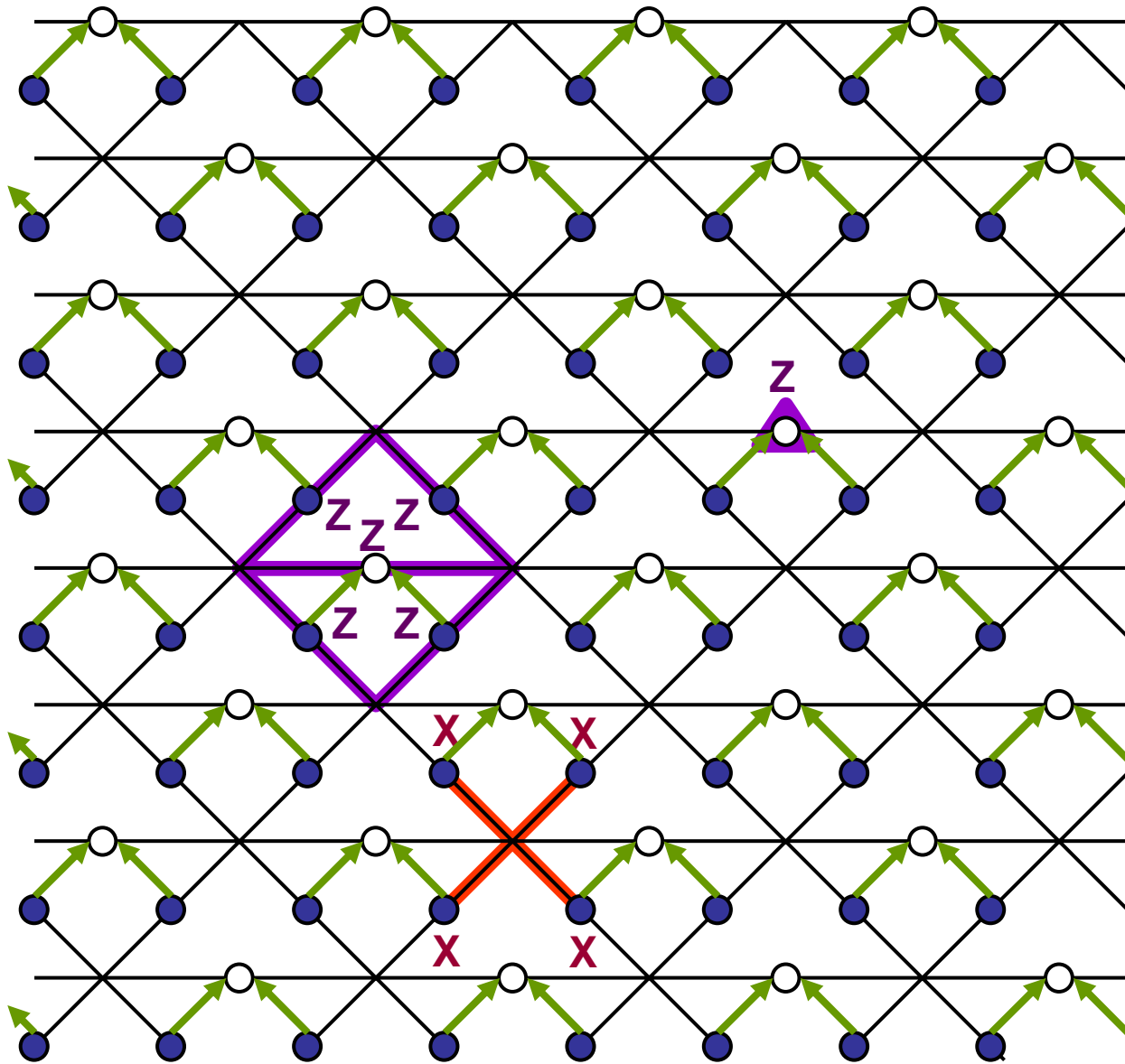




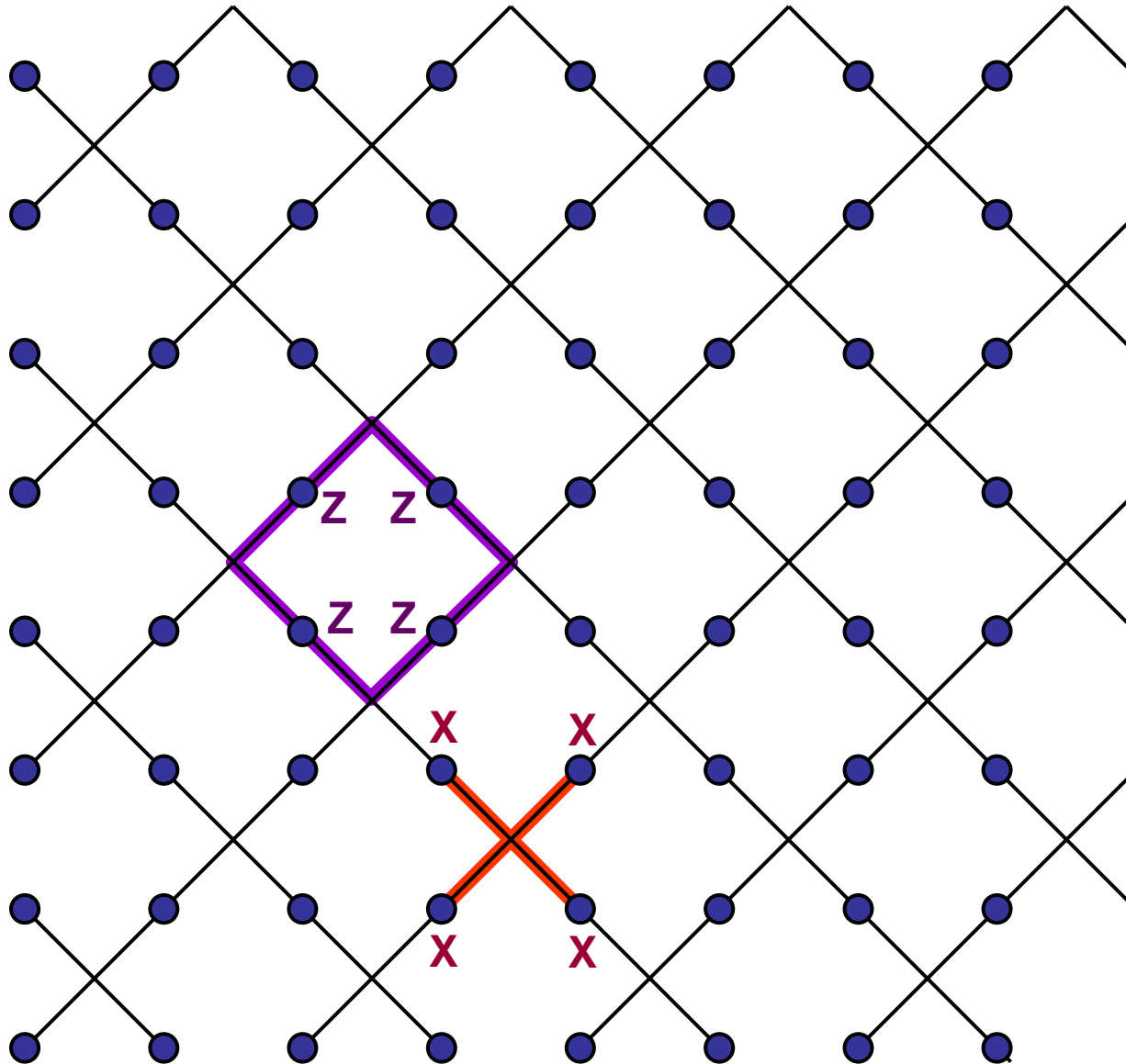
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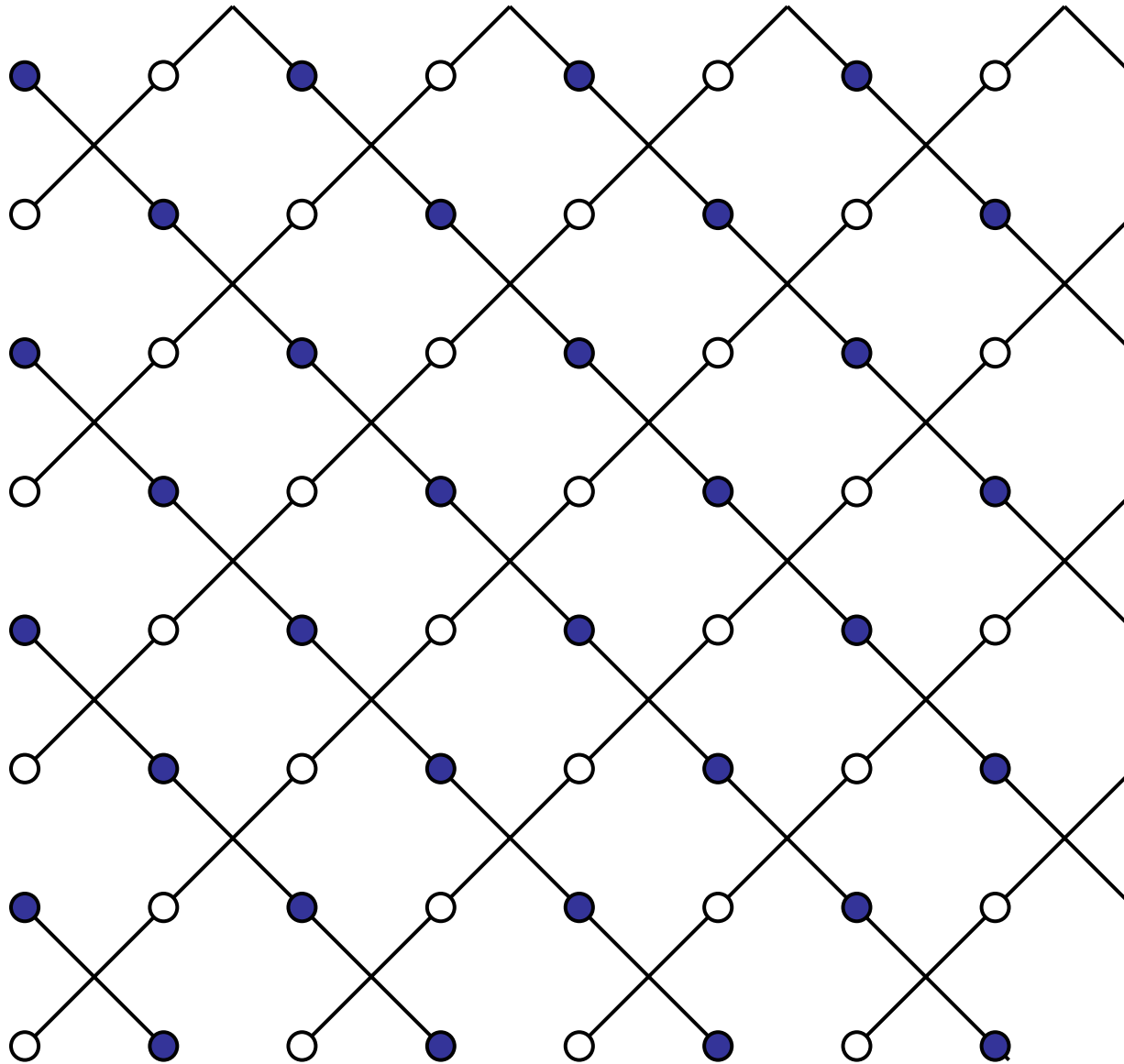
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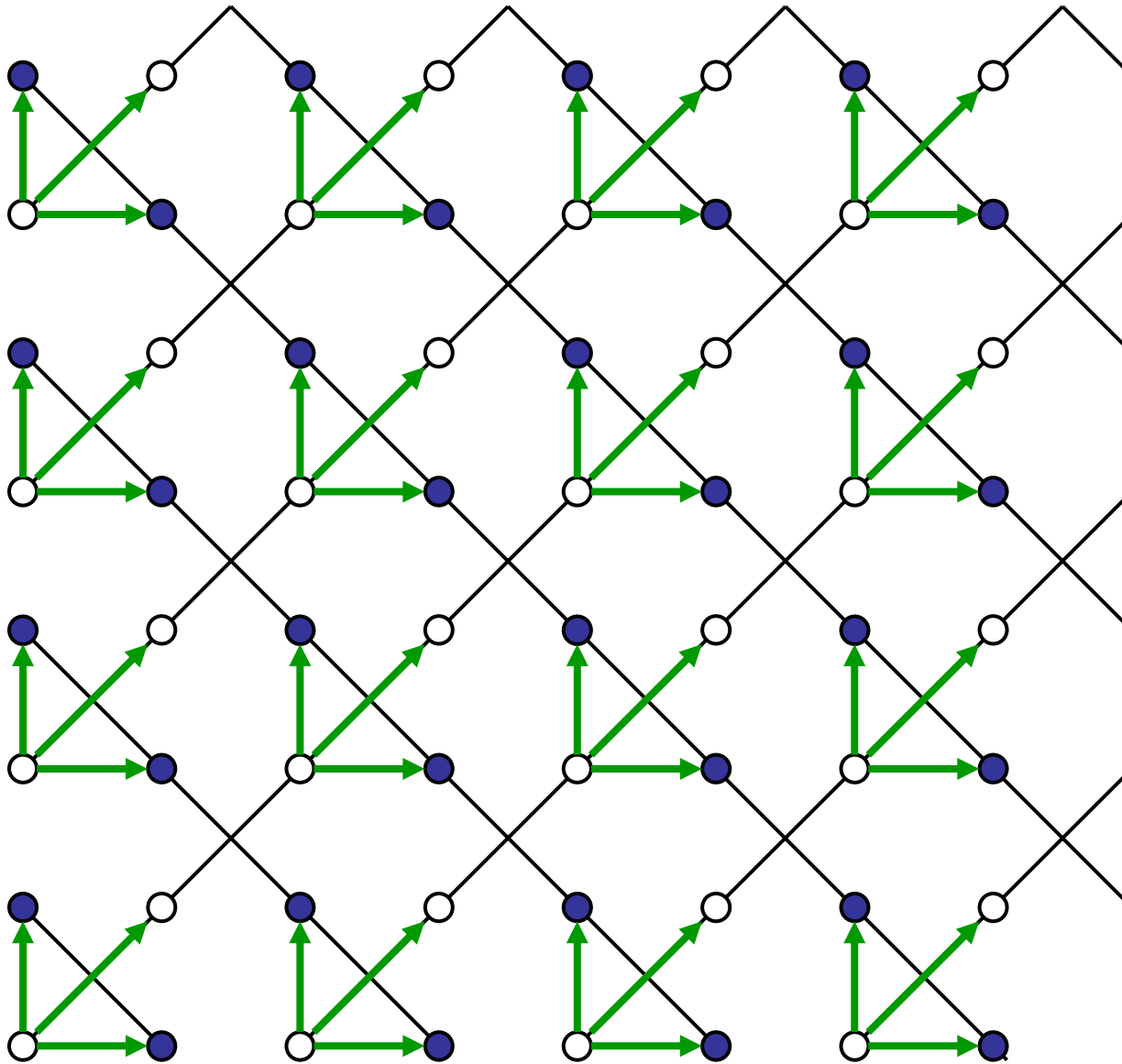
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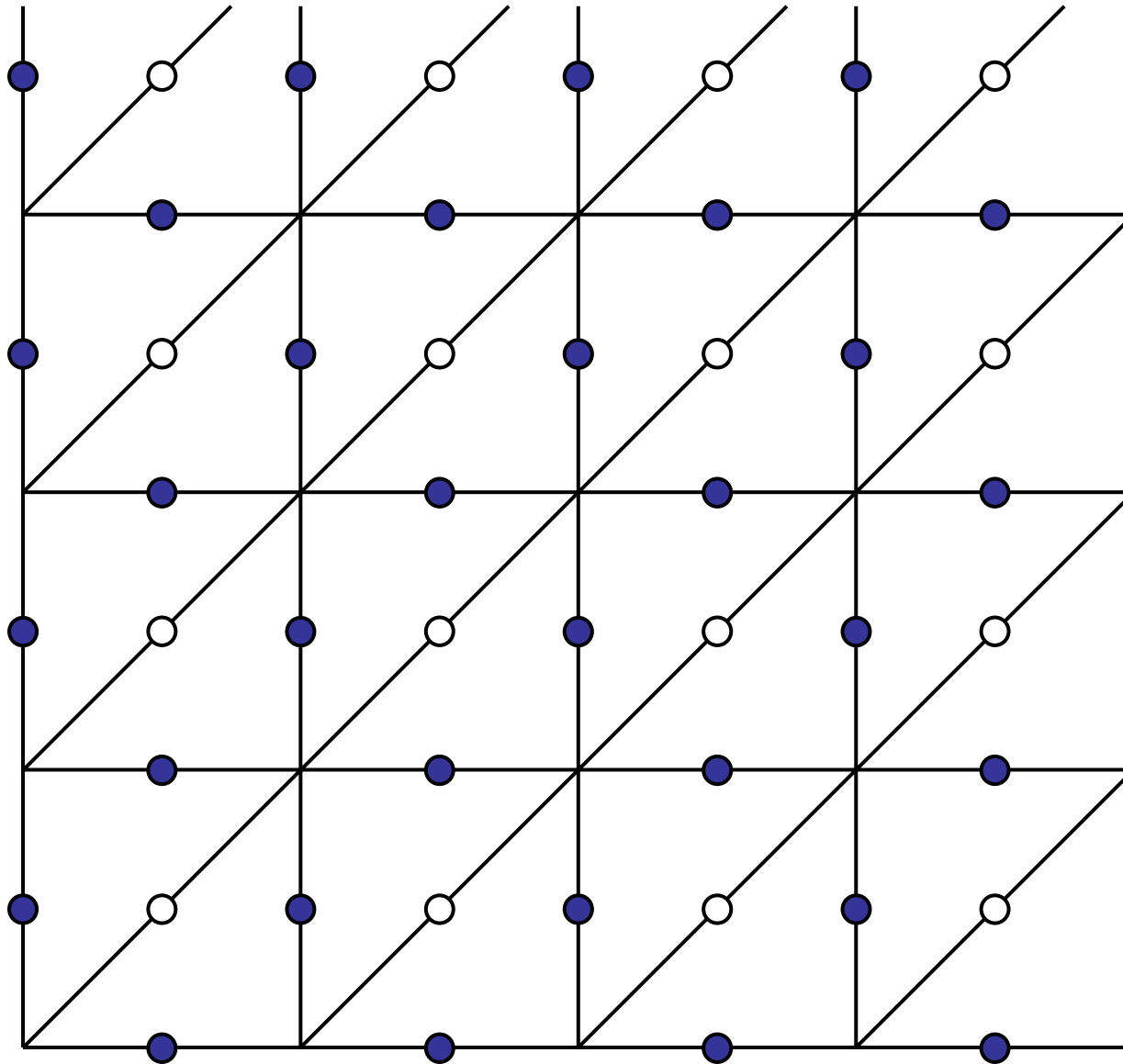
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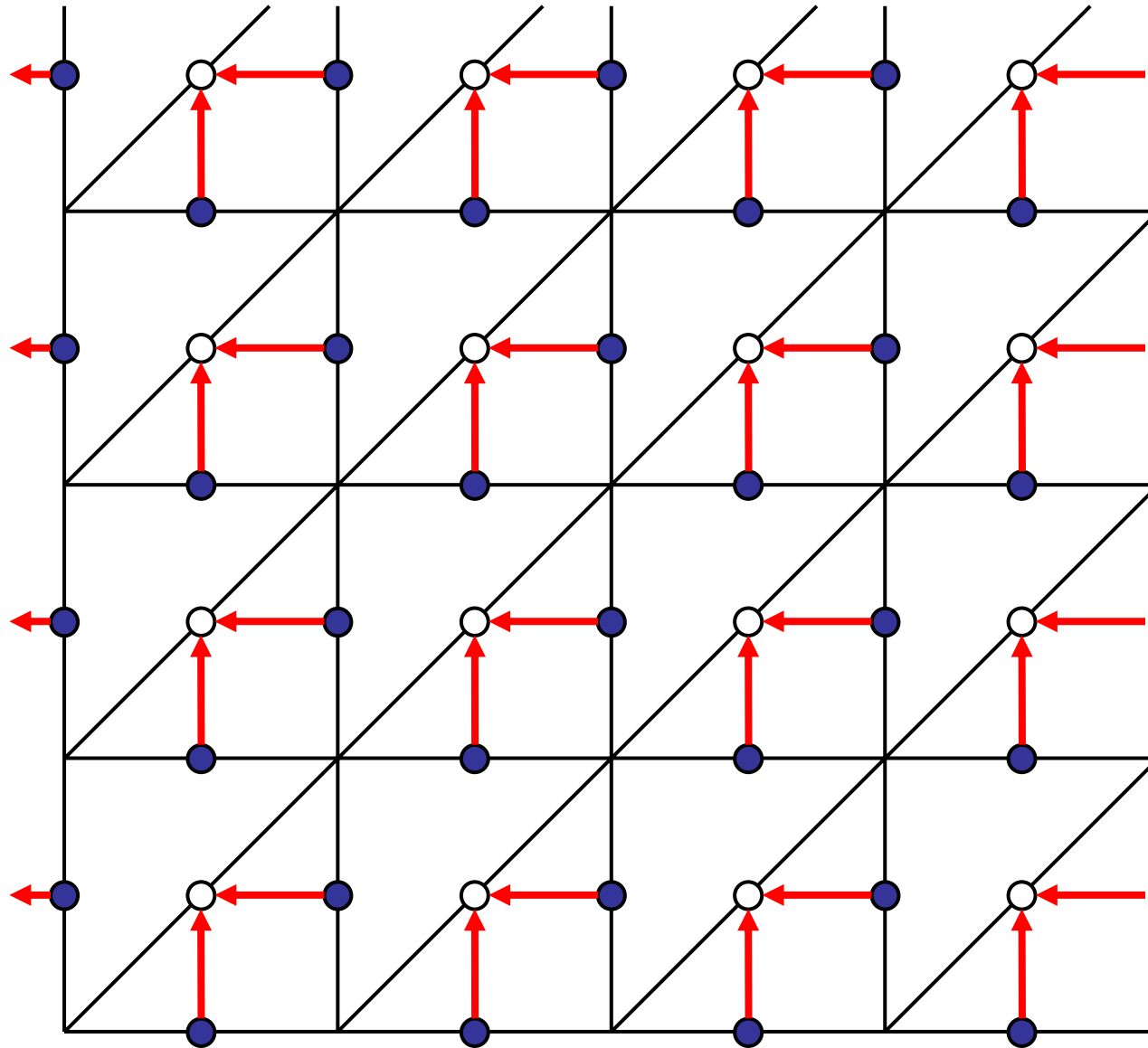
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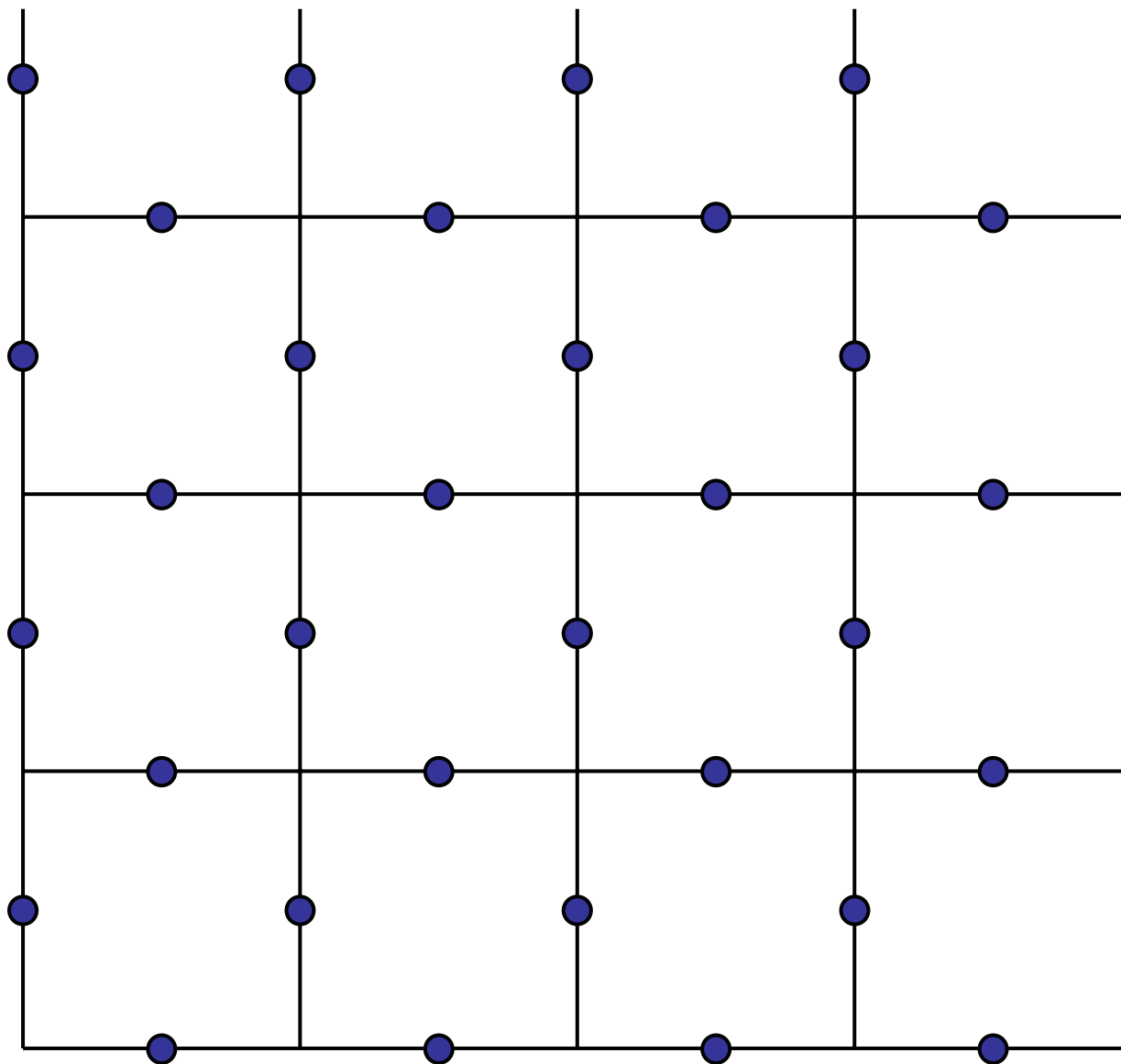
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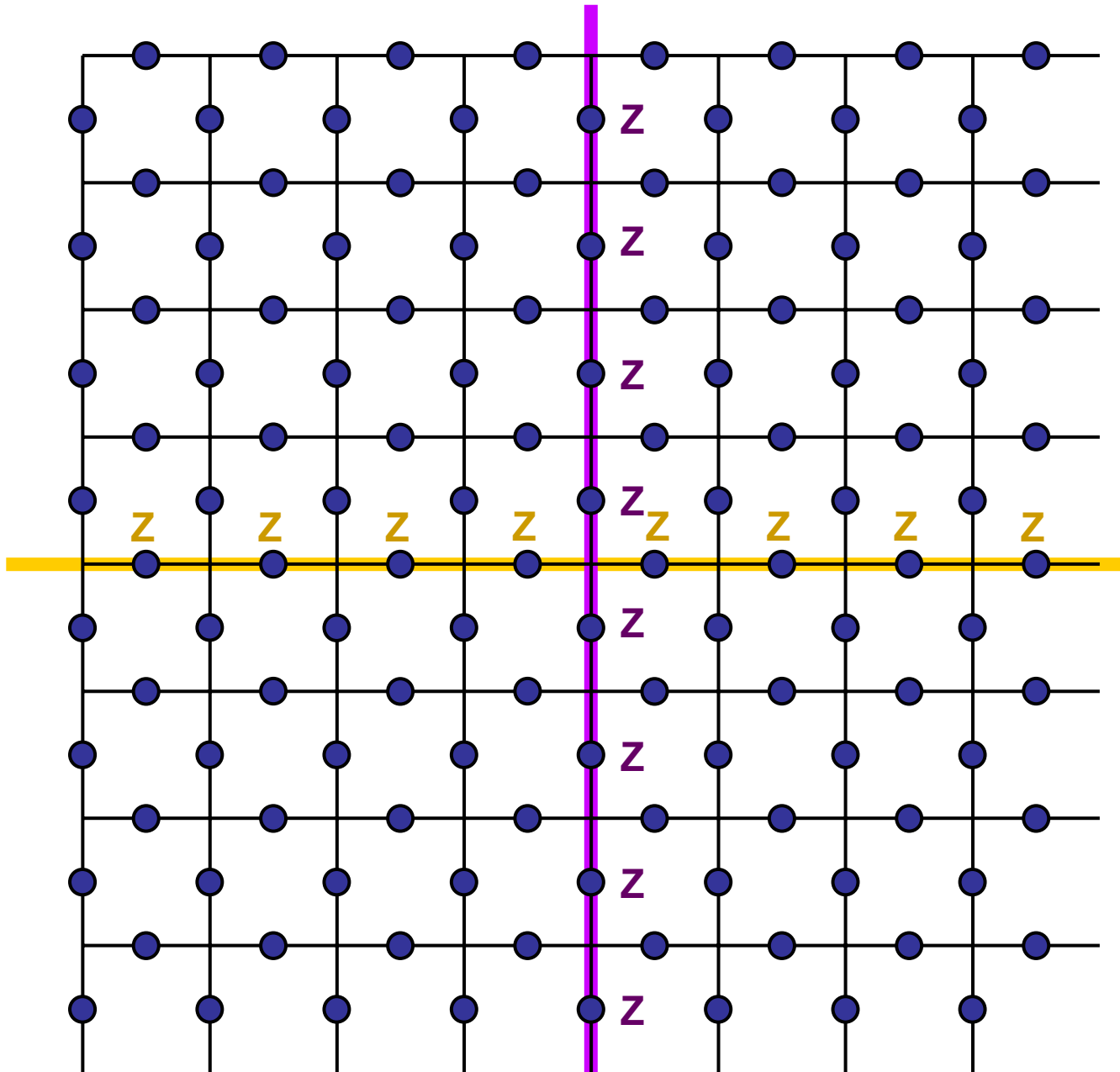
# Kitaev's code on a coarsed-grained torus



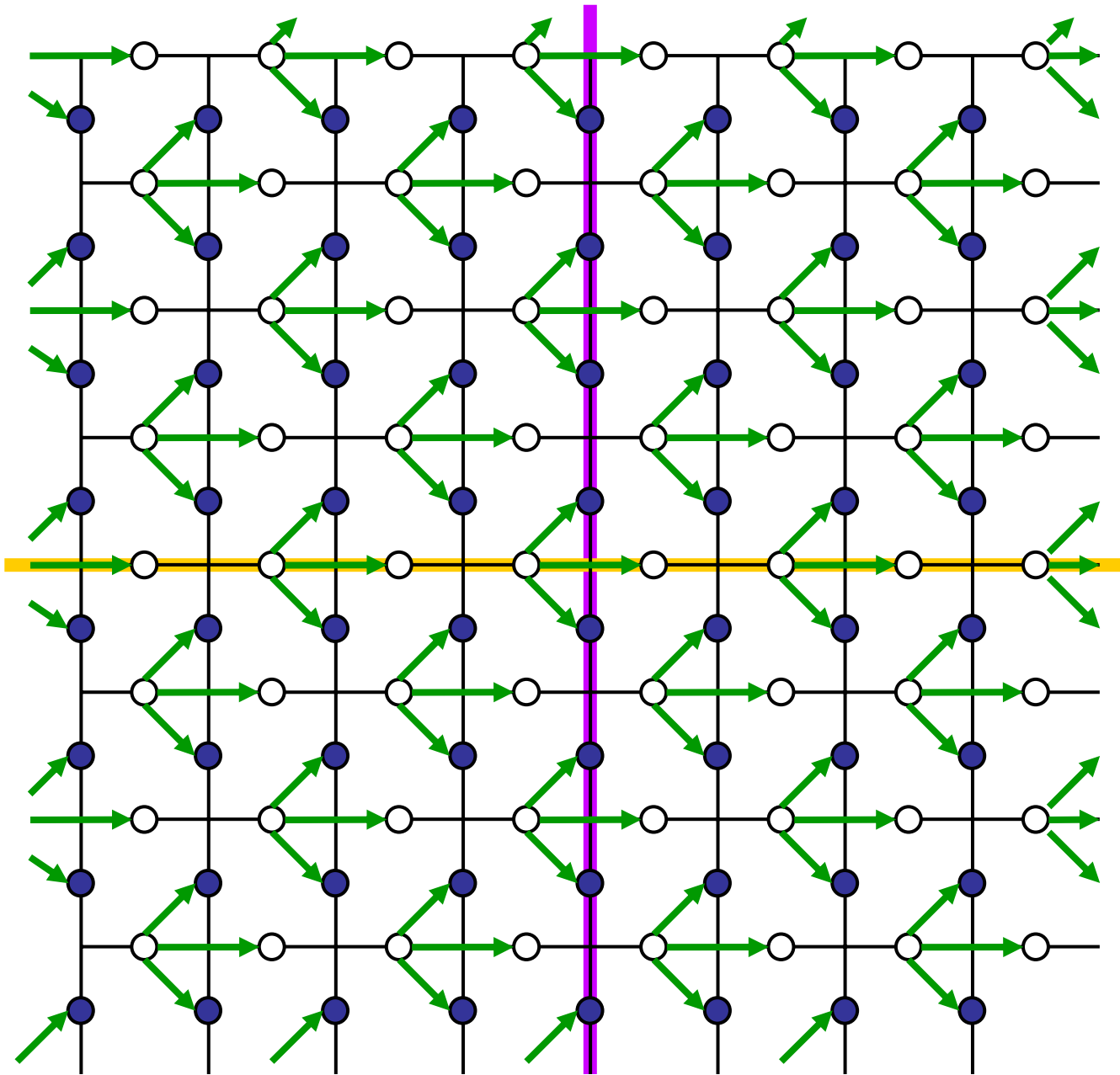


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  - topological charges
- Excitations
- Fixed point of RG flow?

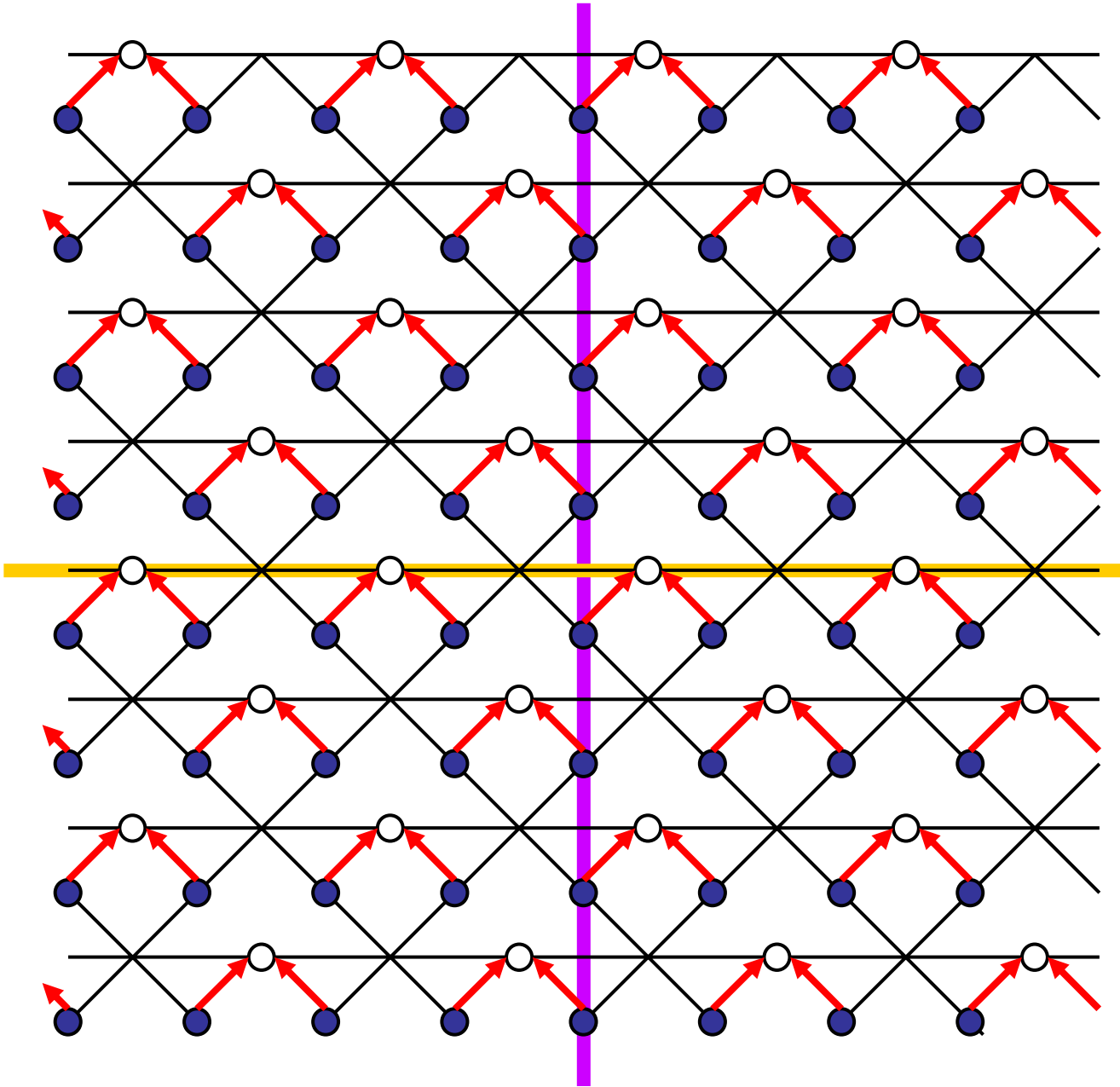
# Charges



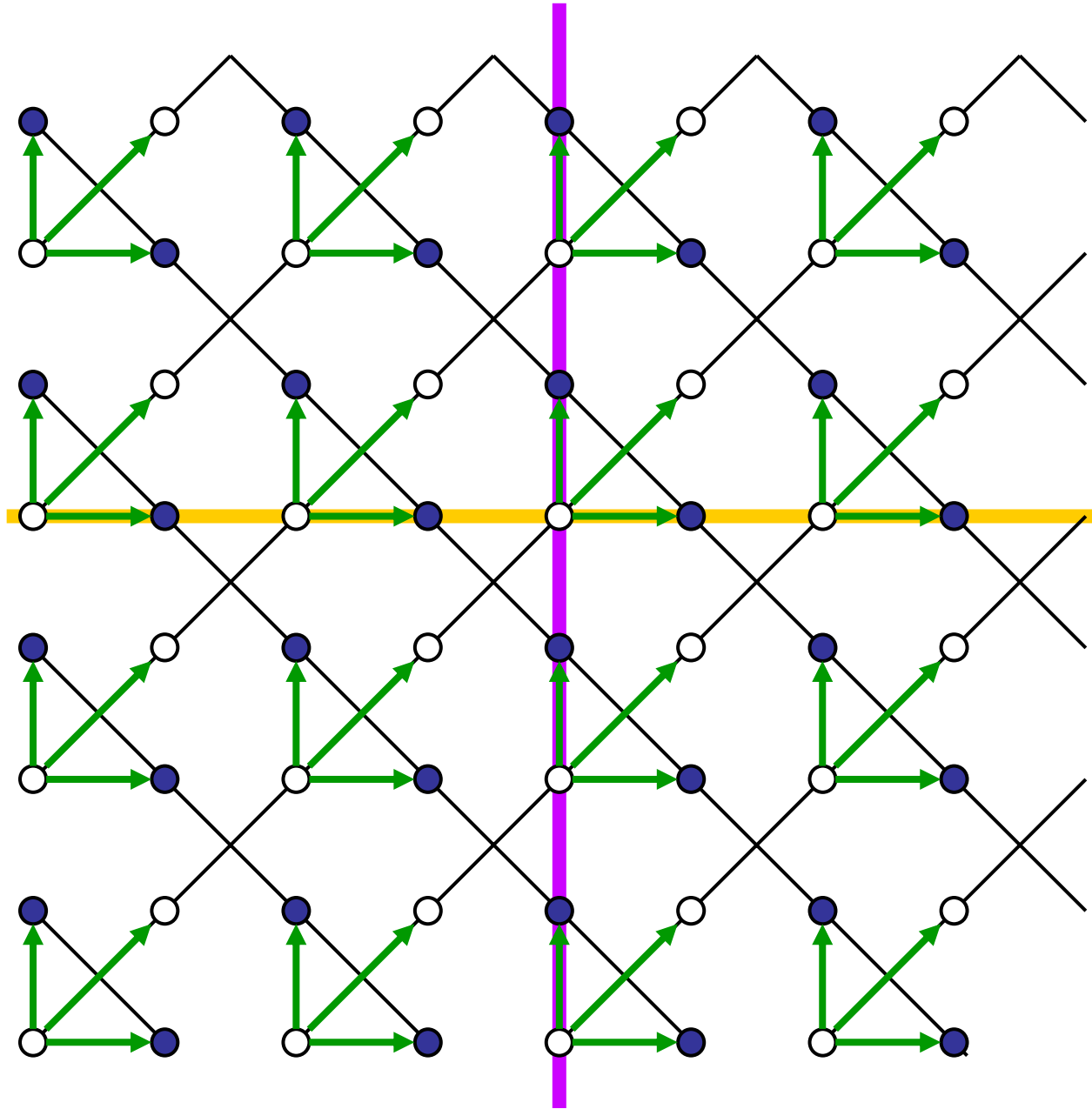
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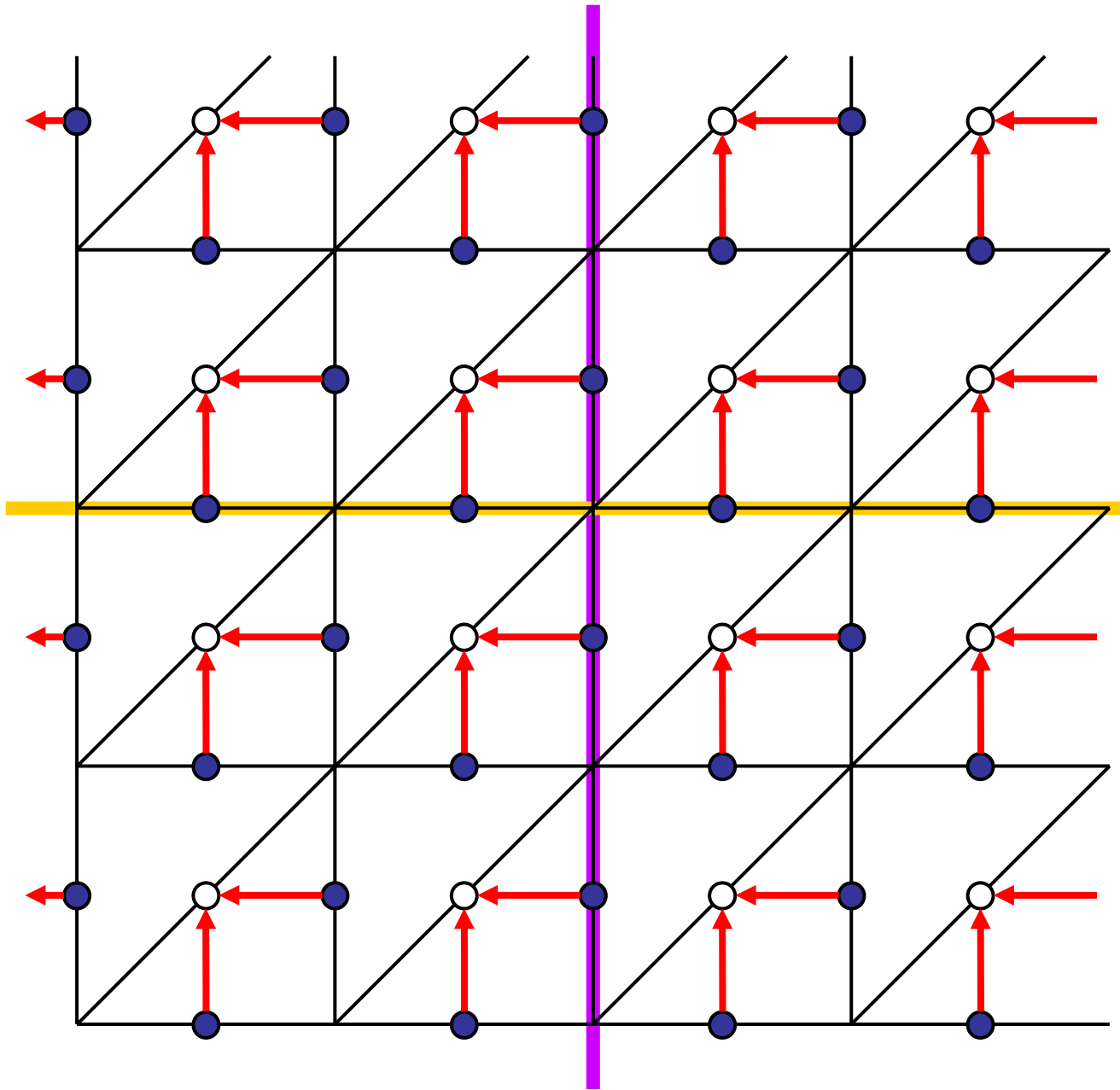
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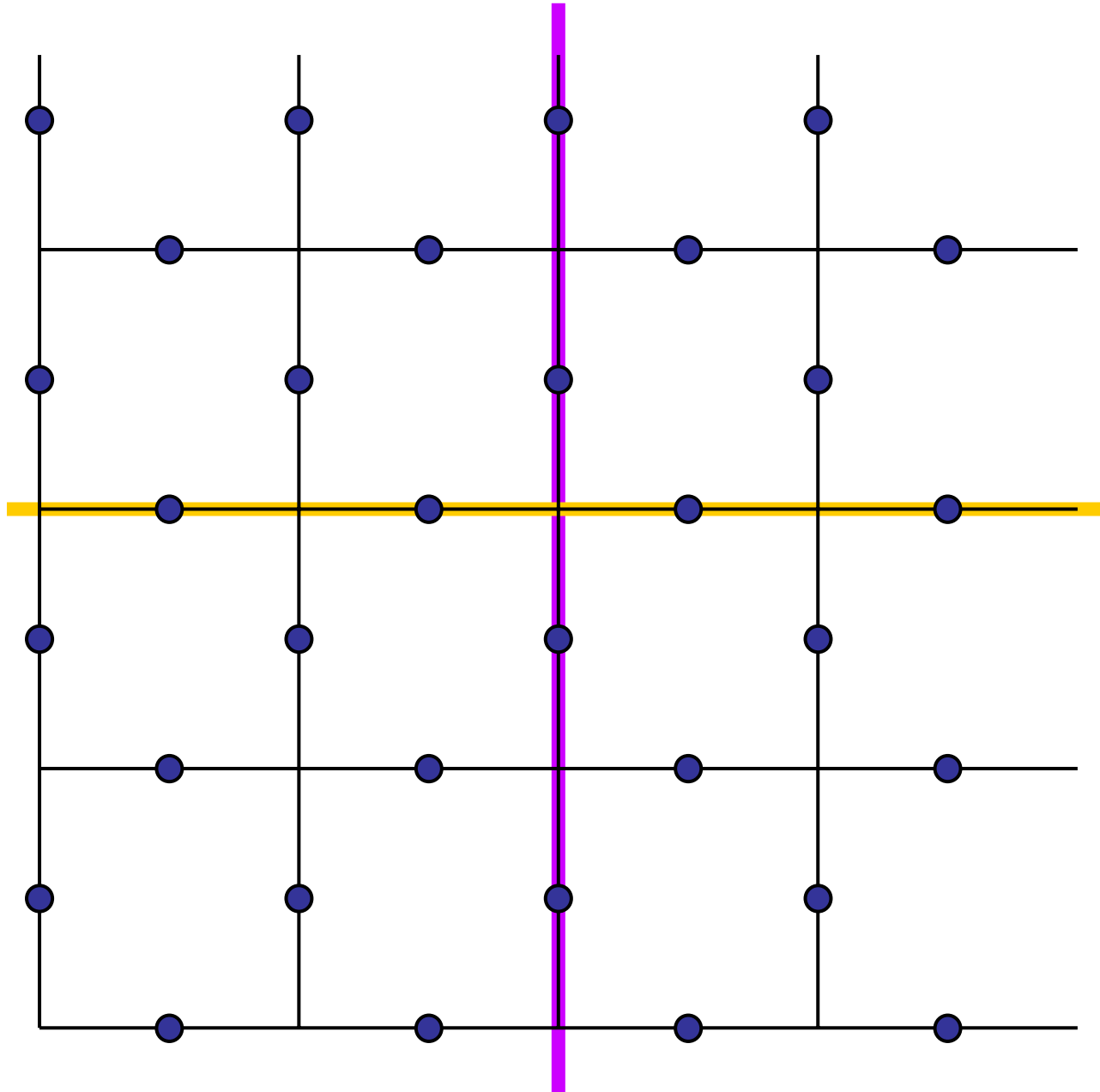
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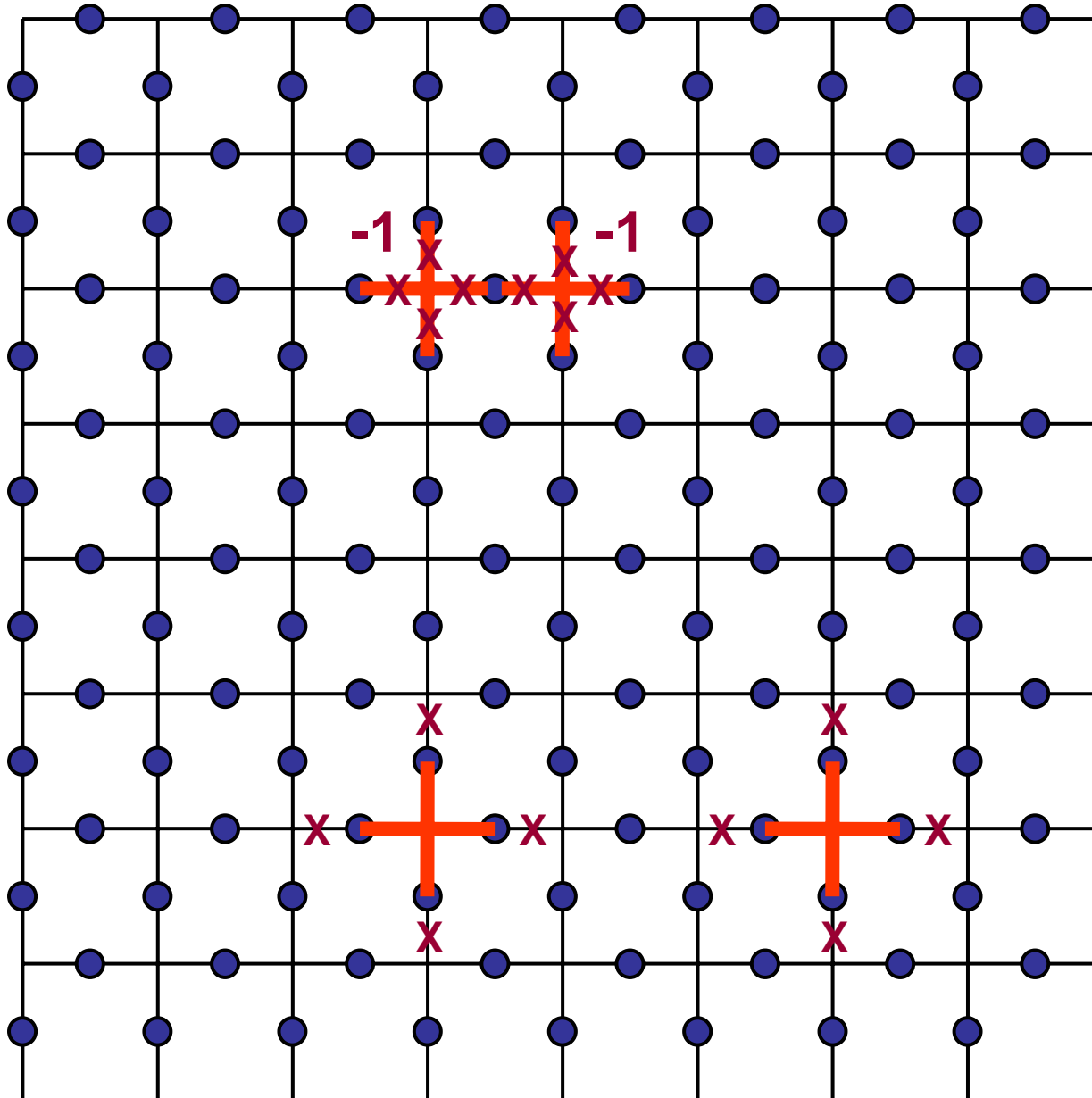
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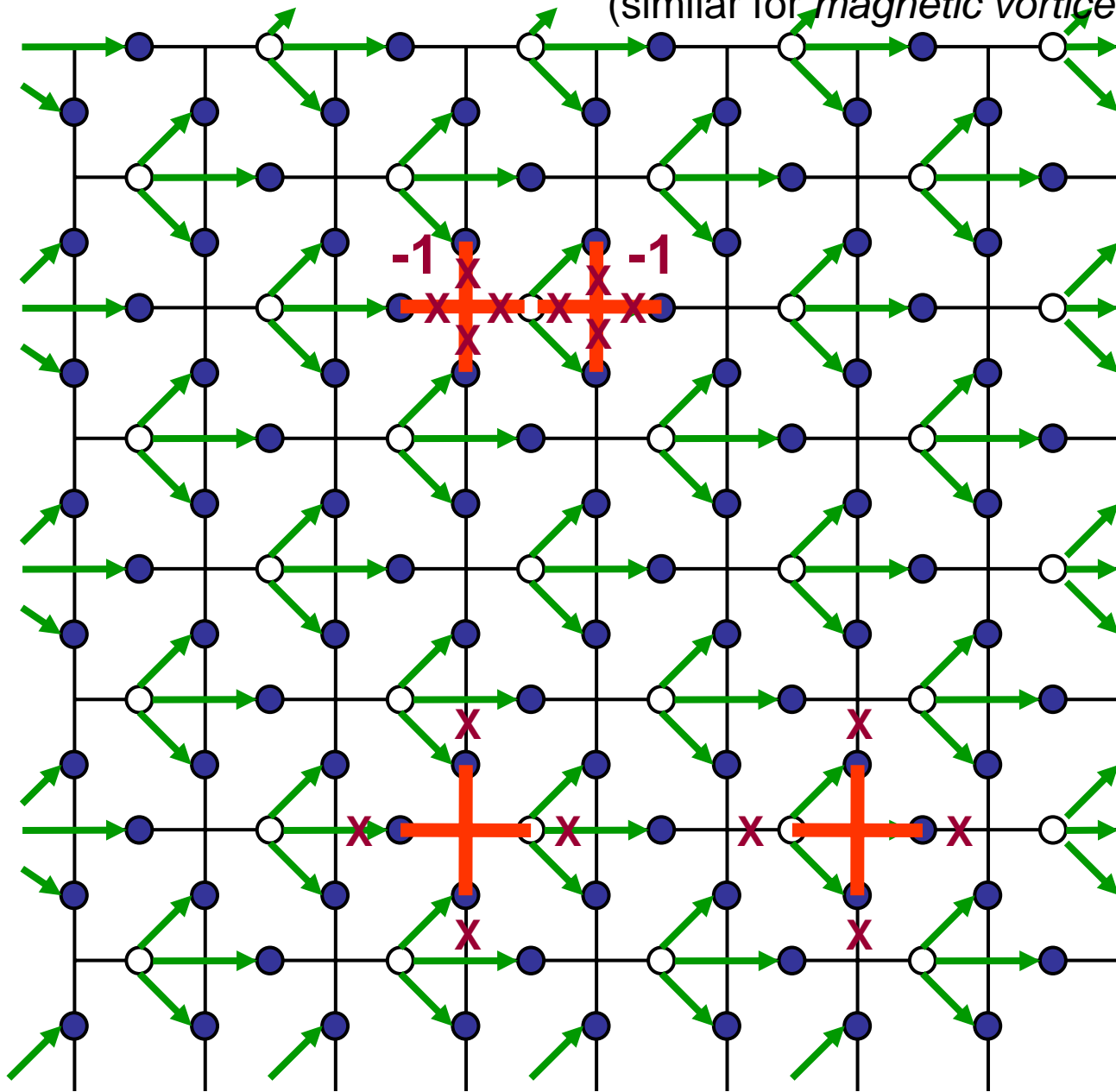
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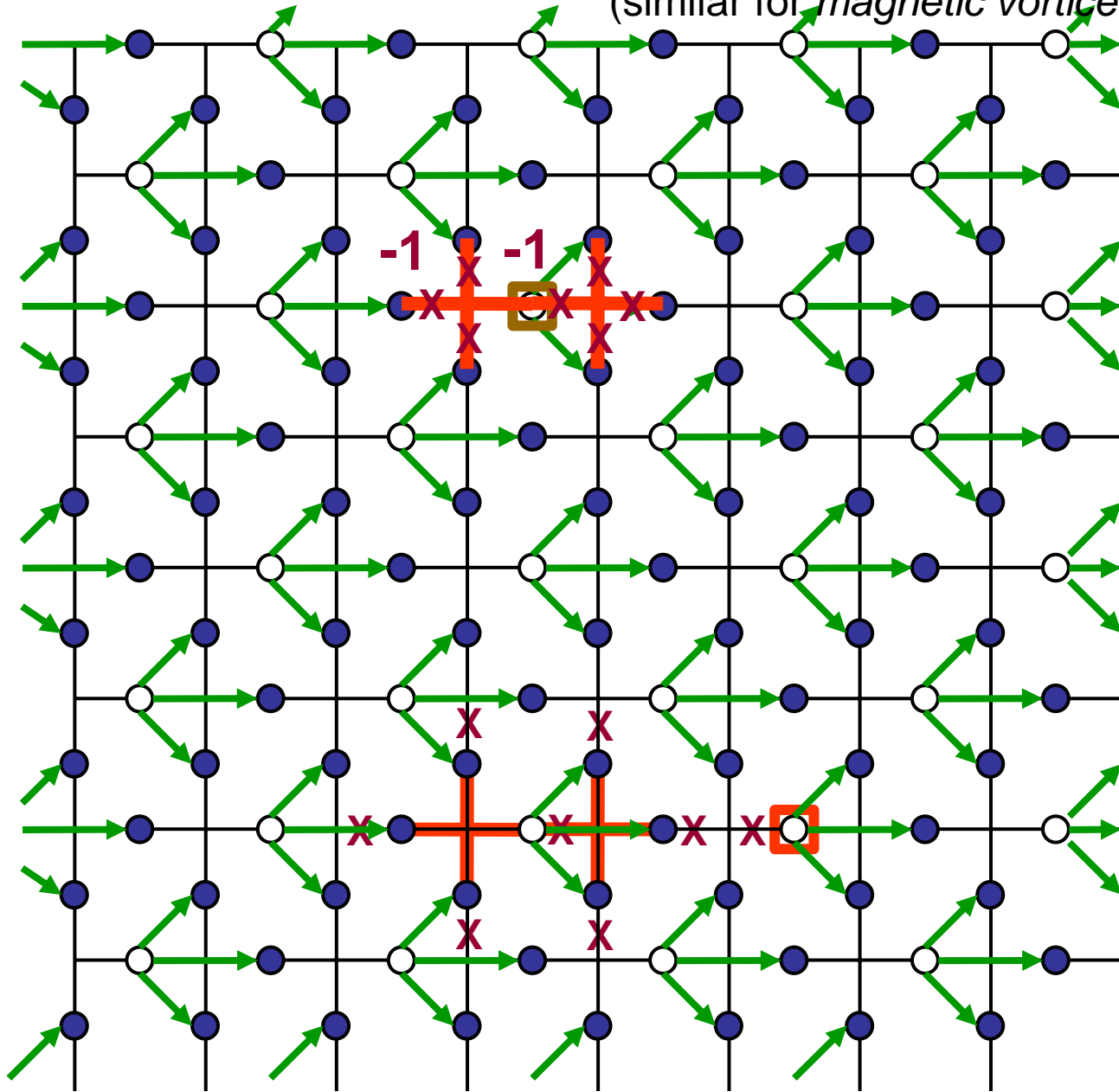
Excitations and ER: annihilation of a pair of *magnetic charges*  
(similar for *magnetic vortices*)



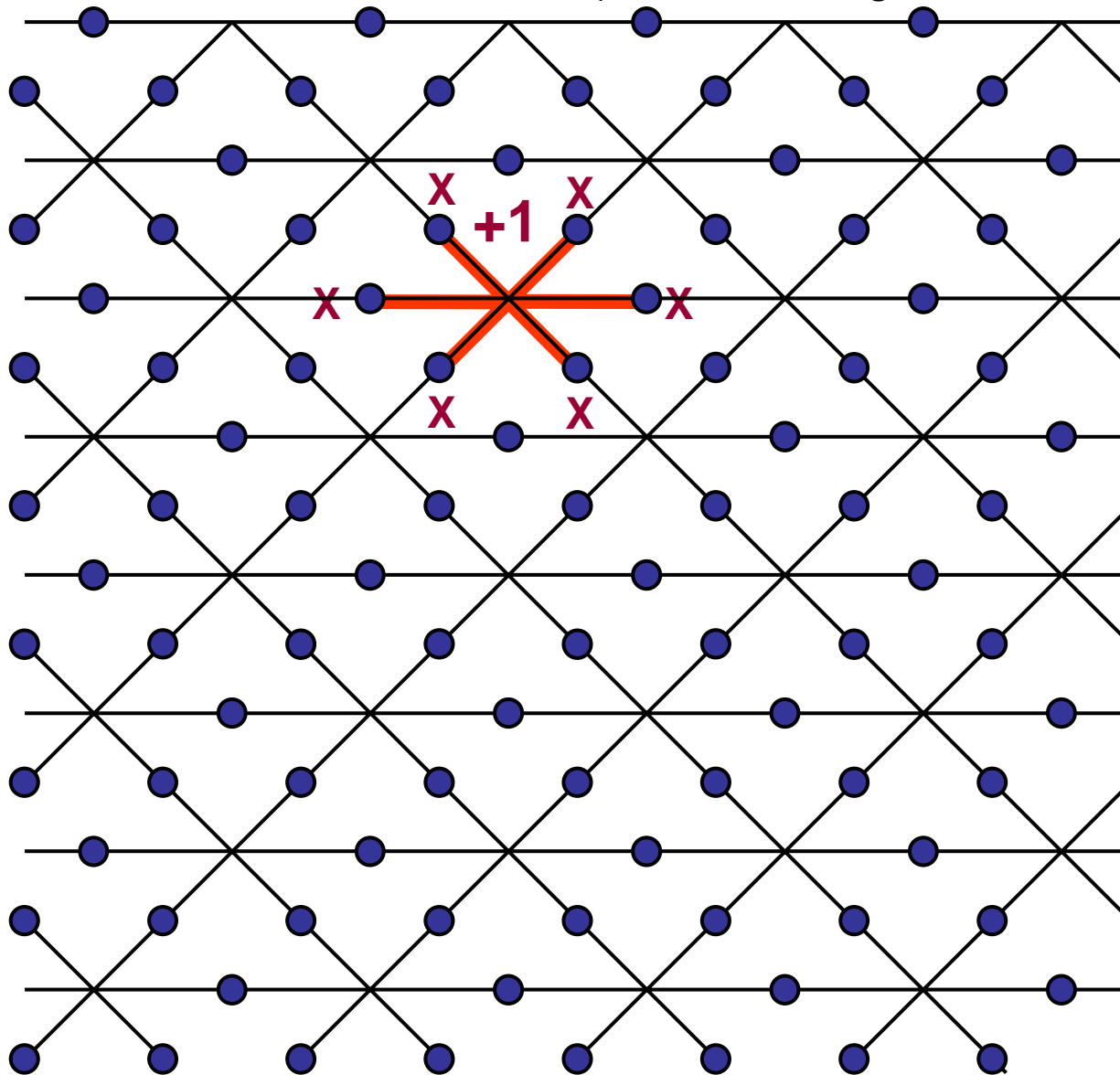
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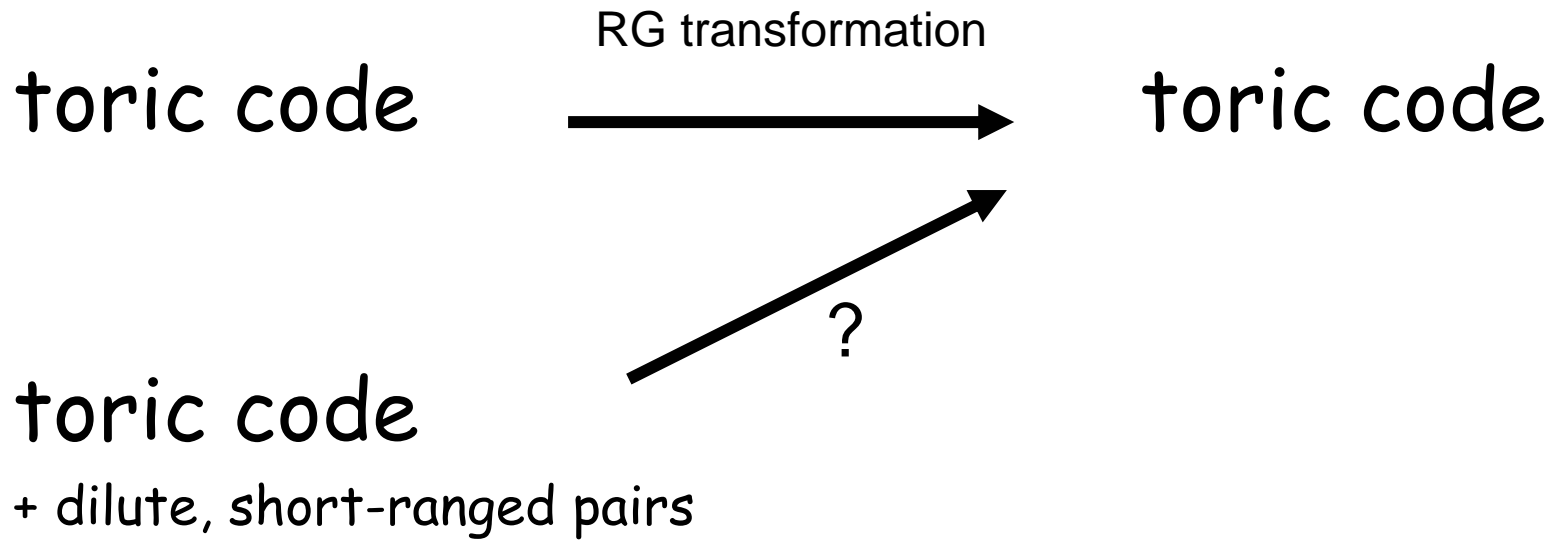
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# Conclusions

key insight from quantum information

- **MERA** (multi-scale entanglement renormalization ansatz)  $\longleftrightarrow$  **Quantum circuit** with bounded width causal cones
  - Renormalization group transformation
  - States can be efficiently prepared

- Good description of
  - Critical ground states  $\longrightarrow$  Quantum phase transitions
  - Kitaev's toric code  $\longrightarrow$  Topological order
  - Classical systems  $\longrightarrow$  Classical statistical mechanics