Soft gluon effects in Higgs production at the LHC

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- Inclusive cross section
  - QCD cross section at NNLO
  - Soft-gluon resummation at NNLL
  - Residual theoretical uncertainty
- Transverse momentum distribution
  - An improved resummation formalism
  - NNLL+NLO results

Direct Higgs production at NNLO

We can identify three kinds of contributions to σ_H^{NNLO} according to their behaviour as $z = M_H^2/\hat{s} \to 1$

- Soft and virtual (SV) contributions: they are the dominant terms as $z \to 1$

  R.Harlander (2000)
  S.Catani, D. de Florian, MG (2001)
  R.Harlander, W.Kilgore (2001)

- Collinear contributions: next-to dominant as $z \to 1$

  M.Kramer, E.Laenen, M.Spira (1998)

- Hard effects: finite as $z \to 1$

  R.Harlander, W.Kilgore (2002)
  C.Anastasiou and K.Melnikov (2002)
  V.Ravindran, J.Smith, W.L. van Neerven (2003)

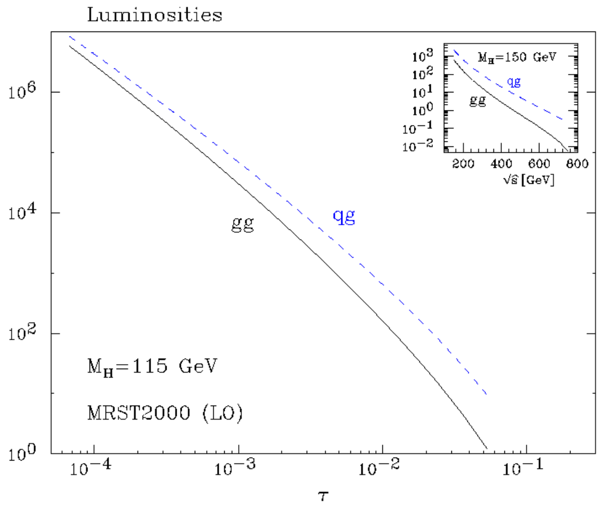
The bulk of the NNLO effect is given by SV(C) contributions

The hard contributions are only about 2% at LHC and 4% at the Tevatron

⇒ This is REASSURING because these are the effects that are most sensible to the top-quark loop !

Why are SV(C) effects so important ?

The hard cross section is convoluted with the parton distributions that are strongly peaked at small x:
$< \hat{s} >=< x_1 x_2 > s \ll s$



⇒The hard cross section is almost always evaluated close to threshold

The $M_H \ll m_{top}$ approximation

For a light Higgs it is possible to use an effective lagrangian in the limit $m_{top} \to \infty$:

$$\mathcal{L}_{eff} = -\frac{1}{4}\left[1 - \frac{\alpha_S}{3\pi}\frac{H}{v}(1+\Delta)\right] Tr G_{\mu\nu} G^{\mu\nu}$$

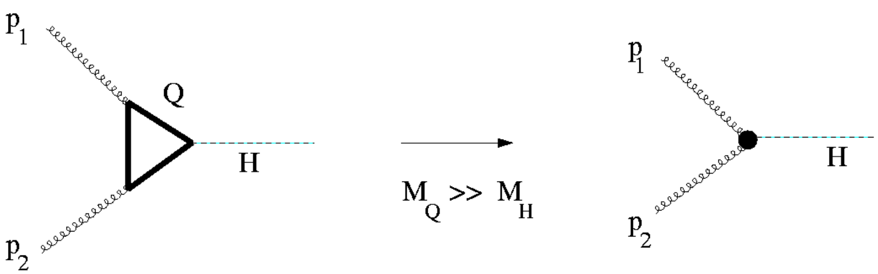
J.Ellis, M.K.Gaillard, D.V.Nanopoulos (1976)
M.Voloshin, V.Zakharov, M.Shifman (1979)

The correction Δ is known up to $\mathcal{O}(\alpha_S^3)$

K.G.Chetyrkin, B.A.Kniehl, M.Steinhauser (1997)

The approximation is correct within 5% if $M_H < 2\, m_{top}$

Effective vertex: one loop less



The approximation works well even for large M_H if full Born result is retained

Soft gluon resummation

S.Catani, D.de Florian, P.Nason, MG (2002)

Inclusive cross section dominated by soft and collinear emission ⇒Multiple soft emission beyond NNLO can be important

In N-space the large logs appear as $\alpha_S^n \log^{2n} N$

This large corrections can be resummed to all orders:

$$G_{gg\to H,N}(\alpha_S) = C(\alpha_S)\exp\Big\{\ln N g_1(\beta_0\alpha_S \ln N) + g_2(\beta_0\alpha_S \ln N) + \alpha_S\, g_3(\beta_0\alpha_S \ln N) + \dots\Big\}$$

The function g_1 controls the LL contributions g_2 the NLL contributions g_3 the NNLL ones and so on

At NNLL three new coefficients appear:

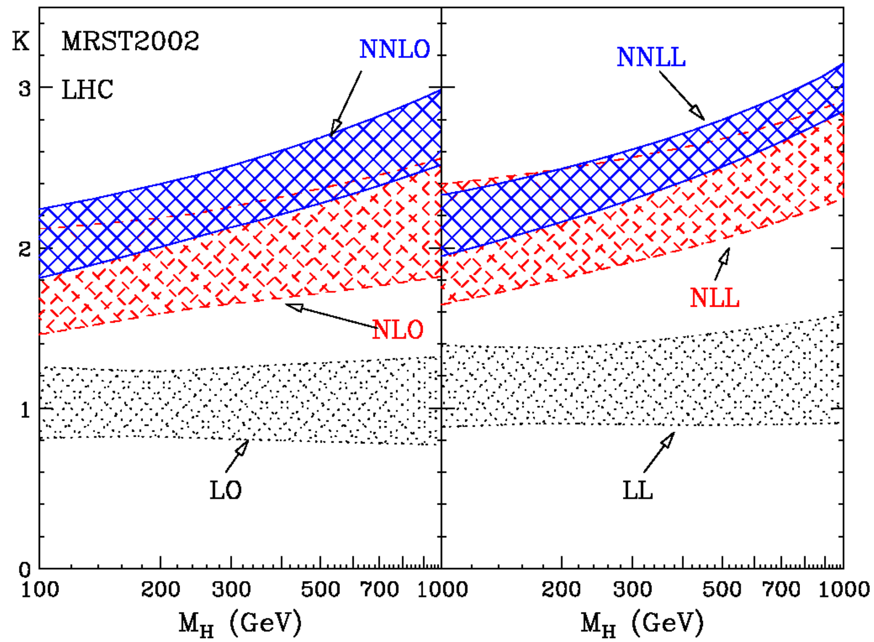
- $D^{(2)}$, $C^{(2)}$ that can be obtained from the NNLO result
- $A^{(3)}$ which is known numerically

A.Vogt (2000)

⇒ We can go to NNLL+NNLO !

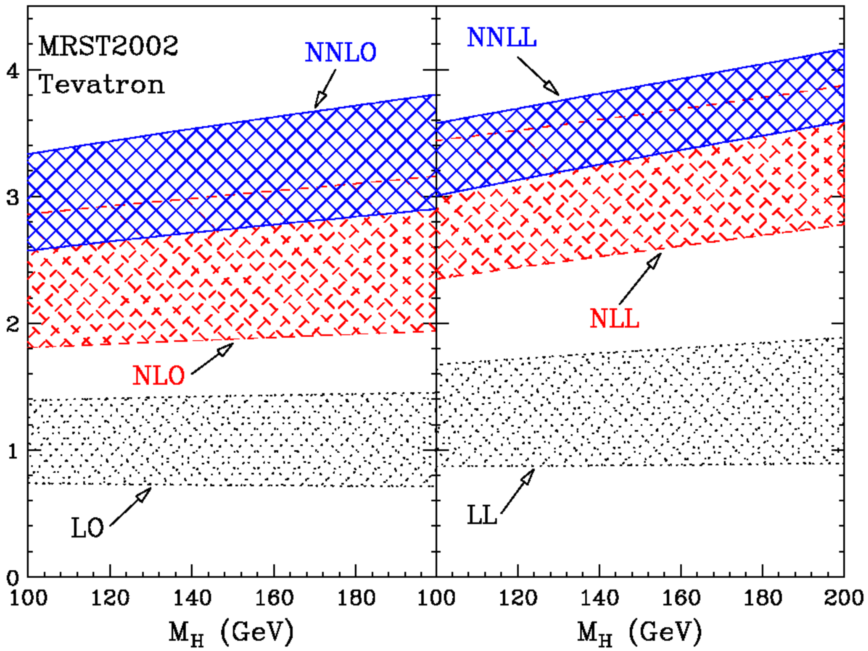
Notice:
This is the first calculation ever performed to this accuracy

Results at the LHC



- Resummed result matched to corresponding fixed order
- K-factors defined with respect to $\sigma^{LO}(\mu_F = \mu_R = M_H)$
  - with $\mu_{F(R)} = \chi_{F(R)} M_H$ and $1/2 \leq \chi_{F(R)} \leq 2$
  - but $1/2 \leq \chi_F/\chi_R \leq 2$
- For a light Higgs:
  - NNLL effect about +6% with respect to NNLO
  - Scale uncertanty at NNLL of about ±8%

Results at the Tevatron



- Effect of about $12-15\%$
- Bands defined as for LHC
- Reduction in scale uncertainty from about $\pm 13\%$ at NNLO to about $\pm 8\%$ at NNLL

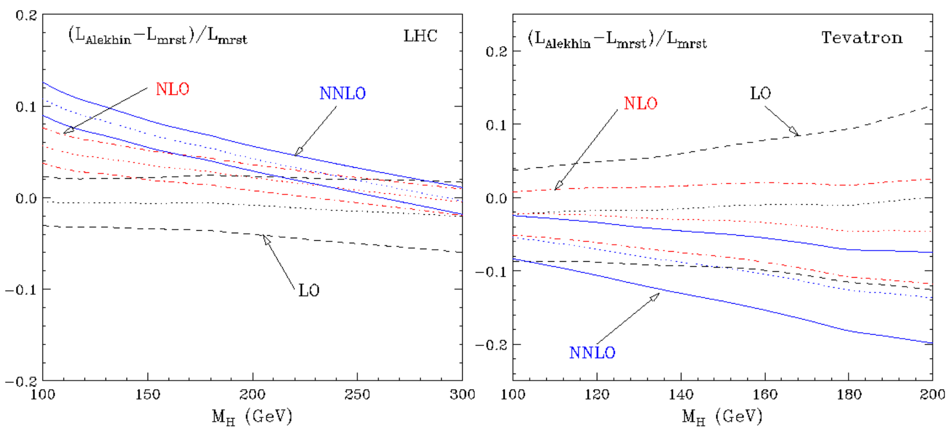
What is the residual theoretical uncertainty on σ_H ?

- scale dependence
- large-m_{top} approximation
  - At NLO the approximation works well BECAUSE the cross section is dominated by soft radiation that is weakly sensible to the heavy quark loop
  - The dominance of soft contributions persists at NNLO $\Rightarrow$it is natural to expect the large-m_{top} approx. work well also at higher orders
  - Message from NLO:
    Use exact Born cross section
    (with m_{top}, m_b dependence) to normalize the result
  - Residual uncertainty from here $\lesssim 5\%$
- Parton distributions
  - At NLO CTEQ6 and MRST2002 results agree reasonably well
  - At the moment only two approximated NNLO pdf sets: MRST, Alekhin

Comparing MRST and Alekhin results we find (relatively) large differences

- LHC: Alekhin results are larger than MRST : difference from 8% at $M_H = 100$ GeV to 2% at $M_H = 200$ GeV
- Tevatron: Alekhin results are smaller than MRST: difference from 7% at $M_H = 100$ GeV to 14% at $M_H = 200$ GeV

Errors only quoted by Alekhin and probably underestimated



The differences are due to the gg luminosities and increase with the order

⇒ A theoretical accuracy of about 10% on σ_H can be attained once problems with NNLO pdf will be solved

The q_T spectrum of the Higgs

G. Bozzi, S. Catani, D. de Florian, MG (2003)

- Signal and background have different shape in q_T ⇒knowledge of q_T distribution can help to improve statistical significance
- Higgs q_T spectrum is expected to be harder than $\gamma\gamma$ background

Studies of the Higgs q_T distribution have been performed at various levels of accuracy

I.Hinchliffe, S.F.Novaes (1988)
R.P.Kauffman (1992)
C.P Yuan (1992)
C.Balazs, C.P Yuan (2000)

Recently (almost) NNLL but still matching to LO

E.L.Berger, J.Qiu (2002)

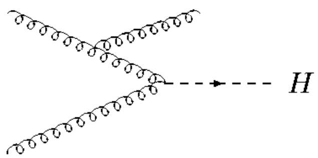
⇓

Our work:

- Include the most complete information which is available at present: NNLL resummation at small q_T and NLO perturbation theory at large q_T
- "Improve" the implementation formalism

The region $q_T \sim M_H$

To have $q_T \neq 0$ the Higgs has to recoil against one parton
$\Rightarrow$The LO is $\mathcal{O}(\alpha_S^3)$



Amplitudes used at LO: $gg \to gH$, $q\bar{q} \to gH$

The calculation shows that the large-M_t approximation works well as long as both q_T and M_H are smaller than M_t

R.K.Ellis, I.Hinchliffe, M.Soldate, J.J.van der Bij (1988)
U.Baur, E.W.Glover (1990)

NLO corrections computed in this limit

D. de Florian, Z. Kunszt, MG (1999)

Amplitudes used at NLO:

- 1 Loop: $gg \to gH$, $q\bar{q} \to gH$

C. Schmidt (1997)

- Bremsstrahlung: $gg \to ggH$, $q\bar{q} \to ggH$, $q\bar{q} \to q\bar{q}H$

R.Kauffman, S.Desai, D.Risal (1997)

The IR singularities were cancelled using the subtraction method

S.Frixione, Z.Kunszt, A.Signer (1996)

Implemented in a parton level Monte Carlo
$\Rightarrow$ HIGGSJET NLO code

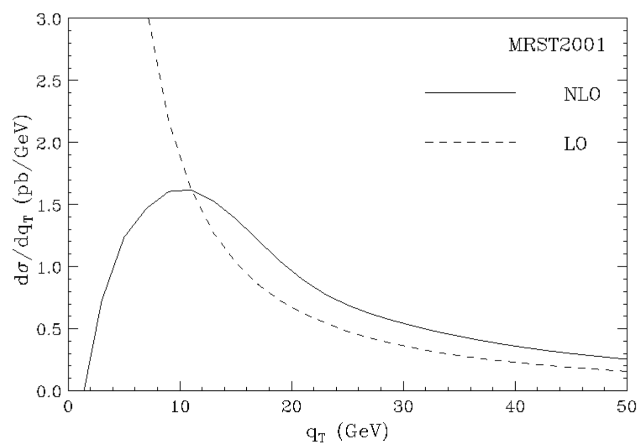
It is possible to compute any infrared safe quantity with Higgs + jet(s)

What happens when $q_T \ll M_H$?

The small q_T region is the most important because it is there that the bulk of events are expected

When $q_T \ll M_H$ large logarithmic corrections appear like $\alpha_S^n \log^{2n} Q^2/q_T^2$ that originate from soft and collinear radiation

$\Rightarrow$The fixed order perturbative calculation is not reliable



LO: $\frac{d\sigma}{dq_T} \to +\infty$

NLO: $\frac{d\sigma}{dq_T} \to -\infty$

At NLO there is an (unphysical) peak due to the compensation of positive leading and negative subleading logarithmic contributions

This is a general problem in the production of systems of high mass Q^2 in hadronic collisions (DY,$\gamma\gamma$...) when $q_T \ll Q$

Resummation

In the region $q_T^2 \ll Q^2$ large logarithmic corrections appear like $\alpha_S^n \log^{2n} Q^2/q_T^2$ that must be resummed to all orders

The q_T distribution can be decomposed as

$$\frac{d\sigma}{dq_T^2 dQ^2} = \frac{d\sigma^{(\text{res.})}}{dq_T^2 dQ^2} + \frac{d\sigma^{(\text{fin.})}}{dq_T^2 dQ^2}$$

- The first term contains all the logarithmically enhanced contributions and has to be resummed at all orders
- The finite part is not singular and can be evaluated at fixed order in PT

The resummation formalism has been developed in the eighties

G.Parisi, R.Petronzio (1979)
Y.Dokshitzer, D.Diakonov, S.I.Troian (1980)
G.Curci, M.Greco, Y.Srivastava (1979)
A.Bassetto, M.Ciafaloni, G.Marchesini (1980)
J.Kodaira, L.Trentadue (1982)
J.C.Collins, D.E.Soper, G.Sterman (1985)

As is customary in QCD resummations one has to work in a conjugate space in order to allow the kinematics of multiple gluon emission to factorize

In this case, to exactly implement transverse momentum conservation the resummation has to be performed in impact parameter b space

The resummation formula is usually written as

$$\frac{d\sigma^{(\text{res.})}}{dq_T^2 dQ^2} = \sum_{a,b,c} \int_0^1 dx_1 \int_0^1 dx_2 \int_0^\infty db \frac{b}{2} J_0(bq_T)\, \sigma^{(LO)}_{c\bar{c}}\, \delta(Q^2 - x_1 x_2 s)$$
$$\cdot \left(f_{a/h_1} \otimes C_{ca}\right)\left(x_1, \frac{b_0^2}{b^2}\right) \left(f_{b/h_2} \otimes C_{\bar{c}b}\right)\left(x_2, \frac{b_0^2}{b^2}\right) S_c(Q,b)$$

where $b_0 = 2e^{-\gamma}$ and $J_0(bq_T)$ have a kinematical origin

The large logarithmic corrections are exponentiated in the Sudakov form factor:

$$S_c(Q,b) = \exp\left\{-\int_{b_0^2/b^2}^{Q^2} \frac{dq^2}{q^2}\left[A_c(\alpha_S(q^2))\ \ln\frac{Q^2}{q^2} + B_c(\alpha_S(q^2))\right]\right\}$$

The coefficients A, B, C can be computed perturbatively

- The coefficients $A^{(1)}$, $A^{(2)}$ and $B^{(1)}$ are known both in the quark and the gluon channels

J.Kodaira, L.Trentadue (1982)
S.Catani, E. d'Emilio, L.Trentadue (1985)

- The coefficient $C^{(1)}$ is known for a variety of processes

C.Davies, W.J.Stirling (1984)
R.Kauffmann (1992)
C.Balazs, E.Berger, S.Mrenna, C.P.Yuan (1998)
D.de Florian, MG (2000)

- The general form of the NNLL coefficient $B^{(2)}$ has been computed recently

D. de Florian, MG (2000)

The standard "CSS" approach has several disadvantages:

a) The coefficients B and C are process dependent

D. de Florian, MG (2000)

⇒ For each process one is interested in (DY, Higgs, $\gamma\gamma$) new resummation coefficients have to be computed

b) The integral over the impact parameter b involves an extrapolation of the parton densities in the NP region

c) The resummation effects are large also at small b

- no control on the normalization
- problems in the matching with PT result
- unjustified higher order terms at large q_T with factorially growing coefficients (artifact of resummation)

S.Frixione, P.Nason, G.Ridolfi (1998)

To cure b) and c) resummation approaches directly in q_T space have been proposed

R.K.Ellis and S.Veseli (1998)
A.Kulesza and W.J.Stirling (1999)

⇒They can be at best approximate (no momentum conservation in transverse space)

Our formalism

A version of the b-space formalism has been proposed that overcomes all these problems

S. Catani, D. de Florian, MG (2000)

$$\frac{d\sigma}{dq_T^2} = \sum_{a,b}\int_0^1 dx_1 \int_0^1 dx_2 f_{a/h_1}(x_1,\mu_F^2) f_{b/h_2}(x_2,\mu_F^2)\, \frac{d\hat\sigma_{ab}}{dq_T^2}$$

$$\frac{d\hat\sigma_{ab}}{dq_T^2} = \frac{d\hat\sigma_{ab}^{(\text{res.})}}{dq_T^2} + \frac{d\hat\sigma_{ab}^{(\text{fin.})}}{dq_T^2}$$

Parton distributions are evaluated at the fact. scale μ_F

$$\frac{d\hat\sigma_{ac}^{(\text{res.})}}{dq_T^2} = \frac{1}{2}\int_0^\infty db\, b\, J_0(bq_T)\, \mathcal{W}_{ac}(b, M_H, \hat s; \alpha_S(\mu_R^2), \mu_R^2, \mu_F^2)$$

$$\mathcal{W}_N(b, M_H; \alpha_S(\mu_R^2), \mu_R^2, \mu_F^2) = \mathcal{H}_N\left(\alpha_S(\mu_R^2); M_H^2/\mu_R^2, M_H^2/\mu_F^2\right) \times \exp\{\mathcal{G}_N(\alpha_S(\mu_R^2), bM_H; M_H^2/\mu_R^2, M_H^2/\mu_F^2)\}$$

where flavour indices should be understood

Then we take $\mu_F \sim \mu_R \sim M_H$ and organize the large logs as

$$\mathcal{G}_N(\alpha_S, bM_H; M_H^2/\mu_R^2, M_H^2/\mu_F^2) = L\, g^{(1)}(\alpha_S L) + g_N^{(2)}(\alpha_S L; M_H^2/\mu_R^2) + \alpha_S\, g_N^{(3)}(\alpha_S L; M_H^2/\mu_R^2, M_H^2/\mu_F^2) + \dots$$

where $L = \ln M_H^2 b^2/b_0^2$ and $\alpha_S = \alpha_S(\mu_R)$

The form factor takes thus the same structure as in e^+e^- event shape variables or in threshold resummation in hadron collisions

⇒A study of scale dependence can be performed as is normally done in fixed order calculation (no need of introducing additional coefficients)

The functions $g^{(n)}(\lambda)$ are defined such that $g^{(n)}(0) = 0$ and:

- $g^{(1)}$ depends on $A^{(1)}$ (LL)
- $g_N^{(2)}$ depends on $A^{(1)}$, $B^{(1)}$ and $A^{(2)}$ (NLL)
- $g_N^{(3)}$ also depends on $C^{(1)}$, $B^{(2)}$ and $A^{(3)}$ (NNLL)

The functions $g_N^{(2)}$ and $g_N^{(3)}$ respectively receive additional contribution from the LO and NLO anomalous dimensions

These modifications are enough to make the resummation formula completely general (process independent)
$\Rightarrow$ Process dependence embodied in the hard coefficient $\mathcal{H}$

What about the normalization ?

Since the large log is $L = \ln\left(M_H^2 b^2/b_0^2\right)$ the form factor is DIVERGENT as $b \to 0$

$\Rightarrow$we perform the replacement

$$L \longrightarrow \widetilde{L} \equiv \ln\left(1 + \frac{M_H^2 b^2}{b_0^2}\right)$$

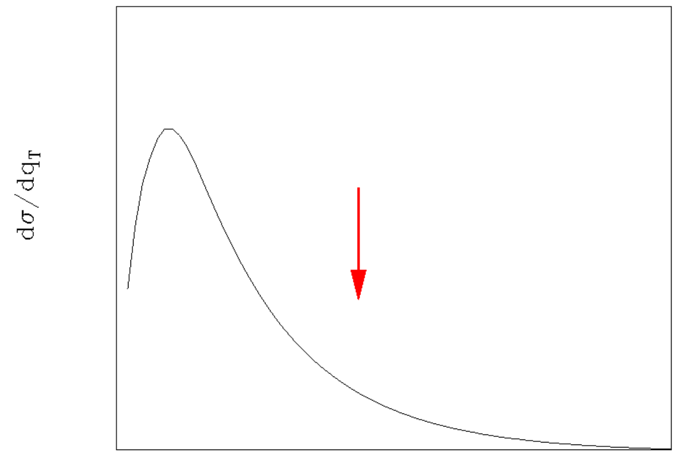
- This replacement is legitimate in the large-b region where $\widetilde{L} \sim L$
- At small b the resummation effects vanish ($\widetilde{L} \to 0$)
- The replacement is inspired by the procedure followed in e^+e^- event shapes

S.Catani, L.Trentadue, G.Turnock, B.R.Webber (1993)

With this modification the total cross section, which is related to the value of the form factor at $b = 0$ is not affected by resummation

$$\frac{d\sigma}{d^2\mathbf{q_T}} \sim \int d^2\mathbf{b}\, e^{i\,\mathbf{q_T}\cdot\mathbf{b}}\, \mathcal{W}(\mathbf{b}) \;\Rightarrow\; \int \frac{d\sigma}{d^2\mathbf{q_T}} d^2\mathbf{q_T} \sim \mathcal{W}(0)$$

But: It is NOT only a problem of normalization



At $q_T \ll M_H$ the effects of resummation dominate

At $q_T \sim M_H$ one trusts the fixed-order calculation

$\Downarrow$

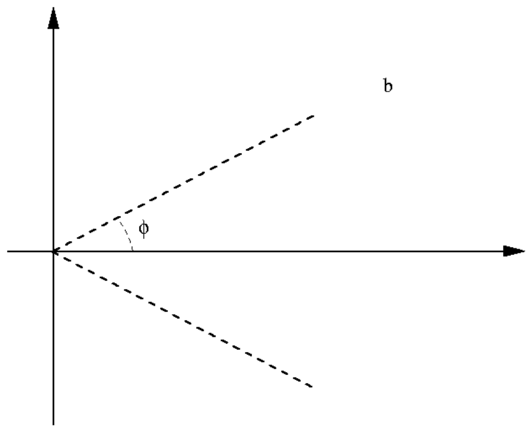
The constraint mainly acts in the intermediate-q_T region

Numerical implementation

The functions $g_i(\lambda)$ are singular as $\lambda = \alpha_S \beta_0 L \to 1$

$\Rightarrow$ We divert the integration contour in the complex plane by introducing two auxiliary functions that provide the necessary convergence

E.Laenen, G.Sterman, W.Vogelsang (2000)



$$h_1(z,v) = \frac{1}{\pi}\int_{-iv\pi}^{-\pi+iv\pi} d\theta e^{-iz\sin\theta}$$

$$h_2(z,v) = \frac{1}{\pi}\int_{\pi+iv\pi}^{-iv\pi} d\theta e^{-iz\sin\theta}$$

These functions obey the constraint

$$h_1(z,v) + h_2(z,v) = 2J_0(z) \text{ for each } v$$

This prescription can be considered as an extension of the minimal prescription used in threshold resummation

S.Catani, M.L.Mangano, P. Nason, L.Trentadue (1996)

Matching

The partonic cross section is decomposed as:

$$\frac{d\hat\sigma_{ab}}{dq_T^2} = \frac{d\hat\sigma_{ab}^{(\text{res.})}}{dq_T^2} + \frac{d\hat\sigma_{ab}^{(\text{fin.})}}{dq_T^2}$$

The resummed and perturbative results have to be properly matched to avoid double counting

$$\frac{d\hat\sigma_{ab}^{(\text{fin.})}}{dq_T^2} = \left[\frac{d\hat\sigma_{ab}}{dq_T^2}\right]_{\text{f.o.}} - \left[\frac{d\hat\sigma_{ab}^{(\text{res.})}}{dq_T^2}\right]_{\text{f.o.}}$$

fix. order contr. exp. of res. component

Both terms are separately divergent as $q_T \to 0$: the singularity has to cancel in the sum

- The fixed order contribution is evaluated with our HIGGSJET NLO program
- The expansion of resummed result is analytically transformed back to q_T space
  - In the standard approach we simply have: $\ln^n M_H^2 b^2/b_0^2 \to 1/q_T^2 \ln^{n-1} M_H^2/q_T^2$
  - In our approach: $\ln^n \left(1 + M_H^2 b^2/b_0^2\right) \to K_1^{(n-1)}(b_0 q_T/M_H)$

$$\text{where} \quad K_1^{(n)}(z) \equiv \left(\frac{\partial K_\nu(z)}{\partial \nu}\right)_{\nu=1}$$

are the n-derivatives of the modified Bessel function of the second kind

Matching works well up to very small q_T

Numerical results

I present NLL results matched to LO (NLL+LO) and NNLL results matched to NLO (NNLL+NLO)

We use MRST2002 pdf

- NLL+LO: LO pdf + 1-loop α_S
- NNLL+NLO: NLO pdf + 2-loop α_S

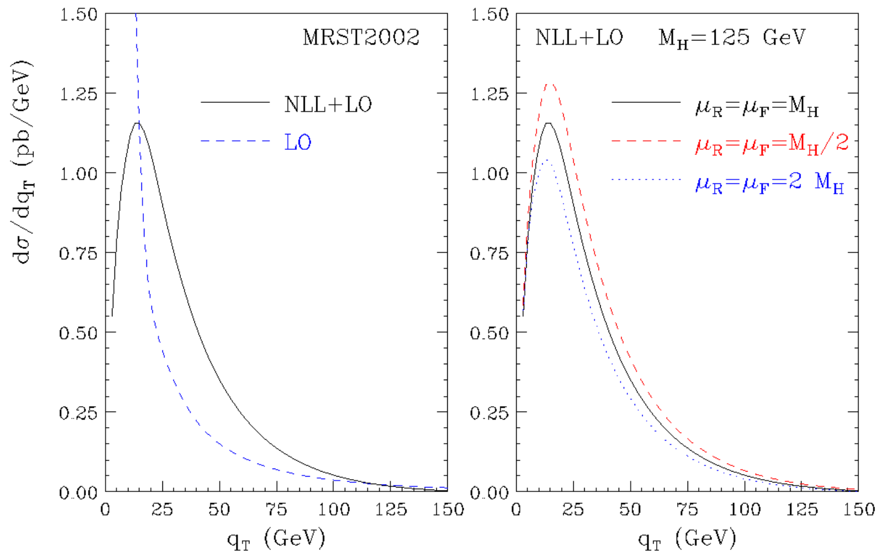
At NNLL+NLO $A^{(3)}$ and $\mathcal{H}^{(2)}$ are not known

- For $A^{(3)}$ we use the numerical estimate available for threshold resummation

  A. Vogt (2000)

- The effect of $\mathcal{H}^{(2)}$ is included in approximated form using the known result for the total NNLO cross section (computed with NLO pdf and 2-loop α_S)

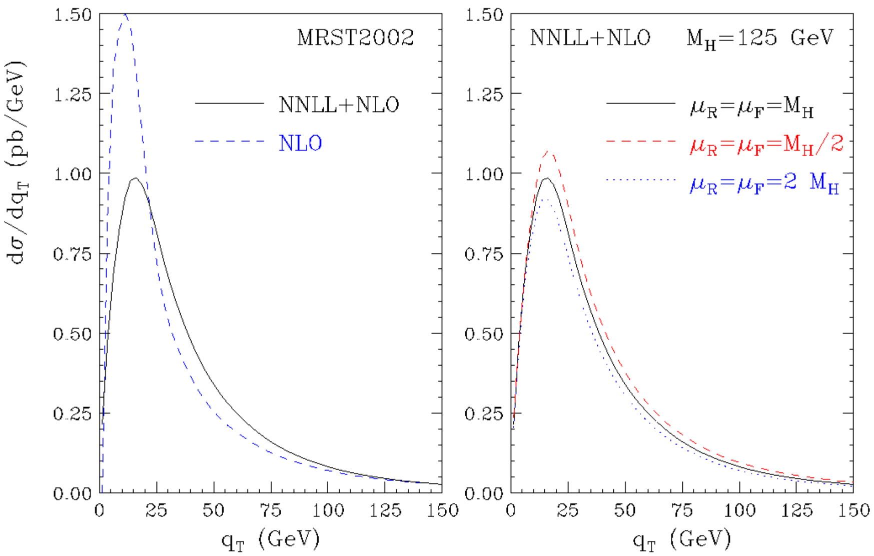
CTEQ6 pdf set gives very similar results



The LO result diverges to $+\infty$ as $q_T \to 0$

The effect of resummation is relevant below $q_T \sim 100$ GeV

Total cross section in good agreement with NLO result computed with LO pdf and 1-loop α_S



The NLO result diverges to $-\infty$ as $q_T \to 0$ (unphysical peak)

The q_T distribution is slightly harder than at NLL+LO ($\langle q_T \rangle \sim 39$ GeV)

The effect of the coefficient $A^{(3)}$ is negligible whereas the coefficient $\mathcal{H}^{(2)}$ increases the cross section by about 20%

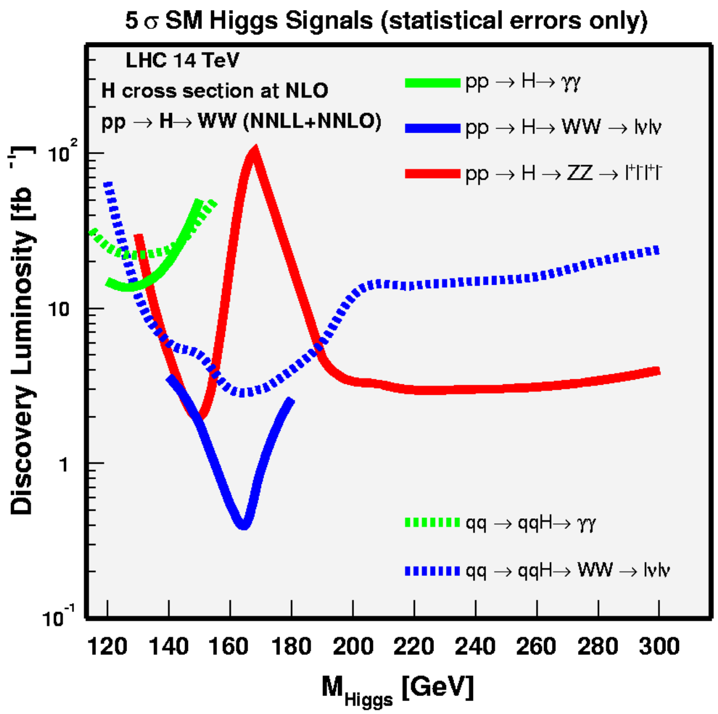
The scale dependence is reduced with respect to NLL+LO: It varies from $\pm 10\%$ at the peak to $\pm 20\%$ at $q_T \sim 100$ GeV

A very recent application in $gg \to H \to WW \to l\nu l\nu$

G.Davatz, G.Dissertori, M.Dittmar, F.Pauss, MG (2004)

Use results for the $gg \to H\ q_T$ spectrum at NNLL+NLO to correct (reweight) signal events from PYTHIA

Apply the resummation formalism to WW pair production $\to$ NLL+LO results used to correct PYTHIA background



Summary

We have evaluated the contribution of multiple soft-gluon emissions to the the total cross section σ_H:

- Effect moderate at LHC: for a light Higgs $+6\%$ with respect to NNLO
- A bit larger at the Tevatron: $+12-15\%$ with respect to NNLO

$\Rightarrow$ Perturbative result under better control now but...
still problems with NNLO pdf !

We have computed the q_T spectrum of the SM Higgs boson at the LHC

- We have implemented the most complete information available at present:
  all-order resummation of logarithmically enhanced contributions at small q_T at NNLL level combined with NLO perturbation theory at large q_T
- Distinctive feature of our approach are:
  - It allows a consistent study of theoretical uncertainties
  - It avoids the introduction of unjustified higher-order corrections in the intermediate-q_T region by using unitarity constraints $\Rightarrow$Normalization OK
- The results appear to be stable
- The method can be used also with other processes (DY, WW...)