# Is perturbation theory applicable to collider physics?

Davison E. Soper
University of Oregon

• Cross sections for hadron colliders are calculated using perturbation theory.

• The particles involved carry strong interactions.

• $\alpha_s(1\text{ fm}) \sim 1$.

## Justification

• $\alpha_s(r) \ll 1$ for $r \ll 1$ fm.

• It is a property of QCD that we can factor the short distance parts from the long distance parts.

• Why?

• What is lacking in the "proof."

If the use of perturbative predictions is based on faith shared among the community of theorists rather than on solid science, perhaps we can apply for additional funding for the LHC experimental program:

“President George W. Bush’s Faith-Based and Community Initiative represents a fresh start and bold new approach to government’s role . . . . ”

The basic factorization formula for $p + \bar{p} \to \mu^+ + \mu^-$

$$\frac{d\sigma}{dQ^2 dy} = \int dx_a \; f_{a/A}(x_a) \int dx_a \; f_{b/A}(x_b) \; \frac{d\widehat{\sigma}_{ab}(x_a, x_b)}{dQ^2 dy} + \mathcal{O}\left(\frac{m^2}{Q^2}\right).$$

Including a measurement function, say for $p+\bar{p} \to \mu^+ + \mu^- +$jets,

$$\begin{aligned} \sigma \;=\; & \int dx_a \; f_{a/A}(x_a) \int dx_a \; f_{b/A}(x_b) \\ & \times \int d^4Q \sum_N \frac{1}{N!} \prod_{i=1}^{N} \left( \int d\vec{p}_i \right) \; \frac{d\widehat{\sigma}_{ab}(x_a, x_b)}{dQ^2 dy \, d\vec{p}_1 \cdots d\vec{p}_N} \\ & \times \mathcal{S}_N(Q^2, y, \vec{p}_1, \cdots, \vec{p}_N) \\ & + \mathcal{O}(m^2/Q^2). \end{aligned}$$

with

$$\begin{aligned} \mathcal{S}_{N+1}(Q^2, y, \vec{p}_1, \cdots, \lambda\vec{p}_N, (1-\lambda)p_N) \;&=\; \mathcal{S}_N(Q^2, y, \vec{p}_1, \cdots, \vec{p}_N) \\ \mathcal{S}_{N+1}(Q^2, y, \vec{p}_1, \cdots, \vec{p}_N, \lambda P_A) \;&=\; \mathcal{S}_N(Q^2, y, \vec{p}_1, \cdots, \vec{p}_N) \\ \mathcal{S}_{N+1}(Q^2, y, \vec{p}_1, \cdots, \vec{p}_N, \lambda P_B) \;&=\; \mathcal{S}_N(Q^2, y, \vec{p}_1, \cdots, \vec{p}_N). \end{aligned}$$

## Comments

$$\frac{d\sigma}{dQ^2dy} = \int dx_a \; f_{a/A}(x_a) \int dx_a \; f_{b/A}(x_b) \; \frac{d\widehat{\sigma}_{ab}(x_a, x_b)}{dQ^2dy} + \mathcal{O}\left(\frac{m^2}{Q^2}\right).$$

1. This is not "leading log." Corrections are power suppressed.

2. Parton distributions have a separate (universal) definition.

3. Parton distributions are non-perturbative.

4. $d\widehat{\sigma}$ has an expansion in powers of $\alpha_s$.

$$\frac{d\sigma}{dQ^2dy} = \int dx_a \; f_{a/A}(x_a) \int dx_a \; f_{b/A}(x_b) \; \frac{d\widehat{\sigma}_{ab}(x_a, x_b)}{dQ^2dy} + \mathcal{O}\left(\frac{m^2}{Q^2}\right).$$

5. The formula is supposed to be true at *any* order of perturbation theory. *What does that mean?*

   (a) Describe initial hadrons using Bethe-Salpeter wave functions.

   (b) Final states are $\mu^+$, $\mu^-$ and quarks and gluons.

   (c) Remaining parts summed up to order $\alpha_s^N$.

$$\frac{d\sigma}{dQ^2dy} = \int dx_a\ f_{a/A}(x_a) \int dx_a\ f_{b/A}(x_b)\ \frac{d\widehat{\sigma}_{ab}(x_a, x_b)}{dQ^2dy} + \mathcal{O}\left(\frac{m^2}{Q^2}\right).$$

6. $d\widehat{\sigma}$ is probably not defined beyond perturbation theory.

   (a) (presumably) $d\widehat{\sigma}^{(N)} \propto N!$.

   (b) Stop when $N\alpha_s \approx 1$.

   (c) An ambiguity remains,

$$N!\,\alpha_s^N = N!\,(1/N)^N \approx e^{-N} = e^{-1/\alpha_s} \approx \left(\Lambda^2/Q^2\right)^{\text{const.}}$$

# Some references


- D. Amati, R. Petronzio and G. Veneziano, "Relating Hard QCD Processes Through Universality Of Mass Singularities," Nucl. Phys. B **140**, 54 (1978); Nucl. Phys. B **146**, 29 (1978).

- S. B. Libby and G. Sterman, "Jet And Lepton Pair Production In High-Energy Lepton - Hadron And Hadron - Hadron Scattering," Phys. Rev. D **18**, 3252 (1978).

- A. H. Mueller, "Cut Vertices And Their Renormalization: A Generalization Of The Wilson Expansion," Phys. Rev. D **18**, 3705 (1978).

- G. Sterman, "Mass Divergences In Annihilation Processes. 1. Origin And Nature Of Divergences In Cut Vacuum Polarization Diagrams," Phys. Rev. D **17**, 2773 (1978).

- R. K. Ellis, H. Georgi, M. Machacek, H. D. Politzer and G. G. Ross, "Perturbation Theory And The Parton Model In QCD," Nucl. Phys. B **152**, 285 (1979). "We prove that for any process which admits a parton-model interpretation, the naive parton model can be modified to include the effects of QCD interactions to all orders in perturbation theory."

- R. Doria, J. Frenkel and J. C. Taylor, "Counter Example To Nonabelian Bloch-Nordsieck Conjecture," Nucl. Phys. B **168**, 93 (1980). Factorization fails beyond the leading power.

- A. V. Efremov and A. V. Radyushkin, “Field Theory Approach To Processes With Large Momentum Transfer” Theor. Math. Phys. **44**, 664 (1981) Theor. Math. Phys. **44**, 774 (1981)

- G. T. Bodwin, S. J. Brodsky and G. P. Lepage, “Initial State Interactions And The Drell-Yan Process,” Phys. Rev. Lett. **47**, 1799 (1981). “It is shown that initial-state interactions violate the usual QCD factorization predictions for massive lepton-pair production in leading twist.”

- J. C. Collins, D. E. Soper and G. Sterman, “Does The Drell-Yan Cross-Section Factorize?,” Phys. Lett. B **109**, 388 (1982). Yes, sort of . . .

- J. C. Collins, D. E. Soper and G. Sterman, "Factorization For One Loop Corrections In The Drell-Yan Process," Nucl. Phys. B **223**, 381 (1983).

- W. W. Lindsay, D. A. Ross and C. T. Sachrajda, "On The Cancellation Of Long Distance, Nonfactorizing Contributions To The Drell-Yan Cross-Section," Nucl. Phys. B **214**, 61 (1983) Factorization works in an example two loop calculation including spectator quarks.

- J. C. Collins, D. E. Soper and G. Sterman, "All Order Factorization For Drell-Yan Cross-Sections," Phys. Lett. B **134**, 263 (1984).

- J. C. Collins, D. E. Soper and G. Sterman, "Factorization For Short Distance Hadron - Hadron Scattering," Nucl. Phys. B **261**, 104 (1985).

- G. T. Bodwin, "Factorization Of The Drell-Yan Cross-Section In Perturbation Theory," Phys. Rev. D **31**, 2616 (1985) [Erratum-ibid. D **34**, 3932 (1986)].

- J. C. Collins, D. E. Soper and G. Sterman, "Heavy Particle Production In High-Energy Hadron Collisions," Nucl. Phys. B **263**, 37 (1986).

- J. C. Collins, D. E. Soper and G. Sterman, "Soft Gluons And Factorization," Nucl. Phys. B **308**, 833 (1988).

- E. Gotsman, E. Levin and U. Maor, "Hard inclusive production and a violation of the factorization theorem," Phys. Lett. B **406**, 89 (1997). An example that demonstrates that earlier work was mistaken.

- J. C. Collins, D. E. Soper and G. Sterman, "Factorization is not violated," Phys. Lett. B **438**, 184 (1998). No it wasn't.

---

- J. C. Collins and D. E. Soper, "Parton Distribution And Decay Functions," Nucl. Phys. B **194**, 445 (1982).

# An instructive exercise

$$\sigma = 1 + \alpha_s I + \mathcal{O}(\alpha_s^2)$$

$$I = \int_0^\infty dk^2 \, \frac{1}{k^2 + m^2} \, \frac{Q^2}{Q^2 + k^2}$$

First, analysis with naive accounting for leading regions.

$$I = \int_{k^2 \ll Q^2} dk^2 \, \frac{1}{k^2 + m^2} \; + \int_{k^2 \sim Q^2} dk^2 \, \frac{1}{k^2} \, \frac{Q^2}{Q^2 + k^2} + \mathcal{O}(m^2/Q^2)$$

So

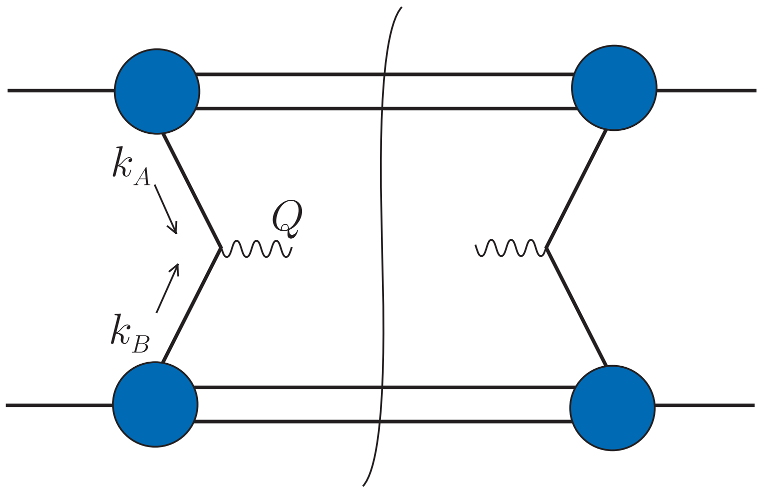
$$\begin{aligned} \sigma &= \left(1 + \alpha_s \int_{k^2 \ll Q^2} dk^2 \, \frac{1}{k^2 + m^2}\right) \left(1 + \alpha_s \int_{k^2 \sim Q^2} dk^2 \, \frac{1}{k^2} \, \frac{Q^2}{Q^2 + k^2}\right) \\ &\quad + \mathcal{O}(\alpha_s^2) + \mathcal{O}(m^2/Q^2) \\ &\equiv f \times \hat{\sigma} + \mathcal{O}(\alpha_s^2) + \mathcal{O}(m^2/Q^2). \end{aligned}$$

Now, analysis with subtractions.

$$
\begin{aligned}
I &= \int_0^\infty dk^2 \left(\frac{k^2}{\mu^2}\right)^\epsilon \frac{1}{k^2+m^2} \times 1 \\
&\quad + \int_0^\infty dk^2 \left(\frac{k^2}{\mu^2}\right)^\epsilon \frac{1}{k^2+m^2} \left(\frac{Q^2}{Q^2+k^2} - 1\right) \\
&= \left[\int_0^\infty dk^2 \left(\frac{k^2}{\mu^2}\right)^\epsilon \frac{1}{k^2+m^2} - \frac{1}{\epsilon}\right] \\
&\quad + \left[\int_0^\infty dk^2 \left(\frac{k^2}{\mu^2}\right)^\epsilon \frac{1}{k^2} \left(\frac{Q^2}{Q^2+k^2} - 1\right) + \frac{1}{\epsilon}\right] \\
&\quad + \mathcal{O}(m^2/Q^2) \\
&= f^{(1)} + \hat{\sigma}^{(1)} + \mathcal{O}(m^2/Q^2)
\end{aligned}
$$

A subtraction scheme is necessary to really control multiloop diagrams. Unfortunately, the existing demonstrations of factorization use the naive method.

For the simplest graph, it's just kinematics.

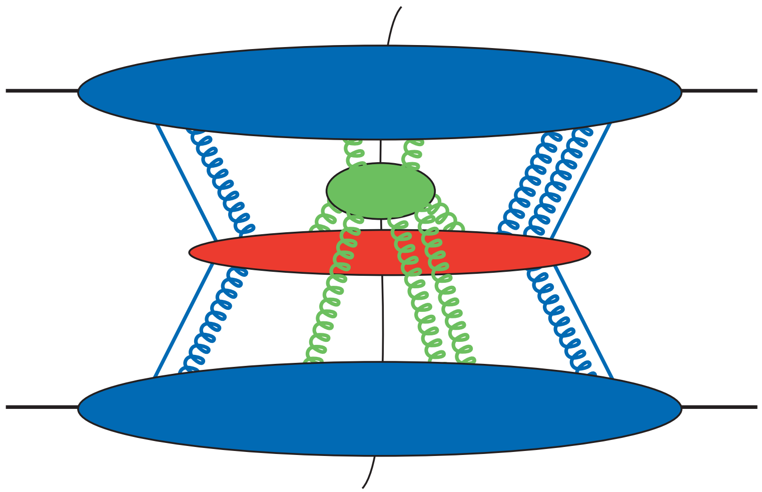



$$\frac{d\sigma}{dQ^2dy} = \int dk_{A,T}\, dk_{B,T} dk_A^- dk_B^+ \; H_{\mu,\nu}(Q^+, Q^-, k_{A,T} + k_{B,T})$$
$$\times \mathrm{Tr}\{\gamma^\mu \Phi_A(Q^+ - k_B^+, k_{A,T}, k_A^-)\gamma^\nu \Phi_B(k_B^+, k_{A,T}, Q^- - k_A^-)\}$$

with $Q^\pm = [Q^2 + (k_{A,T} + k_{B,T})^2]^{1/2} e^{\pm y}/\sqrt{2}$. Simply approximate

$$\begin{aligned} k_{A,T}^2, k_{B,T}^2 &\ll Q^2, \\ k_A^- &\ll Q^-, \\ k_B^+ &\ll Q^+. \end{aligned}$$

Alas, it's not so simple.



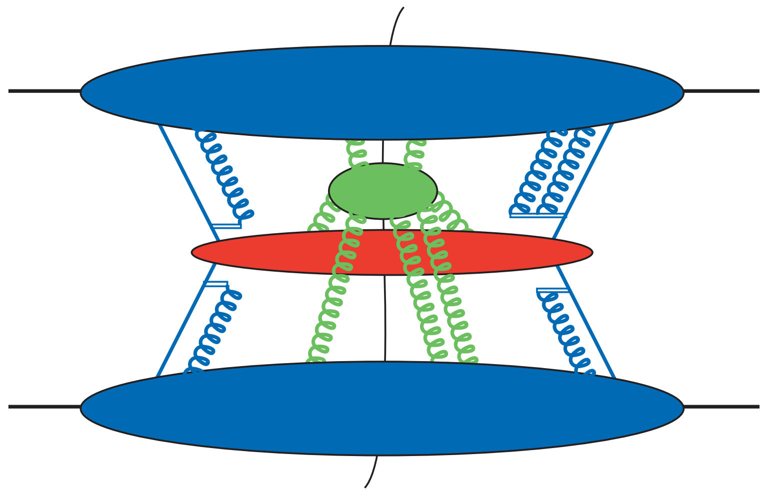
Naive analysis of leading (pinch singular) integration regions gives the following:

Hard (Large $P_T$ or way off shell)

Collinear (to A or to B, small $P_T$)

Soft (All components small, includes "Glauber.")

The extra collinear gluons would be a big problem because the factorization formula contemplates collisions of only one parton from each hadron. But the collinear gluons are OK



- The extra collinear gluons have $\epsilon^\mu \propto k^\mu$.

- There effect can be approximated as shown with eikonal lines, with $u$ in the $-$ direction for hadron A, $u$ in the $+$ direction for hadron B,

$$\text{propagator} = \frac{i}{k \cdot u + i\epsilon}$$
$$\text{vertex} = -igt_a u^\mu$$

# The parton distribution functions

Before I built $\hat{\sigma}$ I'd ask to know
What I was factoring in or factoring out

$$
\begin{aligned}
f_{q/p}(x,\mu) \;&=\; \frac{1}{4\pi}\left(\frac{1}{2}\sum_s\right)\int dy^- e^{ixP^+y^-} \\
&\quad \times \langle P,s|\bar{\psi}(0)E(0)\gamma^+E^\dagger(y^-)\psi(0,y^-,\mathbf{0})|P,s\rangle,
\end{aligned}
$$

where

$$
E^\dagger(y^-) = \mathcal{P}\exp\left(-ig\int_{y^-}^{\infty} dz^- A_a^+(0,z-,0_T)\,t_a\right).
$$

The operator product needs UV renormalization, which is performed using the $\overline{\text{MS}}$ prescription.

$$
\begin{aligned}
f_{q/p}(x,\mu) \;=\; & \frac{1}{4\pi}\left(\frac{1}{2}\sum_s\right)\int dy^- e^{ixP^+y^-} \\
& \times\langle P,s|\bar\psi(0)E(0)\gamma^+E^\dagger(y^-)\psi(0,y^-,\mathbf{0})|P,s\rangle,
\end{aligned}
$$

## Comments

1. The renormalization group equation for $f$ is the DGLAP equation.

2. $f$ does not have a perturbative expansion.

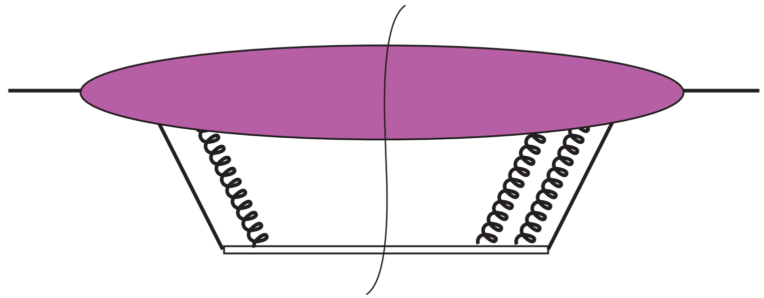
3. One sometimes hears of a "bare" $f$, but I don't know what that is.

4. For consistency with a NNLO calculation, one needs $f$ that obeys the DGLAP equation with the two loop kernel, which is usually called "NLO".

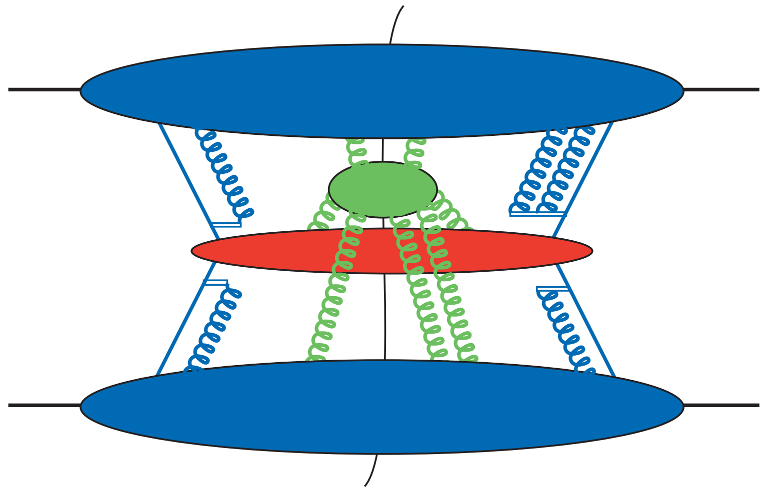
5. In my opinion, using data compared to NLO theory in obtaining $f$ does not "contaminate" $f$ so that it is unsuitable for use with an NNLO calculation.

   - Cf. $\alpha_s(M_z)$, which is obtained from experiments compared to NLO theory and other experiments compared to NNLO theory.

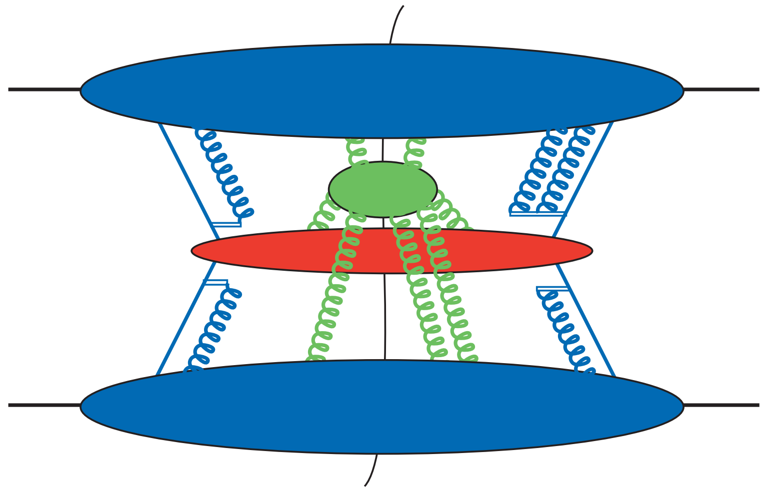
# Parton distribution in diagrams



Compare



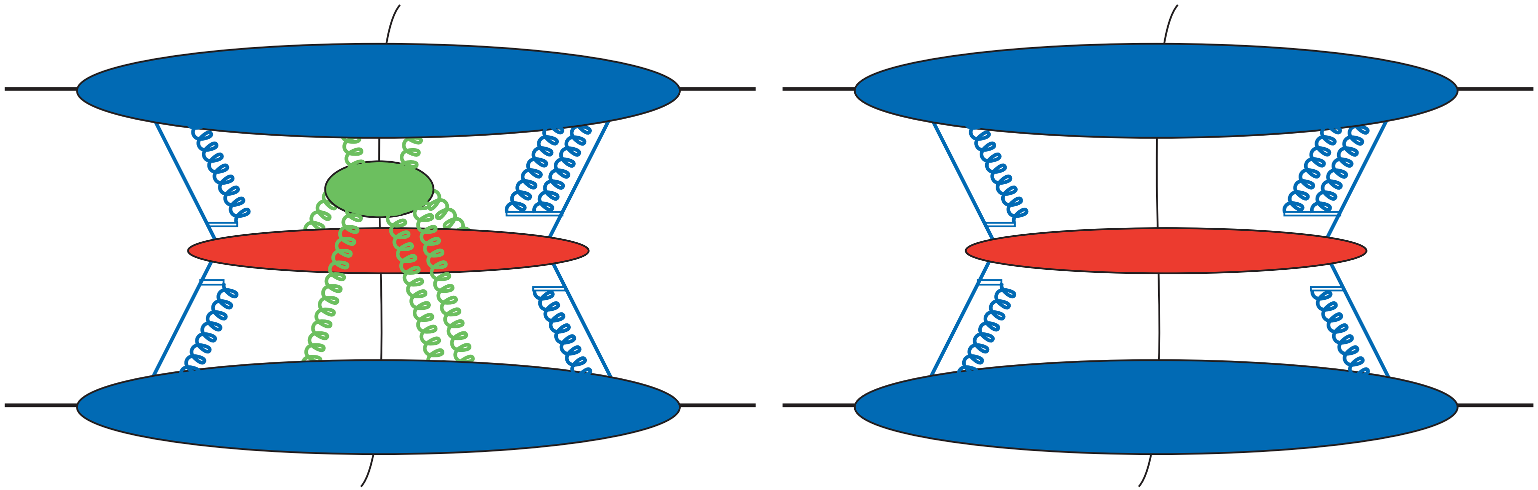
# Soft gluon exchanges



• It seems that a soft gluon exchanged from a spectator quark in hadron A to the active quark in hadron B can rotate the quark's color and thus keep it from annihilating.

• Soft gluon approximations (with eikonal lines) needs $q^{\pm}$ not too small. But $q^{\pm}$ contours can be trapped in "too small" region.

# The soft gluons go away



- This part is quite technical.

- Ingredients: unitarity, causality, gauge invariance.

- We use the fact that the initial state is a color singlet bound state and that we can sum over all final states.

## Comments

1. The proof explicitly uses the facts that the incoming partons are somewhat off-shell and are in a color singlet bound state.

2. One would like also a proof for on-shell colored incoming partons.

    - This is the case in calculations of $\hat{\sigma}$.

3. We need a proof with subtractions.

    - This would give a construction of $\hat{\sigma}$.

4. There is no detailed proof for hadron collisions with a non-trivial measurement function.

   - The perturbative part of this should be pretty simple.
   - But an analysis with proper final state hadron bound states is needed.
   - We need to show that it is correct to apply the same measurement function $\mathcal{S}_N(Q^2, y, \vec{p}_1, \cdots, \vec{p}_N)$ to the partons as to the hadrons.

# An example

• Consider diffractive deeply inelastic scattering.

$$e + p \;\to\; e + p + X$$

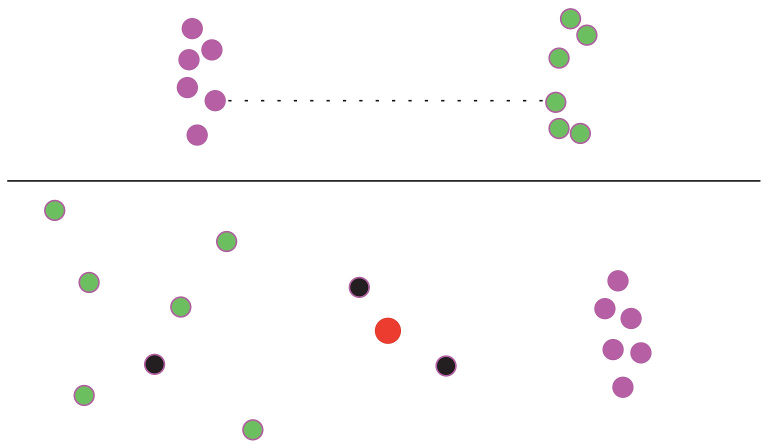
with large $Q^2$, proton loses a small fraction $x_{I\!P}$ of its momentum and gets a small invariant momentum transfer $q^2 \equiv t$.

• Factorization works for this, with new parton distributions $df^{\rm diff}(x)/dx_{I\!P}dt$. (See paper by Collins, also Berera and Soper.)

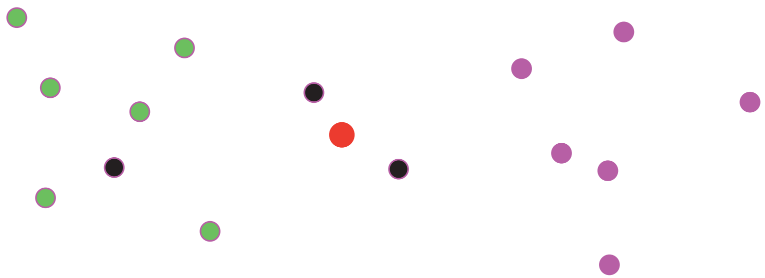
• The hard scattering cross section $d\hat\sigma$ is the same as for inclusive DIS.

- But we expect factorization *not* to work for

$$p + \bar{p} \;\rightarrow\; Z + p + X$$



With spectator scattering, it is much more likely to break up the proton:



• Experimentally, the factorization formula for $p + \bar{p} \rightarrow Z + p + X$ fails by a big factor.