# Inhomogeneous phases driven by competing orders

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# Message

Inhomogeneous states and glassiness may occur spontaneously in systems ***without disorder*** as a result of competing interactions or competing orders.

*S. Brazovskii, 1975; J. Schmalian and P. Wolynes, 2000-2004; ZN, IV, AVB 2004.*

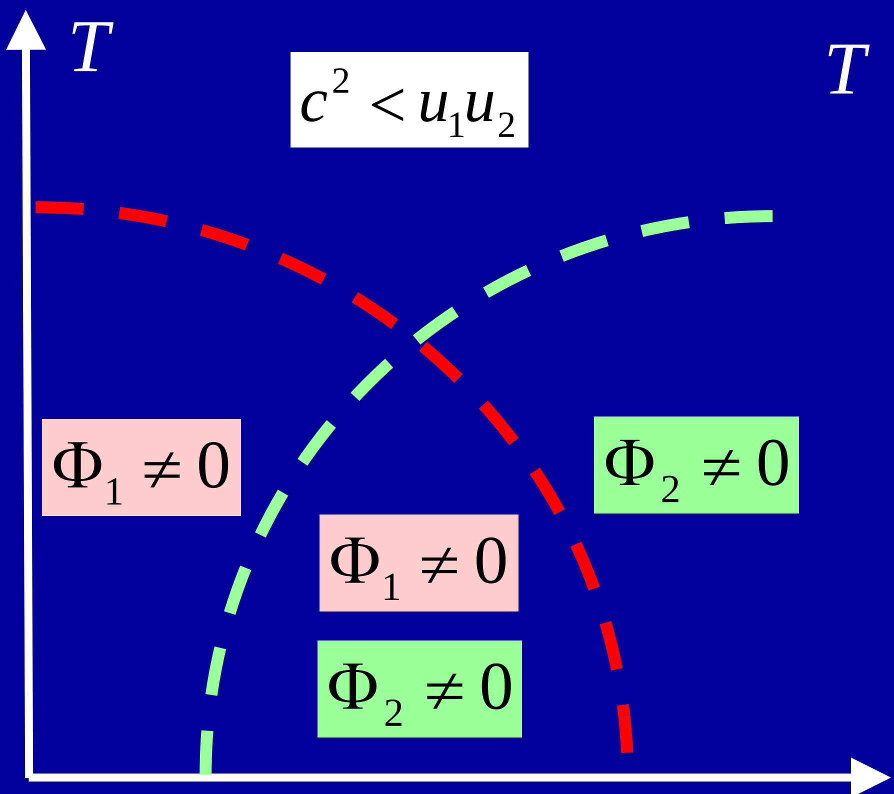
# Inhomogeneous ordered states

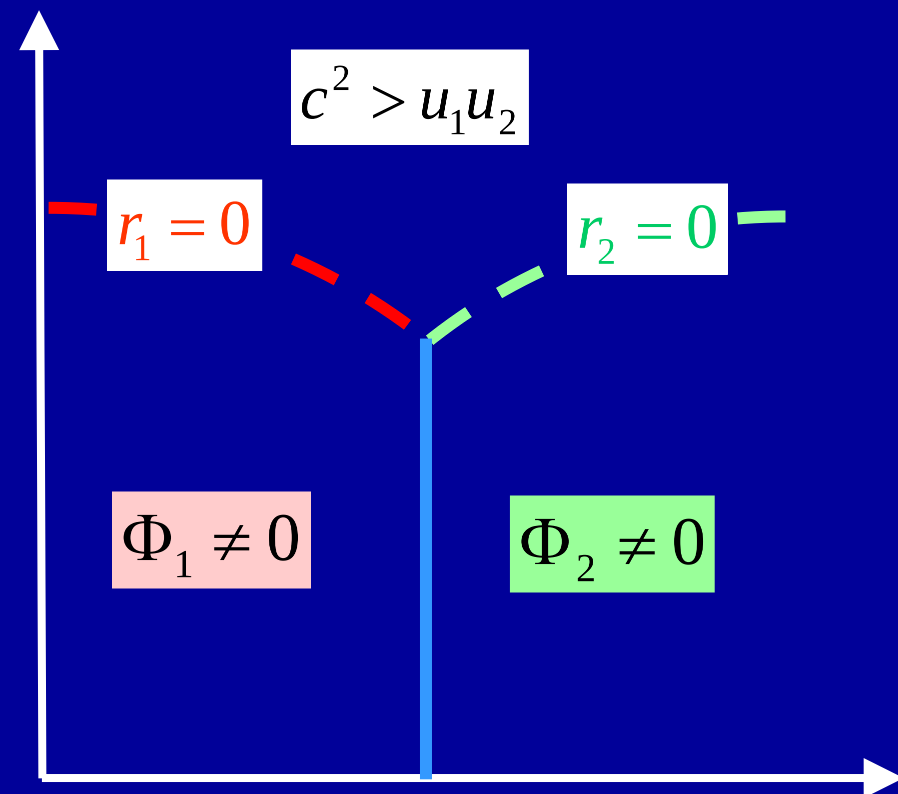
- Coexistence and competition of different orders: *manganites, heavy fermions, cuprates,…*
- Inhomogeneities on the micro-meso-nano- scale
- Microscopic origins: disorder, competing interactions, competing orders
- Description at the level of effective theories (Ginzburg-Landau)?

# Competing orders: a reminder

$$F_0 = \frac{r_1}{2}|\Phi_1|^2 + \frac{u_1}{4}|\Phi_1|^4 + \frac{r_2}{2}|\Phi_2|^2 + \frac{u_2}{4}|\Phi_2|^4 + \frac{c}{2}|\Phi_1|^2|\Phi_2|^2 + \frac{1}{2}(\nabla\Phi_1)^2 + \frac{1}{2}(\nabla\Phi_2)^2$$

*E. M. Lifshitz (1944), K. Wilson and M. Fisher (1972), Liu and Fisher, 1973*

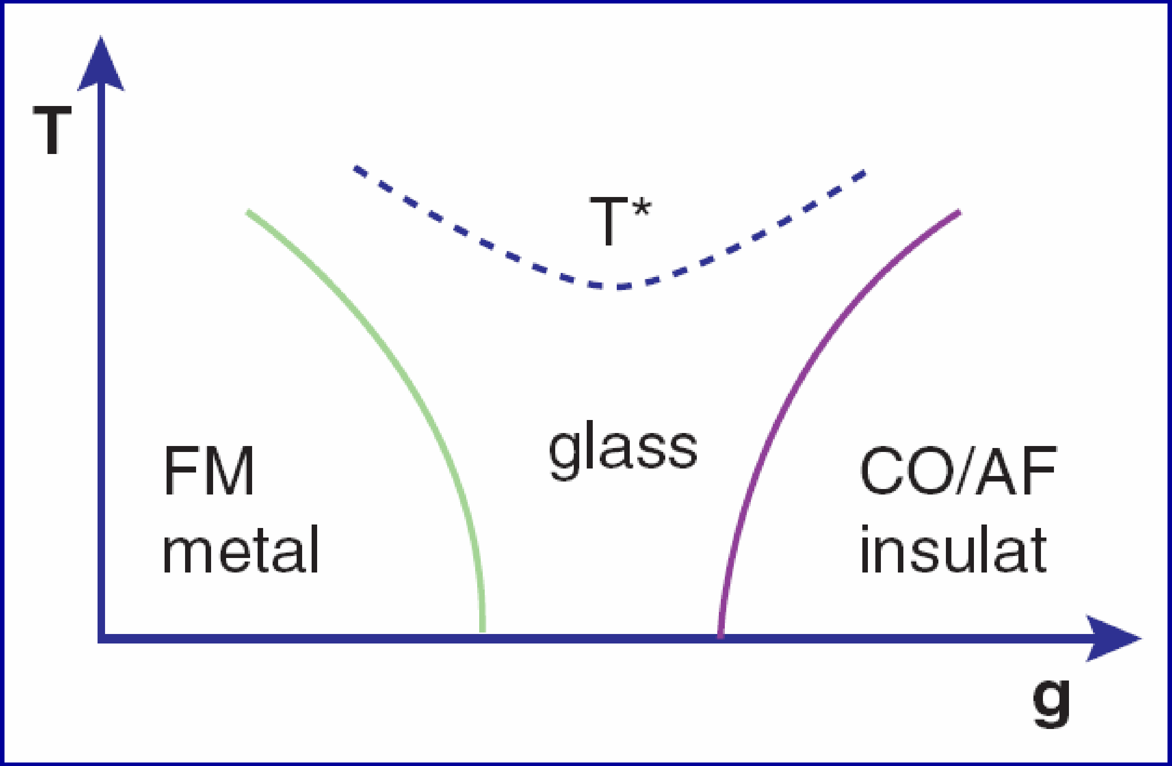


# Disorder-induced glass

Disorder + competing orders

*E. Dagotto et al. 2001-2005;*




**Can we obtain inhomogeneous states in a uniform system?**

# Gradient couplings

- System has a preferred wave vector:

$$F_1 = a\,|\Phi_1|^2|\Phi_2|^2\,(\nabla\varphi_2 - q_0),\ \ \text{where}\quad \Phi_2 = |\Phi_2|\exp(i\varphi_2)$$

**manganites**, *G. Milward, M. Calderon, P. Littlewood, 2005*

- System selects the wave vector from interactions

$$F_1 = a\,|\Phi_2|^2|\nabla\Phi_1|^2 + higher,\ \ a<0$$

***ZN, IV, and AVB, 2004-05***

**e.g. stripes: charges like to sit at magnetic domain walls**

- Inhomogeneity only in the coexistence region

$$F_0 = \frac{r_1}{2}|\Phi_1|^2 + \frac{u_1}{4}|\Phi_1|^4 + \frac{r_2}{2}|\Phi_2|^2 + \frac{1}{4}|\Phi_2|^4 + \frac{c}{2}|\Phi_1|^2|\Phi_2|^2$$

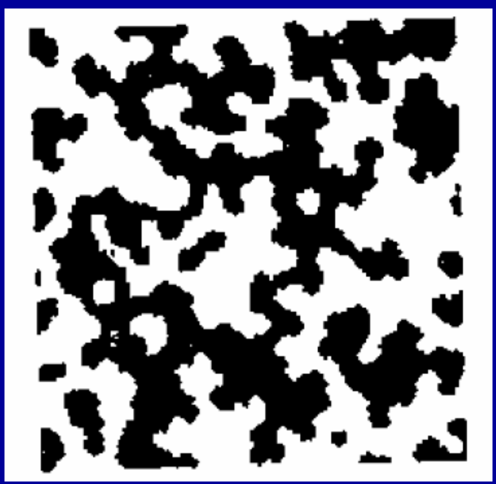
$$F_1 = \frac{a}{2}|\Phi_2|^2|\nabla\Phi_1|^2 + \frac{1}{4}|\nabla\Phi_1|^4 + \frac{1}{2}|\nabla\Phi_1|^2 + \frac{1}{2}|\nabla\Phi_2|^2 + higher$$

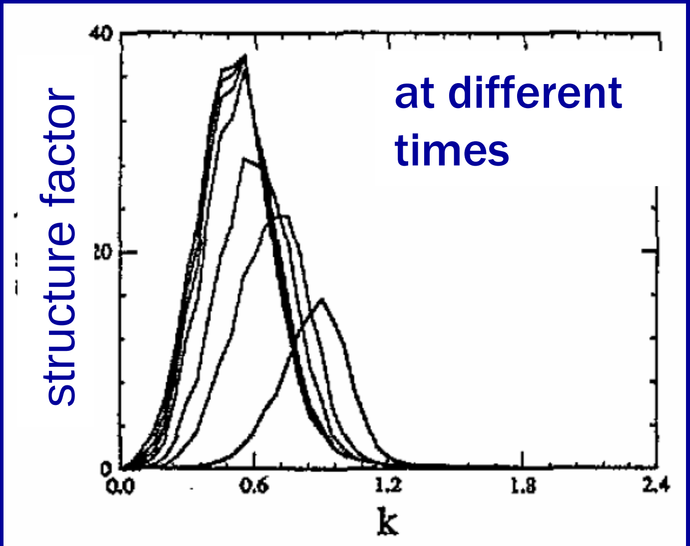
- Example: mean field $T_{c2} > T_{c1}$ :

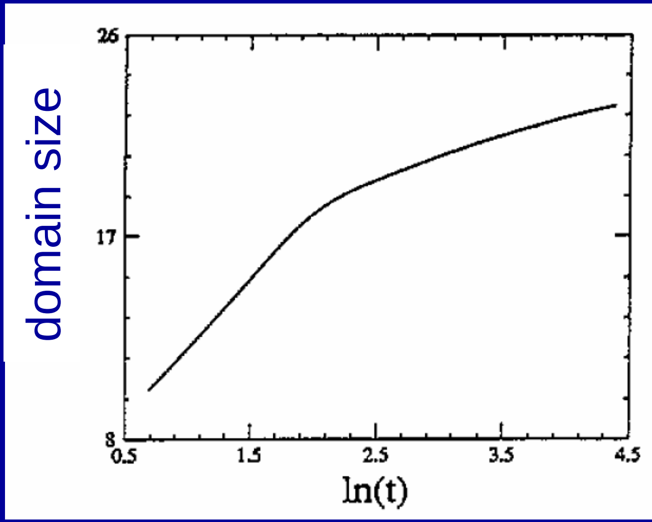
$$F_{11} = -\frac{|a|}{4} |\Phi_2|^2 q^2 |\Phi_1|^2$$

- **Modulated phase** with weakly T-dependent $q_0 \propto \Phi_2$
- **Effective model similar to surfactants in liquids**
- **Complicated inhomogeneous states**

*M. Laradji et al. 1992*





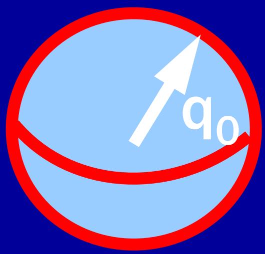

**Inhomogeneous; Slow dynamics; Glassy?**

# Effective theory

*S. Brazovskii, 1975*

$$H = \frac{1}{2}\sum_{\mathrm{k}} v(\mathrm{k})\Phi_1(\mathrm{k})\Phi_1(-\mathrm{k}) + \frac{u}{4}\sum_{\mathrm{k}_1+\mathrm{k}_2+\mathrm{k}_3+\mathrm{k}_4=0} \Phi_1(\mathrm{k}_1)\Phi_1(\mathrm{k}_2)\Phi_1(\mathrm{k}_3)\Phi_1(\mathrm{k}_4)$$

$$v(\mathrm{k}) \approx r_0 + (\mathrm{k}^2 - q_0^2)^2, \qquad r_0 \equiv \xi_0^{-2} = a(T - T_{c1})$$




- Isotropic model - shell of modes $|\mathbf{k}|=q_0$.
- Large phase space for fluctuations:

classical dynamics

$$\langle \Phi^2 \rangle = bTq_0^2 / \sqrt{r_0}$$

or quantum dynamics

$$\langle \Phi^2 \rangle = bq_0^2 \log \frac{E_0}{r_0}$$

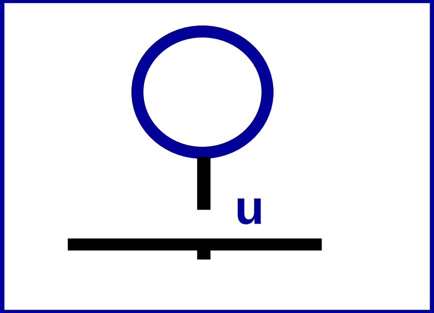
- Drives system away from transition

# $1^{st}$ order transition

***Brazovskii transition**: S. Brazovskii, 1975*

- **Self-consistency (large N)**

$$r = r_0 + u\left\langle \Phi^2 \right\rangle = r_0 + ubTq_0^2/\sqrt{r}$$



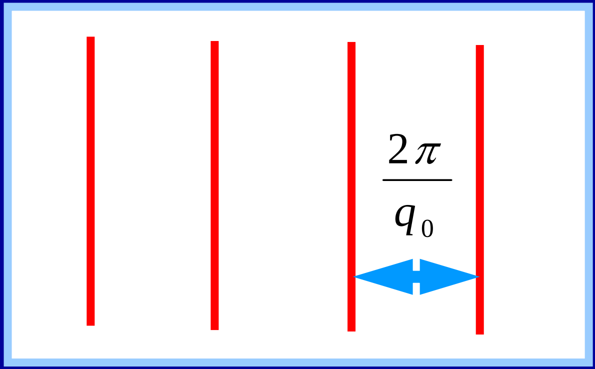

**$2^{nd}$ order transition impossible (large N) ;**
**real transition may occur at $0 < T_c << T_{c1}$)**

- **Fluctuation driven $1^{st}$ order transition**

$$r^{3/2} \approx ubTq_0^2$$

- **Mean field: lamellar phase**

$\Phi_1 = A_1 \cos(q_0 r)$

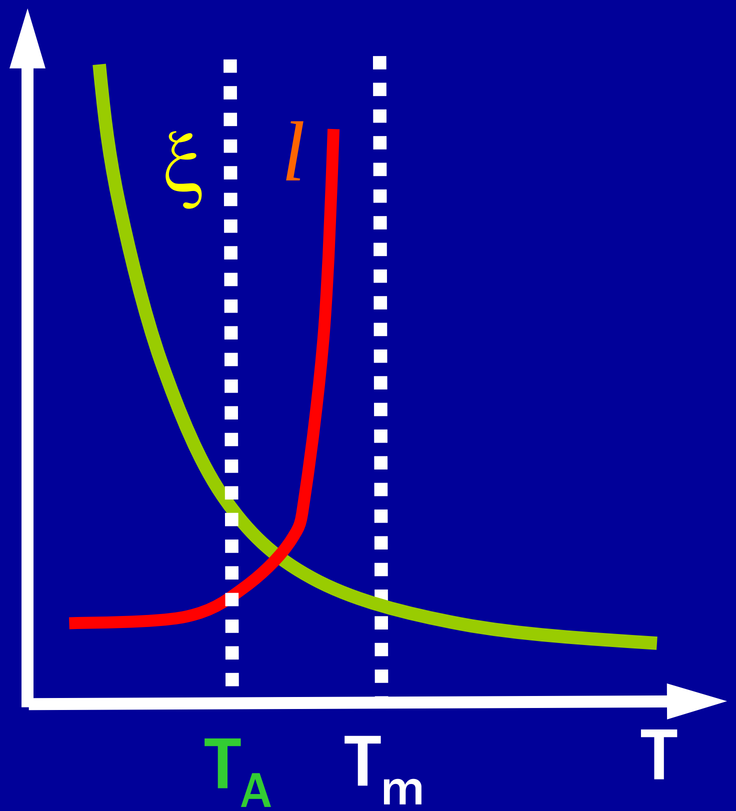


# Two length scales and glassiness

- Competition between

  *Z. Nussinov et al., 1999*

  - correlation length, $\xi^{-2}=r(T)+q_0^2$
  - modulation length $l^{-2}=q_0^2-r(T)$
  - at $q_0^2=r$ ***short range correlations***
  - at $r+q_0^2=0$ **long range order**

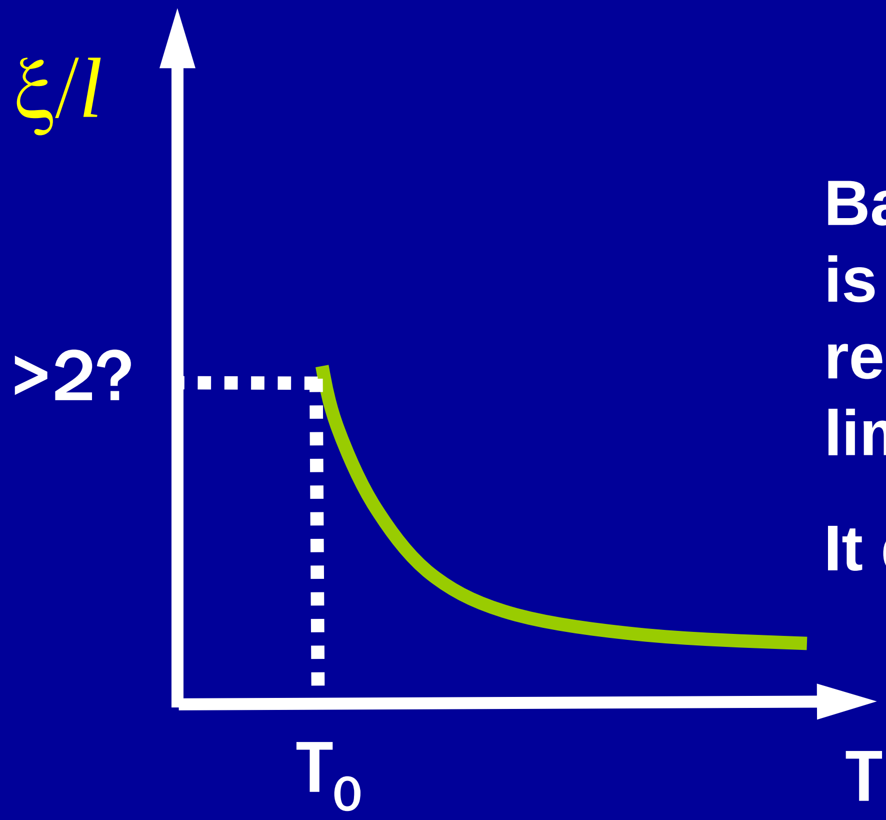



- ***Glass emerges when*** $\xi/l>2$

  *J. Schmalian and P. Wolynes,2000*

  - ***$N\propto\exp(q_0^3V)$ metastable states below $T_A(q_0)$***
  - ***Low cost of creating regions of order parameter ($\xi^{-2}$) correlated over short distance of order $l$***

- If $T_{c2} >> T_{c1}$ : fluctuation-induced first order transition into inhomogeneous or glassy phase at $T_{c2} > T_1 > T_{c1}$ possibly followed by a strongly first order transition to a uniform phase at $T^* << T_{c1}$.
- For mean field $T_{c2} \approx T_{c1}$: 1$^{st}$ order into modulated phase
- Reason: $q_0$ depends on T
- Under investigation




Basic question: is it enough to reach glassy limit?

It depends…

# Summary

- You can get modulated and inhomogeneous states of different orders in nominally uniform systems. *Disorder need not be there.*
- These states may exhibit glassy dynamics.
- For glassiness details matter (unfortunately?)

Open question:

- Hamiltonian from which such GL follows?
- Plan: use mean field miscorscopic theories (see talks by G. Alvarez and W. Atkinson), and check the relevant Ginzburg-Landau theories.