Visiting Newton’s Atelier before the Principia
1679-1684

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This admirable work contains the germs of all the great discoveries that have been made since, about the system of the world: the history of its development by the followers of that great geometer will be at the same time the most useful comment on his work, as well as the best guide to arrive at new discoveries.
Reading Newton became for Chandrasekhar a sustained epiphany:

“The view of science that he exhibit, the clarity with which he writes the number of new things that he finds, manifest a physical and mathematical insight of which there is no parallel in science at any time”

Scientific American, March 1994
NEWTON'S PRINCIPIA.

ANALYTICAL VIEW.

This work is justly considered by all men as the greatest of the monuments of human genius. It contains the exposition of the laws of motion in all its varieties, whether in free space or in resisting media, and of the action exerted by the masses or the particles of matter upon each other, those laws demonstrated by synthetic reasoning; and it unfolds the most magnificent discovery that was ever made by man — the Principle of Universal Gravitation, by which the system of the universe is governed under the superintendence of its Divine Maker.
Meeting on Newtonian Scholarship held at the London Royal Society in 1997
“We now know that neither Principia nor Opticks sprang like Minerva from the head of Jove: they are a palimpsest of investigation and tentative endeavors we have been given glimpses - more is hardly possible- into the way Newton created his sciences . . .”

Rupert Hall in “Review and Reminiscences”, The Foundations of Newtonian Scholarship, pg 201
In his introduction to Newton's Principia, the eminent Newtonian scholar I. B. Cohen asked:

Whatever happened to the work-sheets of the Principia? Do they still exist in some obscure private or public collection? Was this particular set of manuscripts - alone of all the Newton papers - lost or mislaid, either when the Portsmouth Collection was still in Hurstbourne Castle or during the actual transfer to the University Library in Cambridge? Did such work-sheets still exist among Newton's papers at the time of his death? Or were they lost or destroyed - either by chance or design - during Newton's own lifetime?

We may possibly never be certain of the answer to these questions.”

I. B. Cohen, in “Introduction to Newton's Principia"
Certainly there can be no doubt that the peculiar geometrical form in which the exposition of the *Principia* is dressed up bears no resemblance at all to the mental processes by which Newton actually arrived at his conclusions.”

J. Maynard Keynes in “Newton the Man” 1946

Copy of Newton bust at the London Royal Society by Michael Rysbrack
While he was musing in a garden it came into his thought that the power of gravity (which brought an apple from a tree to the ground) was not limited to a certain distance from the earth but that this power must extend to much farther than was usually thought. Why not as high as the moon said he to himself and if so that must influence her motion and perhaps retain her in her orbit.
Newton
Portrait by Sir Godfrey Kneller
at the National Portrait Gallery, London
Michael Nauenberg  
Dept of Physics  
University of Amsterdam  
Valckemert Straat 65  
Amsterdam  
THE NETHERLANDS  

22 April 1996  

Dear Mr Nauenberg,

Thank you for filling out a visitor comment form on 24 March 1996.

The caption about Kneller’s portrait of Sir Isaac Newton was written by my predecessor, who has now left the Gallery, and so I can only presume what his intentions were in writing it. However I am sure that he did not intend to imply that Newton did not develop his law of gravity through a scientific process. Indeed, it is not certain that the incident with the apple ever happened, and I think that in writing that the laws of gravity were ‘traditionally said to have been revealed to him’ when he saw an apple falling from a tree, my predecessor was simply intending to record a tradition that grew up about Newton’s discovery after his death. On the other hand, Newton was, of course, a religious man as well as a scientist, and it may well be that he believed that there was a degree of religious revelation involved in stimulating his discoveries.

However, I agree that the caption is, perhaps, misleading in making Newton seem so passive in relation to his development of the law of gravity. I have amended the text, and I enclose a copy of the new caption. I hope that this meets with your approval.

Thank you for your interest in the Gallery. I hope that in spite of any annoyance caused by the Newton caption you enjoyed your visit.

Yours sincerely,

Catherine MacLeod
“...to avoid being baited by little Smatterers in Mathematics... he designedly made his Principia abstruse; but yet so as to be understood by able Mathematicians’ Newton, as told to William Derham Keynes MS. 133, pg 10
“When Newton’s Principia first appeared only the most advanced mathematicians were able to fathom its depths . . . the work acquiring a reputation as an impenetrable treatise presenting almost divine revelations about Nature.”

On November 24, 1679, Robert Hooke wrote to Newton:

“For my own part I shall take it as a great favour if you please to communicate by Letter your objections against any hypothesis or opinion of mine, and particularly if you will let me know your thoughts of that of compounding the celestially motion of the planetts of a direct motion by the tangents and an attractive motion toward the central body.”
Hooke had elaborated his ideas in a short tract, published in 1674, entitled:

“An attempt to prove the motion of the Earth by observations”.

Hooke argued that attractive gravitational forces were universal. About terrestrial gravitation he wrote:

This propagated Pulse I take it to be the Cause of the descent of bodies towards the Earth

. . . Suppose for Instance there should be 1000 of these Pulses in a second of Time, then must the Grave body receive all those thousand impressions within the space of time of that Second, and a thousand more the next . . .".
Newton’s response to Hooke on Dec 24, 1679:
“Your accute letter having put me upon considering thus far the species of this curve, I might add something about its description by points quam proxime . . .

Newton’s letter to Halley on May 27, 1686
“I then took the simplest case for computation, which was that of Gravity uniform in a medium not Resisting”
Newton’s Graphical Method based on local curvature

Curvature in Orbital Dynamics
American Journal of Physics
73(2005) 340

Newton’s Early Computational Method for Dynamics
Archive for History of Science
46 (1994) 212-221
This letter contains among other mistakes an impossible picture of an orbit . . .

V. I. Arnold in “Huygens & Barrow, Newton & Hooke
According to several accounts originating with Newton, on August 1684 Halley visited him and asked:

"what he thought the curve would be that would be described by planets supposing the force of attraction towards the Sun to be reciprocal to the square of the distance from it".

"Sir Isaac replied immediately that it would be an ellipses . . .” but when asked for his calculation he claimed that he couldn't find it.

Actually Newton could only answer the converse to Halley's question, if the curve is an ellipse, the central force is an inverse square force. It is unlikely that he could have answered Halley's original question.
Hooke graphical calculation of the orbit for a force that depends linearly on the distance from the center

Newton claimed that “Dr. Hook could not perform that which he pretented to: let him give Demonstrations of it: I know he hath not Geometry enough to do it.”

Letter of William Derham to Conduitt Esquire, July 18, 1733
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Hooke graphical calculation of the orbit for a force that depends linearly on the distance from the center

A vortex in the sun's atmosphere, the velocity of the sun's light being $V$, the velocity of light being $L$, and the distance from the sun being $d$.

When the velocity and direction of the portion of the sun's light, which is reflected from the sun's surface, is increased by the sun's light, the body moves in a circle of the gravity to the center of the sun. But if the sun's light be reflected by the sun's surface, the body moves in a circle of the sun's gravity to the center of the sun.
Hooke’s graphical construction of an elliptic orbit for central force depending linearly on the distance from the center.

Drawn on Sept. 1685.
Graphical construction with inverse square force and Hooke’s initial condition
In July 14, 1686, in a letter to Halley Newton admitted his indebtedness to Hooke. He wrote:

“This is true, that his Letters occasioned my finding the method of determining Figures which when I tried it in the Ellipsis”

But then Newton equivocated claiming that:

“I threw the calculation by being upon other studies & so it rested for about 5 years till upon your request I sought for yt paper, & not finding it did it again and reduced it into ye Proposition shown you for Mr. Paget . . .”

In November, 1684, Newton sent to the Royal Society a treatise of 9 pages “On the Motion of Bodies in an Orbit”, that constituted the first draft of the Principia
During his conflict with Leibniz on the development of the Calculus, Newton wrote

“In the end of the year 1679 in answer to a letter from Dr. Hook then secretary of the R.S. . . . I wrote that Whereupon I computed what would be the Orb described by the Planets, for I had found before by the sesquialterate proportion of the tempora periodica of the Planets with respect to their distances from the Sun, that the forces which kept them in their Orbs about the Sun were as the squares of their mean distances from the Sun reciprocally, & I found now that whatsoever was the law of the forces which kept the planets in their Orbs the areas described by a Radius drawn from them to the Sun would be proportional to the times in which they were described. And by the help of these Propositions I found that their Orbs would be such Ellipses as Kepler had described.

MS. Add 3958 b fol. 101
Graphical Construction of orbit for constant central impulses, based on Hooke’s physical concept and Newton’s mathematical implementation.

Let S be the center of force and A the initial position of a body.

\[ S \overline{A} \]
Draw the initial displacement AB were AB = vδt

v = initial velocity

δt = time interval between periodic impulses
To obtain the next point, draw the extension \( Bc = AB \), and the impulse \( BV \) at \( B \) directed along \( SB \).
Obtain the next point $C$ by Newton's parallelogram construction to add velocities \textit{vectorially:} draw $VC$ parallel and equal to extension $Bc$ or draw $Cc$ parallel and equal to impulse $VB$. Then $BC$ is the displacement after the impulse at $B$. 

![Diagram](image-url)
Join S to C and repeat this graphical construction for successive impulses at periodic intervals $\delta t$.
Polygonal orbit $ABCDEF$ obtained after four impulses.
Newton’s diagram for Proposition I in Principia, Book I
Analytic form of Newton’s graphical construction

The velocity $\vec{v}(i)$ before the $ith$ impulse is

\begin{equation}
\vec{v}(i) = \frac{\vec{r}(i) - \vec{r}(i - 1)}{\delta t},
\end{equation}

and after the $ith$ impulse

\begin{equation}
\vec{v}(i + 1) = \vec{v}(i) + \vec{f}(i) \delta t,
\end{equation}

where $\delta t$ is the periodic time interval between impulses,

\begin{equation}
\vec{f}(i) = \frac{\vec{h}(i)}{\delta^2 t},
\end{equation}

$\vec{h}(i)$ is the magnitude of the impulse, and $\vec{f}(i)$ is the corresponding force.

According to Eq. 1,

\begin{equation}
\vec{r}(i + 1) = \vec{r}(i) + \vec{v}(i + 1) \delta t,
\end{equation}

and Eqs. 2 and 4 are the analytic form of Newton’s graphical equations of motion.

These two equations are symplectic (area preserving), and have been rediscovered several times in the past.

In the limit $\delta t \to 0$, Eq. 2 is the well known equation of classical mechanics

\begin{equation}
\vec{f}(t) = m \ddot{\vec{a}}(t),
\end{equation}

where $\ddot{\vec{a}}(t) = \delta \vec{v}/\delta t$ and $m = 1$. 

Early manuscript of Newton’s Principia sent to the Royal Society in 1684

Proposition I diagram

Principia Book I
Newton’s Papers
Cambridge University Library

De Motu
Newton’s preliminary manuscript for the Principia sent to the Royal Society in 1684
Proposition I, Principia, Book I

“The area which bodies made to move in orbits described by radii drawn to an unmoving center of forces lie in unmoving planes and are proportional to the times”
Time Reversal
Conversation between Laplace and Napoleon at a reception in 1802

Napoleon: Newton spoke of God in his book. I perused yours but failed to find his name even once.

Laplace: I had no need for that hypothesis.

Knowledge of the method that has guided a man of genius is no less useful to the progress of science and to his glory than his discoveries; the method is often the most interesting part.”


References


