Dense Light
October 06 2015
KITP, Santa Barbara, USA

Driven Markovian Quantum Criticality

+ Fate of the KT transition

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-> University of Cologne

Collaboration:
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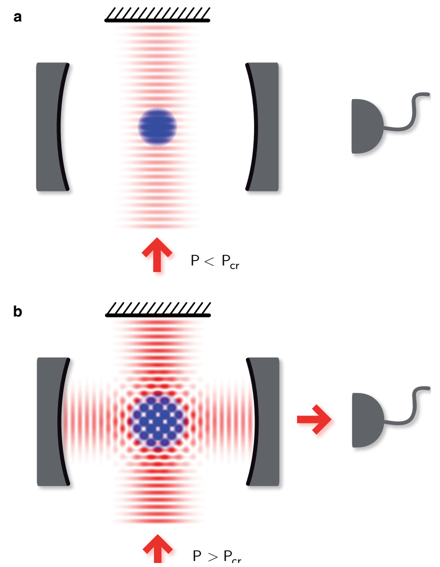
L. Sieberer, Weizmann   E. Altman, Weizmann
G. Wachtel, Weizmann -> Toronto

Motivation: Driven open many-body dynamics

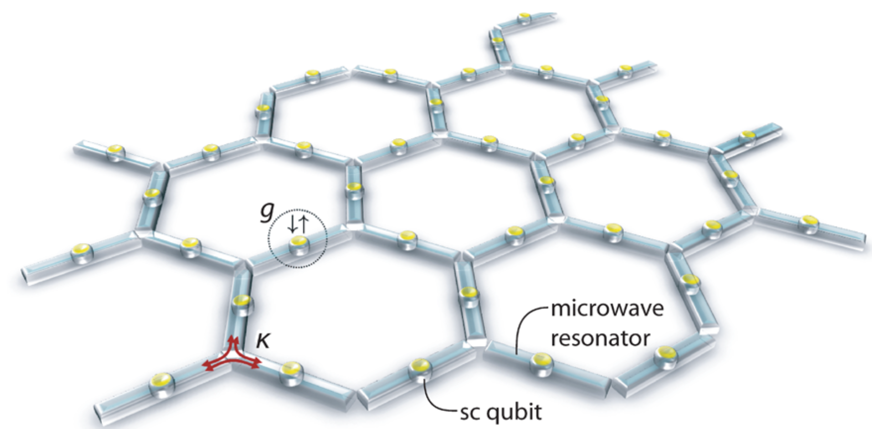
- experimental systems on the interface of quantum optics and many-body physics

  - Driven-open Dicke models



Baumann et al., Nature 2010
Ritsch et al., RMP 2013

  - Coupled microcavity arrays



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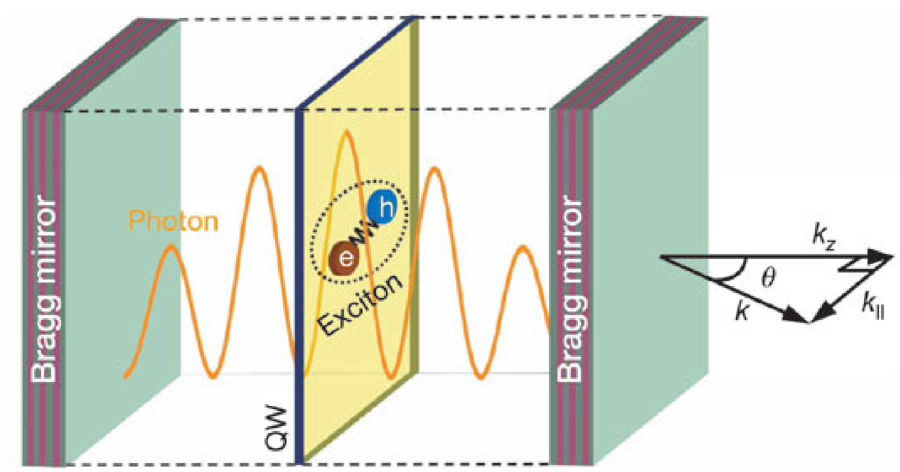
  - exciton-polariton systems in semiconductor quantum wells



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- other platforms (light-matter):
  - ➡ dissipative Rydberg systems — Carr et al. PRL 2013, Marcuzzi et al. PRL 2014
  - ➡ polar molecules — Zhu et al. PRL 2013
  - ➡ photon BECs — Klaers et al. Nature 2010
  - ➡ trapped ions — Barreiro et al. Nature 2011

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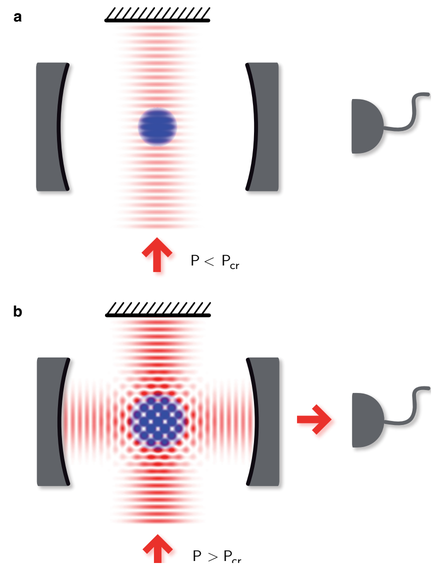

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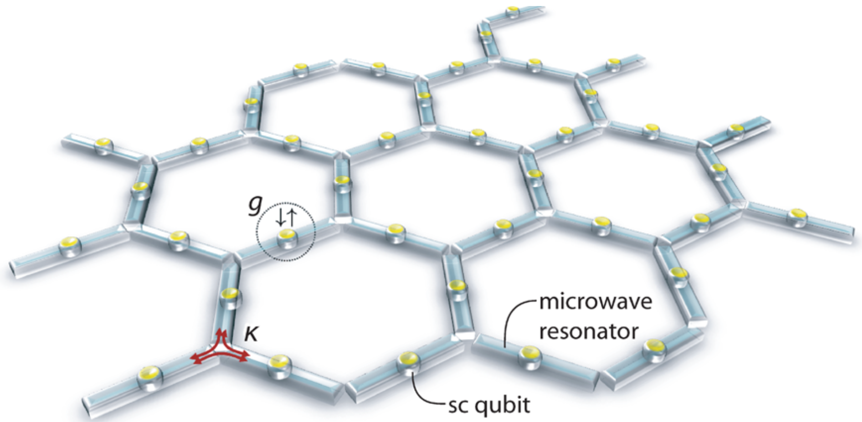

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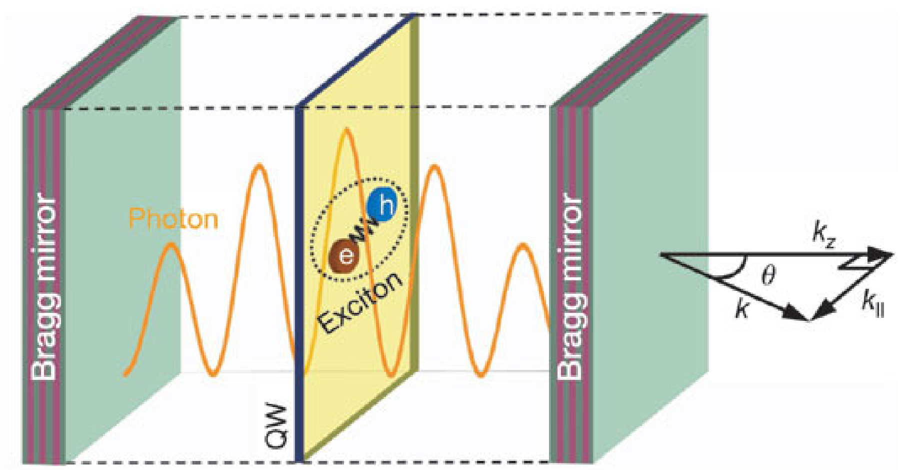

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Non-Equilibrium Physics with Driven Open Quantum Systems

- Interdisciplinary research area: physics at various length scales

Quantum Optics

coherent and driven-dissipative dynamics on equal footing

Many-body physics

continuum of spatial degrees of freedom

Statistical mechanics

Microscopic

“Thermodynamic”

Long wavelength

Non-Equilibrium Physics with Driven Open Quantum Systems

- Interdisciplinary research area: physics at various length scales

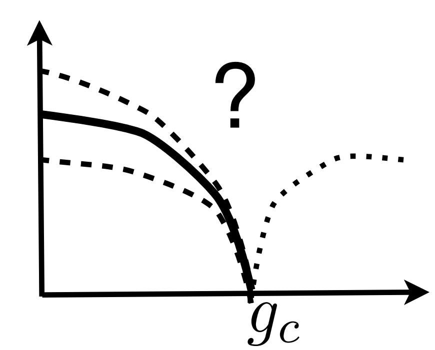
Quantum Optics	Many-body physics	Statistical mechanics
coherent and driven-dissipative dynamics on equal footing	continuum of spatial degrees of freedom	
Microscopic	"Thermodynamic"	Long wavelength

- Challenge to theory: perform the transition from micro- to macrophysics in driven interacting systems

How much "quantum" remains at large distances?

How much "non-equilibrium" remains at large distances?

$$Z[J] = \int \mathcal{D}\varphi \, e^{i(S[\varphi] + \int J\varphi)}$$

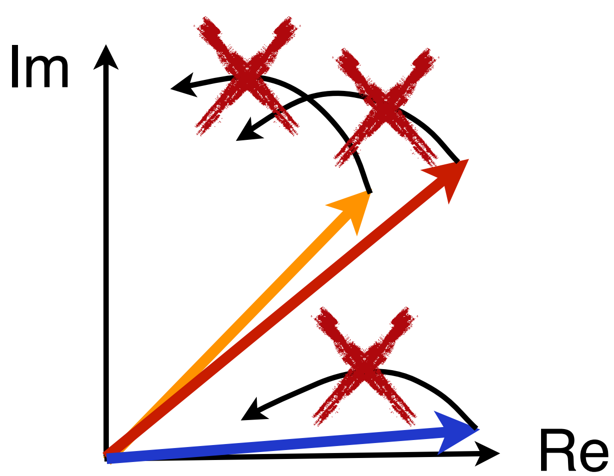


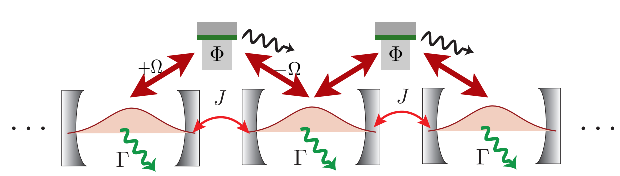
Outline

Driven Open Quantum Systems

How much “quantum”?

Driven Analogue of Quantum Criticality





microcavity arrays

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How much “non-equilibrium”?

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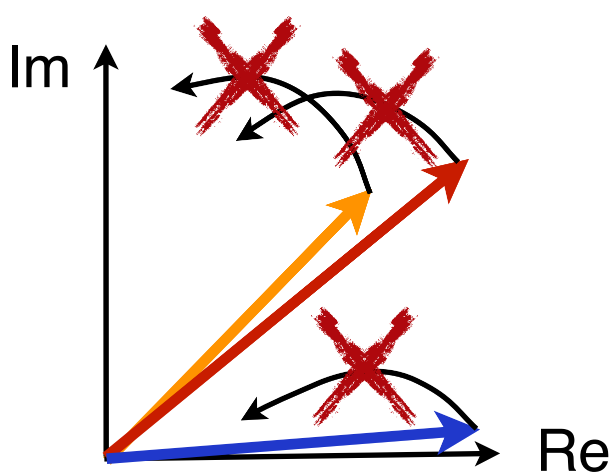
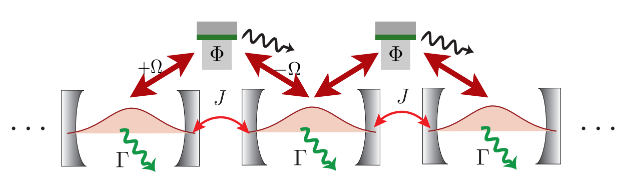
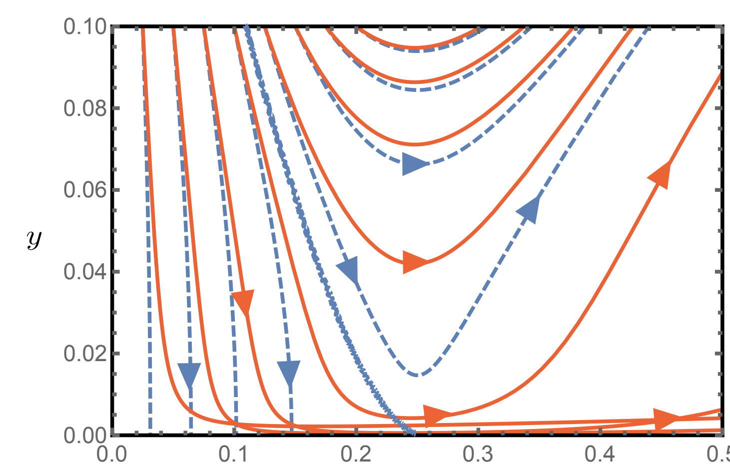
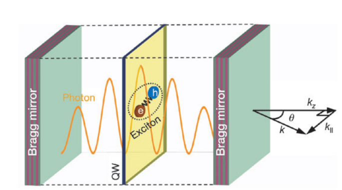




microcavity arrays

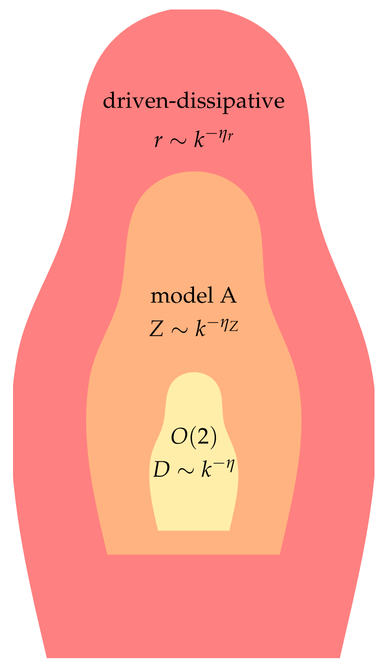
Fate of the KT transition in Driven Systems





exciton-polariton condensates

Dynamical Markovian Quantum Criticality

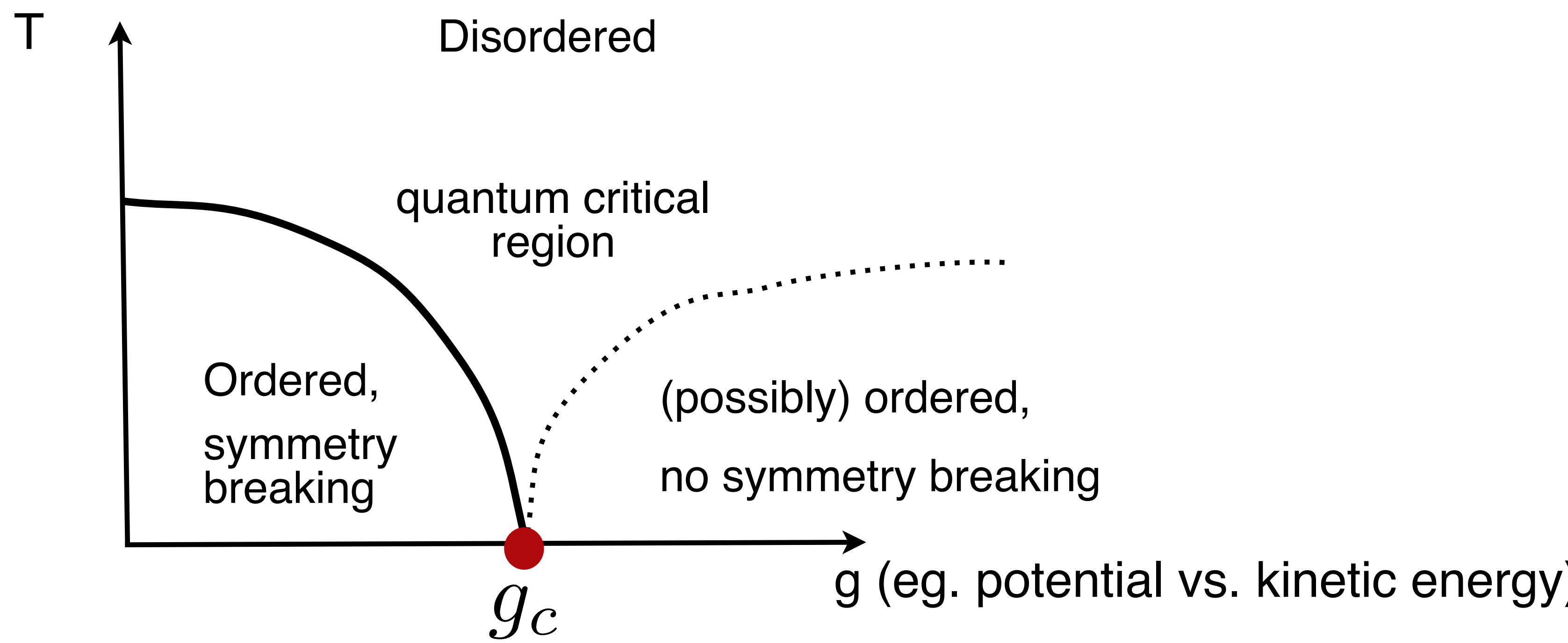


J. Marino, SD, arxiv:1508.02723 (2015)

Microscopic Quantum Optics	“Thermodynamic” Many-body physics	Long wavelength Statistical mechanics

Classical vs. Quantum Criticality

- generic quantum phase diagram



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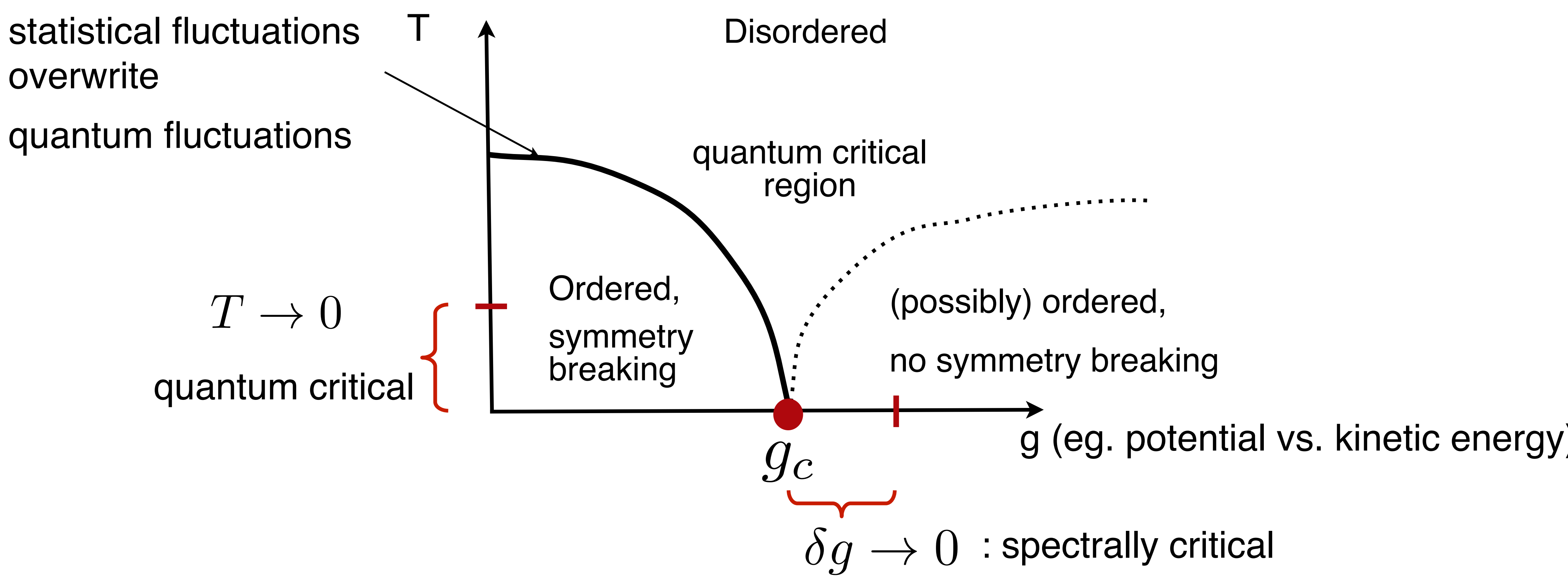

- double fine tuning, temperature is relevant perturbation to the quantum critical point

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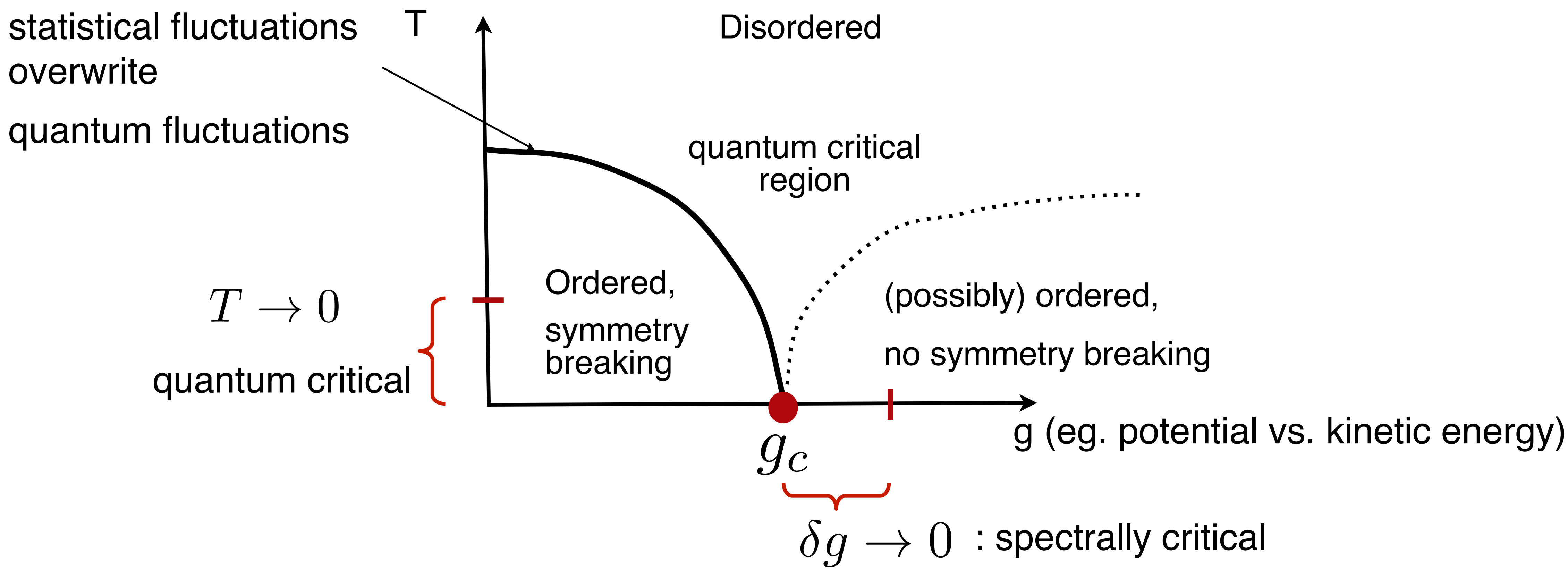


- double fine tuning, temperature is relevant perturbation to the quantum critical point

- quantum critical scaling for

$$T \ll \omega \ll \omega_G$$

quantum

non-gaussian

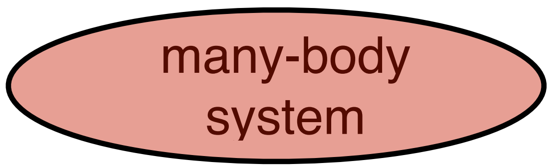
Theoretical Approach

$$e^{i\Gamma[\Phi]} = \int \mathcal{D}\delta\Phi e^{iS_M[\Phi+\delta\Phi]}$$

Microscopic Model

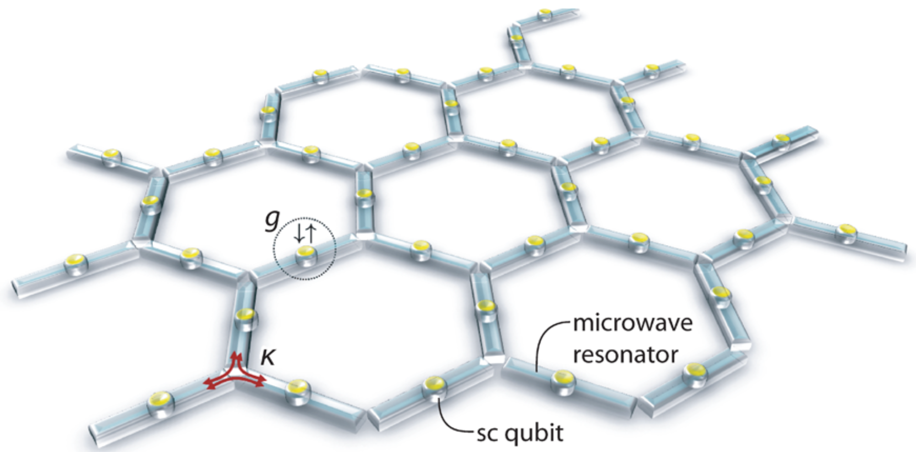
- generic microscopic model: many-body master equation, eg.

$$\partial_t \rho = -i[H, \rho] + \mathcal{D}[\rho] \equiv \mathcal{L}[\rho]$$



$$H = \int_{\mathbf{x}} \hat{\phi}^\dagger_{\mathbf{x}} \left(\frac{\triangle}{2M} - \mu\right) \hat{\phi}_{\mathbf{x}} + \frac{\lambda}{2}(\hat{\phi}^\dagger_{\mathbf{x}} \hat{\phi}_{\mathbf{x}})^2$$

- quantum description of XP systems
- long wavelength limit of microcavity arrays: driven open Bose-Hubbard model (w/ incoherent pump)



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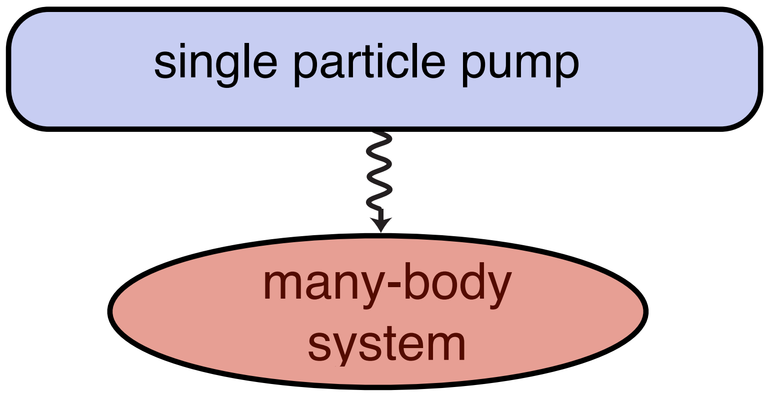

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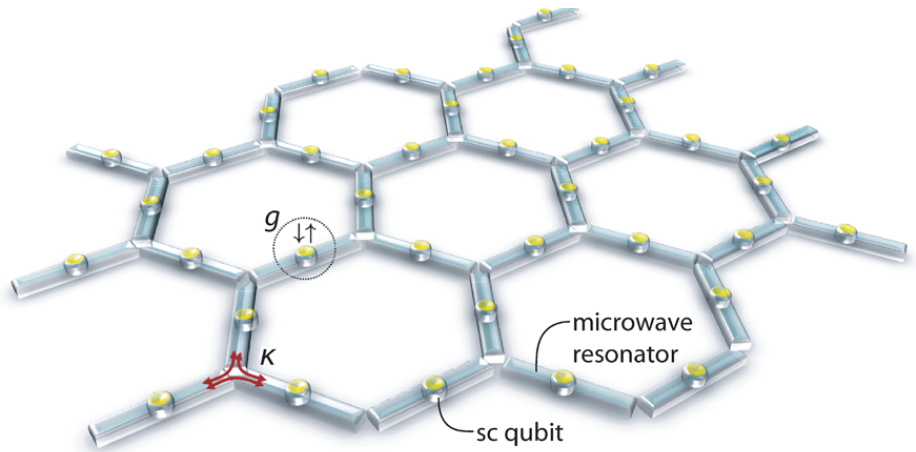

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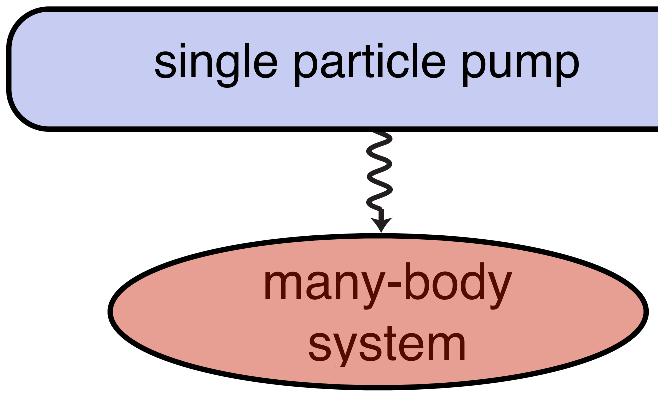

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$$\mathcal{D}[\rho] = \gamma_p \int_{\mathbf{x}} \underbrace{[\hat{\phi}^\dagger_{\mathbf{x}}\, \rho\, \hat{\phi}_{\mathbf{x}} - \tfrac{1}{2}\{\hat{\phi}_{\mathbf{x}} \hat{\phi}^\dagger_{\mathbf{x}}, \rho\}]}_{\text{single particle pump}} \quad +$$

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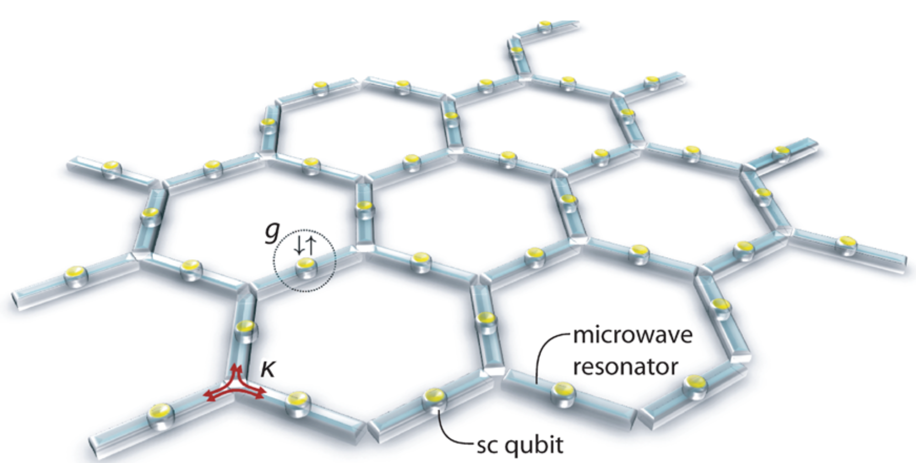

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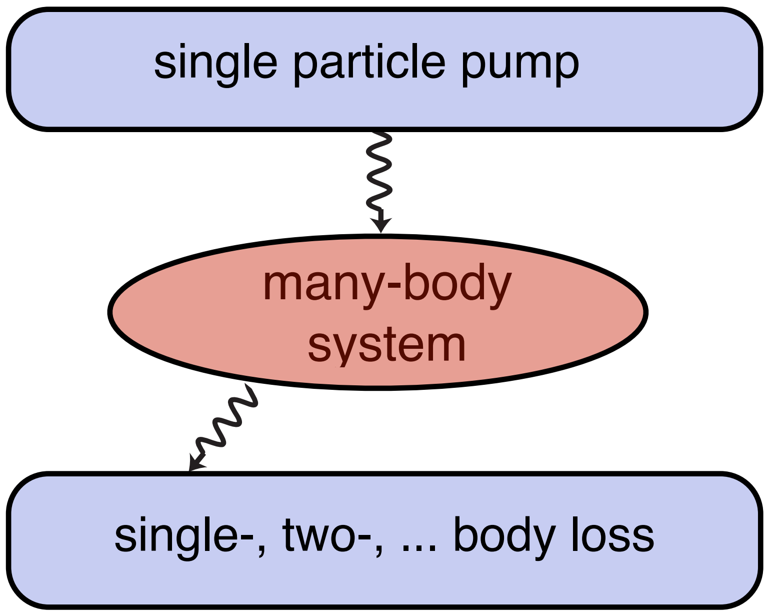

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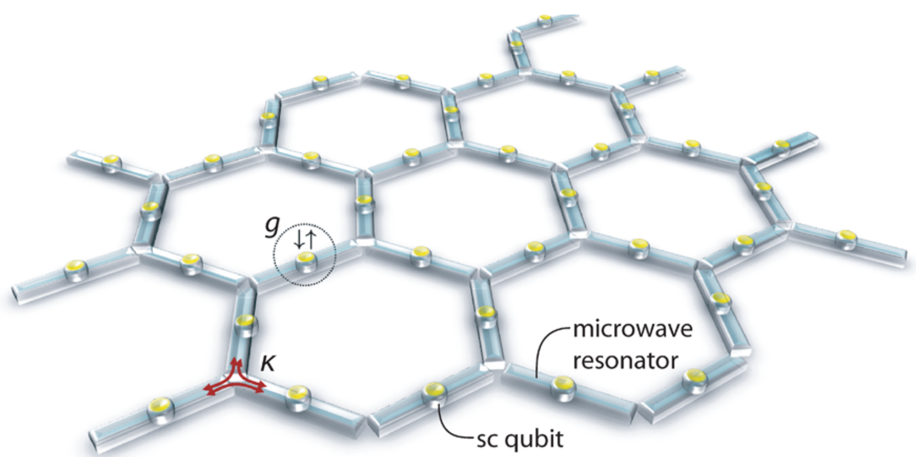

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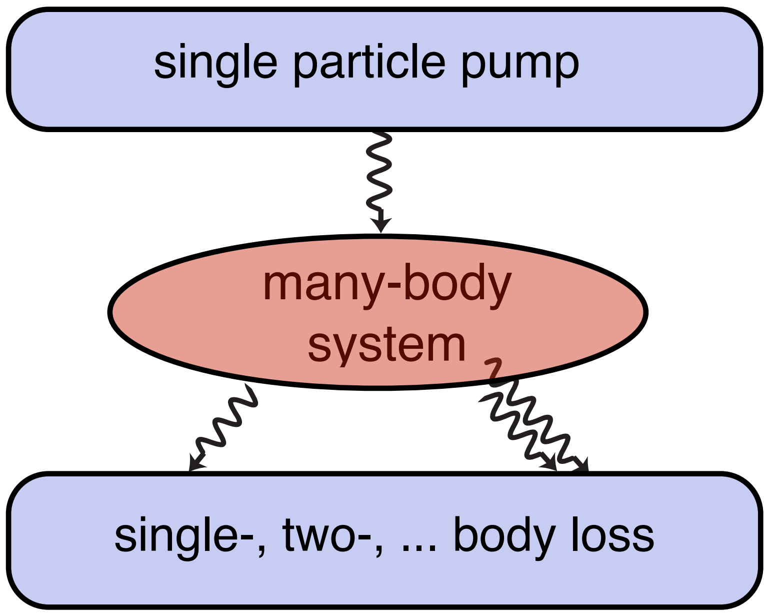

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$$\underbrace{\kappa \int_{\mathbf{x}} [\hat\phi^2_{\mathbf{x}}\,\rho\,\hat\phi^{\dagger\,2}_{\mathbf{x}} - \tfrac{1}{2}\{\hat\phi^{\dagger\,2}_{\mathbf{x}}\hat\phi^2_{\mathbf{x}},\rho\}]}_{\text{two particle loss}}$$

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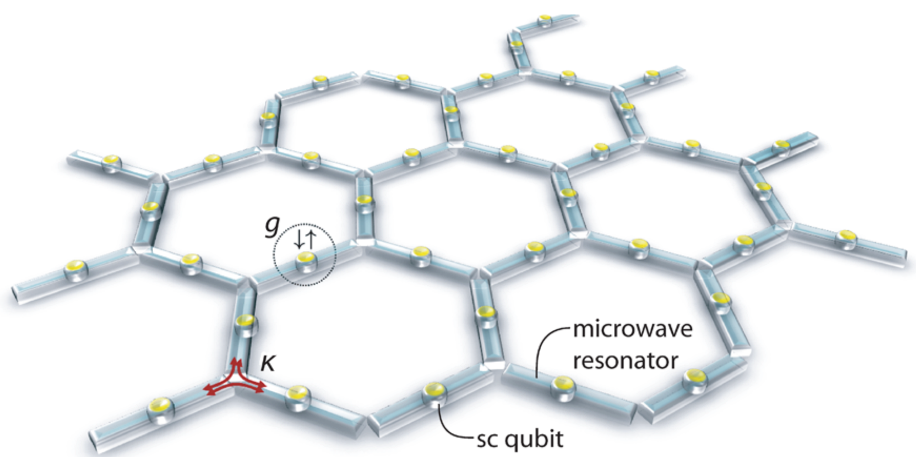

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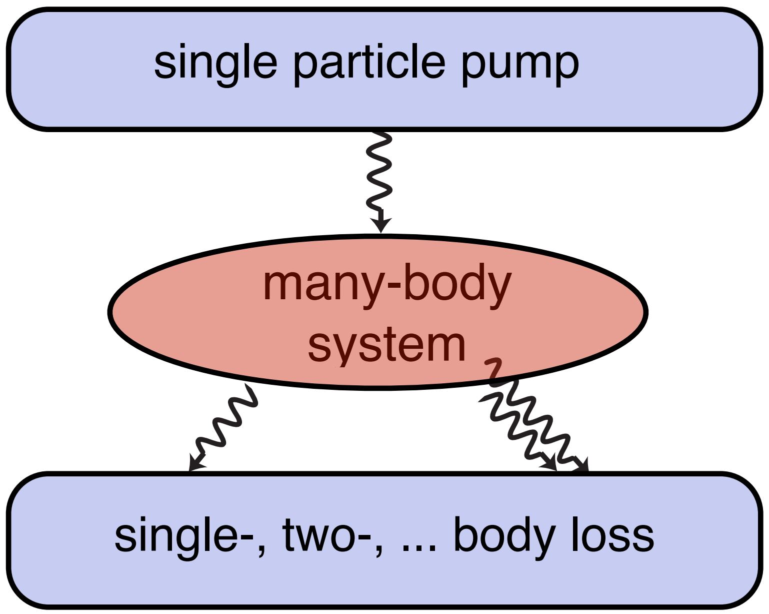


- evaluation strategy:

Many-Body Master Equation → translation table → Microscopic Markovian Dissipative Action → RG flow → Long Wavelength Effective Action

$$\partial_t \rho = -i[H,\rho] + \mathcal{L}[\rho] \quad \Leftrightarrow \quad \mathrm{e}^{\mathrm{i}\Gamma[\Phi]} = \int \mathcal{D}\delta\Phi \mathrm{e}^{\mathrm{i}S_M[\Phi+\delta\Phi]} \quad \Leftrightarrow \quad \partial_k \Gamma_k = \frac{\mathrm{i}}{2}\mathrm{Tr}\left[\left(\Gamma_k^{(2)} + R_k\right)^{-1} \partial_k R_k\right]$$

many-body master equation

Keldysh real time functional integral

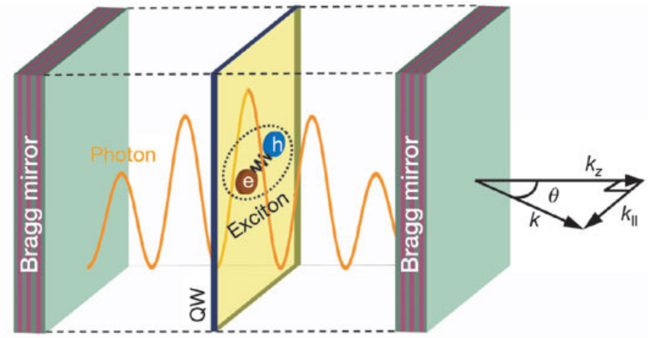
Functional Renormalization Group equation

closed system Keldysh:
Gasenzer, Pawlowski, PLB 08;
Berges, Hoffmeister, Nucl. Phys. B, 09

➡ Opens up the powerful toolbox of quantum field theory to driven-dissipative systems

➡ derivative expansion including all non-irrelevant operators

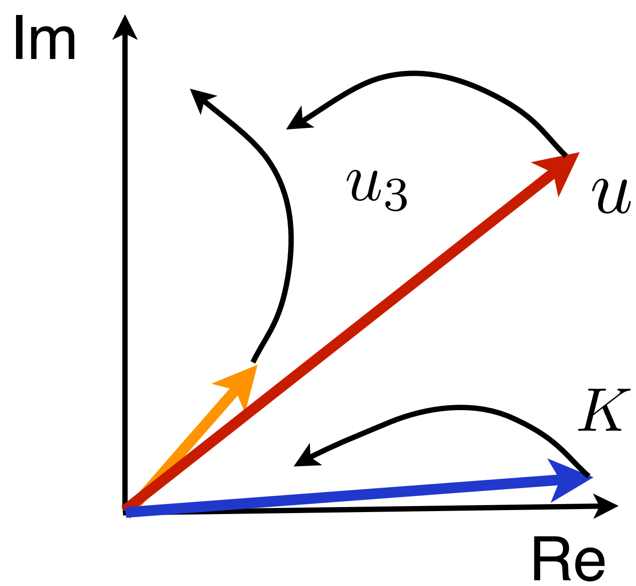
Driven Classical Criticality



L. Sieberer, S. Huber, E. Altman, SD,
PRL 110, 195301 (2013) and PRB 89, 134310 (2014);
U. C. Tauber, SD, PRX 4, 021010 (2014)

Classical driven criticality: Schematic RG flow

- Flow in the complex plane of couplings

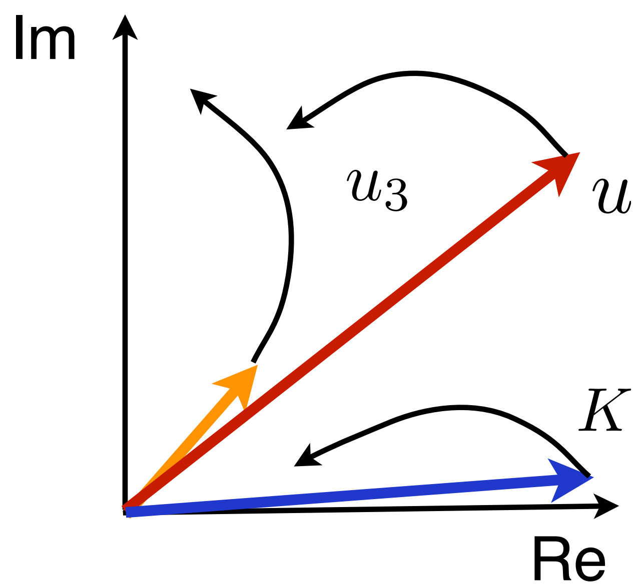


non-linear initial flow

- initial values: $\Gamma_{k\approx\Lambda_0} \approx S$
  - particles propagate
    $$\frac{1}{2m} = \mathrm{Re}[K] \approx 1 \gg D = \mathrm{Im}[K]$$
  - coherent collisions ~ two-body loss

Classical driven criticality: Schematic RG flow

- Flow in the complex plane of couplings



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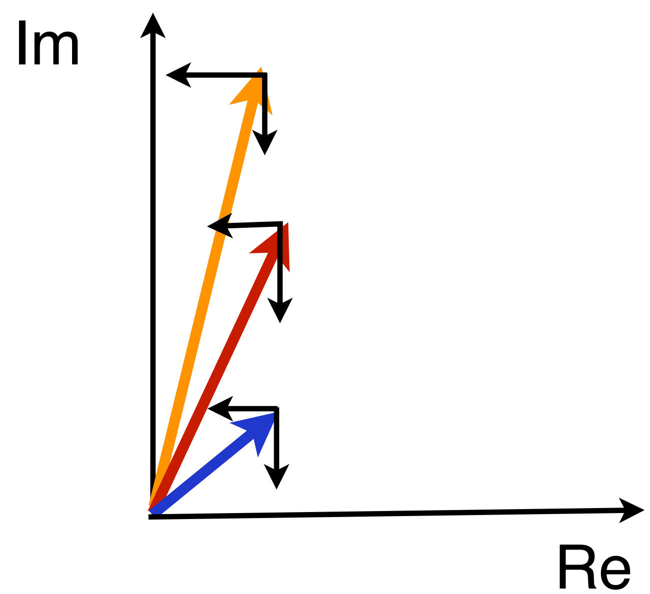

linearized IR flow

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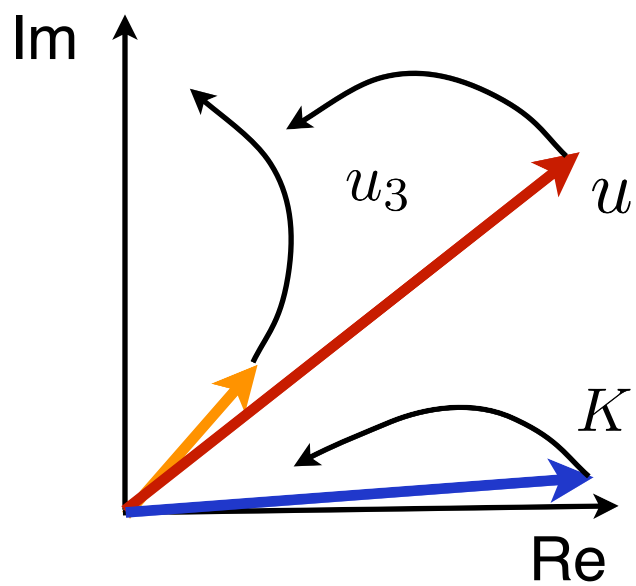
- universal domain encoding universality class
- scaling of running couplings

$$g = ak^{\eta_a} + ibk^{\eta_b}$$

crit. exponent

Classical driven criticality: Schematic RG flow

- Flow in the complex plane of couplings



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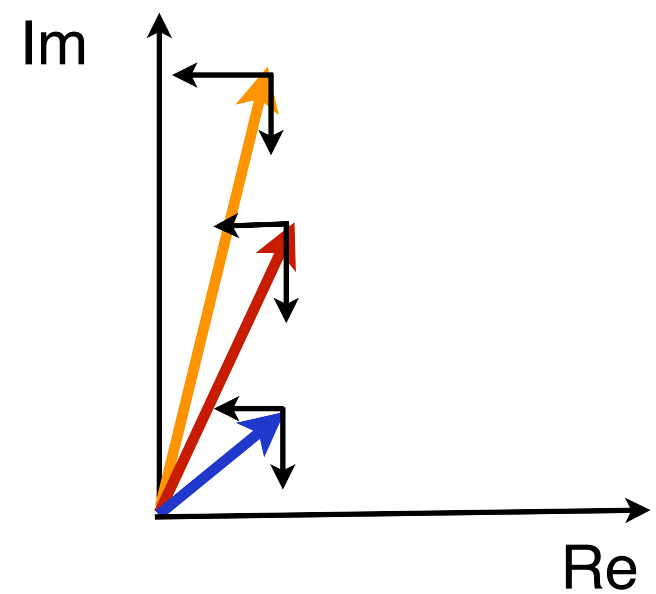

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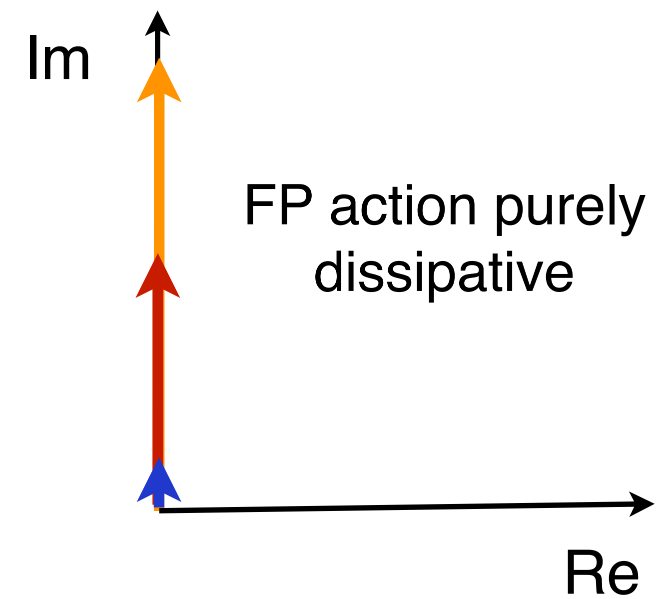


fixed point

- initial values: $\Gamma_{k\approx\Lambda_0} \approx S$
  - particles propagate
    $\frac{1}{2m} = \mathrm{Re}[K] \approx 1 \gg D = \mathrm{Im}[K]$
  - coherent collisions ~ two-body loss
- universal domain encoding universality class ➡ decoherence
- scaling of running couplings

$$g = ak^{\eta_a} + ibk^{\eta_b}$$

crit. exponent

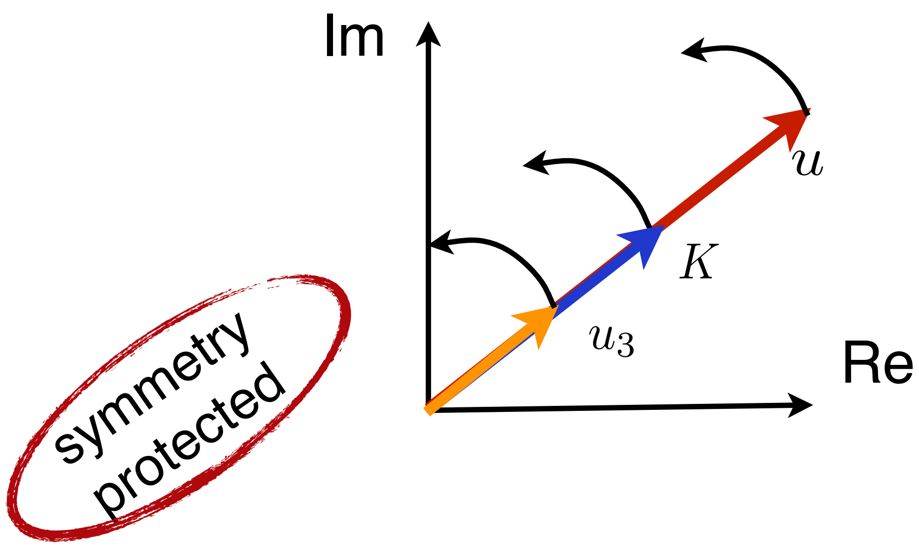
- key results (classical):

➡ universal decoherence (new independent critical exponent)
➡ asymptotic thermalization
➡ reveals equilibrium vs. non-equilibrium fine structure

Universal decoherence, fine structure, and thermalization

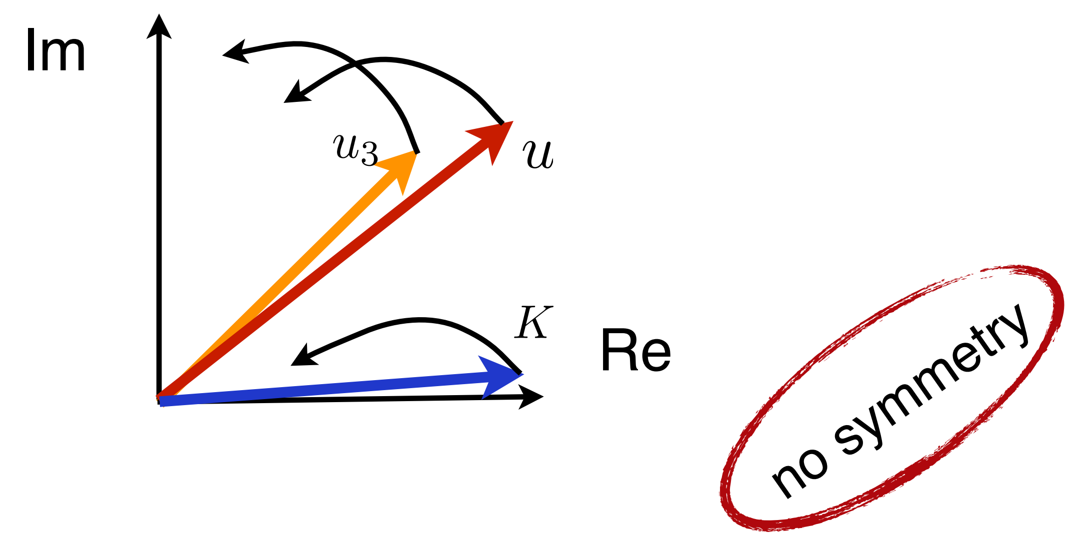
- decoherence <=> purely imaginary fixed point action
- global thermal equilibrium is ensured by symmetry:

equilibrium dynamics



initial flow

non-equilibrium dynamics



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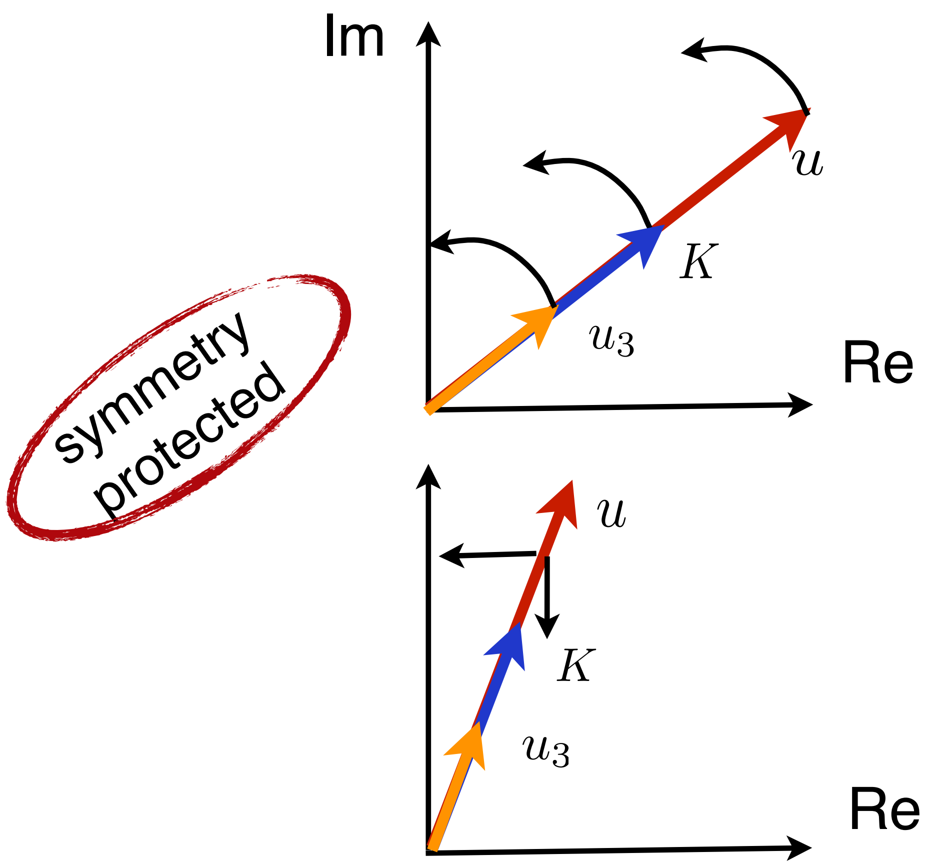
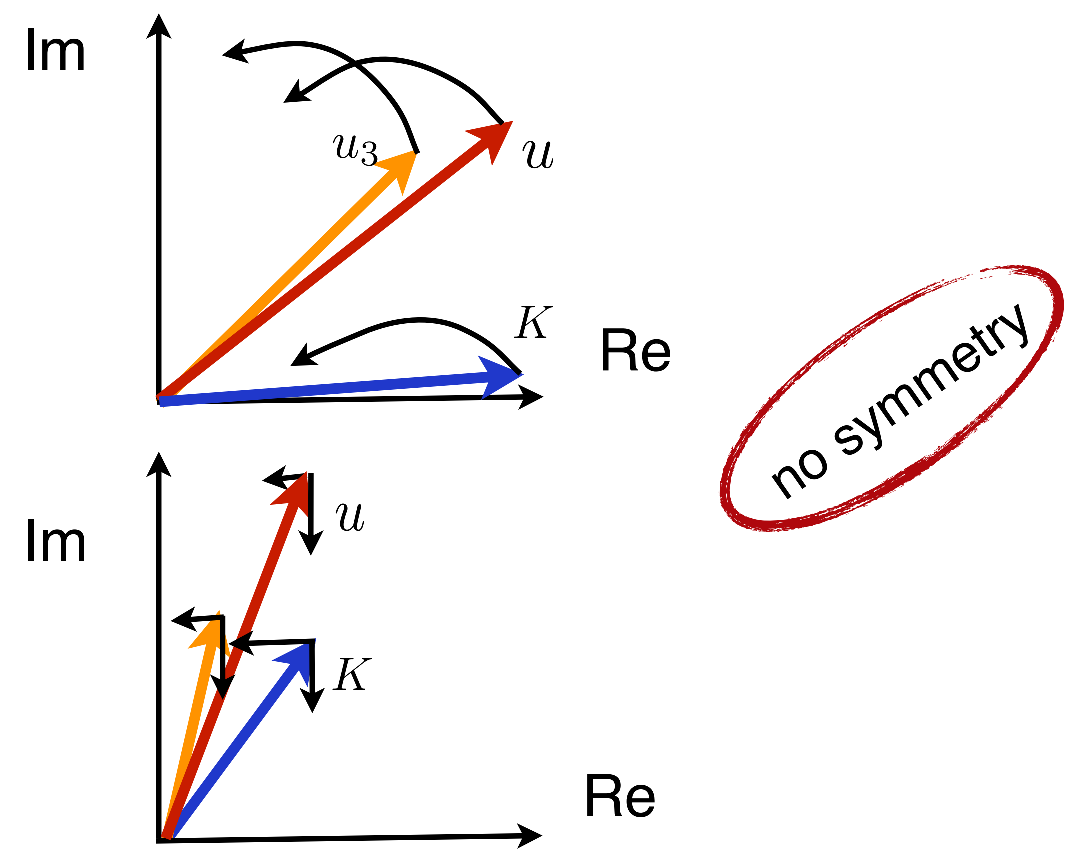


initial flow

infrared flow

non-equilibrium dynamics



- eigenvalue of flow speed

$$\eta_R \approx -0.143$$

- lowest eigenvalue

$$\eta_r \approx -0.101$$

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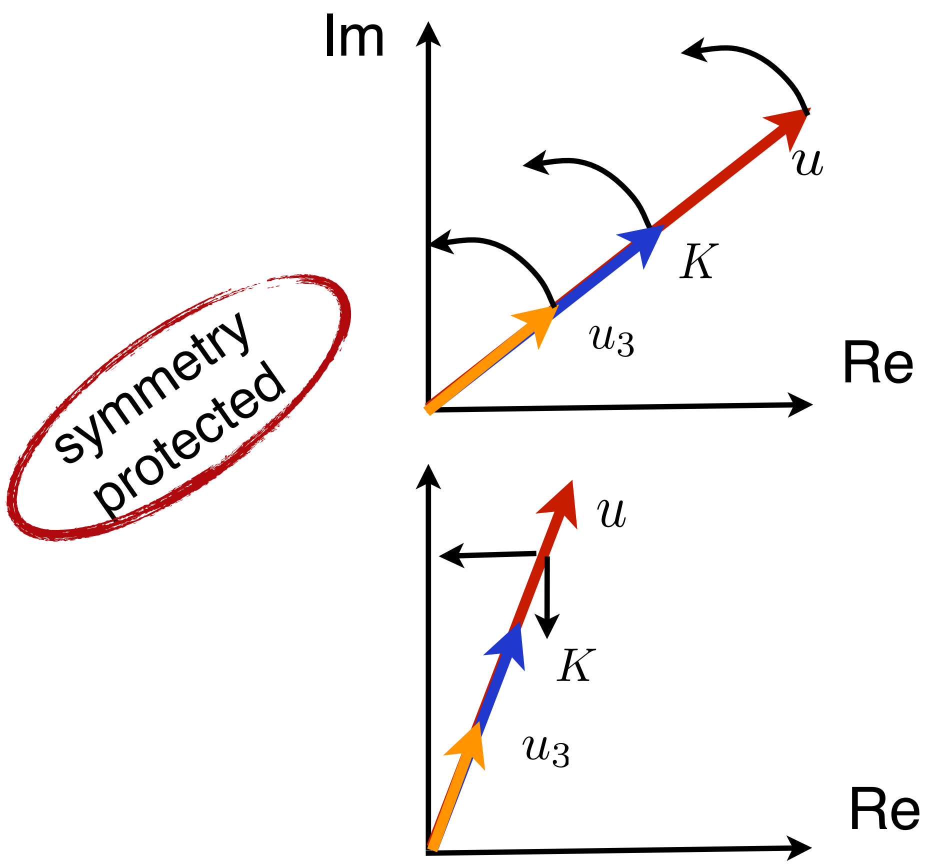

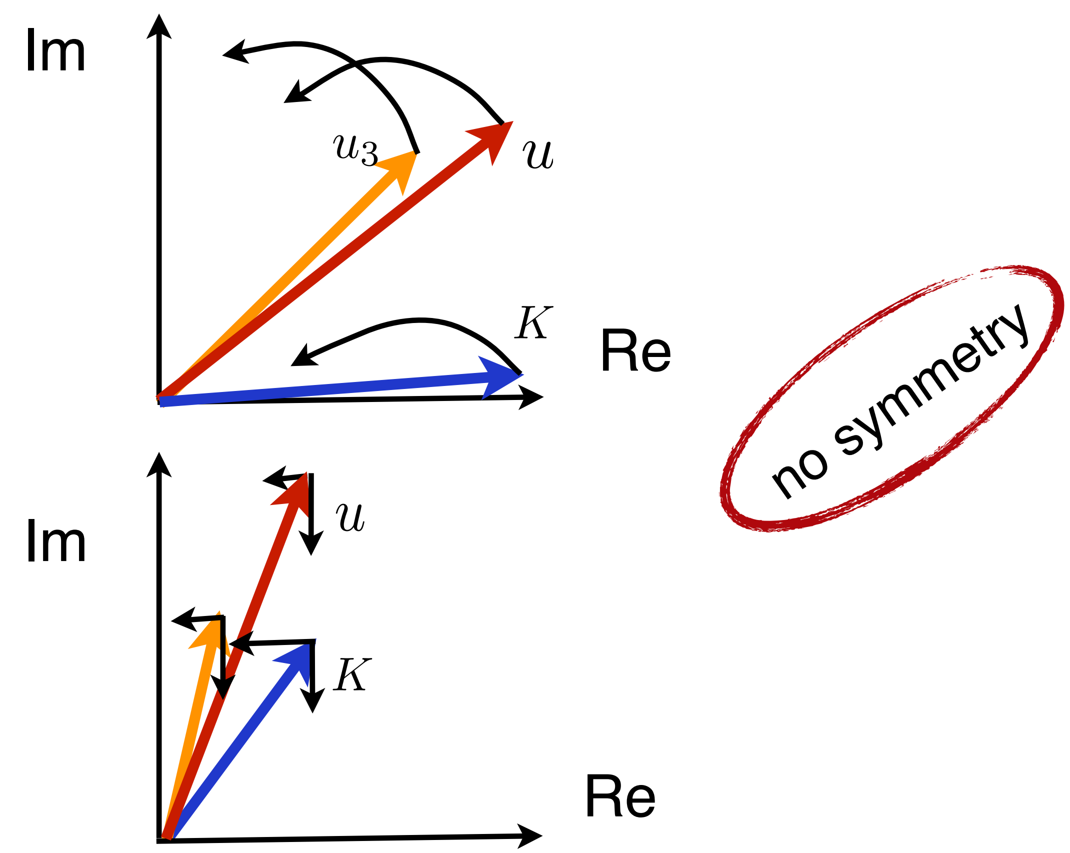

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➡ equilibrium and driven systems are in different universality classes

➡ physical reason: independence of coherent and dissipative dynamics

➡ asymptotic thermalization: all couplings aligned on Im axis

Universal decoherence, fine structure, and thermalization

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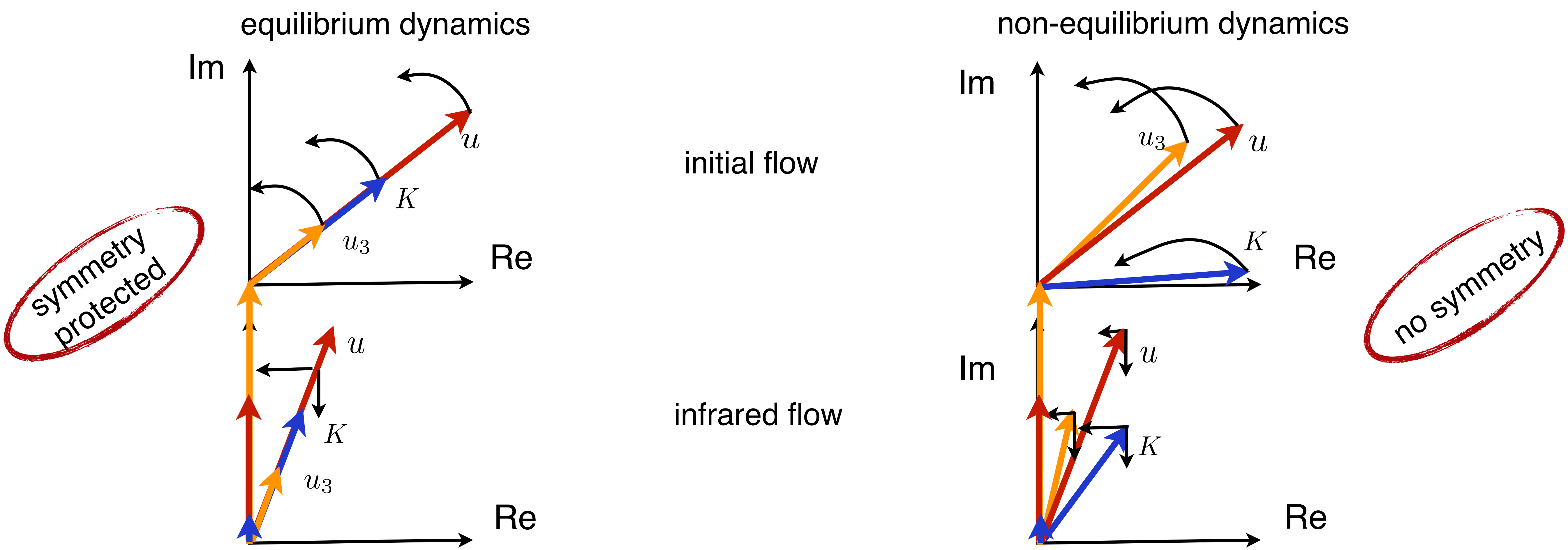
- global thermal equilibrium is ensured by symmetry:



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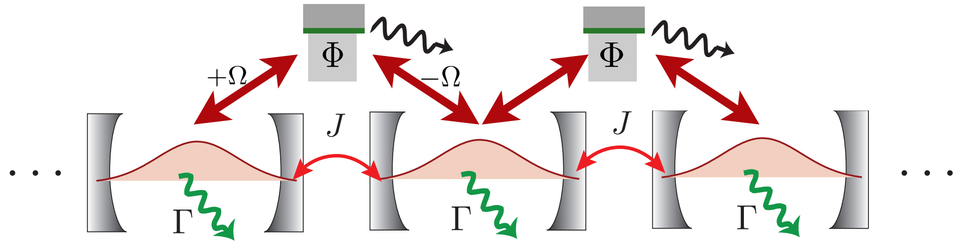
$$\eta_r \approx -0.101$$

➡ equilibrium and driven systems are in different universality classes
➡ physical reason: independence of coherent and dissipative dynamics
➡ asymptotic thermalization: all couplings aligned on Im axis

Driven Quantum Criticality



J. Marino, SD, arxiv:1508.02723 (2015)

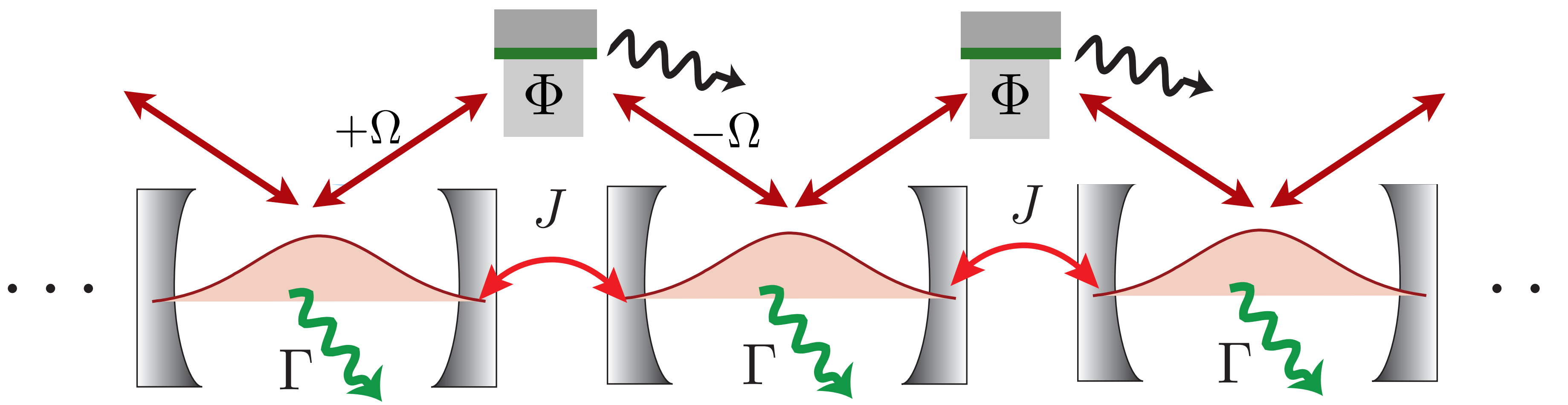
Non-equilibrium analogue of quantum criticality

- Lindblad Master equation with strong quantum diffusion (1D)

$$\gamma_d \int_{\mathbf{x}} [\nabla a(x)\, \rho\, \nabla a^\dagger(x) - \tfrac{1}{2}\{\nabla a^\dagger(x) \nabla a(x), \rho\}]$$

- possible realization: microcavity arrays

cf. D. Marcos et al., NJP (2012)



$$H_c = \Omega \sum_i \sigma_i^+ (a_i - a_{i+1}) + h.c.$$

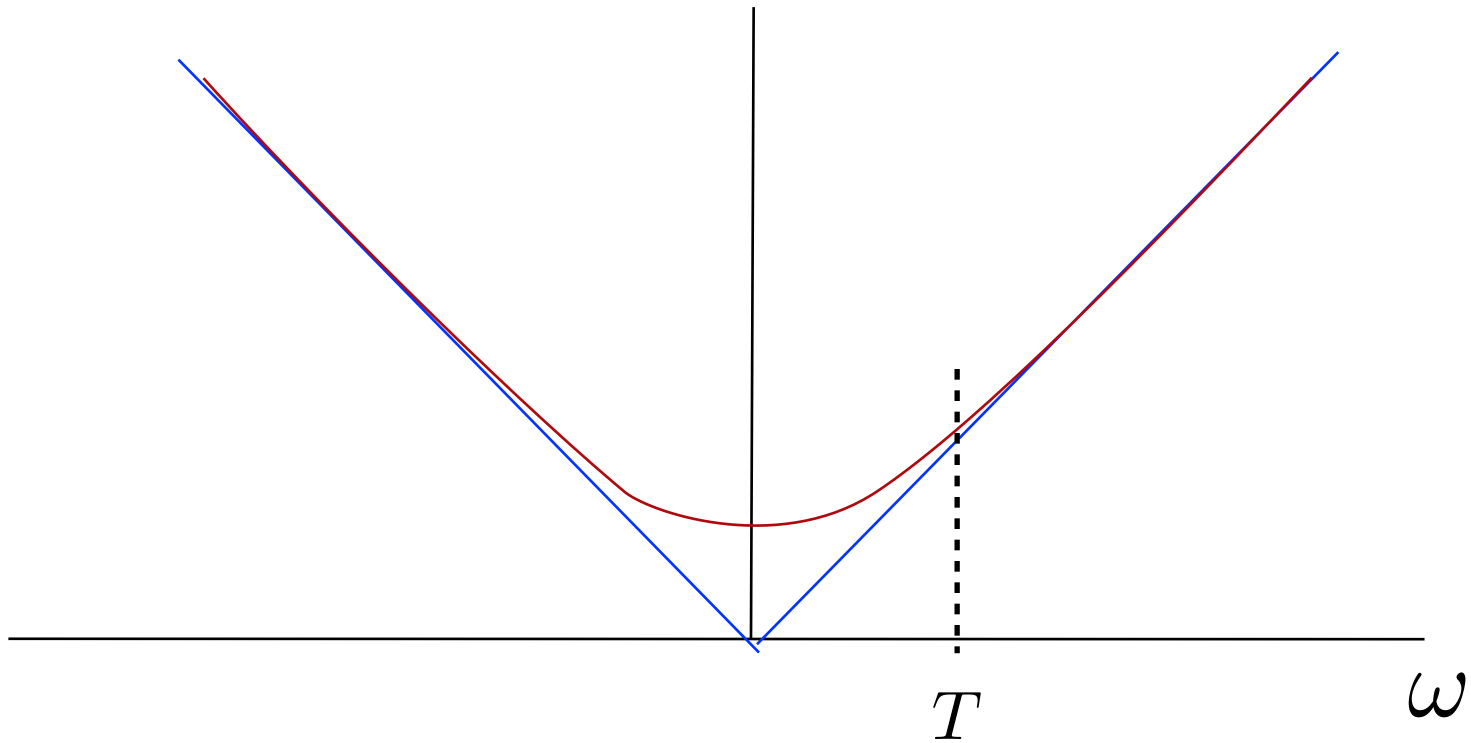
$$\Omega \ll \gamma_q$$

$$\mathcal{D}[\rho] = \gamma_q \sum_i [\sigma_i^- \,\rho\, \sigma_i^+ - \tfrac{1}{2}\{\sigma_i^+ \sigma_i^-, \rho\}]$$

“What is quantum about it?”

- analogy to an equilibrium system: noise level

$$P^K(\omega) \sim \omega \coth \tfrac{\omega}{2T}$$



- two regimes

$$\omega/2T \ll 1: \quad P^K(\omega) \approx 2T, \quad P^K(t-t') \sim \delta(t-t')$$ classical/markovian

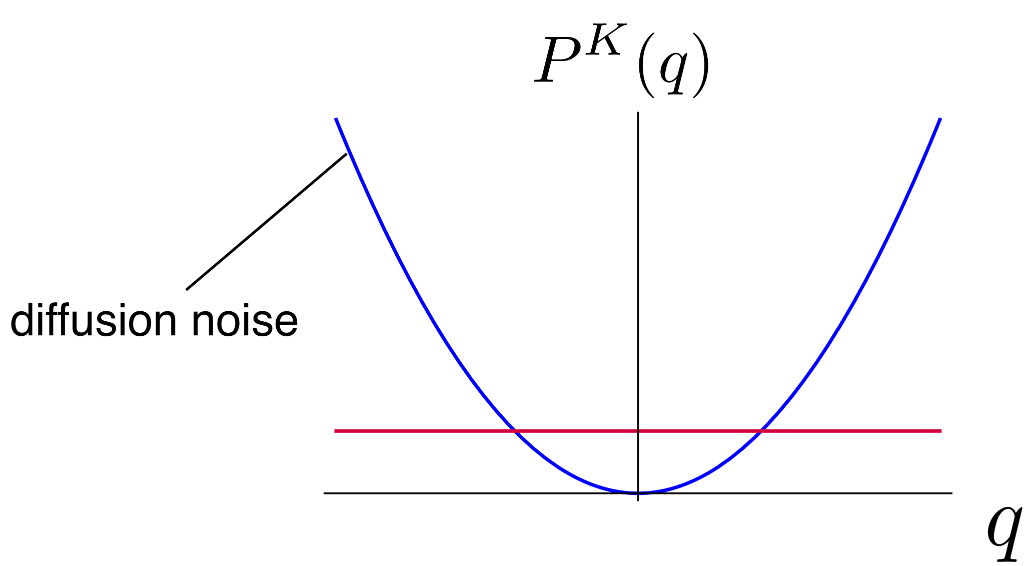
$$\omega/2T \gg 1: \quad P^K(\omega) \approx |\omega|, \quad P^K(t-t') \sim (t-t')^{-2}$$ quantum/non-markovian

➡scaling of the noise level

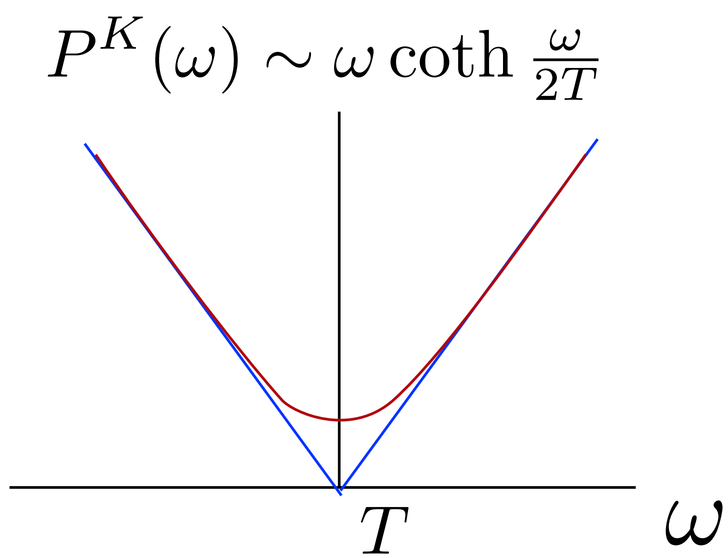
Non-equilibrium analogue of quantum criticality

- strongly momentum dependent noise level

markovian non-equilibrium:
weak noise at long wavelength



equilibrium:
weak noise at long timescales



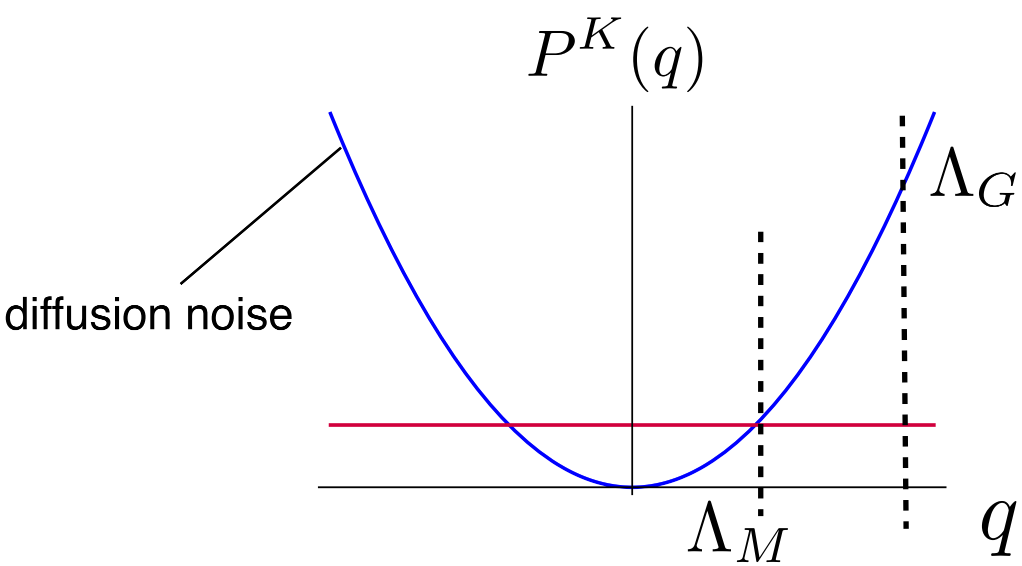
non-eq variant: cf. Dalla Torre et al., Nat Phys. (2010)

- identical canonical scaling to quantum problem for $z = 2 \quad (\omega \sim q^2)$
- but spatial vs. temporal noise

Non-equilibrium analogue of quantum criticality

- strongly momentum dependent noise level

markovian non-equilibrium:
weak noise at long wavelength



equilibrium:
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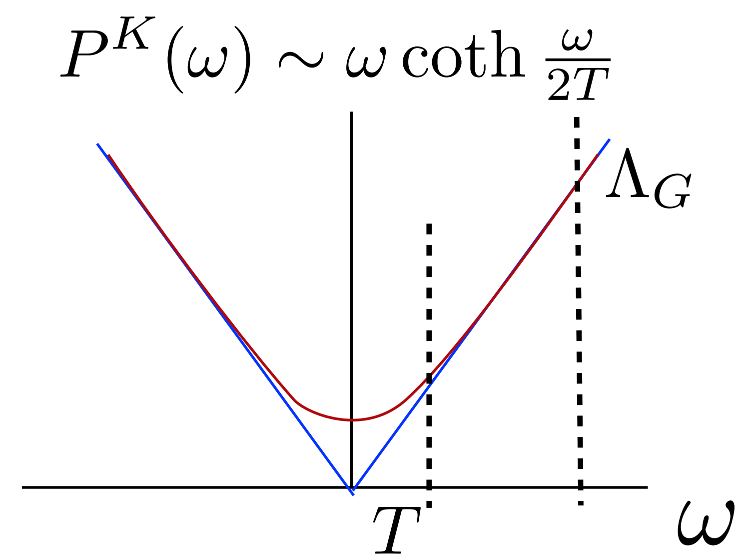


non-eq variant: cf. Dalla Torre et al., Nat Phys. (2010)

- identical canonical scaling to quantum problem for $z = 2 \quad (\omega \sim q^2)$
- but spatial vs. temporal noise

- anomalous scaling regime: two scales

Ginzburg scale

one-loop perturbative

$$\Lambda_G \simeq \frac{\kappa}{\gamma_d}$$

two-body loss

Markov scale

integration of one-loop flow

cf. Chiochetta, Mitra, Gambassi, arxiv (2014)

rescaled Markov noise at FP

$$\Lambda_M \simeq \Lambda_G \left(\frac{\tilde{\gamma}_* + \frac{b_*}{2+a_*}}{2 + \frac{b_*}{2+a_*}} \right)^{\frac{1}{2+a_*}}$$

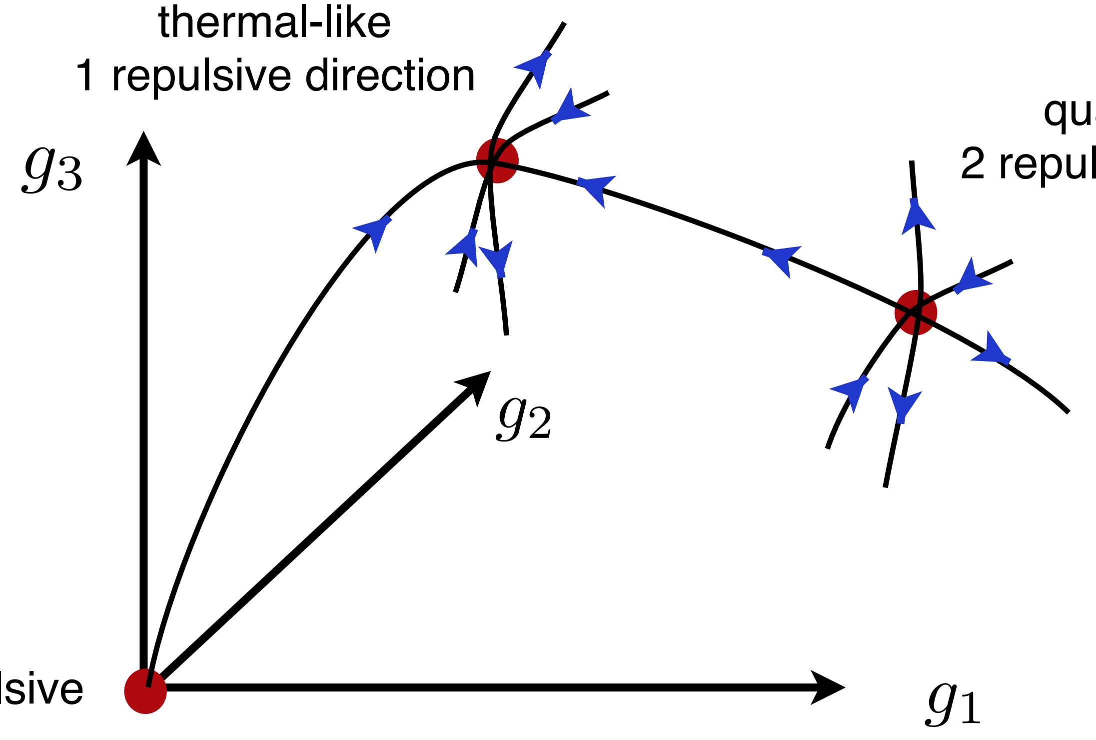
$$a_* \approx 0.3$$

$$b_* \approx 0.2$$

- non-gaussian critical scaling for $\Lambda_M \ll \Lambda_G$

(1) No quantum-classical correspondence

- new fixed point with more repulsive directions (fine tuning of loss rate)



- results for critical exponents

		static			dynamic		noise	
	Crit. Exps.	ν	η_{K_R}	η_{K_I}	η_{ZR}	η_{ZI}	η_{γ_d}	η_{γ}
1+2 dimensions	DD Quantum	0.405	-0.025	-0.025	0.08	0.04	-0.26	$\times$
3 dimensions	DD SC	0.72	-0.22	-0.12	0.16	0	$\times$	-0.16

different degree of divergence of correlations length

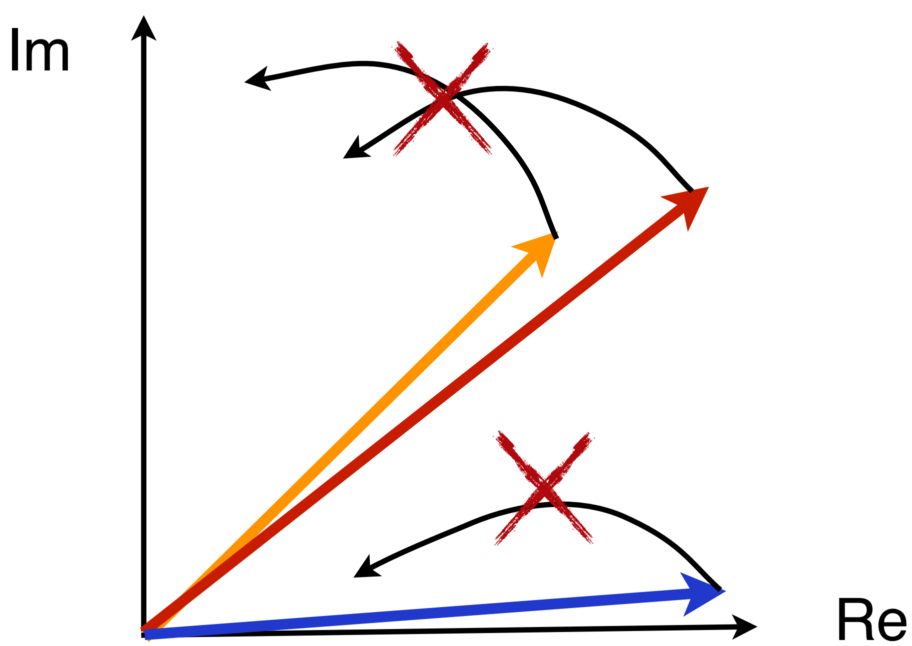
$$\xi \sim (t - t_c)^{-\nu}$$

(2) Absence of Asymptotic Decoherence

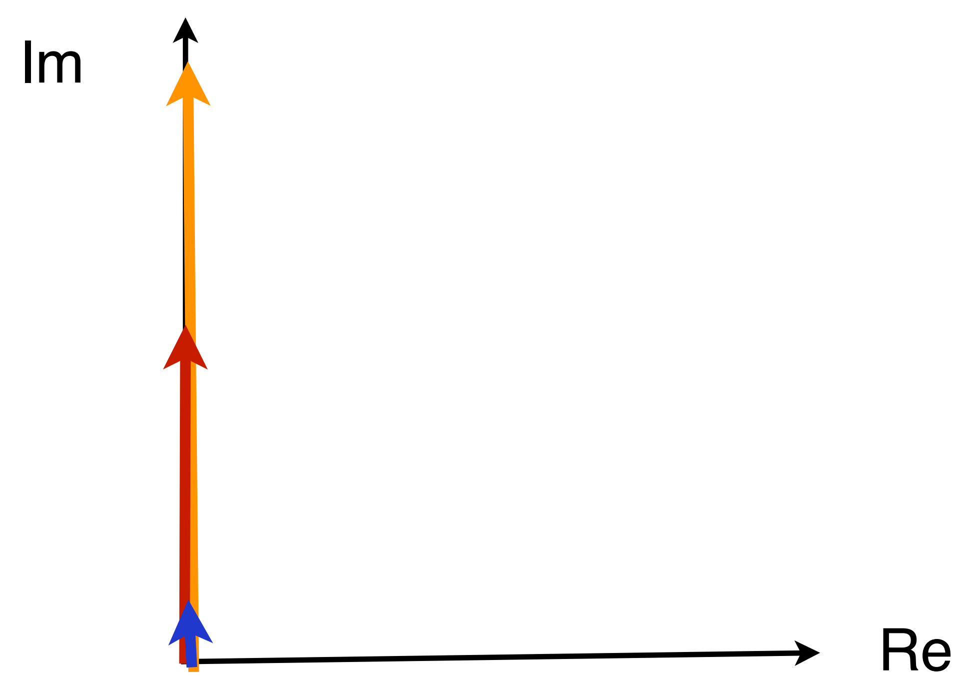
- coherent dynamics does not fade out: $r_i \equiv \dfrac{\mathrm{Im}(g_{i*})}{\mathrm{Re}(g_{i*})} \neq 0$
- exponent degeneracy: $\eta_A = \eta_D = -0.03$ $\quad A \sim k^{\eta_A}, \quad D \sim k^{\eta_D}$

"effective mass" diffusion

quantum



classical



➡ mixed fixed point with finite dissipative and coherent couplings

(3) Absence of Asymptotic Thermalization

- how to detect thermal equilibrium in a quantum system? L. Sieberer, A. Chiochetta, A. Gambassi, U. Tauber, SD, to appear in PRB (2015)

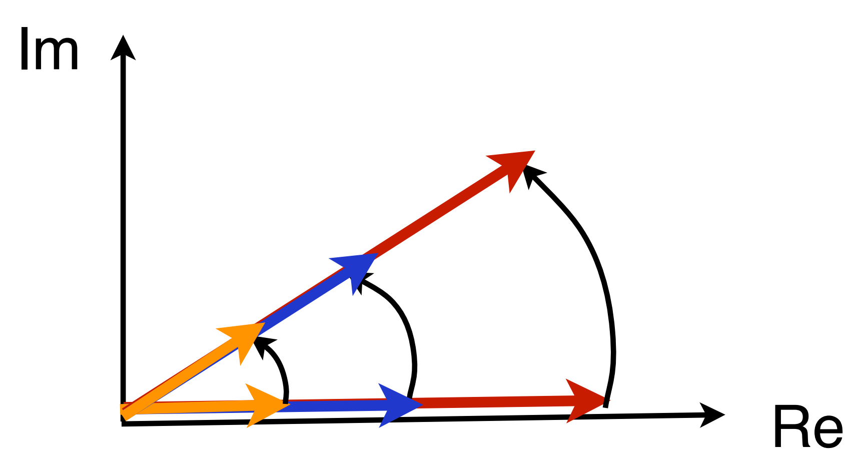
➡ symmetry of Schwinger-Keldysh action under transformation

$$\mathcal{T}_\beta \begin{pmatrix} \Phi_c(\omega, \mathbf{q}) \\ \Phi_q(\omega, \mathbf{q}) \end{pmatrix} = \begin{pmatrix} \sigma_x \cosh(\beta\omega/2) & -\sigma_x \sinh(\beta\omega/2) \\ -\sigma_x \sinh(\beta\omega/2) & \sigma_x \cosh(\beta\omega/2) \end{pmatrix} \begin{pmatrix} \Phi_c(-\omega, \mathbf{q}) \\ \Phi_q(-\omega, \mathbf{q}) \end{pmatrix}$$

$$\Phi_\nu(\omega, \mathbf{q}) = \begin{pmatrix} \phi_\nu(\omega, \mathbf{q}) \\ \phi^*_\nu(-\omega, -\mathbf{q}) \end{pmatrix}$$

- associated “Ward identities” are quantum Fluctuation-Dissipation relations to arbitrary order
- reproduces classical limit for $T \gg \omega$ H. K. Janssen (1976); C. Aron et al, J Stat. Mech (2011)
- present for any microscopically time translation and time reversal invariant Hamiltonian

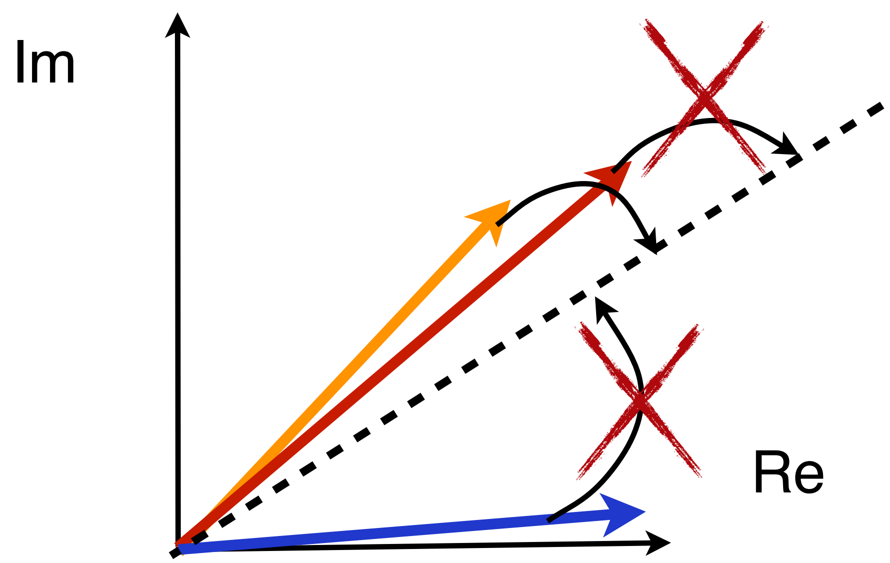
➡ intuition: whenever the dynamics is generated microscopically by a time-independent Hamiltonian, the ensuing irreversible dynamics can be thermal (all scales)



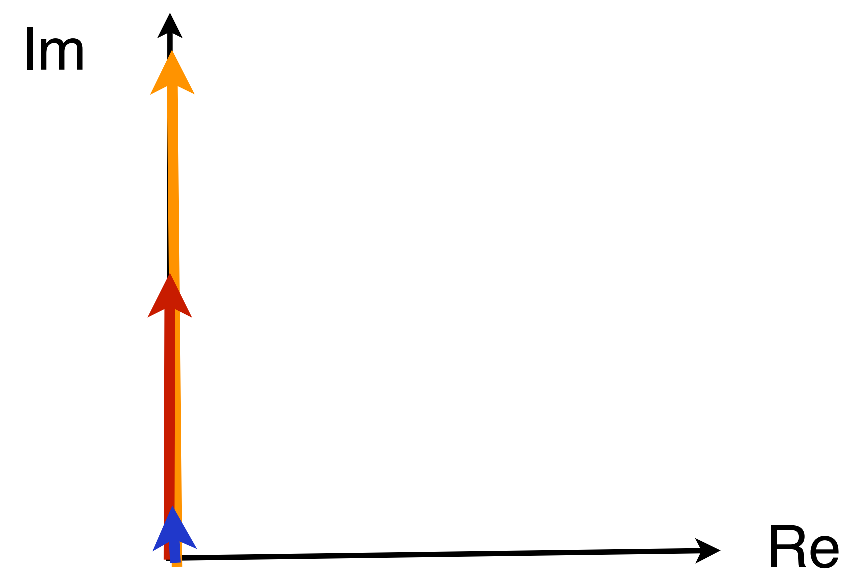
(3) Absence of Asymptotic Thermalization

- practical benefit: symmetry as straightforward diagnostic tool for Schwinger-Keldysh actions
- symmetry explicitly violated microscopically by markovian quantum dynamics
- not emergent:

“quantum”



classical



- formally:

$$\boxed{Z \sim k^{\eta_Z} \quad , \quad \gamma_d \sim k^{\eta_{\gamma_d}}} \qquad Z \sim k^{\eta_Z}, \quad \gamma \sim k^{\eta_\gamma}$$

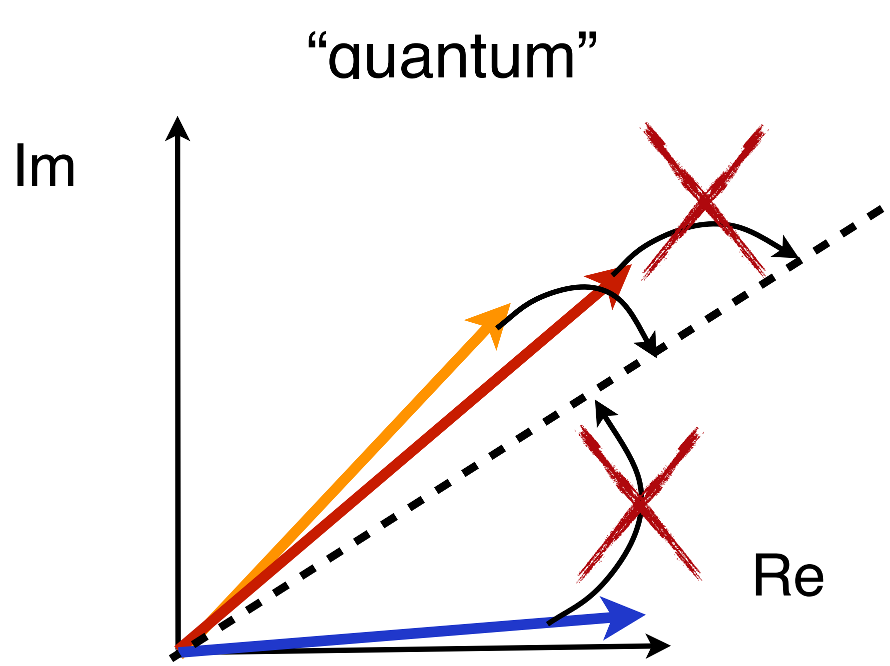
quasiparticle residue — noise level

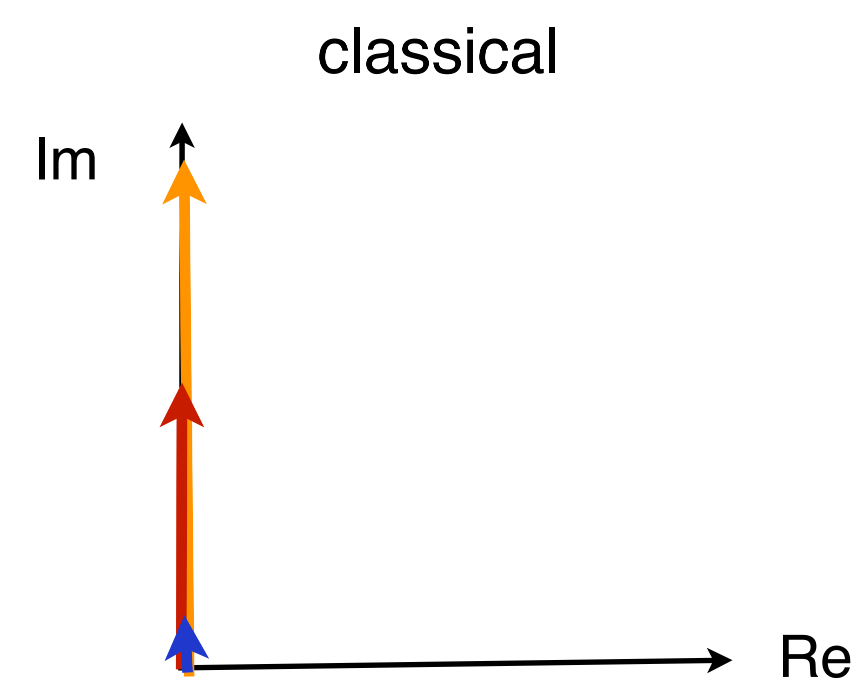
$$\eta_Z = 0.08, \quad , \quad \eta_{\gamma_d} = -0.26 \qquad \eta_Z = \eta_\gamma = 0.16$$

➡ microscopic and universal asymptotic violation of quantum FDR

(3) Absence of Asymptotic Thermalization

- practical benefit: symmetry as straightforward diagnostic tool for Schwinger-Keldysh actions
- symmetry explicitly violated microscopically by markovian quantum dynamics
- not emergent:





- formally:

$$\boxed{Z \sim k^{\eta_Z} e^{\mathrm{i}\eta_Z' \log k/\Lambda}, \qquad \gamma_d \sim k^{\eta_{\gamma_d}}}$$

quasiparticle residue — noise level

$$Z \sim k^{\eta_Z}, \qquad \gamma \sim k^{\eta_\gamma}$$

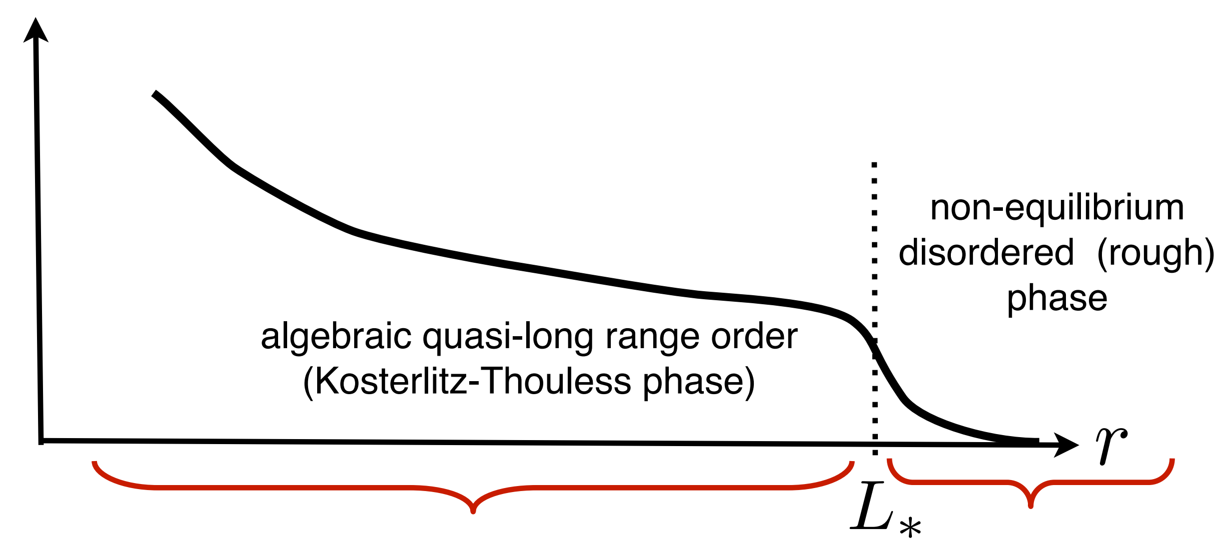
$$\eta_Z = 0.08, \quad \eta_Z' = 0.03, \quad \eta_{\gamma_d} = -0.26$$

$$\eta_Z = \eta_\gamma = 0.16$$

➡ limit-cycle like oscillations with (huge!) period
(observable: spectral density)

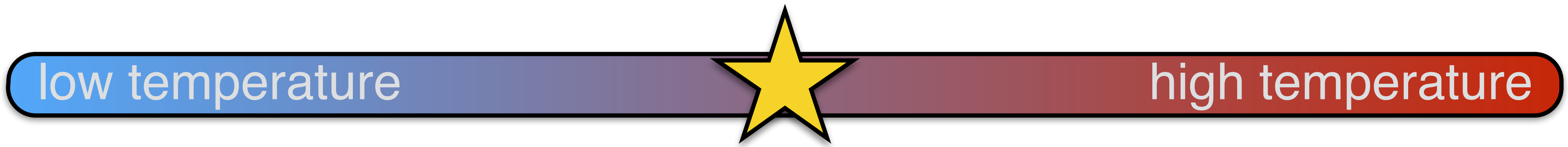
$$\frac{k_{n+1}}{k_n} = e^{\frac{2\pi}{\eta_Z'}}$$

Fate of the Kosterlitz-Thouless transition in Driven Systems



E. Altman, L. Sieberer, L. Chen, SD, J. Toner, PRX (2015)
E. Altman, SD, L. Sieberer, G. Wachtel, in preparation

A paradigm of equilibrium stat mech: (no) BEC in 2D



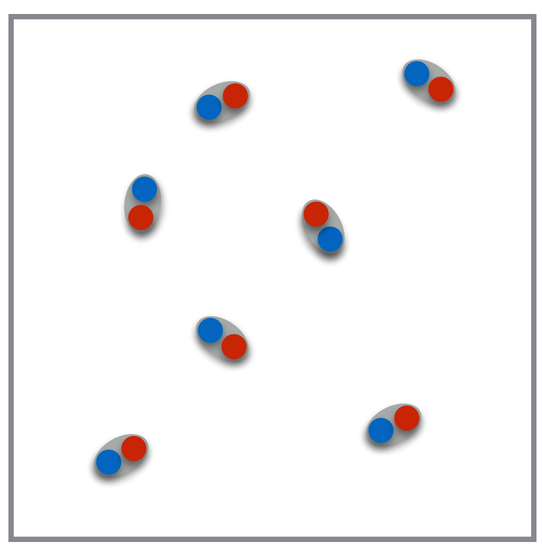
- correlations

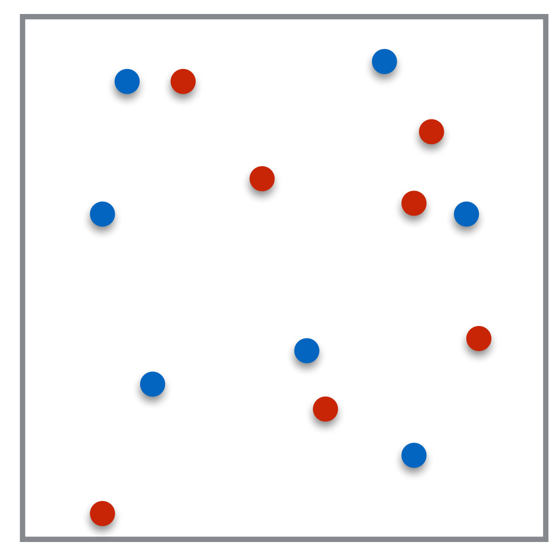
$$\langle \phi(r)\phi^*(0)\rangle \sim r^{-\alpha} \qquad\qquad \sim e^{-r/\xi}$$

- superfluidity

$$\rho_s \neq 0 \qquad\qquad \rho_s = 0$$

- KT transition: unbinding of vortex-antivortex pairs





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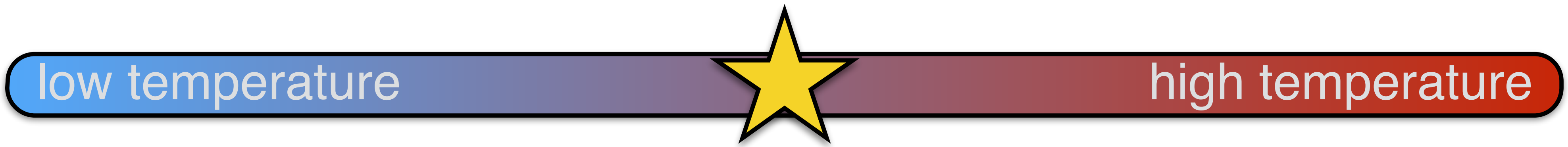

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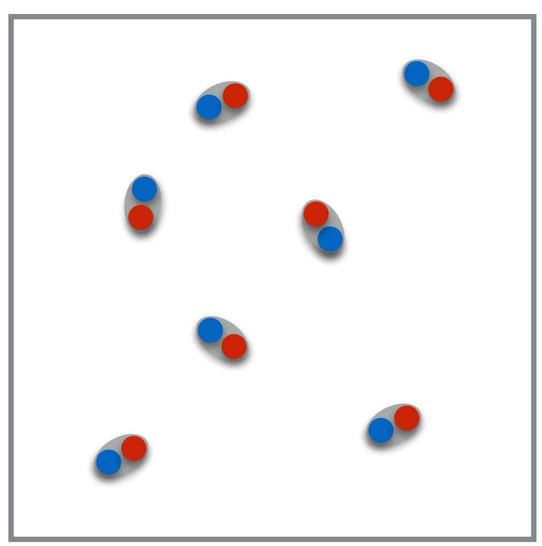
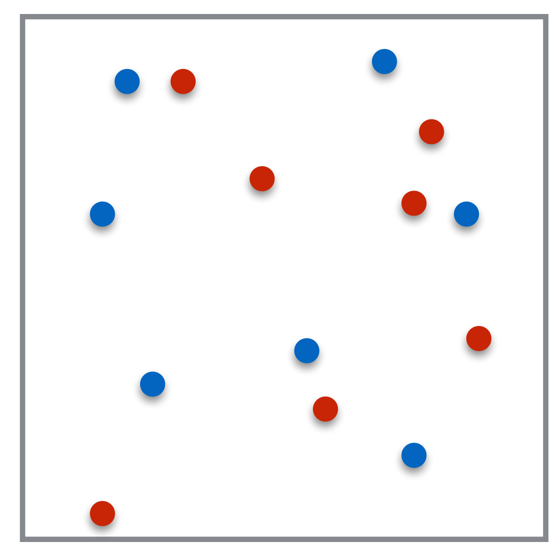




… also for driven-dissipative condensates?

E. Altman, L. Sieberer, L. Chen, SD, J. Toner
PRX (2015)

Fate of correlations in 2D driven systems

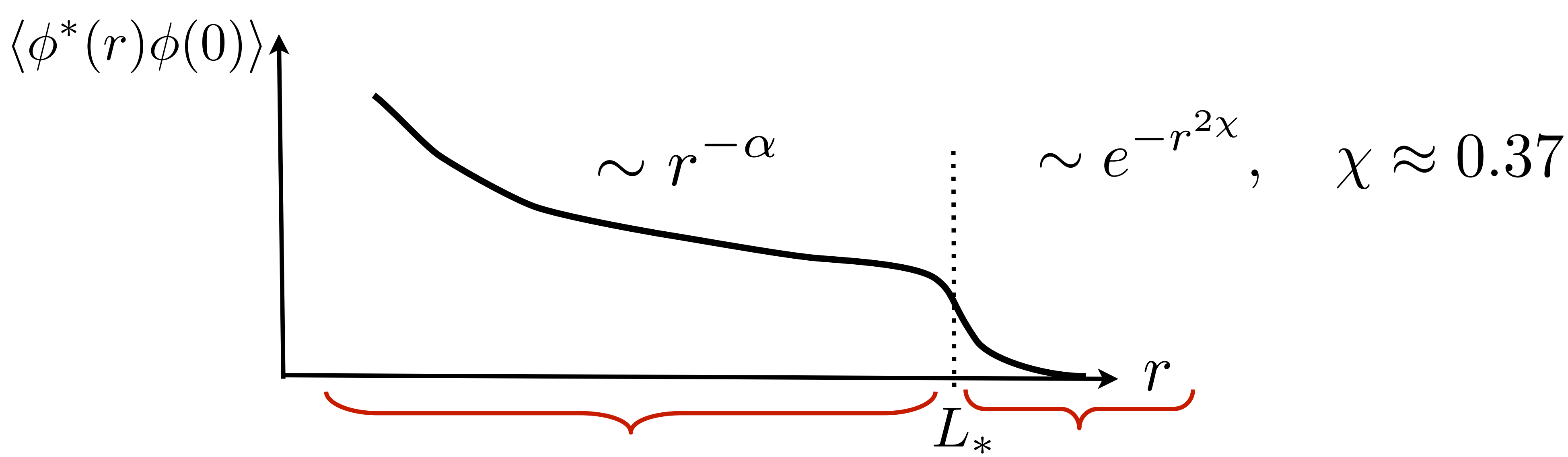
- spin waves become non-linear, described by KPZ equation (surface roughening)

Kardar, Parisi, Zhang, PRL (1986)

$$\partial_t \theta = D\nabla^2\theta + \lambda(\nabla\theta)^2 + \xi$$

absent in equilibrium by symmetry

- implications:
  - a length scale is generated: $L_* = a_0 e^{\frac{2\pi}{\lambda^2}}$
  - beyond this scale, expect KPZ scaling physics



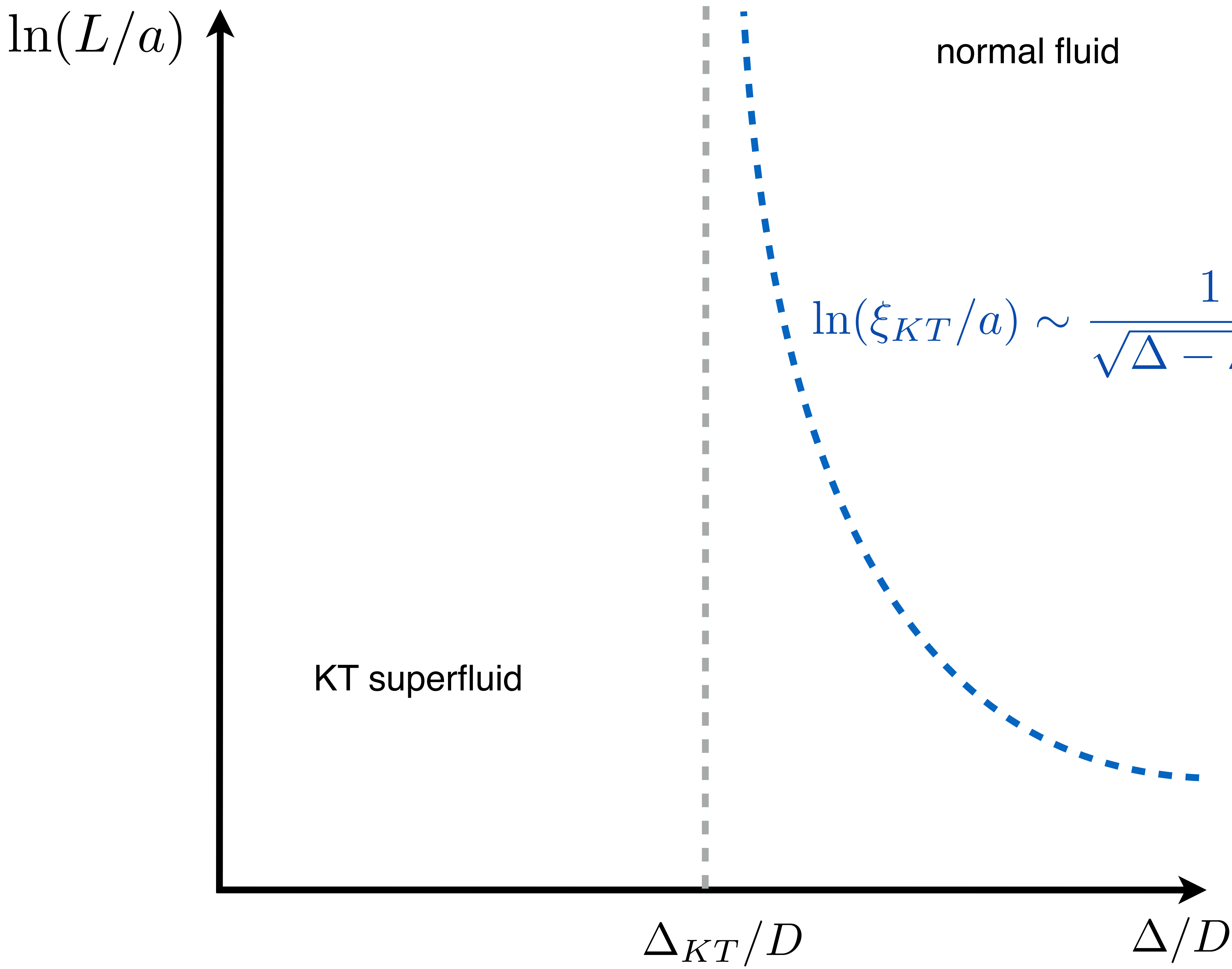
algebraic quasi-long range order (Kosterlitz-Thouless phase)

non-equilibrium disordered (rough) phase

➡ algebraic order absent in any two-dimensional driven open system at the largest distances

Finite-size phase diagram

- (equilibrium) Kosterlitz-Thouless vs. KPZ



Finite-size phase diagram

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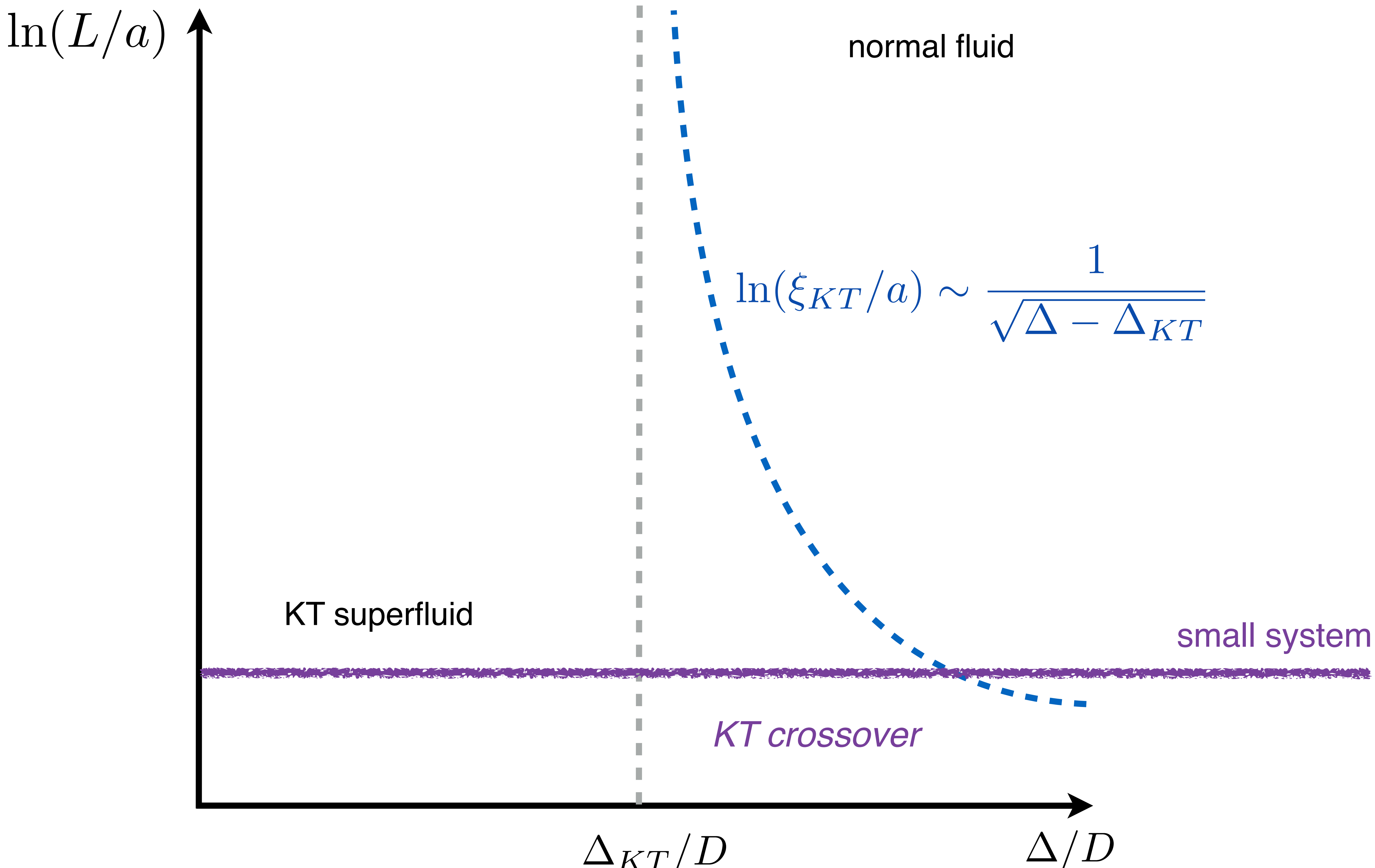

experiments: Yamamoto group, PNAS, 2012 & PRB, 2014
numerics: Dagvadorj et al, arXiv, 2014

Finite-size phase diagram

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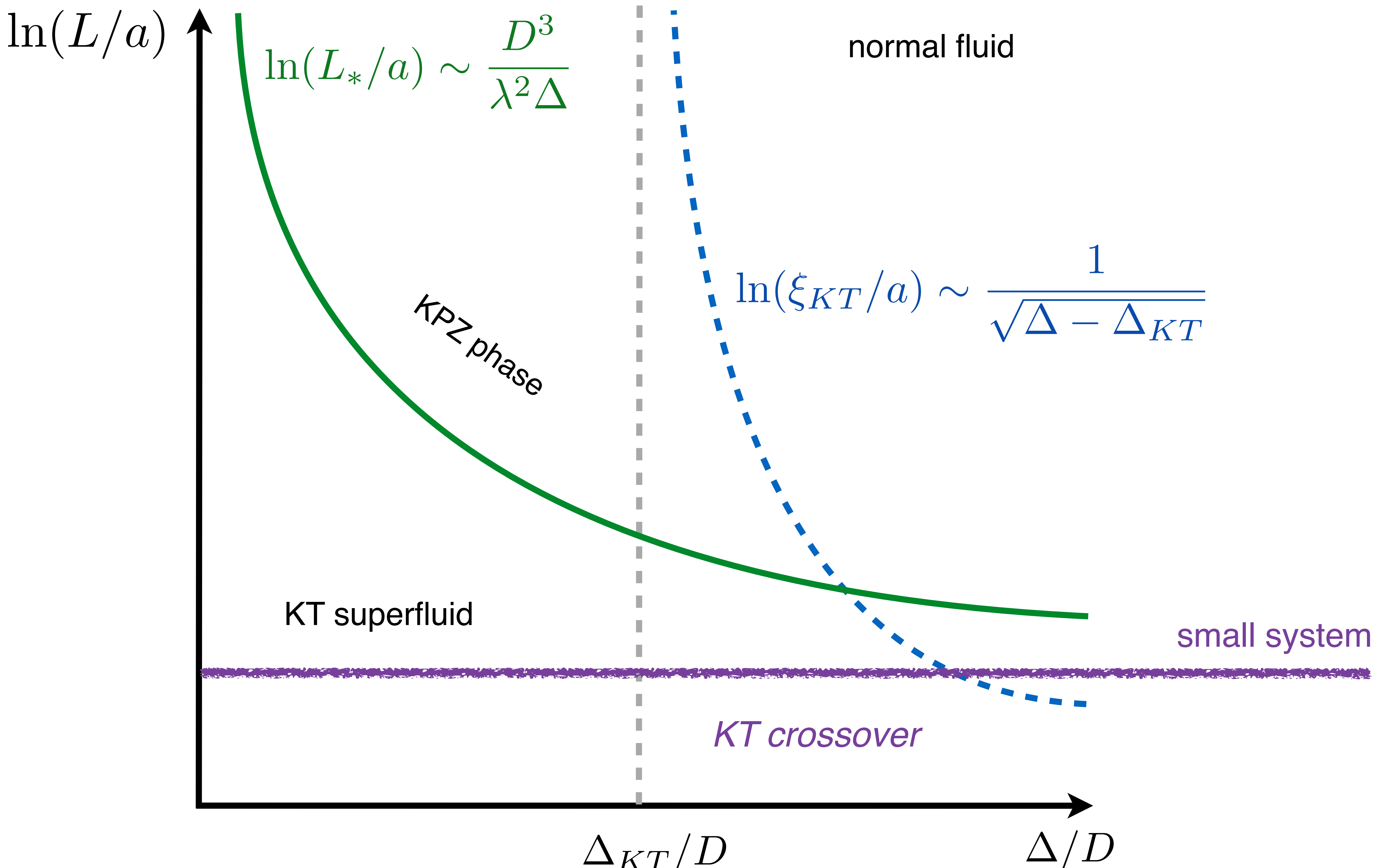

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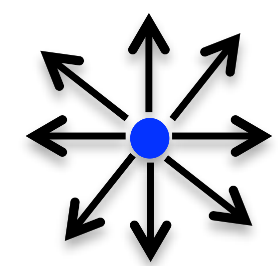
Non-equilibrium Kosterlitz-Thouless

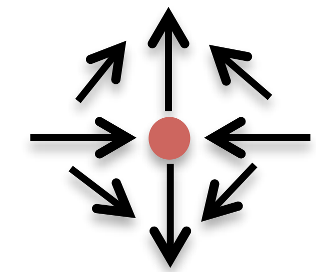
- KPZ equation for phase variable

$$\partial_t \theta = D\nabla^2\theta + \lambda(\nabla\theta)^2 + \xi$$

- compact nature of phase allows for vortex defects in 2D!



vortex

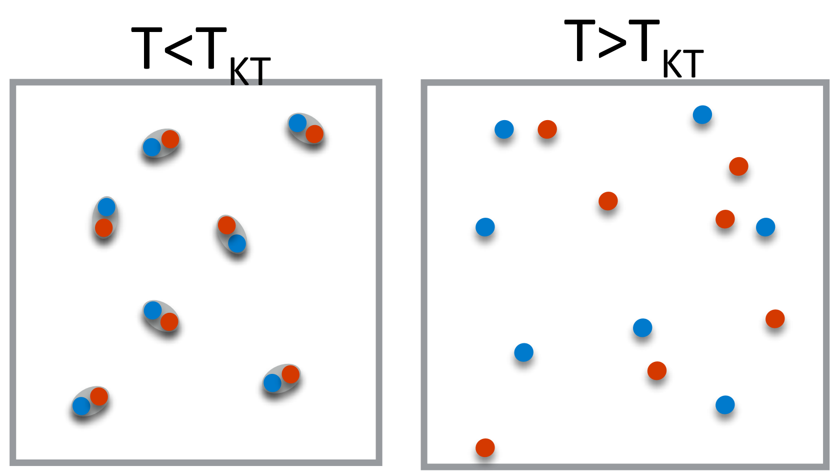


anti-vortex

- in 2D equilibrium: perfect analogy between vortices and electric charges
  - log(r) interactions, $1/(\epsilon r)$ forces
  - dielectric constant ϵ^{-1}= superfluid stiffness

superfluid = dipole gas

$\epsilon^{-1} > 0$



normal fluid = plasma
metallic screening

$\epsilon^{-1} \to 0$

➡ how is this scenario modified in the driven system?

Electrodynamic Duality

- standard identification:

$$\rho - \bar{\rho} \equiv B\hat{\mathbf{z}} \qquad\qquad \hat{\mathbf{z}} \times \nabla\theta \equiv \mathbf{E}$$

- Modified Maxwell equations

irrotational flow

$$\nabla \cdot \mathbf{E} = 2\pi n_v$$

modified continuity eq

$\partial_t \to 1/D$

$$\nabla \times \mathbf{E} + \frac{1}{D}\mathbf{B} = 0$$

phase dynamics (compact KPZ)

$$\nabla \times \mathbf{B} - \frac{\partial \mathbf{E}}{\partial t} = 2\pi \mathbf{J}_v - \hat{\mathbf{z}} \times \nabla \left(\frac{\lambda}{2} E^2 + \bar{\zeta} \right)$$

$$\nabla \cdot \mathbf{B} = 0$$

vortex density & current

KPZ non-linearity and noise

over-damped vortex dynamics (ignoring mag field)

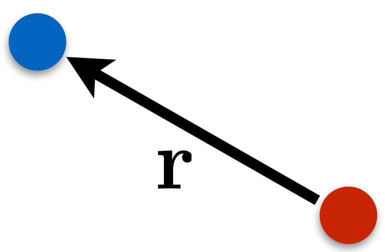
$\ddot{\mathbf{r}}_i \to \dot{\mathbf{r}}_i$

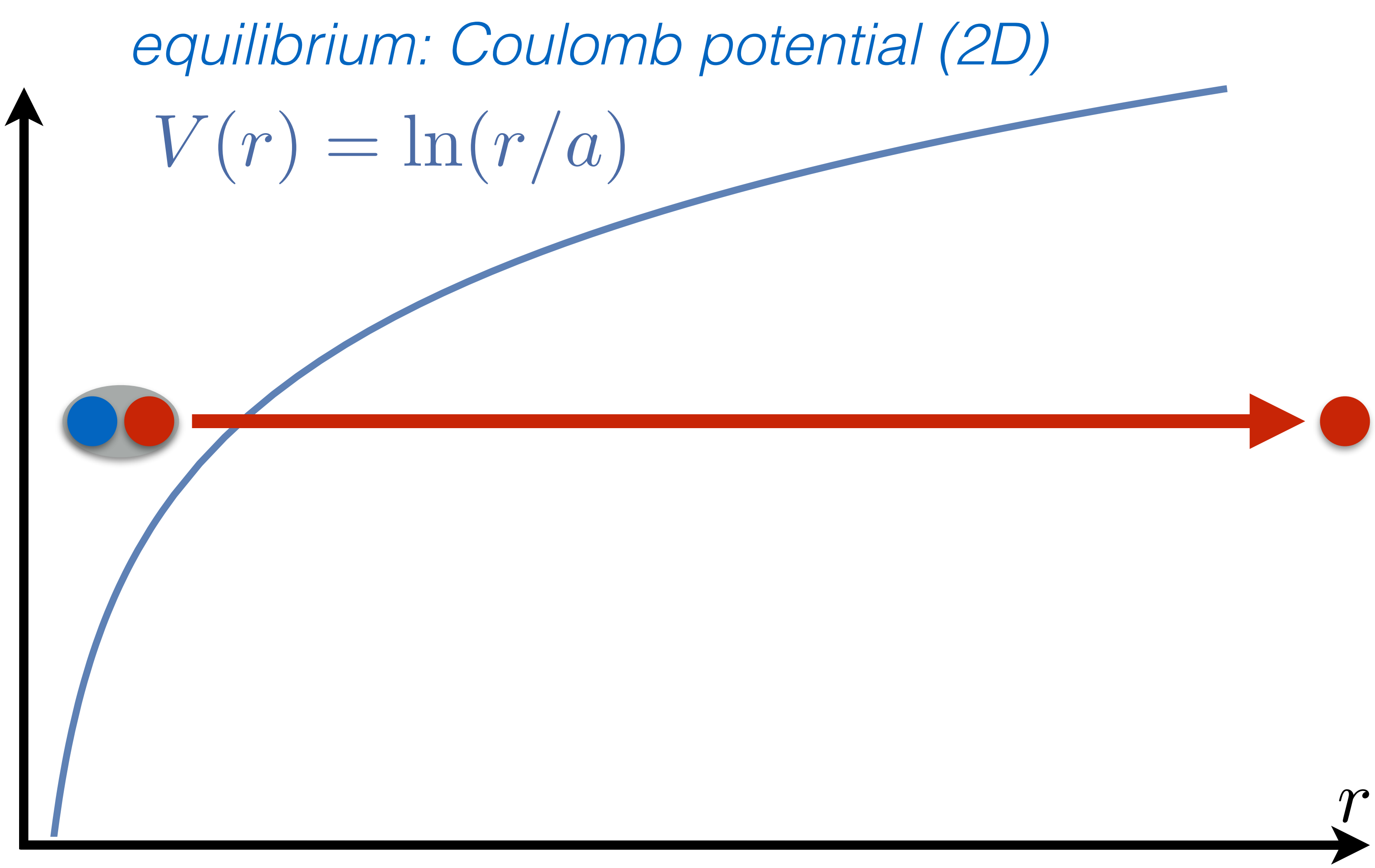
$$\frac{d\mathbf{r}_i}{dt} = \mu n_i \mathbf{E}(t, \mathbf{r}_i) + \boldsymbol{\xi}_i$$

A single vortex-antivortex pair

- close to the transition: dilute gas of vortices
- equation of motion for a single vortex-antivortex pair

$$\frac{d\mathbf{r}}{dt} = -\mu \nabla V(r) + \boldsymbol{\xi}$$





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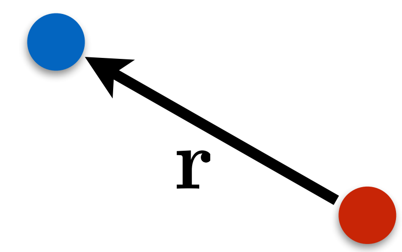

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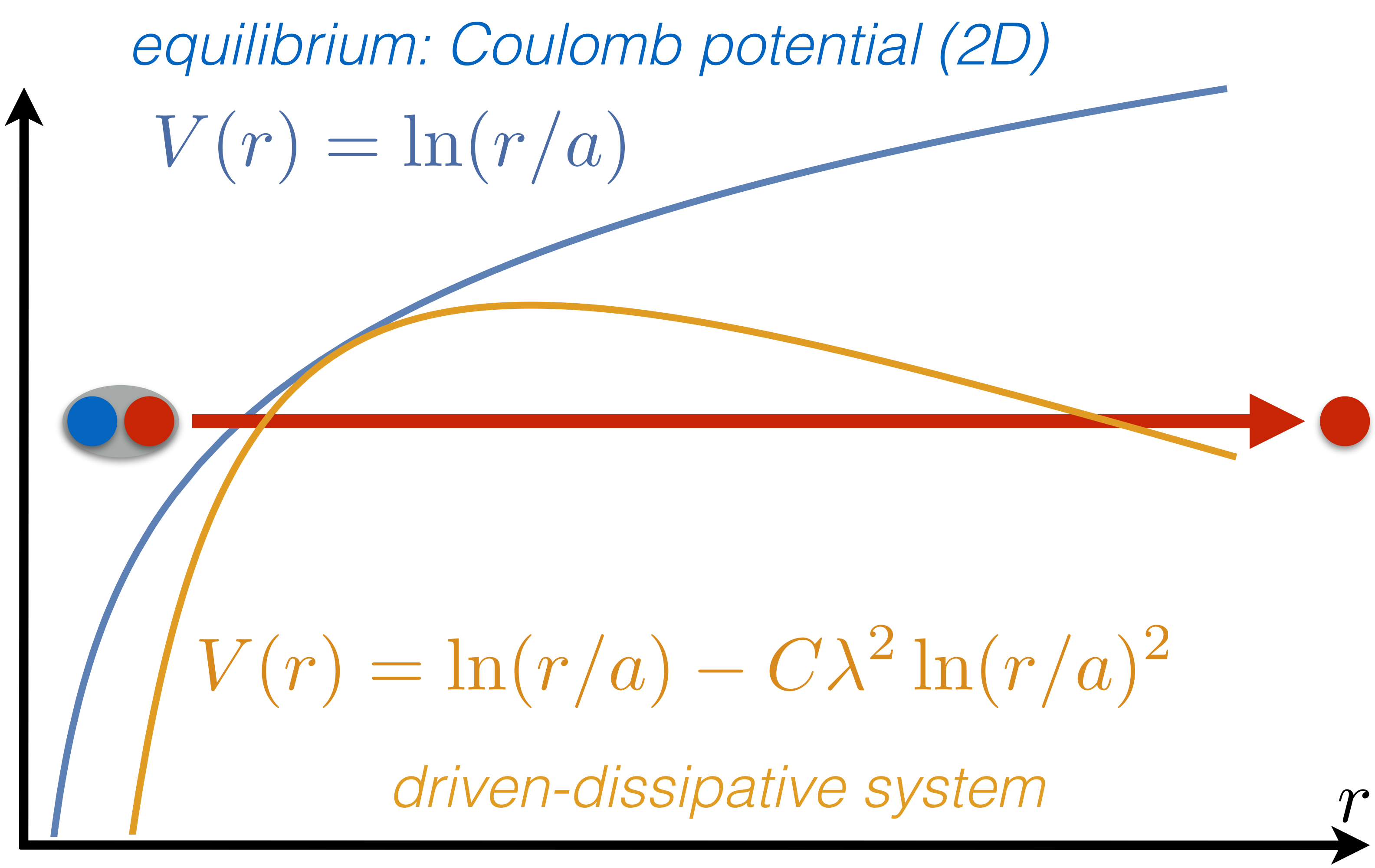

noise-activated unbinding for a single pair (at exp small rate)

Modified Kosterlitz-Thouless RG flow

$$\frac{d\varepsilon}{d\ell} = \frac{2\pi^2 y^2}{\Delta} \qquad \frac{dy}{d\ell} = \left(2 - \frac{1}{2\varepsilon\Delta} + \frac{C\lambda^2}{\varepsilon^2 D^2}\right) y \qquad \frac{d\Delta}{d\ell} = \frac{2C\lambda^2\Delta}{\varepsilon^2 D^2}$$

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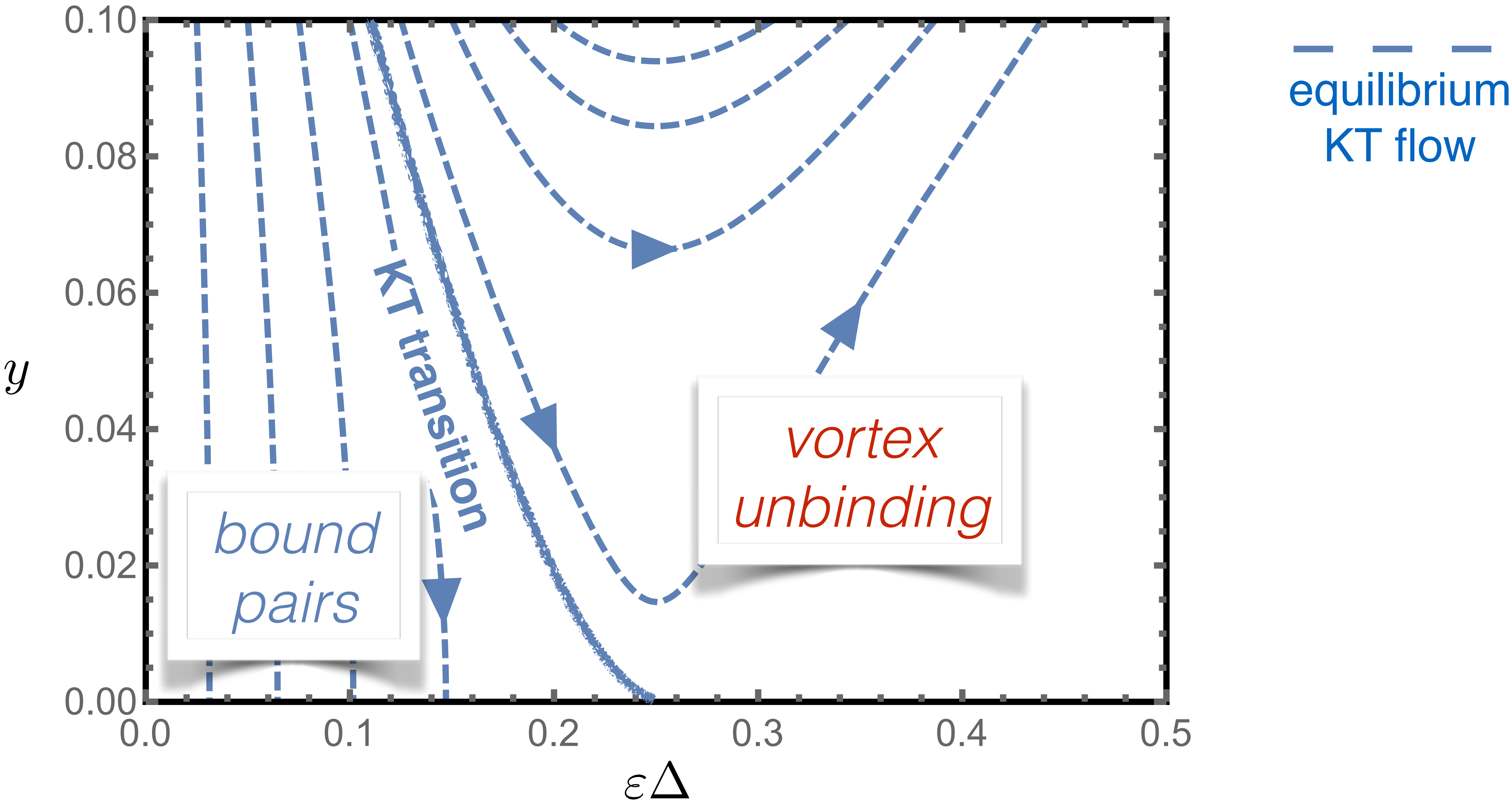

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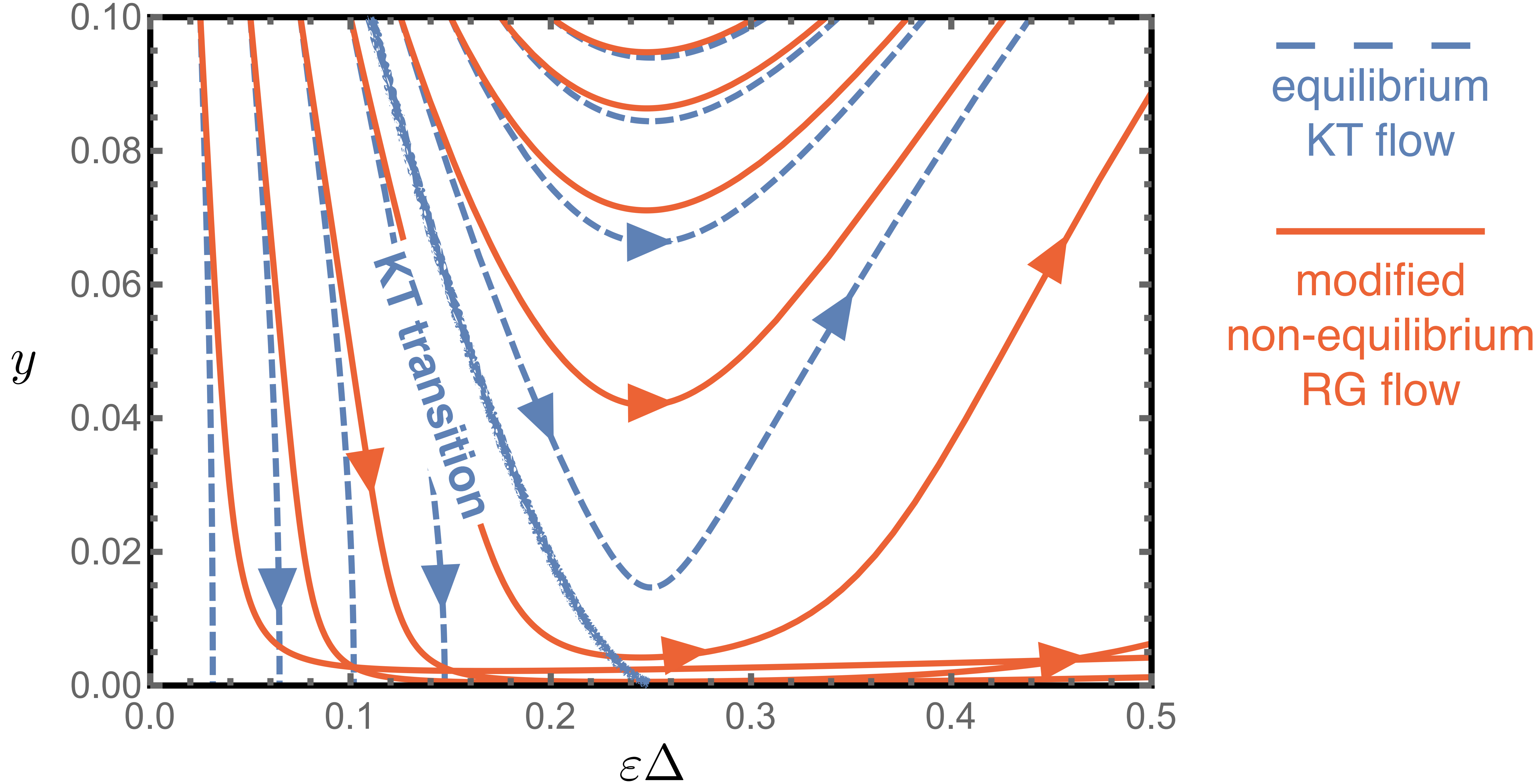

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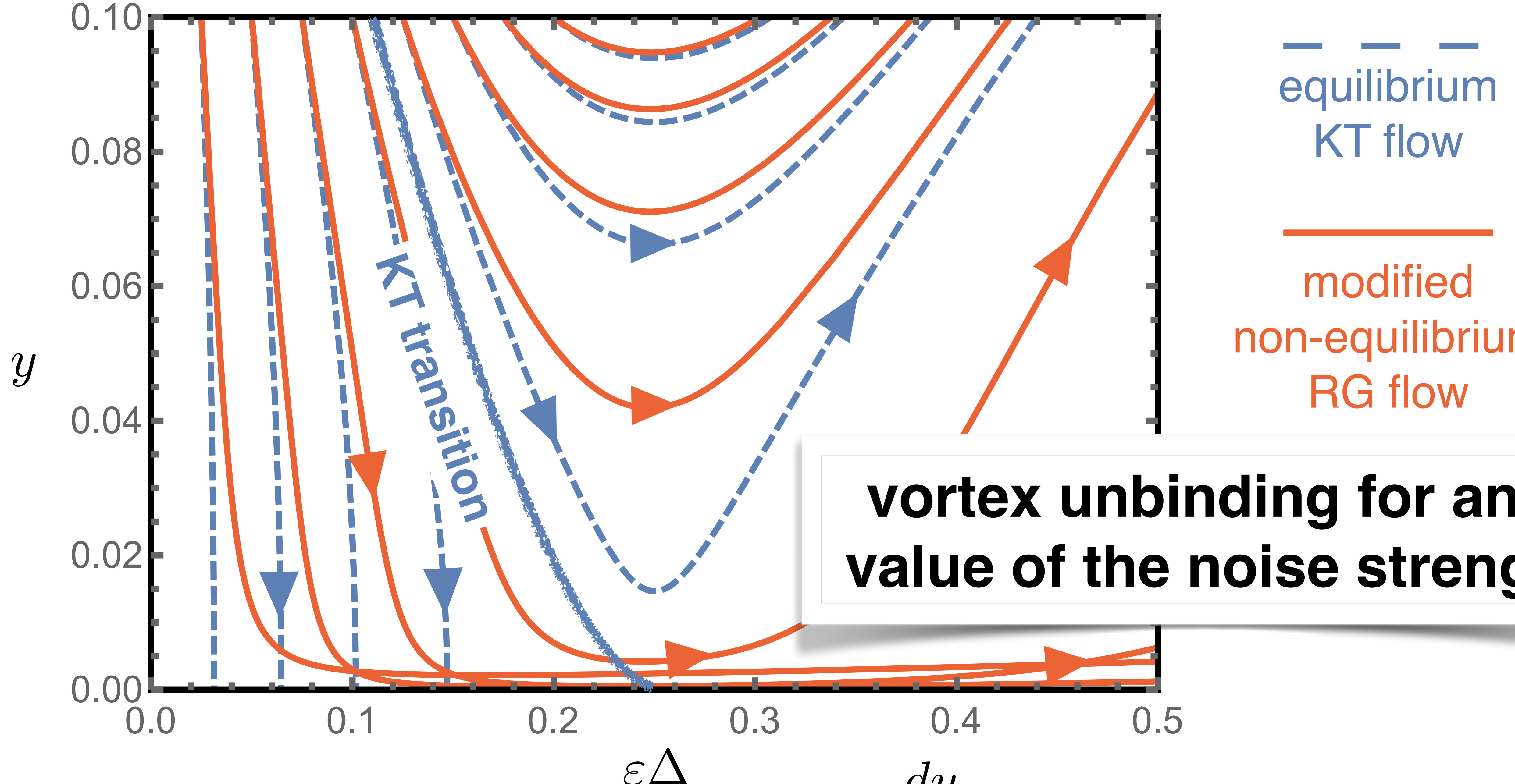
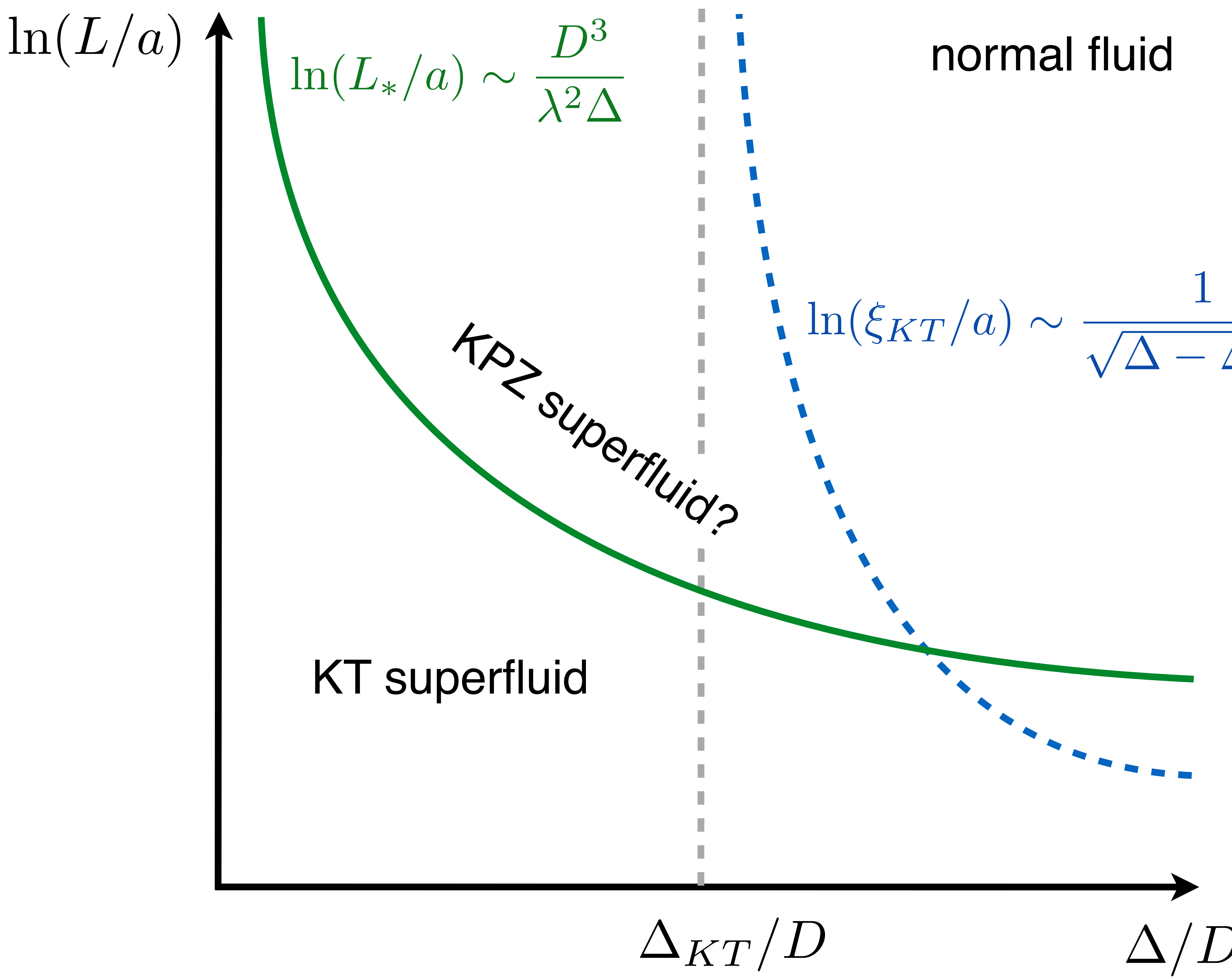


vortex unbinding for any value of the noise strength

$\frac{dy}{d\ell}$ changes sign at a scale L_v

Finite-size phase diagram



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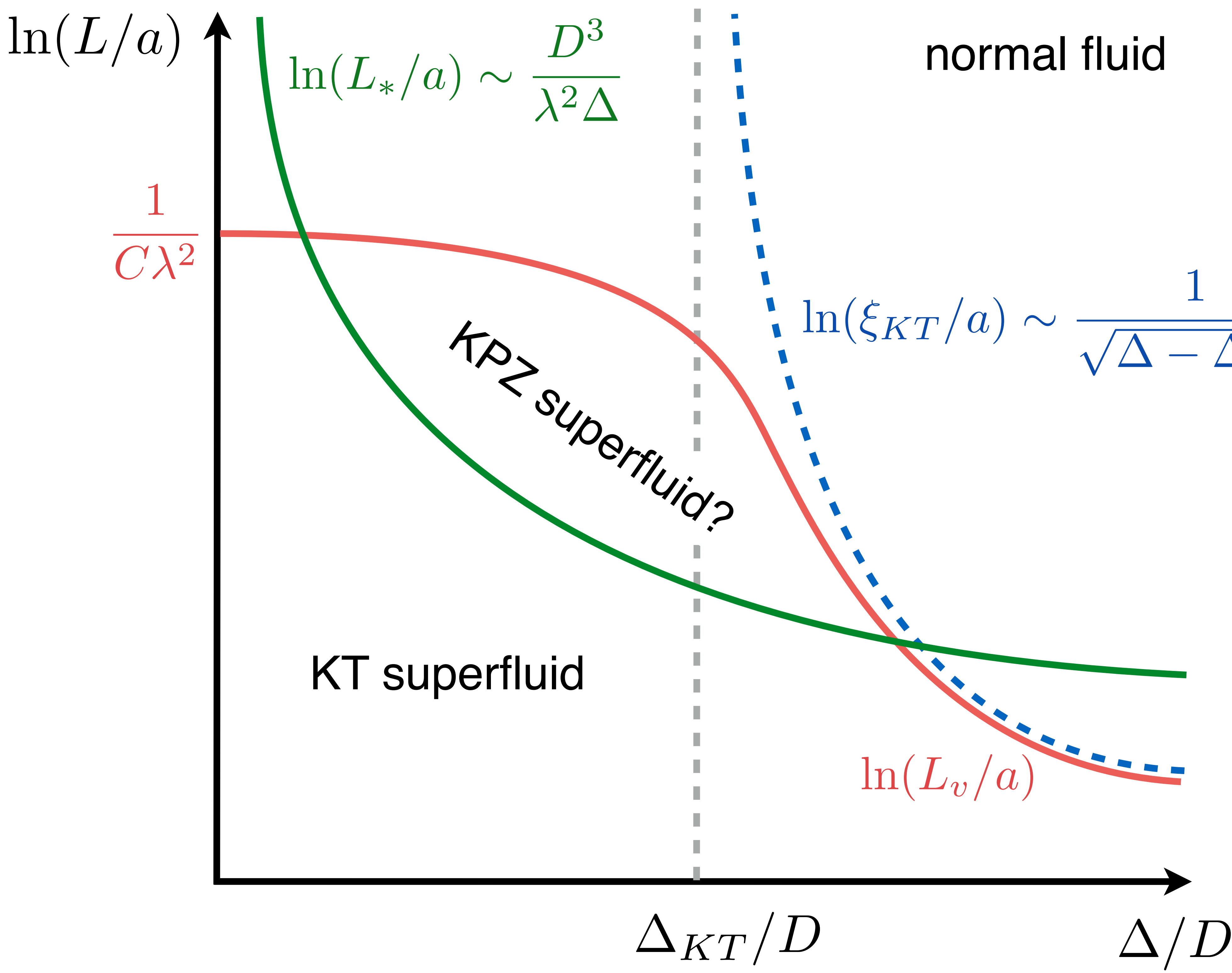

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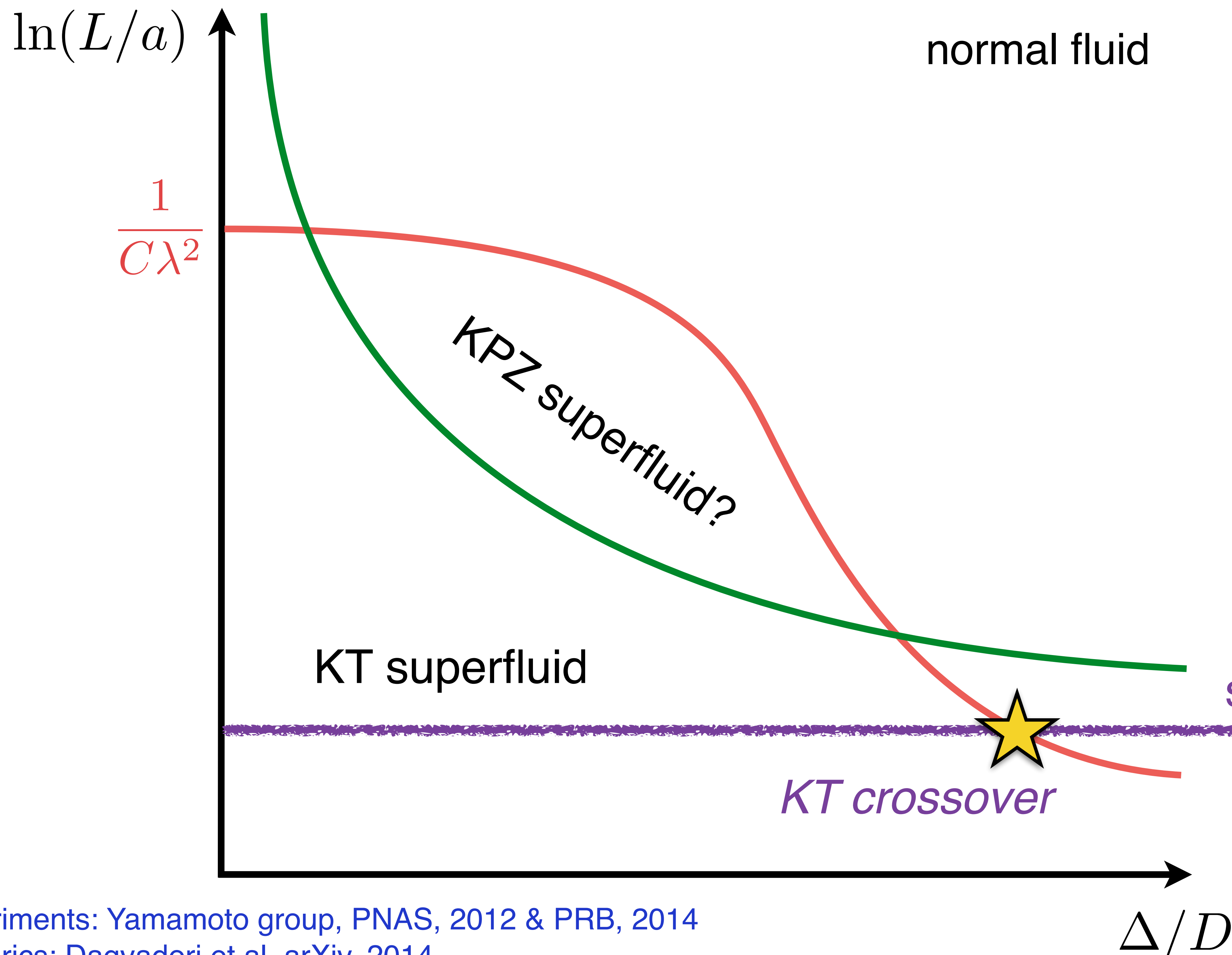

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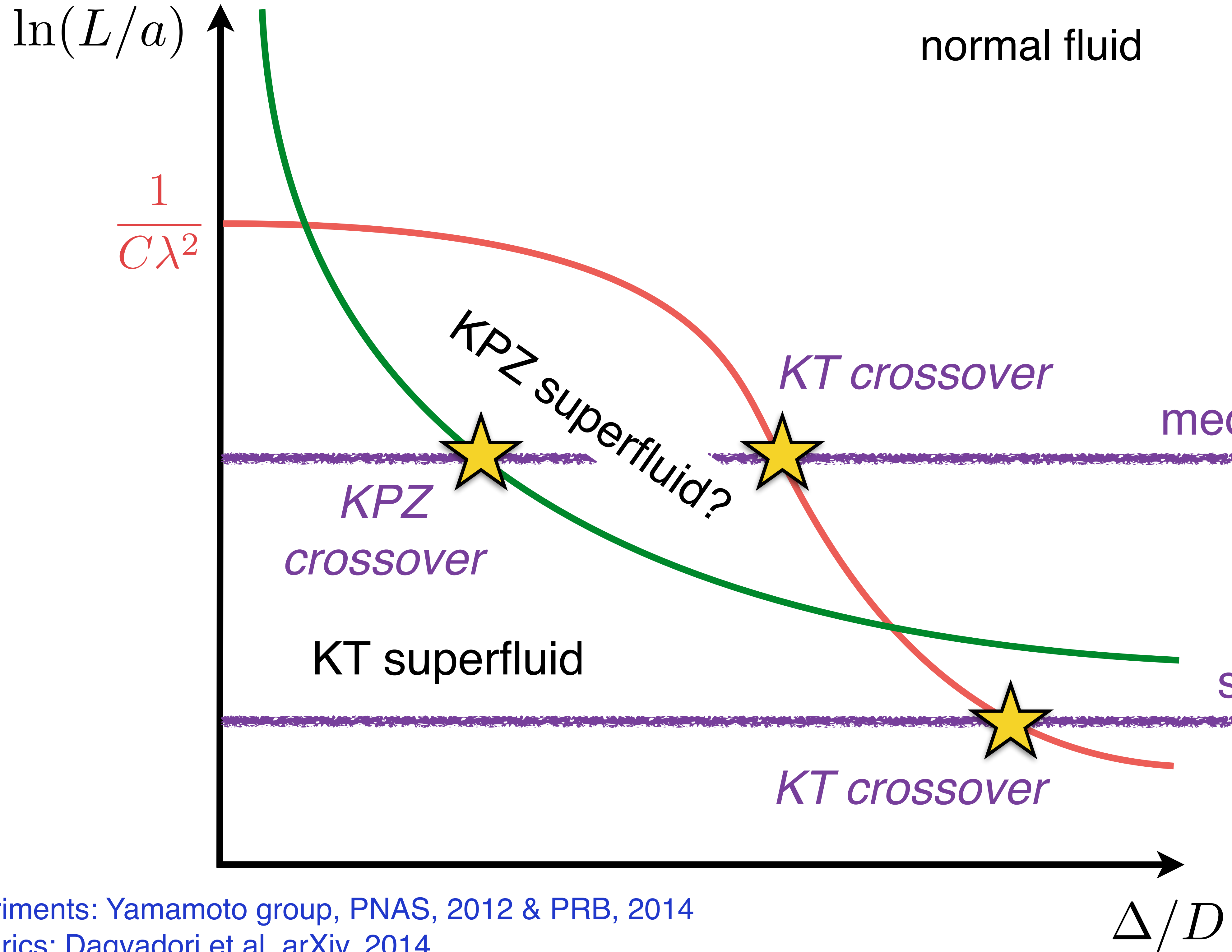

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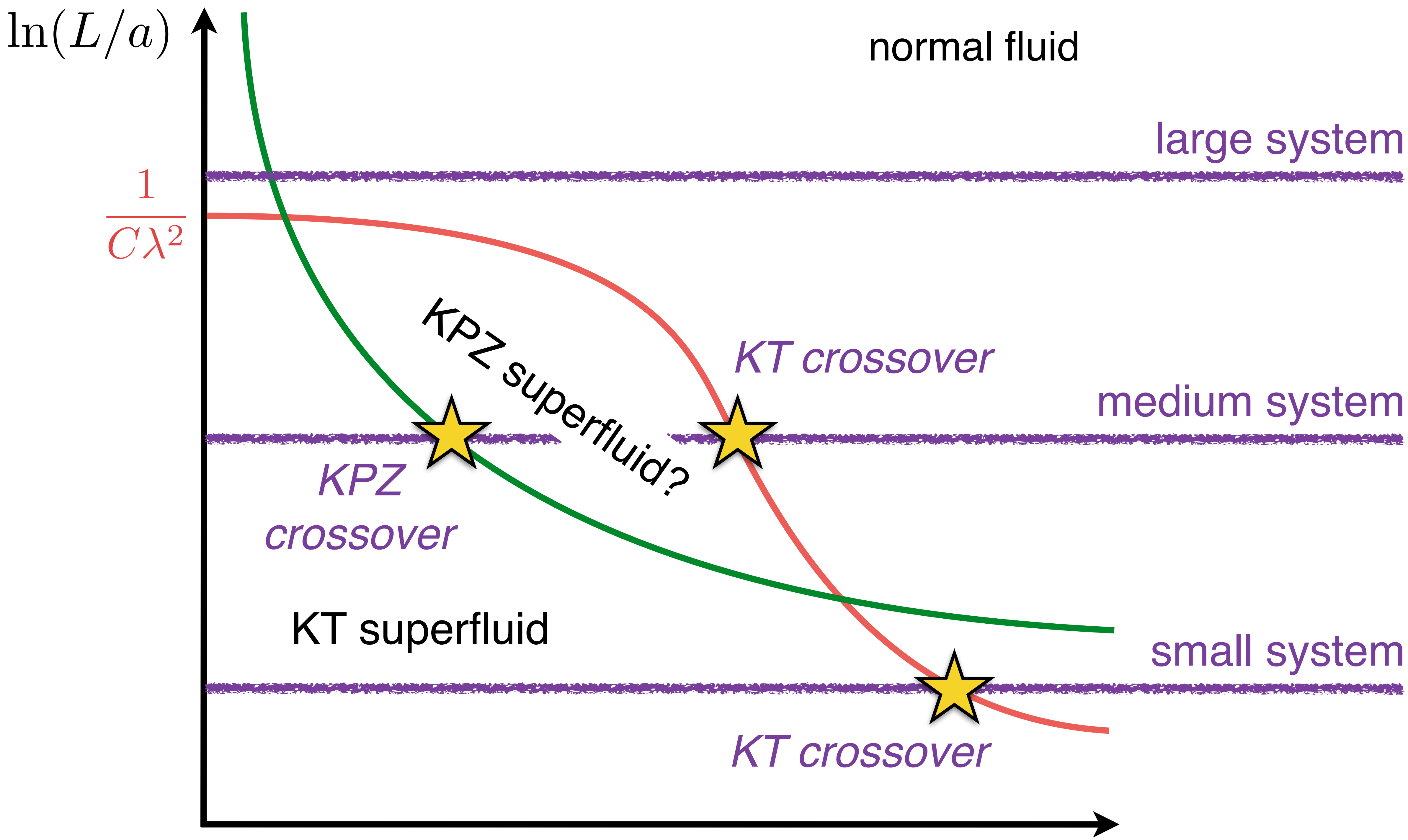
Finite-size phase diagram



experiments: Yamamoto group, PNAS, 2012 & PRB, 2014
numerics: Dagvadorj et al, arXiv, 2014

Summary: Universal non-equilibrium phenomena

1D quantum
New driven universality class

- non-equilibrium persists: no thermalization
- quantum persists: no decoherence
- limit cycle for quasiparticle residue

2D classical
Compact KPZ universality class

- no low-noise ordered phase as in KT
- rich structure of finite size crossovers
  - small systems: eq. like
  - larger systems: non-compact KPZ universality?
  - thermodyn. limit: free vortices

➡ challenge to experiments: universality requires large system sizes!

- PhD & Postdoc positions available within ERC Consolidator grant “Many-body Physics with Driven Open Quantum Systems of Atoms, Light and Solids” (DOQS)!

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