Exotic quantum phases of polar molecules in multicomponent systems

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Outline:

I. Introduction to dipolar atom/molecules
   and dipolar condensate.

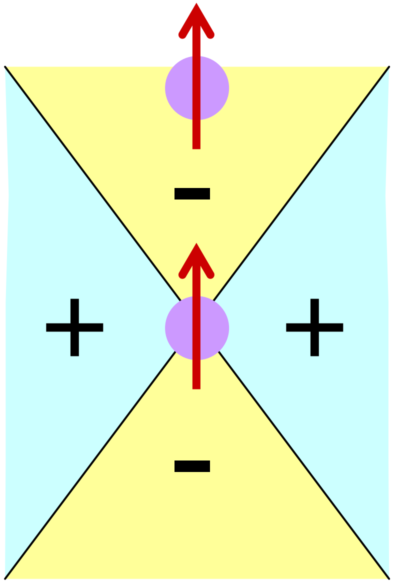
II. New states of dipoles in ***bilayer*** system
   --- Dimer SF and Schrodinger's cat state

III. New states of dipoles in ***multilayer*** system
   --- dipolar chain liquid

IV. New states of dipoles in ***double wire*** system
   --- Spin ferromagnetism

V. Summary

Why dipoles are interesting ?



$$V(r) = D^2 \frac{1-3\cos^2\theta}{r^3}$$

Special features:

(1) Anisotropic interaction

(2) Long-ranged interaction

Seeking for the exotic strongly correlated effects.

Dipoles in nature:

Most available Candidates:

(1) Heteronuclear molecules

(a) Direct molecules

p~ 1-5 D

(b) But difficult to be cooled

(Doyle, Meijer, DeMille etc.)

(2) Atoms with magnetic moment

Small moment $\mu \sim 6\mu_B$ (for Cr)

But it is now ready to go !

(Stuhler, Pfau, etc.)

$$p \sim 1\text{D},\ U_{dd} \sim 10\mu\text{K},\quad \mu = 1\mu_B,\ U_{dd} \sim 1\text{nK}$$

Artificial dipoles:

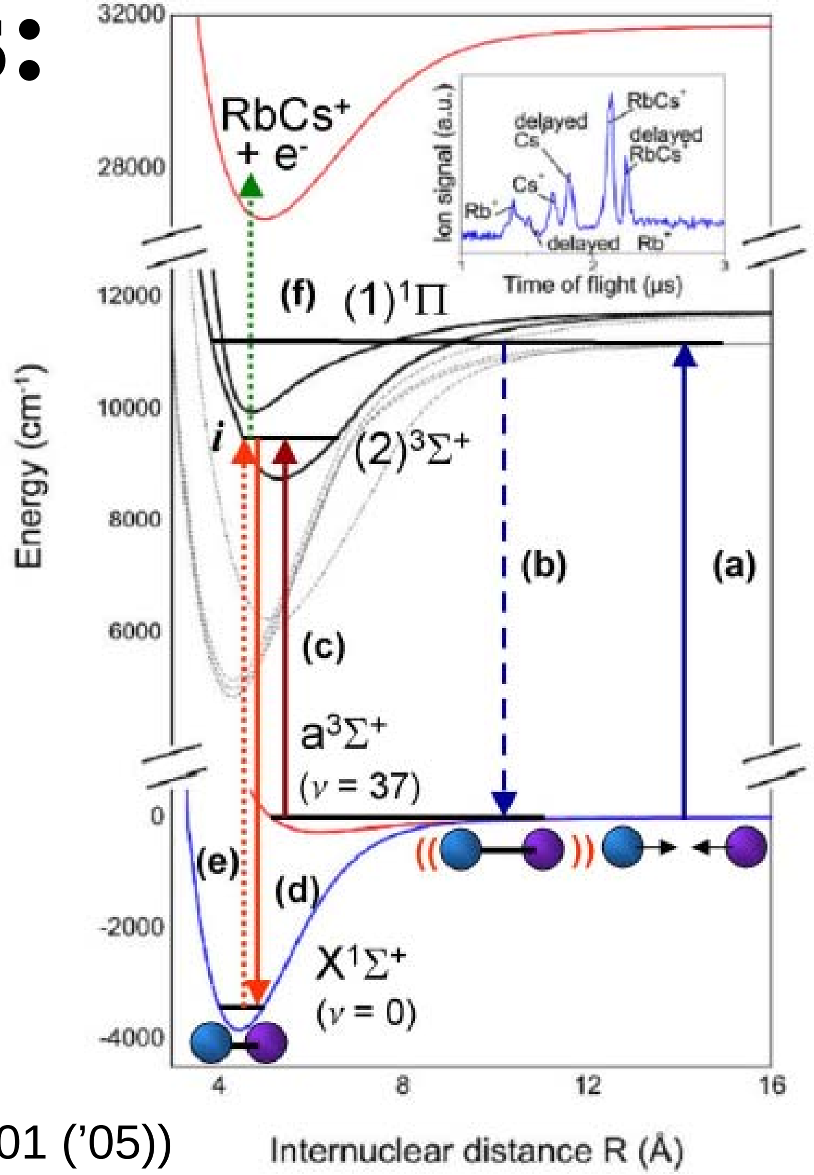
(1)Feshbach resonance

(KRb, JILA, ETH, etc.)

But not in ground state

weak dipole moment

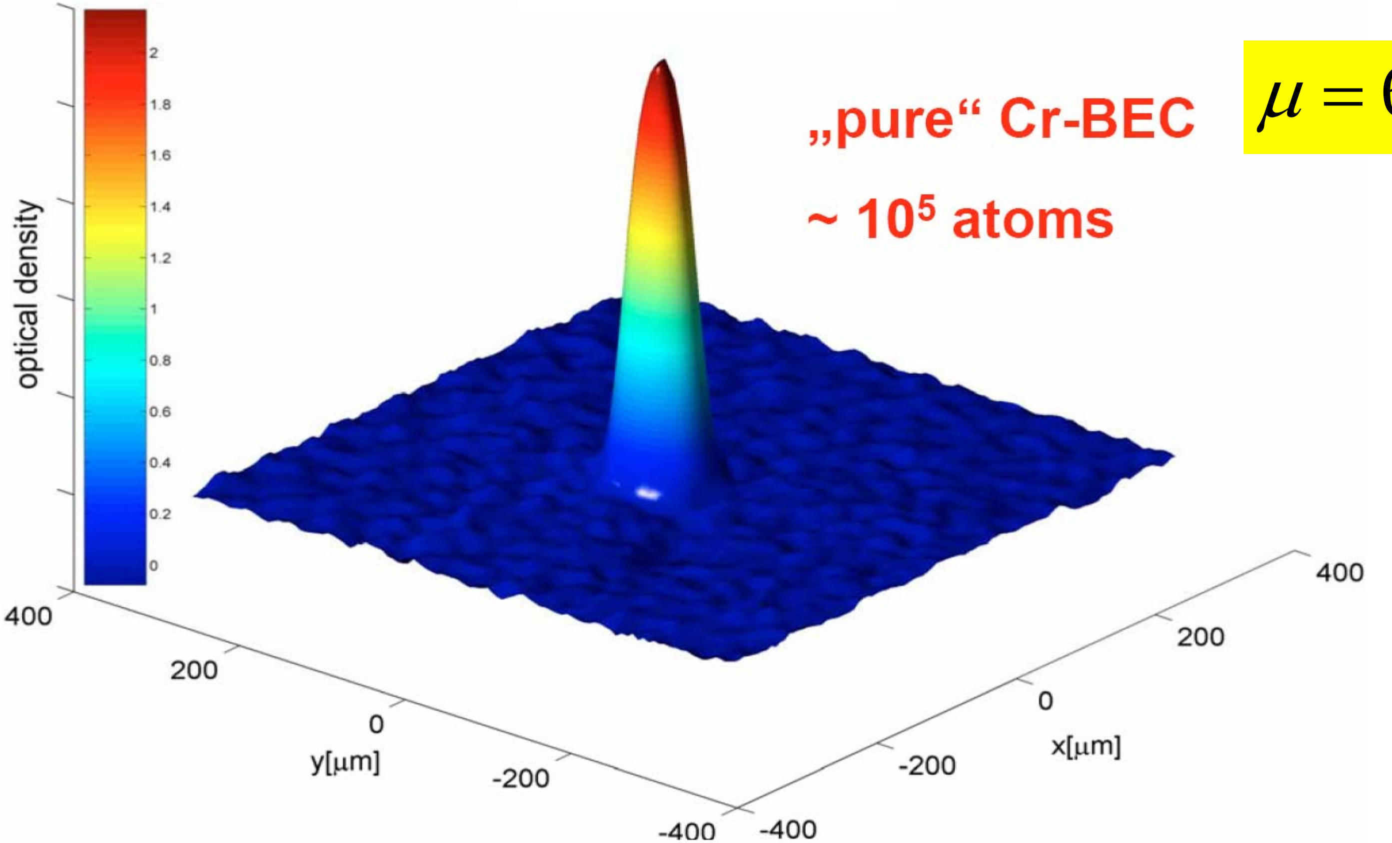
short life time



(J. Sage, et. al., PRL, **94**, 203001 ('05))

Quantum states in single component dipolar system

Condensate (superfluid)



$\mu = 6\mu_B$

A. Griesmaier et al., PRL 94, 160401 (2005)

T_c~700 nK

Theoretical work on dipolar BEC

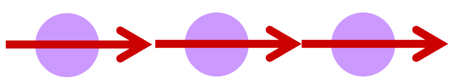
(You, Santos, Lewenstein, Zoller, Bohn, O' Dell, Cooper, etc…)

(1) Pseudo-potential and condensate profile:

$$V_{ps}(r) = \frac{4\pi\hbar^2 a_s}{m}\delta(r) + D^2 \frac{1-3\cos^2\theta}{r^3}$$

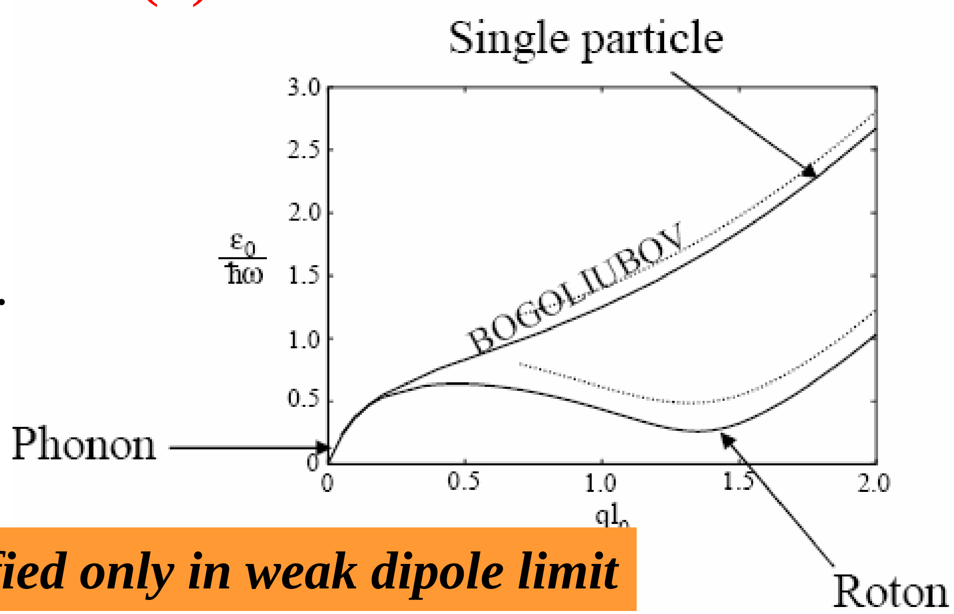
$$n(r) \sim n_0\left(1 - \frac{x^2+y^2}{R_\rho^2} - \frac{z^2}{R_z^2}\right)$$

(2) Phonons and instability in 3D



Instability toward collapse if short-range repulsion is weak.

(3) Roton minimum in 1D & 2D



(4) Vortex of dipolar BEC

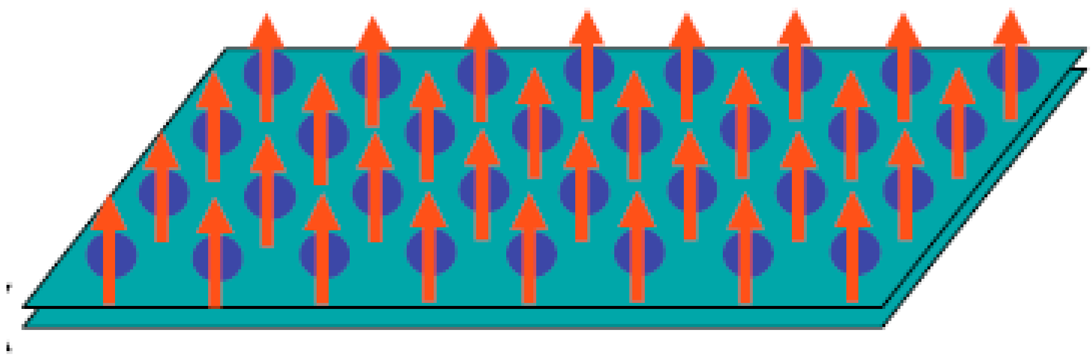
(5) Others….

All above works are justified only in weak dipole limit

Beyond Bose-Eistein condensation

1. Wigner crystal in 2D homogeneous space

(Zoller, Demler, etc…)

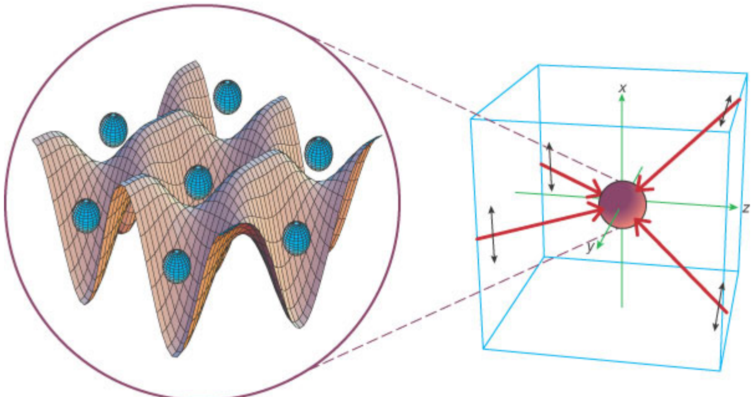


$$V(r) \sim \frac{D^2}{r^3} \Rightarrow \text{ the smaller } r_s = n^{-1/3}\text{, the stronger interaction.}$$

Wigner crystal occurs when $\dfrac{V(r)}{\hbar^2 / 2mr^2} = \dfrac{2mD^2}{\hbar^2 r} \sim 1$

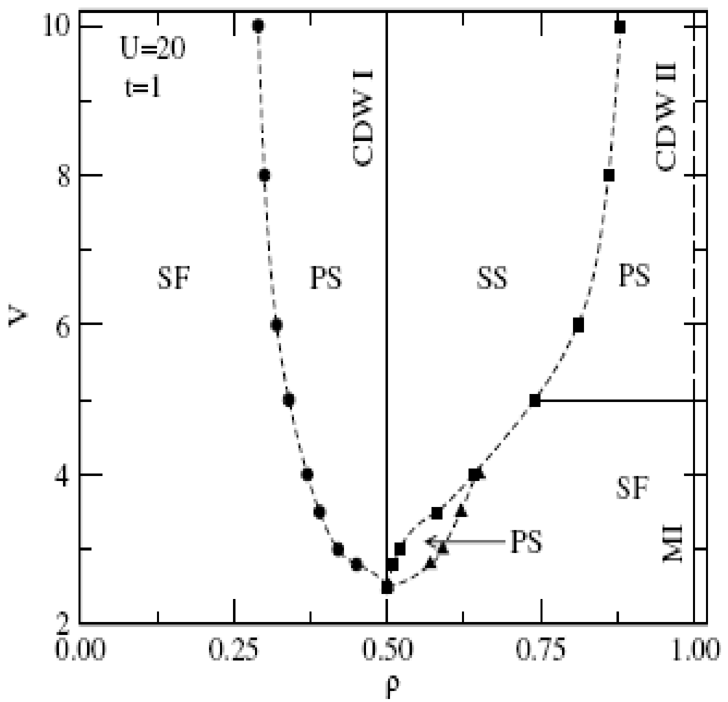
2. Supersolid state in 2D optical lattice

(Troyer, Prokofev, Das Sarma, etc…)



Optical lattice

$$H = -t\sum_{\langle i,j\rangle}(a_i^\dagger a_j + a_j^\dagger a_i) - \mu\sum_i n_i + V\sum_{\langle i,j\rangle} n_i n_j + \frac{U}{2}\sum_i n_i(n_i - 1),$$

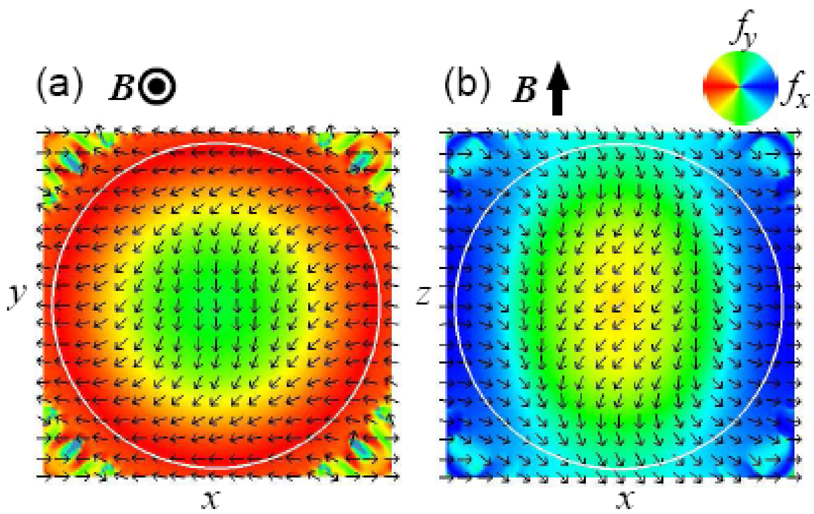


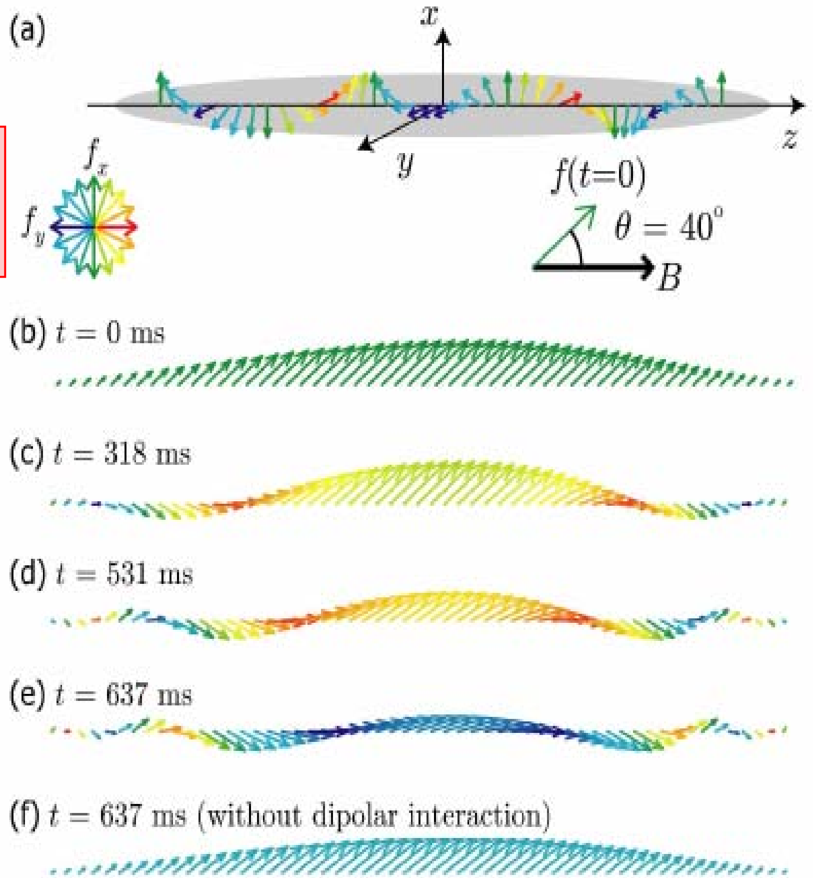
PRL 94, 207202 (2005)

3. Spin texture due to spin-orbital interaction

(Ueda, Ho, et al.)

$$H_{\text{int}} = J\mathbf{S}_1 \bullet \mathbf{S}_2 + D^2\gamma^2 \frac{\mathbf{S}_1 \bullet \mathbf{S}_2 - 3(\mathbf{S}_1 \bullet \mathbf{r}_{12})(\mathbf{S}_2 \bullet \mathbf{r}_{12})}{r_{12}^3}$$

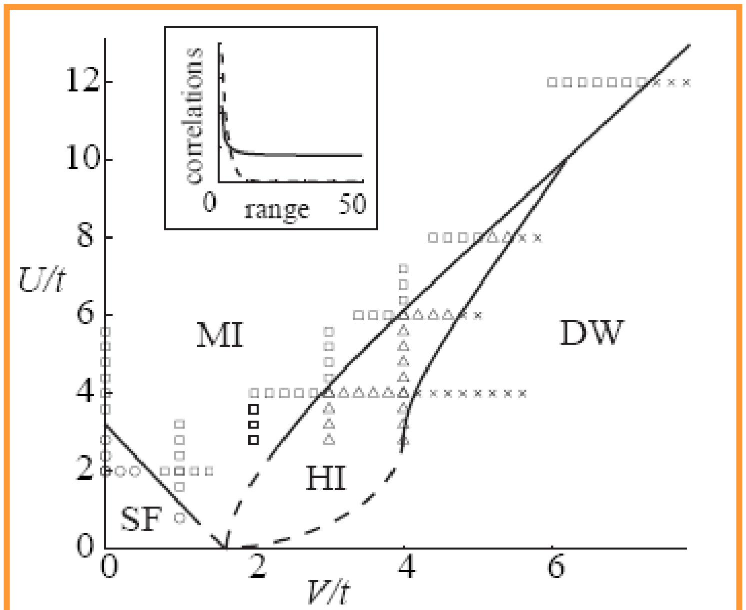




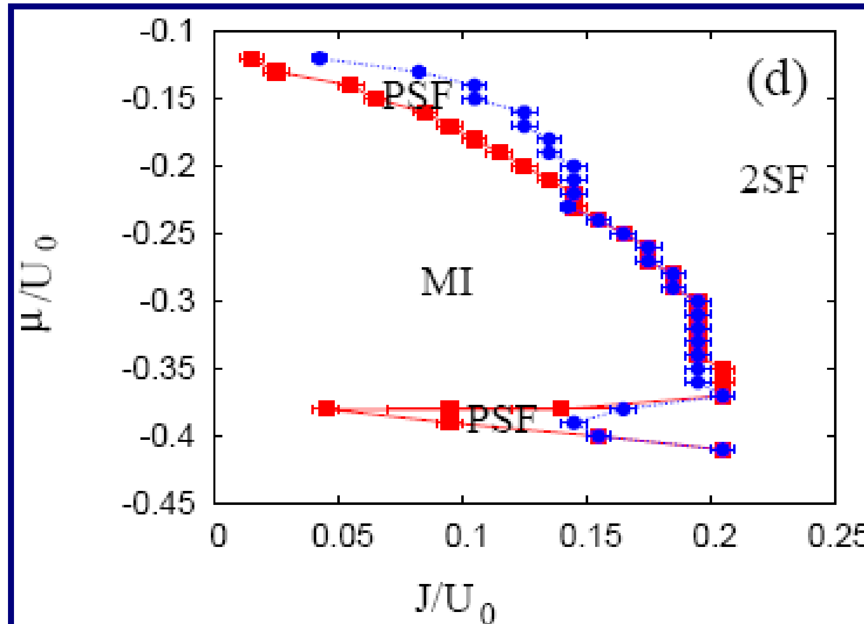
4. Dipoles near Mott state --- 1D system

(Altman, Santos et al.)

String state in 1D optical lattice



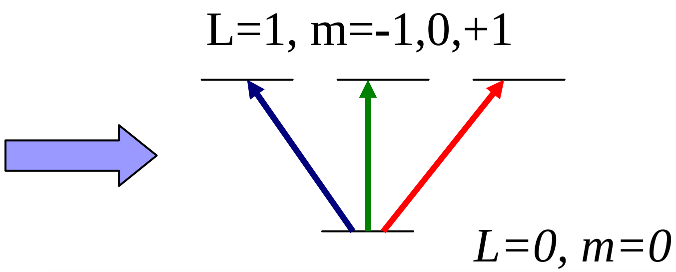
Reentrant MI phase in 1D ladder

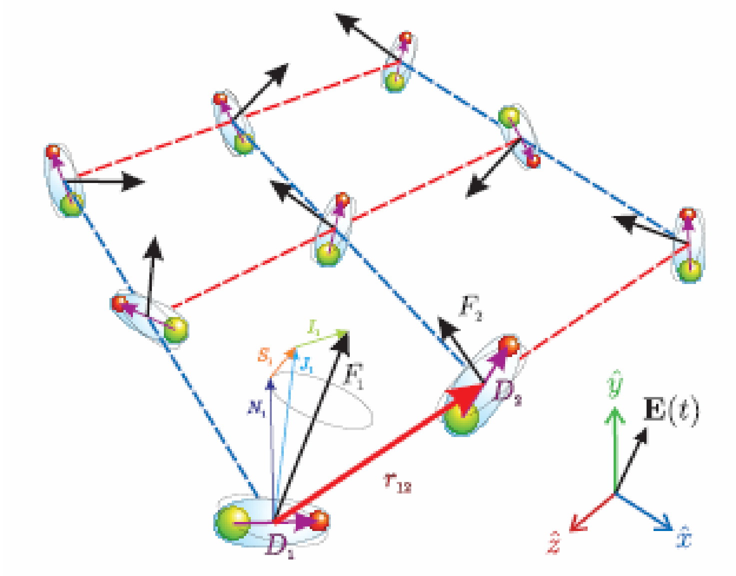


$$R_{\text{string}}\left(|i-j|\right) = \left\langle \delta n_i e^{i\pi \sum_{k=i}^{j} \delta n_k} \delta n_j \right\rangle$$

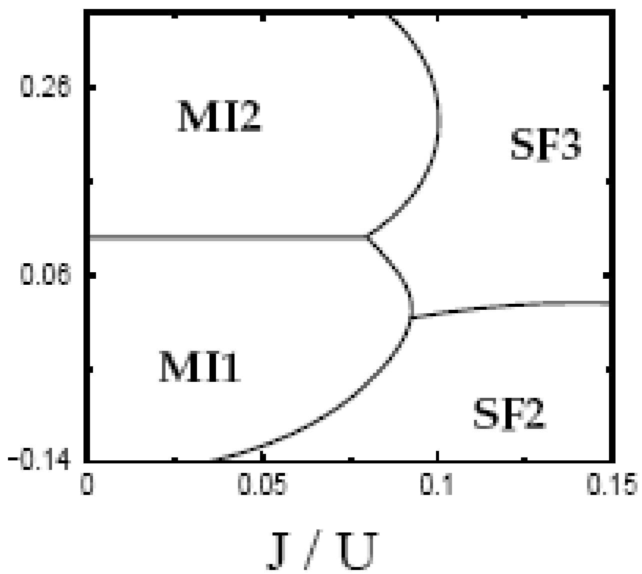
4. Laser induced dipoles in optical lattice

(Zoller, Demler, et al.)





Exotic SF-MI phase transition



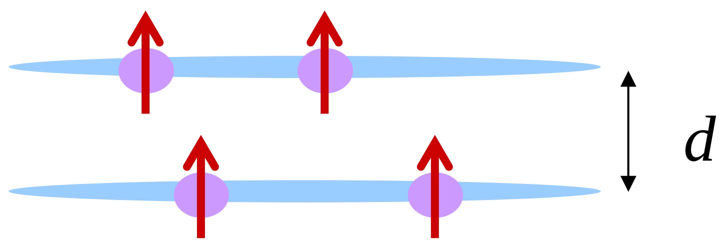
Model S=1/2 and S=1 system

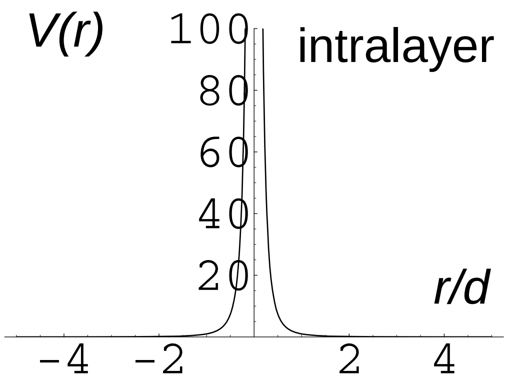
Quantum states of dipoles in double layer systems

Reference:

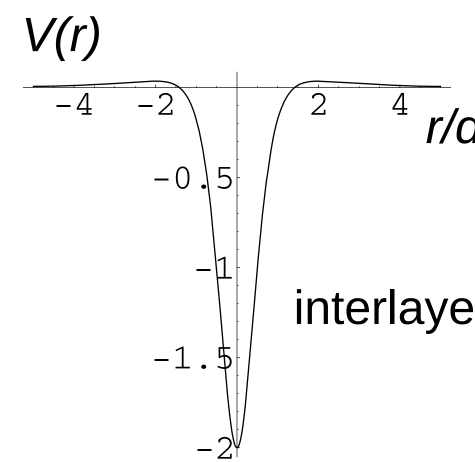
1. DWW, Phys. Rev. Lett. **98**, 060403 (2007).

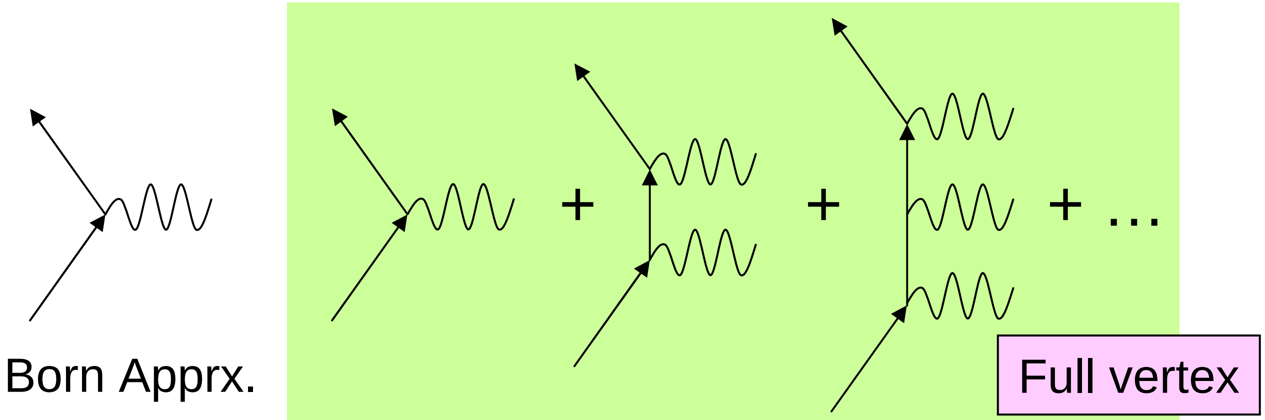
Dipole interaction in bilayer





Within Born approx. intralayer interaction is always much larger than the interlayer. However, this is invalid in 2D system.





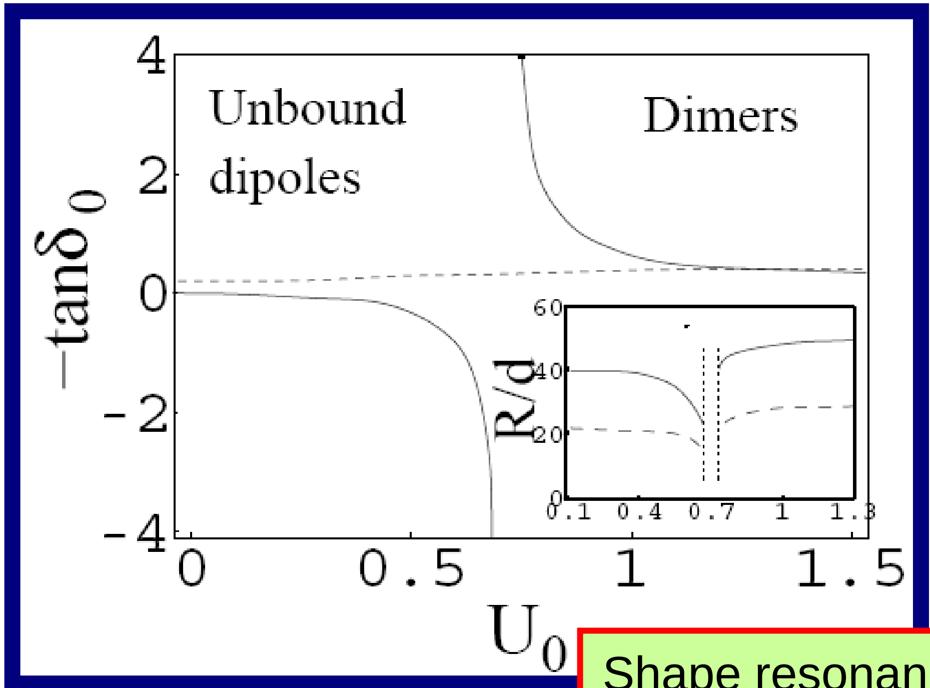
Zero-energy resonance

Pseudo-potential approach (similar to 3D atom gas):

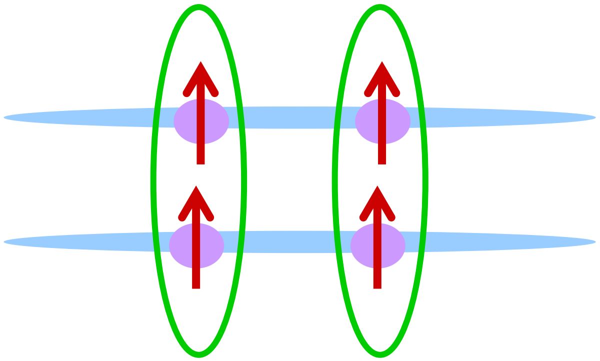
$$\mathcal{V}_{\rm ps}^{(0)/(1)}(\mathbf{r}_\perp) = -\frac{2\hbar^2}{\mu}\tan\delta_0^{(0)/(1)}(k)\cdot\delta(\mathbf{r}_\perp)$$

Correct in the low energy limit
(Huang and Yang, PR 105, 767 (1957))

Phase shift for intra(0)- and inter(1)-layer scattering



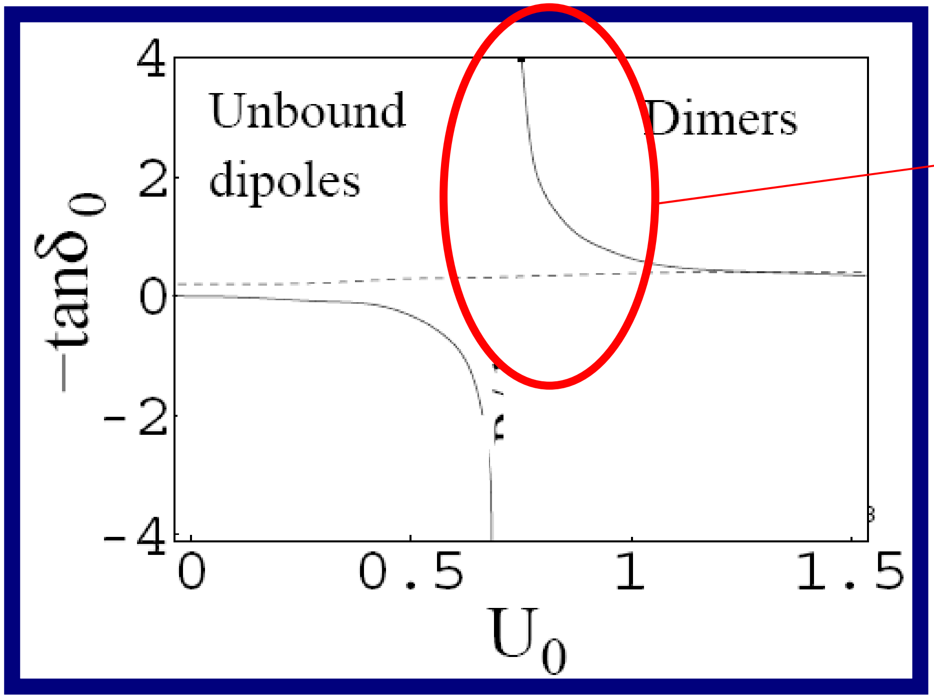
$$U_0 \equiv \frac{mD^2}{\hbar^2 d} = \frac{E_{\rm int}}{E_K}$$



Shape resonance in 2D

Schrodinger Cat State

Superposition of two macroscopic coherent states



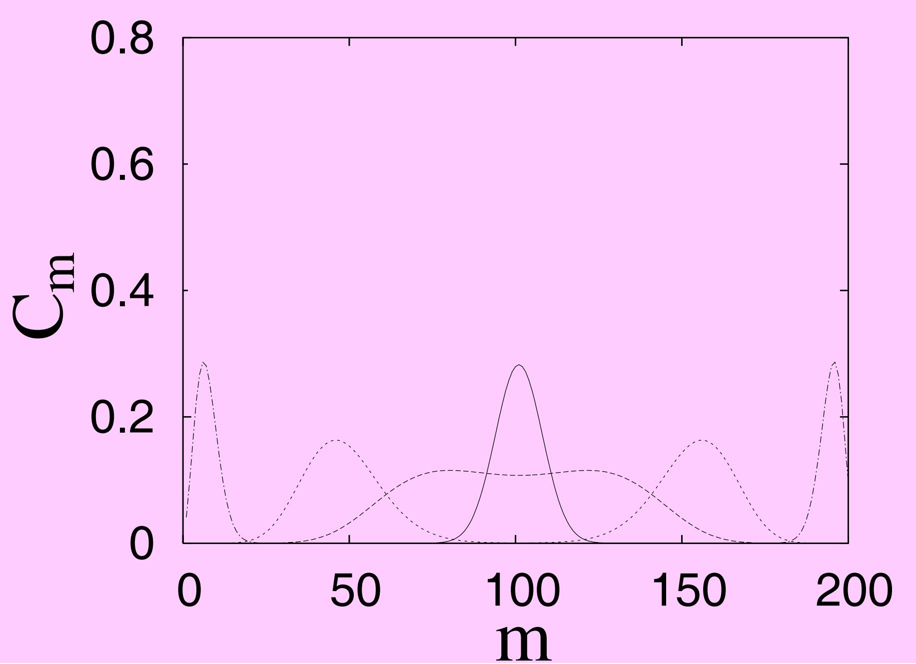
$$g_1 > g_0 > 0$$

Exotic interaction not available
In solid state systems.

Schrodinger Cat State

$$H = -t\left(\hat{a}_0^\dagger \hat{b}_0 + \hat{b}_0^\dagger \hat{a}_0\right) + \frac{g_0}{2N}\left[\hat{n}_a^2 + \hat{n}_b^2\right] + \frac{g_1}{N}\hat{n}_a\hat{n}_b$$
$$= \frac{g_0}{2N}(2N)^2 - t\left(\hat{a}_0^\dagger \hat{b}_0 + \hat{b}_0^\dagger \hat{a}_0\right) + \frac{\Delta g}{N}\hat{n}_a\hat{n}_b,$$

$$\left|\psi_{cat}\right\rangle = \sum_{m=0}^{2N} C_m \left|m, 2N-m\right\rangle$$



$$E_{BEC} = g_0 N^2 + g_1 N^2$$

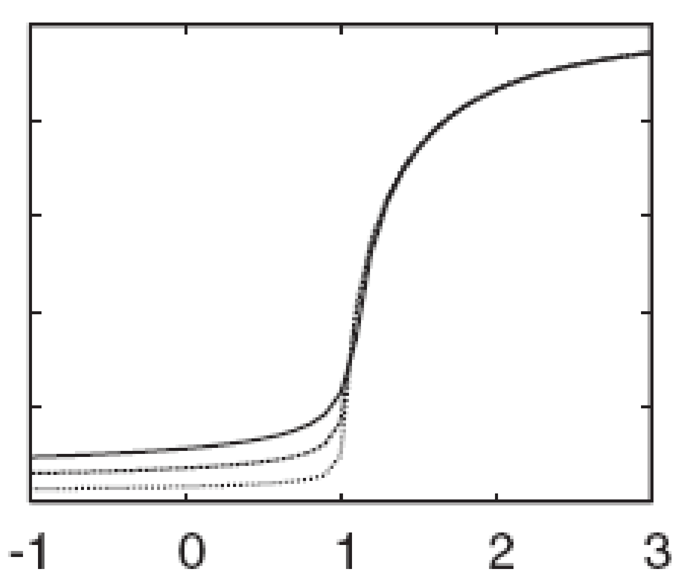
$$E_{cat} = \frac{g_0}{2}(2N)^2 = 2g_0 N^2$$

$$E_{cat} - E_{BEC} = (g_1 - g_0)N^2$$

$$\left|\psi_{cat}\right\rangle = \frac{1}{\sqrt{2}}\left(\left(a^+\right)^{2N} + \left(b^+\right)^{2N}\right)\left|0\right\rangle$$

Phase transition to Cat State

Number variation



$$\sqrt{\langle \Delta N^2 \rangle / 2N}$$

$$\Delta g / t$$

$$\hat{S}_{\pm} = \frac{1}{2}\left(a_0^+ b_0 \pm b_0^+ a_0\right),$$

$$\hat{S}_z = \frac{1}{2}\left(a_0^+ a_0 - b_0^+ b_0\right),$$

$$\mathbf{S}^2 = \frac{1}{4}(n_a + n_b)(n_a + n_b + 2)$$

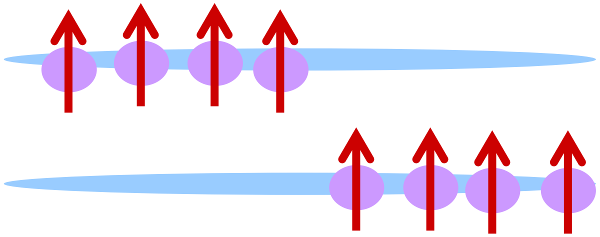
$$H_{eff} = -2t\hat{S}_x - \frac{\Delta g}{N}\hat{S}_z^2$$

$$E/N = -2t\sin\theta\cos\phi - \Delta g\cos^2\theta$$

$$= -2t + (t - \Delta g)(\theta - \pi/2) + \tfrac{1}{3}(\Delta g - t/4)(\theta - \pi/2)^2 + \ldots$$

Q&A about Cat State

Q: Phase separation ?



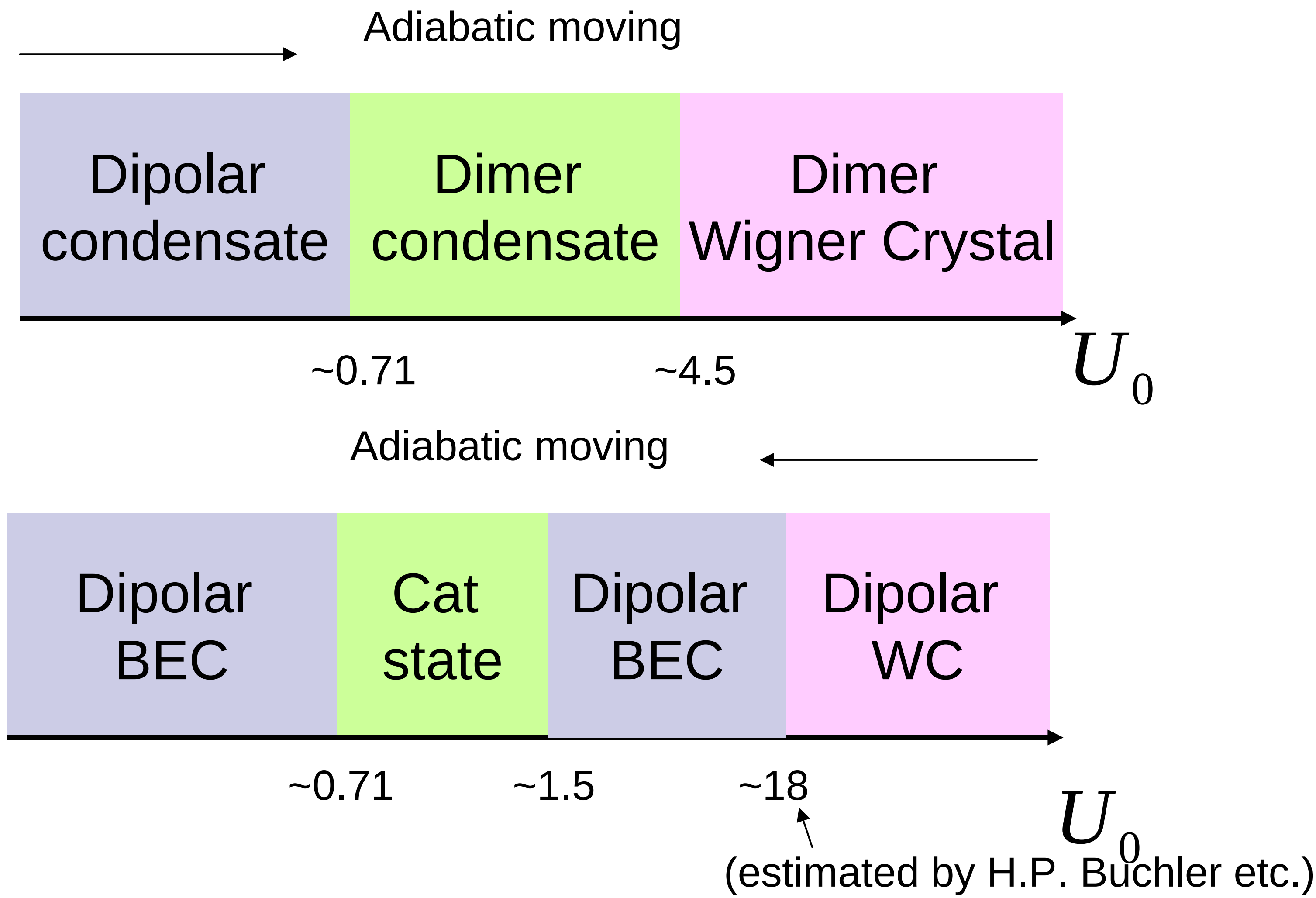
A: No, because it causes too much surface energy. Besides, interlayer pseudo-potential also becomes weaker in large *k*.

Q: How to observe ?

$$|\psi_{cat}\rangle = \frac{1}{\sqrt{2}}\left(\left(a^{+}\right)^{2N} + e^{i\phi}\left(b^{+}\right)^{2N}\right)|0\rangle$$

A: Since quantum measurement will break the Wavefunction into one of the two macroscopic state, ***no intereference pattern*** even in single shot TOF.

Phase diagram for bilayer



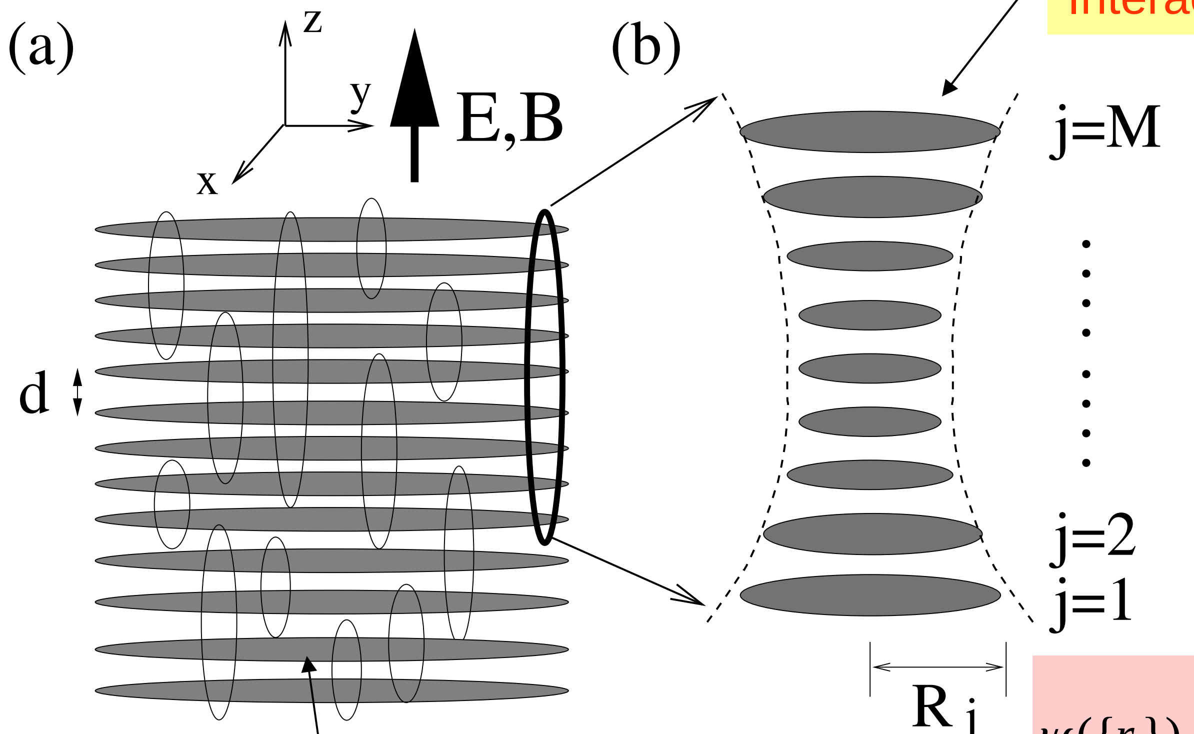
Quantum states of dipoles in multi-layer systems

Reference:

DWW, M. D. Lukin, and E. Demler, Phys. Rev. Lett. **97**, 180413 (2006).

Chaining in multilayer system ?

Larger size due to weaker Interaction from other dipoles



Dipolar chain liquid (DCL)

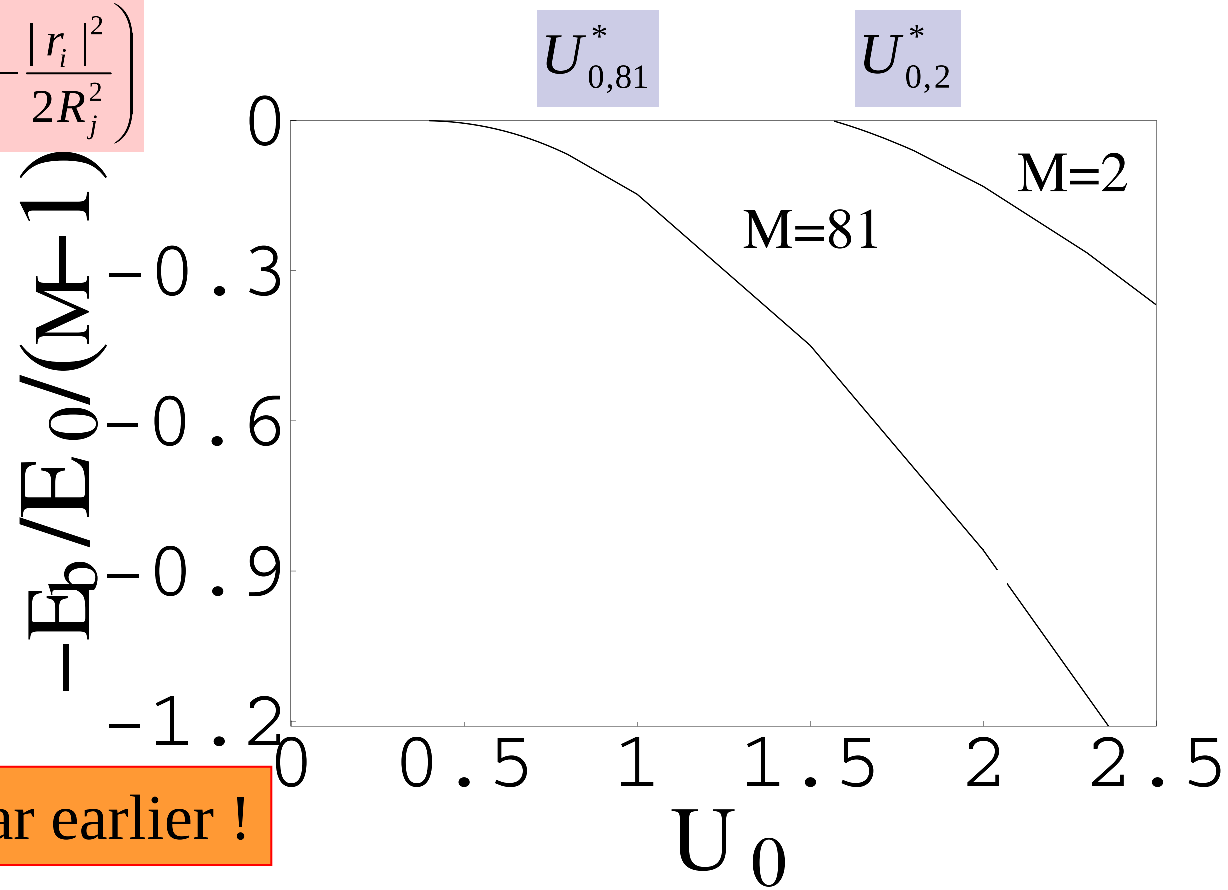
Trial wavefunction

$$\psi(\{r_i\}) = \prod_{j=1} \frac{1}{\sqrt{\pi}R_j} \exp\left(-\frac{|r_i|^2}{2R_j^2}\right)$$

Binding energy of a single chain

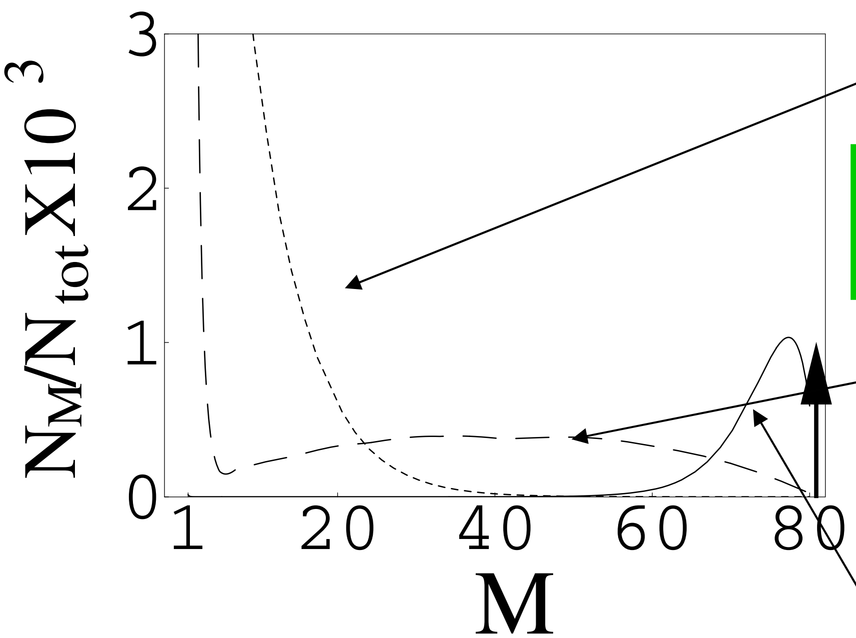
Trial wavefunction

$$\psi(\{r_i\}) = \prod_{j=1} \frac{1}{\sqrt{\pi} R_j} \exp\left(-\frac{|r_i|^2}{2R_j^2} \right)$$



Longer chains appear earlier !

Nonmonotonic distribution of chains in finite temperature



High temperature:
2D version of Saha’s equation

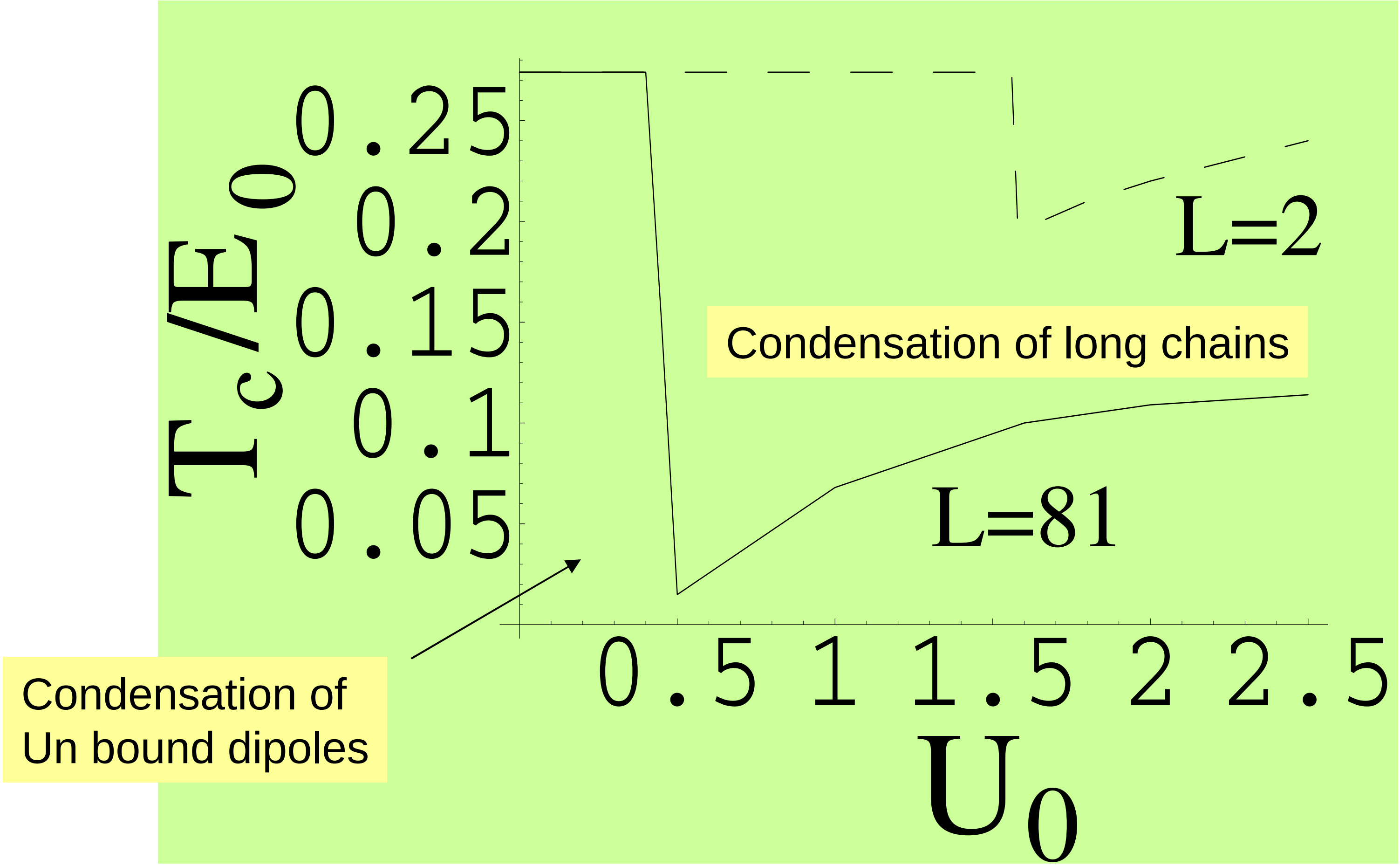
$$N_M \sim \left(\frac{N_{\rm tot}}{L}\right)^M \left(\frac{\hbar\omega}{k_B T}\right)^{2M-2} e^{-E_{b,M}/k_B T}$$

Mediate temperature:
nonmonotonic ditribution

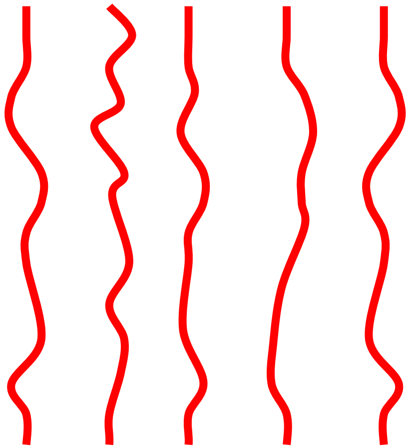
Low temperature:
Condensation of the longest chains

Dipolar chain liquid (DCL)

Condensate Tc of chain liquid



Excitations of dipolar chains



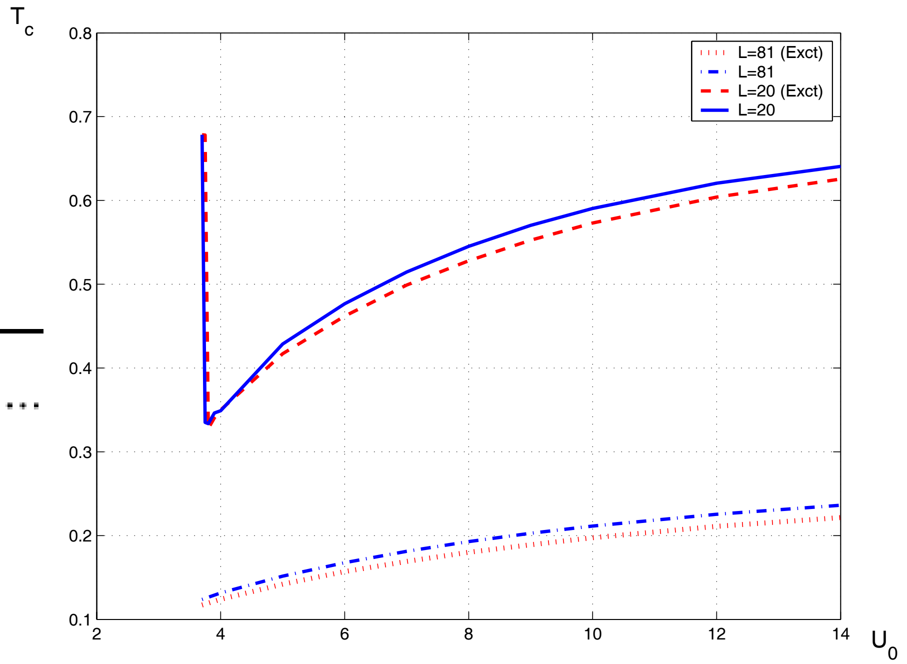
Expansion in large dipole moment limit:

$$H_{dd} = \frac{D^2}{d^3}\{-2\sum_{i<j}^{M}\frac{1}{|i-j|^3} + 6\sum_{i<j}^{M}\frac{1}{|i-j|^5}\frac{s_i^2+s_j^2-2s_is_j\langle s_i,s_j\rangle}{d^2}\}$$

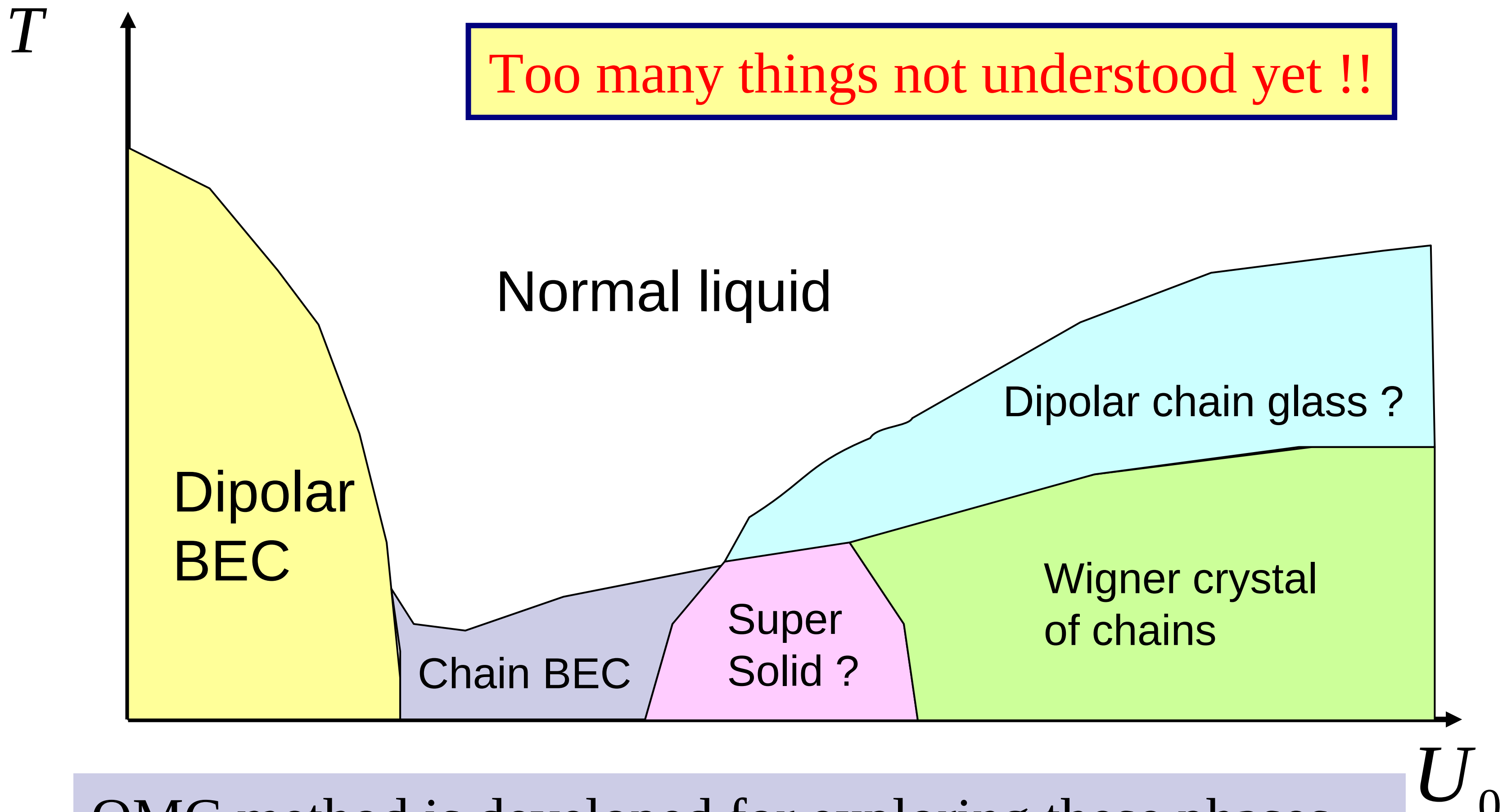
Excitation spectrum:

$$\omega(k) = \frac{2c}{d}\sqrt{sin^2(\frac{kd}{2}) + \frac{1}{2^5}sin^2(kd) + ...}$$

$$c = \sqrt{12D^2/md^3}$$



Possible phase diagram for multilayer systems



QMC method is developed for exploring these phases…

Quantum states of dipoles in double wire system

(ongoing project with Chi-Ming Chang and Po-Chung Chen)

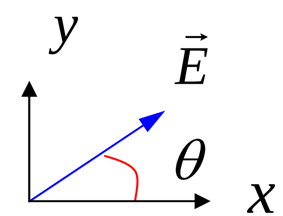
Fermionic polar molecules are loaded in the double wires potential.

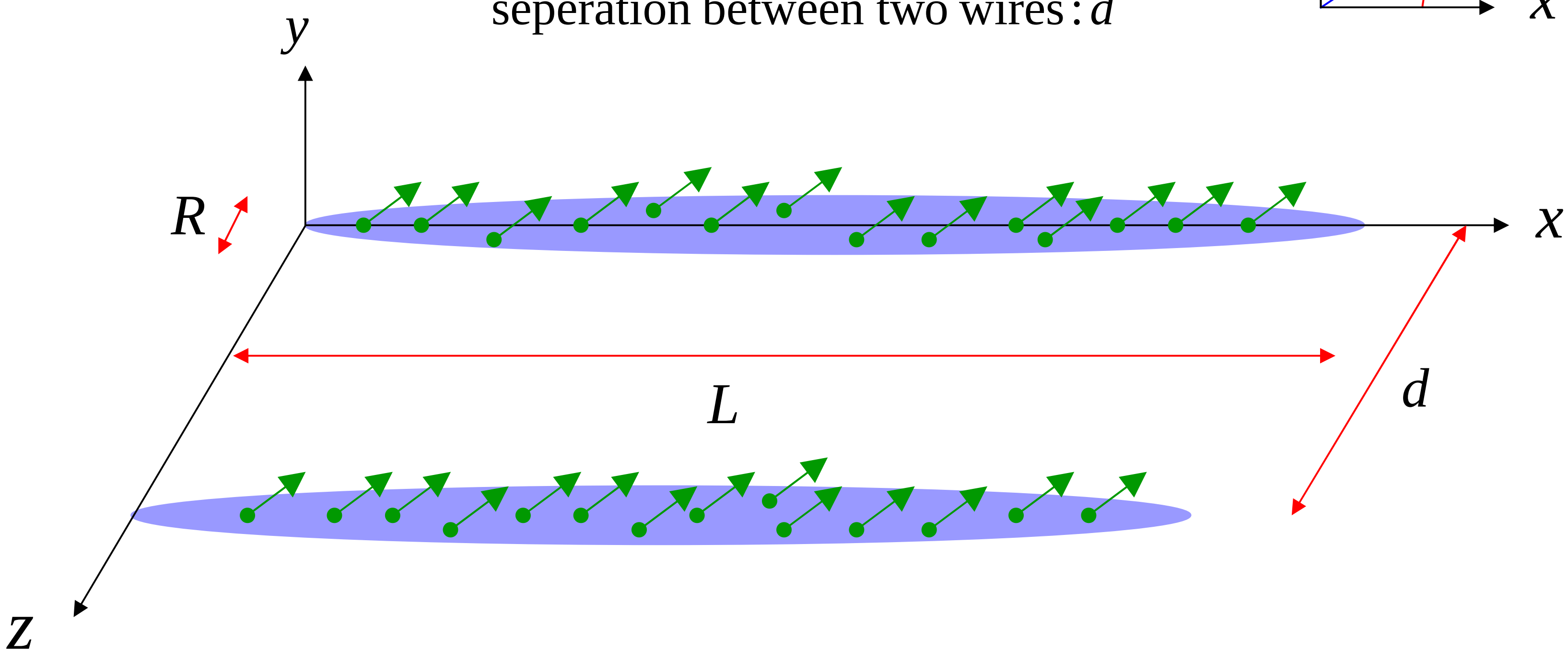
$R << d << L$

wire radius : R

wire length : L

seperation between two wires : d





Why this system is interesting ?

We may have : **Interwire interaction > intrawire interaction**

intrawire interaction: $V_{//} \approx \frac{D^2}{\left|x_1 - x_2\right|^3}\left(1 - 3\cos^2\theta\right)$

interwire interaction: $V_{\perp} \approx \frac{D^2}{\left[d^2 + \left(x_1 - x_2\right)^2\right]^{3/2}} \frac{d^2 + \left(x_1 - x_2\right)^2\left(1 - 3\cos^2\theta\right)}{d^2 + \left(x_1 - x_2\right)^2}$

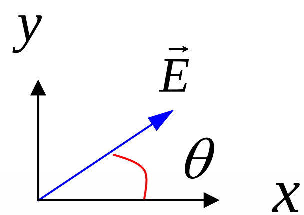
where $x_1 - x_2$ is the seperaction of the molecular in x direction.

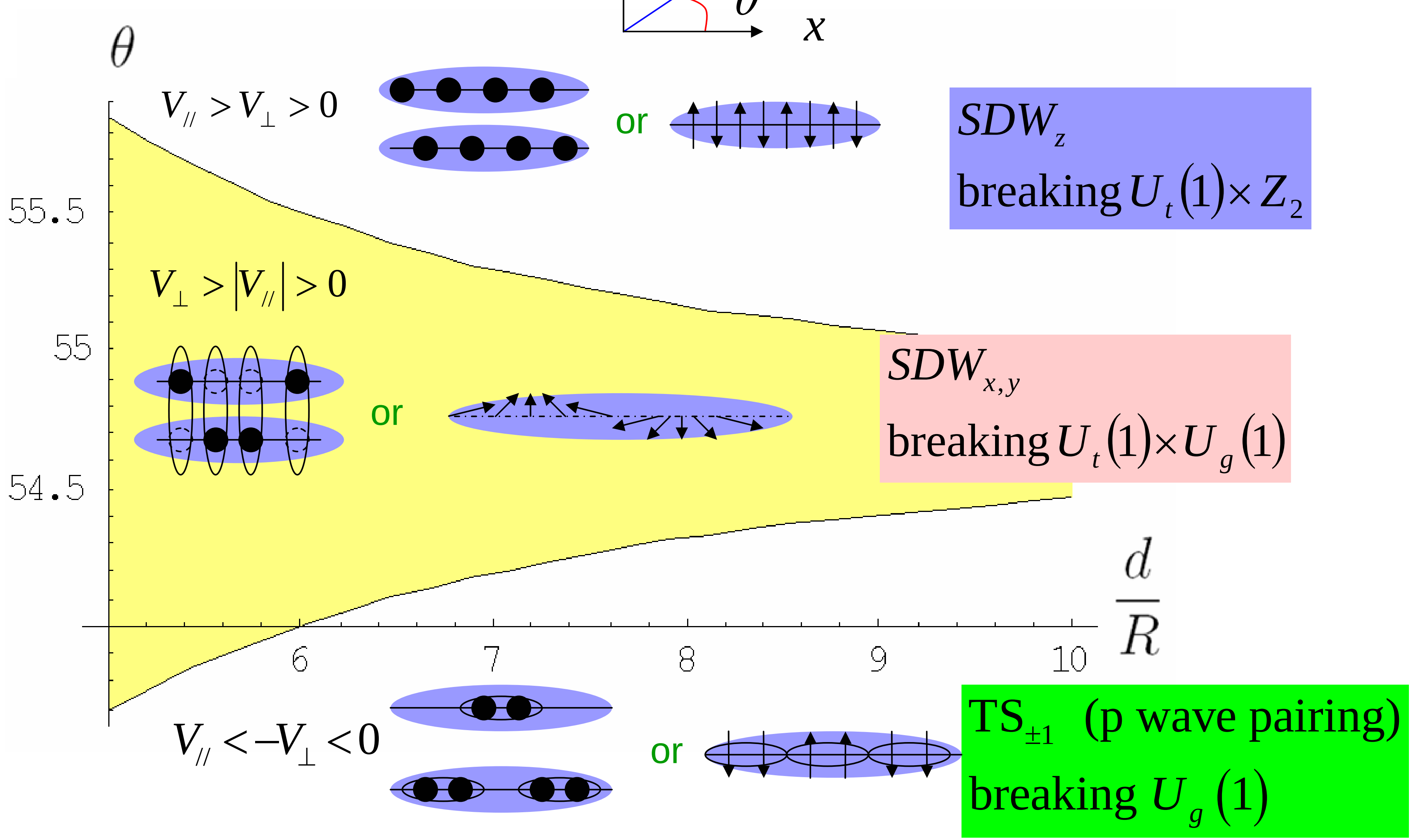
when $\cos^2\theta = \frac{1}{3}, V_{//} = 0, V_{\perp} > 0.$

Such exotic interaction can not be realized in semiconductor wires.

Phase diagram

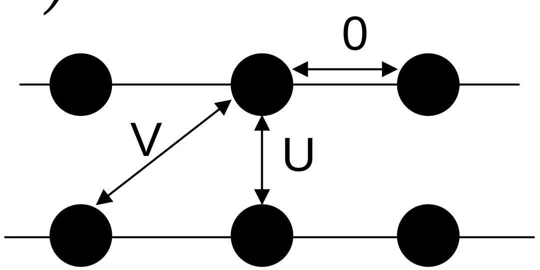
$U_t(1)$: tranlation in x direction
$U_g(1)$: gauge symmetry
Z_2 : two wires symmetry





Strong coupling limit $\left(\theta \sim 55^\circ\right)$

For simplicity, we consider molecules in 1D optical lattice



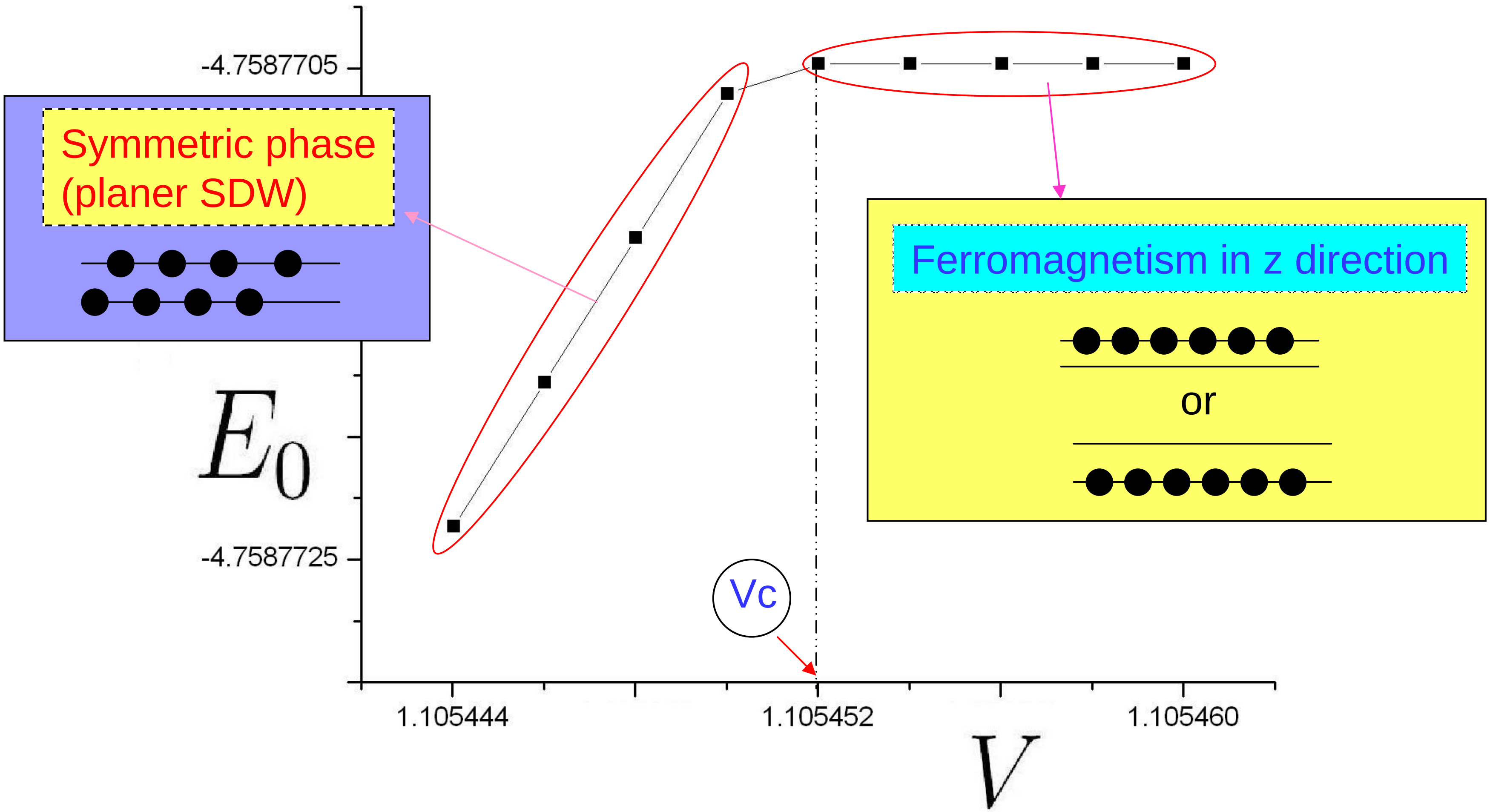
$$H = -t\sum_{i=1}^{L-1}\left(a_i^\dagger a_{i+1} + b_i^\dagger b_{i+1} + h.c.\right) + U\sum_{i=1}^{L} n_i^\uparrow n_i^\downarrow + \frac{V}{2}\sum_{i=1}^{L-1}\left(n_i^\uparrow n_{i+1}^\downarrow + n_i^\downarrow n_{i+1}^\uparrow\right)$$

not extended Hubbard model!!

We use **exact diagonalization** to study the ground state energy.

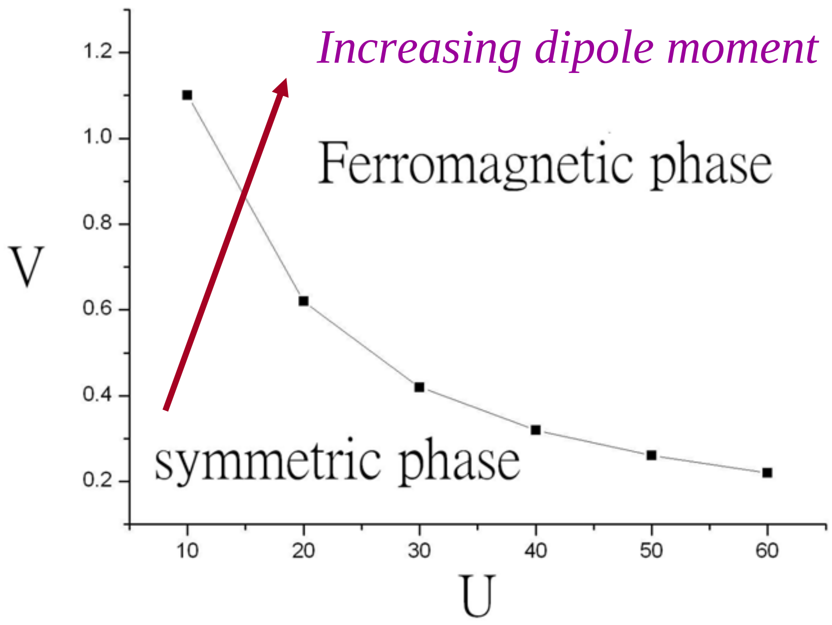
First order phase transition to Ferromagnetic Phase

Fixed L=8,N=4,t=1,U=10

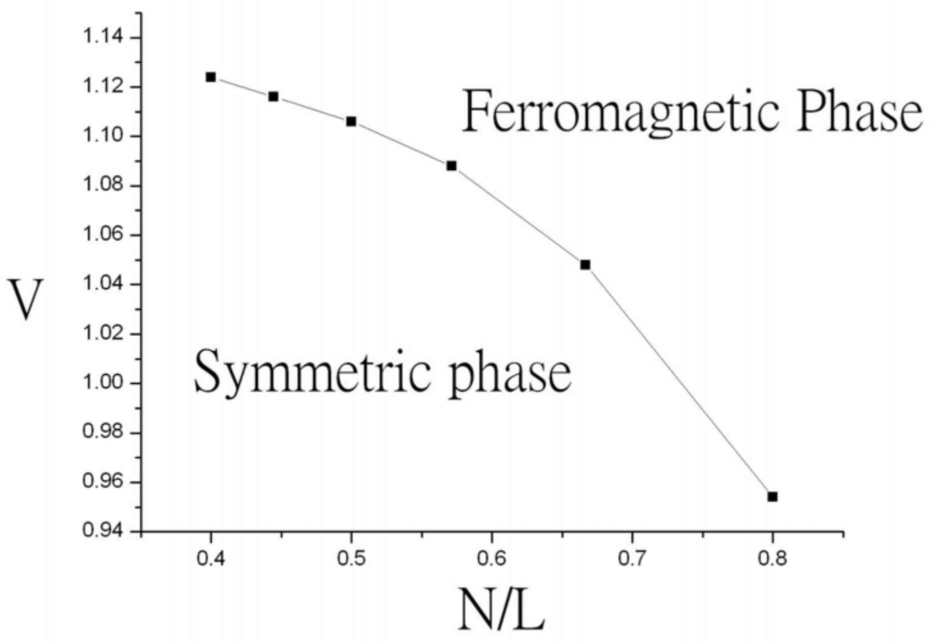


Phase diagram

L=8,N=4,t=-1



N=4,t=1,U=10

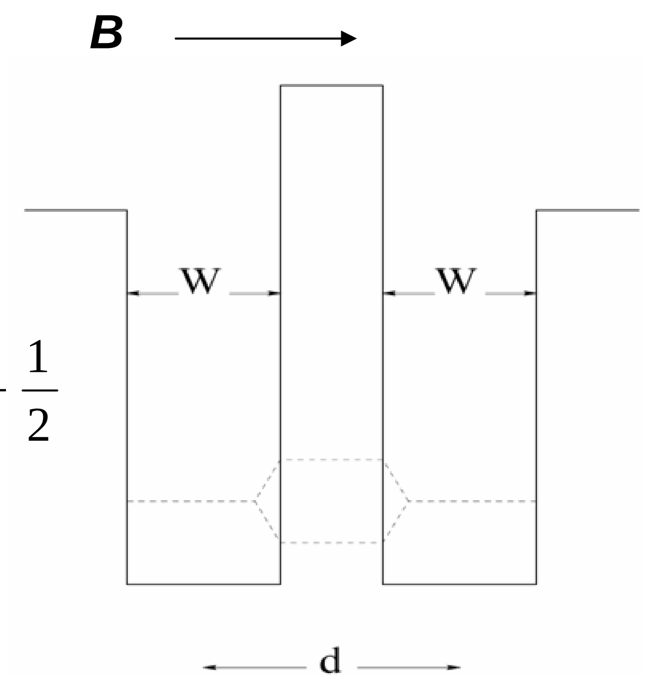


DMRG calculation for large N system is in process, but **we believe our results are still qualitatively correct even in the thermodynamic limit.**

Future study: Can we have 1D planar Ferromagnetism in double wire systems ?

Also called interlayer coherence or exciton condensate

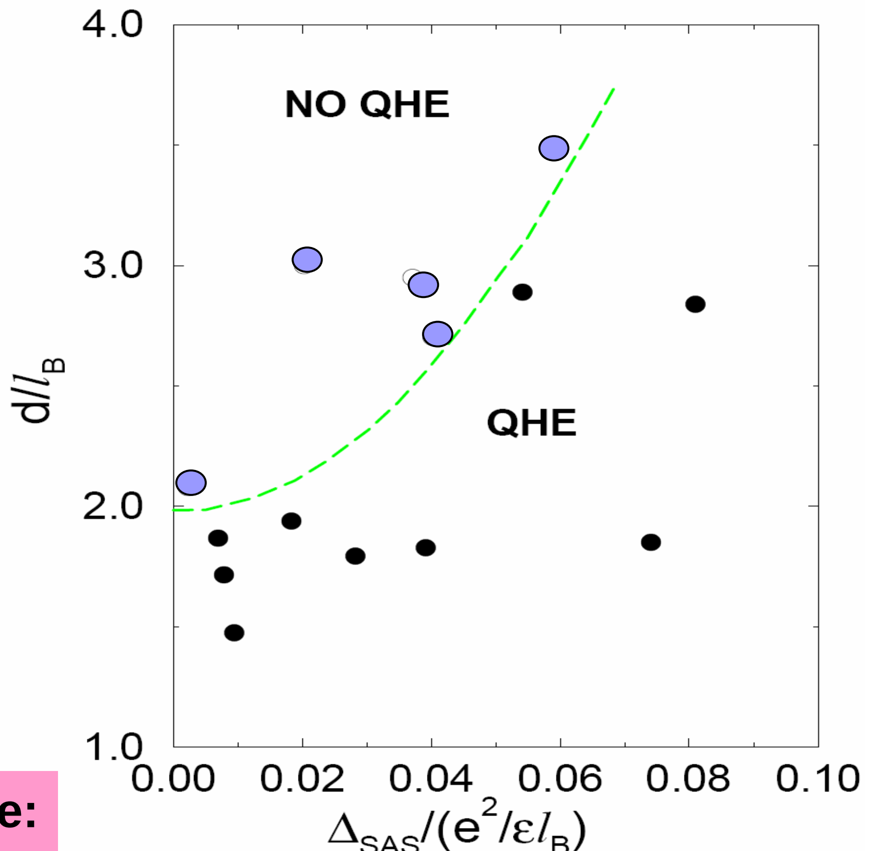
$$\nu = \frac{1}{2} + \frac{1}{2}$$



Double well potential

Spontaneous interlayer coherence:

$$\sum_k \left\langle a_k^+ b_k \right\rangle \neq 0, \text{ even } t_\perp \to 0$$



J. Eisenstein et al.

Summary:

Ultracold dipolar atoms/molecules are fantastic systems for studying interesting many-body phenomena.

Multicomponent dipolar systems may mimic some strongly correlated systems in solid states.

Theoretically proposed quantum states are expected to be realized and observed experimentally within a few years.