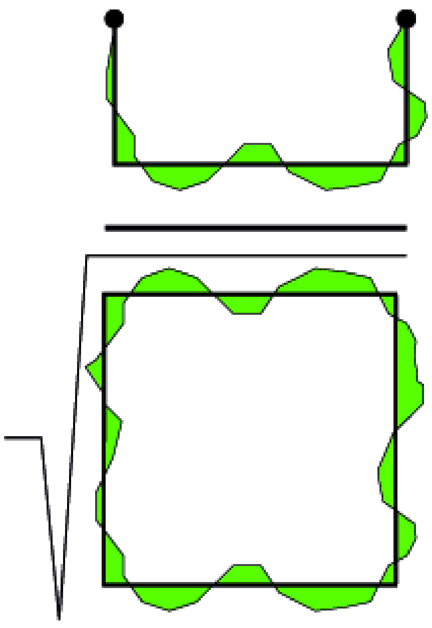
On intrinsic and emergent gauge structures: from irrational charge to deconfinement diagnostics



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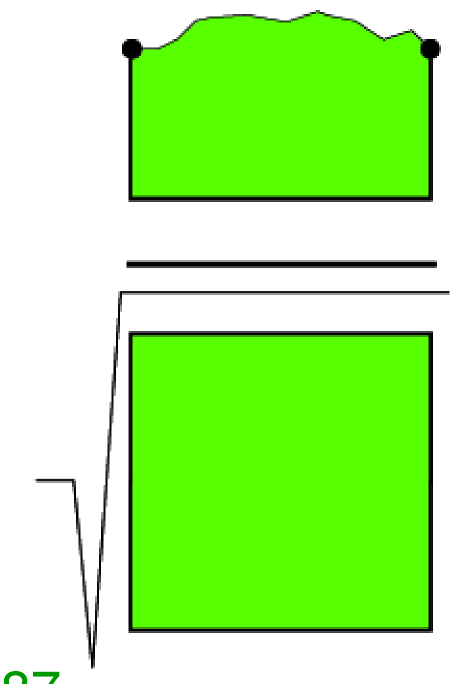


arXiv:1004.2154,1011.4187

Outline

Introduction

- emergent gauge structures and spin liquids

Intrinsic and emergent gauge charges

- fractional, and irrational, charge

Diagnosing deconfinement

- equal-time diagnostice (‘non-local order parameter’)
- application to some known instances

Exotic phases with emergent gauge fields

No definition (necessary and sufficient criteria) available which covers all cases of interest and is reasonable

- low-energy physics: emergent weakly fluctuating gauge field

gapped spin liquids

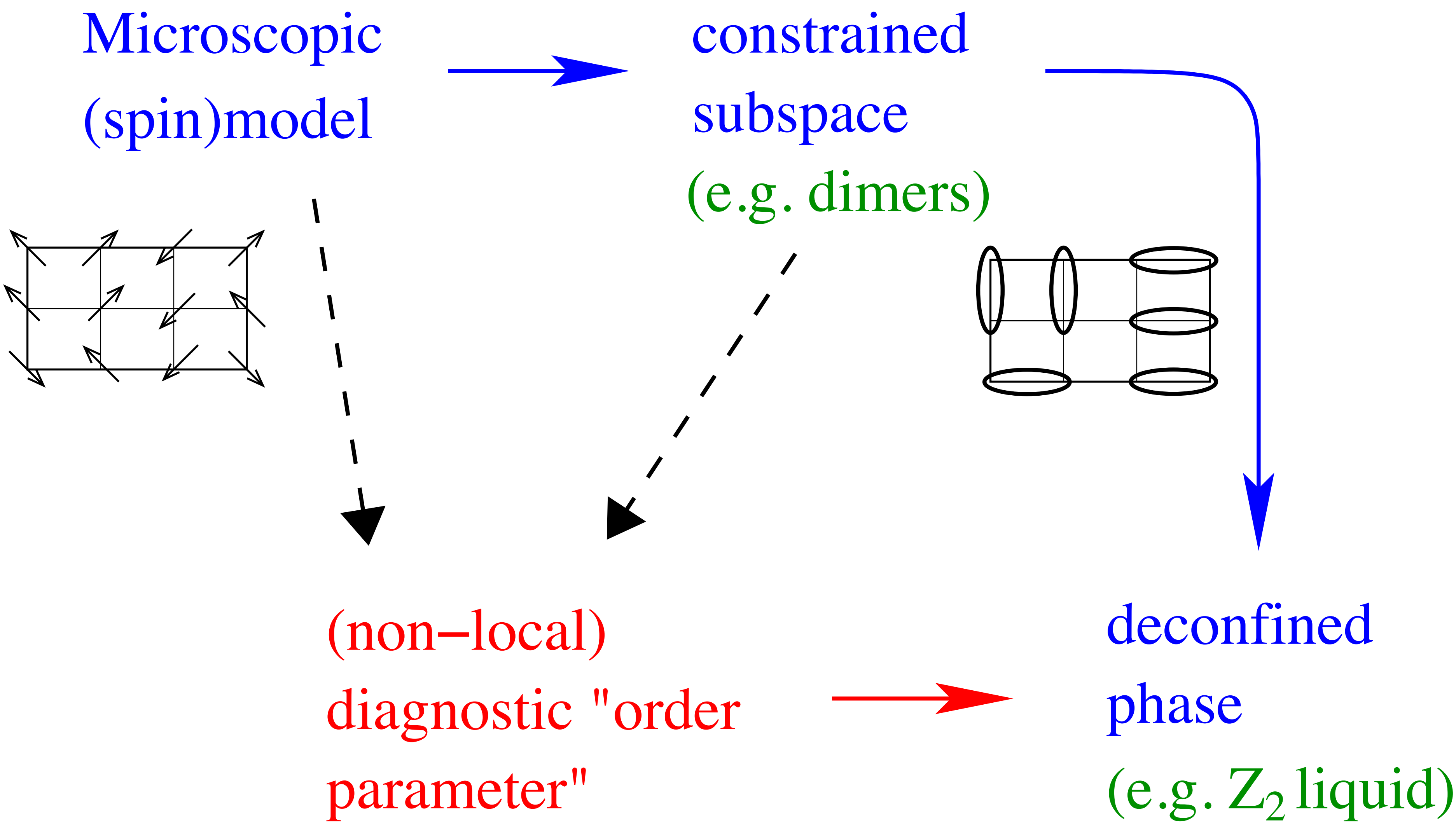
gapless spin liquids

spin ice

quantum dimer models

quantum Hall effect

Wishlist



Microscopic
(spin)model
constrained
subspace
(e.g. dimers)
(non–local)
diagnostic "order
parameter"
deconfined
phase
(e.g. Z_2 liquid)

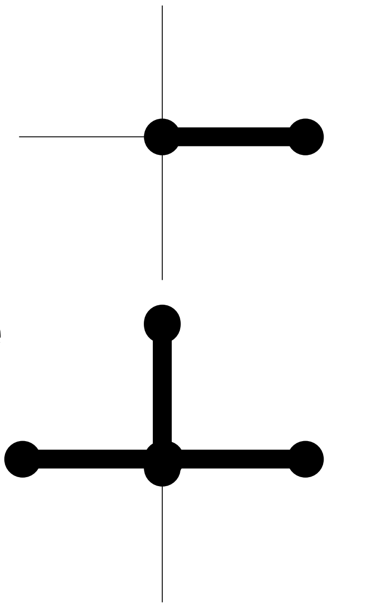
Where emergent gauge fields can appear

Constraints on energetics or Hilbert space

- ‘exclusive’ singlet formation $\Rightarrow$ hardcore dimer constraint
- double-occupancy constraint
- slave-particle constraint: $f^\dagger f + b^\dagger b = 1$

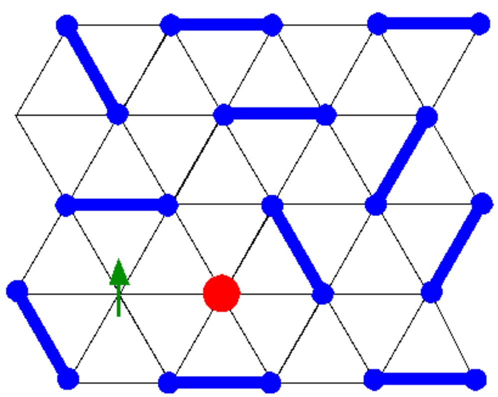
Constraints can give rise to Gauss’ law / gauge transformations

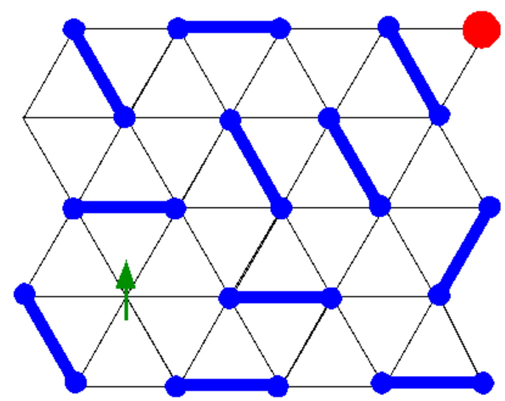
- in some cases, explicit gauge construction based on lattice model available
- often appears when considering fluctuations around mean-field saddle point



Emergent Z_2 gauge theory: the quantum dimer model

emergent Z_2 gauge field + deconfined (topological) phase





- topological order and quantum number fractionalisation go hand in hand (e.g. spin charge separation)
- no local symmetry breaking

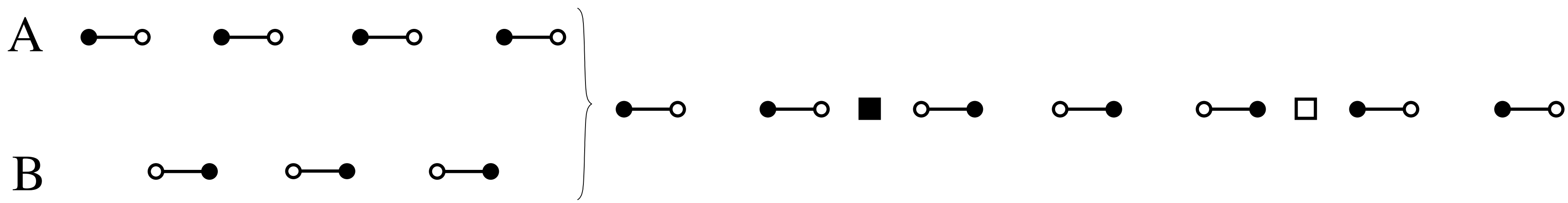
Existence of various gapped liquids well established

- gapped Z_2 liquid in $d \geq 2$; gapless U(1) liquid in $d \geq 3$

Reliable construction based on $SU(2)$ spins is messy

Quantum number fractionalisation : $d = 1$

solitons (polyacetylene) in dimerised chain Su,Schrieffer,Heeger



removing one electron creates two 'unpaired sites'

- pair can be separated at finite energy cost ("deconfinement")
- in presence of sublattice symmetry, each soliton has charge

$$Q = e/2$$

Quantum number fractionalisation : $d = 1$

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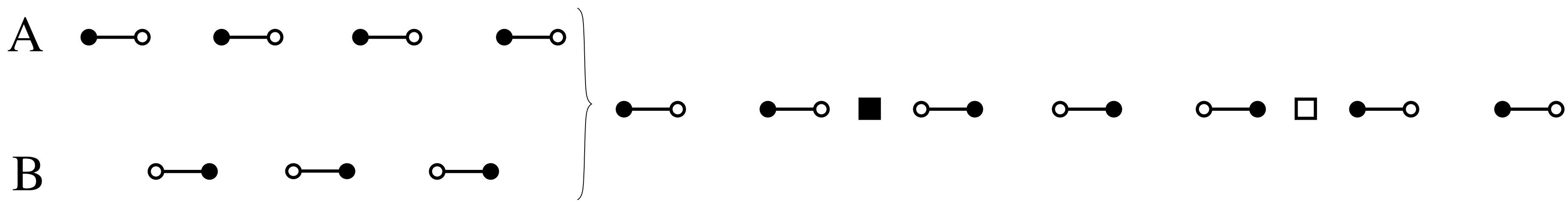

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$$Q = e/2$$

- Without sublattice symmetry, know only: $Q_A + Q_B = e$
  Brazovskii; Rice,Mele

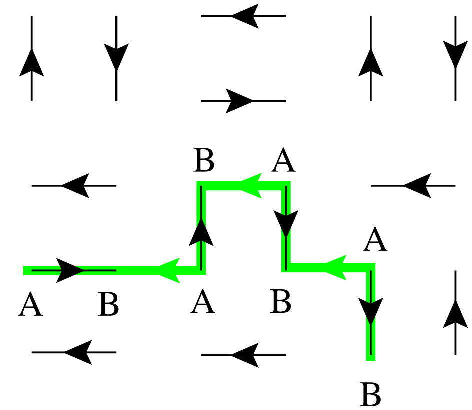
$\Rightarrow$ $Q_{A,B}$ can be irrational

Mechanism: inverting strings of (oriented!) dipoles

$$V(r) = \frac{P}{a}\int_\Lambda d\vec{r'} \cdot \vec{\nabla}\frac{1}{|r-r'|} = Q(\frac{1}{|r-r_a|} - \frac{1}{|r-r_b|})$$

Potential due to a string of dipoles

- same as two charges at end of string
- $Q = P/a =$ moment per unit length
- reversing string of dipoles creates (tunable) charges



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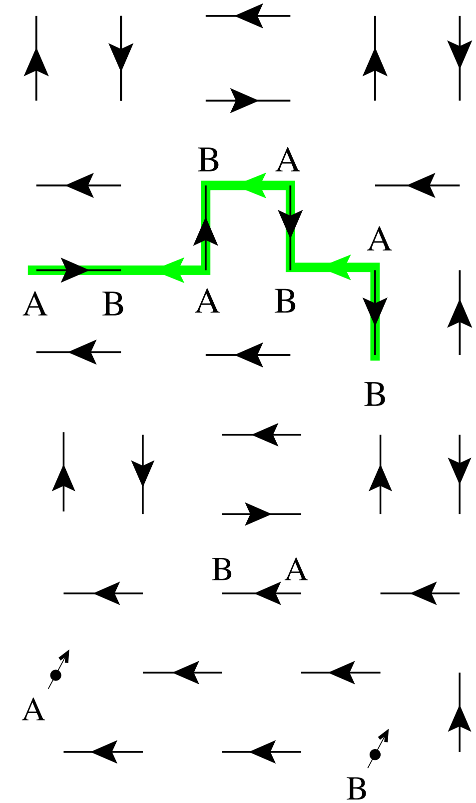
Potential due to a string of dipoles

- same as two charges at end of string
- $Q = P/a =$ moment per unit length
- reversing string of dipoles creates (tunable) charges
- works both for $d = 1$ and $d \geq 2$
- Examples: several models; (spin) ice



Emergent vs. intrinsic charge

Intrinsic charge

- is almost accidental
- can be irrational, ie not a sharp quantum number
- dipoles not only mechanism, cf. magnetoelectric effect Zhang

Emergent charge

- for cases of irrational charge, have e.g. sublattice index
- more ‘fundamental’?

Uses as diagnostic

- Electric (intrinsic) charge not necessarily sharp

Diagnosing topological order/deconfinement

Diagnostic should satisfy:

- require knowledge of ground or Gibbs state only
- be independent of Hamiltonian
- work in presence of dynamical matter / at finite temperature

$\Longrightarrow$ Need non-local "order parameter":

- generalisation of Wilson loop   Fredenhagen + Marcu; Huse + Leibler
- measures effective 'line tension'

Apply to some known phase diagrams

Related work

Entangled entropy Levin-Wen;Kitaev-Preskill

- (subdominant) term signals topological nature of phase $(d = 2)$
- no simple interpretation in terms of correlations

Wilson loop "zero law" Hastings-Wen

- "undress" wavefunction to revert to ideal strong-coupling point
- construction depends on Hamiltonian

What's hard about diagnosing deconfinement?

No local order parameter Wegner

- Wilson loop area vs. perimeter law indicates phase transition

Example: (pure) Z_2 gauge theory

$$S_0 = K \sum_{\square} \sigma\sigma\sigma\sigma \Longrightarrow Z_0 \propto \mathrm{Tr}_{\{\sigma\}} \prod_{\square} [1 + (\tanh K)\sigma\sigma\sigma\sigma]$$

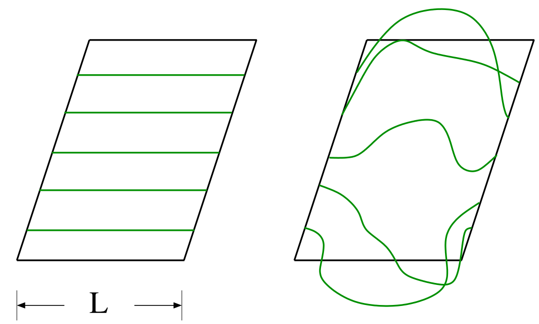
- theory of surfaces
- Wilson loop diagnoses absence/presence of surface tension

$$\langle W \rangle = \left\langle \prod_{\square} \sigma \right\rangle$$

Wilson loop, and theory of surfaces with edges

Area/perimeter law diagnose surface tension:

$$\langle W \rangle = \mathrm{Tr}_{\{\sigma\}} \left[W \prod_{\square} [1 + (\tanh K)\sigma\sigma\sigma\sigma] \right] / Z$$



Wilson loop, and theory of surfaces with edges

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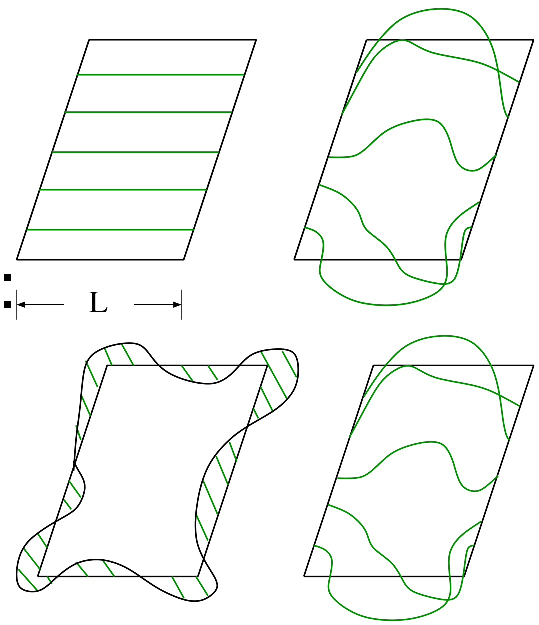

Not diagnostic when dynamical matter is added:

$$\begin{aligned} S_1 &= S_0 + J \sum \tau\sigma\tau \Longrightarrow \\ Z &\propto Z_0 \cdot \prod [1 + (\tanh J)\tau\sigma\tau] \end{aligned}$$

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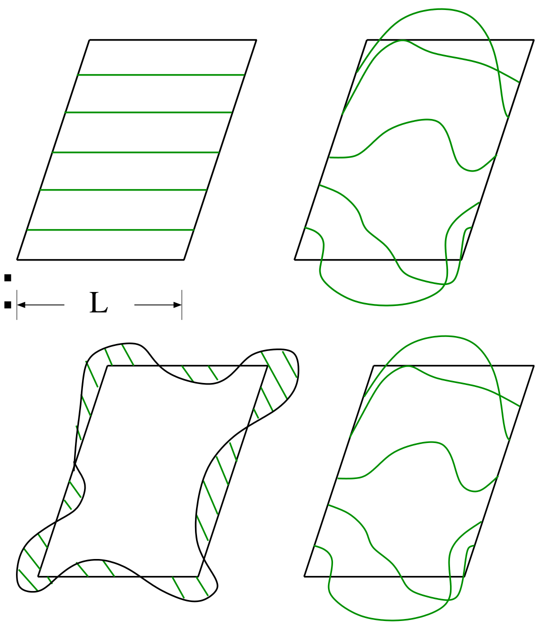

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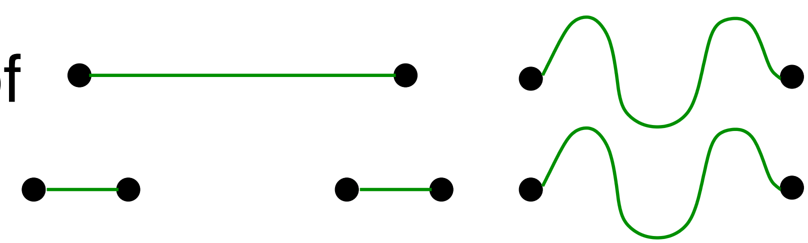
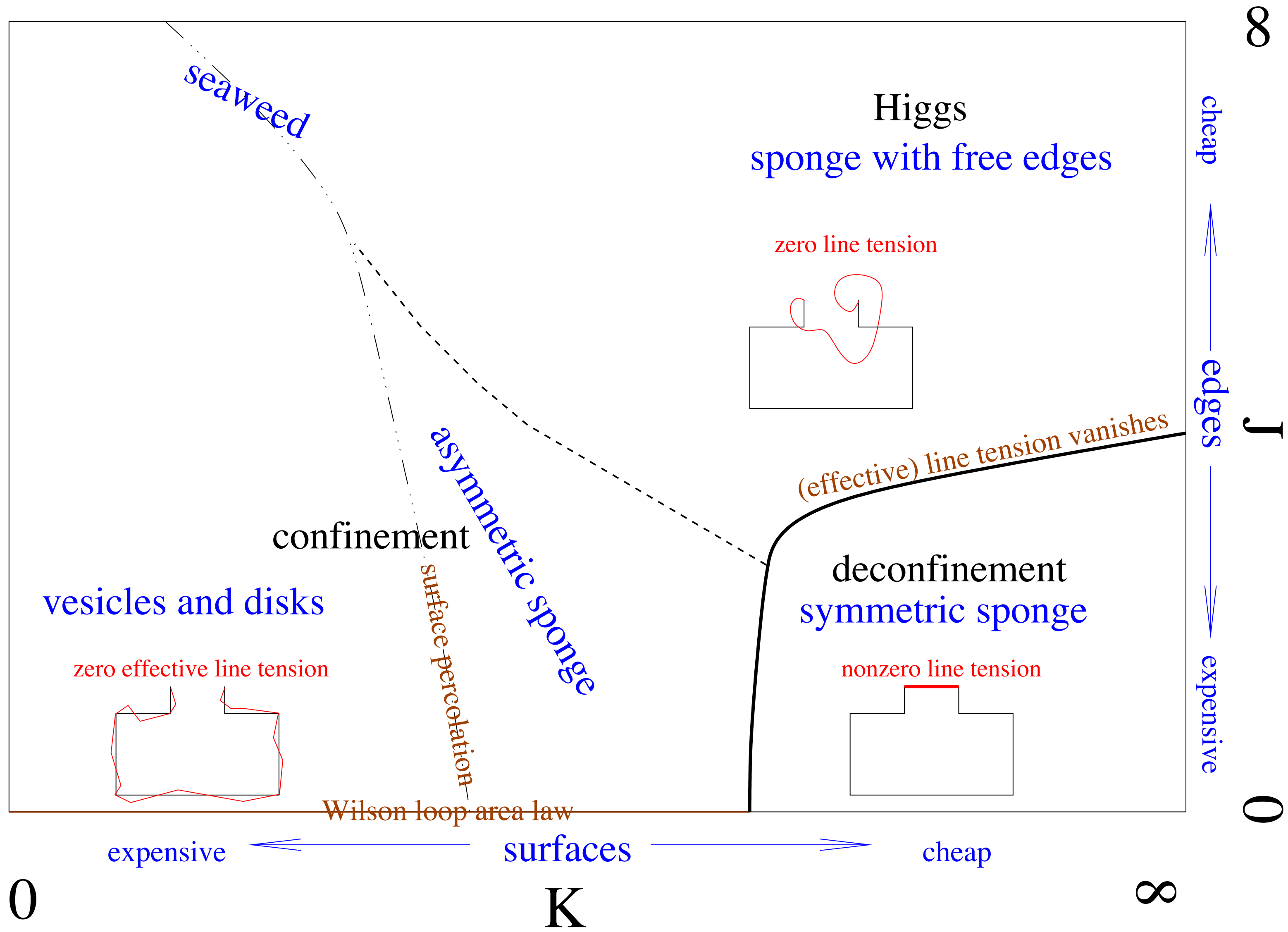
Always perimeter law (cf. breaking of flux string): $\langle W \rangle = e^{-\alpha L}$



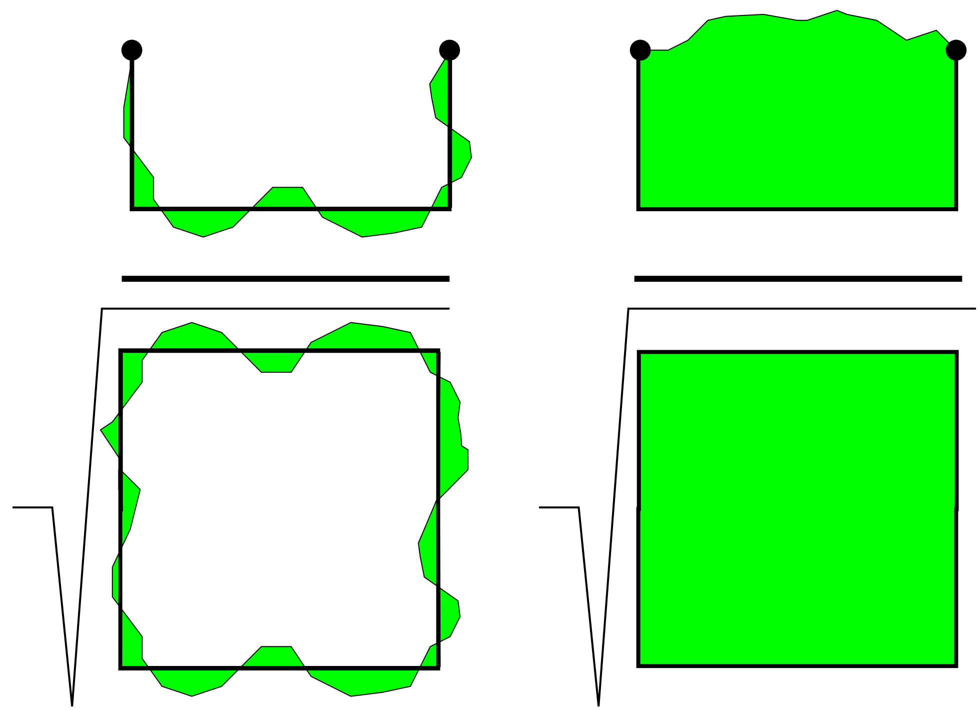
$\Longrightarrow$ need to diagnose 'underlying surface tension'

The Huse-Leibler horseshoe: effective line tension



The Fredenhagen-Marcu order parameter

$$R(L) = \frac{W_{1/2}(L)}{\sqrt{W(L)}} = \frac{\langle \tau_s (\prod_{l \in C_{1/2}} \sigma_l) \tau_{s'} \rangle}{\sqrt{\langle \prod_{l \in C} \sigma_l \rangle}}$$



This diagnoses deconfinement

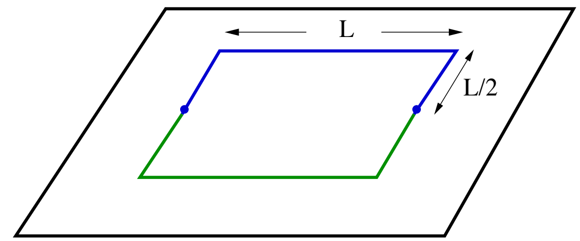
$\lim_{L\to\infty} R(L) = 0$ deconfined phase

$\lim_{L\to\infty} R(L) \neq 0$ otherwise

Space-time inerpretations

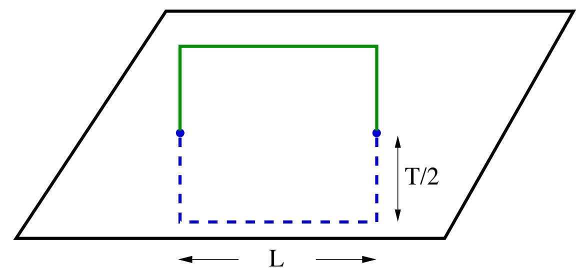
Equal-time diagnostic

$$R(L) \equiv \frac{W_{1/2}(L)}{\sqrt{W(L)}} = \frac{\langle G|\tau^z_s (\prod_{l\in C_{1/2}} \sigma^z_l)\tau^z_{s'}|G\rangle}{\sqrt{\langle G|\prod_{l\in C}\sigma^z_l|G\rangle}}$$



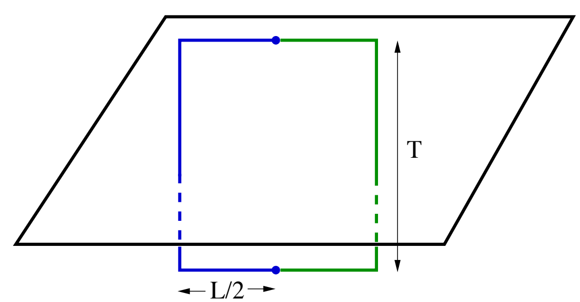
Fredenhagen-Marcu order parameter

$$R(L) = \frac{\langle G|ss'\rangle}{\sqrt{\langle ss'|ss'\rangle}}; \; |ss'\rangle = \tau^z_s \tau^z_{s'} \prod_{l\in C_{ss'}} \sigma^z_l(-T/2)|G\rangle$$



Spinon-delocalisation diagnostic

$$R(L) = \frac{e^{-(E_{\rm defect}+E_{\rm spinon})T}}{\sqrt{e^{-(E_{\rm defect}+E_{\rm defect})T}}}$$



Some small print

Finite temperature topological order

Senthil+Fisher;Nussinov+Ortiz;Castelnovo+Chamon,KG et al.

- In $d = 3$ (but not in $d = 2$), topological order persists to finite temperature for Z_2

Fluctuating constraints

- In emergent context, unphysical sector is physical but high-energy (no Lorentz invariance)

$\Rightarrow$ not all orientations are equivalent

U(1) gauge theories with charge q matter Fradkin-Shenker

$-S = K \sum_p \prod_{l \in \partial p} U_{\tilde{l}(l)} + J \sum_{l;s,s' \in \partial l} \tau_s U_l \tau_{s'} + \text{c.c.}$

Fields now of form $U = \exp(iA_{ij})$, $\tau = \exp(i\phi_i)$; $A, \phi \in [0, 2\pi($.

$$W_q(L) = \langle \prod_{l \in C} U^q_{\tilde{l}(l)} \rangle$$

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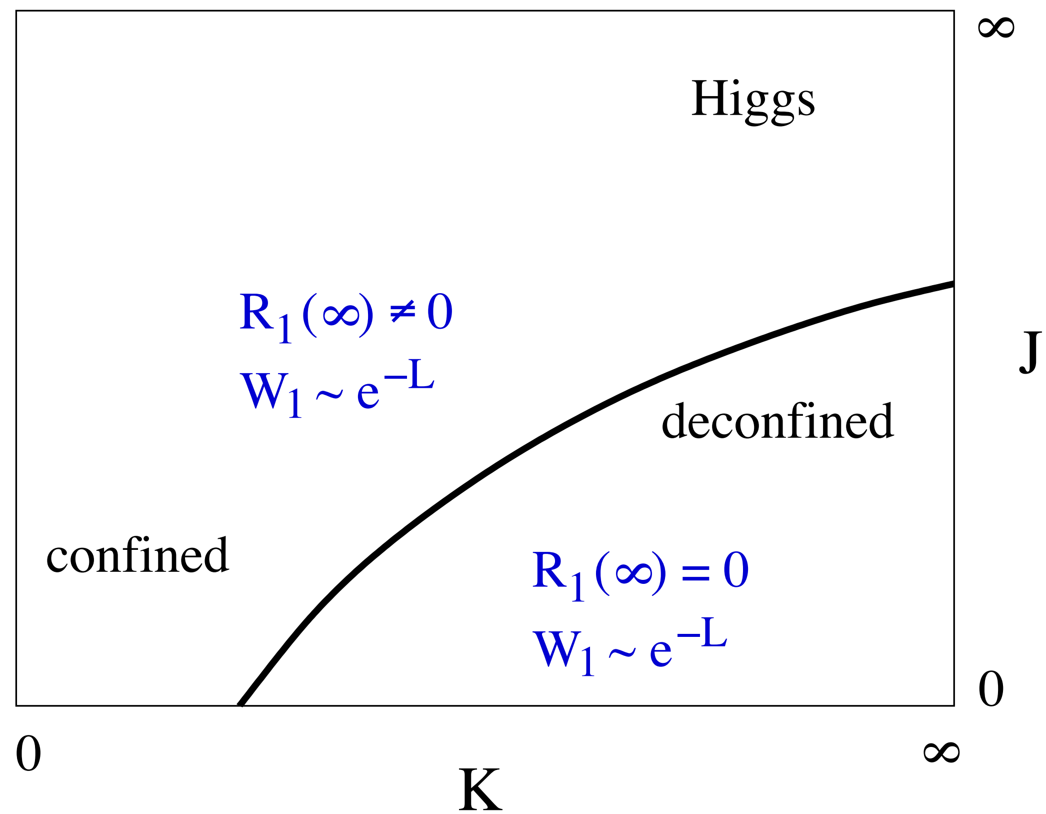

$q = 1$: confined = Higgs

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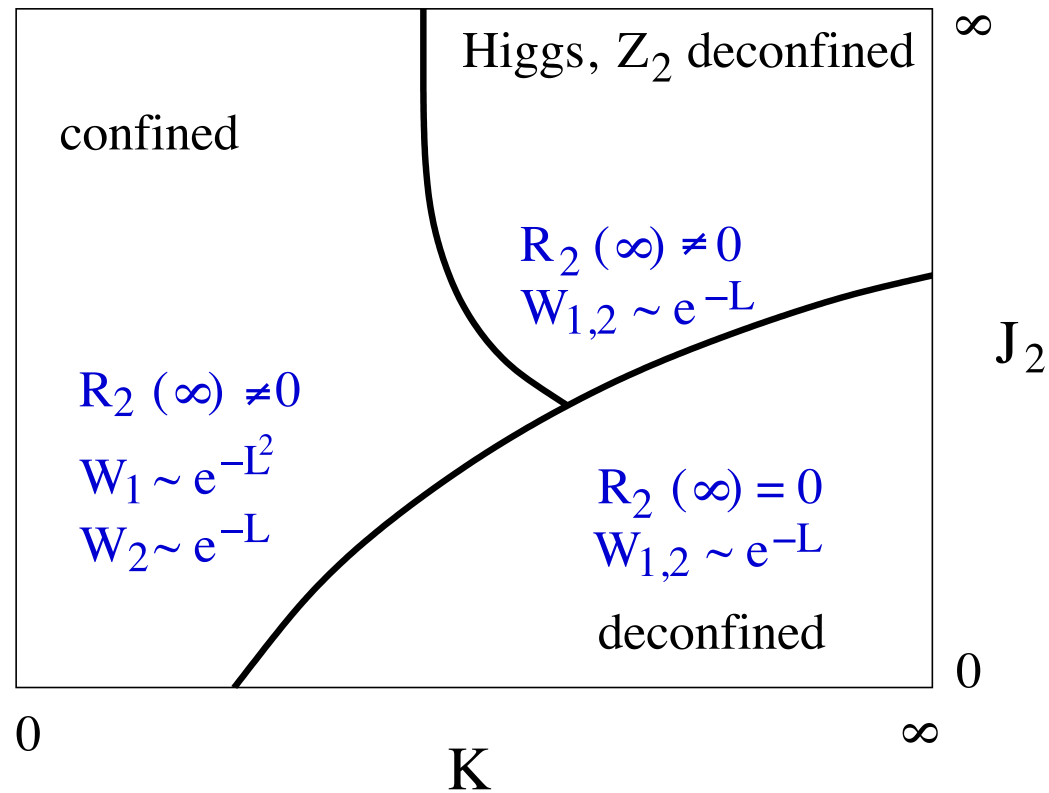
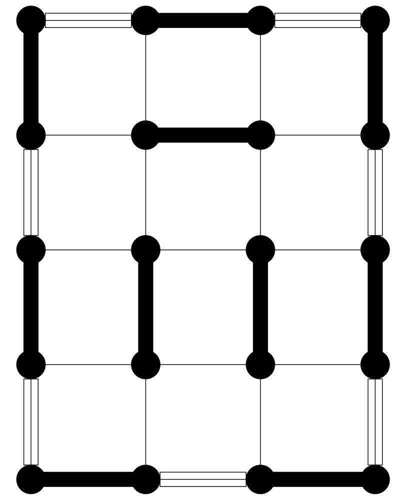


$q = 1$: confined = Higgs ; $q = 2$: R_2, W_1 act as diagnostic

Gauge theories for quantum magnets

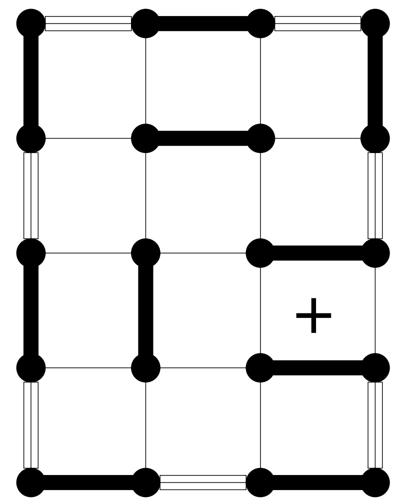
Wilson loop (in dimer variables, $\sigma_z = \pm 1$): $W_\Box = \sigma^+\sigma^-\sigma^+\sigma^- \cdots$



In deconfined phase, $|\phi\rangle \sim (1/N_c) \sum_c |c\rangle$

- Only configurations with appropriate dimerisation contribute
- amounts to restricting dimer configuration in volume $L\xi$ in gapped case

$\Rightarrow$ perimeter law $W \sim \exp(-\varsigma \xi L)$



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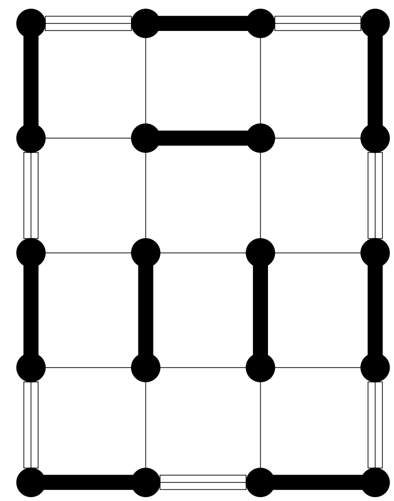

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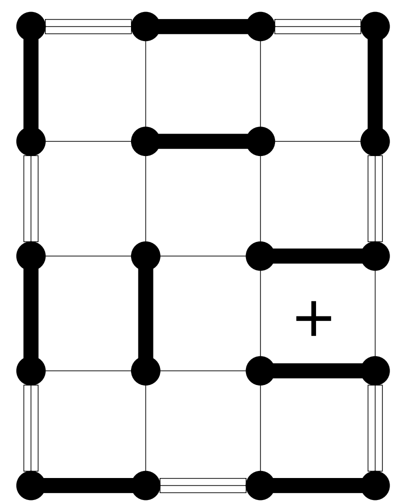
- With gapped spinful monomers Balents et al. (‘spinons’, $\tau_z = \pm 1$) :

  W fails but R should work

- full $SU(2)$ case: work in progress



Conclusions and outlook

Gauge theories from a condensed matter viewpoint

- origin and occurrences
- emergent vs. intrinsic charges
- irrational charge

Diagnostics in the presence of dynamical matter

- generalisation of Wilson loop, $R(L)$
- application to examples: U(1) gauge theory; QDM

Work in progress

- SU(2) magnets
- broader class of gauge theories