Superfluidity of Attractive Hubbard Model in Optical Lattice

Chih-Chun Chien, Qijin Chen, Yan He and K. Levin
University of Chicago

Hamiltonian

\[ H = -t \sum_{\langle i,j \rangle, \sigma} (c_{i,\sigma}^\dagger c_{j,\sigma} + c_{j,\sigma}^\dagger c_{i,\sigma}) + U \sum_i n_{i,\uparrow} n_{i,\downarrow} \quad (U < 0) \]

- Weak coupling (|U|/t << 1): BCS superfluid
  Pairing in momentum space
  \[ E_k = \sqrt{\epsilon_k - \mu}^2 + \Delta^2 \quad \Delta = -U \sum_k \langle c_k | c_k \rangle \]
  \[ \epsilon_k = -2t \sum_i \cos k_i \]
- Strong coupling (|U|/t >> 1): “Composite bosons”
  Pairing on one lattice site
  \[ b_i = c_{i,\uparrow}^\dagger c_{i,\downarrow}^\dagger \quad [b_i, b_j^\dagger] = 1 - 2n_i, \quad n_i = b_i b_i^\dagger \]
  Pair hopping ~ \( t^2 / U \)
  Effective nearest-neighbor repulsion ~ \( t^2 / |U| \)

R. Micnas et al., Rev. Mod. Phys. 62, 113

** New phase may exist when density is high. **

Finite Temperature: Pseudogap Effect

- Pairs form at one temperature \( T^* \)
- Pairs condense at lower temperature \( T_c \)
- New form of condensate excitation—pair excitations below \( T_c \).

T-matrix Formalism

- Consistent with the BCS-Leggett ground state
- T-matrix:
  \[ t(Q) = \cdots = |\begin{array}{ccc} S & \cdots & S \\ Q \cdot k & \cdots & Q \cdot k \end{array}| + \cdots \]
- Fermion self-energy:
  \[ \Sigma = -\Delta^2 G_0 \quad \Delta^2 = \Delta_{pg}^2 + \Delta_{sc}^2 \]

Self-consistent Equations

- Gap equation: (Thouless criterion)
  \[ 1 + U \sum_k \frac{1 - 2f(E_k)}{2E_k} = 0 \quad \mu_{pair} = 0 \]
- Pseudogap equation: Pair density
  \[ \Delta_{pg}^2 = -\sum_{Q \neq 0} t(Q) \]
- Number equation
  \[ n_\sigma = \sum_K G_\sigma(K) \]

Behavior of \( T_c \) and \( T^* \)

\[ T_c \text{ drops to zero when density is too high.} \]

Superfluid Density

Linear response theory:

\[ J_\mu(Q) = \left[ P_{\mu\nu}(Q) - \frac{n_\mu}{m} \right] A_\nu(Q) \]

Diagrams for the electromagnetic vertex

The diagrams and vertex guarantee that non-condensed pairs do not contribute to Meissner effect when \( T < T_c \):

\[ \Lambda' = \lambda + \delta \Lambda_{pg} - \delta \Lambda_{sc} \]

\[ \frac{n_\mu}{m} = \frac{2}{3} \sum_k \frac{\Delta_{sc}^2}{E_k^2} \left[ 1 + 2f(E_k) + f'(E_k) \right] (\nabla_k \xi_k)^2 \]

(ns/m) is well-behaved between \( T=0 \) and \( T=T_c \).

New Phase: Localized Pair States

Possible scenario of pair density wave

Localization causes incoherence and destroys superfluidity.

d-Wave Case: Cuprate Phase Diagram

Apply crossover theory to d-wave lattice case

Mignod et al., Physica B 169,58; Oda et al., Physical C 281,135