Testing String Theory with Cosmological Observations

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   - Review of Inflationary Cosmology
   - Problems of Inflationary Cosmology
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   - Moduli stabilization in SGC

3. String Gas Cosmology and Structure Formation
   - Review of the Theory of Cosmological Perturbations
   - Overview
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   - Signatures in CMB anisotropy maps

4. Conclusions

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The **Inflationary Universe Scenario** is the current paradigm of early universe cosmology.

**Successes:**

- Solves horizon problem
- Solves flatness problem
- Solves size/entropy problem
- Provides a causal mechanism of generating primordial cosmological perturbations (Chibisov & Mukhanov, 1981).
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Moduli stabilization
Credit: NASA/WMAP Science Team
Fig. 1a. Diagram of gravitational instability in the 'big-bang' model. The region of instability is located to the right of the line $H(t)$; the region of stability to the left. The two additional lines of the graph demonstrate the temporal evolution of density perturbations of matter: growth until the moment when the considered mass is smaller than the Jeans mass and oscillations thereafter. It is apparent that at the moment of recombination perturbations corresponding to different masses correspond to different phases.

Fig. 1b. The dependence of the square of the amplitude of density perturbations of matter on scale. The fine line designates the usually assumed dependence $\delta^2(\nu) \sim \nu^{2n}$. It is apparent that fluctuations of relic radiation should depend on scale in a similar manner.
Challenges for the Current Paradigm

In spite of the phenomenological successes, current realizations of the inflationary scenario suffer from several conceptual problems.

In light of these problems we need to look for input from new fundamental physics to construct a new theory which will overcome these problems.

Question: Can Superstring theory lead to a new and improved paradigm?

Question: Can this new paradigm be tested in cosmological observations?
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Question: Can this new paradigm be tested in cosmological observations?
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Review of Inflationary Cosmology

Context:

- **General Relativity**
- **Scalar Field Matter**

**Metric:**
\[ ds^2 = dt^2 - a(t)^2 dx^2 \]  \( (1) \)

**Inflation:**
- phase with \( a(t) \sim e^{tH} \)
- requires matter with \( p \sim -\rho \)
- requires a slowly rolling scalar field \( \varphi \)
- in order to have a potential energy term
- in order that the potential energy term dominates sufficiently long
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Time line of inflationary cosmology:

\[ a(t) = e^{tH} \]

- \( t_i \): inflation begins
- \( t_R \): inflation ends, reheating
Space-time sketch of inflationary cosmology:

Note:
- \( H = \frac{\dot{a}}{a} \)
- curve labelled by \( k \): wavelength of a fluctuation
inflation renders the universe large, homogeneous and spatially flat

- classical matter redshifts → matter vacuum remains
- quantum vacuum fluctuations: seeds for the observed structure [Chibisov & Mukhanov, 1981]
- sub-Hubble → locally causal
Conceptual Problems of Inflationary Cosmology

- Nature of the scalar field $\varphi$ (the “inflaton”)
- Conditions to obtain inflation (initial conditions, slow-roll conditions, graceful exit and reheating)
- Amplitude problem
- Trans-Planckian problem
- Singularity problem
- Cosmological constant problem
- Applicability of General Relativity
Trans-Planckian Problem

Success of inflation: At early times scales are inside the Hubble radius → causal generation mechanism is possible.

Problem: If time period of inflation is more than $70H^{-1}$, then $\lambda_p(t) < l_p$ at the beginning of inflation → new physics MUST enter into the calculation of the fluctuations.
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- **Success of inflation**: At early times scales are inside the Hubble radius \( \rightarrow \) causal generation mechanism is possible.
- **Problem**: If time period of inflation is more than \( 70H^{-1} \), then \( \lambda_p(t) < l_{pl} \) at the beginning of inflation
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**Problem**: If time period of inflation is more than $70H^{-1}$, then $\lambda_p(t) < l_{pl}$ at the beginning of inflation $\rightarrow$ new physics MUST enter into the calculation of the fluctuations.
If evolution in Period I is non-adiabatic, then scale-invariance of the power spectrum will be lost [J. Martin and RB, 2000]

→ Planck scale physics testable with cosmological observations!
Singularity Problem

- **Standard cosmology:** Penrose-Hawking theorems $\rightarrow$ initial singularity $\rightarrow$ incompleteness of the theory.
- **Inflationary cosmology:** In scalar field-driven inflationary models the initial singularity persists [Borde and Vilenkin] $\rightarrow$ incompleteness of the theory.

**Penrose-Hawking theorems:**
- Ass: i) Einstein action, 2) weak energy conditions $\rho > 0, \rho + 3p \geq 0$
- $\rightarrow$ space-time is geodesically incomplete.
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Penrose-Hawking theorems:
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- → space-time is geodesically incomplete.
Cosmological Constant Problem

- Quantum vacuum energy does not gravitate.
- Why should the almost constant $V(\varphi)$ gravitate?

$$\frac{V_0}{\Lambda_{obs}} \sim 10^{120} \quad (2)$$
In all approaches to quantum gravity, the Einstein action is only the leading term in a low curvature expansion. Correction terms may become dominant at much lower energies than the Planck scale. Correction terms will dominate the dynamics at high curvatures. The energy scale of inflation models is typically \( \eta \sim 10^{16}\text{GeV} \). \( \eta \) too close to \( m_{pl} \) to trust predictions made using GR.
Zones of Ignorance

String Cosmology
R. Brandenberger

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Message I

- Current realizations of inflation have conceptual problems.
- We need a new paradigm of very early universe cosmology based on new fundamental physics.
- **Hypothesis:** New paradigm based on Superstring Theory.
- The new paradigm of early universe cosmology may not involve inflation.
- New cosmological model motivated by superstring theory: String Gas Cosmology (SGC) [R.B. and C. Vafa, 1989]
- New structure formation scenario emerges from SGC [A. Nayeri, R.B. and C. Vafa, 2006].
String Gas Cosmology makes testable predictions for cosmological observations

- Blue tilt in the spectrum of gravitational waves [R.B., A. Nayeri, S. Patil and C. Vafa, 2006]
- Line discontinuities in CMB anisotropy maps [N. Kaiser and A. Stebbins, 1984]
- Line discontinuities may have junctions
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- Line discontinuities may have junctions
- **Line discontinuities in B-mode CMB polarization maps** [R. Danos, R.B. and G. Holder, 2010]
Idea: make use of the new symmetries and new degrees of freedom which string theory provides to construct a new theory of the very early universe.

Assumption: Matter is a gas of fundamental strings

Assumption: Space is compact, e.g. a torus.

Key points:

- New degrees of freedom: string oscillatory modes
- Leads to a maximal temperature for a gas of strings, the Hagedorn temperature
- New degrees of freedom: string winding modes
- Leads to a new symmetry: physics at large $R$ is equivalent to physics at small $R$
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T-Duality

- Momentum modes: $E_n = n/R$
- Winding modes: $E_m = mR$
- Duality: $R \rightarrow 1/R$ $(n, m) \rightarrow (m, n)$
- Mass spectrum of string states unchanged
- Symmetry of vertex operators
- Symmetry at non-perturbative level $\rightarrow$ existence of D-branes
Temperature-size relation in string gas cosmology

\[ T \]

\[ T_H \]

\[ \ln R \]
Temperature-size relation in standard cosmology

![Temperature-size relation graph]

Singularity Problem in Standard and Inflationary Cosmology
Dynamics

Assume some action gives us $R(t)$
Dynamics II

We will thus consider the following background dynamics for the scale factor $a(t)$:

![Graph showing $a(t)$ vs. $t$ with $\sim t^{1/2}$]
Dimensionality of Space in SGC

- Begin with all 9 spatial dimensions small, initial temperature close to $T_H \rightarrow$ winding modes about all spatial sections are excited.

- Expansion of any one spatial dimension requires the annihilation of the winding modes in that dimension.

- Decay only possible in three large spatial dimensions.

$\rightarrow$ dynamical explanation of why there are exactly three large spatial dimensions.

Note: this argument assumes constant dilaton [R. Danos, A. Frey and A. Mazumdar]
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Moduli Stabilization in SGC

**Size Moduli** [S. Watson, 2004; S. Patil and R.B., 2004, 2005]

- winding modes prevent expansion
- momentum modes prevent contraction

\[ \rightarrow V_{\text{eff}}(R) \text{ has a minimum at a finite value of } R, \rightarrow R_{\text{min}} \]

- in heterotic string theory there are enhanced symmetry states containing both momentum and winding which are massless at \( R_{\text{min}} \)

\[ \rightarrow V_{\text{eff}}(R_{\text{min}}) = 0 \]

\[ \rightarrow \text{size moduli stabilized in Einstein gravity background} \]

**Shape Moduli** [E. Cheung, S. Watson and R.B., 2005]

- enhanced symmetry states

\[ \rightarrow \text{harmonic oscillator potential for } \theta \]

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**Shape Moduli** [E. Cheung, S. Watson and R.B., 2005]
- enhanced symmetry states
- \( \rightarrow \) harmonic oscillator potential for \( \theta \)
- \( \rightarrow \) shape moduli stabilized
The only remaining modulus is the dilaton

Make use of gaugino condensation to give the dilaton a potential with a unique minimum

→ dilaton is stabilized

Dilaton stabilization is consistent with size stabilization [R. Danos, A. Frey and R.B., 2008]
Cosmological fluctuations connect early universe theories with observations

- Fluctuations of matter $\rightarrow$ large-scale structure
- Fluctuations of metric $\rightarrow$ CMB anisotropies
- N.B.: Matter and metric fluctuations are coupled

Key facts:

1. Fluctuations are small today on large scales
   $\rightarrow$ fluctuations were very small in the early universe
   $\rightarrow$ can use linear perturbation theory
2. Sub-Hubble scales: matter fluctuations dominate
   Super-Hubble scales: metric fluctuations dominate
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Quantum Theory of Linearized Fluctuations

Step 1: Metric including fluctuations

\[ ds^2 = a^2[(1 + 2\Phi)d\eta^2 - (1 - 2\Phi)dx^2] \]  \hspace{2cm} (3)

\[ \varphi = \varphi_0 + \delta\varphi \]  \hspace{2cm} (4)

Note: \( \Phi \) and \( \delta\varphi \) related by Einstein constraint equations

Step 2: Expand the action for matter and gravity to second order about the cosmological background:

\[ S^{(2)} = \frac{1}{2} \int d^4x ((v')^2 - v_i v^i + \frac{z''}{z} v^2) \]  \hspace{2cm} (5)

\[ v = a(\delta\varphi + \frac{z}{a}\Phi) \]  \hspace{2cm} (6)

\[ z = a\frac{\varphi_0'}{\mathcal{H}} \]  \hspace{2cm} (7)
Step 1: Metric including fluctuations

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\begin{align*}
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(7)
Step 3: Resulting equation of motion (Fourier space)

\[ v''_k + (k^2 - \frac{z''}{z})v_k = 0 \] (8)

Features:

- oscillations on sub-Hubble scales
- squeezing on super-Hubble scales \( v_k \sim z \)

Quantum vacuum initial conditions:

\[ v_k(\eta_i) = (\sqrt{2k})^{-1} \] (9)
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\[ v_k'' + (k^2 - \frac{Z''}{Z})v_k = 0 \]  \hspace{1cm} (8)

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Quantum vacuum initial conditions:

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Structure formation in inflationary cosmology

N.B. Perturbations originate as quantum vacuum fluctuations.
Background for string gas cosmology

![Graph showing scale factor a as a function of time t]

- **Moduli Stabilization in SGC**

- **String gas**
  - Principles
  - Features
  - Moduli stabilization in SGC

- **Structure**
  - Perturbations
  - Overview
  - Analysis
  - Signatures in CMB anisotropy maps

- **Conclusions**

- **Moduli Stabilization**
N.B. Perturbations originate as thermal string gas fluctuations.
Method

- Calculate matter correlation functions in the Hagedorn phase (neglecting the metric fluctuations)
- For fixed \( k \), convert the matter fluctuations to metric fluctuations at Hubble radius crossing \( t = t_i(k) \)
- Evolve the metric fluctuations for \( t > t_i(k) \) using the usual theory of cosmological perturbations
Extracting the Metric Fluctuations

Ansatz for the metric including cosmological perturbations and gravitational waves:

\[ ds^2 = a^2(\eta)((1 + 2\Phi)d\eta^2 - [(1 - 2\Phi)\delta_{ij} + h_{ij}]dx^i dx^j) . \]

Inserting into the perturbed Einstein equations yields

\[ \langle |\Phi(k)|^2 \rangle = 16\pi^2 G^2 k^{-4} \langle \delta T^0_0(k)\delta T^0_0(k) \rangle , \]

\[ \langle |h(k)|^2 \rangle = 16\pi^2 G^2 k^{-4} \langle \delta T^i_j(k)\delta T^i_j(k) \rangle . \]
Key ingredient: For thermal fluctuations:

\[
\langle \delta \rho^2 \rangle = \frac{T^2}{R^6} C_V. \tag{13}
\]

Key ingredient: For string thermodynamics in a compact space

\[
C_V \approx 2 \frac{R^2 / \ell_S^3}{T (1 - T / T_H)}. \tag{14}
\]
Power Spectrum of Cosmological Perturbations

Key ingredient: For thermal fluctuations:

$$\langle \delta \rho^2 \rangle = \frac{T^2}{R^6} C_V .$$ \hspace{1cm} (13)

Key ingredient: For string thermodynamics in a compact space

$$C_V \approx 2 \frac{R^2 / \ell_s^3}{T \left(1 - T / T_H \right)} .$$ \hspace{1cm} (14)
Power spectrum of cosmological fluctuations

\[ P_\Phi(k) = 8G^2k^{-1} \langle |\delta \rho(k)|^2 \rangle \quad (15) \]
\[ = 8G^2k^2 \langle (\delta M)^2 \rangle_R \quad (16) \]
\[ = 8G^2k^{-4} \langle (\delta \rho)^2 \rangle_R \quad (17) \]
\[ = 8G^2 \frac{T}{\ell_s^3} \frac{1}{1 - T/T_H} \quad (18) \]

Key features:

- scale-invariant like for inflation
- slight red tilt like for inflation
Power spectrum of cosmological fluctuations

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\[ = 8G^2k^2 < (\delta M)^2 >_R \quad (16) \]
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Key features:

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Evolution for $t > t_i(k)$: $\Phi \simeq \text{const}$ since the equation of state parameter $1 + w$ stays the same order of magnitude unlike in inflationary cosmology.

Squeezing of the fluctuation modes takes place on super-Hubble scales like in inflationary cosmology $\rightarrow$ acoustic oscillations in the CMB angular power spectrum

In a dilaton gravity background the dilaton fluctuations dominate $\rightarrow$ different spectrum [R.B. et al, 2006; Kaloper, Kofman, Linde and Mukhanov, 2006]
\[ P_h(k) = 16\pi^2 G^2 k^{-1} < |T_{ij}(k)|^2 > \]  
\[ = 16\pi^2 G^2 k^{-4} < |T_{ij}(R)|^2 > \]  
\[ \sim 16\pi^2 G^2 \frac{T}{\ell_s^3} (1 - T / T_H) \]

Key ingredient for string thermodynamics

\[ < |T_{ij}(R)|^2 > \sim \frac{T}{\ell_s^3 R^4} (1 - T / T_H) \]

Key features:
- scale-invariant (like for inflation)
- slight blue tilt (unlike for inflation)
Spectrum of Gravitational Waves

\[ P_h(k) = 16\pi^2 G^2 k^{-1} < |T_{ij}(k)|^2 > \]  
\[ = 16\pi^2 G^2 k^{-4} < |T_{ij}(R)|^2 > \]  
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Key features:
- scale-invariant (like for inflation)
- slight blue tilt (unlike for inflation)
1. static Hagedorn phase (including static dilaton) $\rightarrow$ new physics required.

2. $C_V(R) \sim R^2$ obtained from a thermal gas of strings provided there are winding modes which dominate.

3. Cosmological fluctuations in the IR are described by Einstein gravity.

Note: Specific higher derivative toy model: T. Biswas, R.B., A. Mazumdar and W. Siegel, 2006
Requirements

1. static Hagedorn phase (including static dilaton) → new physics required.

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Remnant of the Hagedorn phase: network of cosmic superstrings

This string network will be present at all times and will achieve a scaling solution like cosmic strings forming during a phase transition.

Scaling Solution: The network of strings looks statistically the same at all times when scaled to the Hubble radius.
Kaiser-Steppbins Effect

Space perpendicular to a string is conical with deficit angle

$$\alpha = 8\pi G\mu, \quad (23)$$

Photons passing by the string undergo a relative Doppler shift

$$\frac{\delta T}{T} = 8\pi \gamma(v)vG\mu, \quad (24)$$

→ network of line discontinuities in CMB anisotropy maps.

N.B. characteristic scale: comoving Hubble radius at the time of recombination → need good angular resolution to detect these edges.
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10° x 10° map of the sky at 1.5’ resolution (South Pole Telescope specifications)
Cosmic string temperature map

$10^0 \times 10^0$ map of the sky at 1.5’ resolution
This signal is superimposed on the Gaussian map. The relative power of the string signature depends on $G_{\mu}$ and is bound to contribute less than 10% of the power.
Cosmic string $\rightarrow$ wake
**Wake**: overdensity of free electrons

- → rectangle in the sky with extra polarization.
- Since the direction of the string is uncorrelated with the axis of the CMB quadrupole, statistically an equal contribution of E and B modes is predicted.
- Signal is strongest from wakes produced by strings close to $t_{\text{rec}}$
- → typical length scale is $1^\circ$
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\( G_\mu = 3 \times 10^{-7} \), string signal multiplied by \( 10^2 \), “noise” is due to the (dominant) Gaussian fluctuations.
Temperature map Gaussian + strings

**Challenge:** pick out the string signature from the Gaussian "noise" which has a much larger amplitude
### CANNY edge detection algorithm

- **New technique:** use CANNY edge detection algorithm [Canny, 1986]
- **Idea:** find edges across which the gradient is in the correct range to correspond to a Kaiser-Stebbins signal from a string

**Step 1:** generate "Gaussian" and "Gaussian plus strings" CMB anisotropy maps: size and angular resolution of the maps are free parameters, flat sky approximation, cosmic string toy model in which a fixed number of straight string segments is laid down at random in each Hubble volume in each Hubble time step between $t_{rec}$ and $t_0$.

**Step 2:** run the CANNY algorithm on the temperature maps to produce edge maps.

**Step 3:** Generate histogram of edge lengths

**Step 4:** Use Fisher combined probability test.
Preliminary Results

- For South Pole Telescope (SPT) specification: limit \( G_\mu < 2 \times 10^{-8} \) can be set [A. Stewart and R.B., 2008, R. Danos and R.B., 2008]
- Anticipated SPT instrumental noise only insignificantly affects the limits [A. Stewart and R.B., 2008]
- WMAP data: limit \( G_\mu < 2 \times 10^{-7} \) can be set [E. Thewalt, in prep.]
Conclusions:

- **String Gas Cosmology**: Model of cosmology of the very early universe based on new degrees of freedom and new symmetries of superstring theory.
- **SGC → nonsingular cosmology**
- **SGC → natural explanation of the number of large spatial dimensions.**
- **SGC → new scenario of structure formation**
- **Scale invariant spectrum of cosmological fluctuations (like in inflationary cosmology).**
- **Spectrum of gravitational waves has a small blue tilt (unlike in inflationary cosmology).**
Conclusions II

- SGC leaves behind a network of cosmic superstrings.
- These cosmic superstrings give rise to line discontinuities in CMB anisotropy maps which can be probed using a CANNY edge detection algorithm.
- A specific signature are string junctions.
- Line discontinuities in CMB B-mode polarization maps.
- String theory testable in cosmological observations.
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Action: Dilaton gravity plus string gas matter

\[ S = \frac{1}{\kappa} (S_g + S_\phi) + S_{SG}, \quad (25) \]

\[ S_{SG} = - \int d^{10}x \sqrt{-g} \sum_{\alpha} \mu_\alpha \epsilon_\alpha, \quad (26) \]

where

- \( \mu_\alpha \): number density of strings in the state \( \alpha \)
- \( \epsilon_\alpha \): energy of the state \( \alpha \).

Introduce comoving number density:

\[ \mu_\alpha = \frac{\mu_{0,\alpha}(t)}{\sqrt{g_s}}, \quad (27) \]
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Energy-Momentum Tensor

Ansatz for the metric:

\[ ds^2 = -dt^2 + a(t)^2 d\vec{x}^2 + \sum_{a=1}^{6} b_a(t)^2 dy_a^2, \]  

(28)

Contributions to the energy-momentum tensor

\[ \rho_\alpha = \frac{\mu_{0,\alpha}}{\epsilon_\alpha \sqrt{-g}} \epsilon_\alpha^2, \]  

(29)

\[ p^i_\alpha = \frac{\mu_{0,\alpha}}{\epsilon_\alpha \sqrt{-g}} \frac{p_d^2}{3}, \]  

(30)

\[ p^a_\alpha = \frac{\mu_{0,\alpha}}{\epsilon_\alpha \sqrt{-g} \alpha'} \left( \frac{n_a^2}{b_a^2} - w_a^2 b_a^2 \right). \]  

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Single string energy

\( \epsilon_\alpha \) is the energy of the string state \( \alpha \):

\[
\epsilon_\alpha = \frac{1}{\sqrt{\alpha'}} \left[ \alpha' p_d^2 + b^{-2}(n, n) + b^2(w, w) \\
+ 2(n, w) + 4(N - 1) \right]^{1/2},
\]

where

- \( \vec{n} \) and \( \vec{w} \): momentum and winding number vectors in the internal space
- \( \vec{p}_d \): momentum in the large space
Single string energy

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(32)

where

- $\vec{n}$ and $\vec{w}$: momentum and winding number vectors in the internal space
- $\vec{p}_d$: momentum in the large space
Background equations of motion

Radion equation:

$$\ddot{b} + b \left( 3 \frac{\dot{a}}{a} + 5 \frac{\dot{b}}{b} \right) = \frac{8\pi G \mu_{0,\alpha}}{\alpha' \sqrt{\hat{G}_a \epsilon_\alpha}}$$

$$\times \left[ \frac{n_a^2}{b^2} - w_a^2 b^2 + \frac{2}{(D-1)} \left[ b^2 (w, w) + (n, w) + 2(N - 1) \right] \right]$$

Scale factor equation:

$$\ddot{a} + \dot{a} \left( 2 \frac{\dot{a}}{a} + 6 \frac{\dot{b}}{b} \right) = \frac{8\pi G \mu_{0,\alpha}}{\sqrt{\hat{G}_i \epsilon_\alpha}}$$

$$\times \left[ \frac{p_\alpha^2}{3} + \frac{2}{\alpha' (D - 1)} \left[ b^2 (w, w) + (n, w) + 2(N - 1) \right] \right]$$
Special states

Enhanced symmetry states

\[ b^2(w, w) + (n, w) + 2(N - 1) = 0. \]  \hspace{1cm} (35)

Stable radion fixed point:

\[ \frac{n_a^2}{b^2} - w_a^2 b^2 = 0. \]  \hspace{1cm} (36)
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Add a single non-perturbative ingredient - **gaugino condensation** - in order to fix the remaining modulus, the dilaton.

Kähler potential: (standard)

\[
\mathcal{K}(S) = -\ln(S + \bar{S}), \quad S = e^{-\Phi} + ia. \tag{37}
\]

where \( \Phi = 2\phi - 6 \ln b \) is the 4-d dilaton, \( b \) is the radion and \( a \) is the axion.

Non-perturbative superpotential (from gaugino condensation):

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W = M_P^3 \left( C - Ae^{-a_0 S} \right) \tag{38}
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Yields a potential for the dilaton (and radion)

\[ V = \frac{M_P^4}{4} b^{-6} e^{-\Phi} \left[ \frac{C^2}{4} e^{2\Phi} + A C e^\Phi \left( a_0 + \frac{1}{2} e^\Phi \right) e^{-a_0 e^{-\Phi}} ight. \\
+ A^2 \left( a_0 + \frac{1}{2} e^\Phi \right)^2 e^{-2a_0 e^{-\Phi}} \right] . \]  

(39)

Expand the potential about its minimum:

\[ V = \frac{M_P^4}{4} b^{-6} e^{-\Phi_0} a_0^2 A^2 \left( a_0 - \frac{3}{2} e^{\Phi_0} \right)^2 e^{-2a_0 e^{-\Phi_0}} \times \left( e^{-\Phi} - e^{-\Phi_0} \right)^2 . \]  

(40)
Dilaton potential I

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V = \frac{M_P^4}{4} b^{-6} e^{-\Phi} \left[ \frac{C^2}{4} e^{2\Phi} + A C e^{\Phi} \left( a_0 + \frac{1}{2} e^{\Phi} \right) e^{-a_0 e^{-\Phi}} \right. \\
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\]

(40)
Lift the potential to 10-d, redefining $b$ to be in the Einstein frame.

$$V(b, \phi) = \frac{M_{10}^{16}}{4} \hat{V} e^{-\phi_0} a_0^2 A^2 \left( a_0 - \frac{3}{2} e^{\phi_0} \right)^2 e^{-2a_0 e^{-\phi_0}}$$

$$\times e^{-3\phi/2} \left( b^6 e^{-\phi/2} - e^{-\phi_0} \right)^2 .$$ (41)

Dilaton potential in 10d Einstein frame

$$V \simeq n_1 e^{-3\phi/2} \left( b^6 e^{-\phi/2} - n_2 \right)^2$$ (42)
Analysis including both string matter and dilaton potential I

Worry: adding this potential will mess up radion stablilization
Thus: consider dilaton and radion equations resulting from the action including both the dilaton potential and string gas matter.

Step 1: convert the string gas matter contributions to the 10-d Einstein frame

\[
\begin{align*}
g^E_{\mu\nu} &= e^{-\phi/2} g^S_{\mu\nu} \tag{43} \\
b_s &= e^{\phi/4} b_E \tag{44} \\
T^E_{\mu\nu} &= e^{2\phi} T^S_{\mu\nu} \tag{45}
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\[ T^E_{\mu\nu} = e^{2\phi} T^S_{\mu\nu} \]  
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Step 2: Consider both dilaton and radion equations:

\[- \frac{M_{10}^8}{2} \left( 3a^2 \dot{a} b^6 \dot{\phi} + 6a^3 b^5 \dot{b} \phi + a^3 b^6 \ddot{\phi} \right)\]

+ \[\frac{3}{2} n_1 a^3 b^6 e^{-3\phi/2} \left( b^6 e^{-\phi/2} - n_2 \right)^2\]

+ \[a^3 b^{12} n_1 e^{-2\phi} \left( b^6 e^{-\phi/2} - n_2 \right)\]

+ \[\frac{1}{2} e^{\phi/4} \left( -\mu_0 \epsilon^2 + \mu_0 |p_d|^2 \right)\]

+ \[6\mu_0 \left( \frac{n_2^2}{\alpha'} e^{-\phi/2} b^{-2} - \frac{w^2}{\alpha'} e^{\phi/2} b^2 \right)\]

= 0, \quad (46)
Joint analysis III

\[
\ddot{b} + 3 \frac{\dot{a}}{a} \dot{b} + 5 \frac{\dot{b}^2}{b} = - \frac{n_1 b}{M_{10}^8} e^{-3\phi/2} \left( b^6 e^{-\phi/2} - n_2 \right)^2 
- \frac{2n_1}{M_{10}^8} b^7 e^{-2\phi} \left( b^6 e^{-\phi/2} - n_2 \right) 
+ \frac{1}{2 - D} \left[ - \frac{10b}{M_{10}^8} n_1 e^{-3\phi/2} \left( b^6 e^{-\phi/2} - n_2 \right)^2 
- \frac{12n_1}{M_{10}^8} b^7 e^{-2\phi} \left( b^6 e^{-\phi/2} - n_2 \right) \right] 
+ \frac{8\pi G_D \mu_0}{\alpha' \sqrt{\hat{G}_a \epsilon}} e^{2\phi} \left[ n_1^2 b^{-2} e^{-\phi/2} - w_a^2 b^2 e^{\phi/2} \right] 
+ \frac{2}{D - 1} \left( e^{\phi/2} b^2 w^2 + n \cdot w + 2(N - 1) \right) 
\]
Joint analysis IV

**Step 3: Identifying extremum**
- Dilaton at the minimum of its potential **and**
- Radion at the enhanced symmetry state

**Step 4: Stability analysis**
- Consider small fluctuations about the extremum
- Show stability (tedious but straightforward)

Result: Dilaton and radion stabilized simultaneously at the enhanced symmetry point.
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