BEC-BCS crossover, phase transitions and phase separation in polarized resonantly-paired superfluids

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Overview

• Bose-Einstein condensation (BEC) of dilute vapors of alkali atoms
  – All bosons in same quantum state
  – Superfluidity
  – Condensed matter physics in an atomic physics setting

• Recent interest: condensation of two types of atomic fermion
  
  – Fermionic superfluid

• Relies on strong attraction between fermions: Feshbach resonance
  – Novel experimental knob: Tune interaction strength
  – Crossover from BEC to BCS superfluidity

• Recent Work: Apply spin polarization to fermion superfluid
  – Usual case: Equal numbers of $\uparrow$ and $\downarrow$
  – Polarization: More $\uparrow$ than $\downarrow$
  – Exotic phases, phase separation

Next: Fermionic superfluids…
Fermionic pairing of cold atoms
Regal et al PRL 04; Zwierlein et al ibid; Kinast et al ibid, Bartenstein et al ibid, Bourdel et al ibid; Partridge et al ibid 05…

• Fermionic superfluidity: atomic fermions $^{40}$K, $^6$Li
  
  Ultracold: $\sim$10-100 nK
  Dilute: $\sim 10^{10}$-10$^{13}$ cm$^{-3}$

• Confined to a parabolic (harmonic) trap -- typically optical

  Atoms attracted to laser beam

  – Pairing of two distinguishable fermion species
  – “Spin” -- different atomic hyperfine states

  (e.g. $^{40}$K: $f$=9/2, $m_f$ = 7/2,9/2)
  
  total atom spin

• Novel feature: Interactions experimentally tunable
  
  – Feshbach resonance: interactions enhanced by applied magnetic field $B$

  $a_s \propto \frac{1}{B - B_0}$

  $B_0$ “resonance position”

  Next: Feshbach resonance
One channel model of Feshbach Resonance

• Near resonance:
  \[ a_s \propto -\frac{1}{B - B_0} \]
  \( B \) is the magnetic field, \( B_0 \) is the resonance field.

  e.g., Regal & Jin PRL 90, 230404 (2003)

• One-channel model:
  \[ H = \sum_{p,\sigma} (\varepsilon_p - \mu) c_{p\sigma}^\dagger c_{p\sigma} + g \sum_{p,q,k} c_{k\uparrow}^\dagger c_{p\downarrow} c_{k+q\downarrow} c_{p-q\uparrow} \]
  \( \sigma = \uparrow, \downarrow \)
  \( g < 0 \)
  \( g \) is the scattering length.

  Strong attractive interactions

• Reproduces correct two-body physics
  – Vacuum scattering length:
    \[ a_s \propto \frac{1}{|g| - \frac{2\pi^2}{m\Lambda}} \]
    \( \Lambda \) is the ultraviolet (UV) cutoff.
  – Gas at density \( n \): Mean-field theory based on BCS wavefunction
    – Quantitatively valid for weak coupling (small \( g \))
    – Qualitatively valid for any coupling

Next: BEC-BCS crossover

Holland et al PRL 01
Ohashi & Griffin PRL 02
Andreev et al PRL 04
**BEC-BCS crossover: Mean-field theory**

- **Theory:** smooth crossover between BEC and BCS limits
  - Assume: Variational BCS ground state characterized by the pairing \( \Delta = \left\langle c_k \uparrow c_{-k} \downarrow \right\rangle \)
  - Minimize variational ground-state energy

**Positive detuning: BCS regime** \( (a_s < 0) \)
- Weakly attractive interactions
  
  \[
  \mu > 0 \\
  \Delta \ll \varepsilon_F
  \]

  Neutral BCS superconductor!!

  Cooper Pair size \( \gg \) Interparticle spacing

**Negative detuning: BEC regime** \( (a_s > 0) \)
- Strong attractive interactions
  
  \[
  \mu < 0 \\
  n_m \propto |\Delta|^2 \approx n/2
  \]

  Tightly bound Molecular BEC

  Molecule size \( \ll \) Interparticle spacing

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**Chemical potential** \( \mu / \varepsilon_F \)

**Pairing** \( \Delta / \varepsilon_F \)

Next: Validity
Validity of BEC-BCS mean-field theory

- Quantitatively valid at large positive detuning (BCS) regime $a_s < 0$, $|a_s| << k_F^{-1}$
  - Fluct’s around mean field theory grow with reduced detuning

- Asymptotic negative detuning (BEC) regime $a_s > 0$, $|a_s| << k_F^{-1}$
  - Gas of repulsive bosons
    - BCS wavefn: $a_m = 2a_s$
    - Exact analysis: $a_m = 0.6a_s$
    - Born approx. result
    - Petrov et al PRL 04; Levensen & Gurarie PRA 06
    - Confirms qualitative validity (and quantitative invalidity) of BCS w.f.

- Unitary regime: $|a_s| >> k_F^{-1}$
  - Universal: Only energy scale is $\varepsilon_F = \frac{k_F^2}{2m}$
    - No small parameter
    - Monte Carlo
    - Introduce artificial small parameter
      - Narrow resonance - Andreev et al PRL 2004
      - Epsilon expansion (spatial dimension) - Nishida & Son PRL 2006
        also Nikolic & Sachdev 2007

Next: Exp’ts
Experiments: Smooth crossover & superfluidity

- Measure **condensation** directly: Occupation of lowest state
  - Regal et al PRL 2004
  - Zwierlein et al PRL 2004

- Measure binding energy of pairs/molecules
  - Increases with reduced detuning
  - Chin et al Science 2004
  - Partridge et al PRL 2005

- Collective oscillations: consistent with superfluidity
  - Indirect measure of superfluidity
  - Kinast et al PRL 2004

- Rotation of cloud: Vortices across resonance
  - Zwierlein et al Nature 2005
  - Direct measure of superfluidity
  - Vortices in a Bose-Einstein condensate
  - Vortices in a neutral BCS superconductor

Next: Spin Polarization
Applied spin polarization

• Recent work*: Explore changing relative number of $\downarrow$, $\uparrow$
  – Additional experimental “knob” for cold-atom experiments

  Polarization: $P = \frac{N_{\uparrow} - N_{\downarrow}}{N_{\uparrow} + N_{\downarrow}}$

  Aim: Extend phase diagram to polarized case $P \neq 0$

  – Analogous to applying a Zeeman magnetic field that favors more $\uparrow$ than $\downarrow$
    • “Pauli limiting” (Clogston limit) magnetic field in superconductors
    • FFLO state: Exotic inhomogeneous superconductor $\Delta(r) \propto \cos[Q \cdot r]$

  – Strongly-interacting fermions with density imbalance
    • crystalline color superconductivity Alford et al PRD 01, Liu & Wilczek PRL 03
  
  – Smooth crossover fractured into rich phase diagram

  Phase transitions, polarized superfluidity, polarized Fermi liquid, phase separation...

*Theory: DS & Leo Radzihovsky, PRL 06, Ann. Phys. 07, PRB 07, Bedaque et al PRL 03, Carlson & Reddy PRL 05, Cohen PRL 05
Pao et al PRB 06, Son & Stephanov PRA 06, Chien et al PRL 06, Parish et al Nat. Phys. 07,....

Exp’t: Zwierlein et al Science 06, Partridge et al Science 06,....

Next: Model
Model of a polarized superfluid

Model: \[ H = \sum_{p,\sigma} (\varepsilon_p - \mu_\sigma) c_{p\sigma}^\dagger c_{p\sigma} + g \sum_{p,q,k} c_{k\uparrow}^\dagger c_{p\downarrow}^\dagger c_{k+q\downarrow} c_{p-q\uparrow} \]

- Different chemical potentials \( h = \mu_\uparrow - \mu_\downarrow \) induce (polarization) \( P \)

- BCS superconductor under applied Zeeman field \( h \): First-order transition

- Phenomenology of first-order transitions
  - Phase separation, metastable, unstable solutions
  - Cannot detect transitions locally
    - Local criteria determines spinodals

- Must globally minimize ground-state energy (or free energy) to correctly obtain phase diagram
  - Sarma 1963: Solutions to gap equation that do not minimize \( E_G \)
    \[ 0 = \frac{dE_G}{d\Delta} \]
Global phase diagram

\[ P = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow} \]

Top axis: \( N_\downarrow = 0 \)
(Pure Normal)

Polarized Fermi gas

Bottom axis: Smooth BEC-BCS crossover phase

“Magnetic superfluid”: Tightly-bound molecules and spin-polarized Fermi sea

See also: Gu et al cond-mat 06, Parish et al 07
BCS regime zero-T phase diagram

- **Grand-canonical ensemble:**
  - Impose $\mu_\uparrow, \mu_\downarrow$, minimize $E_G(\Delta) = \langle \Psi|H|\Psi \rangle$
  - Adjust $\mu_\uparrow, \mu_\downarrow$ to achieve correct $N_\uparrow, N_\downarrow$ where $N_\sigma = \langle \Psi|\hat{N}_\sigma|\Psi \rangle$
  - Certain regimes: cannot achieve correct $N_\uparrow, N_\downarrow$…Maxwell construction

- **Positive-detuning BCS regime:**
  - $h = 0$:
    - Usual BCS pairing
  - $h \neq 0$:
    - Second local minimum @ $\Delta = 0$
    - Global min. unaltered
    - Transition to state at $\Delta = 0$
    - Unpaired N state stable!
  - $h = h_c \equiv \Delta_0/\sqrt{2}$:
    - Clogston 1962
    - Unpaired N state stable!

- Minimum at:
  - $\Delta_0 \approx 8e^{-2}\mu \exp\left[\frac{\pi}{2k_Fa_s}\right]$
  - Usual BCS pairing
  - BCS won’t polarize! Sarma 1963

- True solution is phase separated in such regimes
First-order BCS-to-N transition

- Polarization (P) and pairing (\(\Delta\)) jump at first-order BCS-N transition

**BCS regime phase diagram:**

- Paired BCS phase: \(P = 0\)
- Phase sep. for arbitrarily small \(P\)
- Below solid line: N phase unstable via first-order transition \(P_{c2} \propto \Delta_0\)
- Thin window of inhomogeneous FFLO state (FFLO-N continuous trans.)

**BCS**

\(n_\uparrow = n_\downarrow\)

**Normal**

\(n_\uparrow > n_\downarrow\)

- Polarization \(P < P_c\): cannot be attained for a homogeneous phase
- Phase separation to achieve imposed polarization
**FFLO state**

- Excess spin $\uparrow$: Larger Fermi surface $p_{F\uparrow} \propto n_{\uparrow}^{1/3}$, $p_{F\downarrow} \propto n_{\downarrow}^{1/3}$

- Pairing of low-energy states near Fermi surface: $Q \equiv p_{F\uparrow} - p_{F\downarrow}$
  
  Cooper pairs have finite momentum! $\Delta(r) \propto \cos[Q \cdot r]$ 
  - Breaks rotational and translational symmetry

- Evaded observation in condensed-matter systems
  - Disorder
  - Coupling of physical magnetic field to orbital electron motion
  - Possibly observed in CeCoIn$_5$ Radovan et al Nature 2003 Bianchi et al PRL 2003

- Motivation: Observe FFLO in cold-atom experiment?
  - Perfectly clean; Purely Zeeman coupling
  - Spontaneous crystalline order observable in time-of-flight exp’ts

Fulde & Ferrell PR 1964; Larkin & Ovchinnikov JETP 1965
Predictions for FFLO regime

- Simplest FFLO-type state: \( \Delta(r) = \Delta_Q \exp[iQ \cdot r] \)

  - More generally: \( \Delta(r) = \sum_Q \Delta_Q \exp[iQ \cdot r] \)

  Bowers & Rajagopal PRD 02

- \( P_{c2} < P_{FFLO} \) are \( \sim \exp \left[ -\frac{c}{k_F |a_s|} \right] \)

- Large detuning: \( \frac{P_{FFLO}}{P_{c2}} \approx 1.07 \)

- \( P_{FFLO} \) crosses \( P_{c2} \) at \( -\frac{1}{k_F a_s} \approx 0.5 \)

  FFLO no longer globally stable

- Critical polarization \( P_{FFLO} \approx \frac{3}{2} \frac{\eta \Delta}{\varepsilon_F} \)

  Estimate using exp’t parameters

  Chin et al Science 2004

- FFLO wavevector: \( Q \approx 2\eta \lambda \frac{\Delta}{\hbar \varepsilon_F} \)

- Phase separation: SF-FFLO coexistence underneath \( P_{FFLO} \)

  Still observable in time of flight
Mean-field theory predictions: Positive detuning

- $P_{c1}=0$: Phase Separation for any small $P$
  - Polarization above which system phase sep.

- $P_{c2}=0.93$ at unitarity
  - Polarization below which system phase sep.

- Phase separated regime: Two phases in chemical equilibrium with different densities
  - Harmonic trap: higher density phase (superfluid) falls to the center

Shell structure: Imposed polarization goes to the edge, center paired!
Evidence for shell structure in phase separation regime

- Partridge et al Science 2006: Density data
  - Integrated in one direction
  - Highly prolate trapping potential
    \[ N_\uparrow = 8.6 \times 10^4 \]
    \[ N_\downarrow = 6.5 \times 10^4 \]

- Shin et al PRL 2006: Shrinking BCS core with increasing polarization

Plots: Integrated Magnetization
\[ M(r) = n_\uparrow(r) - n_\downarrow(r) \]

- Quantitative understanding: Go beyond local density approximation to handle trap
  Kinnunen et al, Yi & Duan, Chevy, De Silva & Mueller, Imambekov et al, …

Next: BEC regime
BEC superfluid under applied $h$

- $h = 0$: BEC superfluid
  - Paired molecular bosons
  - Fermions gapped: $\mu < 0$

  $$\mu = -\frac{\hbar^2}{2ma_s^2}$$

  - vacuum of fermions; BEC of pairs

- Apply $h$ to induce polarization $P$

  $$\mu_\uparrow = \mu + h$$

  $$\mu_\downarrow = \mu - h$$

  Tilt Fermion bands!

  - Upper fermion band dips below mol. level
  
  - Pairs break up into spin-$\uparrow$ fermions

  - Coherent mixture of BEC and spin-$\uparrow$ Fermi gas

  Magnetic superfluid ($\text{SF}_M$)

Next: $\text{SF}_M$
Magnetic superfluid ($S_{FM}$) phase

- **Negative detuning:** BEC tolerates small polarization
  - Unlike BCS regime
  - Minority spins pair; excess majority form Fermi sea

- **Spin-up fermion & BEC are miscible fluids**
  - Analogous to $^3$He-$^4$He mixtures!

- **First-order transition to phase sep. with increasing $P$**
  - Compute molecular scattering length vs. $\hbar$
    \[ a_m(\hbar) = 2a_s F(h/|\mu|) \]
    \[ F(0) = 1 \]
  - $F(x)$ vanishes at $x = 1.30$

- **Stability requires $a_m > 0$**
  - Instability at: $h_{c1} \approx 1.30 \frac{\hbar^2}{2ma_s^2}$
  - determined more accurately by minimizing $E_G$
BEC regime phase diagram

• SFM phase: **First-order** transition into regime of phase separation
  
  – $P_{c1}$: first-order trans. to phase sep.
  
  – Deep BEC: 2nd-order trans. to fully polarized
  
  – **Red** dot: tricritical point
  
  – $P_{c2} = 1$ except close to unitarity

  Add one spin $\downarrow$ to Fermi sea of $\uparrow$: Forms Pair

• Bogoliubov sound velocity in SF$_M$: Driven to zero near transition at $P_{c1}$

  – Relation between $a_m$ and sound vel.
  
  – Transition to phase sep. precedes vanishing

Next: LDA in BEC regime
Magnetic superfluid in a trap: LDA

Small $P$: Interior $\text{SF}_M$ shell

Three shells:

Large $P$: $\text{SF}_M$ surrounded by Normal

Two shells:

$n_m(r) = \text{molecular density}$

$M(r)$

$\frac{r}{R_{TF}}$

SF, $\text{SF}_M$, N
Global phase diagram

\[ P = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow} \]

Top axis: \( N_\downarrow = 0 \)
(Pure Normal)

Bottom axis: Smooth BEC-BCS crossover phase

"Magnetic superfluid": Tightly-bound molecules and spin-polarized Fermi sea

- 1st order
- 2nd

FFLO – \( N \) 2nd

\( P_{c1}, P_{c2} \)

Recent other work: Finite T: Parish et al, Chien et al…
Including trap: Kinnunen et al, Duan et al, …
Dynamics of phase separation: Lamacraft et al

Next: Tricritical
Tricritical point at nonzero T

Extension to finite T: (Parish et al, Chien et al)

$P_{c1}$  $P_{c2}$  FFLO  

$P$  tricritical point  

$\frac{1}{k_Fa_s}$

$\delta_c$  $\delta_M$  $\delta_s$  0  1  2  

SF$_M$  PS  N  

By analogy:

$T$  

$2nd$  

$SF_M$  PS  

$P_{c1}$  $P_{c2}$  

$1st$  

$3He-4He$ mixtures


$3He$ fraction

Other possibilities:

Blume Emery Griffiths 70
Concluding remarks

• Cold-atom experiments studying superconductivity of paired fermions
  New context for strongly-correlated condensed-matter physics

• Other examples of interacting fermion systems: Superconductors, quark matter, nuclear matter, ...

• Experiments already observed crossover between BEC and BCS states

• Different numbers of \( \uparrow \), \( \downarrow \): Simple crossover “fractured”
  – Phase transitions
  – Phase separation
  – Magnetic superfluidity
  – Fulde-Ferrell-Larkin-Ovchinnikov states

• Future work:
  – Finite-T phase diagram near unitary point
  – Vicinity of the tricritical point
  – Strongly coupled normal state (recent MIT experiments)
  – Experimental signatures of FFLO and SF\(_M\)