

# Unconventional Magnetism: Dynamic Generation of Spin-orbit Coupling and Electron Liquid Crystal States

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C. Wu and S. C. Zhang, PRL 93, 36403 (2004);

C. Wu, K. Sun, E. Fradkin, and S. C. Zhang, PRB 75, 115103 (2007).

# Collaborators

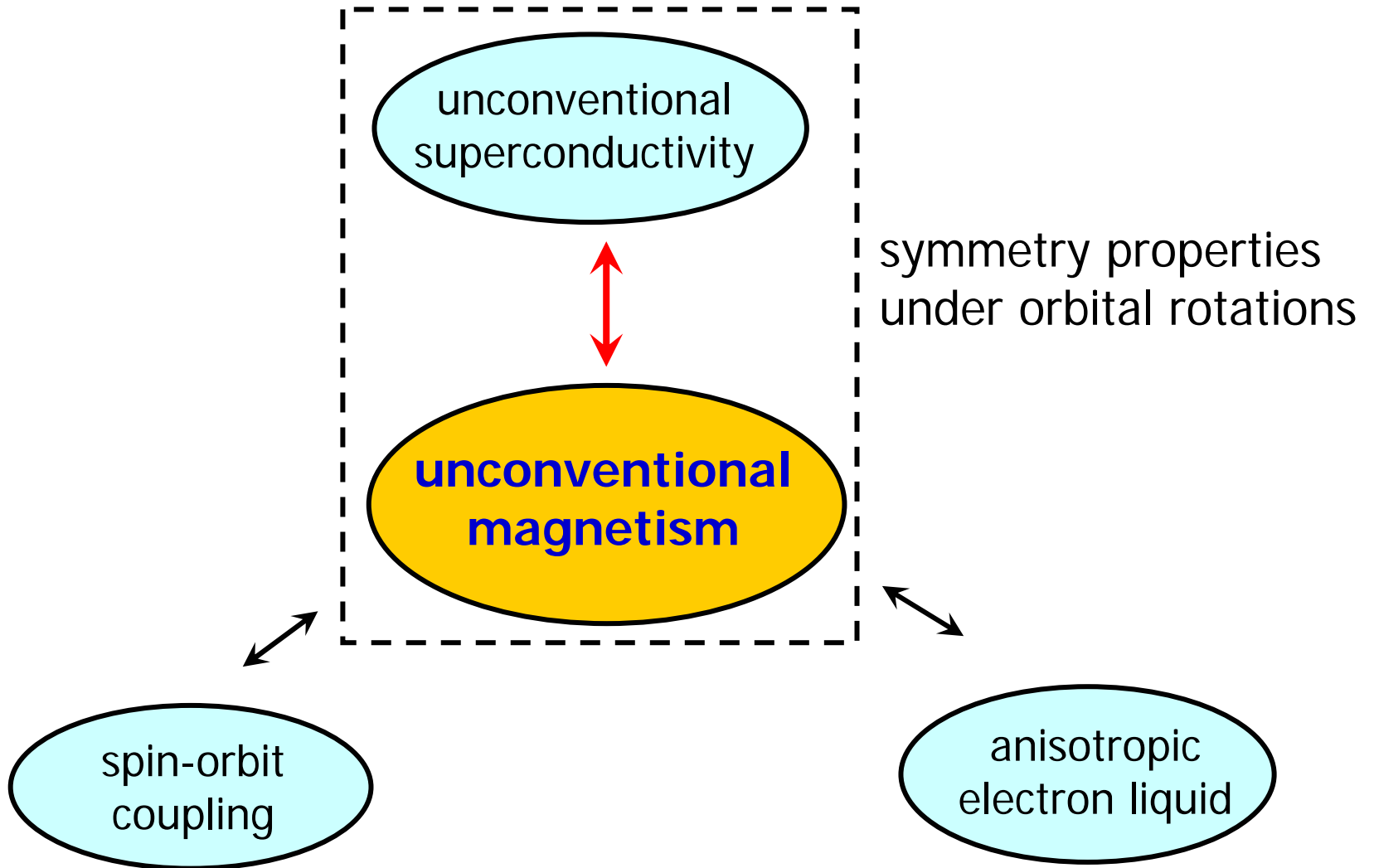
- E. Fradkin, UIUC.
- K. Sun, UIUC.
- S. C. Zhang, Stanford.

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# Outline

- **Introduction.**
  - What is unconventional magnetism?
  - Its deep connections to several important research directions in condensed matter physics.
- Mechanism for unconventional magnetic phase transitions.
- Low energy collective modes.
- Possible experimental realization and detection methods.

# Introduction



# The early age of ferromagnetism



司南

World's first compass:  
south-pointer

Thales (624-546 BC) says that a stone (lodestone) has a soul because it causes movement to iron.

----*De Anima*, Aristotle (384-322 BC)

The lodestone attracts iron.

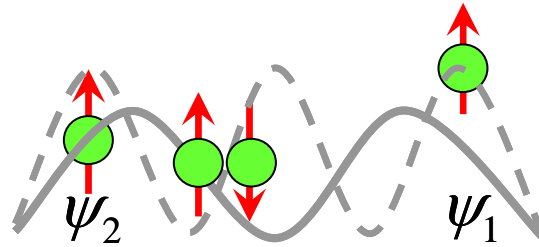
---- *Guiguzi* (鬼谷子) (4<sup>th</sup> century BC)

# Ferromagnetism: many-body collective effect



E. C. Stoner

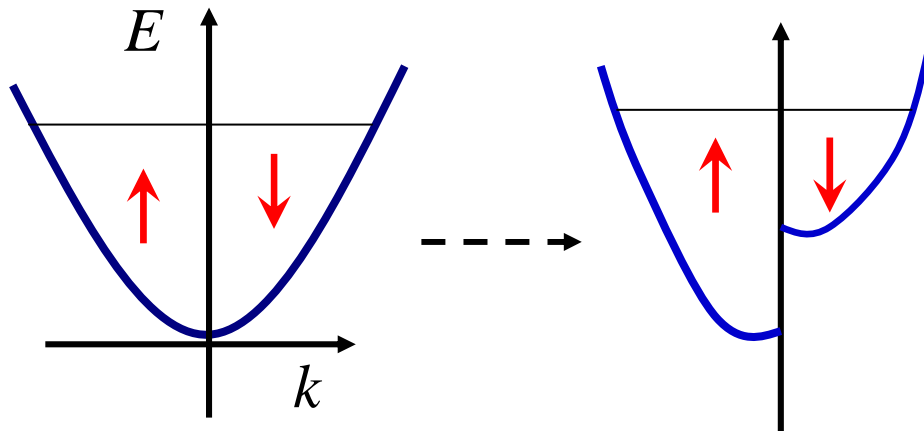
- Driving force: **exchange interaction among electrons.**



$$E_{\uparrow\uparrow} < E_{\uparrow\downarrow}$$

- Stoner criterion:

$$UN_0 > 1$$



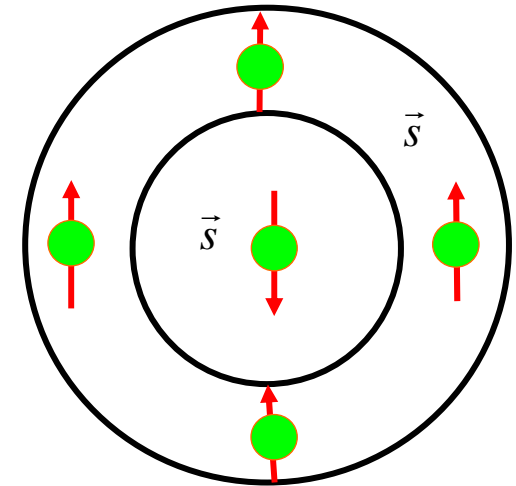
$U$  – average interaction strength;  $N_0$  – density of states at the Fermi level

Fe	Co	Ni
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# Ferromagnetism: **s-wave** magnetism

- Spin rotational symmetry is broken.

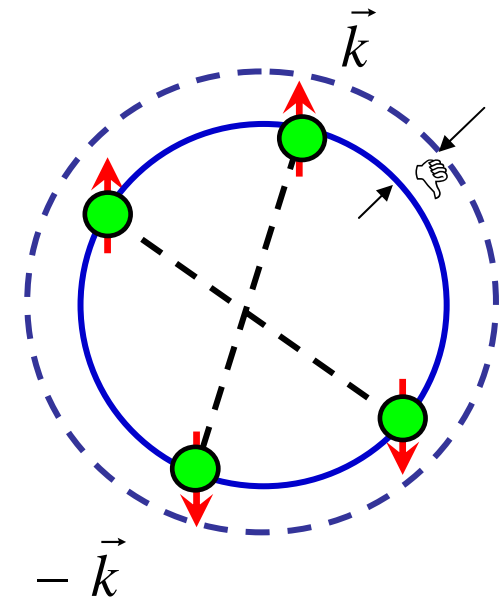
• Orbital rotational symmetry is **NOT** broken: spin polarizes along a **fixed direction** on the Fermi surface.



- *cf.* conventional superconductivity.

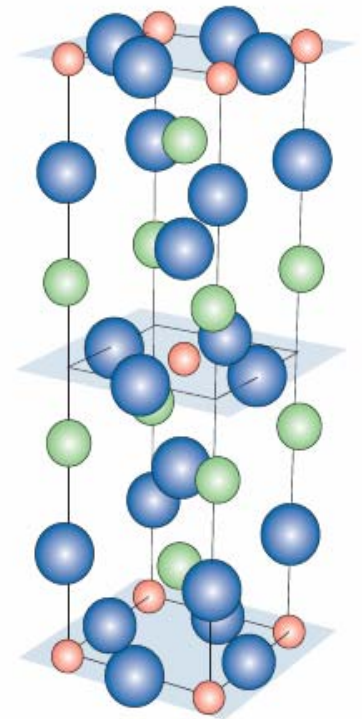
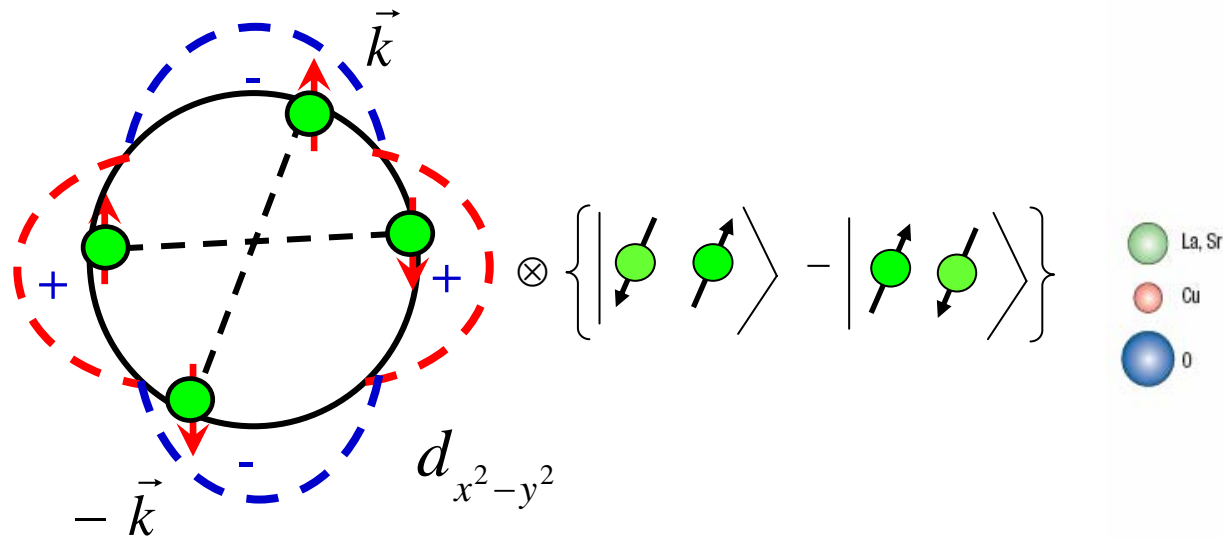
Cooper pairing between electrons with opposite momenta.

s-wave: pairing amplitude does not change over the Fermi surface.



# cf. Unconventional superconductivity

- High partial wave channel Cooper pairings (e.g.  $p$ ,  $d$ -wave ...).
- $d$ -wave: high  $T_c$  cuprates. Pairing amplitude changes sign in the Fermi surface.



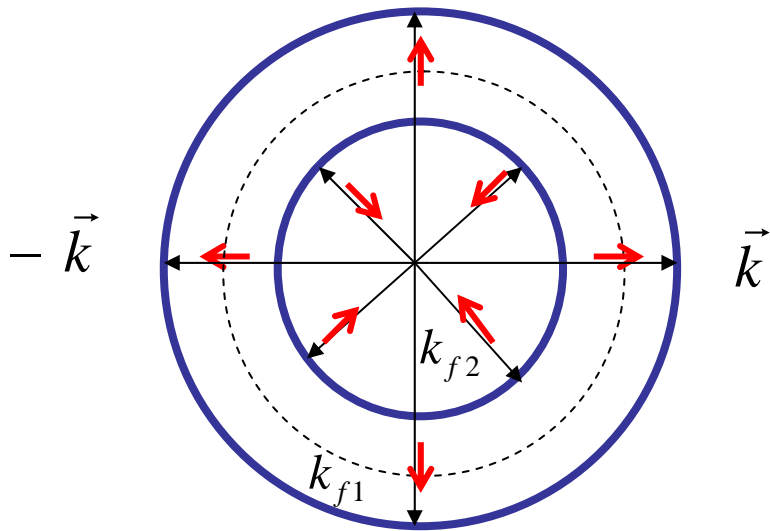
- $p$ -wave:  $\text{Sr}_2\text{RuO}_4$ ,  $^3\text{He-A}$  and  $B$ .

D. J. Van Harlingen, Rev. Mod. Phys. 67, 515 (1995); C. C. Tsuei et al., Rev. Mod. Phys. 72, 969 (2000).



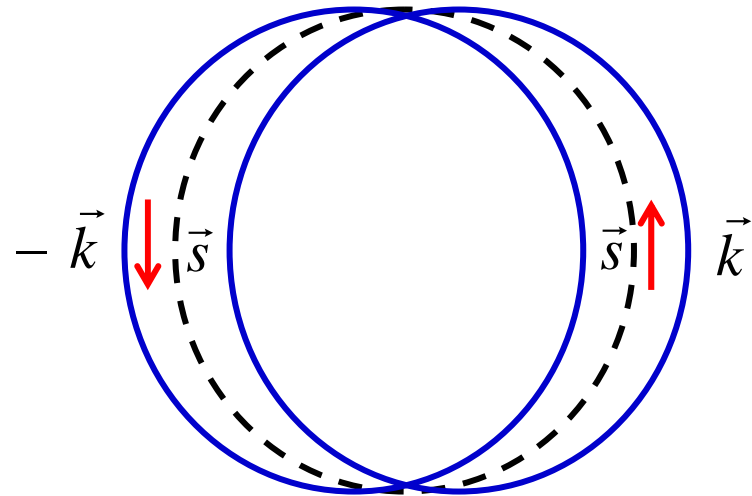
# New states of matter: unconventional magnetism!

- High partial wave channel generalizations of ferromagnetism (e.g.  $p$ ,  $d$ -wave...).
- Spin polarization varies over the Fermi surface.



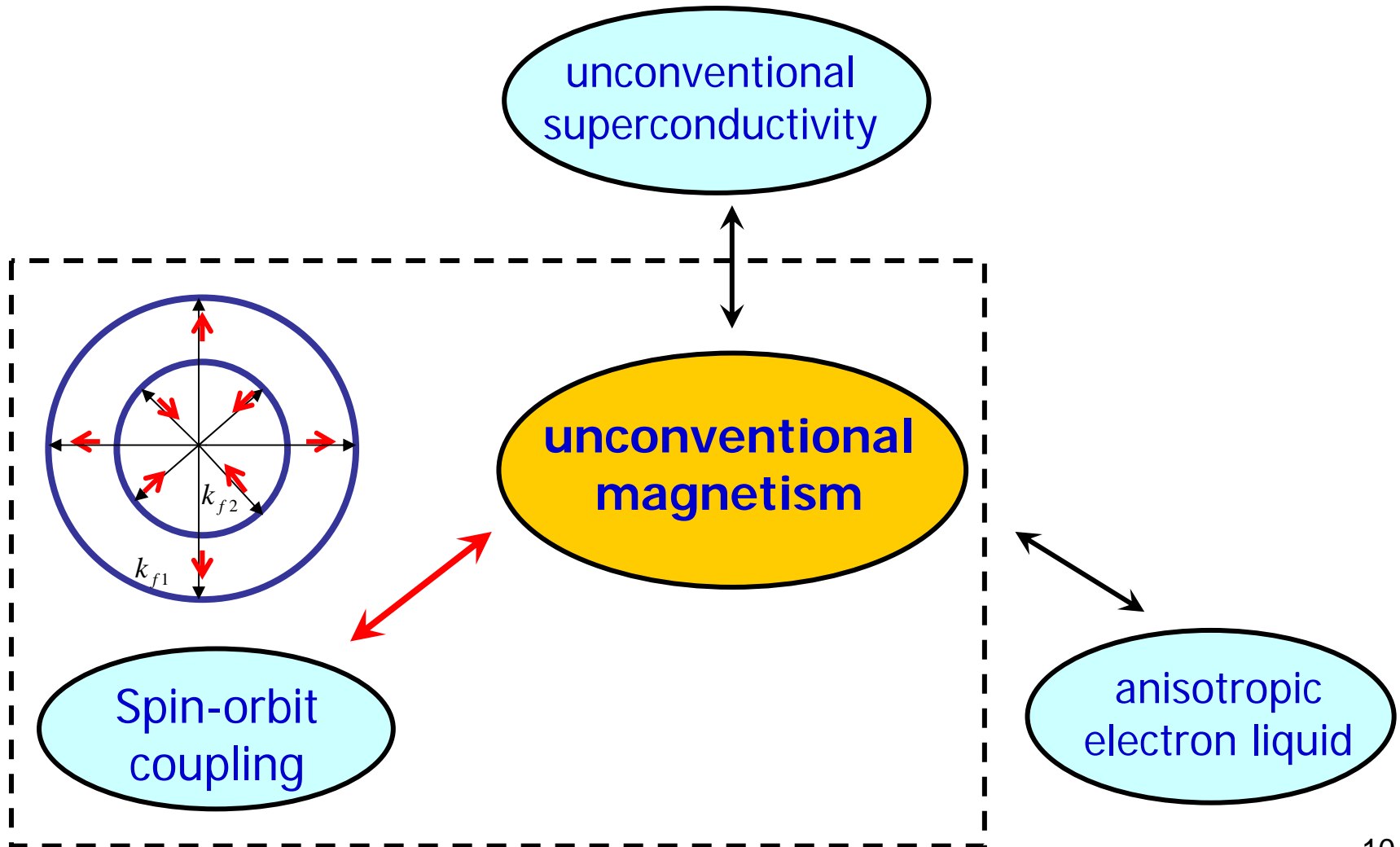
isotropic  $p$ -wave magnetic state

spin flips the sign as  $\vec{k} \rightarrow -\vec{k}$ .



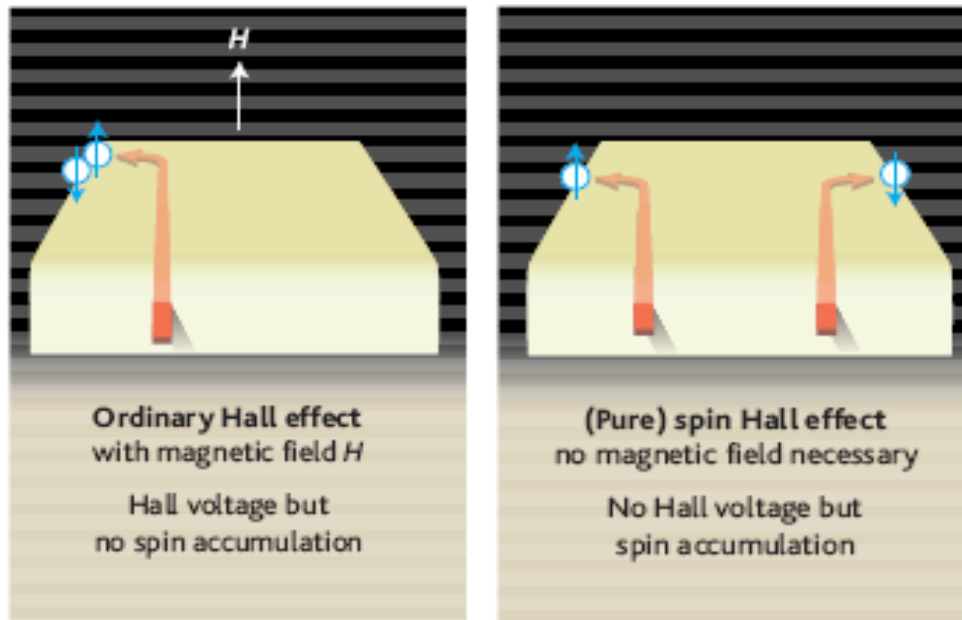
anisotropic  $p$ -wave magnetic state

# Introduction: dynamic generation of spin-orbit coupling



# Spintronics: controlling spin degree of freedom

- Electric fields manipulation, rather than magnetic fields.
- **Spin Hall effect**: electric fields induced transverse spin accumulations due to **spin-orbit coupling**.



Science 309, 2004 (2005).

Theory:

S. Murakami et al., *Science* **301**, 1348 (2003); J. Sinova et al., *PRL* **92**, 126603 (2004); J. Hirsch, *PRL* **83**, 1834 (1999).....

Experiment:

Y. K. Kato, et al., *Science* **306**, (2004); J. Wunderlich, et al., *PRL* **94**, 047204 (2005); N. P. Stern et al., *PRL* **97**, 126603 (2006) .....

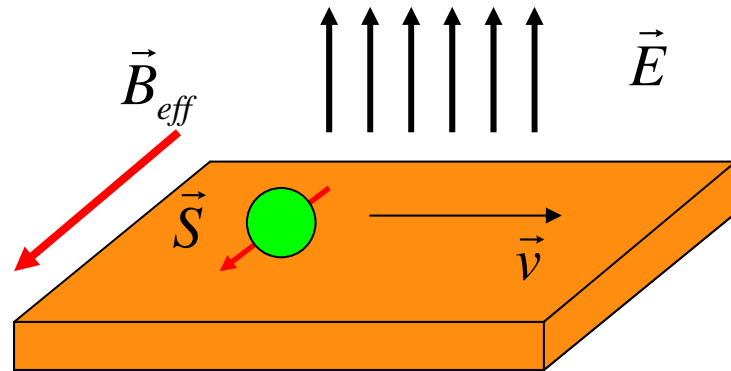
# Microscopic origin of spin-orbit coupling

- Spin-orbit coupling originates from relativity.

- An electron moving in an  $\vec{E}$  field. In the co-moving frame, the  $\vec{E}$  field is moving.

- Due to relativity, an internal effective  $\vec{B}$  field is induced and couples to electron spin.

$$\vec{B}_{eff} = \frac{\vec{v}}{c} \times \vec{E}$$
$$H_{so} \propto -\vec{S} \cdot \vec{B}_{eff}$$



# Unconventional magnetism: dynamic generation of spin-orbit (SO) coupling without relativity!

- Conventional mechanism:

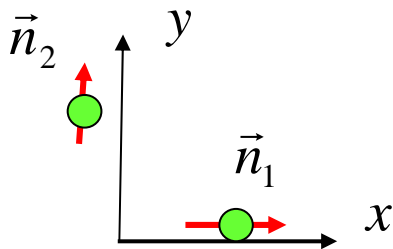
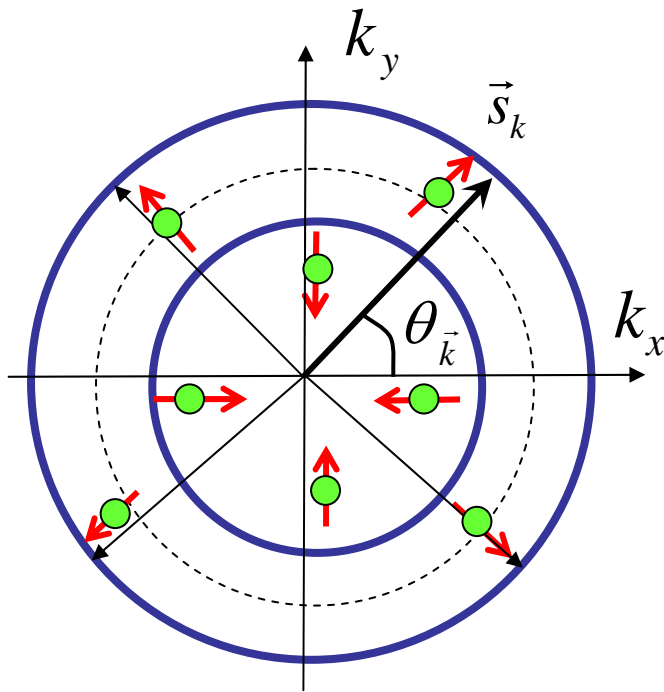
A **single-body** effect; not directly related to many-body interactions.

- **New mechanism (many-body collective effect):**

Dynamically generate SO coupling through **unconventional magnetic phase transitions**.

- **Advantages:** tunable SO coupling by varying temperatures; new types of SO coupling.

# The isotropic $p$ -wave magnetic phase



- Spin is not conserved; helicity  $\vec{\sigma} \cdot \vec{k}$  is a good quantum number; no net spin-moment.
- Order parameter: spin dipole moment in momentum space (not in the coordinate space).

$$\vec{n}_1 = \sum_{\vec{k}} \vec{s}_k \cos \theta_k, \quad \vec{n}_2 = \sum_{\vec{k}} \vec{s}_k \sin \theta_k$$

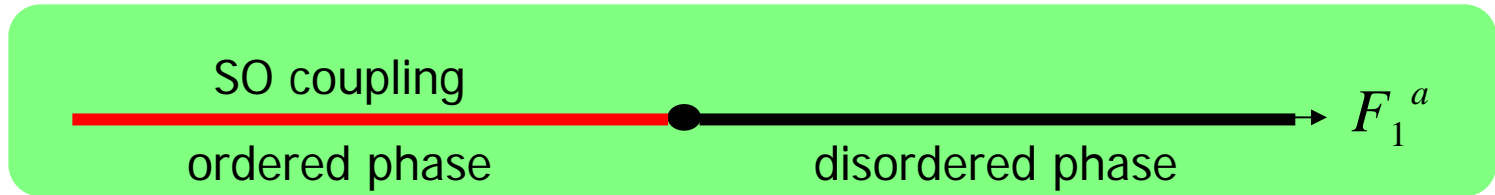
- **Isotropic phase with spin-orbit coupling.**

$$H_{MF} = H_0 + \bar{n} \sum_k \psi_{\alpha}^+ \vec{\sigma}_{\alpha\beta} \cdot \vec{k} \psi_{\beta}$$

$$\bar{n} = |\vec{n}_1| = |\vec{n}_2|$$

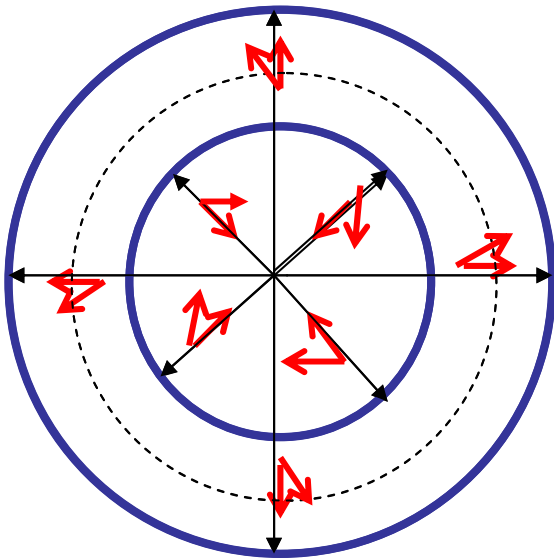
C. Wu et al., PRL 93, 36403 (2004);  
C. Wu et al., cond-mat/0610326.

# The subtle symmetry breaking pattern



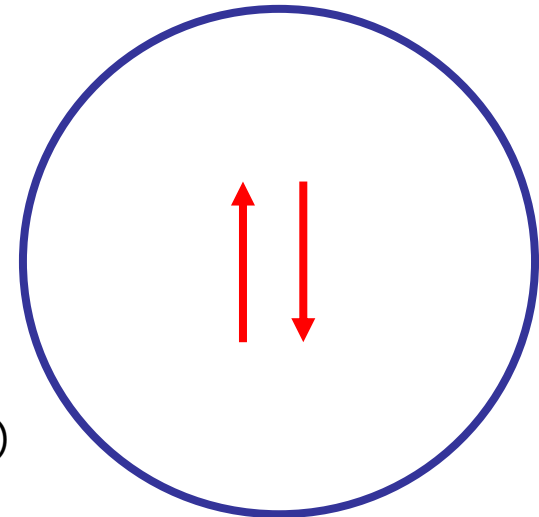
- $\vec{J}$  is conserved, but  $\vec{L}$ ,  $\vec{S}$  are not separately conserved.

- **Independent** orbital and spin rotational symmetries.



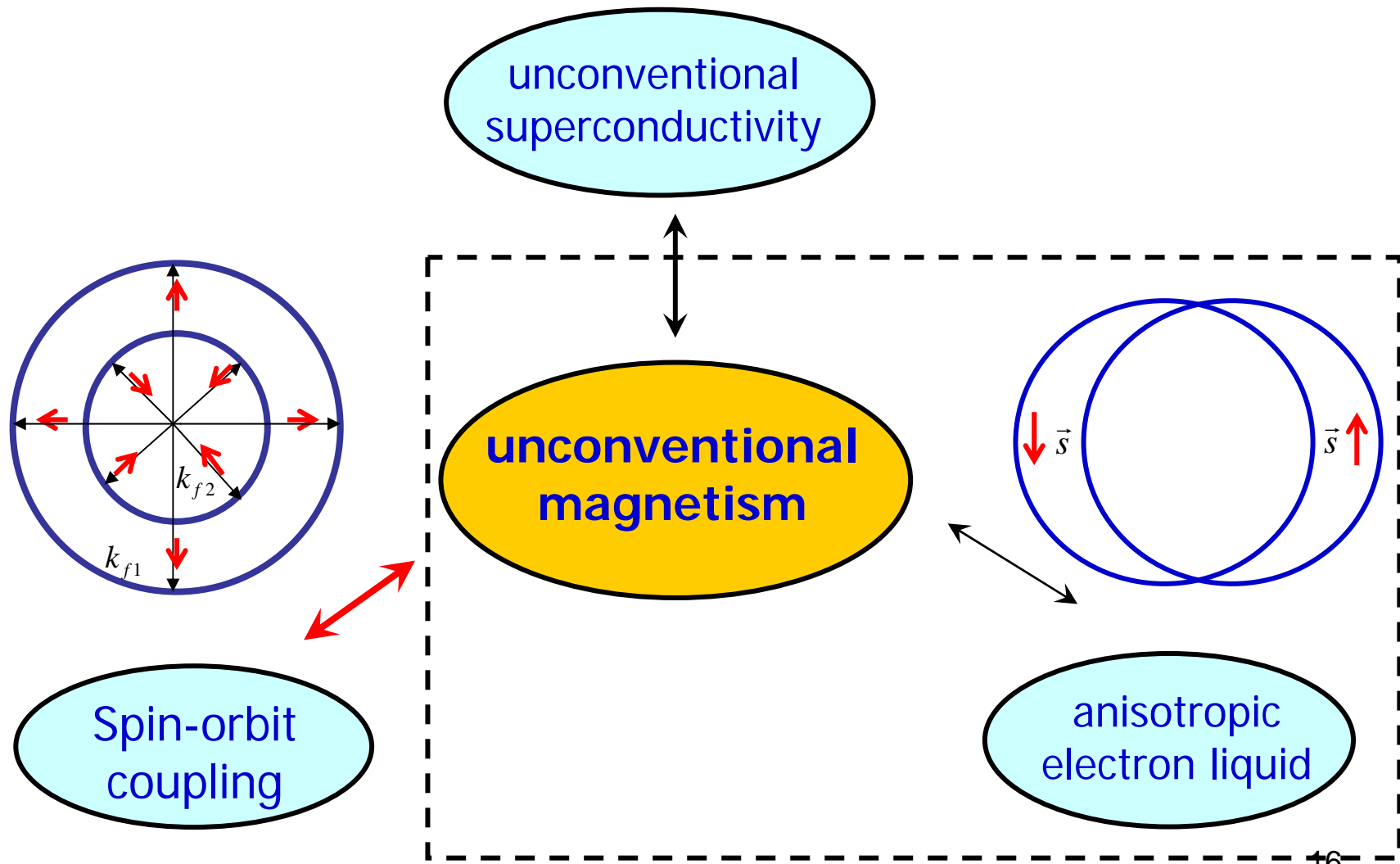
$$\vec{J} = \vec{L} + \vec{S}$$

Leggett, Rev. Mod. Phys **47**, 331 (1975)



- **Relative spin-orbit** symmetry breaking.

# Introduction: electron liquid crystal phase with spin

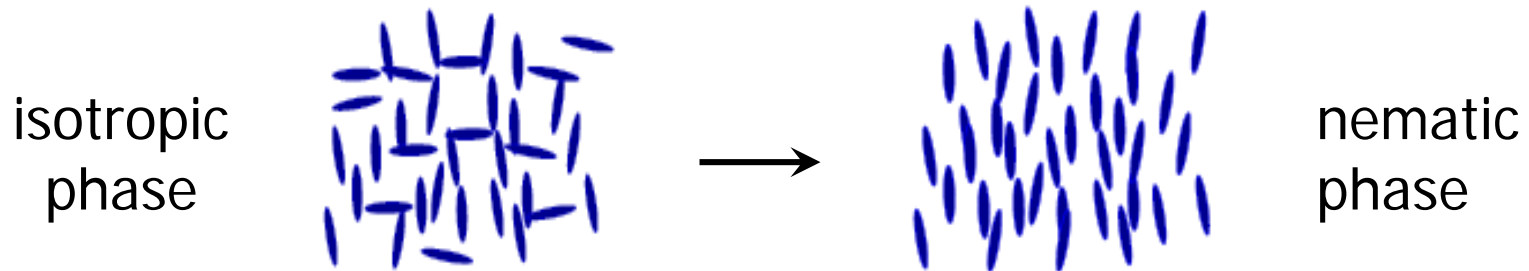




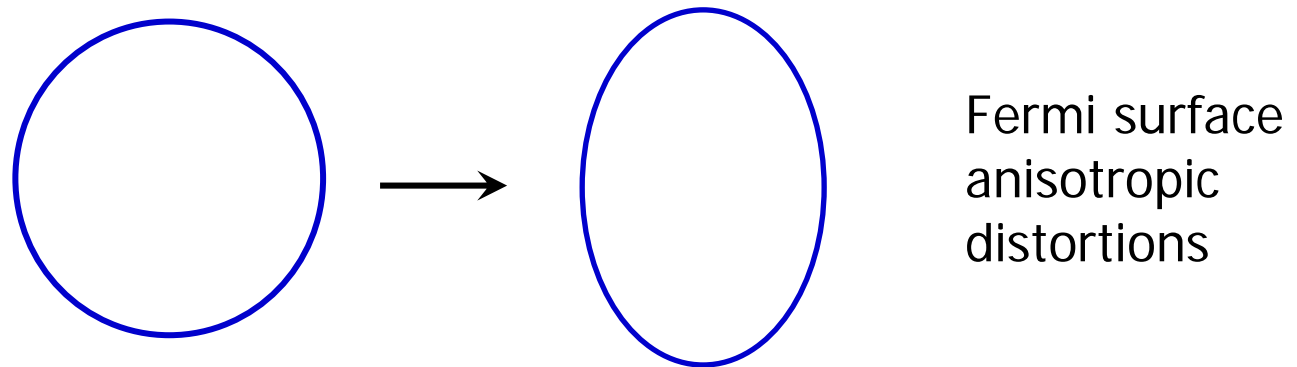
# Anisotropy: liquid crystalline order

- Classic liquid crystal: LCD.

Nematic phase: rotational anisotropic but translational invariant.

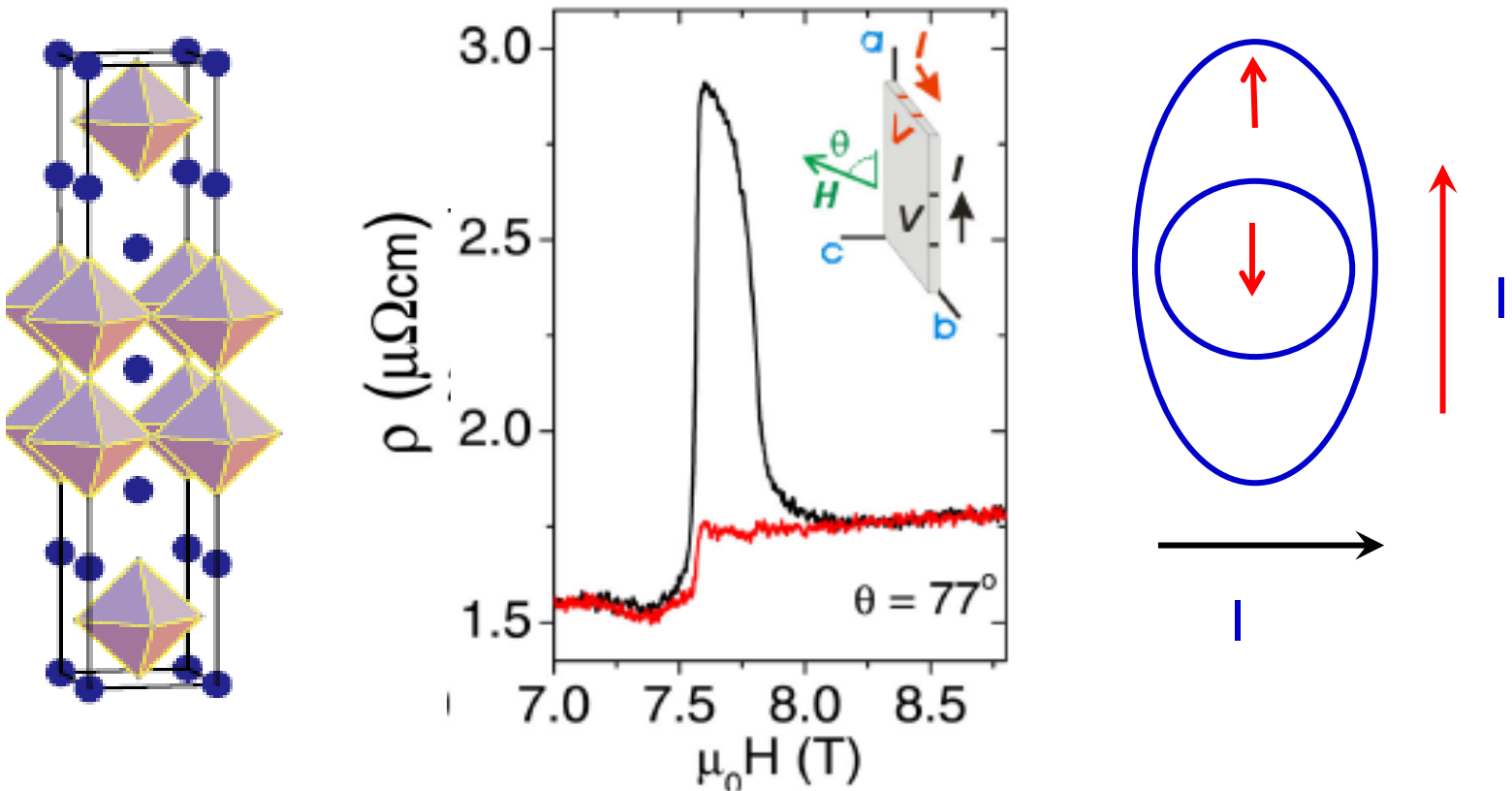


- Quantum version of liquid crystal: **nematic electron liquid.**

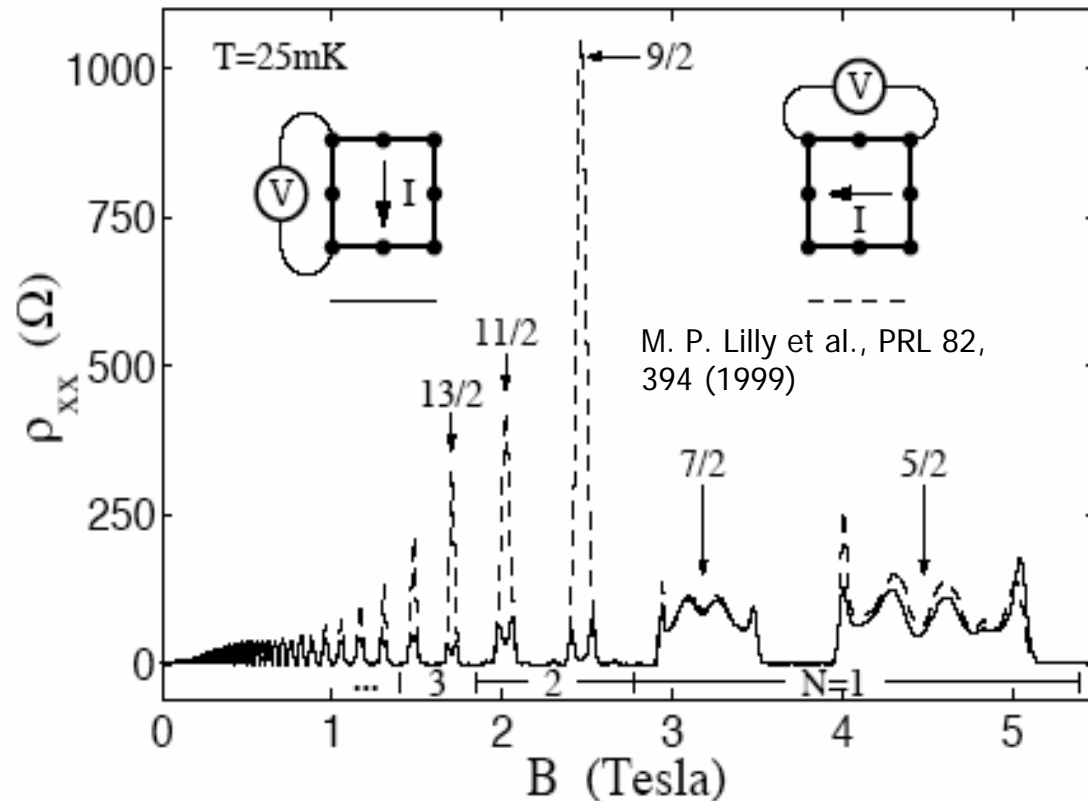


# Nematic electron liquid in $\text{Sr}_3\text{Ru}_2\text{O}_7$ at high B fields

- Quasi-2D system; resistivity **anisotropy** at 7~8 Tesla.
- Fermi surface nematic distortions.

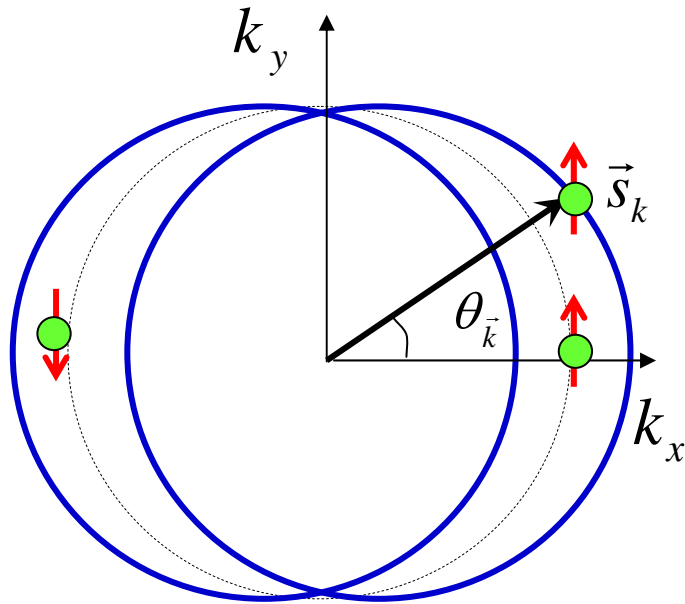


# Nematic electron liquid in 2D GaAs/AlGaAs at high B fields



M. M. Fogler, et al, PRL 76 ,499 (1996), PRB 54, 1853 (1996); E. Fradkin et al, PRB 59, 8065 (1999), PRL 84, 1982 (2000); L. Radzihovsky et al., PRL 88, 216802 (2002).

# Unconventional magnetism: electron liquid crystal phases with spin!



**anisotropic  $p$ -wave magnetic phase**

- $p$ -wave distortion of the Fermi surface.

- No net spin-moment:  $\vec{S} = \sum_{\vec{k}} \vec{s}_k = 0$

- Spin dipole moment in momentum space (not in coordinate space).

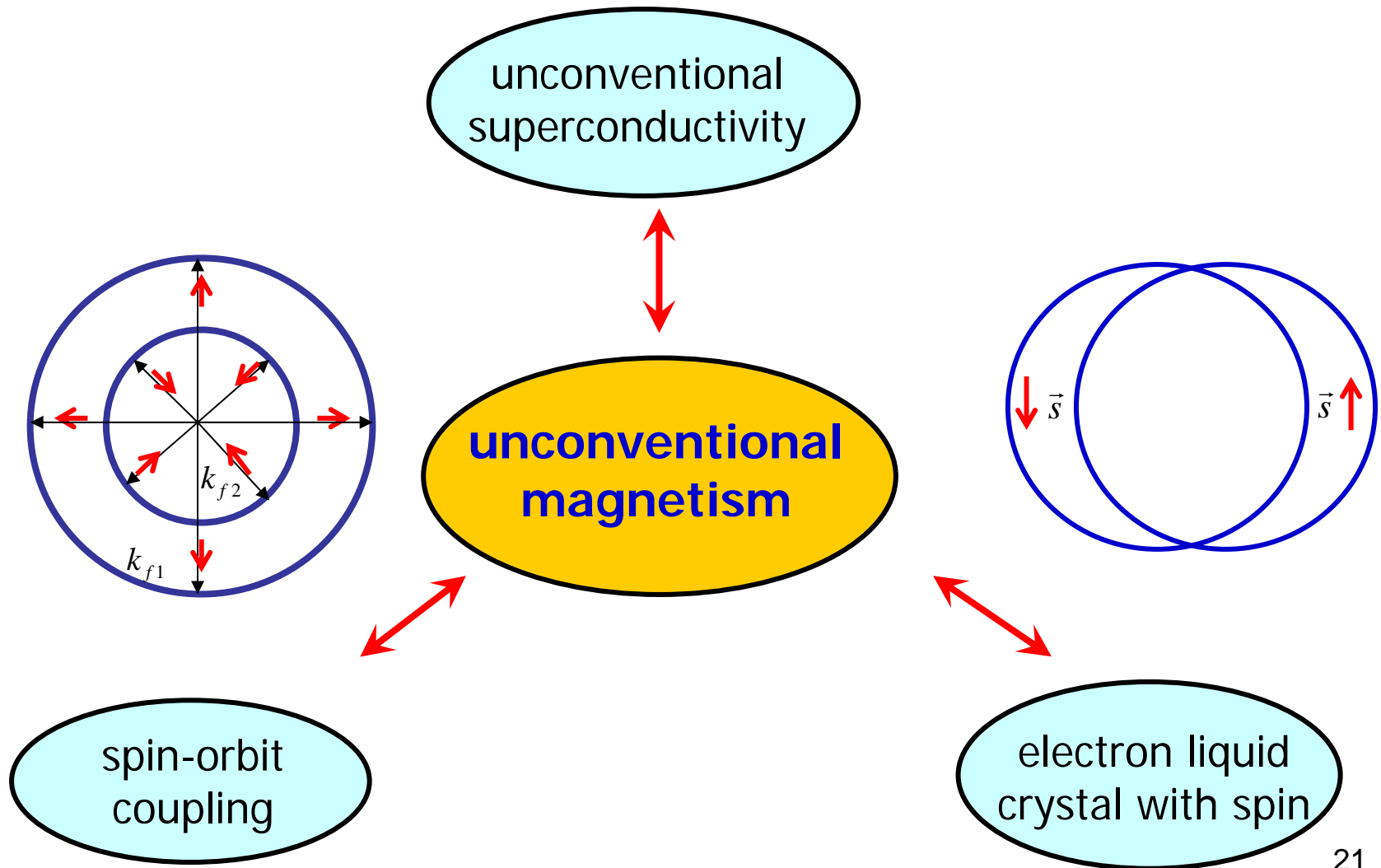
$$\vec{n}_1 = \sum_{\vec{k}} \vec{s}_k \cos \theta_k \neq 0$$

- Both orbital and spin rotational symmetries are broken.

**spin-split state** by J. E. Hirsch, PRB 41, 6820 (1990); PRB 41, 6828 (1990).

V. Oganesyan, et al., PRB 64,195109 (2001).  
C. Wu et al., PRL 93, 36403 (2004); Varma et al., Phys. Rev. Lett. 96, 036405 (2006)

# Summary of the introduction part



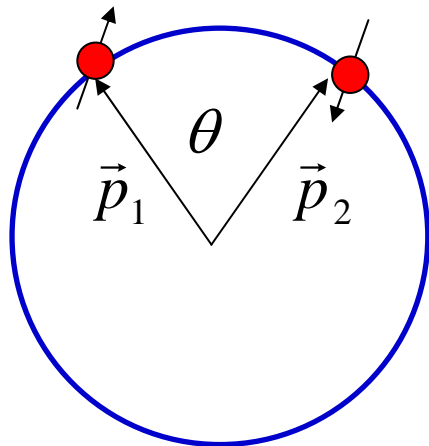
# Outline

- Introduction.
- **Mechanism for unconventional magnetic phase transitions.**
  - Fermi surface instability of the Pomeranchuk type.
  - Mean field phase structures.
- Low energy collective modes.
- Possible directions of experimental realization and detection methods.

# Landau Fermi liquid (FL) theory



L. Landau



- The existence of Fermi surface. Electrons close to Fermi surface are important.
- Interaction functions (no SO coupling):

$$f_{\alpha\beta,\gamma\delta}(\hat{p}_1, \hat{p}_2) = f^s(\hat{p}_1, \hat{p}_2) \quad \text{density} \\ + f^a(\hat{p}_1, \hat{p}_2) \vec{\sigma}_{\alpha\beta} \cdot \vec{\sigma}_{\gamma\delta} \quad \text{spin}$$

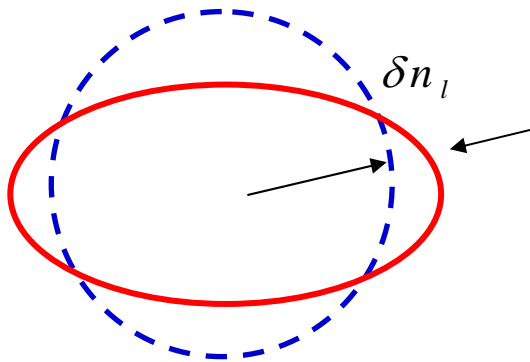
- Landau parameter in the  $l$ -th partial wave channel:

$$F_l^{s,a} = N_0 f_l^{s,a} \quad N_0 : \text{DOS}$$

# Pomeranchuk instability criterion



I. Pomeranchuk



- Fermi surface: elastic membrane.

- Stability: 
$$\Delta E_K \propto (\delta n_l^{s,a})^2$$
$$\Delta E_{\text{int}} \propto \frac{F_l^{s,a}}{2l+1} (\delta n_l^{s,a})^2$$

- Surface tension vanishes at:

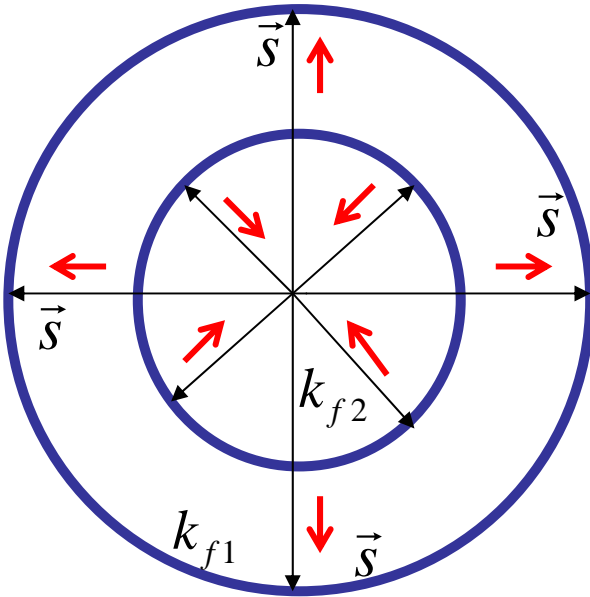
$$F_l^{s,a} < -(2l+1)$$

- Ferromagnetism: the  $F_0^a$  channel.
- Nematic electron liquid: the  $F_2^s$  channel.

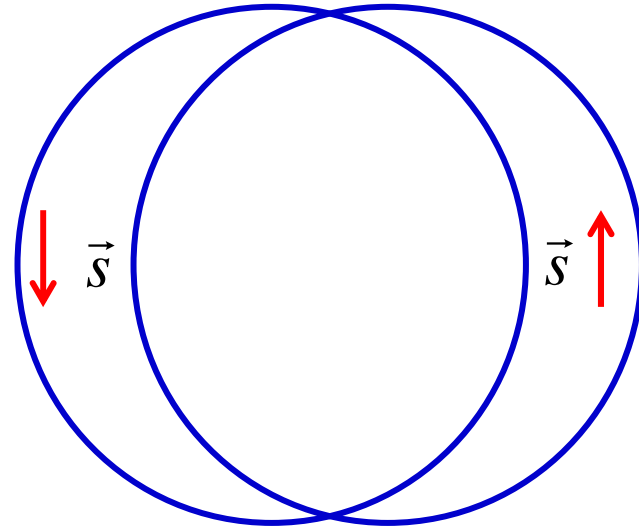


# Unconventional magnetism: Pomeranchuk instability in the spin channel

$F_1^a$



$\beta$ -phase

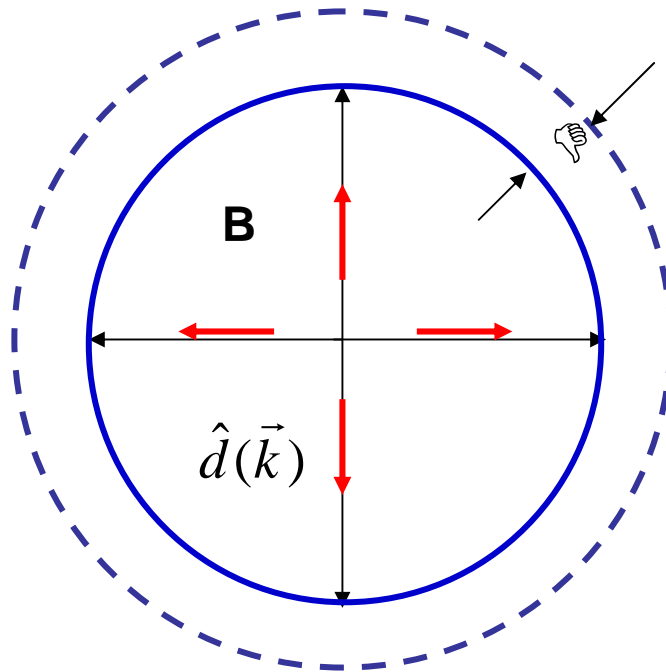


$\alpha$ -phase

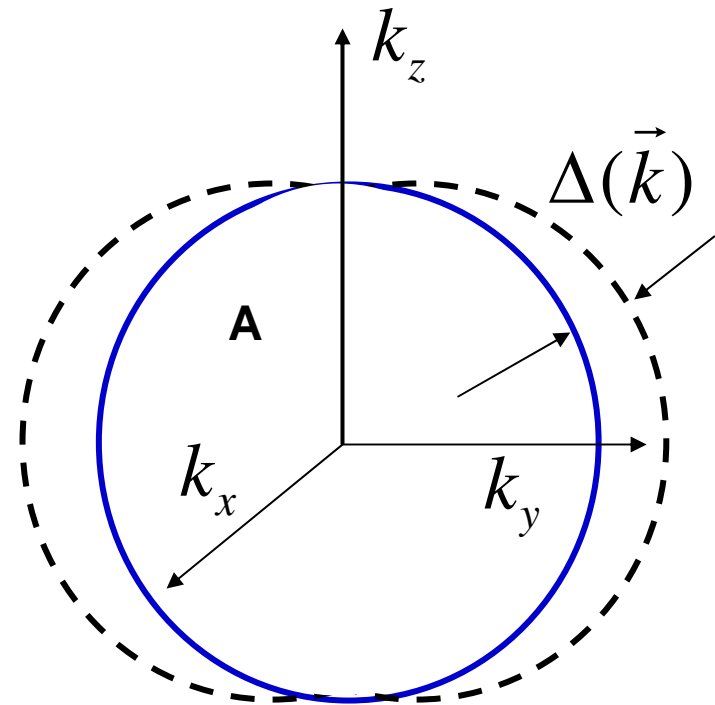
- An analogy to superfluid  $^3\text{He-B}$  (isotropic) and A (anisotropic) phases.

## cf. Superfluid $^3\text{He-B}$ , A phases

- $p$ -wave triplet Cooper pairing.



$$\vec{\Delta}(\vec{k}) = \Delta \hat{d}(\vec{k}) = \Delta \hat{k}$$



$$\vec{\Delta}(\vec{k}) = \Delta \hat{d}(\hat{k}_x + i\hat{k}_y)$$

- $^3\text{He-B}$  (isotropic) phase.

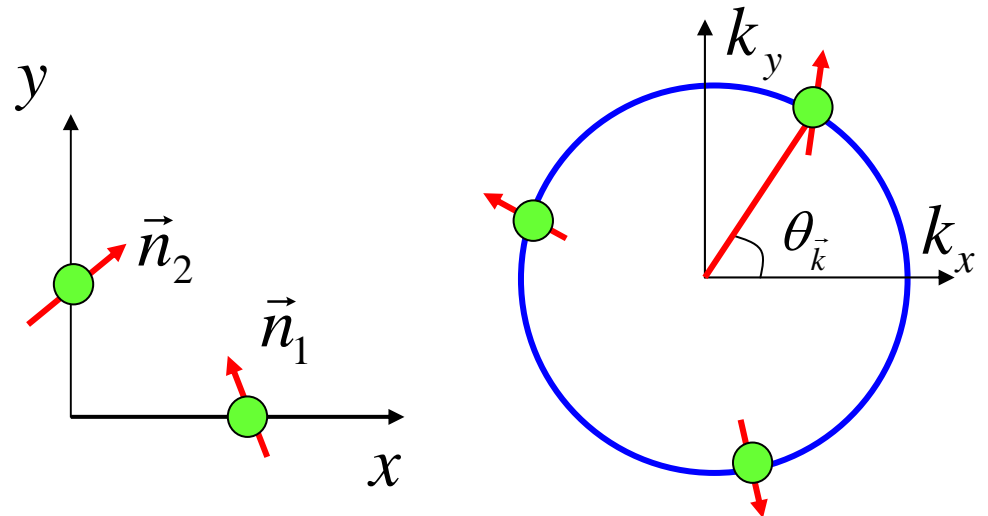
- $^3\text{He-A}$  (anisotropic) phase.

# The order parameters: the 2D $p$ -wave channel

- $F_1^a$  : Spin currents flowing along x and y-directions, or **spin-dipole moments in momentum space**.

$$\vec{n}_1 = \sum_{\vec{k}} \psi_k^+ \vec{\sigma} \psi_k \cos \theta_k$$

$$\vec{n}_2 = \sum_{\vec{k}} \psi_k^+ \vec{\sigma} \psi_k \sin \theta_k$$



- *cf.* Ferromagnetic order (s-wave):  $\vec{s} = \sum_{\vec{k}} \psi_k^+ \vec{\sigma} \psi_k$

- Arbitrary partial wave channels: spin-multipole moments.

$$F_l^a : \cos \theta_k \rightarrow \cos l\theta_k; \sin \theta_k \rightarrow \sin l\theta_k$$

# Mean field theory and Ginzburg-Landau free energy

- The simplest non-s-wave exchange interaction:

$$F_1^a \quad H_{\text{int}} = \sum_q f_1^a(\vec{q}) \{ \vec{n}_1(\vec{q}) \cdot \vec{n}_1(\vec{q}) + \vec{n}_2(\vec{q}) \cdot \vec{n}_2(\vec{q}) \}$$

$$H_{MF} = \sum_k \psi^\dagger(k) [ \varepsilon(k) - \mu - (\vec{n}_1 \cos\theta_k + \vec{n}_2 \sin\theta_k) \cdot \vec{\sigma} ] \psi(k)$$

- Symmetry constraints: rotation (spin, orbital), parity, time-reversal.

$$F(\vec{n}_1, \vec{n}_2) - F(0) = r(|\vec{n}_1|^2 + |\vec{n}_2|^2) + v_1(|\vec{n}_1|^2 + |\vec{n}_2|^2)^2 + v_2 |\vec{n}_1 \times \vec{n}_2|^2$$

$$r = \frac{N_0}{2} \frac{1 + F_1^a / 2}{|F_1^a|}$$

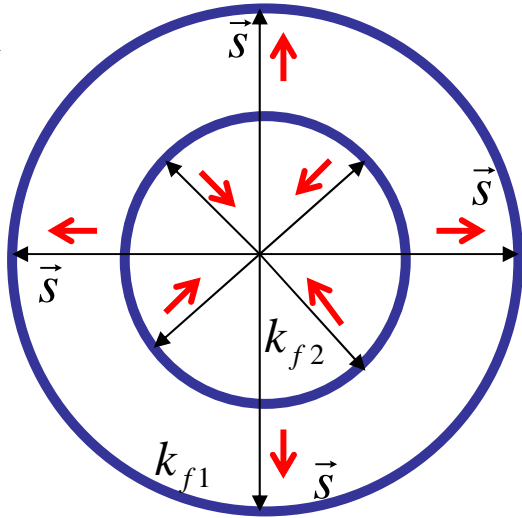
$$F_1^a < -2$$



instability!

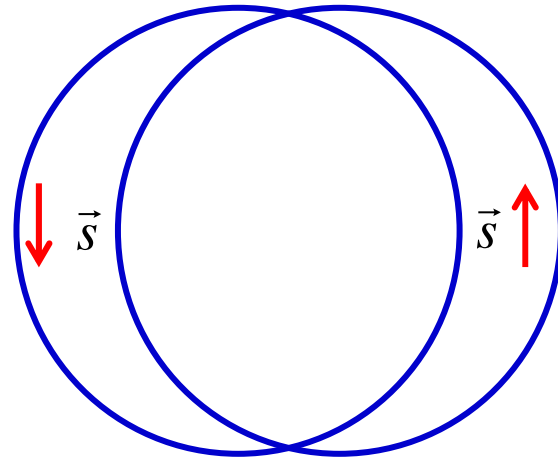
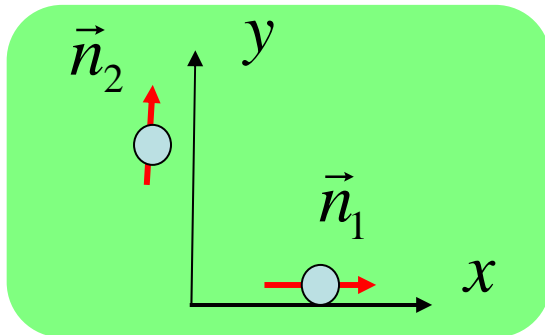
# $\beta$ and $\alpha$ -phases ( $\rho$ -wave)

$F_1^a$



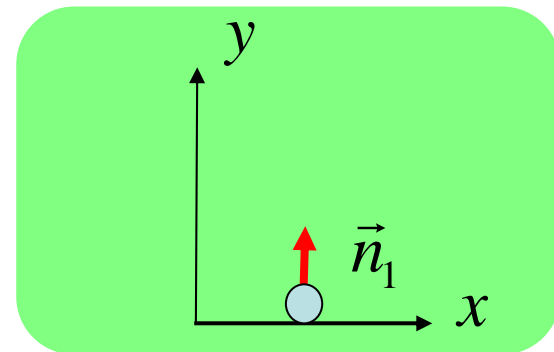
$v_2 < 0$ :  $\beta$ -phase

$\vec{n}_1 \perp \vec{n}_2$  and  $|\vec{n}_1| = |\vec{n}_2|$

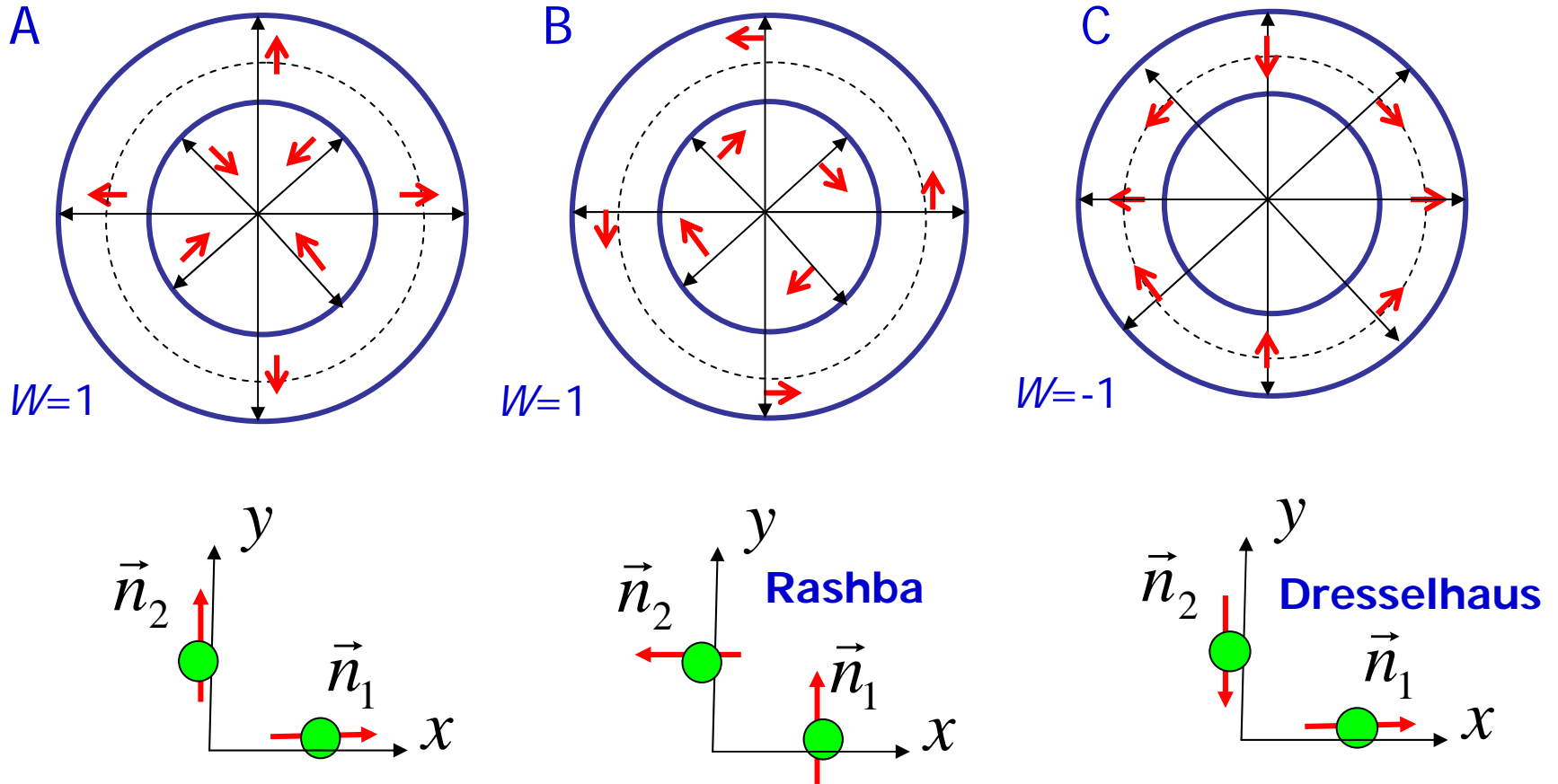


$v_2 > 0$ :  $\alpha$ -phase

$\vec{n}_1 \parallel \vec{n}_2$ ;  $|\vec{n}_2| \parallel |\vec{n}_1|$  arbitrary

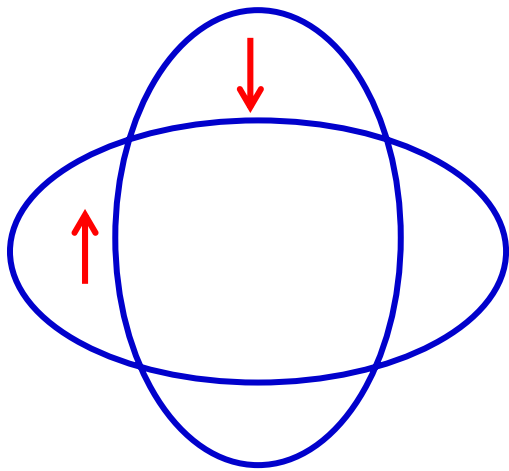


# The $\delta\Omega$ -phases: vortices in momentum space

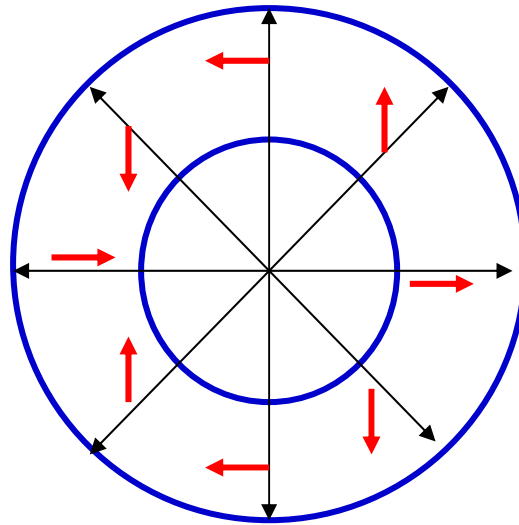


- Perform global spin rotations,  $A \rightarrow B \rightarrow C$ .

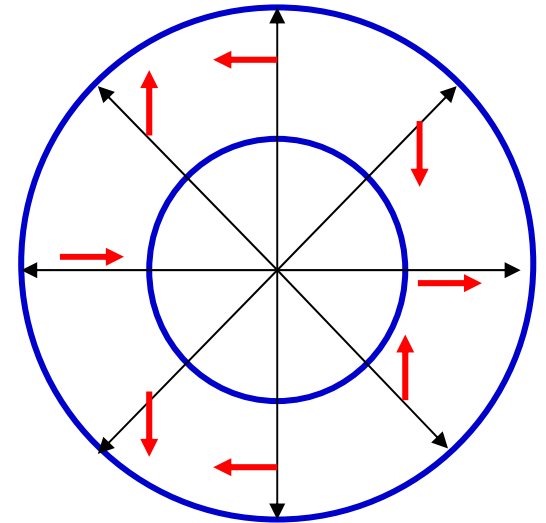
## 2D $d$ -wave $\alpha$ and $\beta$ -phases



$\alpha$ -phase



$\beta$ -phase:  $w=2$



$\beta$ -phase:  $w=-2$

# Outline

- Introduction.
- Mechanism for unconventional magnetic states.
- **Collective excitations.**
  - Symmetry spontaneously breaking: low energy Goldstone modes; Neutron scattering spectra.
  - Ground state chiral instability.
- Possible experimental realization and detection methods.



# The $\alpha$ -phases: orbital & spin channel Goldstone (GS) modes

- Orbital channel GS mode: FS oscillations (intra-band transition).

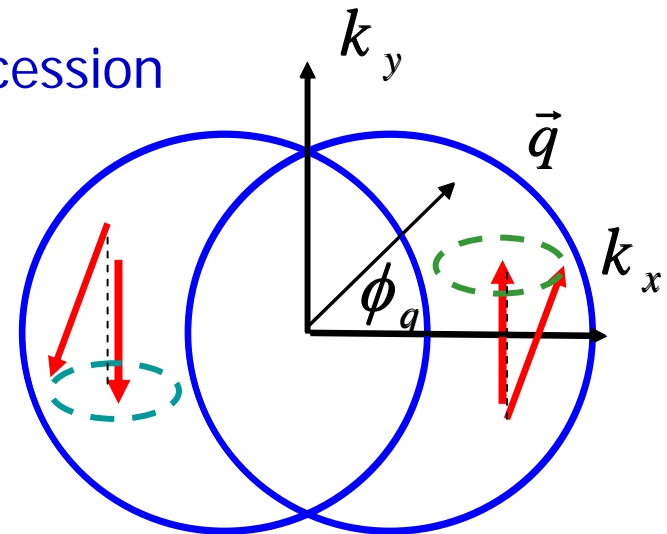
$$L_{FS}^{\alpha}(\vec{q}, \omega) = N_0 \left\{ \frac{(q\xi)^2}{|F_l^a|} - i \frac{\omega}{2v_f q} (1 + \cos 2\phi_q) \right\}$$

Anisotropic overdamping: The mode is maximally overdamped for  $q$  along the  $x$ -axis, and underdamped along the  $y$ -axis ( $l=1$ ).

- Spin channel GS mode: “spin dipole” precession (spin flip transition).

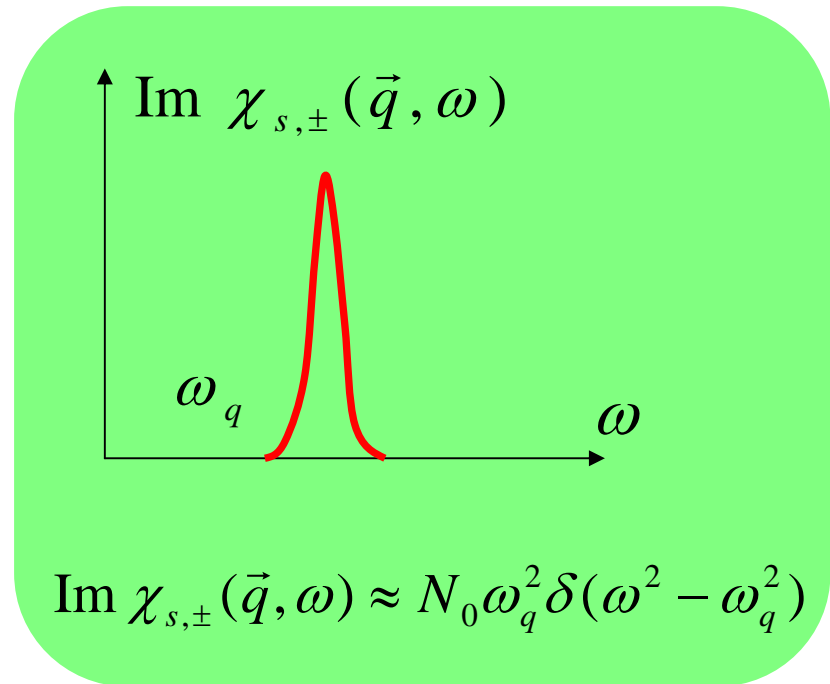
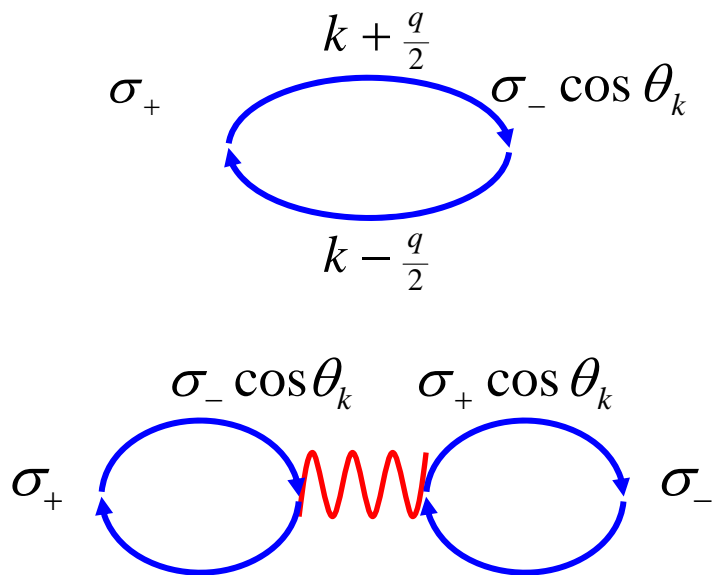
$$\omega_{x\pm iy}^2 = \frac{\bar{n}^2}{|F_l^a|} (q\xi)^2$$

Nearly isotropic, underdamped and linear dispersions at small  $q$ .



# The $\alpha$ -phases: neutron spectra

- *Elastic* neutron spectra: no Bragg peaks. Order parameters do not couple to neutron moments directly.
- *In-elastic* neutron spectra: **resonance peaks** develop at  $T < T_c$  due to the coupling between GS modes and spin-waves (spin-flip channel).



# The $\beta$ -phases: GS modes

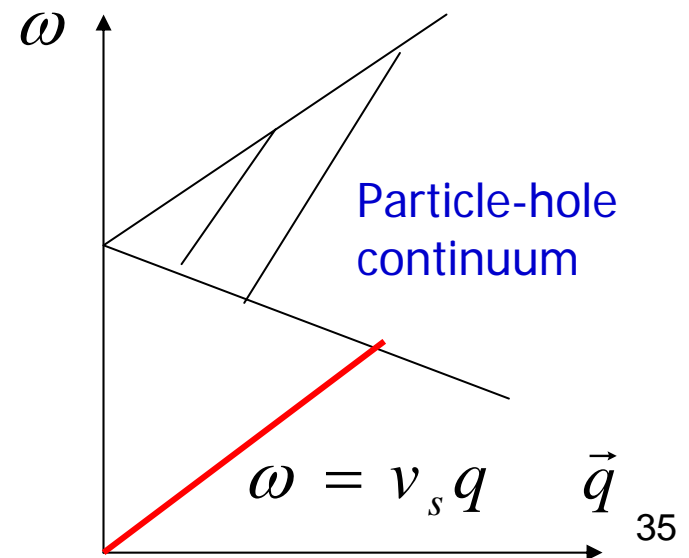
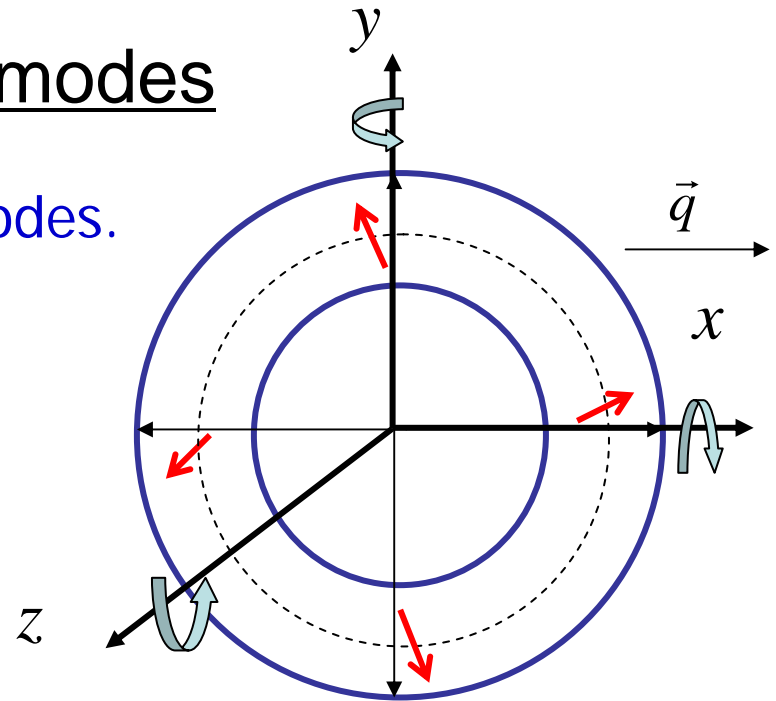
- 3 branches of relative spin-orbit modes.

$$O_z = \frac{1}{\sqrt{2}}(n_2^x - n_1^y);$$

$$O_x = -n_2^z; \quad O_y = n_1^z;$$

- For  $l \geq 2$ , these modes are with linear dispersion relations, and underdamped at small  $q$ .

- Inelastic neutron spectra: GS modes also couple to spin-waves, and induce resonance peaks in both spin-flip and non-flip channels.



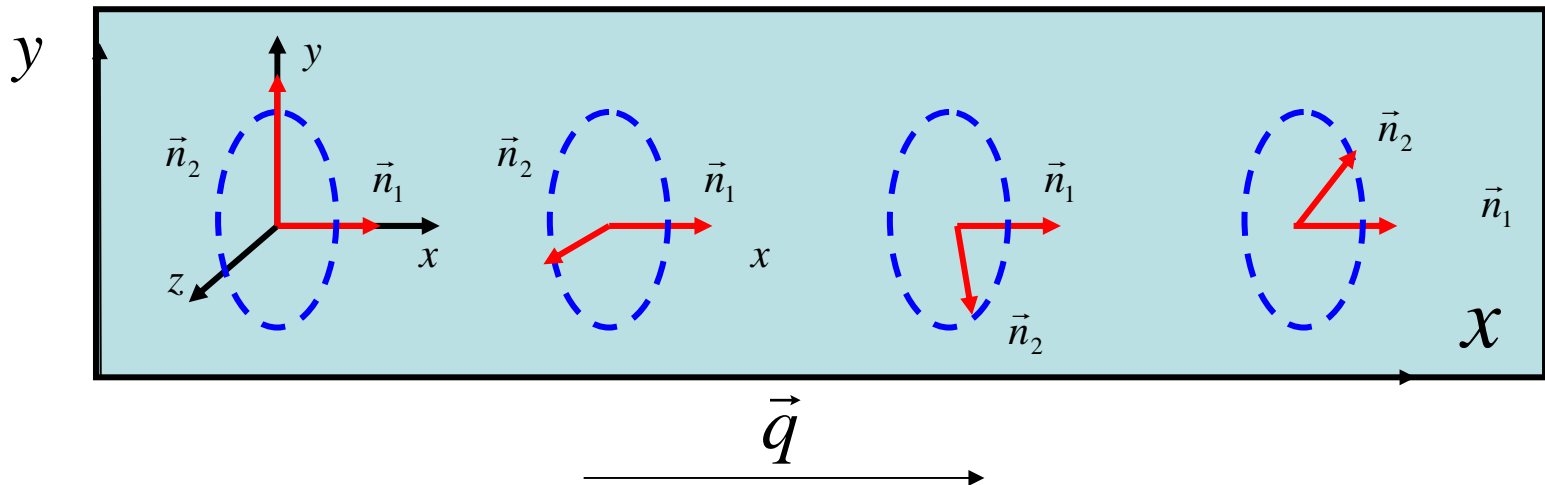
# Spontaneous chiral instability: the $p$ -wave $\beta$ -phase

- The uniform state can not be the true ground state.
- Linear spatial derivative term satisfying all the symmetry constraints.

$$F_{grad}(\partial_b \vec{n}_a) = \gamma_1 \partial_a \vec{n}_b \cdot \partial_a \vec{n}_b + \gamma_2 (\partial_x \vec{n}_2 - \partial_y \vec{n}_1) \cdot (\vec{n}_1 \times \vec{n}_2)$$

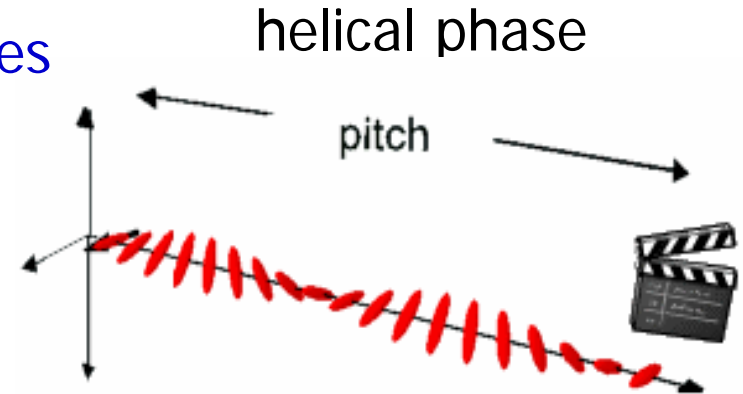
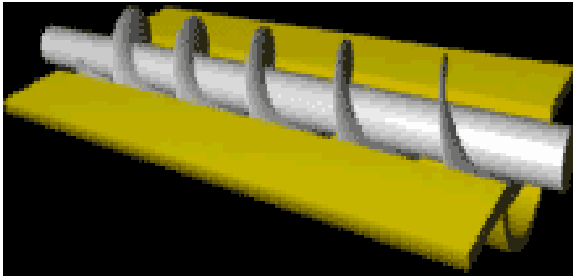
- Spiral of spin-dipole moments.

$$F_{grad}(q) = \gamma_1 \bar{n}^2 q^2 - \gamma_2 \bar{n}^3 q \quad q_c = \gamma_2 \bar{n} / (2\gamma_1)$$



# cf. Chirality: explicit parity breaking

- Cholesteric liquid crystals. Molecules are chiral.

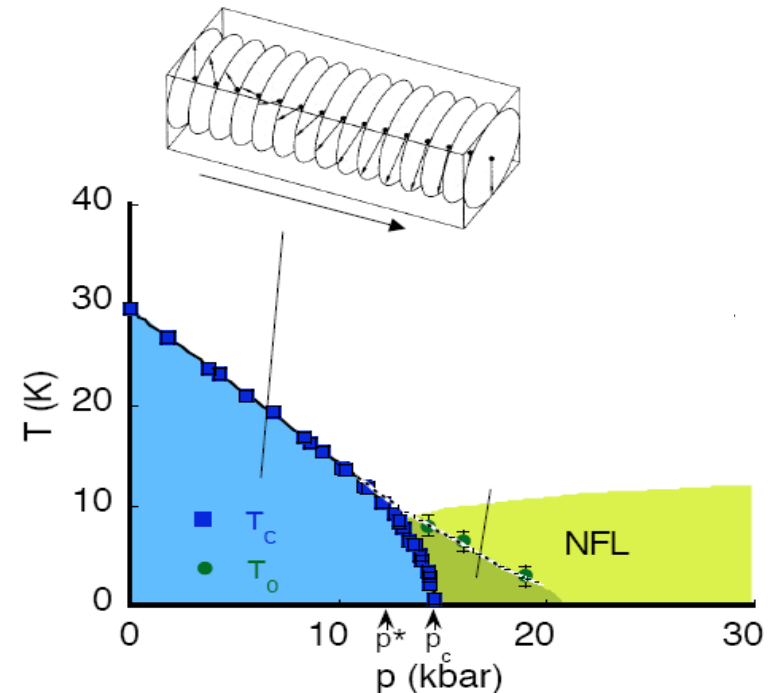


- Helical magnet: MnSi. Lattices lack of the inversion center.

Dzyaloshinskii-Moriya interaction:

$$H_{DM} = \gamma \vec{S} \cdot (\vec{\nabla} \times \vec{S})$$

C. Pfleiderer et al., Nature 427, 227 (2004).



# The $p$ -wave $\beta$ -phase: spontaneous chiral instability

- **Chiral instability** in the **originally non-chiral** system.
- Dynamic generation of Dzyaloshinskii-Moriya-like spin-orbit coupling.

$$\gamma_2(\partial_x \vec{n}_2 - \partial_y \vec{n}_1) \cdot (\vec{n}_1 \times \vec{n}_2)$$

No spin-moments; spirals of spin-dipole moment.

- This chiral instability does not occur in the ferromagnetic transition and in any other channel Pommeranchuk instability.

# Outline

- Introduction.
- Mechanism for unconventional magnetic phase transitions.
- Low energy collective modes.
- **Possible experimental realization and detection methods.**

# A natural generalization of ferromagnetism

- The driving force is still exchange interactions, but in **non-s-wave** channels.

	<i>s</i> -wave	<i>p</i> -wave	<i>d</i> -wave
SC/SF	Hg, 1911	<sup>3</sup> He, 1972	high T <sub>c</sub> , 1986
magnetism	Fe, ancient time	?	?

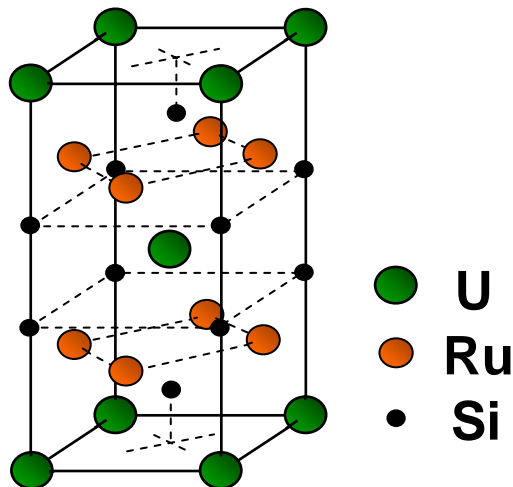
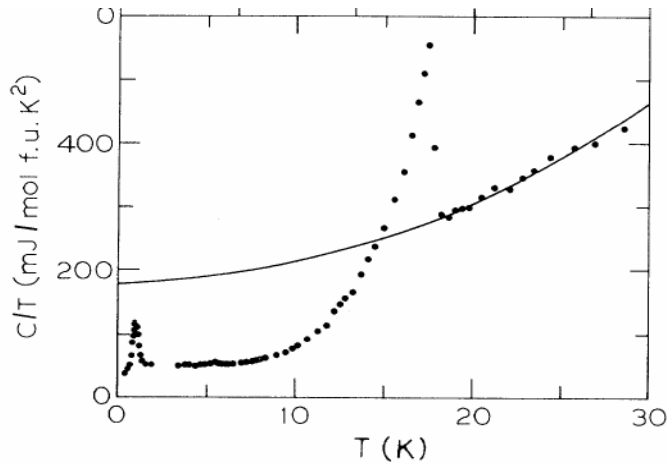
- Optimistically, unconventional magnets are probably not rare.

cf. Antiferromagnetic materials are actually very common in transition metal oxides. But they were not well-studied until neutron-scattering spectroscopy was available.

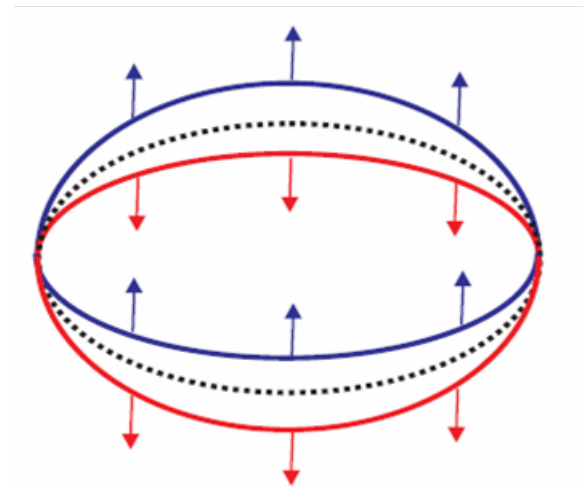


# Search for unconventional magnetism (I)

- $\text{URu}_2\text{Si}_2$ : hidden order behavior below 17.5 K.



T. T. M. Palstra et al., PRL 55, 2727 (1985); A. P. Ramirez et al., PRL 68, 2680 (1992)

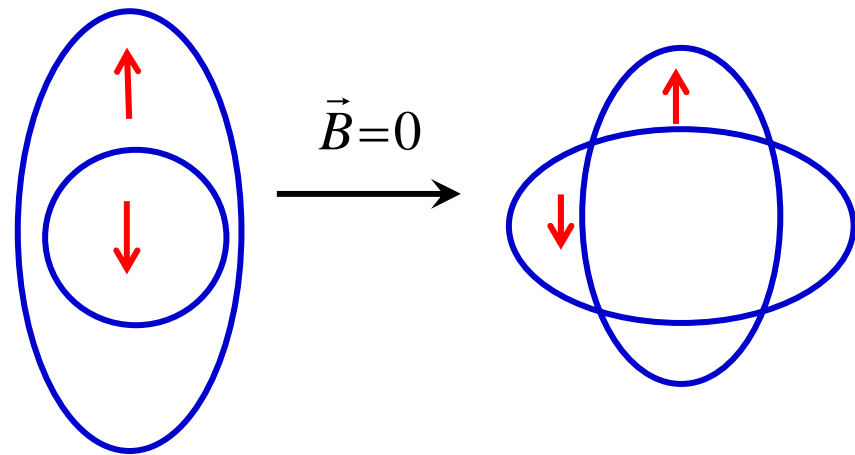
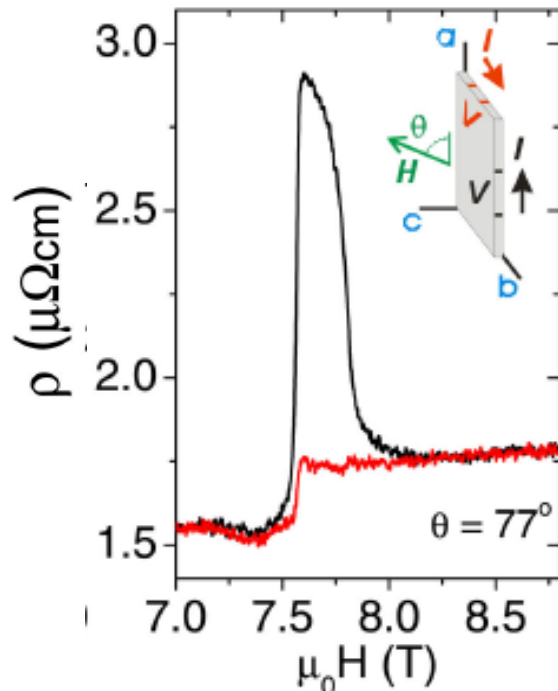


**helicity order (the  $p$ -wave  $\alpha$ -phase);**

Varma et al., Phys. Rev Lett. 96, 036405 (2006)

# Search for unconventional magnetism (II)

- $\text{Sr}_3\text{Ru}_2\text{O}_7$  (large external B field): a mixture of the instabilities in the density and spin channels.
- We need an instability purely in the spin channel with zero field.



# Search for unconventional magnetism (III)

- 2D electron gas. Quantum Monte-Carlo simulation of Landau parameters.

Y. Kwon, D. M. Ceperley, and R. M. Martin, PRB 50, 1684 (1994).

- $F_1^a$  is negative and decreases as the density is lowered.

	$r_s = 1.0$	$r_s = 2.0$	$r_s = 3.0$	$r_s = 5.0$
$F_0^s$	-0.60(1)	-0.99(1)	-1.63(1)	-3.70(3)
$F_0^a$	-0.34(3)	-0.41(8)	-0.49(7)	-0.5(1)
$F_1^s$	-0.14(2)	-0.10(1)	-0.03(1)	0.12(2)
$F_1^a$	-0.19(2)	-0.24(1)	-0.26(1)	-0.27(2)
$F_2^s$	-0.07(2)	-0.16(3)	-0.27(3)	-0.50(5)
$F_2^a$	0.01(2)	0.07(3)	0.14(3)	0.32(5)
$m^*/m$	0.93(1)	0.95(1)	0.99(1)	1.06(1)
$g^*/g$	1.52(6)	1.7(2)	2.0(3)	1.9(4)

$$n \propto r_s^{-2}$$

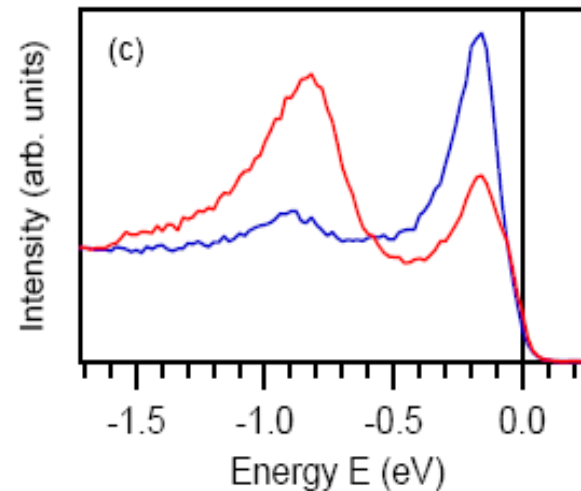
- It would be great if the simulation can be further extended into even lower density regime.

# Detection (I): ARPES

- Angular Resolved Photo Emission Spectroscopy (ARPES).

ARPES in spin-orbit coupling systems ( Bi/Ag surface), Ast et al., cond-mat/0509509.

**band-splitting for two spin configurations.**



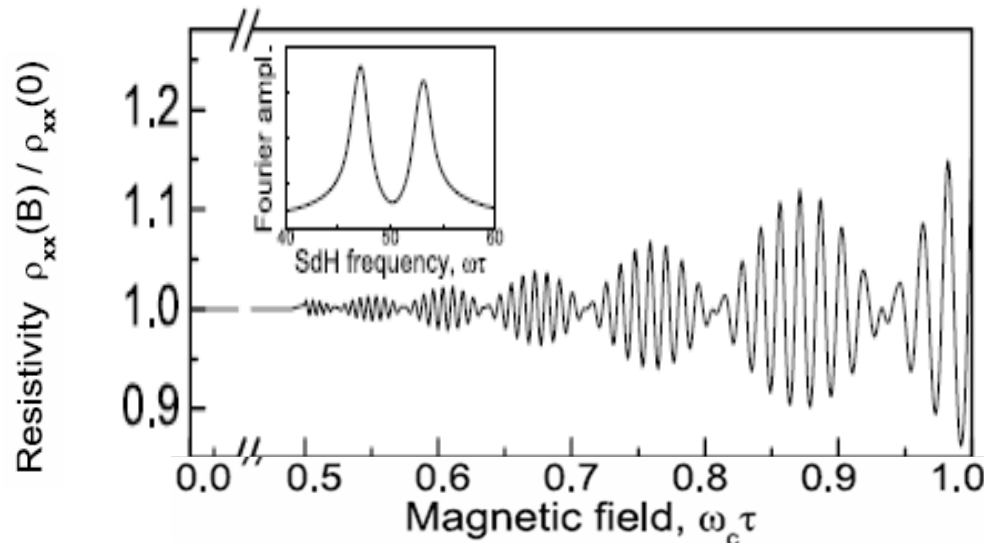
- $\alpha$  and  $\beta$ -phases (dynamically generated spin-orbit coupling):

The band-splitting is proportional to order parameter, thus is temperature and pressure dependent.

# Detection (II): neutron scattering and transport

- Elastic neutron scattering: no Bragg peaks;  
Inelastic neutron scattering: resonance peaks below  $T_c$ .
- Transport properties.

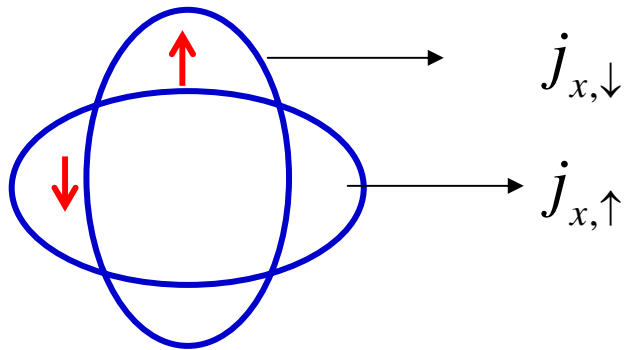
$\beta$ -phases: Temperate dependent beat pattern in the Shubnikov - de Hass magneto-oscillations of  $\rho(B)$ .



N. S. Averkiev et al.,  
Solid State Comm. 133,  
543 (2004).

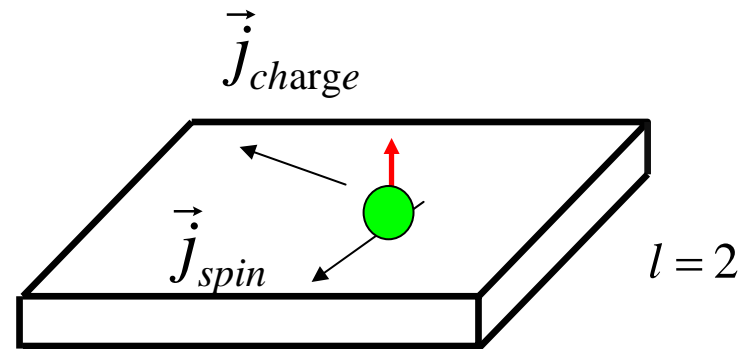
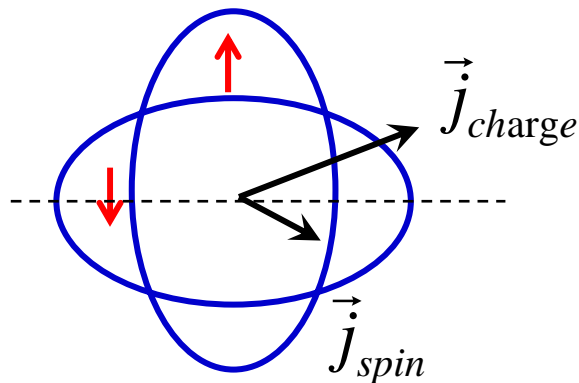
# Detection (III): transport properties

- Spin current induced from charge current (d-wave). The directions of charge and spin currents are symmetric about the x-axis.



$$j_{x,charge} = j_{x,\uparrow} + j_{x,\downarrow}$$

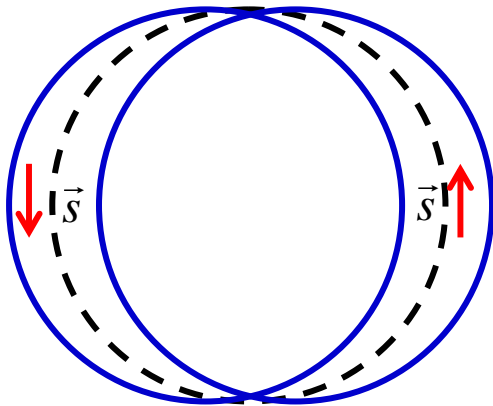
$$j_{x,spin} = j_{x,\uparrow} - j_{x,\downarrow}$$



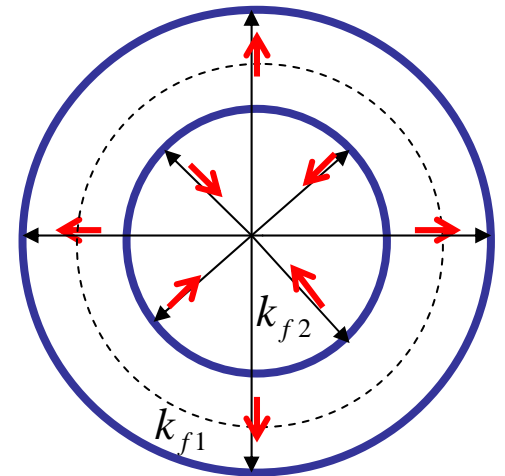
# Summary

unconventional  
superconductivity

$\alpha$ -phase



$\beta$ -phase



unconventional  
magnetism

electron liquid crystal  
with spin

spin-orbit  
coupling

# Two dimensional metal-insulator transition?

Conjecture:

