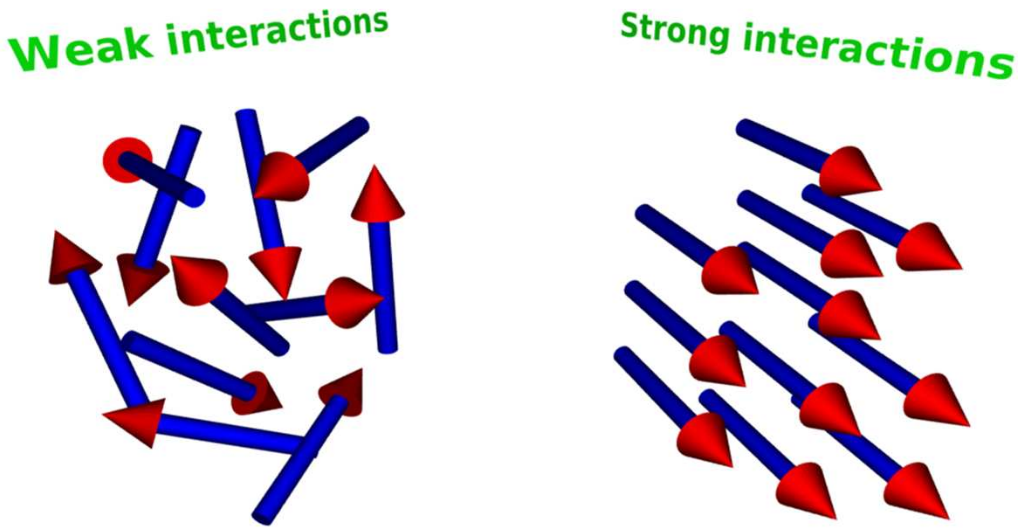
Non equilibrium Ferromagnetism and Stoner transition in an ultracold Fermi gas



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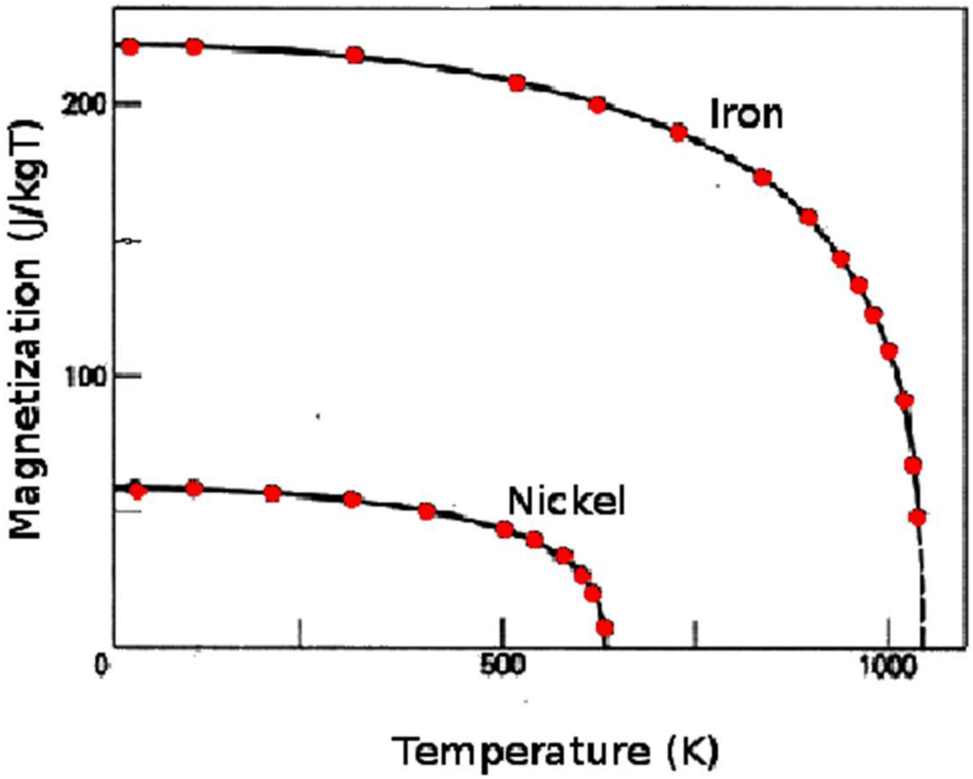
See: Phys. Rev. A 82, 043603 (2010) and arXiv: 0911.2839

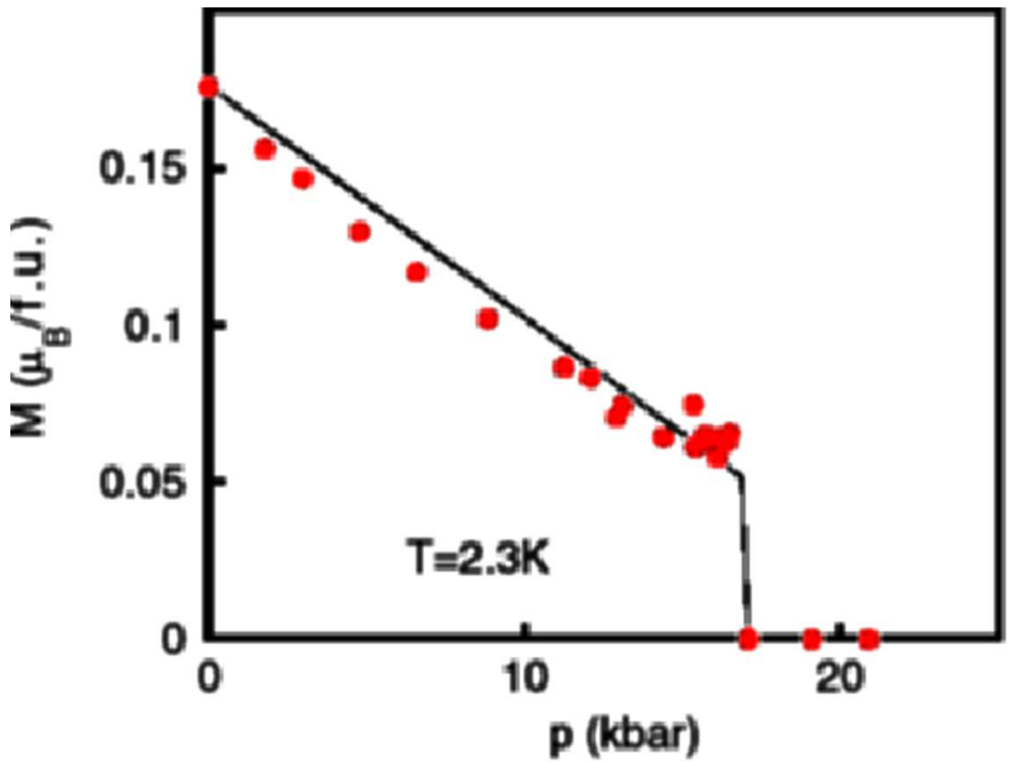
Disentangling Quantum Many-Body Systems, KITP, Nov 1, 2010

Ferromagnetism in solid state

Second order in iron & nickel

First order in $ZrZn_2$





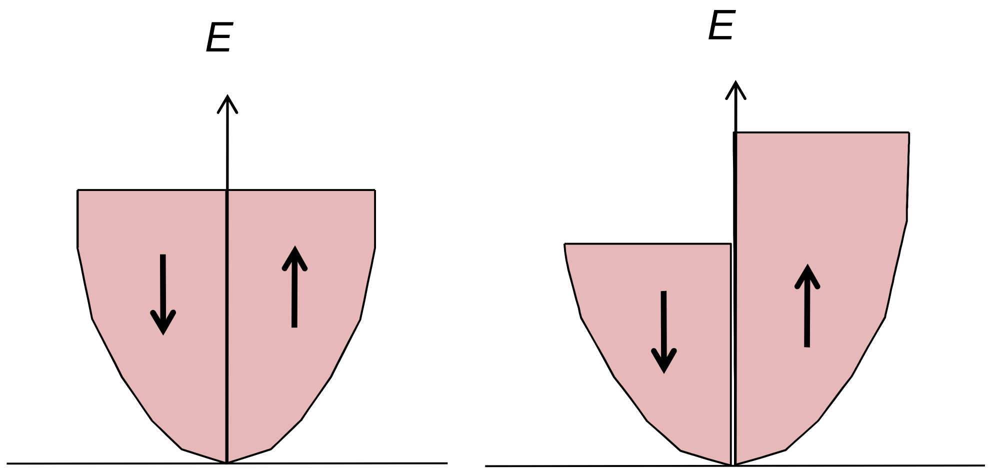
Uhlarz *et al.*, PRL 2004

Reminder: what is the stoner transition

E. Stoner, Phil. Mag. 15:1018 (1933)

$$H = \int d^d x \sum_\sigma \psi_\sigma^\dagger \left(\frac{\hat{\mathrm{p}}^2}{2m} - \mu \right) \psi_\sigma + g \psi_\uparrow^\dagger \psi_\downarrow^\dagger \psi_\downarrow \psi_\downarrow$$

Spontaneous spin polarization reduces interaction energy but costs kinetic energy.



Expect a transition to an antiferromagnet when gν ~ 1:

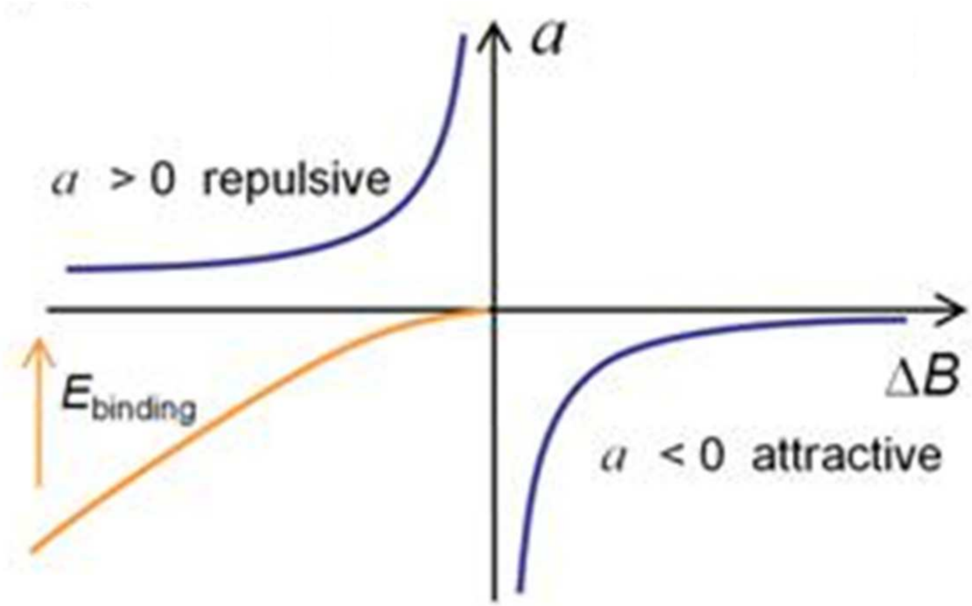
How to study ferromagnetism with ultracold atoms

Salasnich et. al. (2000); Sogo, Yabu (2002); Duine, MacDonald (2005); Conduit,Simons (2009); LeBlanck et al. (2009);

<u>Basic idea:</u>

Tune effective contact interaction with a Feshbach resonance.

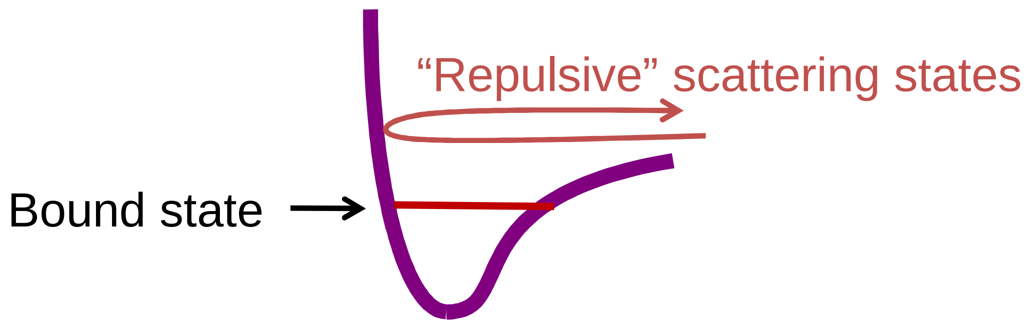
$$U_{eff} = \frac{ma}{4\pi}$$

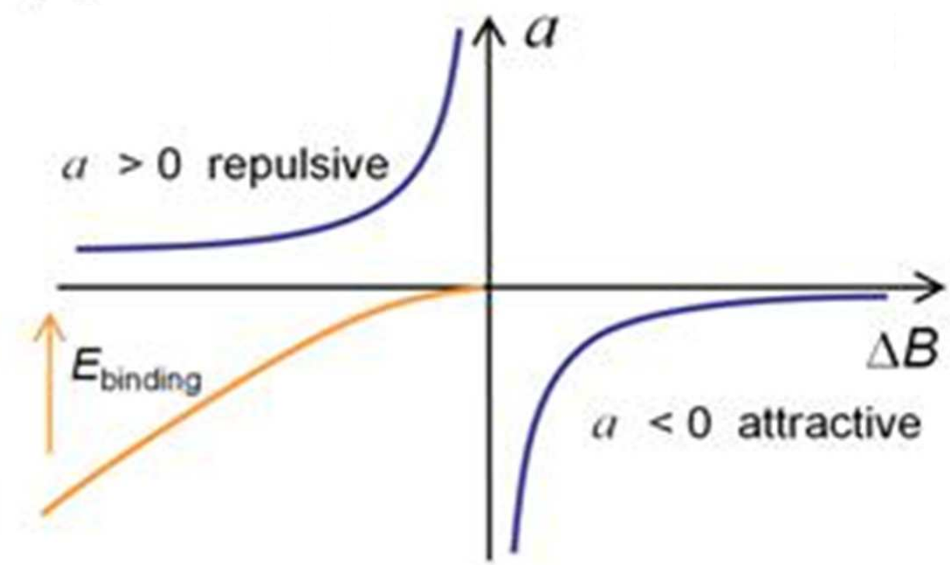


Expect a transition around $k_F a \sim 1$

Complication: repulsive interactions are not the whole story!

The underlying 2-body potential is **attractive**.





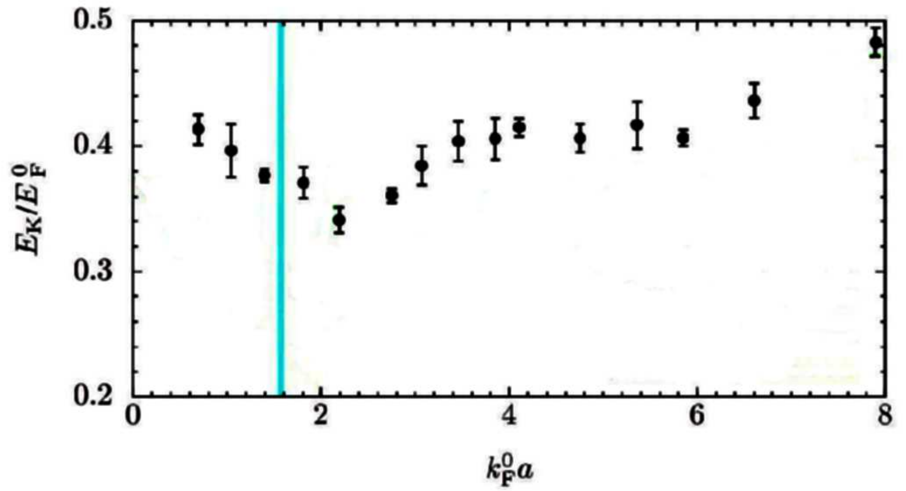
The purely repulsive model does not account for the bound state

Particles can decay coherently or incoherently into bound molecules

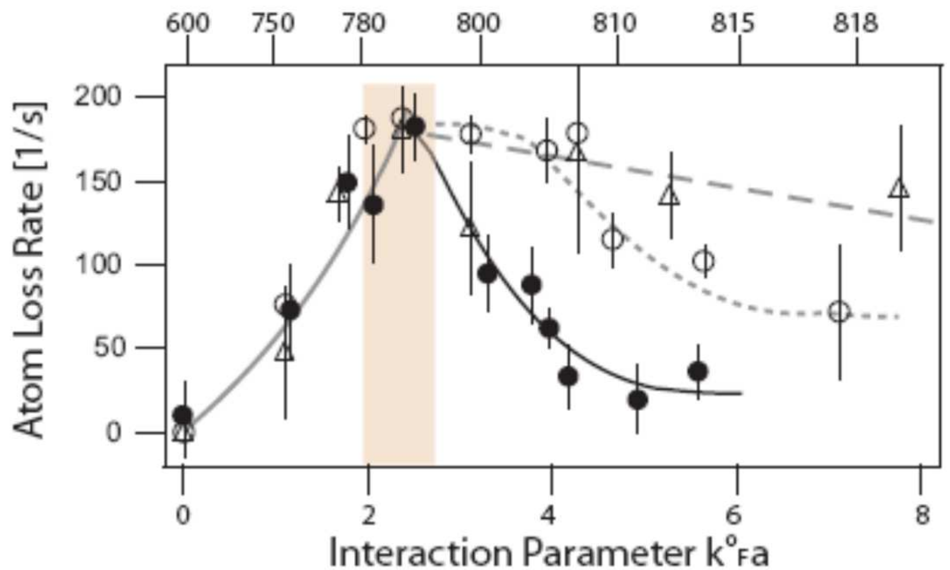
Experimental evidence for ferromagnetism

Jo et. al. (Ketterle's group), Science (2009)

Kinetic energy:



Loss:

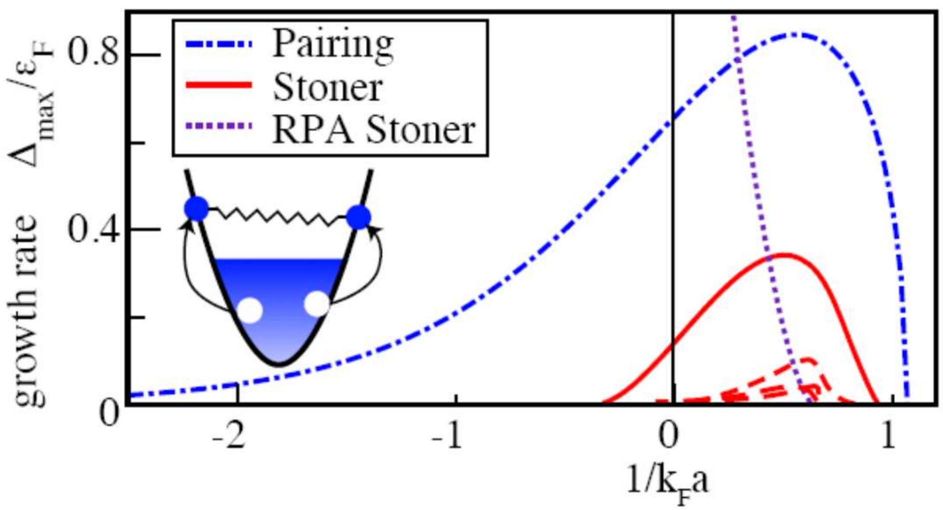


Observed transition point $k_F a_c$ ~ 2.2 disagrees with QMC and RPA results ($k_F a_c$ ~ 0.85 – 1)

Questions

1. Effect of the non-equilibrium conditions (Loss) on the transition?
   - Effect on the critical interaction strength?
   - Order of transition?
   - Collective modes?
   - Does the transition survive at all ?

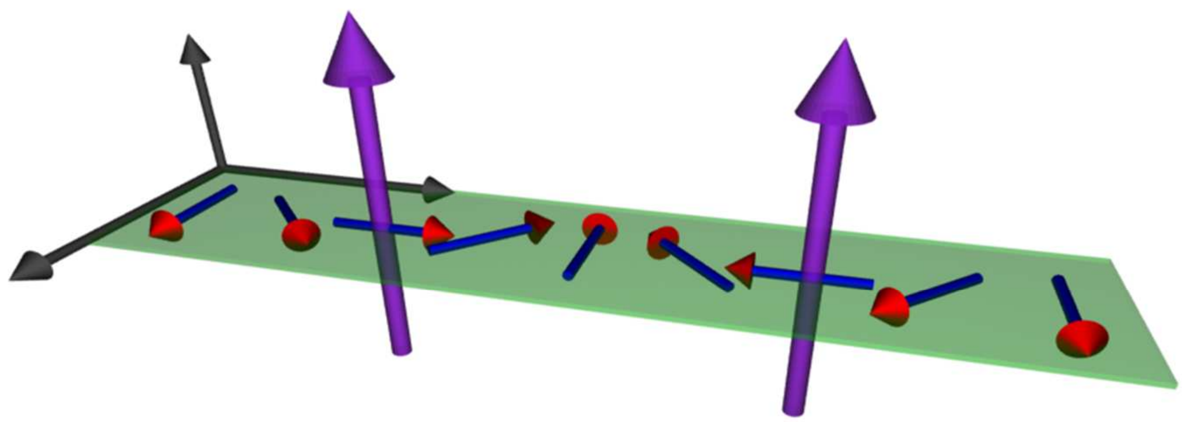
Pekker, Demler et. al. arXiv:1005.2366: Some of the experimental results can be explained by a competing pairing instability



2. How to circumvent the problem of loss?

Outline

- Circumventing the loss problem: instabilities of a spin spiral



- Stoner transition in the presence of atom loss
  - A novel quantum effect of loss: generates a dissipative interaction
  - Shifts transition to $k_F a_c \sim 2$ and makes it 2^{nd} order

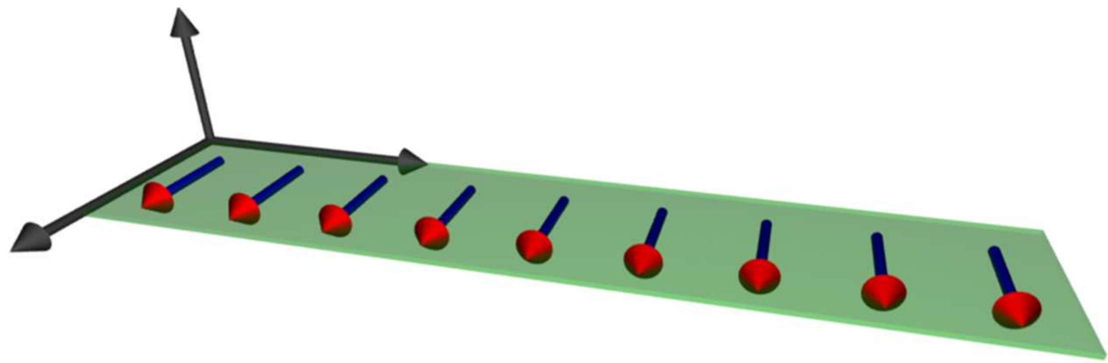
Circumventing loss: dynamics of the spin spiral

Proposed experimental scheme:

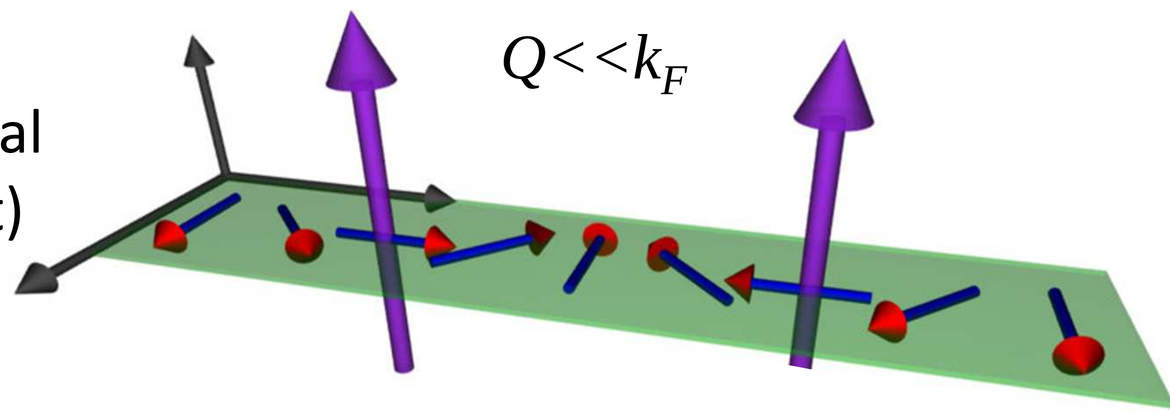
1. Prepare a polarized Fermi gas

   exact eigenstate of the hamiltonian

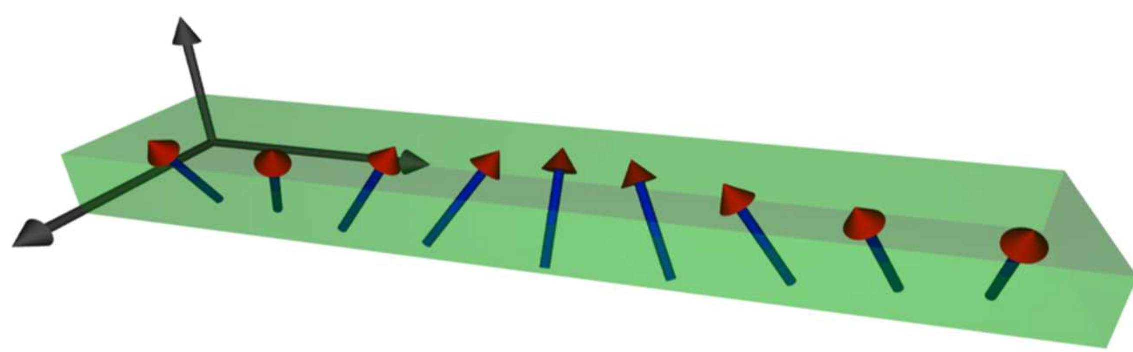
   ➡ No dynamical instability or loss



2. Induce a small pitched spin spiral
   (Using a magnetic field gradient)



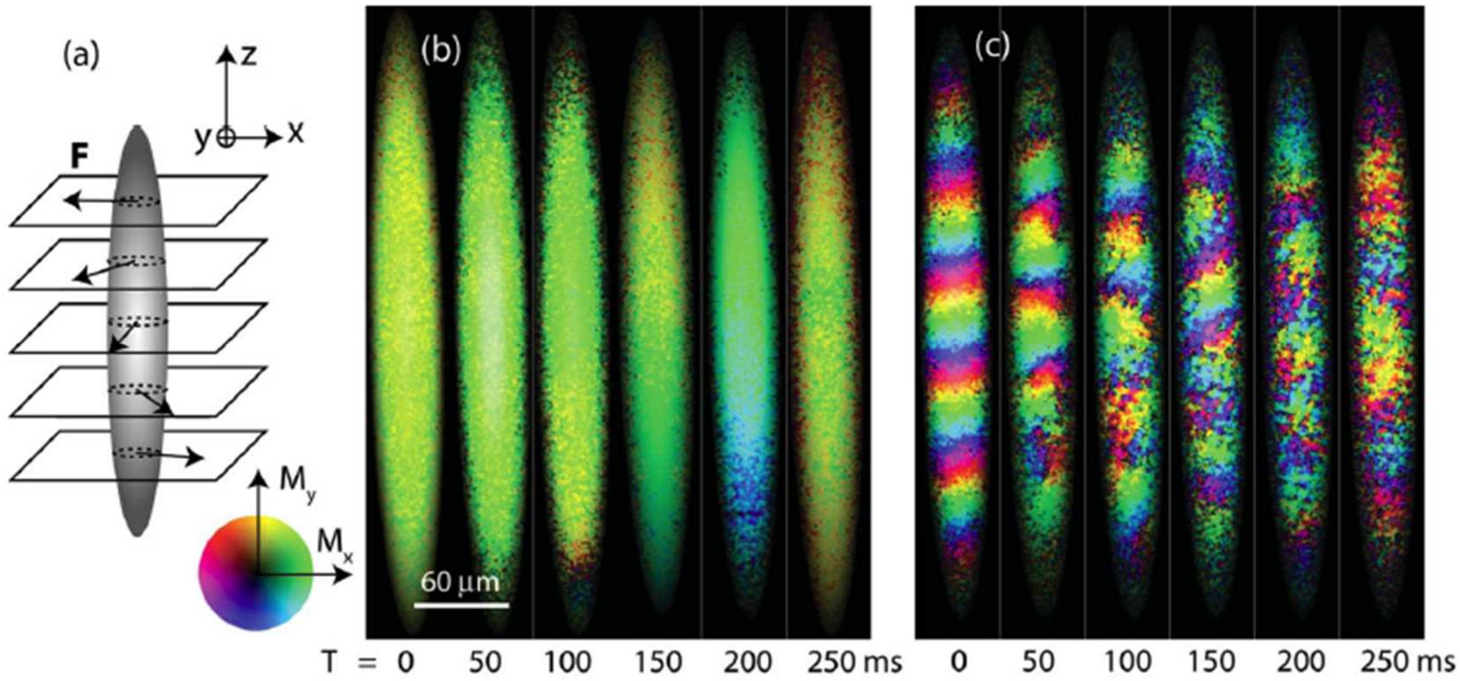
3. Observe the time evolution



How do the dynamical instabilities
depend on the interaction parameter?
paramagnetic vs. ferromagnetic regime?

A similar scheme was used to study Ferromagnetic spinor condensates

M. Vengalattore,[1] S. R. Leslie,[1] J. Guzman,[1] and D. M. Stamper-Kurn[1,2] PRL **100,** 170403 (2008)



Theory: spinor GP equation

Cherng et. al. PRL (2008), Lamacraft PRA (2008)

Only in the fermion case we can ask: how do the instabilities change as one tunes across the paramagnet to ferromagnet transition?

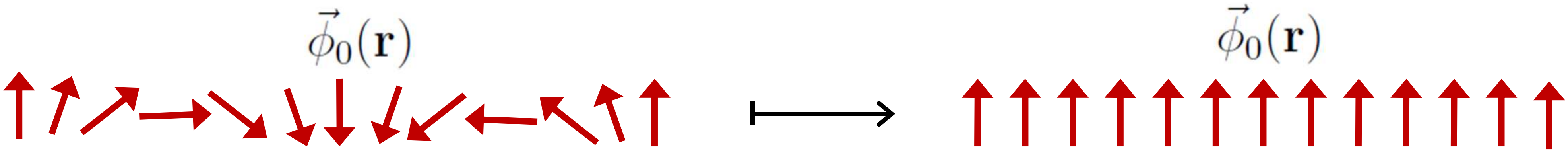
Theoretical approach

Action of a Fermi gas with contact interactions:

$$S = \sum_{k,\sigma} \bar{\psi}_{\mathbf{k},\sigma} \left(\partial_\tau + \epsilon_{\mathbf{k}} - \mu\right) \psi_{\mathbf{k},\sigma} + g \int d^3x \; \bar{\psi}_\uparrow \bar{\psi}_\downarrow \psi_\downarrow \psi_\uparrow$$

<u>Strategy:</u> expand in small fluctuations about the ordered spin spiral configuration

Trick: map spiral to uniform magnetization using gauge transformation $\psi(\mathbf{r}) \mapsto e^{\frac{i}{2}\sigma_x \mathbf{Q}\cdot\mathbf{r}} \psi(\mathbf{r})$

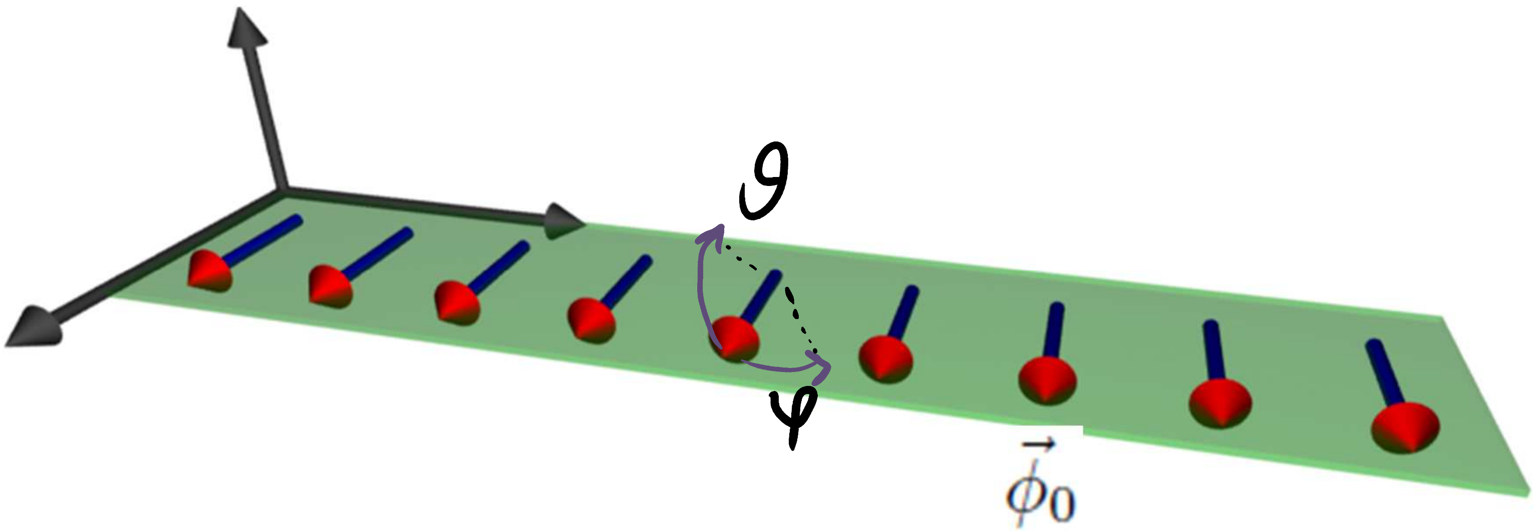


Hubbard -Stratonovich decoupling:

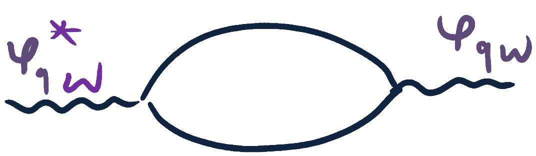
$$S[\vec{\phi}, \bar{\psi}, \psi] = \int d^3x \left[g\phi^2 + \bar{\psi} \left(\frac{\hat{\mathrm{p}}^2}{2m} \nabla^2 + \frac{1}{2}\sigma_x \mathbf{Q} \cdot \hat{\mathrm{p}} - g\vec{\sigma} \cdot \vec{\phi} \right) \psi \right]$$

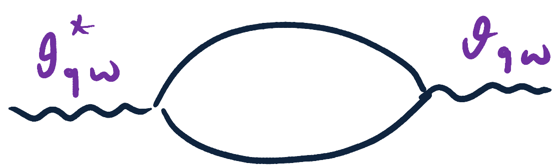
Price paid for “unwinding” the spin is a spin dependent vector potential

Linearized collective modes (spinwaves)



Integrate out the Fermions and expand action
to quadratic order in the spin fluctuations:





$$S = \sum_{\mathbf{k},\omega} \begin{pmatrix} \theta^\star_{\vec{q},\omega} & \varphi^\star_{\vec{q},\omega} \end{pmatrix} \begin{pmatrix} \chi[q^2 - Q^2] & \mathrm{i}\omega \\ -\mathrm{i}\omega & \chi q^2 \end{pmatrix} \begin{pmatrix} \theta_{\vec{q},\omega} \\ \varphi_{\vec{q},\omega} \end{pmatrix}$$

$$\chi = \frac{1}{2}\left(1 - \frac{a_0}{a}\right)$$

Linear spinwave instability

$$S=\sum_{\mathbf{k},\omega}\begin{pmatrix}\theta^\star_{\vec{q},\omega} & \varphi^\star_{\vec{q},\omega}\end{pmatrix}\begin{pmatrix}\chi[q^2-Q^2] & \mathrm{i}\omega \\ -\mathrm{i}\omega & \chi q^2\end{pmatrix}\begin{pmatrix}\theta_{\vec{q},\omega} \\ \varphi_{\vec{q},\omega}\end{pmatrix}$$

$$\chi=\frac{1}{2}\left(1-\frac{a_0}{a}\right)$$

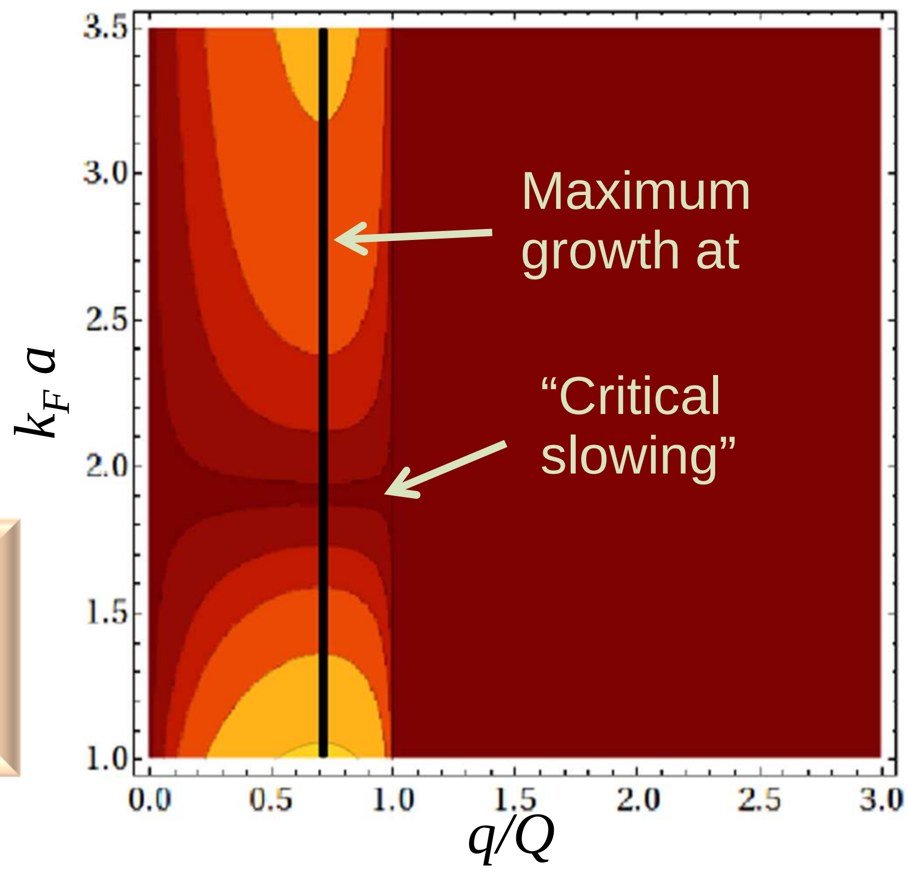
Collective modes:

$$\omega=\chi q\sqrt{q^2-Q^2}$$

$$\theta_q(t)=\theta_q(0)e^{-i\omega t}=\theta_q(0)e^{\Gamma t}$$

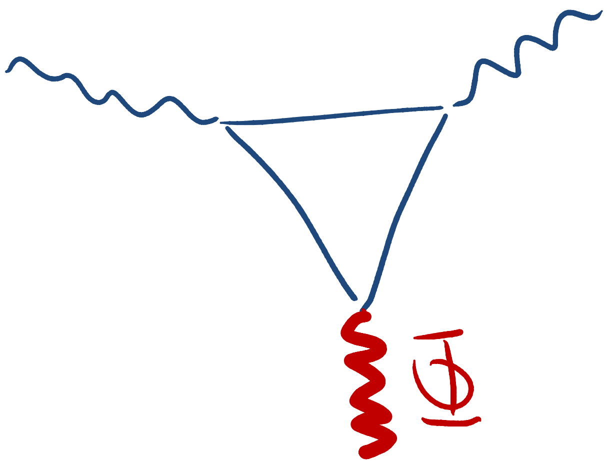
Exponential growth of modes at $q<Q$

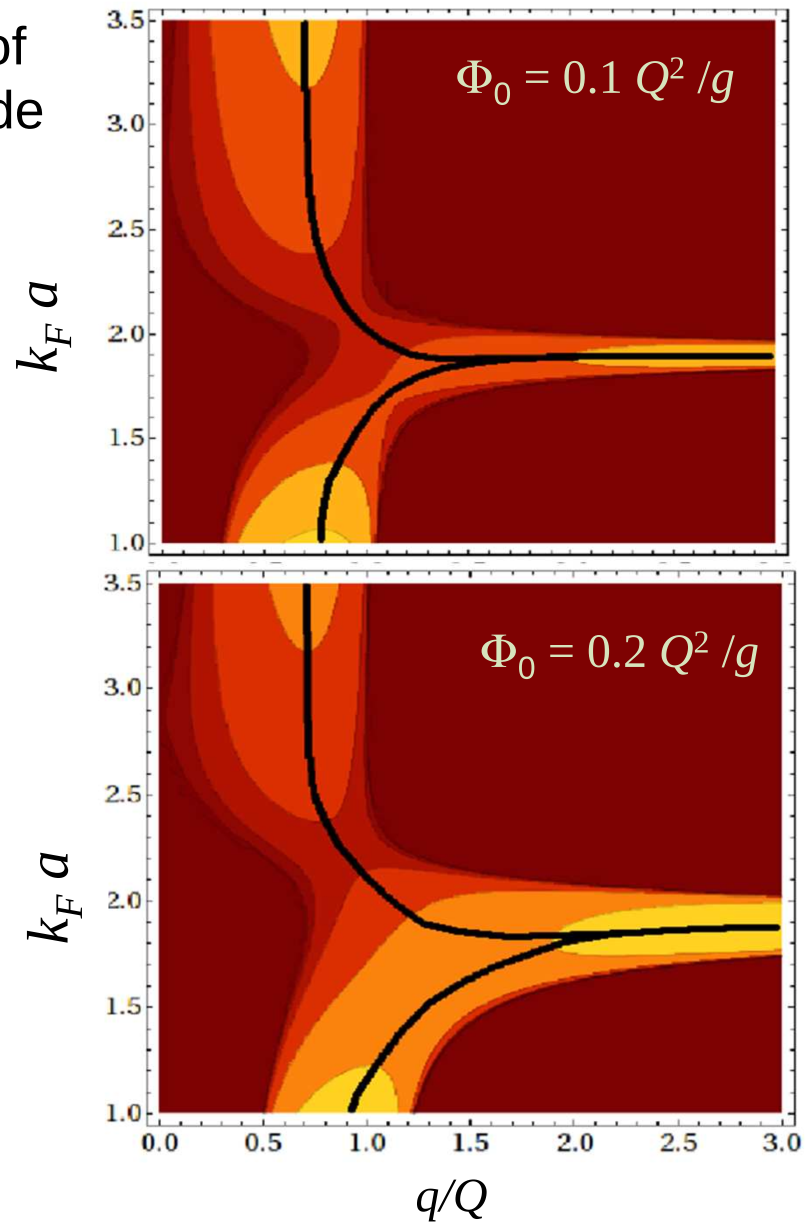
- Spectrum of instabilities the same on both sides of the transition
- Critical slowing down close to the transition



Instabilities beyond the linear analysis

Spectrum of instabilities renormalized by scattering of fluctuations on the fastest exponentially growing mode (Self consistent calculation)

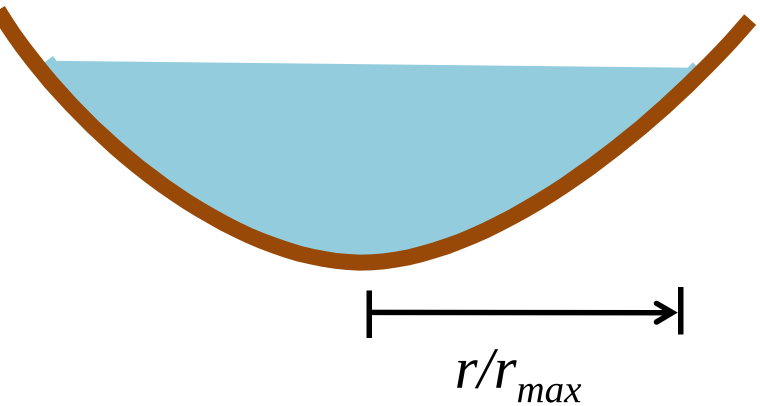


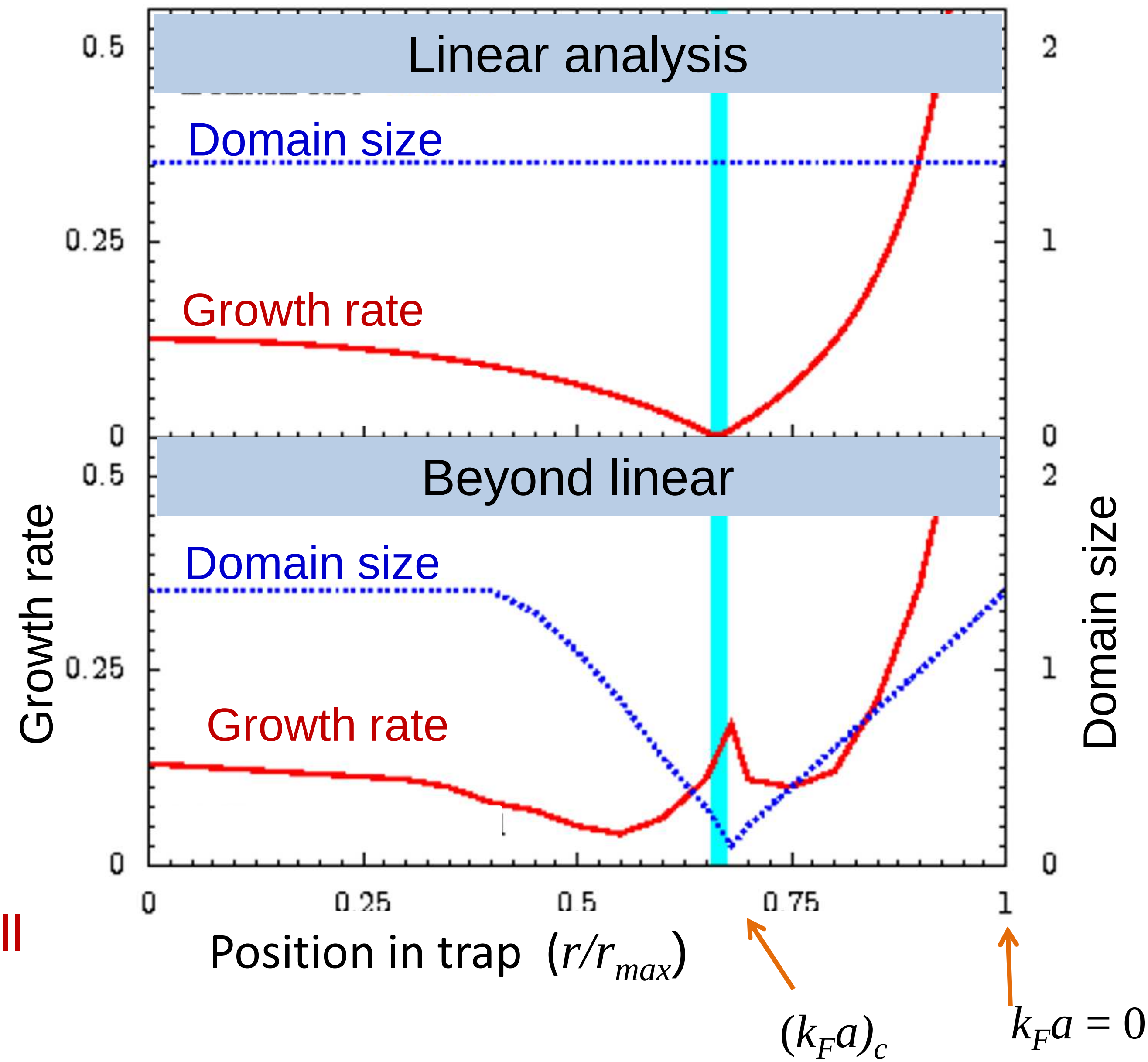


<u>Main features:</u>

- Result depends on seed fluctuation.
- Critical slowing down is eliminated
- New branch of short wavelength instabilities

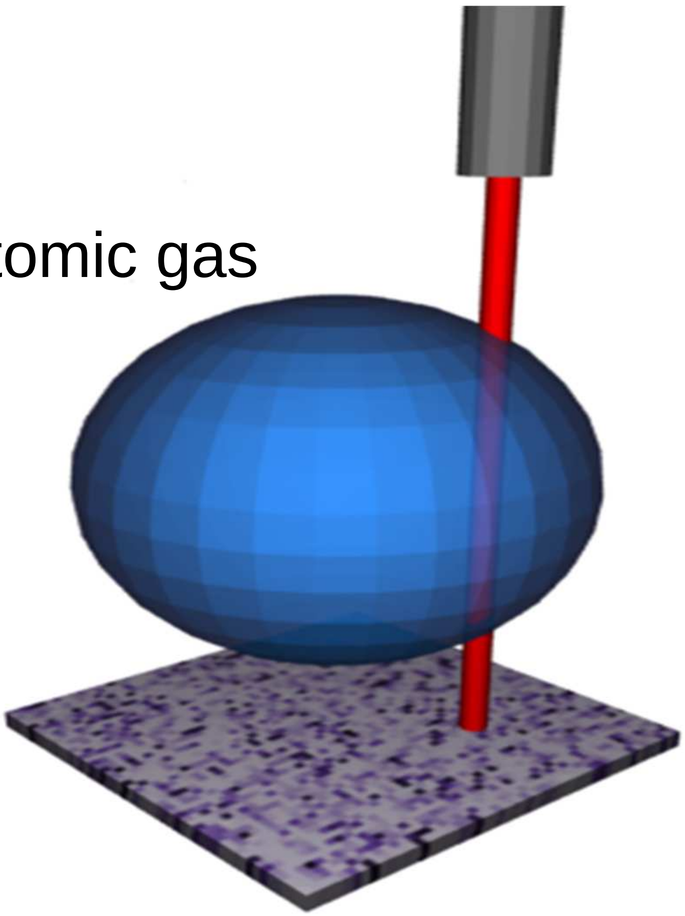
Effect of the harmonic trap





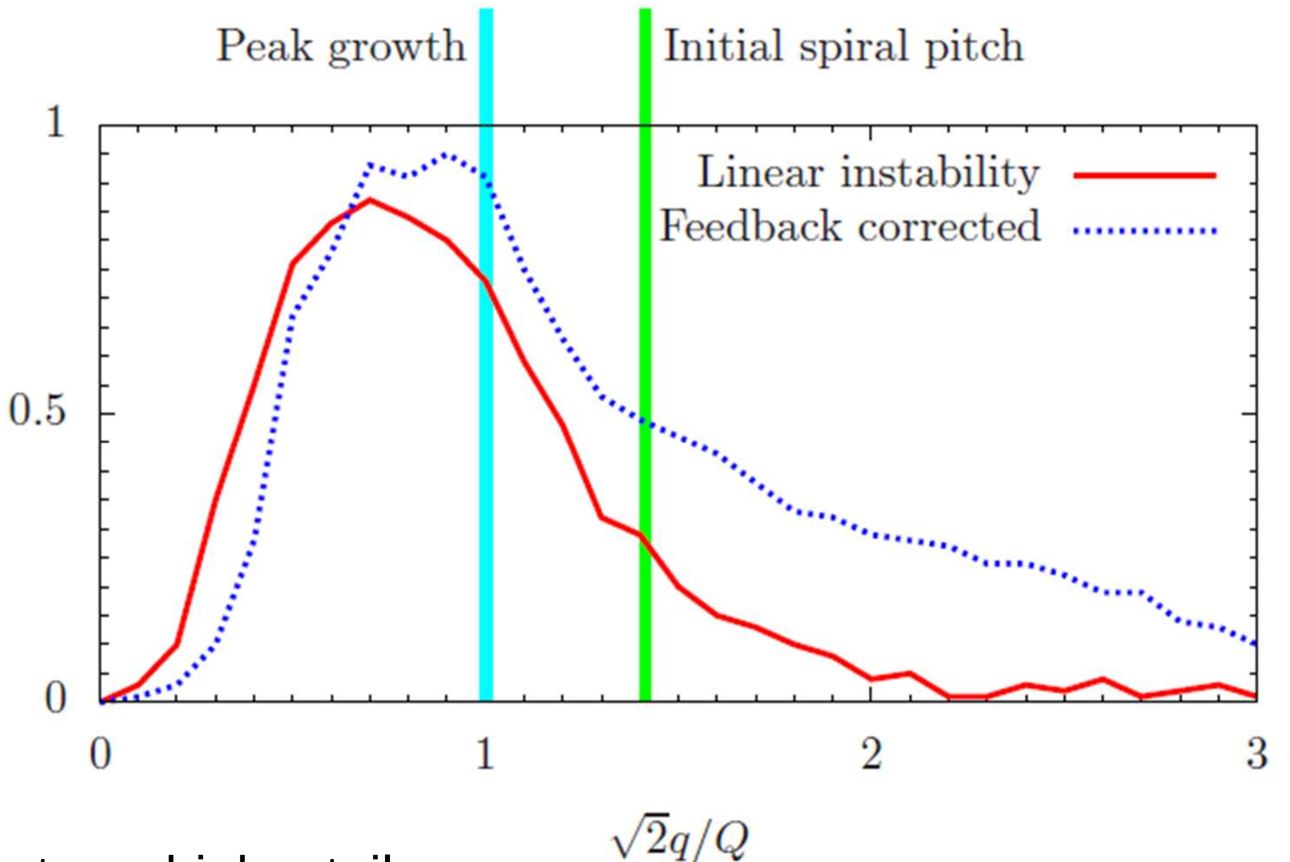
Domain sizes are very small around critical radius

Global probe: Phase-contrast imaging



Column integrated map of z-magnetization

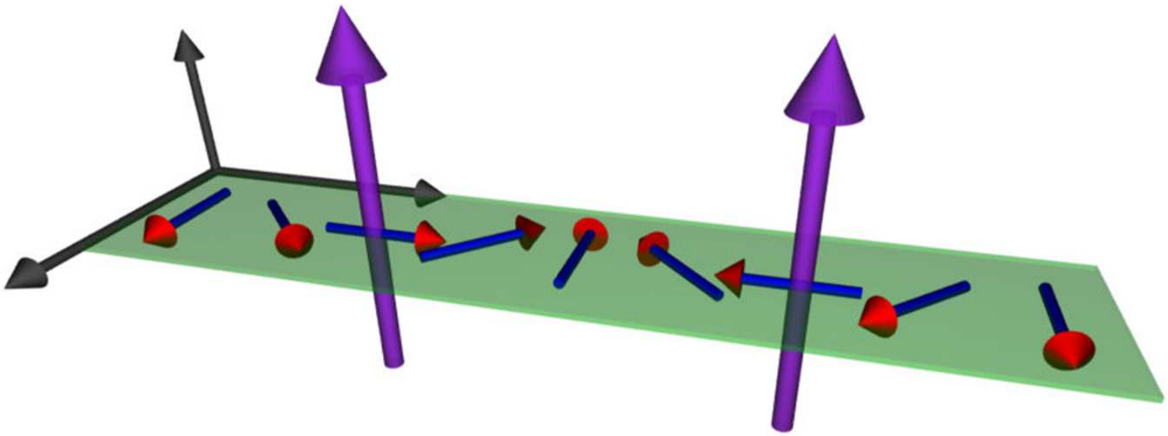
Spatial fluctuation Fluctuation spectrum:



The nonlinear feedback generates a high q tail
Can tune the strength of this tail using seed size (e.g. preparation field fluctuation)

Outline

- Circumventing the loss problem: instabilities of a spin spiral



- Stoner transition in the presence of atom loss

  - A novel quantum effect of loss: generates a dissipative interaction
  - Shifts transition to $k_F a_c \sim 2$ and makes it 2^{nd} order

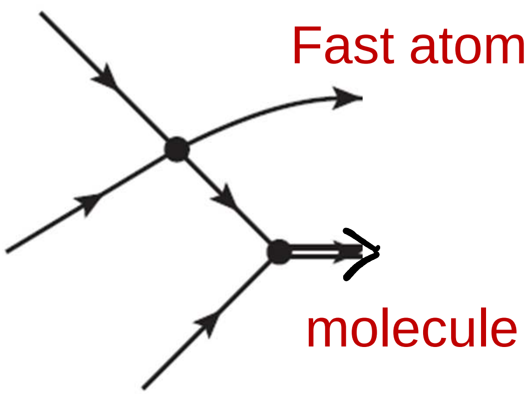
Mechanism of atom loss in the dilute regime

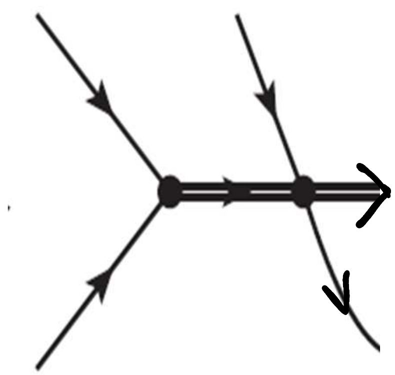
We approach the problem from weak coupling (=dilute limit) and hope that the results hold beyond their strict regime of validity to $k_F a \sim 1$

(In the spirit of usual Hartree-Fock or RPA treatment of the Stoner model)

In this limit Loss occurs in a 3-body collision (see Petrov 2003):

3 low energy atoms $\longrightarrow$ Molecule + fast atom



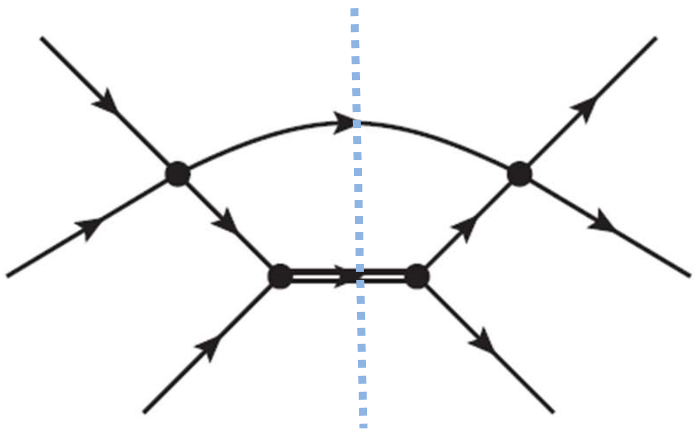


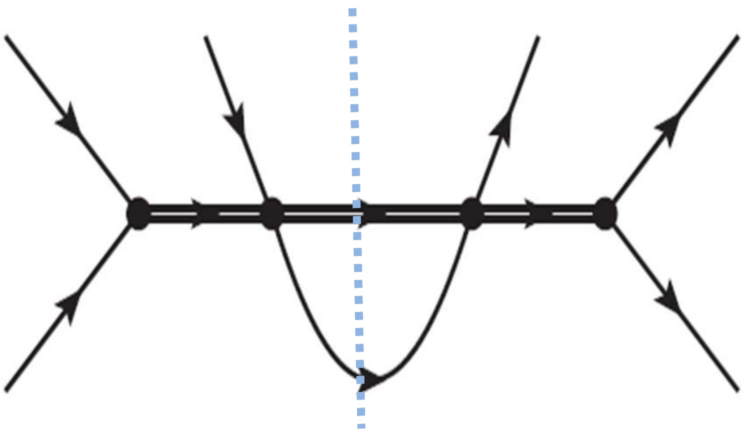
Classical rate equation:

$$\frac{dn}{dt} = -\tilde{\lambda} n^3$$

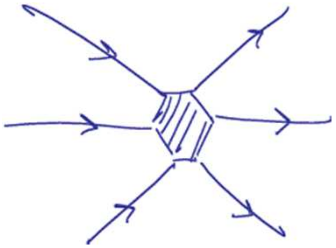
Quantum effect of loss

Integrate out the bound Feshbach molecules and the out-going fast atoms to obtain an effective interaction (T-matrix) for the 3-body collision.



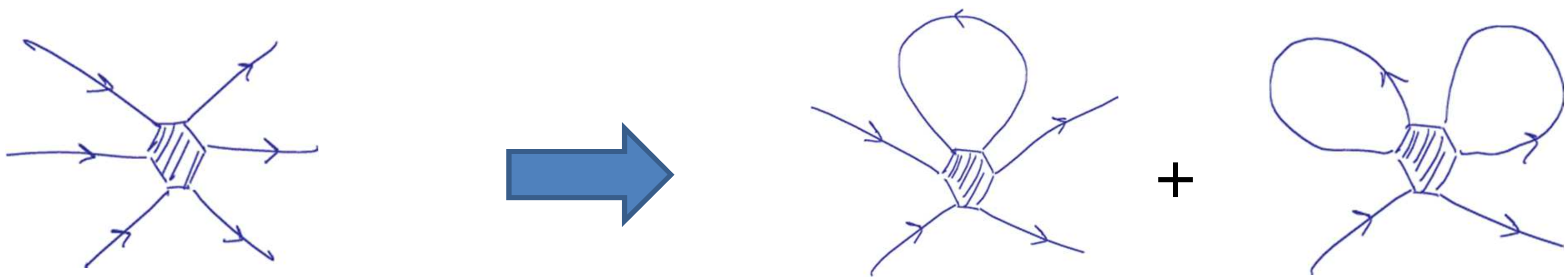


Imaginary 3-body interaction:



Effective action

Further renormalization toward the Fermi surface
Generates imaginary 2-body and q-body terms:



$$S = \int_0^\beta \mathrm{d}\tau \mathrm{d}\mathrm{r} \left[\sum_\sigma \bar{\psi}_\sigma \left(\partial_\tau + \epsilon_{\mathbf{k}} + \mathrm{i}\lambda \bar{\rho}_\uparrow \bar{\rho}_\downarrow k^2 - \mu \right) \psi_\sigma + \left[g - \mathrm{i}\lambda (\bar{\rho}_\uparrow \mu_\uparrow + \bar{\rho}_\downarrow \mu_\downarrow) \right] \bar{\psi}_\uparrow \bar{\psi}_\downarrow \psi_\downarrow \psi_\uparrow \right]$$

Treat the imaginary 2-body interaction on the same footing as the real interaction

Generalized RPA analysis

Apply usual Hubbard-Stratonovic decoupling (in density and spin channels) to derive a Ginsburg-Landau Free energy

<u>No loss (λ=0):</u>

$$F = F_0 + \frac{1-g\nu}{2\nu} m^2 + u m^4 + v m^6 + g^2 \left(r\, m^2 + w\, m^4 \ln|m| \right)$$

Effect of quantum fluctuations:
Shift the transition to lower interaction strength and turn it first order

Belitz *et al.* Z. Phys. B 1997, Abrikosov 1958

<u>With loss:</u>

$$F = F_0 + \frac{1-g\nu}{2\nu} m^2 + u m^4 + v m^6 + \left(g^2 - \lambda^2 \bar{N}^2 \right) \left(r\, m^2 + w\, m^4 \ln|m| \right)$$

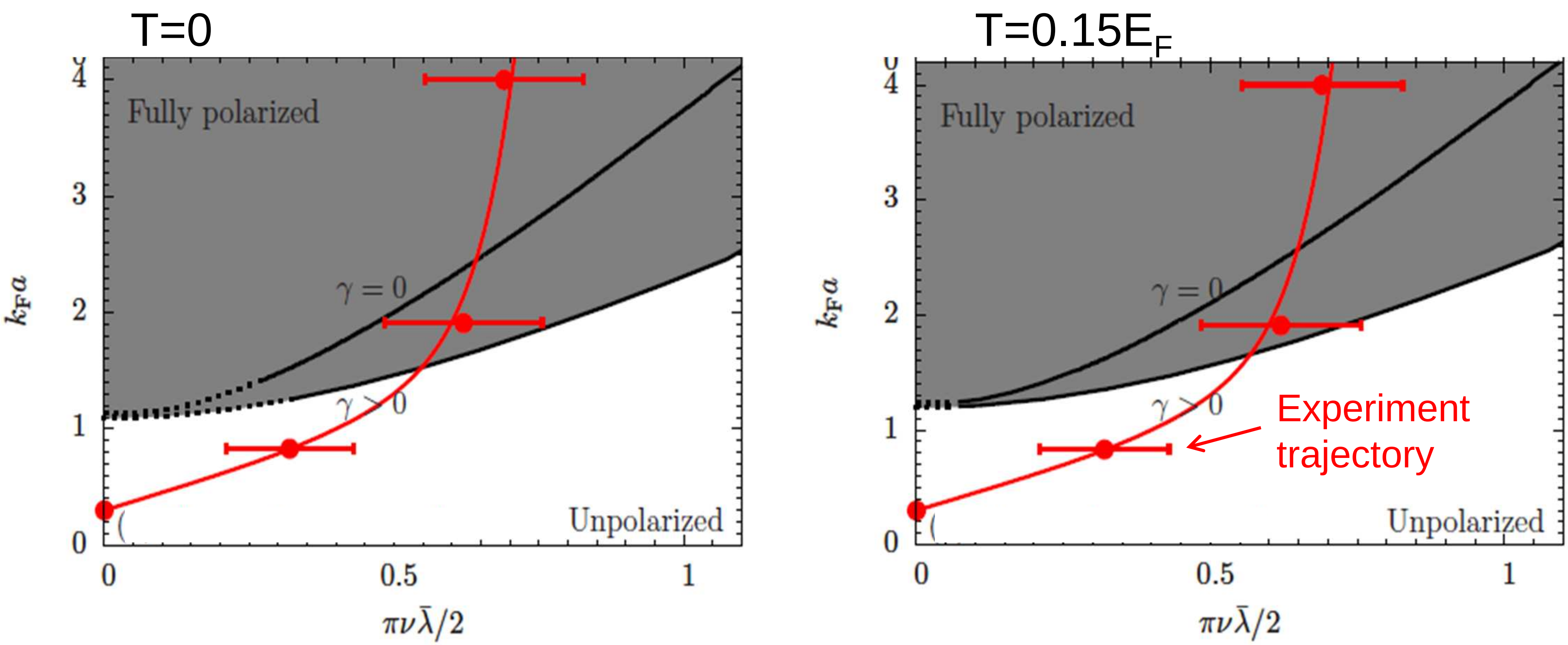
(+ neglected imaginary part , which encapsulates actual loss)

Loss acts to damp quantum fluctuations:

1. Shifts transition to higher interactions
2. Recover the second order transition

Phase boundary with atom loss

Conduit & Altman, arXiv: 0911.2839



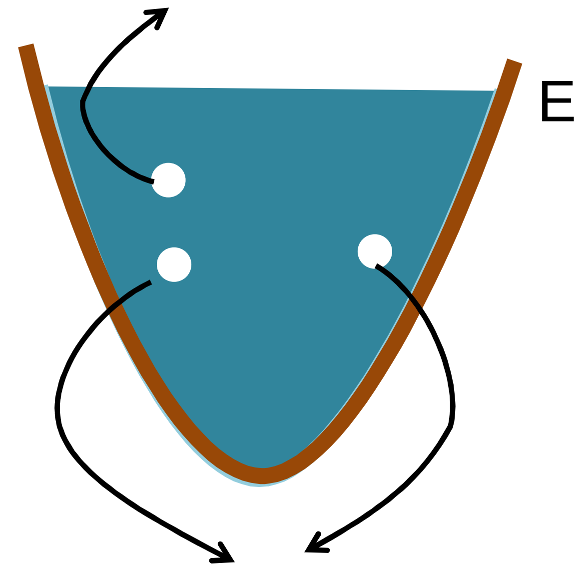
- Atom loss raises the interaction strength required for ferromagnetism

Note: our analysis is strictly valid only in the dilute limit ($k_F a \ll 1$).
So predictions for the transition at $k_F a \sim 2$ should be treated with care.

The fact that loss induces an effective dissipative interaction is probably robust

Why are we justified using the equilibrium theory in presence of loss?

Loss creates holes in the Fermi sea
(Even very deep)



But the deeper the holes the faster they relax

Kinetic equation:

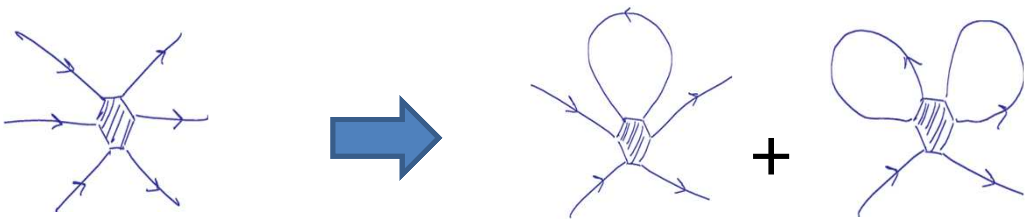
$$\frac{\mathrm{d}n_{\mathbf{k},\sigma}}{\mathrm{d}t} = \frac{4(\epsilon-\epsilon_{\mathrm{F}})^2(k_{\mathrm{F}}a)^2}{\hbar\epsilon_{\mathrm{F}}}\left(N^{\mathrm{eq}}_{\mathbf{k},\sigma}-n_{\mathbf{k},\sigma}\right)-2\Gamma_0(k_{\mathrm{F}}a)^6\bar{n}^2 n_{\mathbf{k},\sigma}$$

In the dilute limit the relaxation term is much larger than the loss!

Solution: quasi equilibrium distribution with a slowly time dependent temperature and average density.

$$T=\sqrt{T_0^2-\frac{2\mu_0^2}{15\pi^2 k_{\mathrm{B}}^2}\left[\left(1+4\Gamma_0(k_{\mathrm{F}}a)^6\bar{n}^2 t\right)^{-2/3}-1\right]}$$

More general outlook:
What is the nature of a lossy Fermi-liquid?



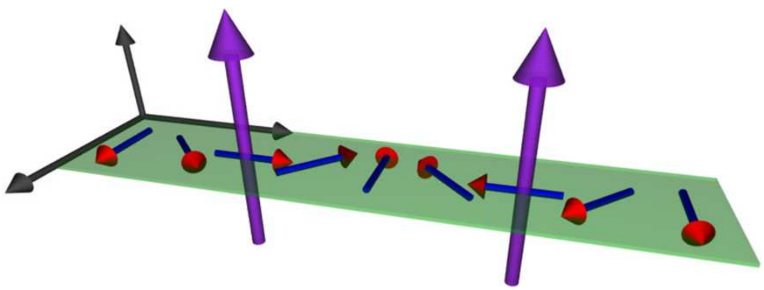
$$S = \int_0^{\beta} \mathrm{d}\tau \mathrm{d}\mathrm{r} \left[\sum_{\sigma} \bar{\psi}_{\sigma} \left(\partial_{\tau} + \epsilon_{\mathbf{k}} + \mathrm{i}\lambda \bar{\rho}_{\uparrow} \bar{\rho}_{\downarrow} k^2 - \mu \right) \psi_{\sigma} + \left[g - \mathrm{i}\lambda (\bar{\rho}_{\uparrow} \mu_{\uparrow} + \bar{\rho}_{\downarrow} \mu_{\downarrow}) \right] \bar{\psi}_{\uparrow} \bar{\psi}_{\downarrow} \psi_{\downarrow} \psi_{\uparrow} \right]$$

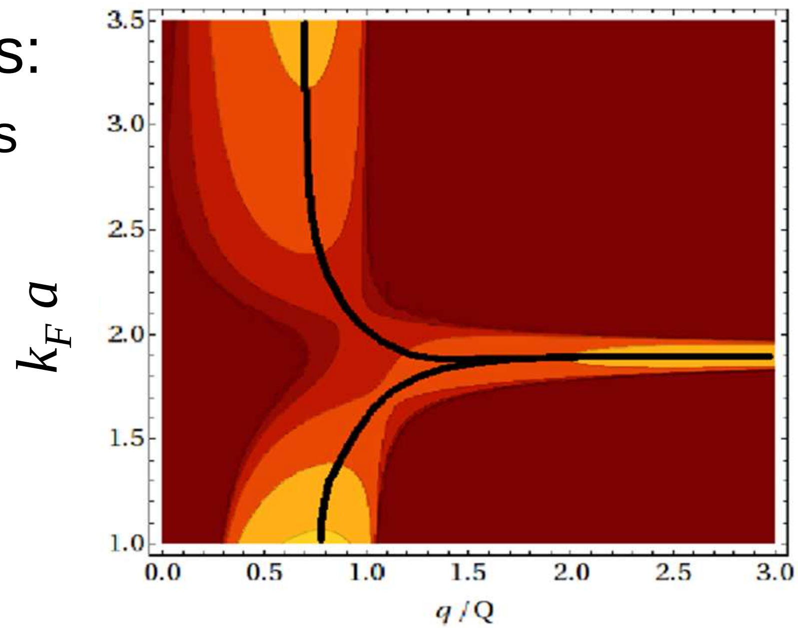
Continue to renormalize toward lower energies (cut off by the loss rate).

What is the effect of loss on FL instabilities?

Summary

- Instabilities of a spin spiral in a Fermi gas:
  - Probe of the Stoner transition insensitive to loss
  - Novel dynamical magnetic phenomena





- How the Stoner transition is modified by loss: