Exotic gapless Bose metals and insulators on multi-leg ladders

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Outline

- Introduction to the "*d*-wave Bose liquid"

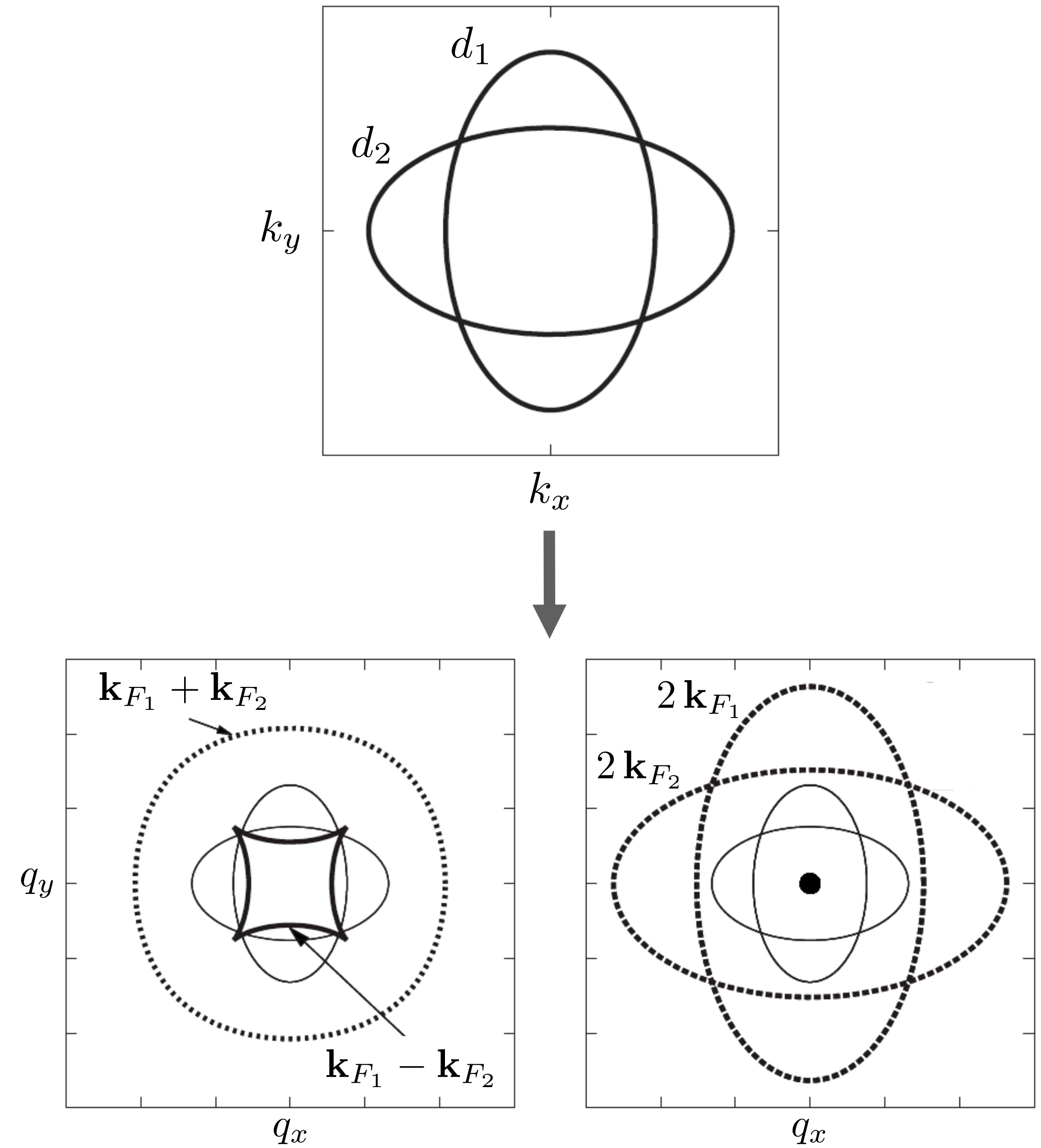
- Where, how, and why?

- Gapless Mott *insulator* on the 3-leg ladder
  - arXiv:1008.4105

- Gapless Bose *metals* on 3- and 4-leg ladders
  - Still in progress...

Overview: *d*-wave Bose liquid (DBL)

- System in mind
  - Itinerant hard-core bosons
  - 2D square lattice
- Important properties
  - *d*-wave correlations (nontrivial signs)
  - No broken symmetries
  - Gapless excitations on "*Bose surfaces*" in momentum space
- Construction
  - Gutzwiller projected product of filled Fermi seas (FFSs):



$$\Psi_b(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_N) = \underbrace{\Psi_{d_1}(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_N)\Psi_{d_2}(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_N)}$$

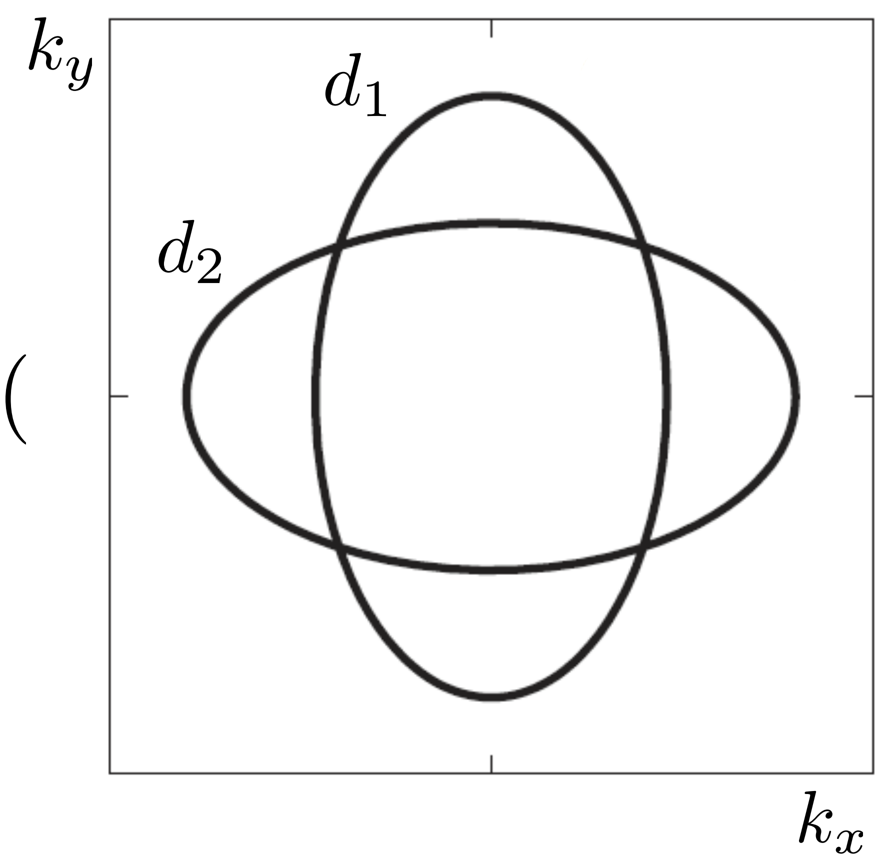
Slater determinants with FFSs compressed in *x* and *y* directions

Motrunich and MPAF, PRB **75**, 235116 (2007)

Variational wave functions and gauge theory

- Projected Fermi sea wave functions

$$\Psi_b = \mathcal{P}_G\Big(\quad \Big)$$



Gutzwiller projection:

$$\mathcal{P}_G : n_{d_1}(\mathbf{r}) = n_{d_2}(\mathbf{r}) \ \forall \ \mathbf{r}$$
$$n_{d_\alpha} \in \{0, 1\}$$

- Gauge theory description: $\hat{b}^\dagger = \hat{d}_1^\dagger \hat{d}_2^\dagger$
  - d_1 and d_2 hopping on square lattice coupled to U(1) gauge field
  - Strong coupling limit realizes $\hat{b}^\dagger \hat{b} = \hat{d}_1^\dagger \hat{d}_1 = \hat{d}_2^\dagger \hat{d}_2$

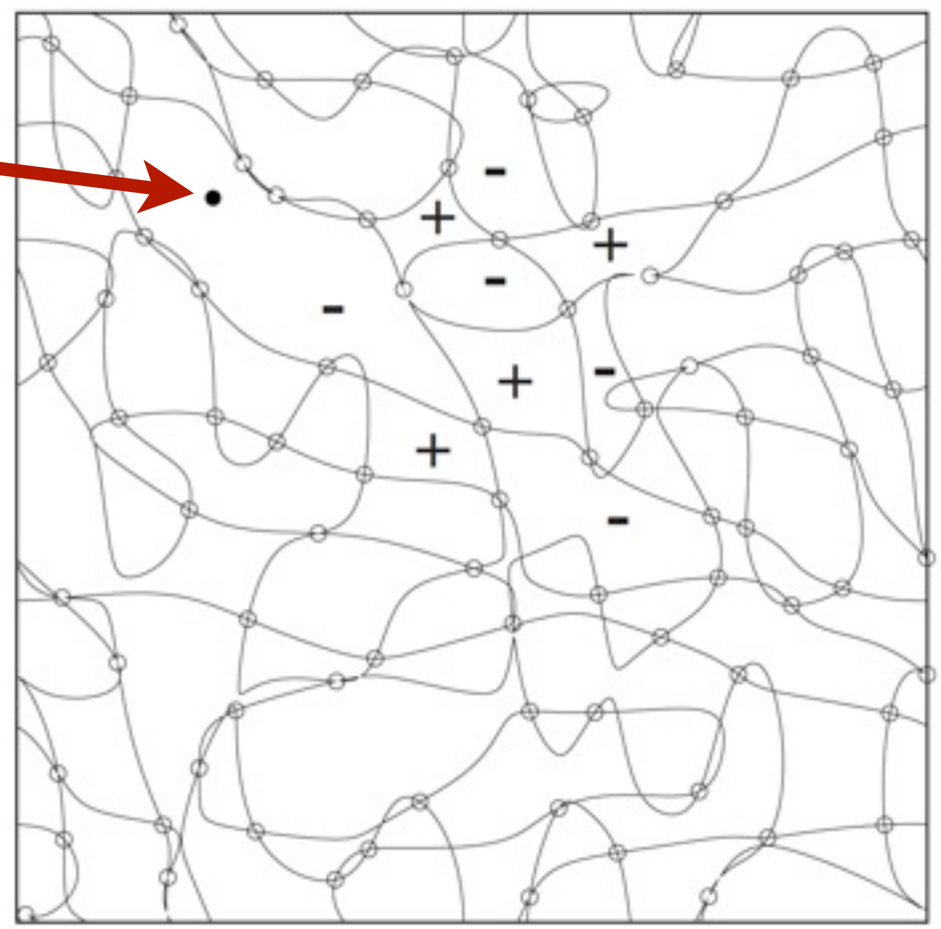
Sign structure (why "d-wave"?)

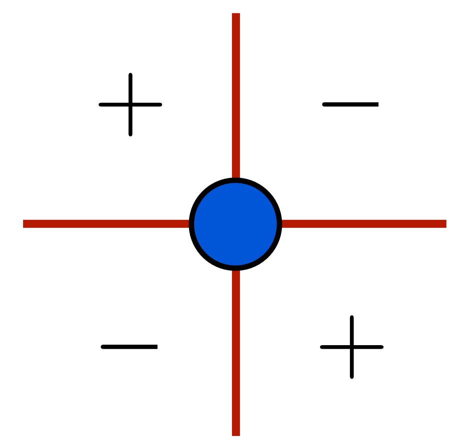
- $\Psi_b(\mathbf{r}_1, \ldots, \mathbf{r}_N) = \Psi_{d_1}(\mathbf{r}_1, \ldots, \mathbf{r}_N)\Psi_{d_2}(\mathbf{r}_1, \ldots, \mathbf{r}_N) = (\det)_x \times (\det)_y$
- Expect for 2-particle correlations:

$$\Phi_b(\mathbf{r}) \sim (x - x_i)(y - y_i), \text{ where } \Phi_b(\mathbf{r}) \equiv \Psi_b(\mathbf{r}, \mathbf{r}_2, \ldots, \mathbf{r}_N)$$

Test particle

$\text{sign}[\Phi_b(\mathbf{r})] =$



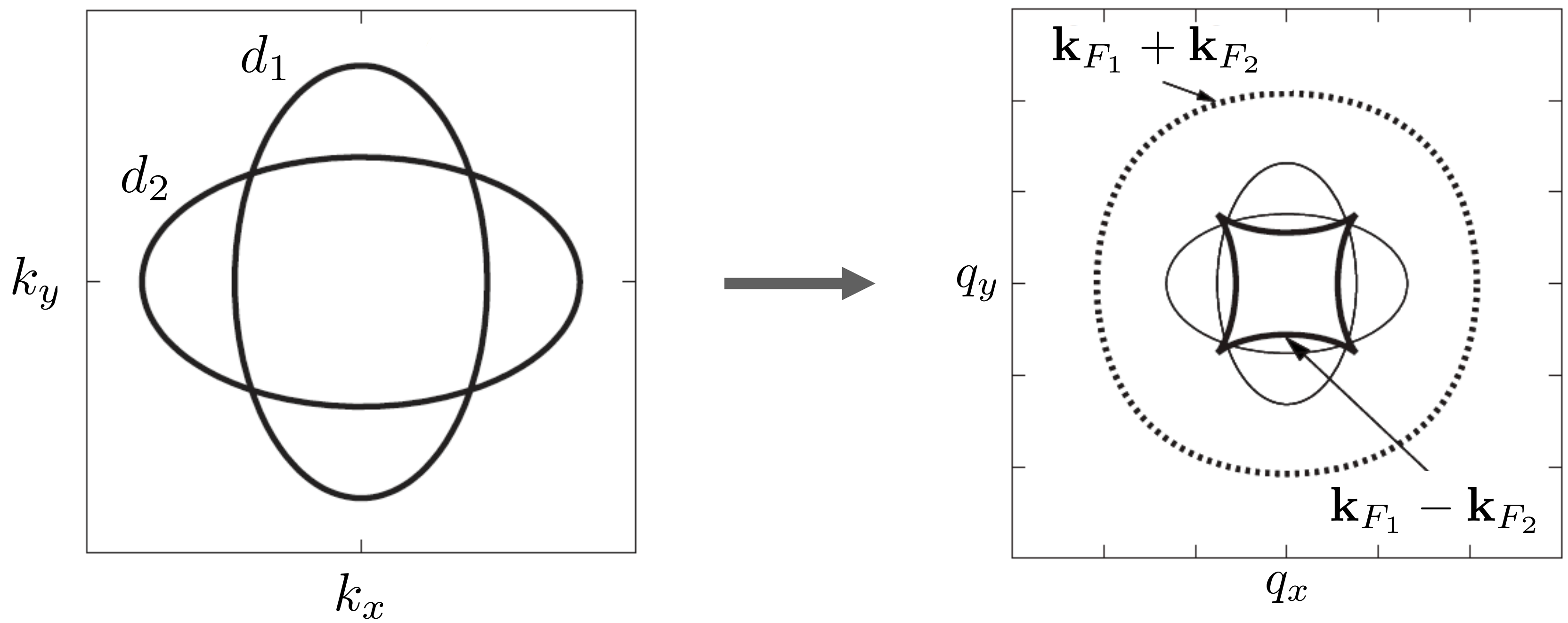


So what are these "Bose surfaces"?

- Result of oscillatory power law correlations in real space
- Boson Green's function: $G_b(\mathbf{r}) \equiv \langle \hat{b}^\dagger_{\mathbf{r}} \hat{b}_{\mathbf{0}} \rangle$
  - Mean-field: $G_b^{MF}(\mathbf{r}) = G_{d_1}^{MF}(\mathbf{r}) G_{d_2}^{MF}(\mathbf{r})/\nu$

$$G_{d_\alpha}^{MF}(\mathbf{r}) \approx \frac{1}{2^{1/2}\pi^{3/2}} \frac{\cos(\mathbf{k}_{F_\alpha} \cdot \mathbf{r} - 3\pi/4)}{c_\alpha^{1/2} |\mathbf{r}|^{3/2}}, \quad \mathbf{k}_{F_\alpha} = \mathbf{k}_{F_\alpha}(\hat{\mathbf{r}})$$

  - Singularities in boson momentum distribution at $k_{F_1}(\hat{\mathbf{r}}) \pm k_{F_2}(\hat{\mathbf{r}})$



Another singular surface ...

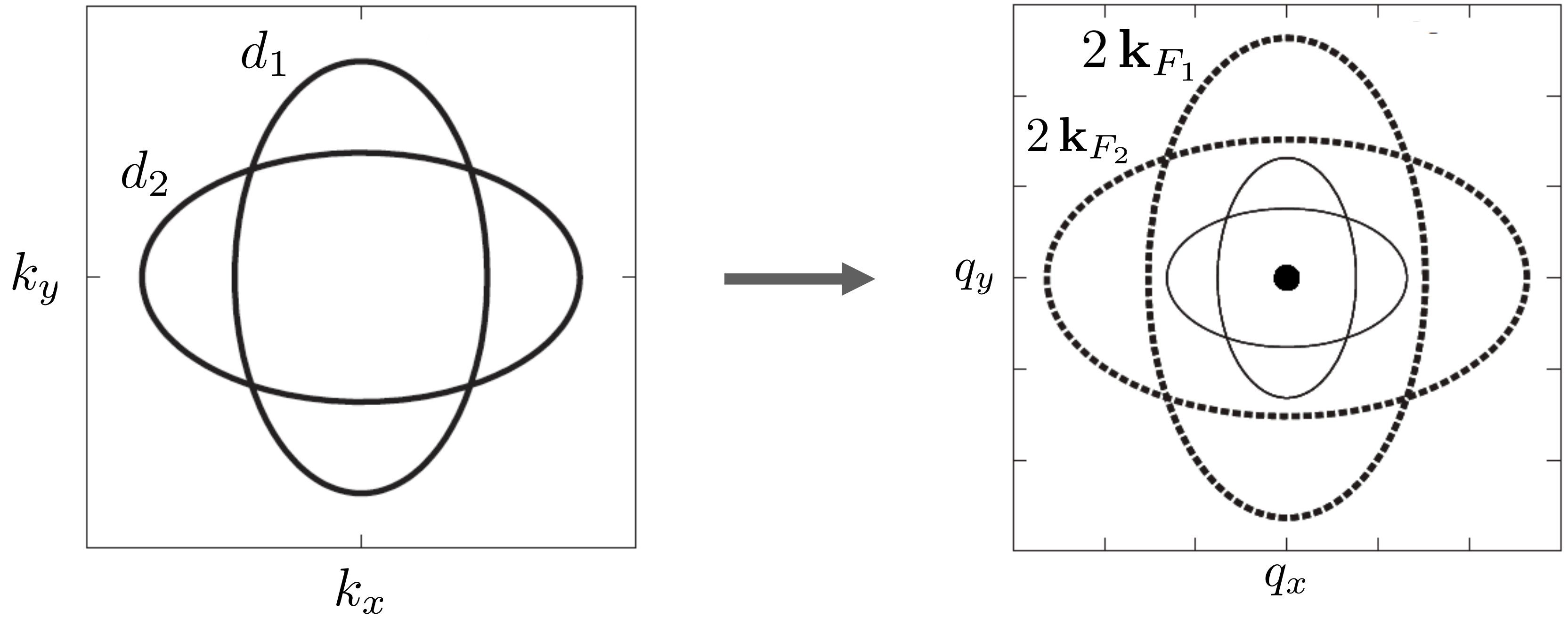
- Density-density correlation function:

$$D_b(\mathbf{r}) \equiv \langle(\hat{n}_\mathbf{r} - \nu)(\hat{n}_\mathbf{0} - \nu)\rangle, \ \nu = \text{filling factor}$$

  - Mean-field: $D_b^{MF}(\mathbf{r}) \approx \frac{1}{2}[D_{d_1}^{MF}(\mathbf{r}) + D_{d_2}^{MF}(\mathbf{r})]$

$$D_{d_\alpha}^{MF}(\mathbf{r}) = -|G_{d_\alpha}^{MF}(\mathbf{r})|^2 \sim -\frac{1+\cos[2\mathbf{k}_{F_\alpha}\cdot\mathbf{r} - 3\pi/2]}{c_\alpha|\mathbf{r}|^3}$$

  - Singularities in density-density structure factor at $2k_{F_\alpha}(\hat{\mathbf{r}})$



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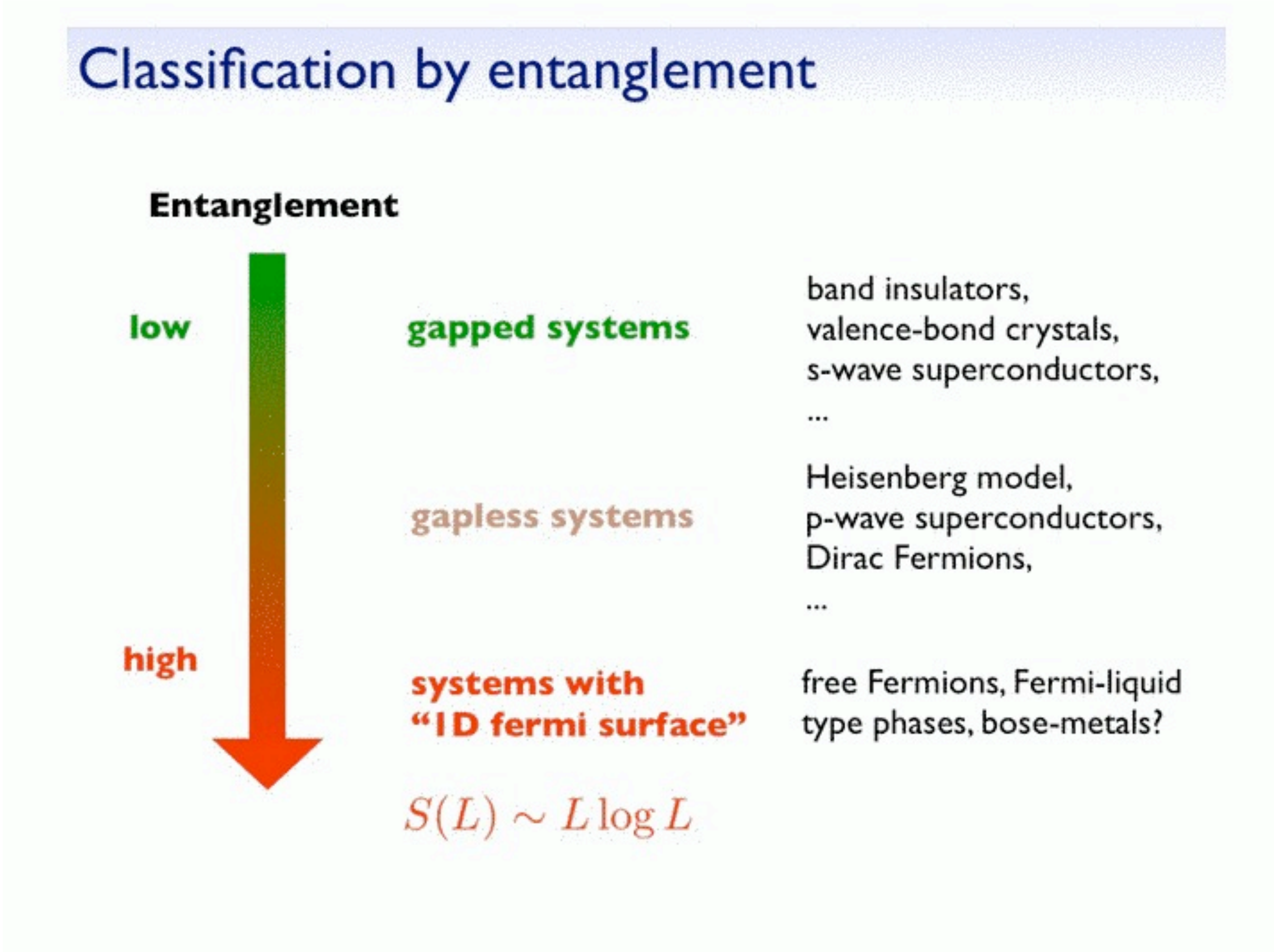
Philippe Corboz, CompQCM, 12/02/2010:



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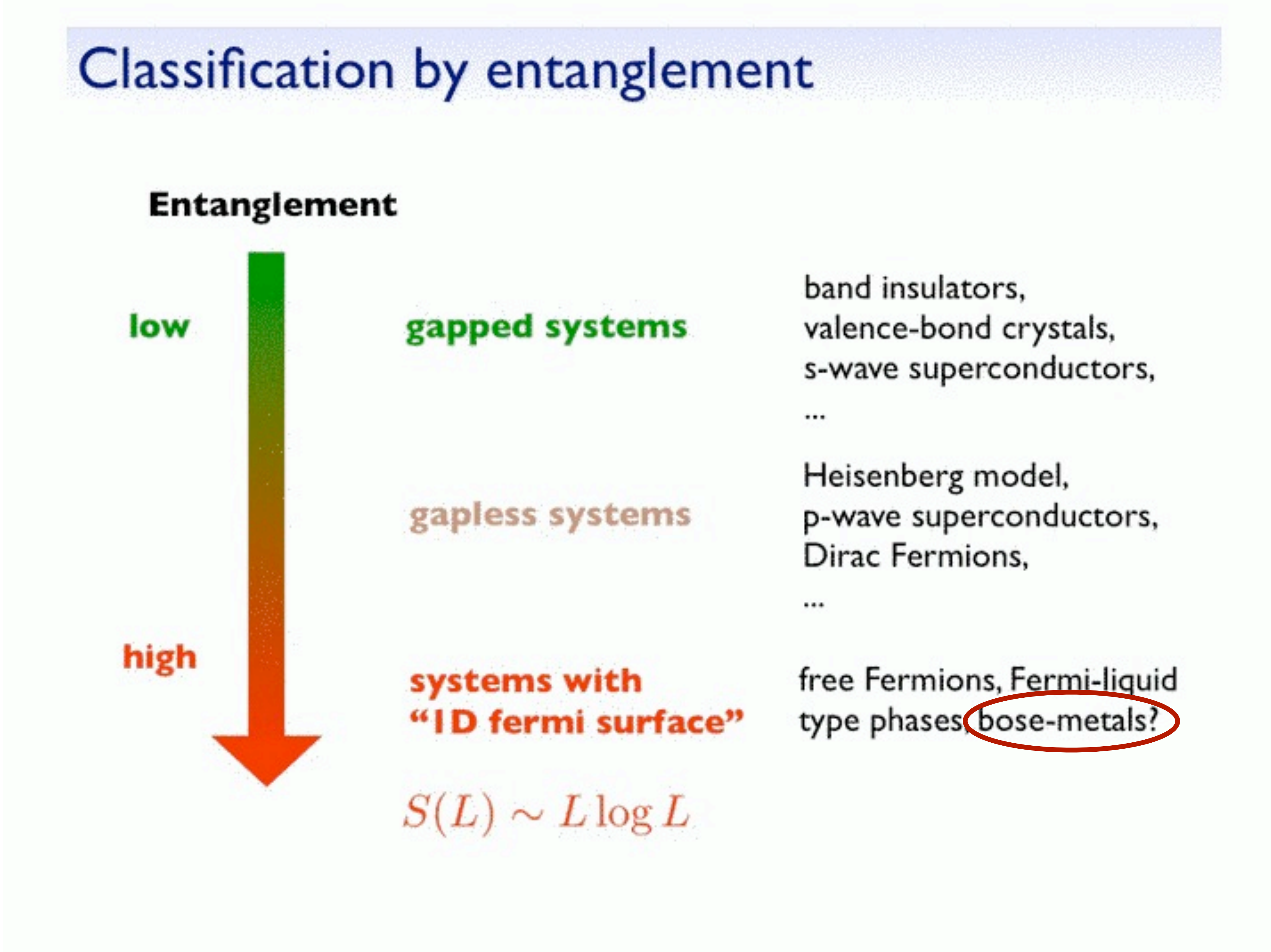
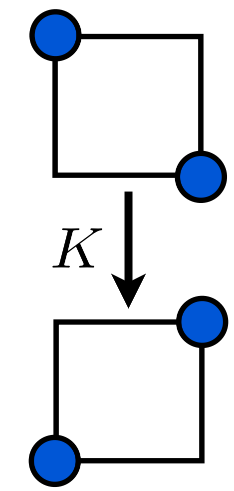


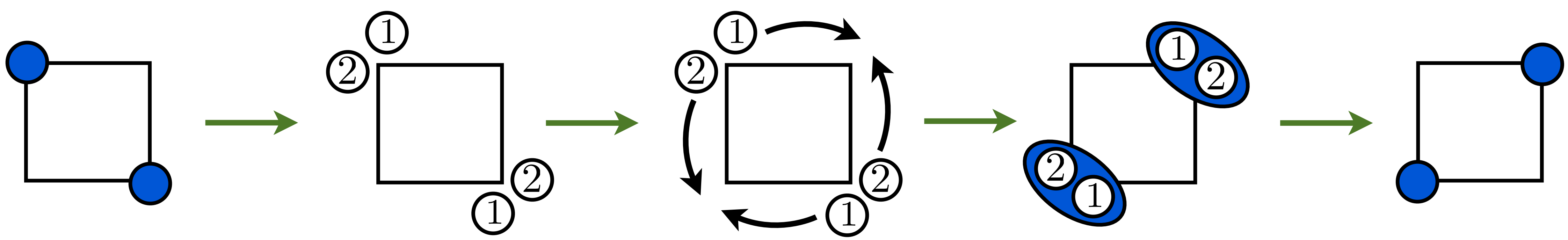
Where (to look)?

- Frustrated 4-site ring-exchange "*J-K* model"

$$\hat{H}_{JK} = -J\sum_{\mathbf{r}}(\hat{b}^\dagger_{\mathbf{r}}\hat{b}_{\mathbf{r}+\hat{\mathbf{x}}} + \text{h.c.}) - J_\perp\sum_{\mathbf{r}}(\hat{b}^\dagger_{\mathbf{r}}\hat{b}_{\mathbf{r}+\hat{\mathbf{y}}} + \text{h.c.}) + K\sum_{\mathbf{r}}(\hat{b}^\dagger_{\mathbf{r}}\hat{b}_{\mathbf{r}+\hat{\mathbf{x}}}\hat{b}^\dagger_{\mathbf{r}+\hat{\mathbf{x}}+\hat{\mathbf{y}}}\hat{b}_{\mathbf{r}+\hat{\mathbf{y}}} + \text{h.c.})$$



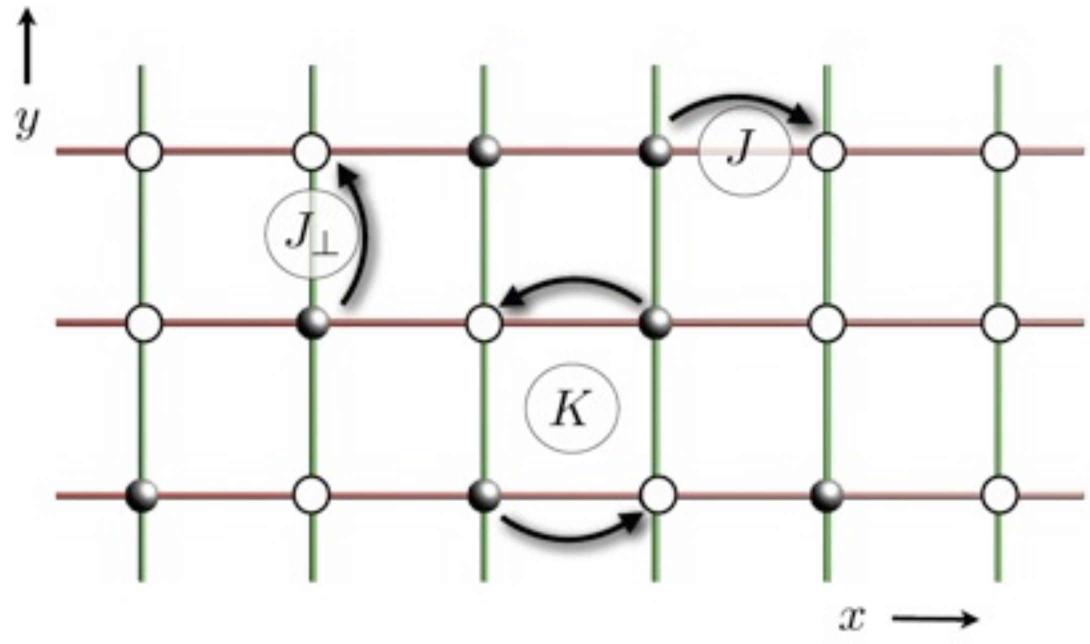
  - Strong coupling limit of gauge theory for DBL
  - Anisotropic hopping of two fermion species + Gutzwiller projection

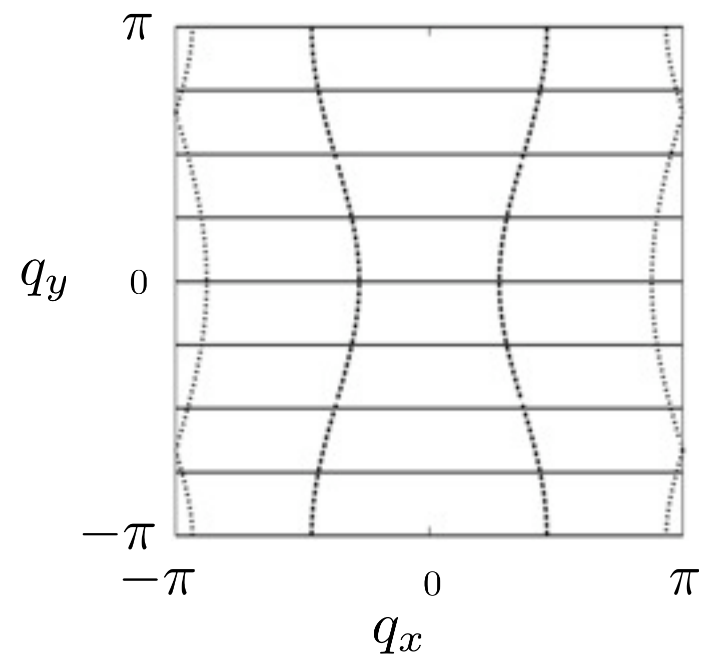


- Unfrustrated ($K < 0$) case known from QMC
  - Sandvik *et al.*, PRL (2002); Melko *et al.*, PRB Rapid (2004)

How (to access)?

- Sign problem …
  - But … DBL has singular *surfaces* in momentum space
  - So … can controllably be studied on the *N*-leg ladder





- Methods of attack: DMRG, VMC, ED, bosonization
- 2-leg ladder already thoroughly investigated
  - Sheng *et al.*, PRB **78**, 054520 (2008)

Why (should anyone care)?

- DBL is very much a non-Fermi liquid
  - New uncondensed (non-superfluid) phase of itinerant bosons
  - Lack of long-lived quasiparticles
  - DBL as piece of "*d*-wave metal" for model wave function of the strange metal in high-T_c? (See MPAF's talks)

$$\Psi_{NFL} = \mathcal{P}_G[\Psi_f^{FF} \times \Psi_b^{DBL}]$$

- Cold atom realizations?
  - Two species of fermions + anisotropic hopping + attraction
    - Feiguin and MPAF, PRL (2009); Feiguin and MPAF, arXiv:1007.5251
  - Engineer ring-exchange Hamiltonian [Buchler *et al.*, PRL (2005)]
- U(1) limit of exotic state of SU(2) spins in Zeeman field?

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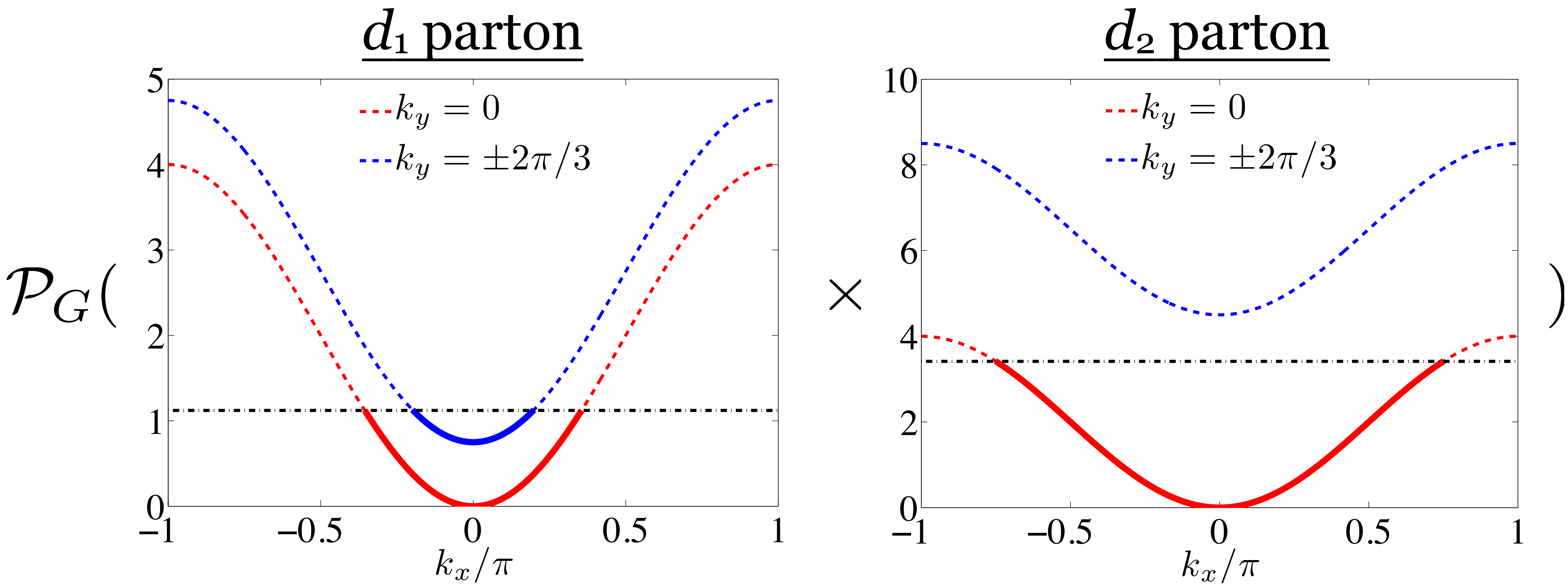
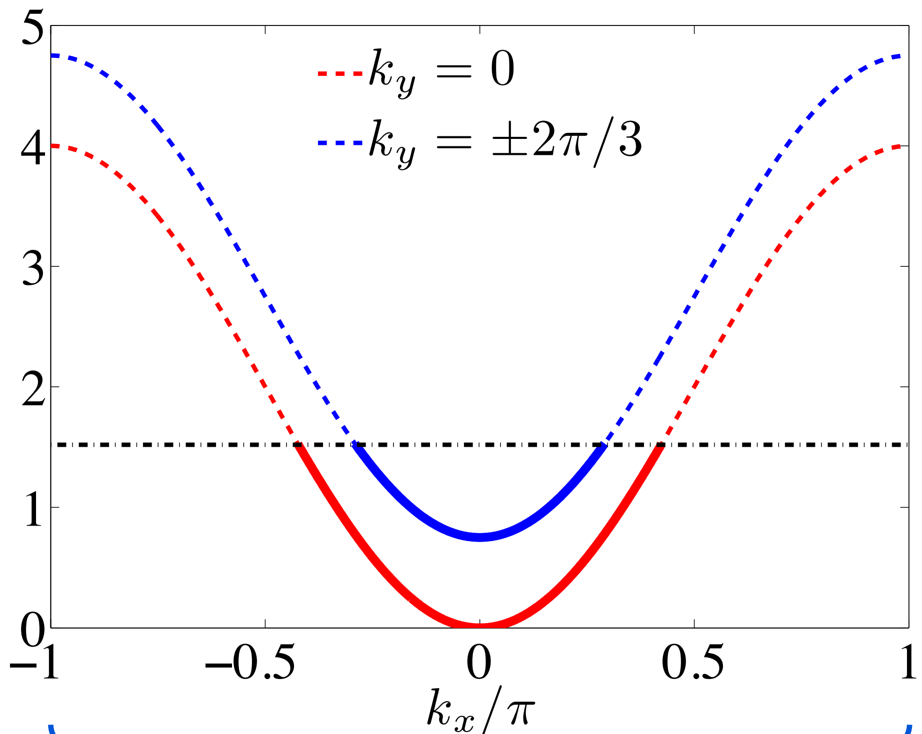
DBL wave functions on the 3-leg ladder



- **Wave function properties**
  - Depends only on filled momenta, not dispersions
  - Oscillatory power law correlations: fingerprints of "Bose surfaces"
    - Fermi points of partons → "Bose points"
- **Notation: DBL[n, m]**
  - n, m partially filled d_1, d_2 bands
  - Example above: DBL[3,1]

DBL[3,0]: Exotic gapless Bose insulator

$$\mathrm{DBL}[3,0] = \mathcal{P}_G\Big($$



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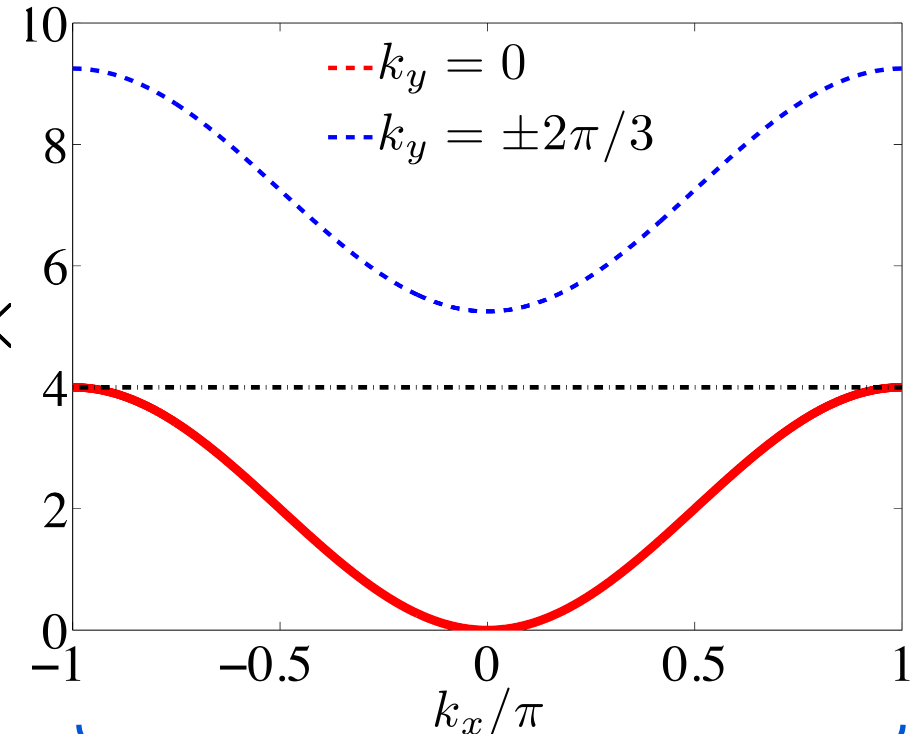


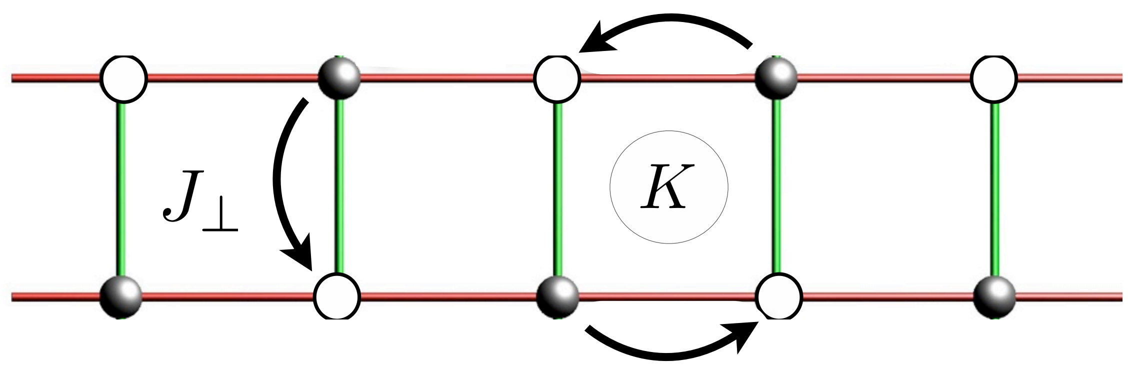
$\Big)$

3 partially filled d_1 bands (metal) 1 fully filled d_2 band (insulator)

- Filling factor: $\nu = 1/3 = 1/L_y$
- Features
  - Filled d_2 band → exactly 1 boson per rung (1D insulator)
  - Still 3 - 1 = 2 gapless 1D modes
  - $2k_F$ wave vectors from d_1 visible in boson density-density structure factor

Aside: What about the 2-leg ladder at 1/2 filling?

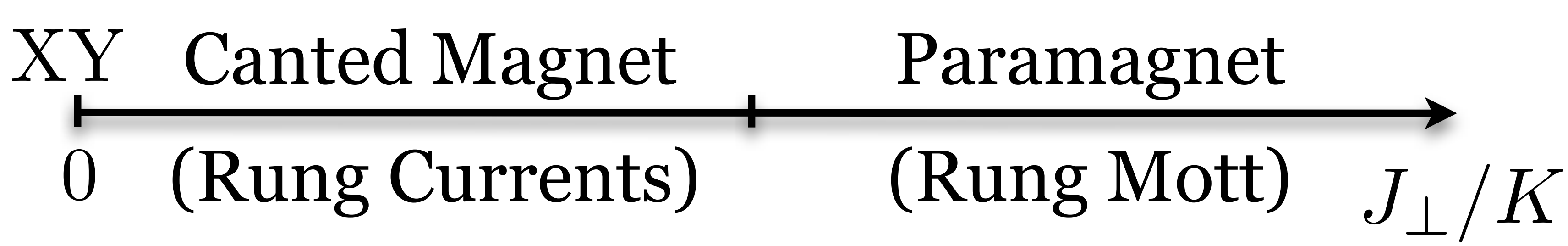
- First focus on case $J = 0$
  - One boson per rung



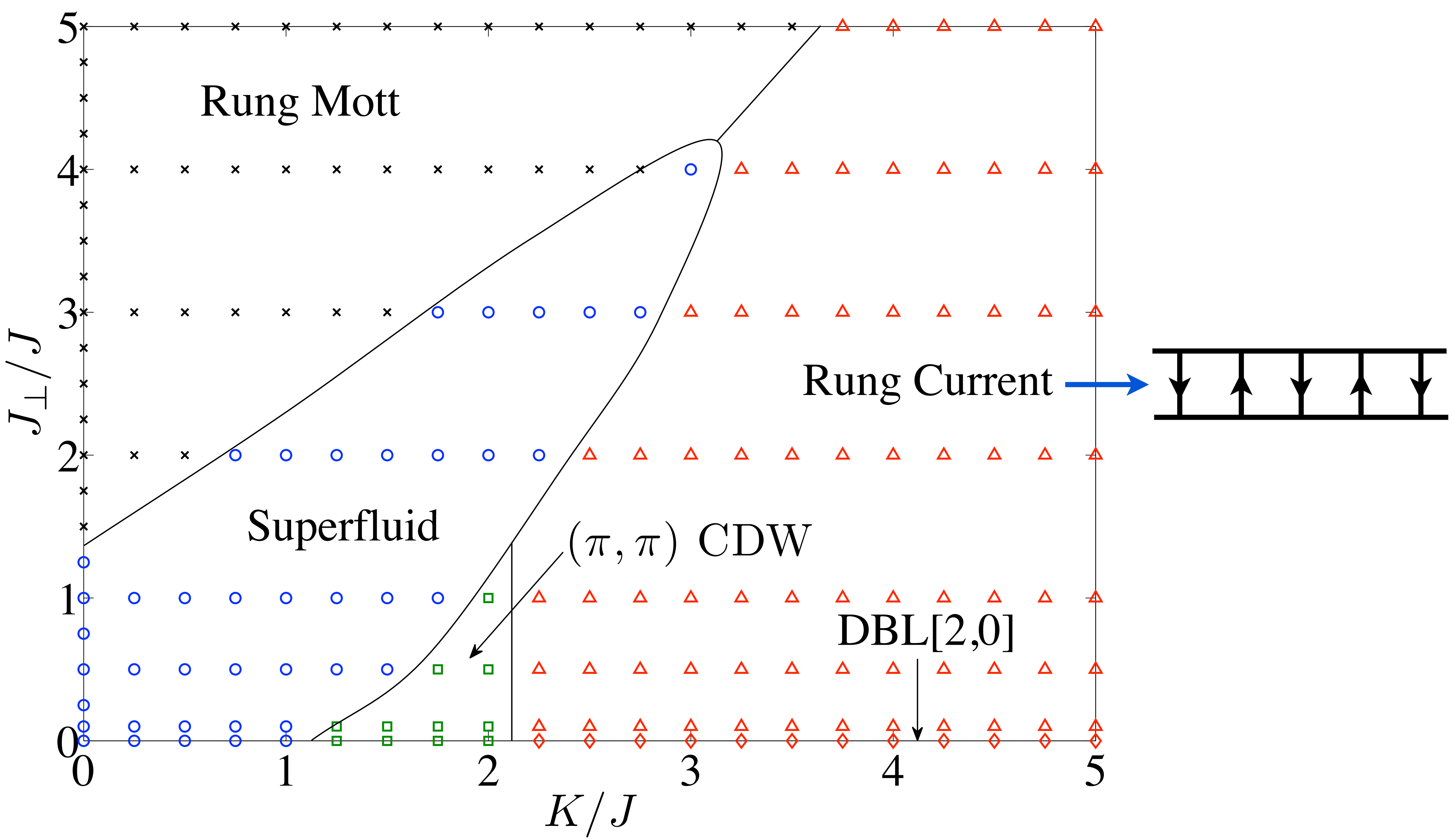
  - Maps onto 1D XY model in an *in-plane* field

$$\hat{H}_{\mathrm{XY}} = K \sum_i (\hat{\sigma}_i^+ \hat{\sigma}_{i+1}^- + \mathrm{h.c.}) - J_\perp \sum_i \hat{\sigma}_i^x$$

  - Phase diagram



Full 2-leg, 1/2-filling phase diagram (DMRG)



No exotic gapless insulating phase (except boring “[2,0]” at $J_\perp = 0$) ...

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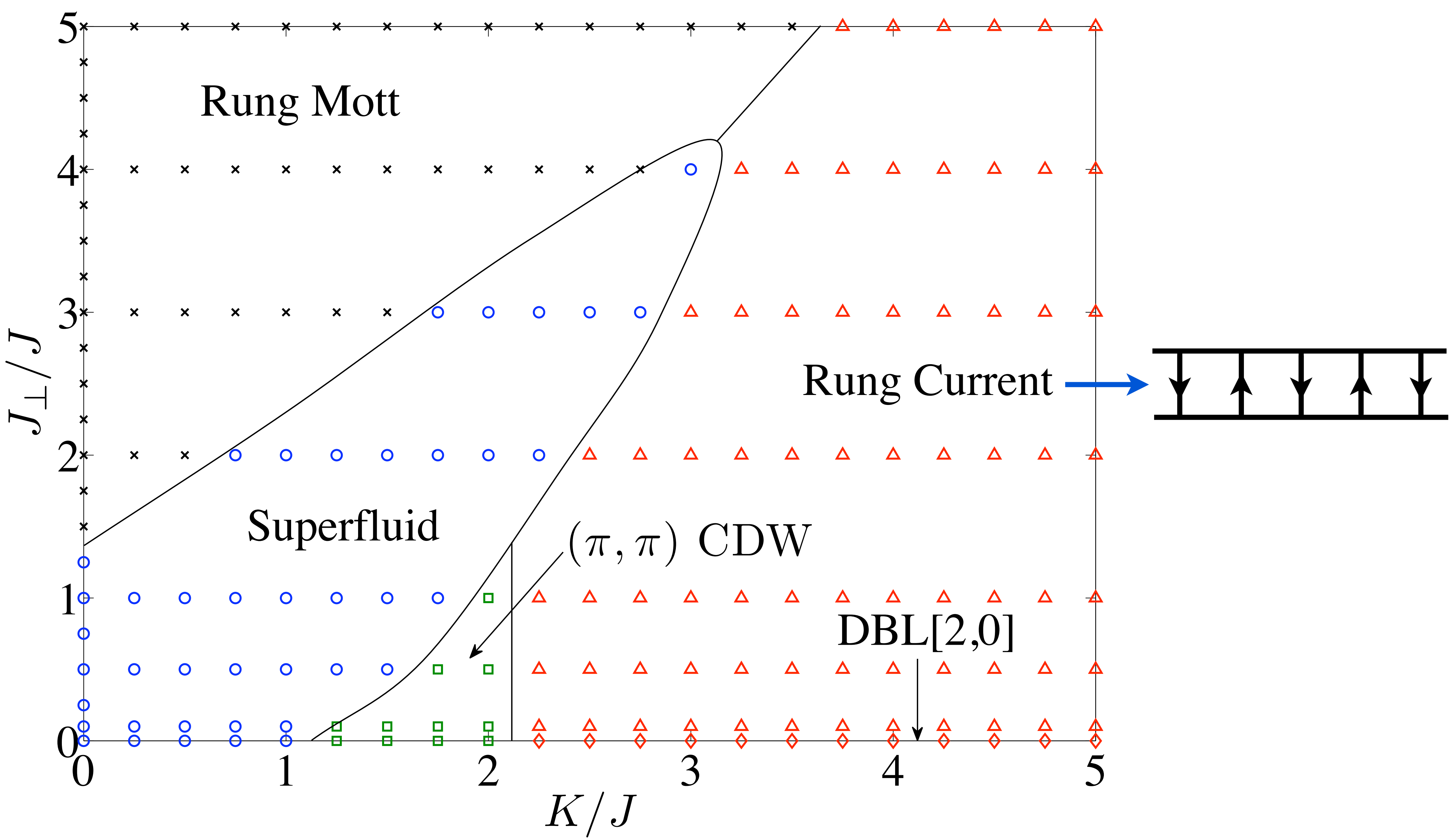
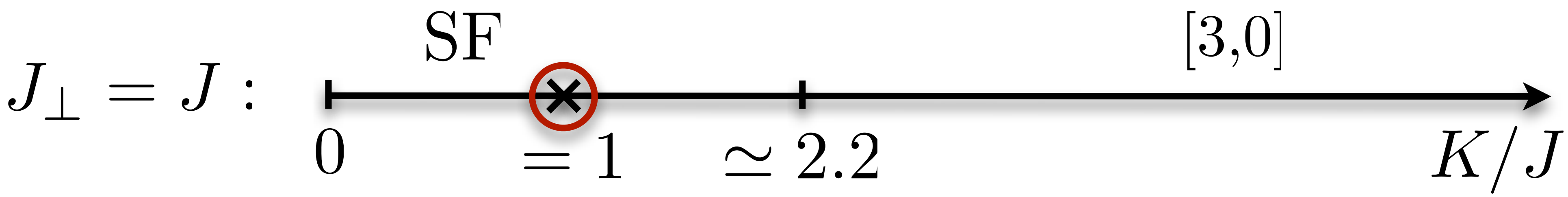


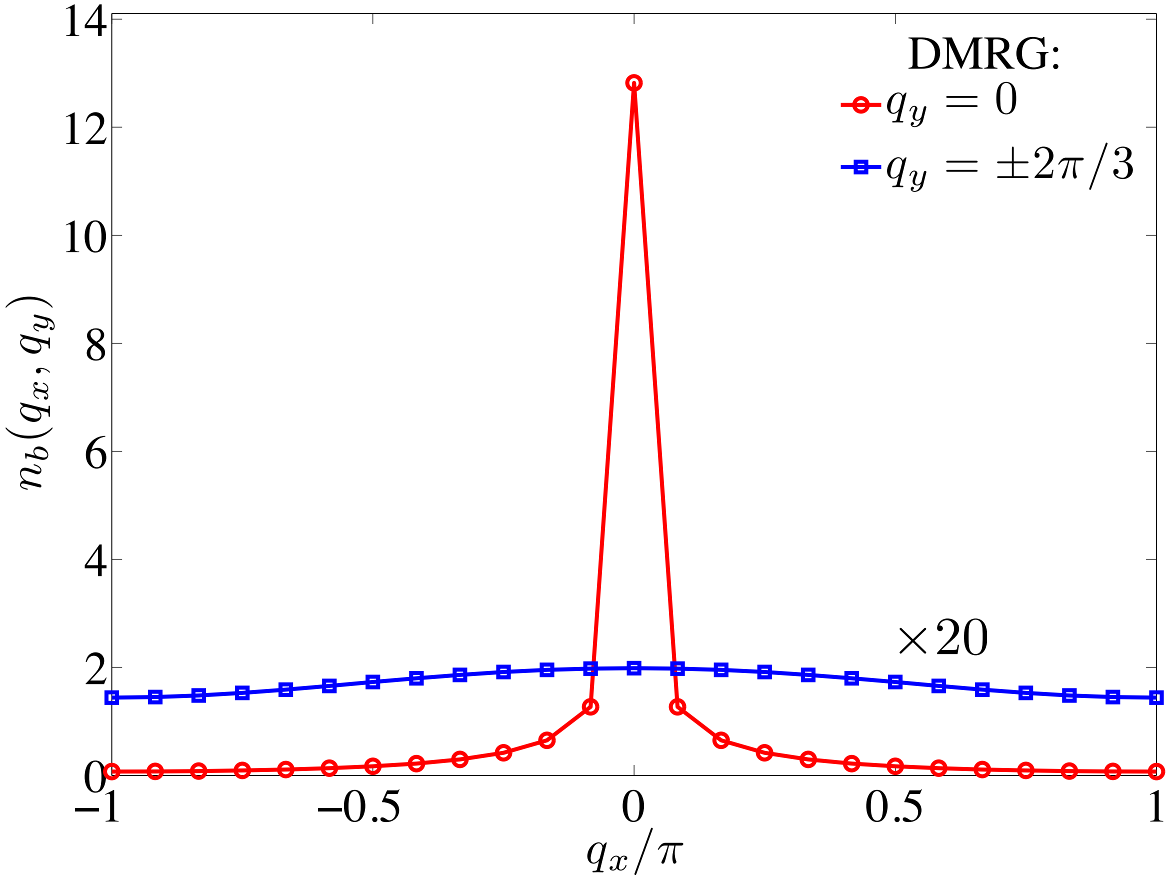
No exotic gapless insulating phase (except boring "[2,0]" at $J_\perp = 0$) ...

But ... DBL[3,0] is a stable phase of the 3-leg *J*-*K* model

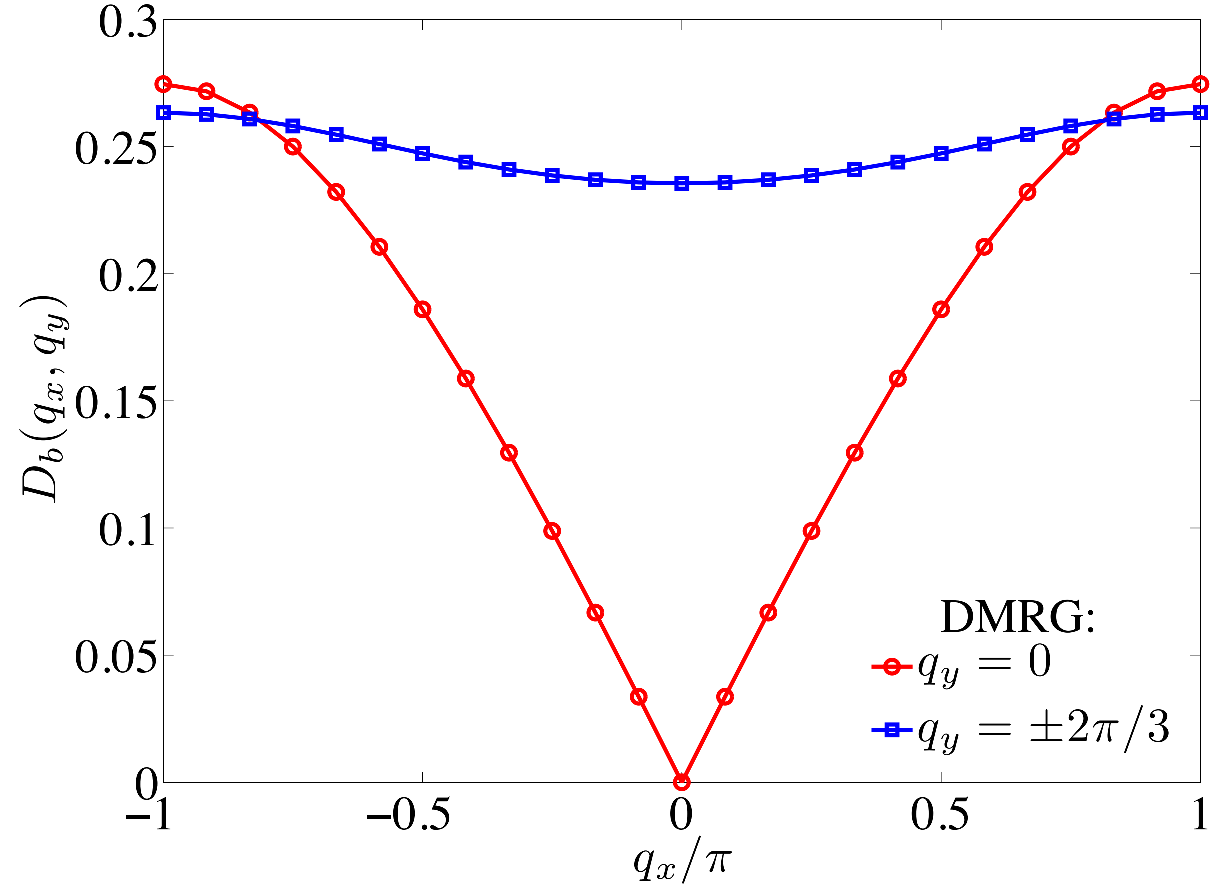
Momentum space correlators: Superfluid



<u>Boson momentum distribution</u>



<u>Density-density structure factor</u>



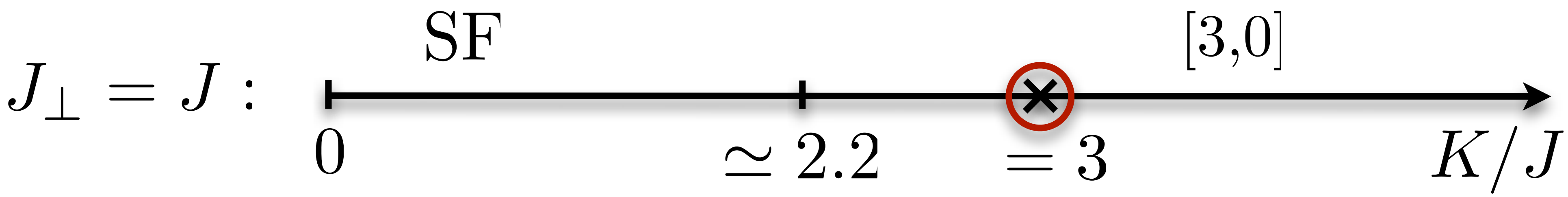
$n_b(\mathbf{q}) \equiv \langle \hat{b}^\dagger_{\mathbf{q}} \hat{b}_{\mathbf{q}} \rangle$:

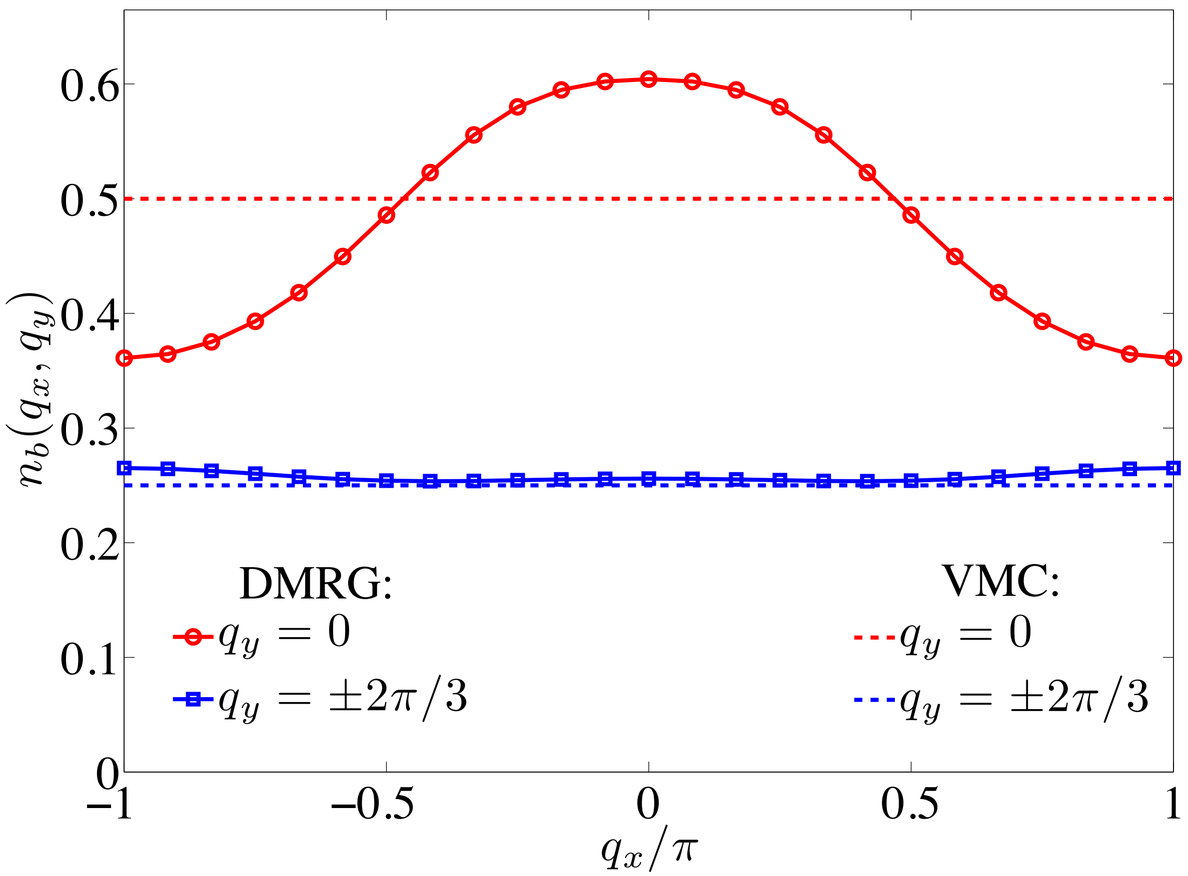
- Bose condensate at $\mathbf{q} = 0$
- 1D Quasi-ODLRO

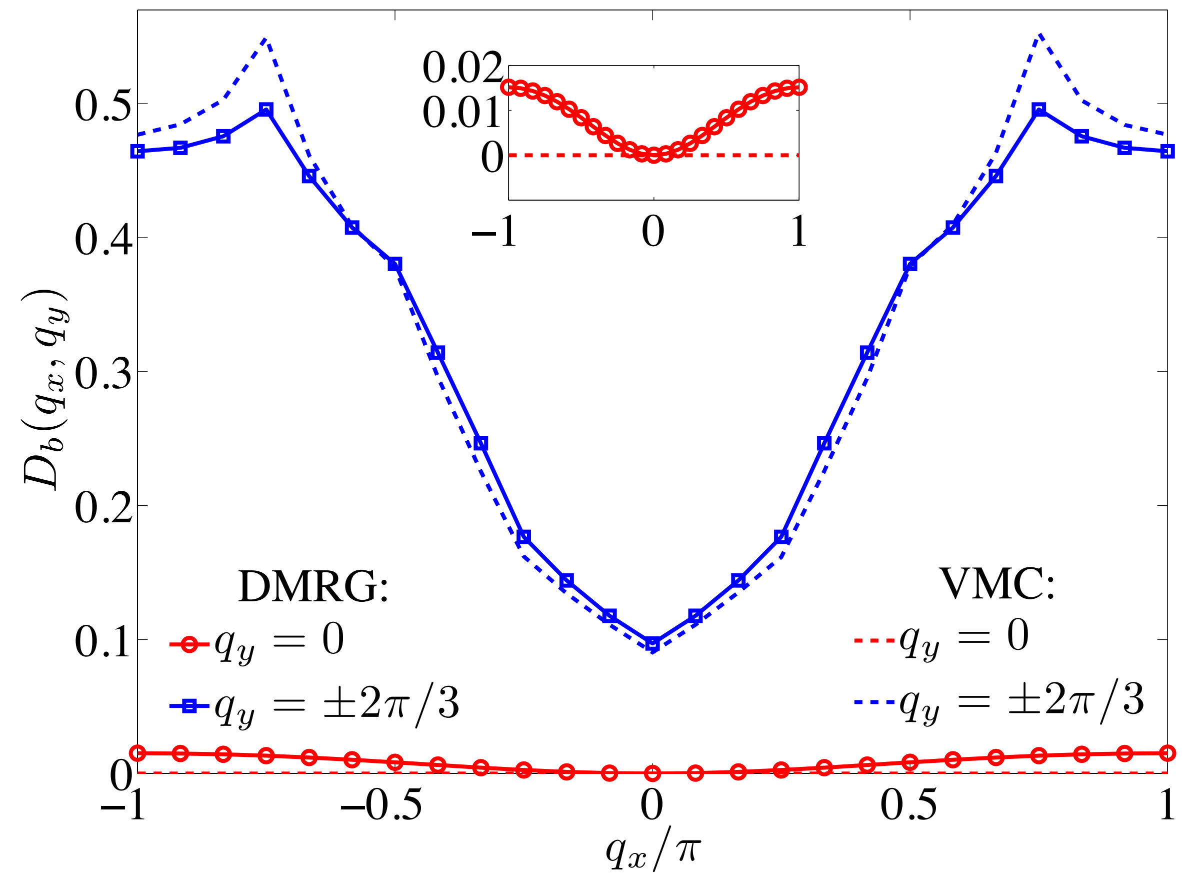
$D_b(\mathbf{q}) \equiv \langle \delta\hat{n}_{\mathbf{q}} \delta\hat{n}_{-\mathbf{q}} \rangle$:

- $|q_x|$ dependence around $q_x = 0$ at $q_y = 0$

Momentum space correlators: DBL[3,0]







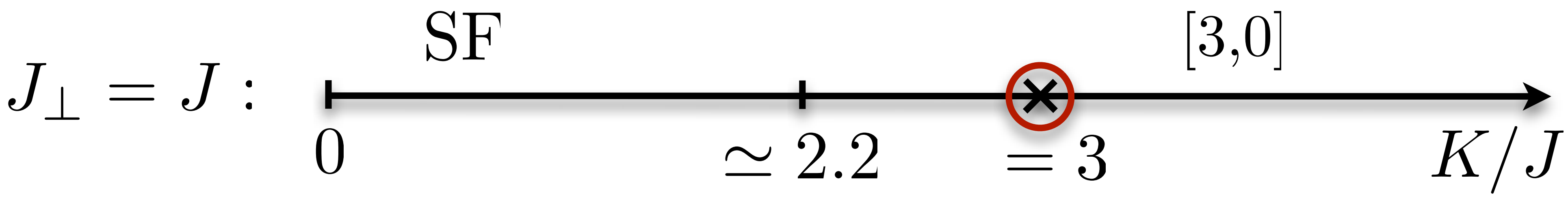
$n_b(\mathbf{q}) \equiv \langle \hat{b}^\dagger_{\mathbf{q}} \hat{b}_{\mathbf{q}} \rangle$:

- Featureless due to filled band
- Signature of 1D insulator

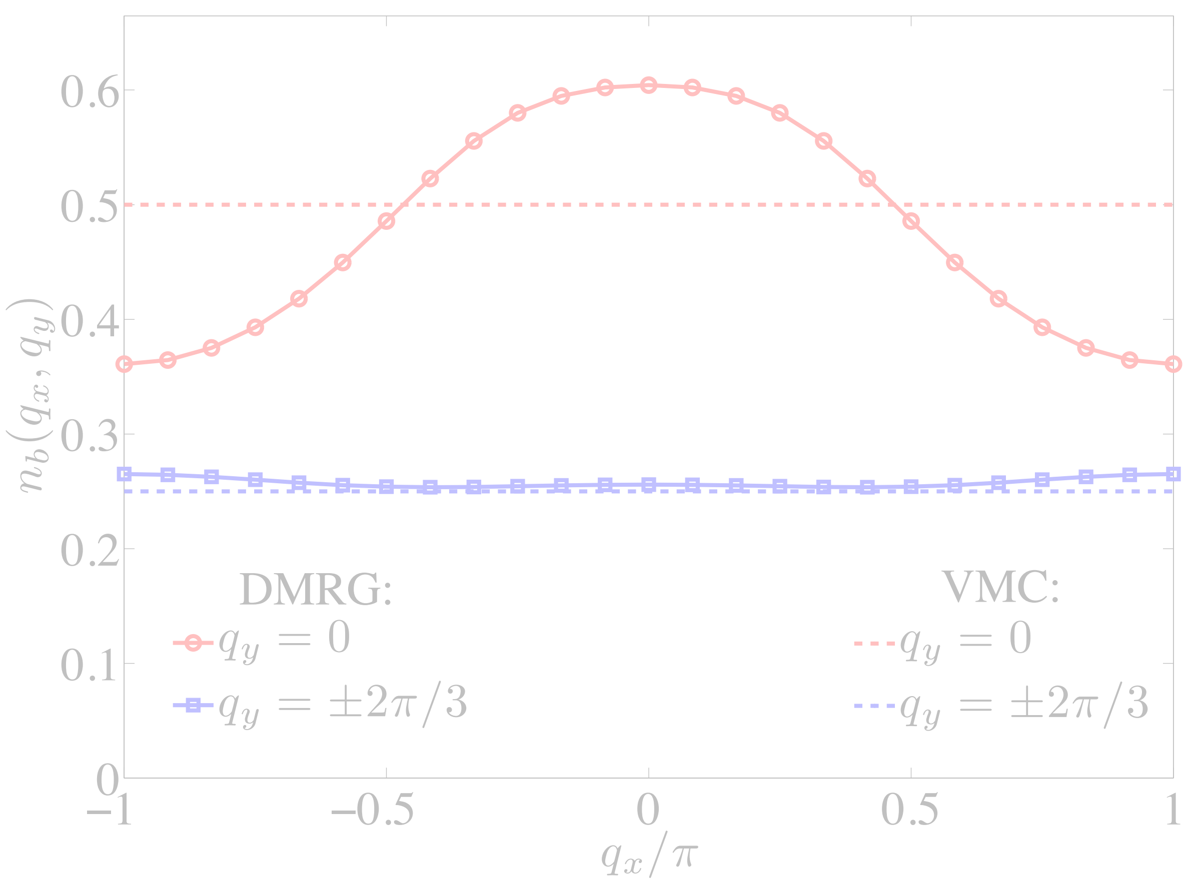
$D_b(\mathbf{q}) \equiv \langle \delta\hat{n}_{\mathbf{q}} \delta\hat{n}_{-\mathbf{q}} \rangle$:

- Singularites at $2k_F$ wave vectors
- Signature of state's "gaplessness"

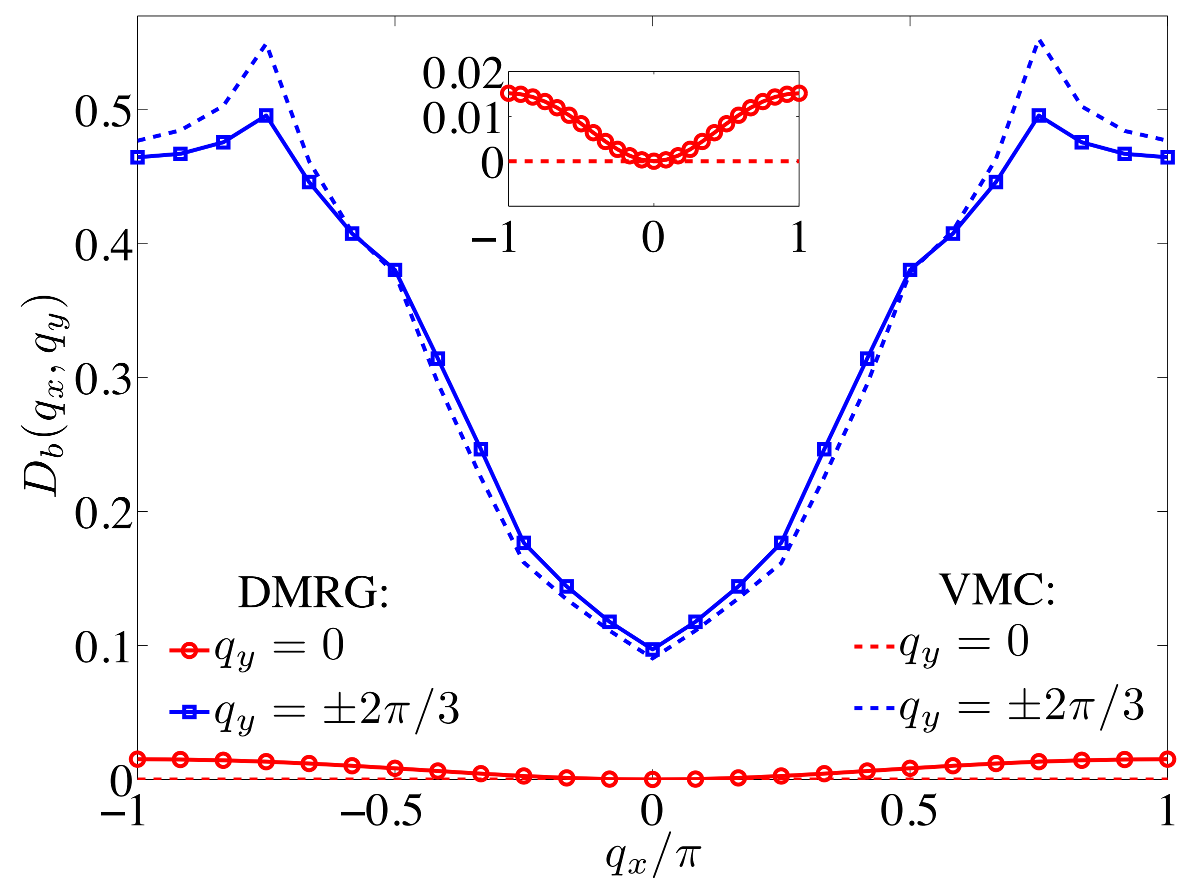
Momentum space correlators: DBL[3,0]

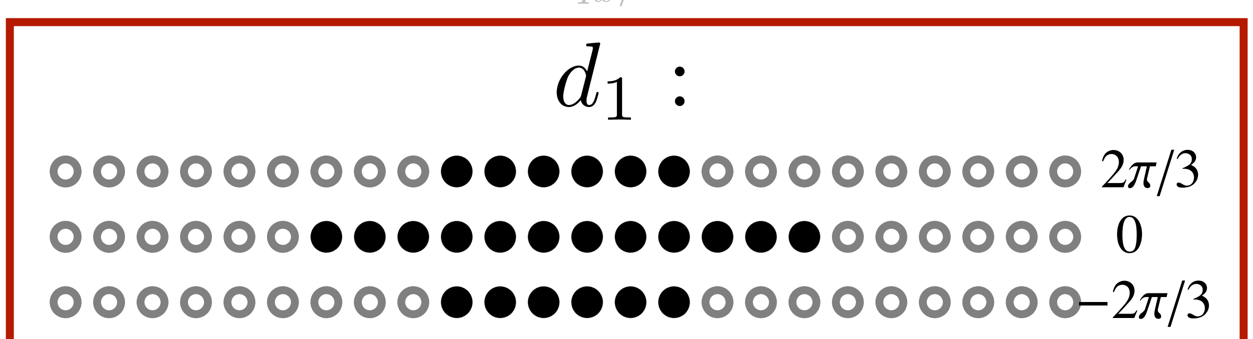


Boson momentum distribution



Density-density structure factor

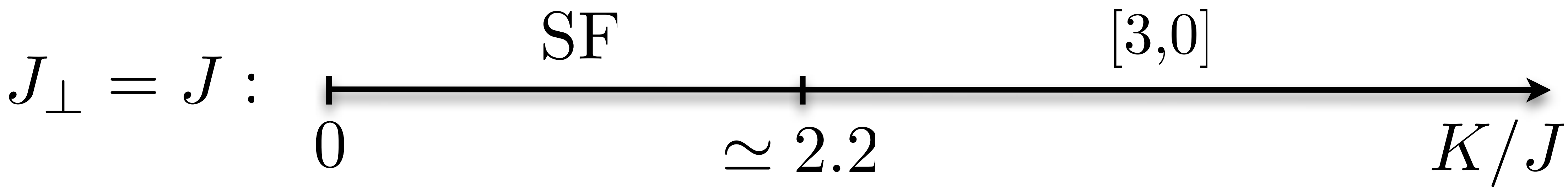




$$D_b(\mathbf{q}) \equiv \langle \delta\hat{n}_{\mathbf{q}} \delta\hat{n}_{-\mathbf{q}} \rangle :$$

- Singularites at $2k_F$ wave vectors
- Signature of state's "gaplessness"

Evolution of "Bose surfaces"



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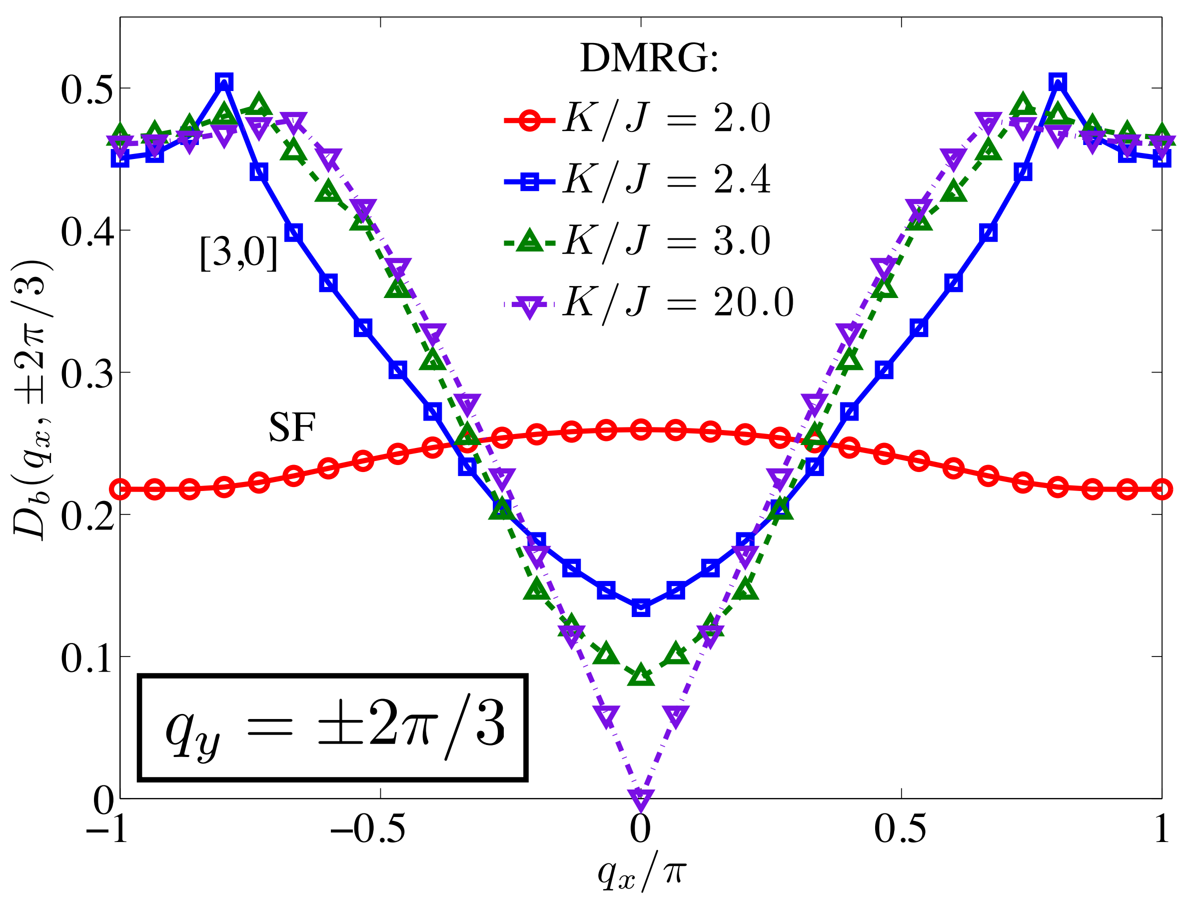
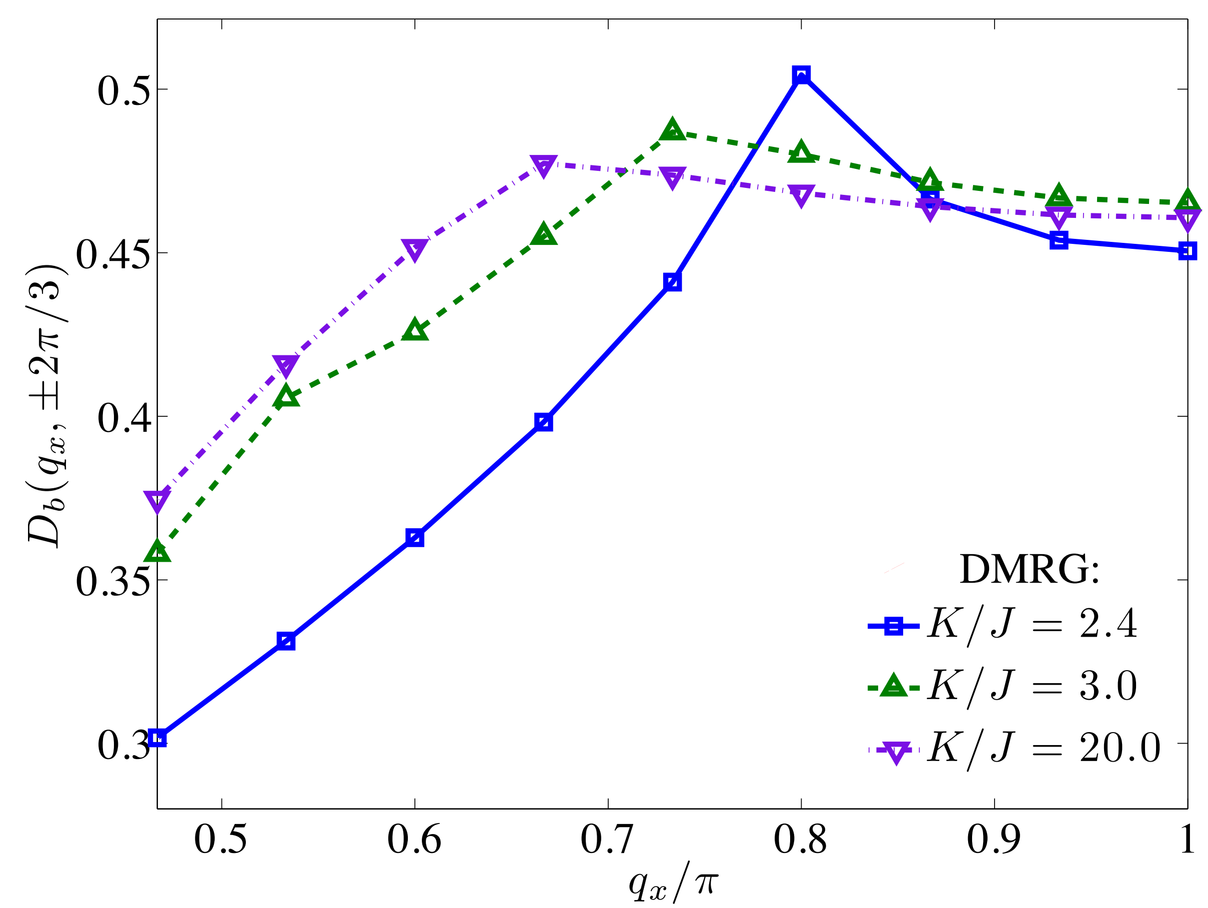


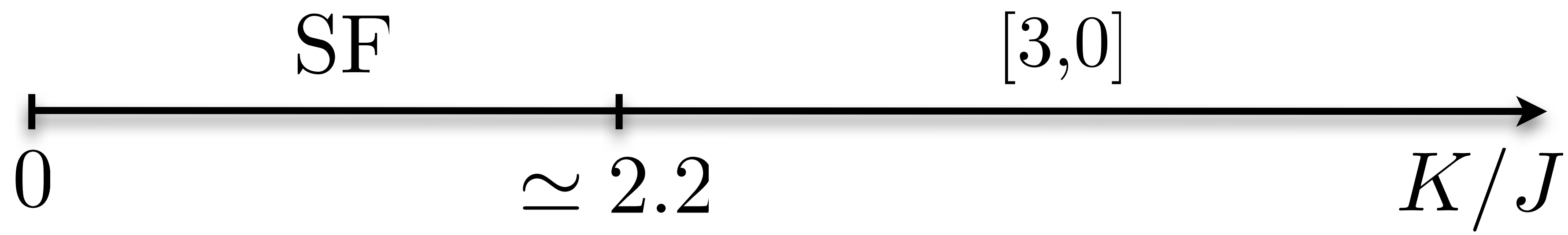
<u>Zooming in...</u>



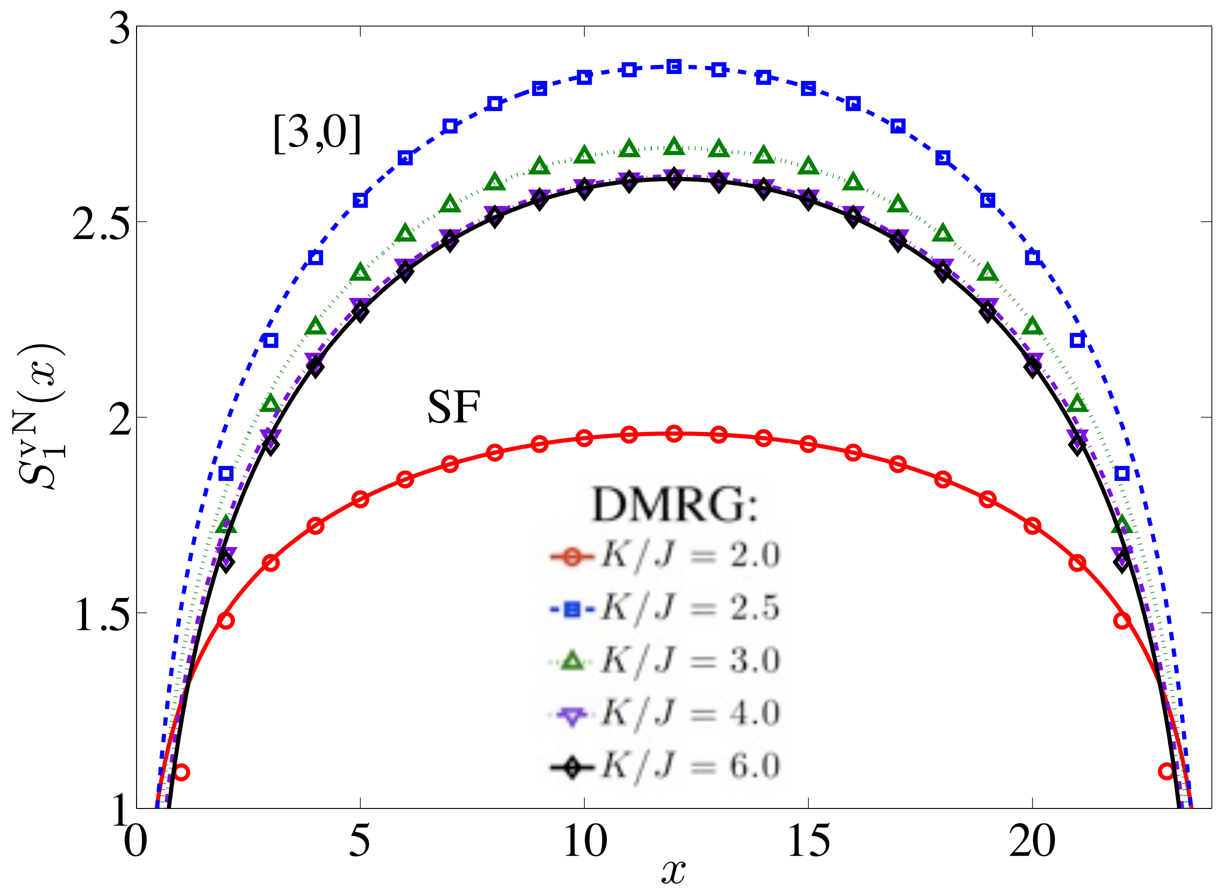
Evolution of peaks consistent with different d_1 band fillings

Evolution of entanglement entropy

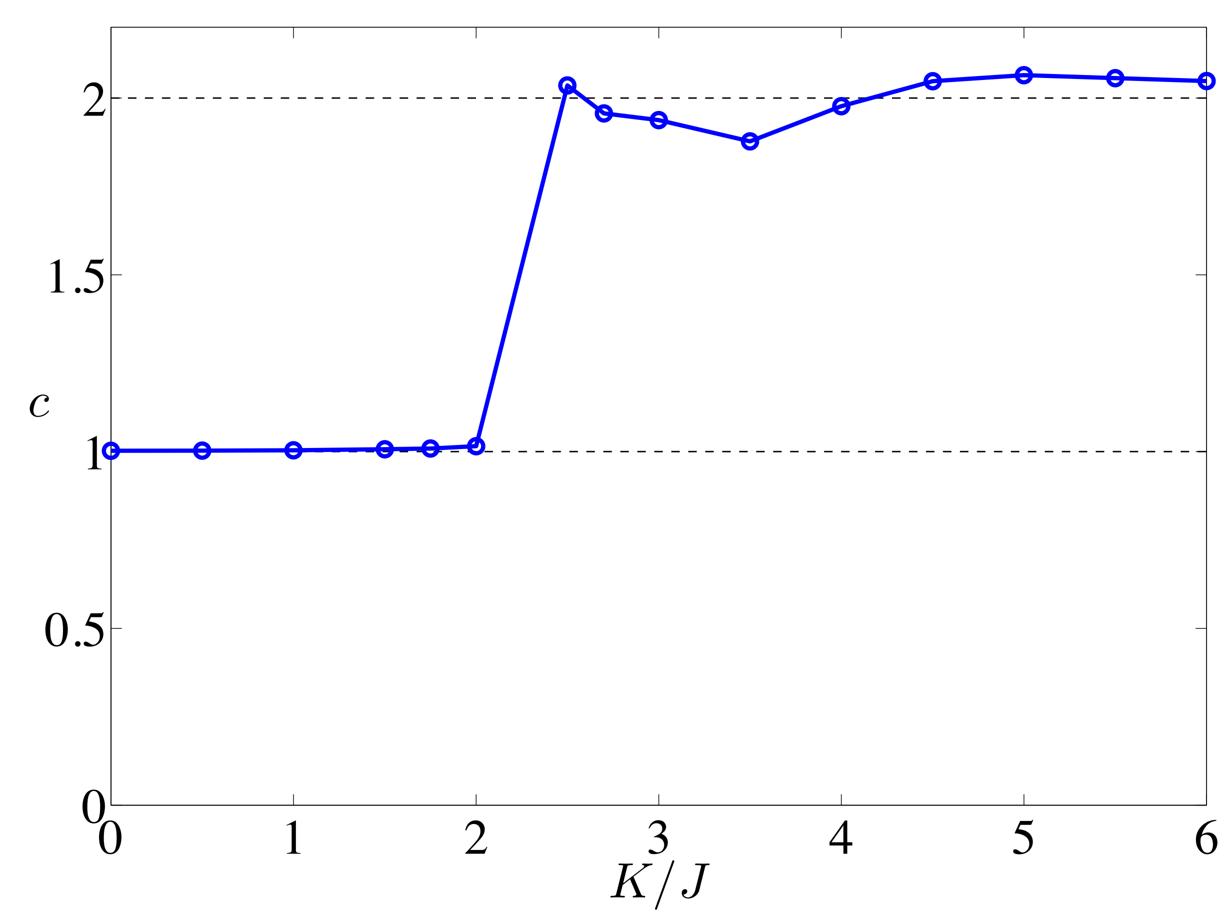
$J_\perp = J :$



<u>Entropy vs. subsystem length</u>



<u>Central charge vs. *K/J*</u>



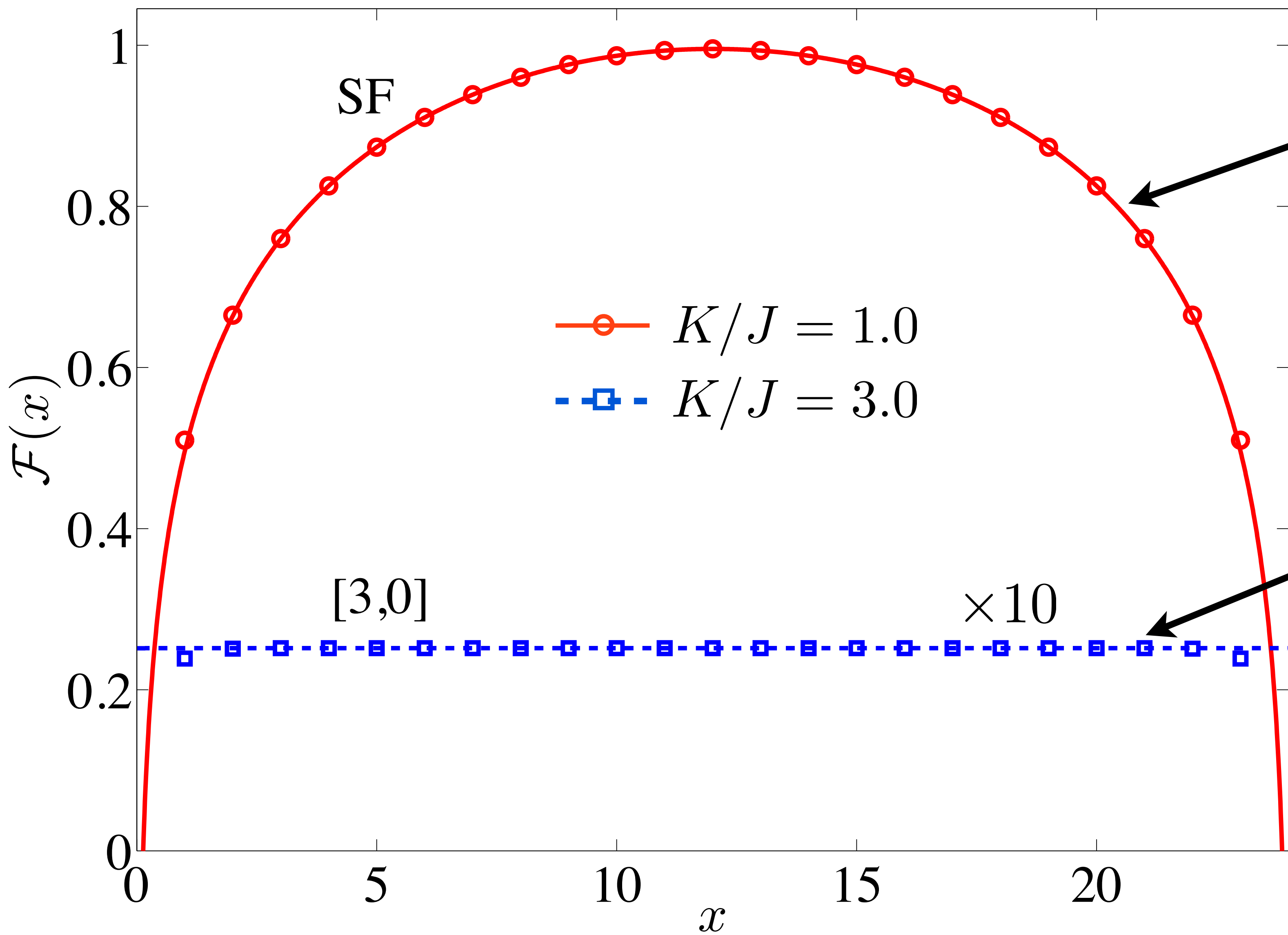
Effective central charge, *c*, from scaling of entanglement entropy:

$$S^{\mathrm{vN}}(x) = \frac{c}{3} \ln \left[\frac{L_x}{\pi} \sin\left(\frac{\pi x}{L_x} \right) \right] + A,$$

$c \simeq$ # of 1D gapless modes

Scaling of bipartite number fluctuations

Number fluctuations vs. subsystem length



Fits well to LL/CFT scaling form:

$$g \ln\left[\frac{L_x}{\pi}\sin\left(\frac{\pi x}{L_x}\right)\right] + B$$

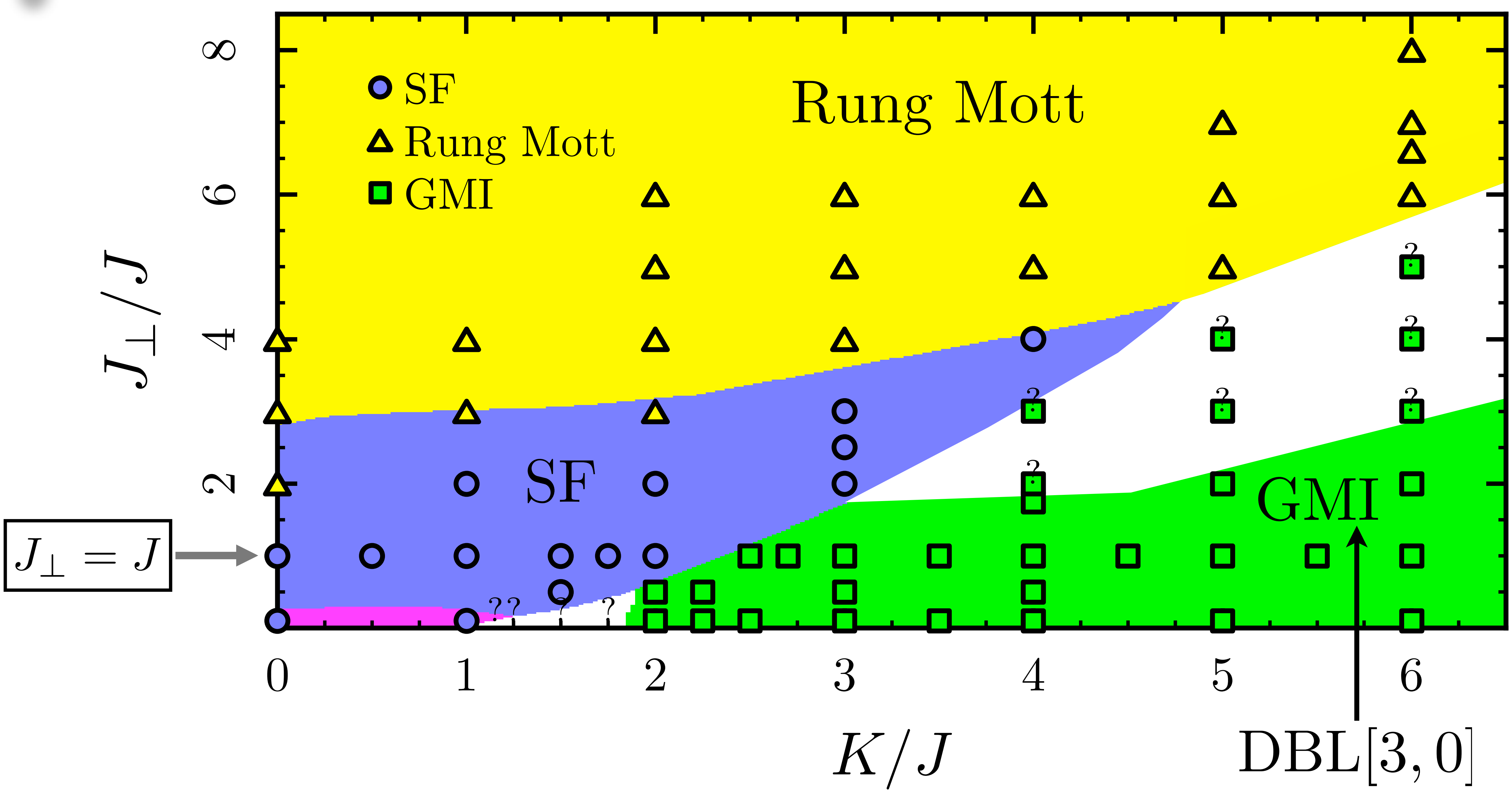
$g \propto$ compressibility

Saturates to a constant: $g \to 0$
1D insulator!

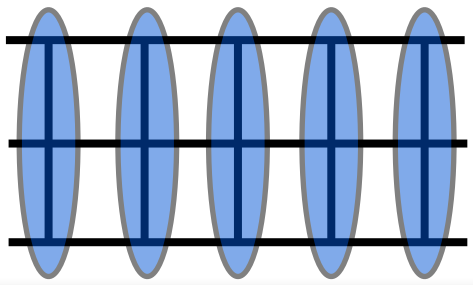
$\mathcal{F}(x) \equiv \langle(\hat{N}_A - \langle\hat{N}_A\rangle)^2\rangle$, $A =$ leftmost contiguous block of size $3x = L_y x$

Cf. recent work relating bipartite entanglement and fluctuations in 1D:
Song, Rachel, and Le Hur, PRB **82**, 012405 (2010)

Full 3-leg, 1/3-filling phase diagram (DMRG and VMC)



Rung Mott = conventional 1D Mott insulator:



Extensions of DBL[3,0] to 2D

- Relative (nephew?) of "extremal DLBL" in 2D case
- Quasi-1D
  - DBL[N,0] on the N-leg ladder at $\nu = m/N$?
- Quasi-2D
  - Gapless Bose insulator on an N-*layered* system at filling $\nu = m/N$?

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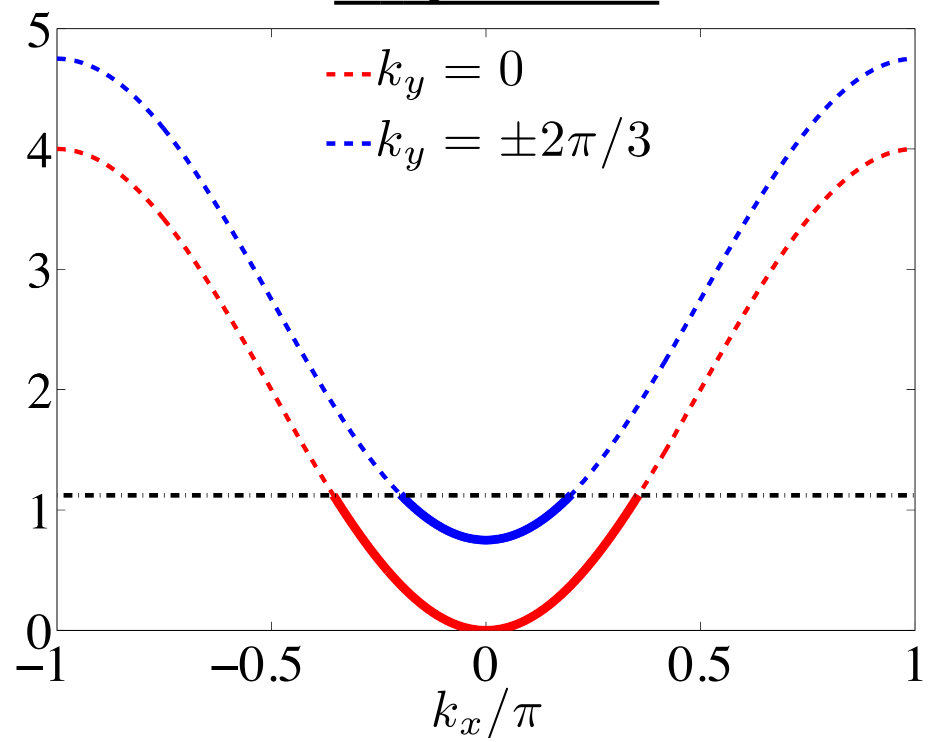
Evidence for *d*-wave Bose *metals* on 3 legs?

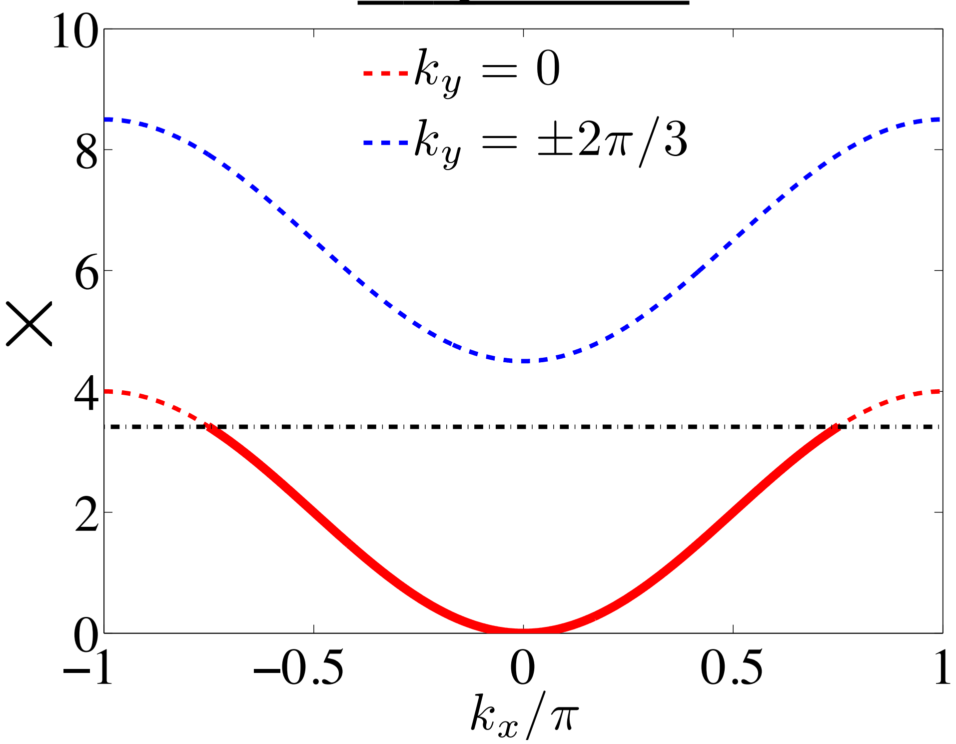
- Yes … DBL[3,1] for $\nu < 1/3$ (hole-doped DBL[3,0]):

$$\mathrm{DBL}[3,1] = \mathcal{P}_G\Big(\quad d_1 \text{ parton} \quad \times \quad d_2 \text{ parton} \quad \Big)$$

<u>*d*$_1$ parton</u>



<u>*d*$_2$ parton</u>

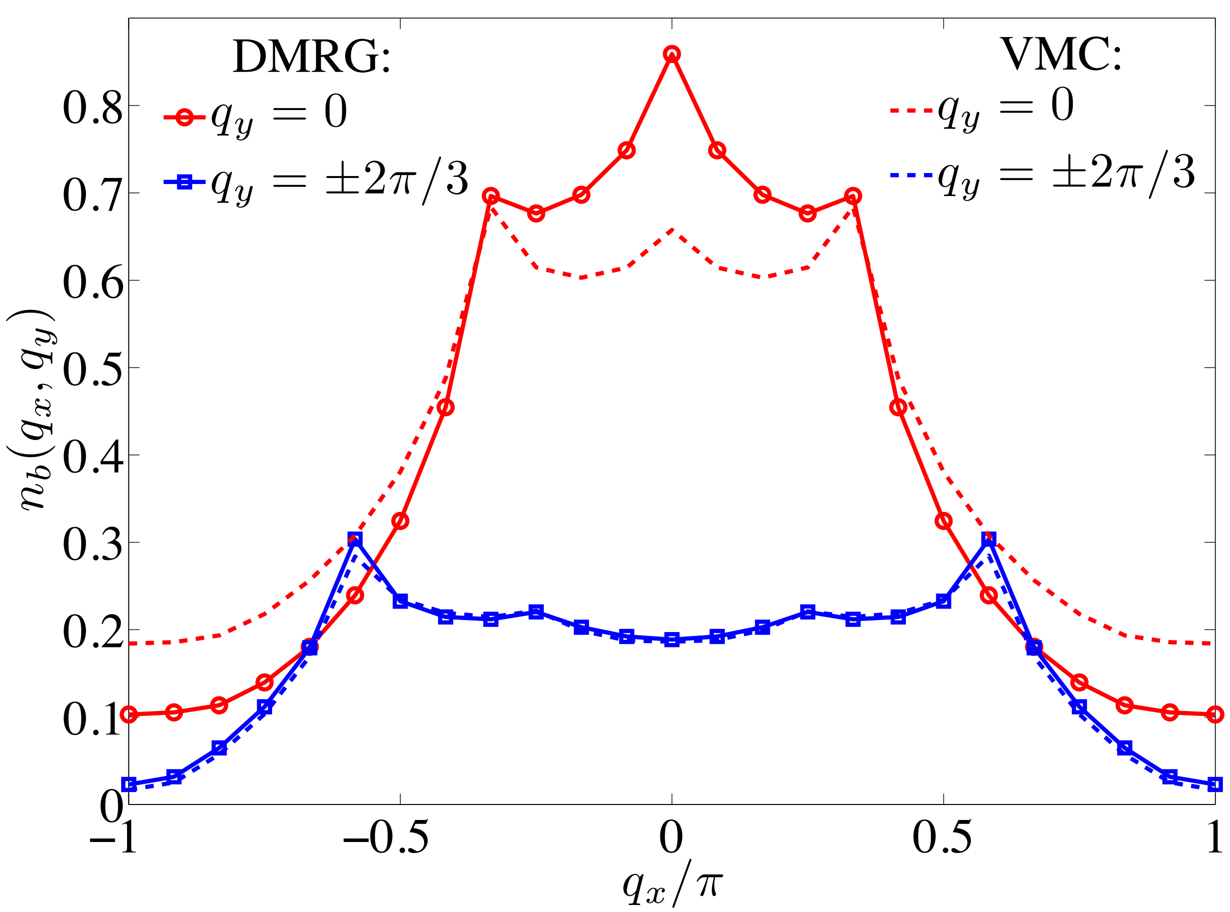


- But …
  - Simple extension of DBL[2,1] on 2 legs
  - d_2 acts like a Jordan-Wigner transformation
  - Only exists for $\nu < 1/3$ in rather small region of phase diagram

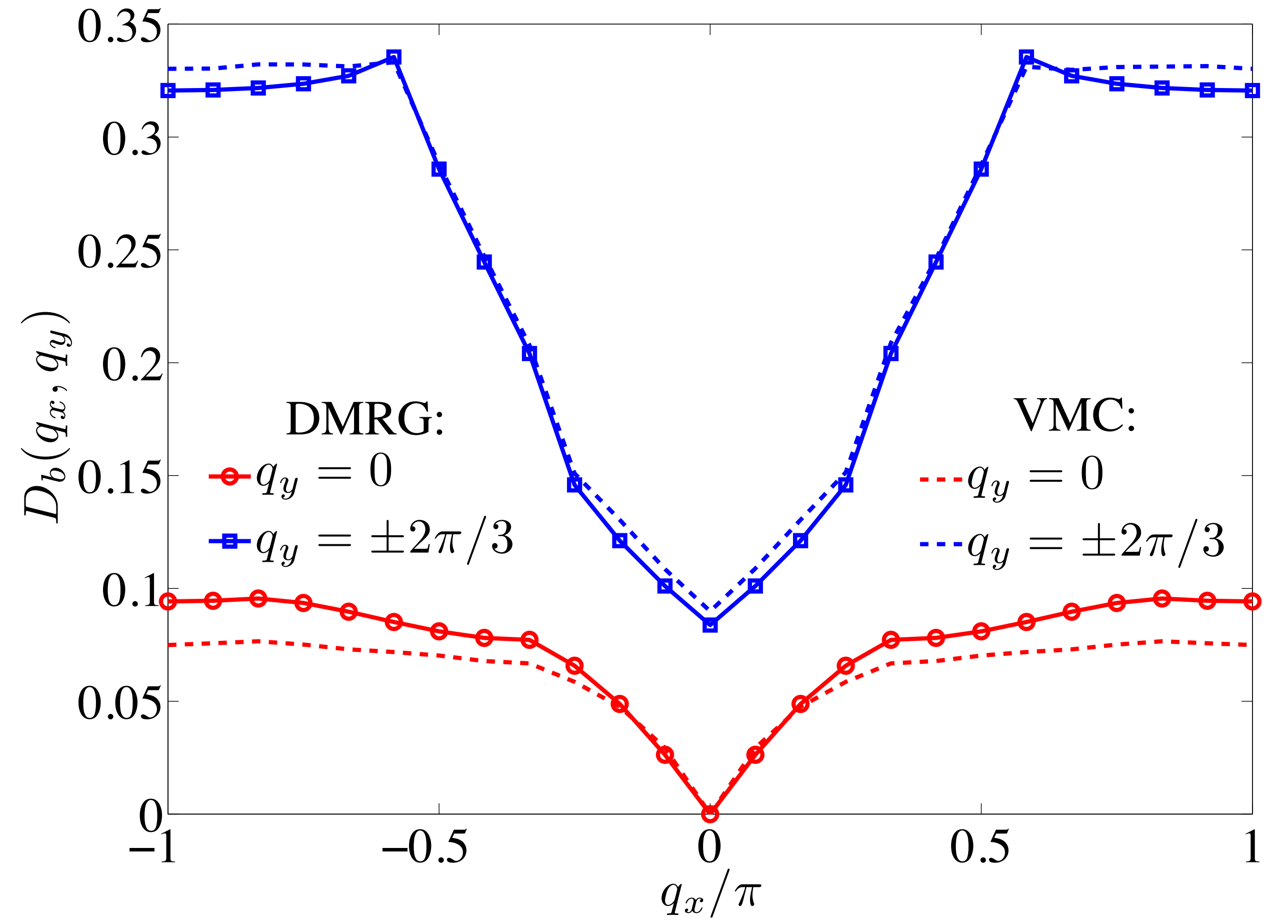
Still, impressive DMRG-VMC comparisons

$$\nu = 1/4: \;\; K/J = 2.25, J_\perp/J = 0.5$$

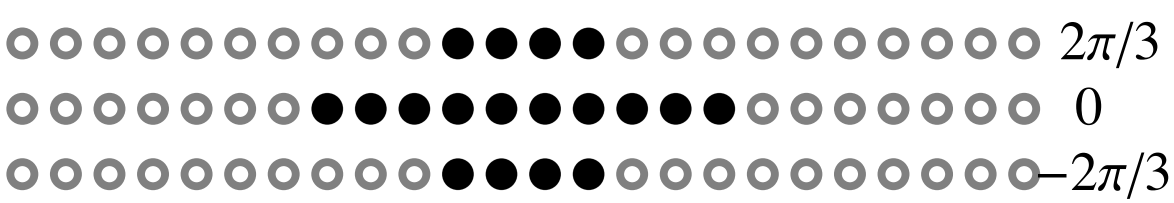
Boson momentum distribution



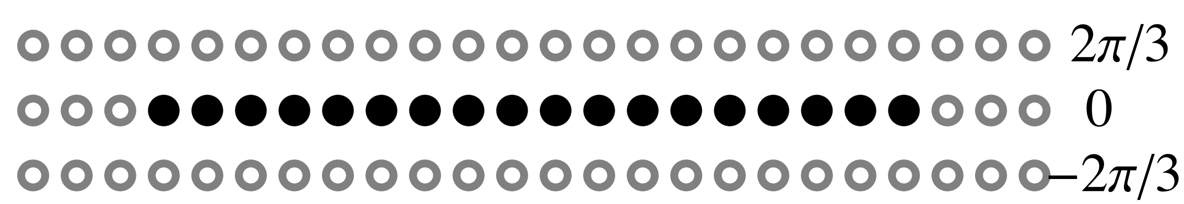
Density-density structure factor



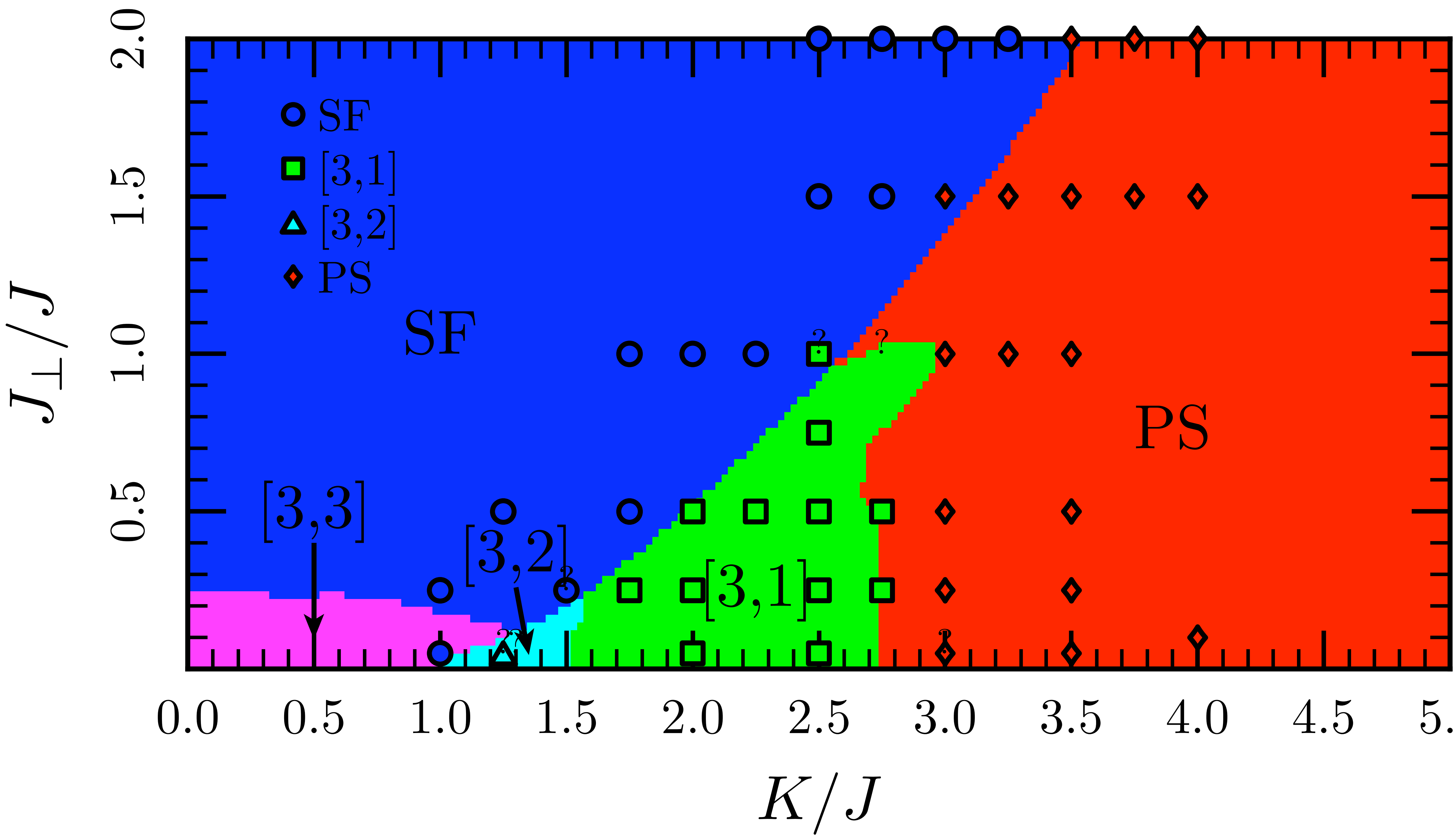
d_1 :



d_2 :



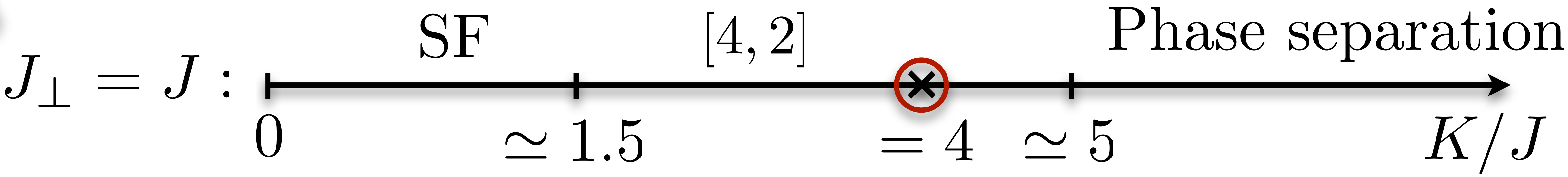
Full 3-leg, 1/4-filling phase diagram (DMRG and VMC)



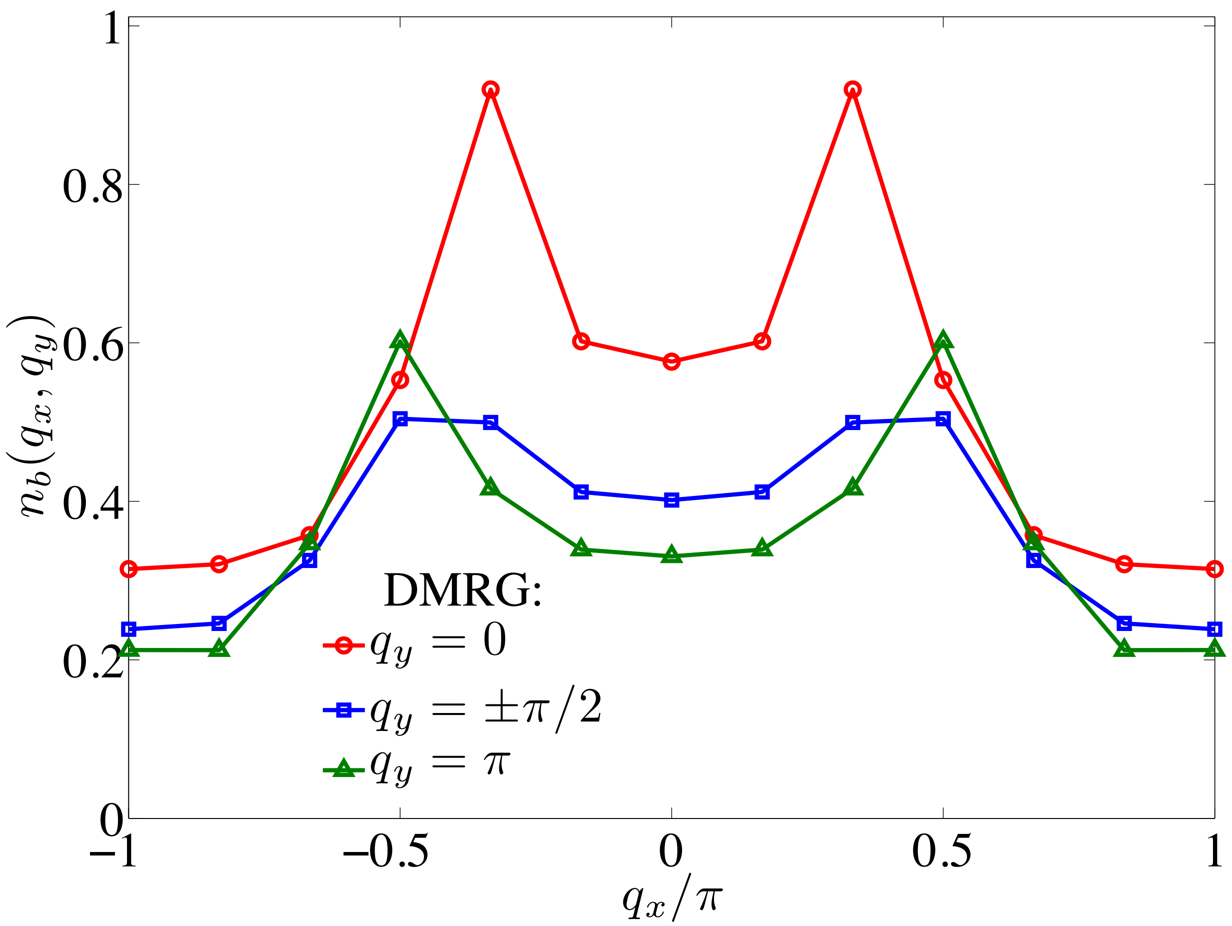
What about $\nu > 1/3$ on 3 legs?

- DBL[3,2] seems to be unstable in *J*-*K* model
  - Very stable compressible, non-DBL phase at $\nu = 1/2$
    - Quasi-condensate at zero momentum?
    - Static order in rung currents at $\mathbf{q} = (\pm\pi, \pm 2\pi/3)$
  - Some evidence for another variant of DBL[3,1] at $\nu = 5/12$
    - d_2 has one *fully* filled band and one partially filled band
    - Only exists at small $J_\perp$
- Main goals for approaching 2D
  - Find stable metallic DBL[m, n] phase with $n > 1$
  - Find DBL phase with $c > L_y$
  - NB: DBL[3,1] has $n = 1$ and “only” $c = 3 = L_y$ gapless modes
  - On to 4 legs!

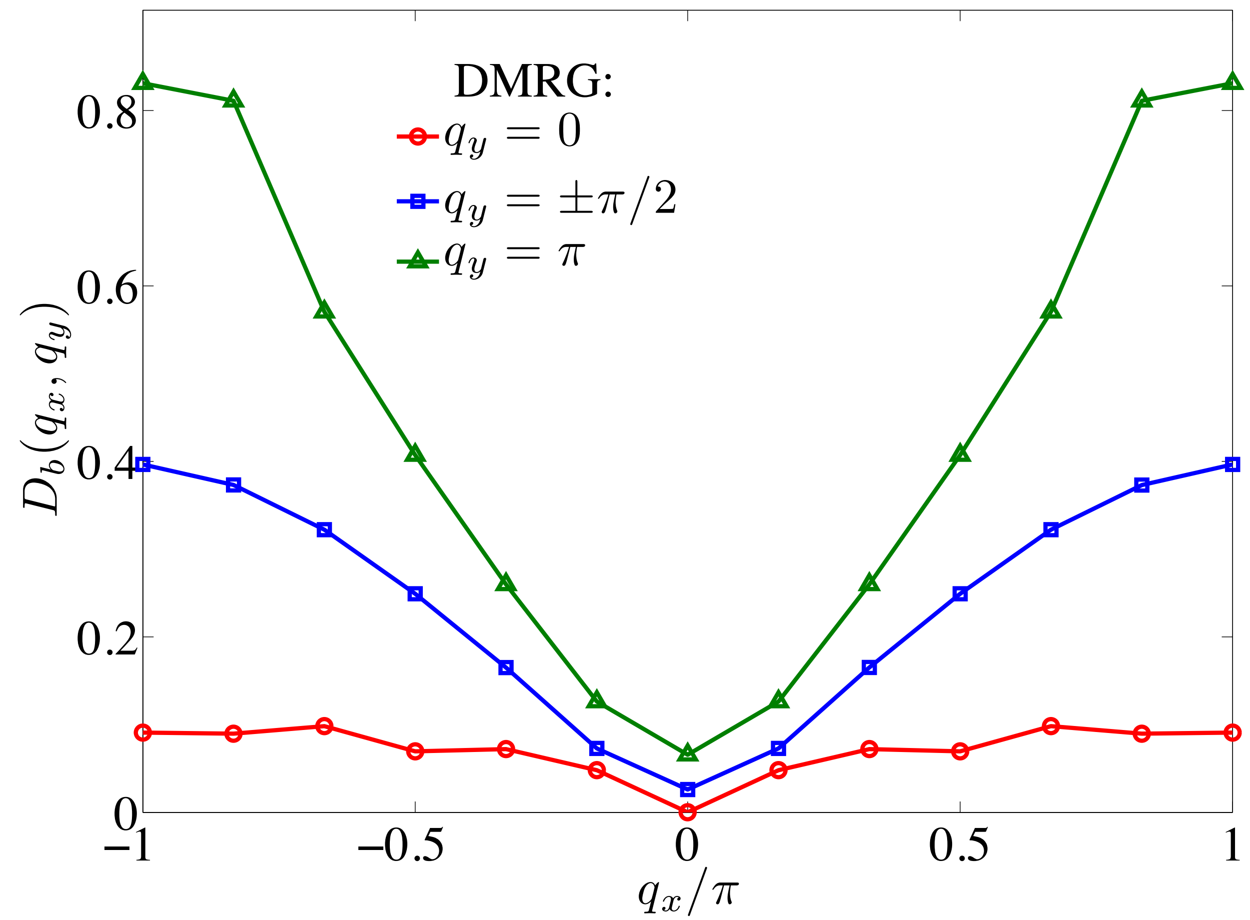
Encouraging evidence for DBL[4,2] on 4 legs, $\nu = 5/12$



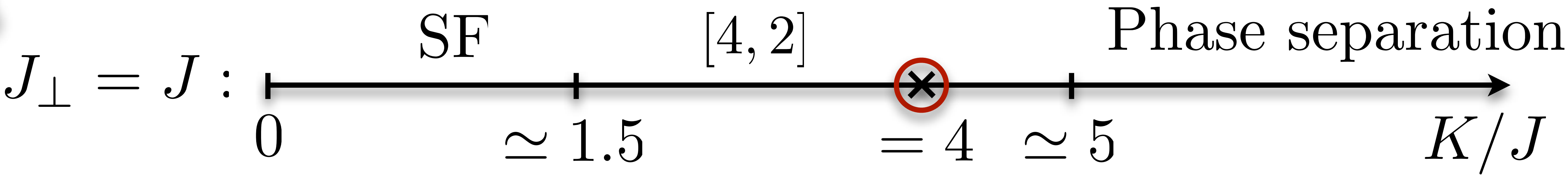
Boson momentum distribution



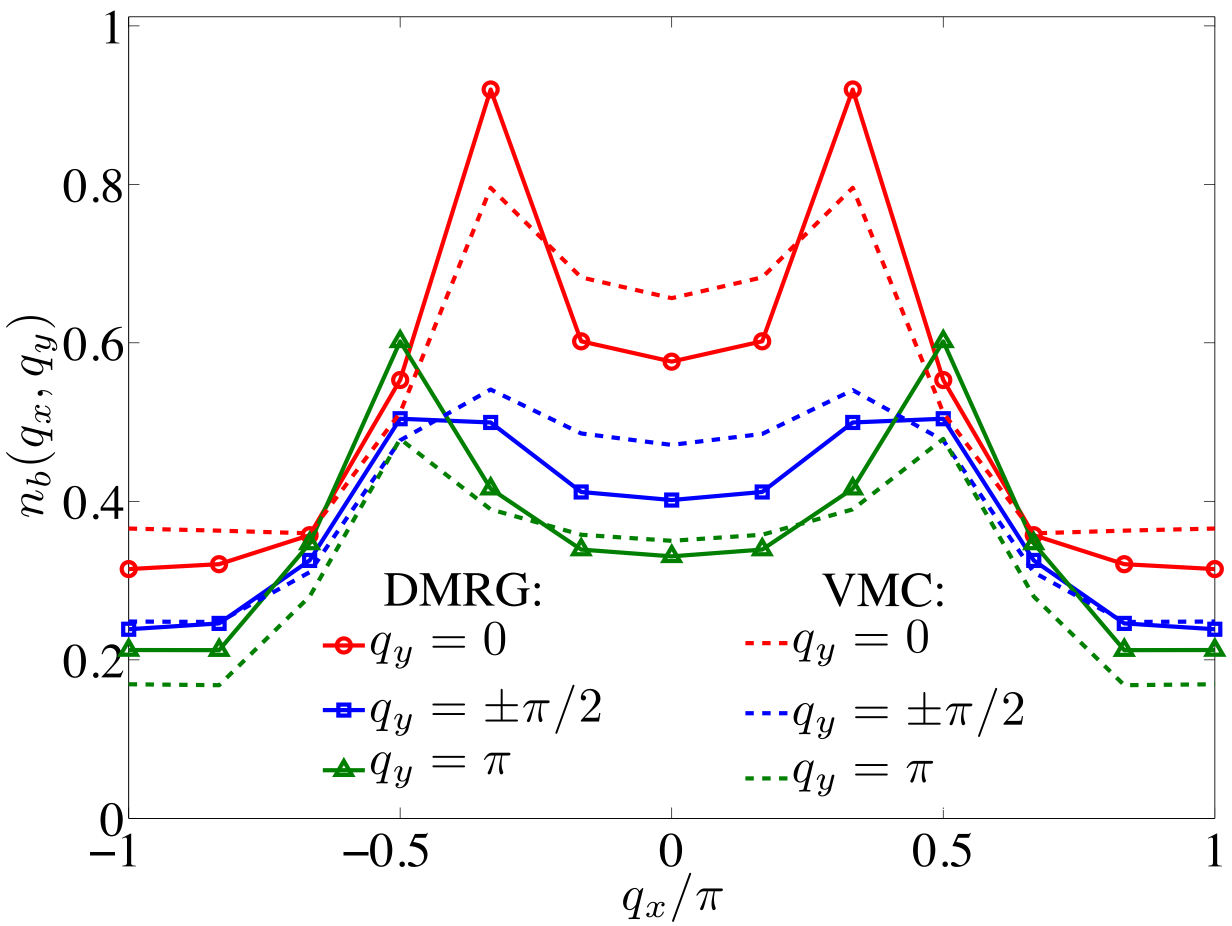
Density-density structure factor



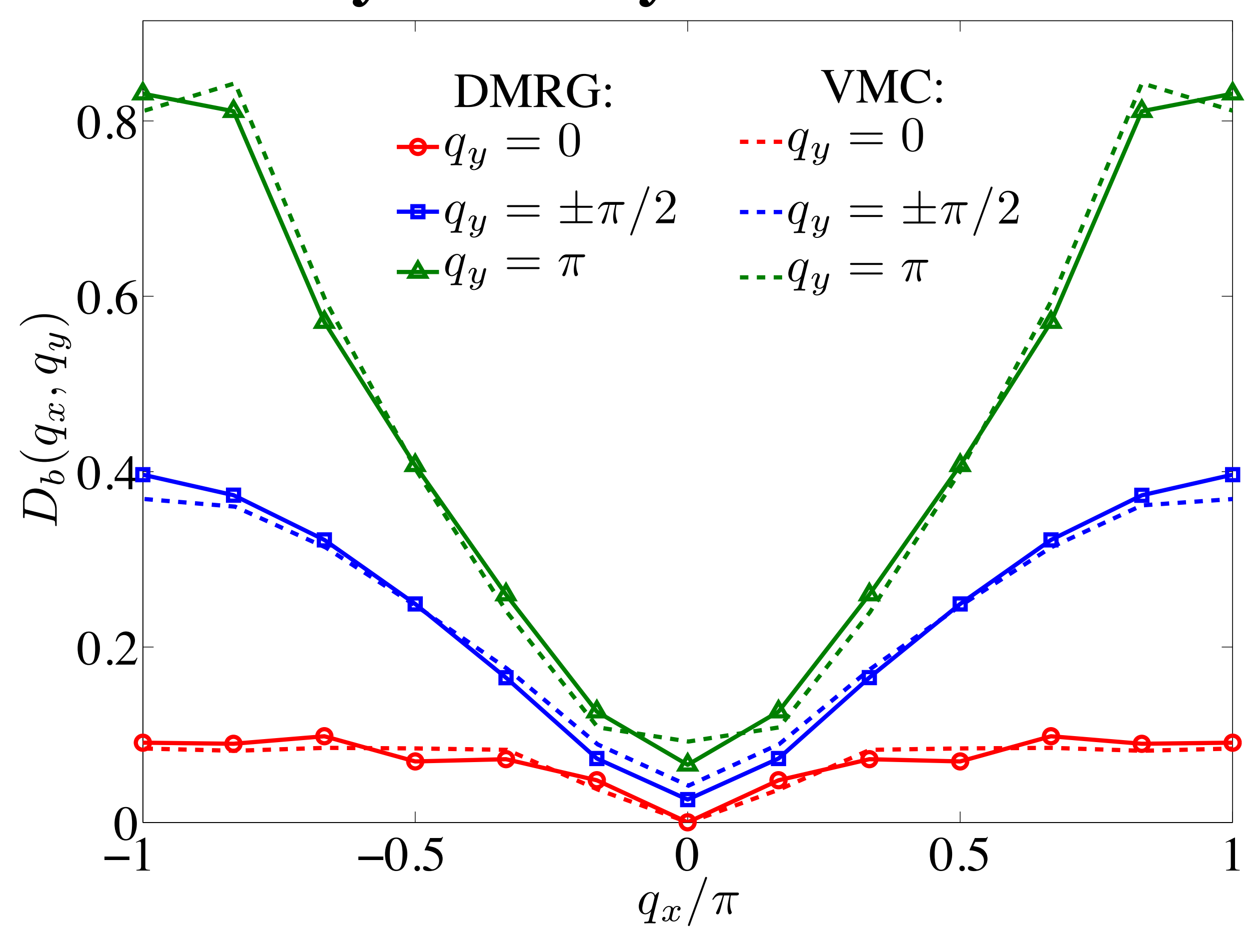
Encouraging evidence for DBL[4,2] on 4 legs, $\nu = 5/12$



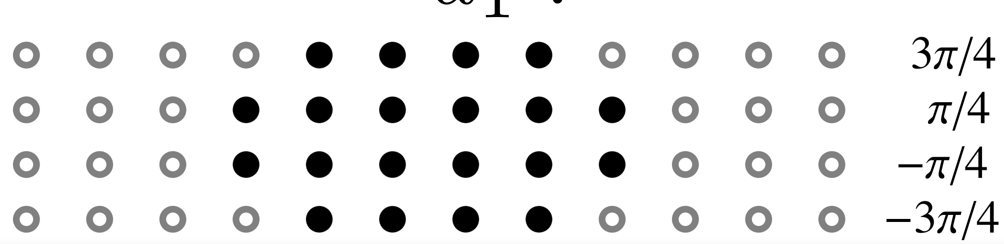
Boson momentum distribution



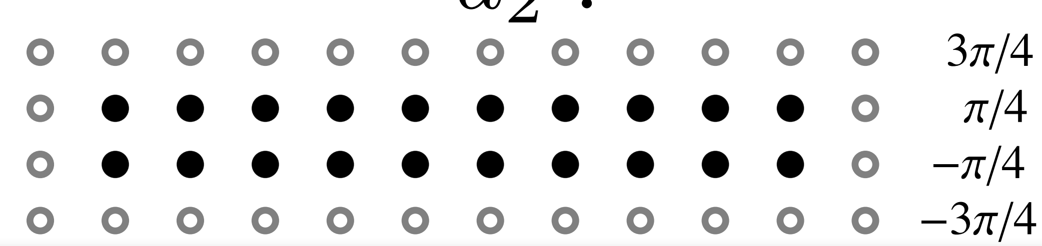
Density-density structure factor



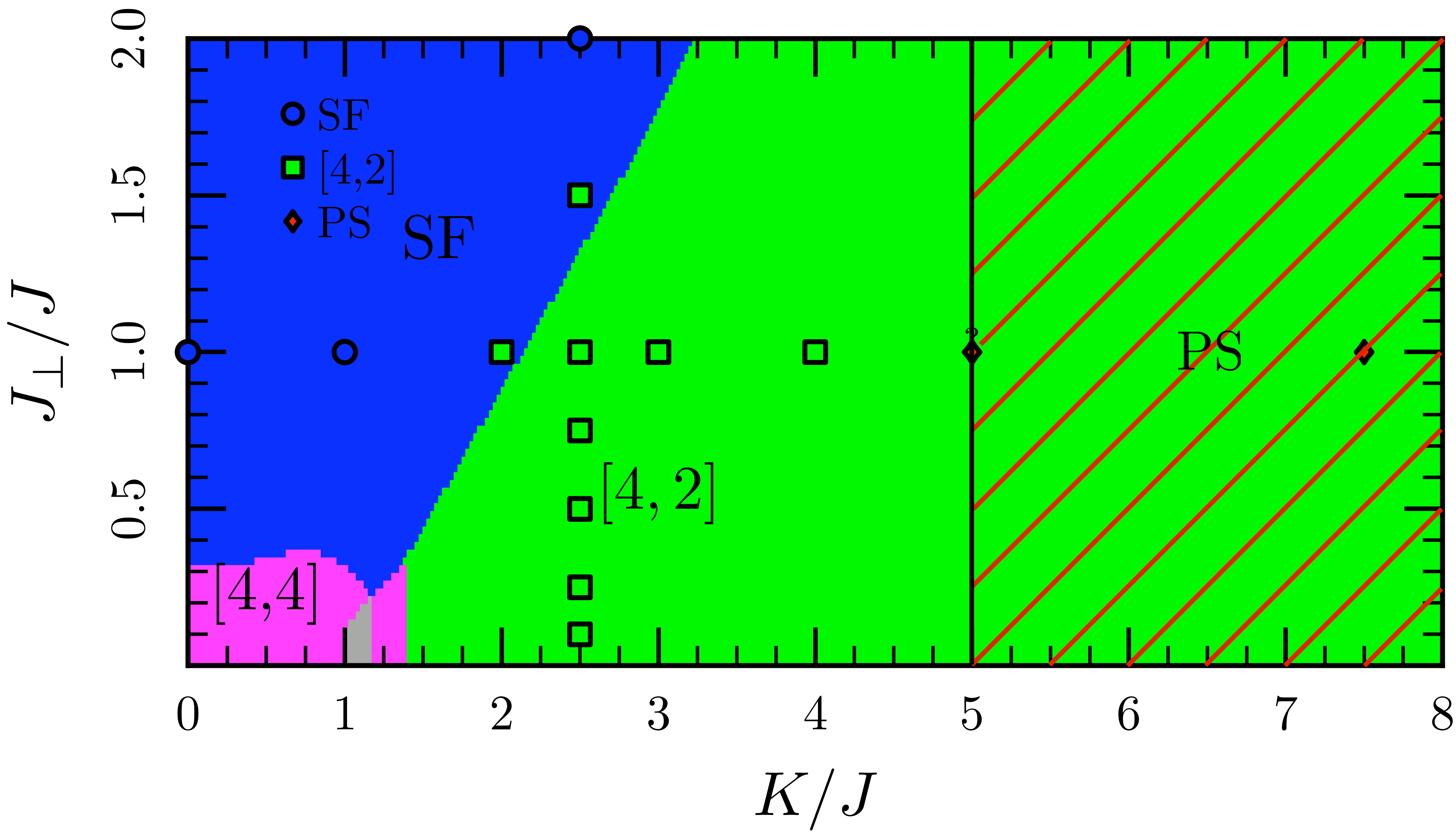
d_1 :



d_2 :



Preliminary 4-leg, 5/12-filling phase diagram



A few random remarks

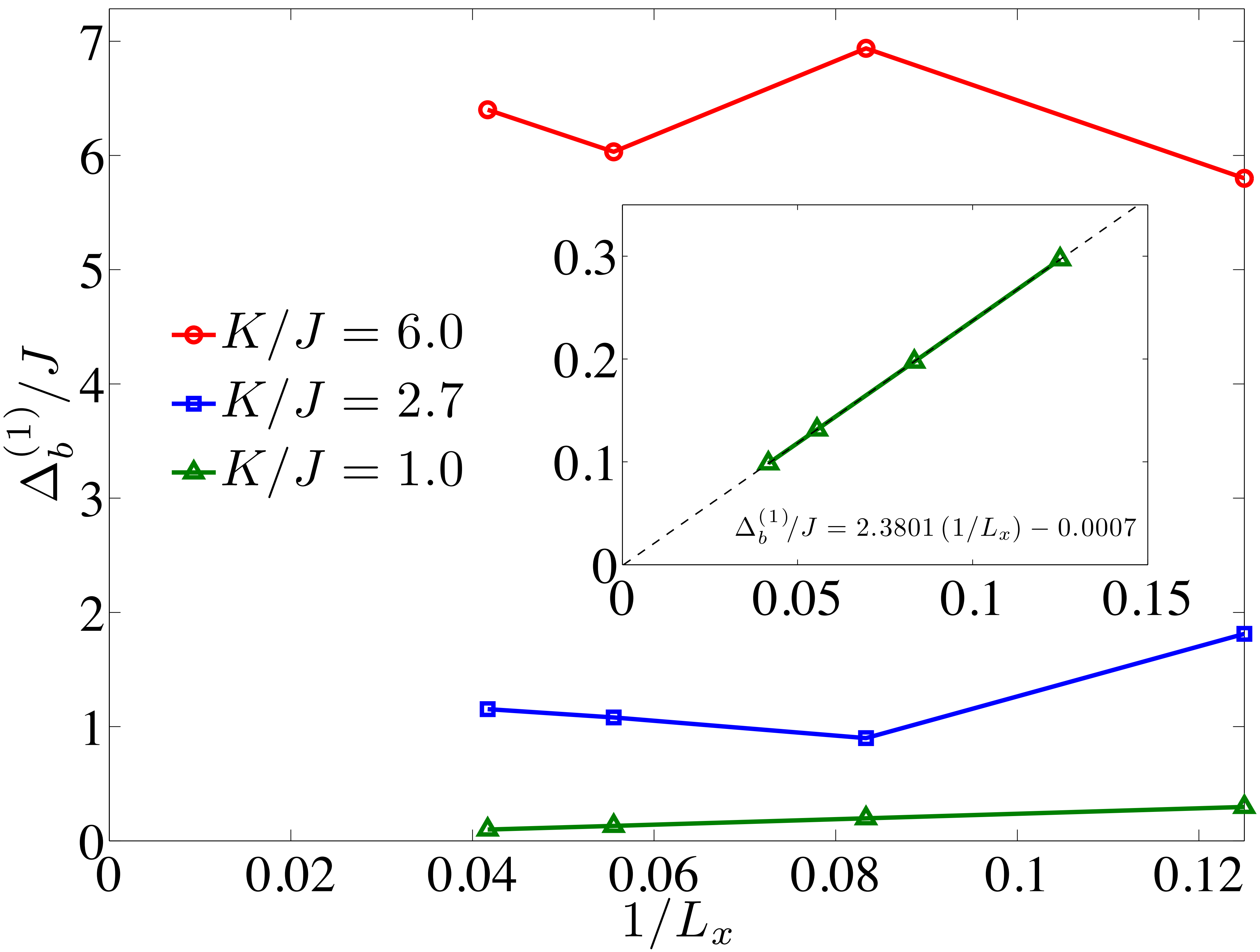
- No conventional Jordan-Wigner description of DBL[4,2]
- $c = 5 > L_y$ gapless modes in DBL[4,2]
  - Impossible to obtain via entanglement entropy in DMRG with PBC
  - May be doable with OBC
- DBL[3,2] on 3 legs with *anti-periodic* BCs in y direction?
  - From DBL[4,2] structure, (PBC for d_1) x (ABC for d_2) seems natural
- Gapless Mott insulator on 4 legs?
  - System phase separates right out of superfluid at $\nu = 1/4$ filling
    - DBL[3,0] special for $1/L_y$ gapless Mott insulators
  - Situation at $\nu = 1/2$ on 4 legs still unclear

Summary

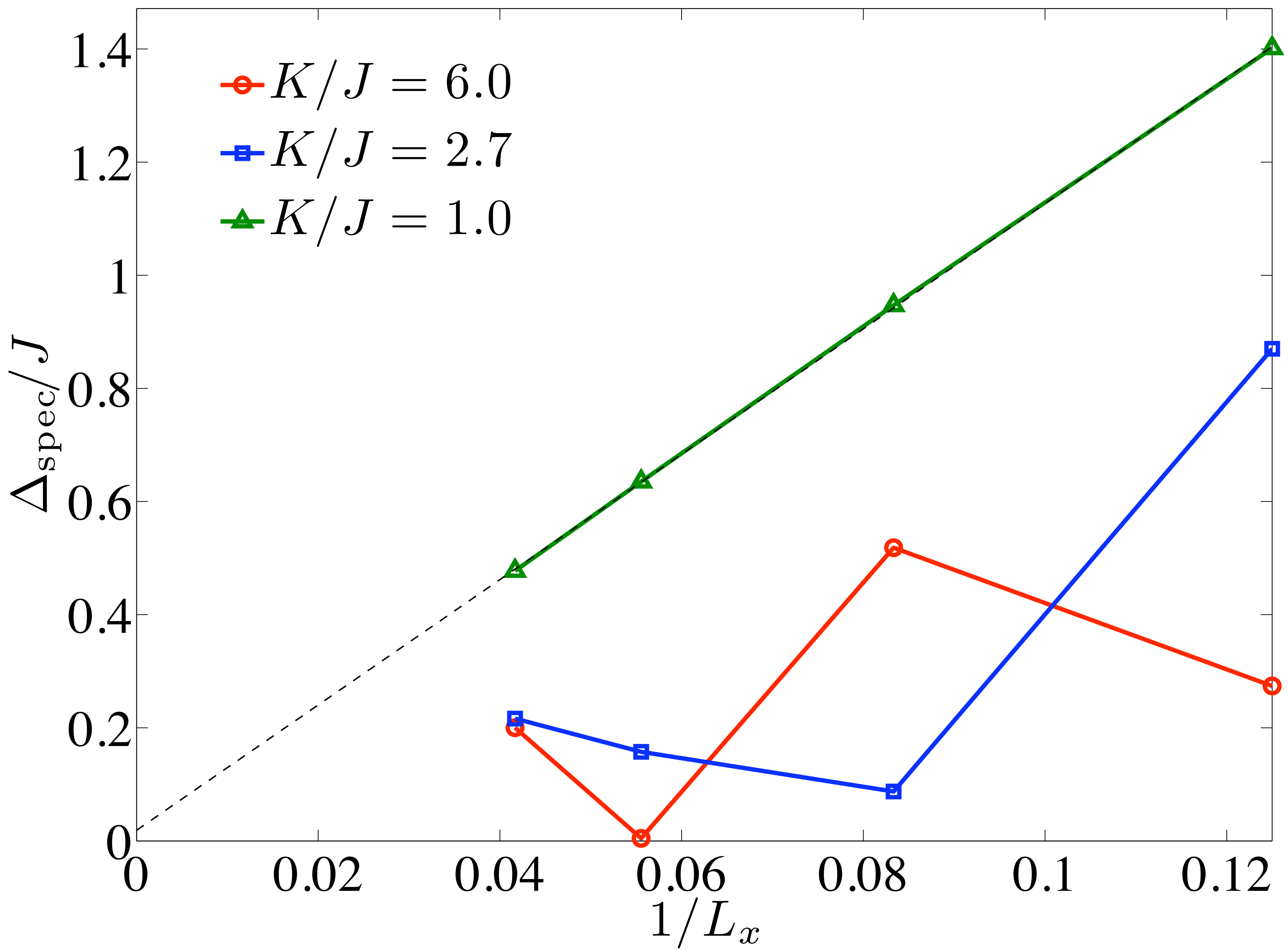
- Study of the frustrated *J*-*K* model on the 3- and 4-leg ladder
  - Pushing towards 2D
  - Main tools = DMRG and VMC
- Gapless Mott insulator on the 3-leg ladder
  - Incompressible phase
  - Power law density-density correlations at incommensurate wave vectors
  - Fundamentally quasi-1D phase with 2 gapless modes
- Gapless Bose metals on 3- and 4-leg ladders
  - DBL[3,1] phase on 3 legs, but no DBL[3,2]
  - Evidence for DBL[4,2] phase on 4 legs
    - More gapless modes than number of legs
    - Encouraging for existence of the 2D *d*-wave Bose metal

That’s all!

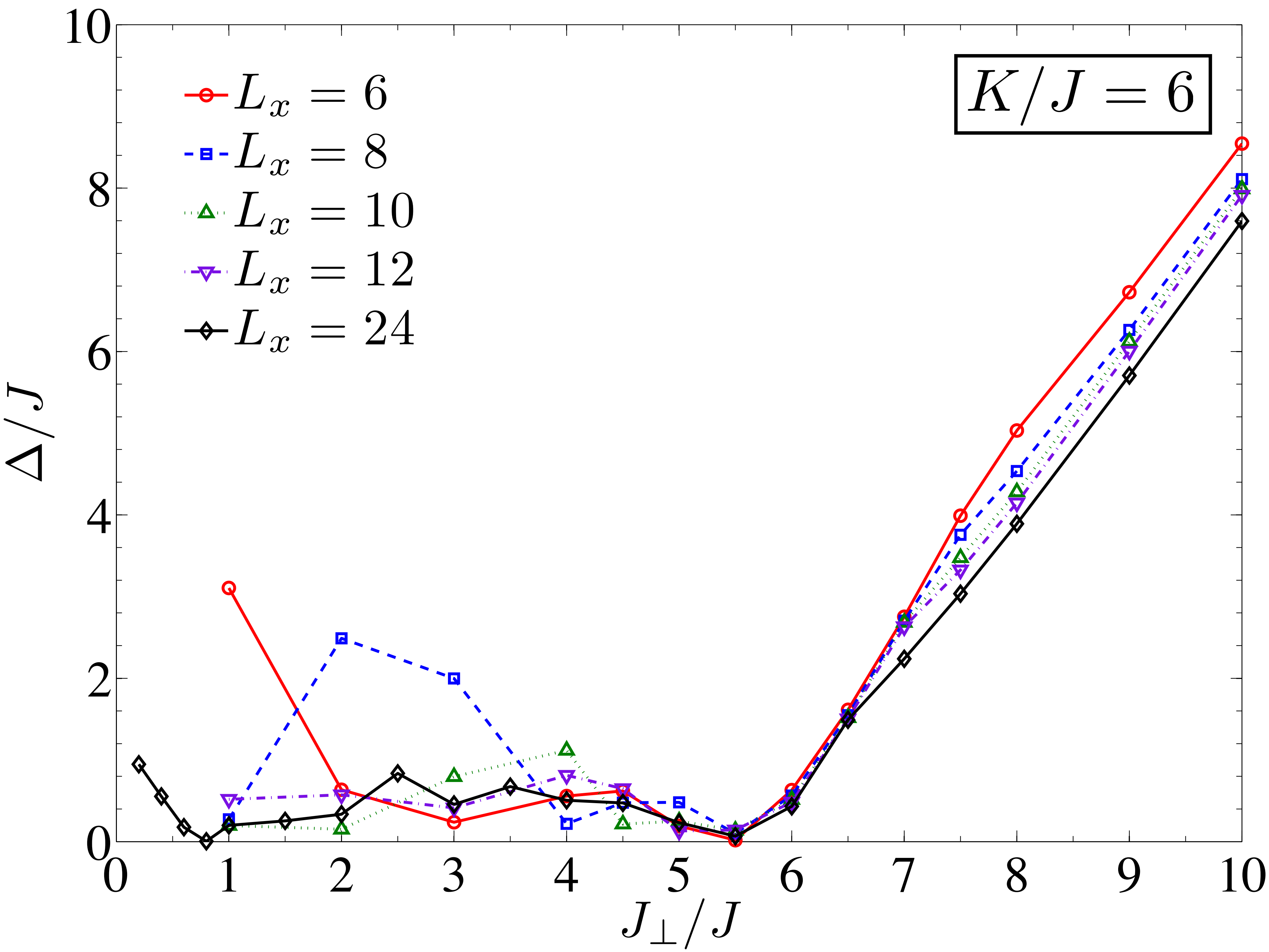
One-boson gap: [3,0] and SF



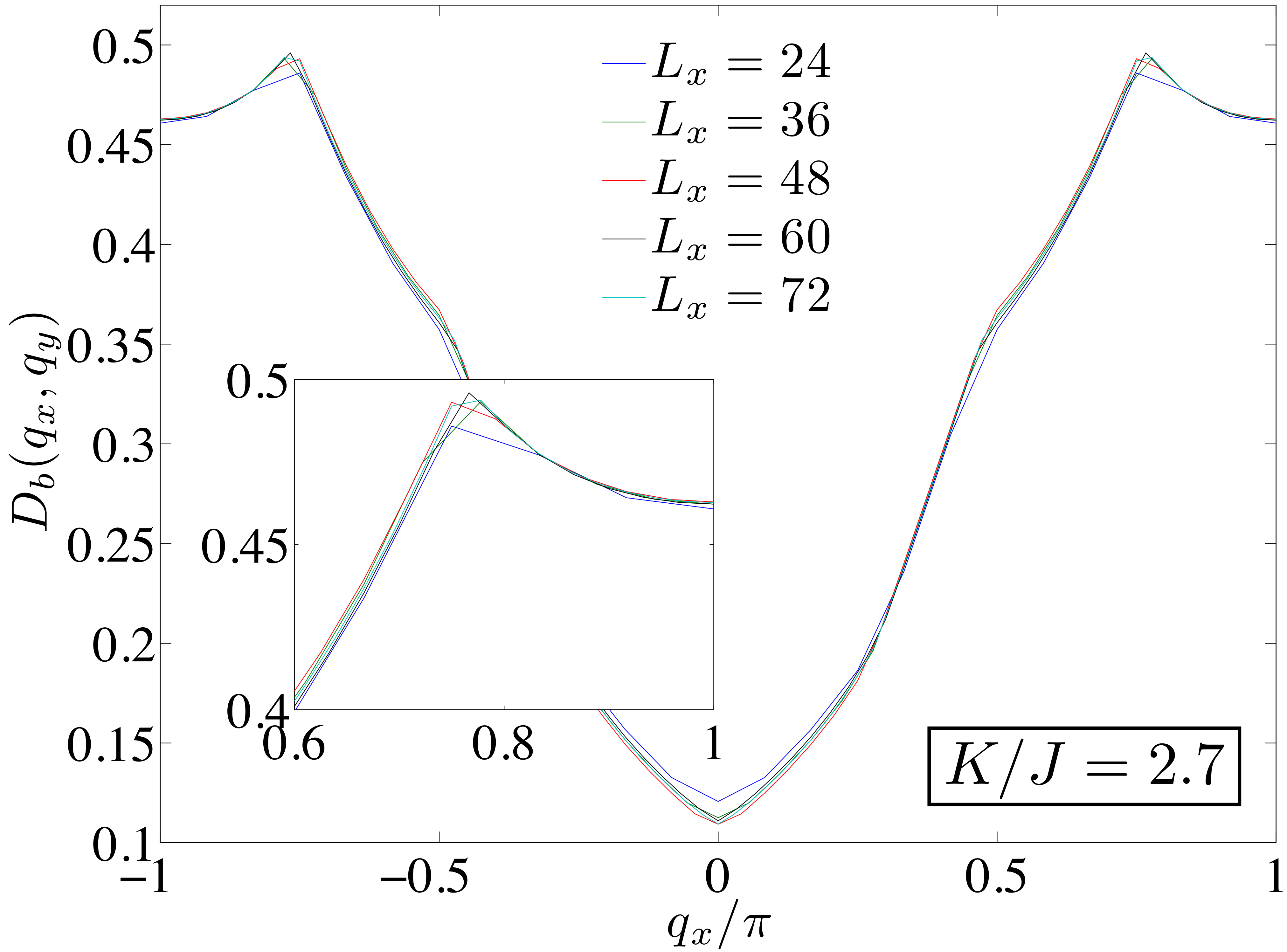
Spectral gap at fixed boson number: [3,0] and SF



Spectral gap across [3,0] to [0,0] transition

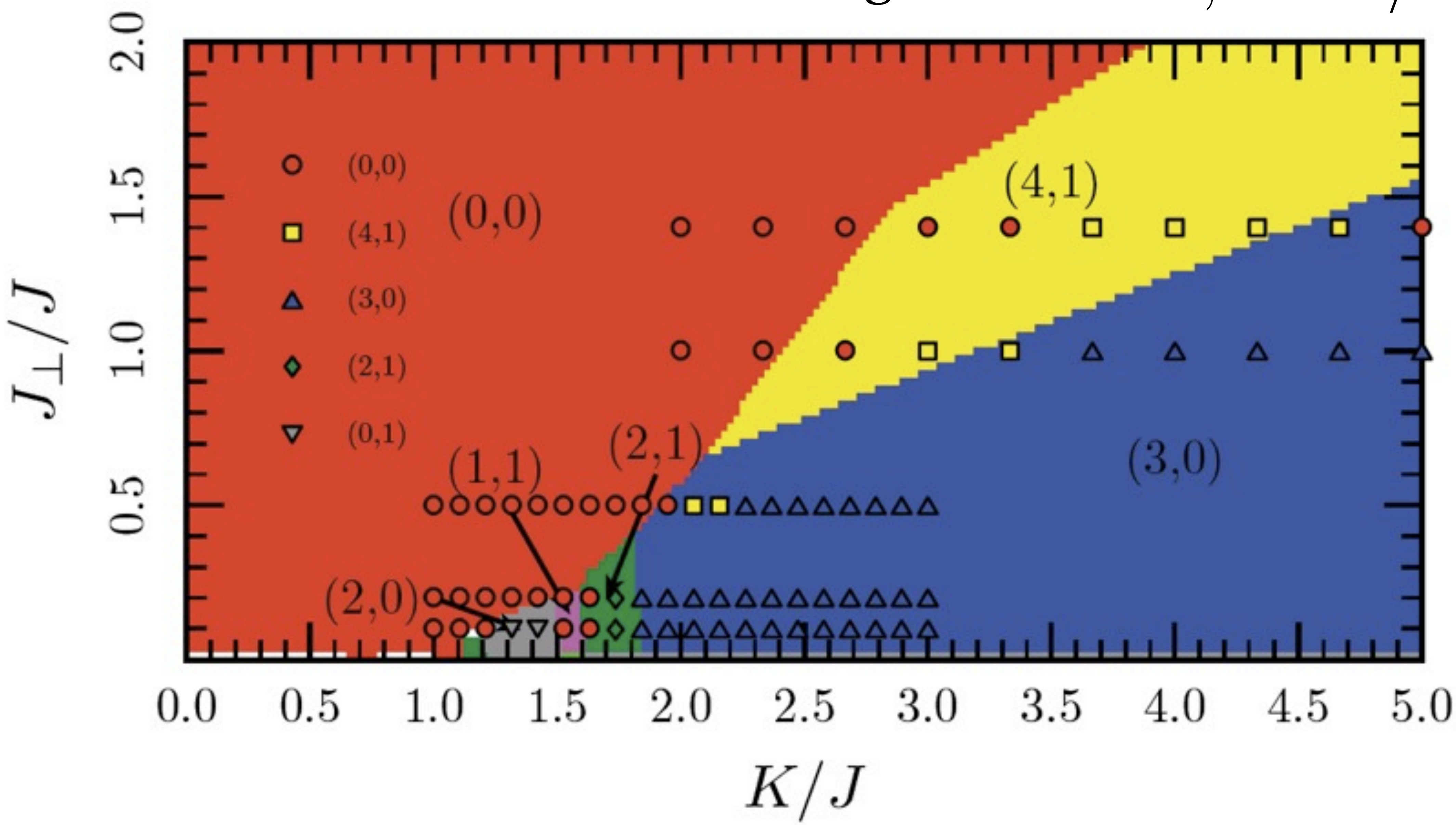


Dependence of [3,0]'s Bose surfaces on L_x (OBC)



Ground state momenta: VMC vs. ED

<u>Ground state momentum diagram:</u> $3 \times 10,\ \nu = 1/3$



$$(n, m):\ \text{ground state momentum} = (q_x, q_y) = \left(n\frac{2\pi}{L_x}, m\frac{2\pi}{L_y}\right)$$

Precise definitions of measures

- Boson momentum distribution
  - $n_b(\mathbf{q}) \equiv \frac{1}{L_x L_y} \sum_{\mathbf{r},\mathbf{r}'} \langle \hat{b}^\dagger_{\mathbf{r}} \hat{b}_{\mathbf{r}'} \rangle e^{i\mathbf{q}\cdot(\mathbf{r}-\mathbf{r}')}$
  - In DBL, singularities at $k_{F1}^{(k_y)} \pm k_{F2}^{(k_y')}$
- Density-density structure factor
  - $D_b(\mathbf{q}) \equiv \frac{1}{L_x L_y} \sum_{\mathbf{r},\mathbf{r}'} \langle (\hat{n}_{\mathbf{r}} - \nu)(\hat{n}_{\mathbf{r}'} - \nu) \rangle e^{i\mathbf{q}\cdot(\mathbf{r}-\mathbf{r}')}$
  - In DBL, singularities at "$2k_F$" : $k_{F\alpha}^{(k_y)} \pm k_{F\alpha}^{(k_y')}$
- von Neumann entanglement entropy
  - $S^{\mathrm{vN}}(x) \equiv -\mathrm{Tr}\left(\hat{\rho}_A \ln \hat{\rho}_A\right)$