Entanglement and coherence in many-body dipolar systems

Susanne Yelin

Rui Li, Guin-Dar Lin, Gray Putnam, Efi Shahmoon, Elie Wolfe

University of Connecticut

Harvard University

KITP, Many-body physics with light, Oct 8, 2015

Entanglement and coherence in many-body dipolar systems

superradiant

Susanne Yelin

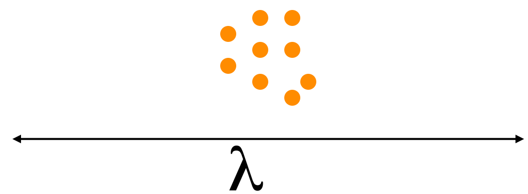
Rui Li, Guin-Dar Lin, Gray Putnam,
Efi Shahmoon, Elie Wolfe

University of Connecticut

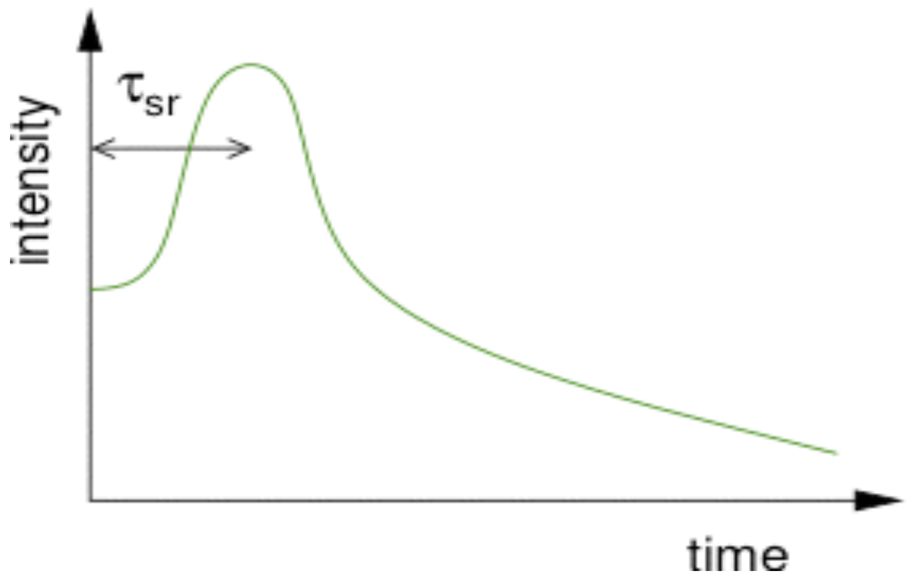
Harvard University

$$: ARO, NSF

Atom-atom correlations in superradiance: Classic example

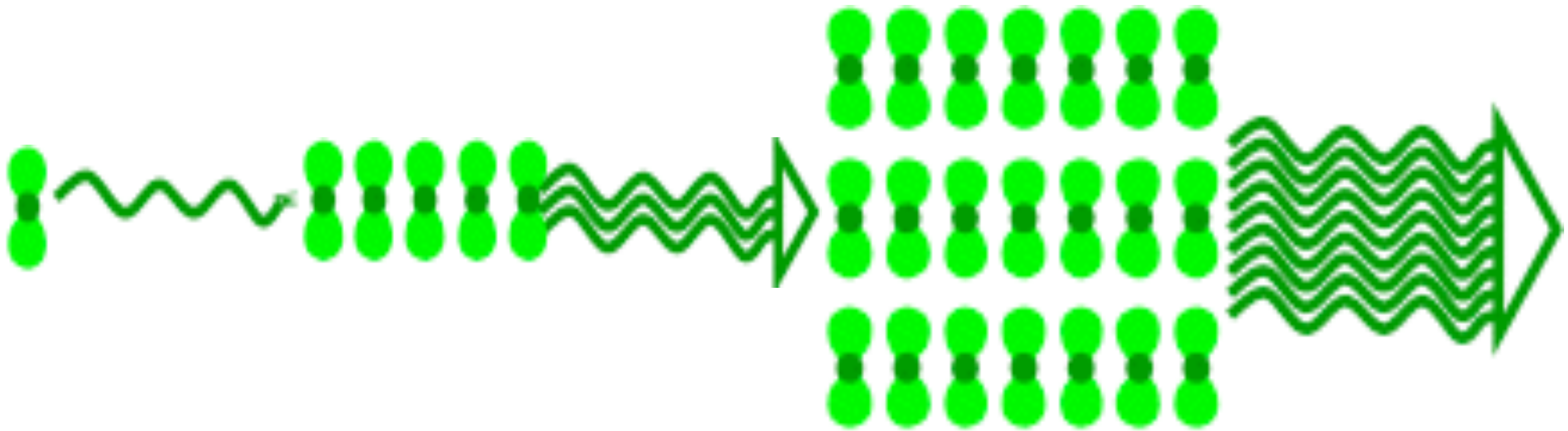


- Superradiance



Gross, Haroche, Phys. Rep. **93**, 301 ('82)

What is super in superradiance?



- Superradiance (superfluorescence) $\propto N^2$ (particle number)
- Build-up of collective dipoles

What is "superradiance"?

1. Everything that involves Dicke states
   - (e.g., collective √N effects,
   - bad-cavity limit,
   - …)
2. Only systems involving cooperative (and nonlinear) effects
   - i.e., effect of exchange int
   - more than single excitation

only for purists

Questions - guideline

- Superradiance - What? Why?
- How do we calculate it (better)?
- Is there a collective (Lamb) shift?
- How does entanglement come into the picture?

Questions - guideline

- Superradiance - What? Why?

so far…

- ideal (Dicke)

PHYSICAL REVIEW VOLUME 93, NUMBER 1 JANUARY 1, 1954

Coherence in Spontaneous Radiation Processes

R. H. Dicke
Palmer Physical Laboratory, Princeton University, Princeton, New Jersey
(Received August 25, 1953)

By considering a radiating gas as a single quantum-mechanical system, energy levels corresponding to certain correlations between individual molecules are described. Spontaneous emission of radiation in a transition between two such levels leads to the emission of coherent radiation. The discussion is limited first to a gas of dimension small compared with a wavelength. Spontaneous radiation rates and natural line breadths are calculated. For a gas of large extent the effect of photon recoil momentum on coherence is calculated. The effect of a radiation pulse in exciting "super-radiant" states is discussed. The angular correlation between successive photons spontaneously emitted by a gas initially in thermal equilibrium is calculated.

Dicke states

Fully symmetric state of n excitations in N particles, for example

$$|\mathbf{2}\rangle_4 = \frac{1}{\sqrt{6}}\left(|1100\rangle + |1010\rangle + |1001\rangle + |0110\rangle + |0101\rangle + |0011\rangle\right)$$

N-particle Dicke states decay with up to N^2 speedup

so far…

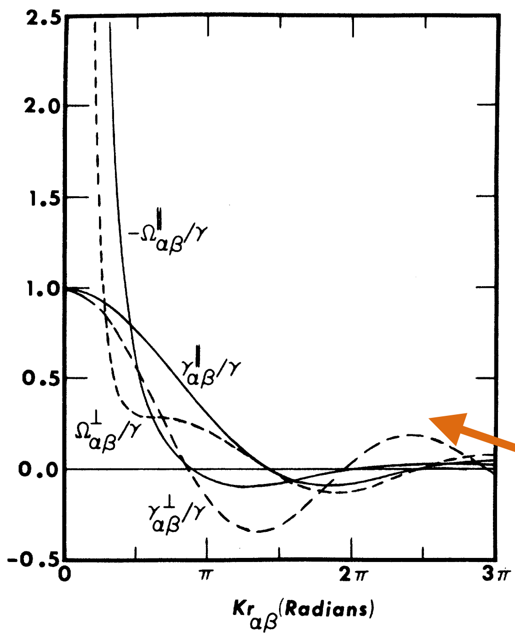
- ideal (Dicke)
- classica

PHYSICAL REVIEW A VOLUME 2, NUMBER 3 SEPTEMBER 1970

Radiation from an N-Atom System. I. General Formalism

R. H. Lehmberg
U. S. Naval Air Development Center, Warminster, Pennsylvania
(Received 19 November 1969)

We consider the radiation from a system of N identical two-level atoms coupled to a con-
ıuum of quantized em modes, and possibly, to an external driving field near resonance. The
oms can be distributed over a region large in comparison to the resonant wavelength, but
ıaller than the spontaneous pulse length. Radiation rates and correlation functions are ex-
essed in terms of expectation values of time-dependent atomic operators, which are shown
satisfy coupled first-order differential equations involving similar atomic operators and
e *initial* radiation operators. The corresponding equations for the expectation values sim-
ify considerably if no driving field is present. Similar results are derived for a model in
ıich each atom is replaced by a harmonic oscillator.



collective shift

so far...

- ideal (Dicke)
- classical (Friedberg e
- single-photon (Svidzi

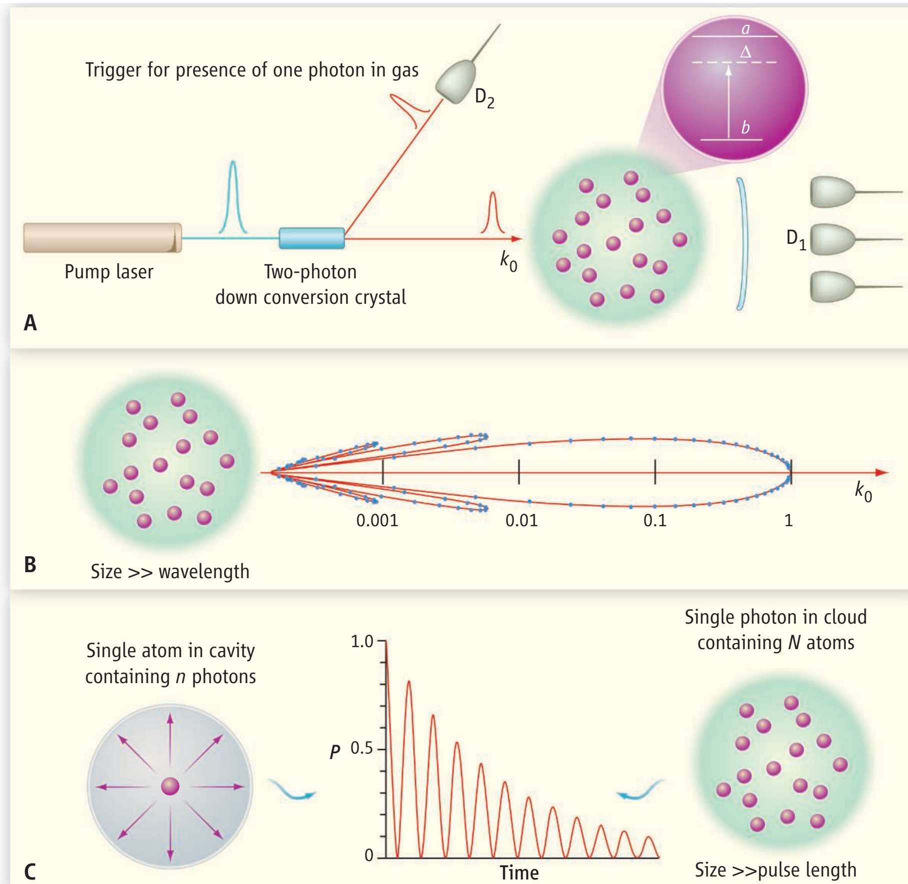
PHYSICS

The Super of Superradiance

Marlan O. Scully[1,2] and Anatoly A. Svidzinsky[1]

In 1954, Robert Dicke introduced the

An even more interesting kind c

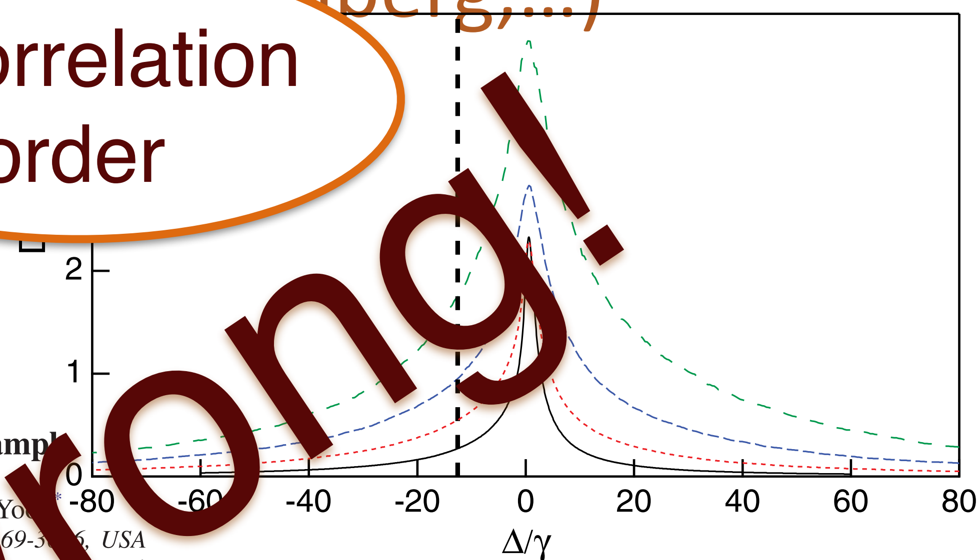


so far…

- ideal (Dicke)
- classical (Friedbe… …mberg,…)
- single-photon
- linear (Javanainen)

in correlation order

wrong!



Shifts of a Resonance Line in a Dense Atomic Sampl…

Juha Javanainen,[1] Janne Ruostekoski,[2] Yi Li,[1] and Sung-M… Yo…*

[1]Department of Physics, University of Connecticut, Storrs, Conne…cut 0…69-3…6, USA

[2]Mathematical Sciences, University of Southampton, Southampton S… …7 1BJ, Unite… …ingdom

(Received 28 August 2013; published 21 March …01…)

We study the collective response of a dense atomic sample to light …en…ally e…ly using classical-electrodynamics simulations. In a homogeneously broadened atomic sa… there is no overt Lorentz-Lorenz local field shift of the resonance, nor a collective Lamb shift. However, the addition of inhomogeneous broadening restores the usual mean-field phenomenology.

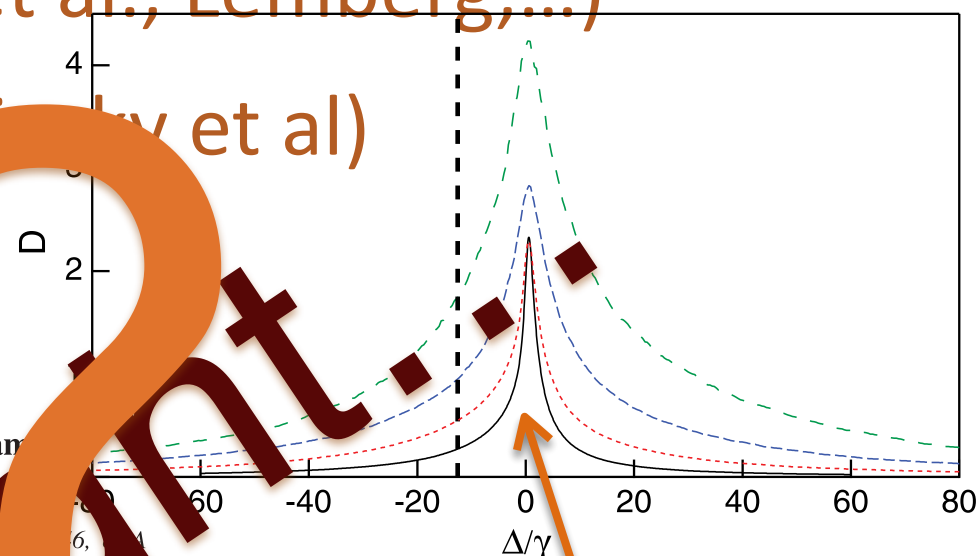
so far…

- ideal (Dicke)
- classical (Friedberg et al., Lemberg,…)
- single-photon (Svid…ky et al)
- linear (Javanaine…

Shifts of a Resonance Line in a Dense Atomic Sam…

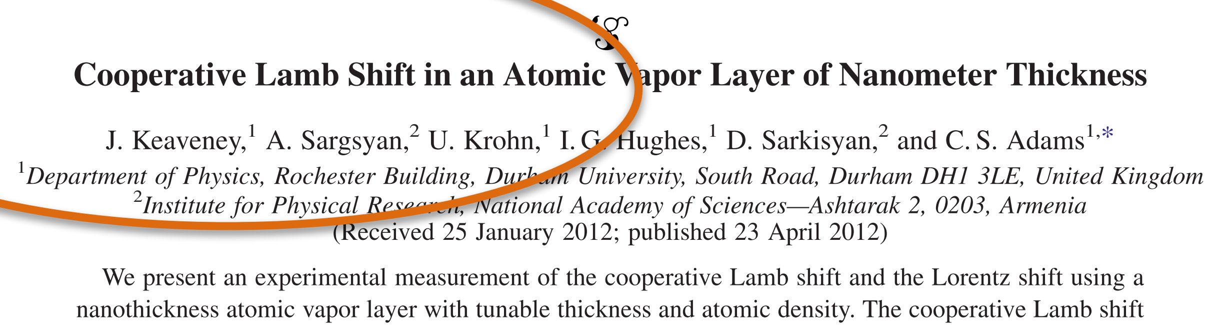
Juha Javanainen,[1] Janne Ruostekoski,[2] Yi Li,[1] and …ung-Mi…
[1]Department of Physics, University of Connecticut, Storrs, Connect…ut 06…6, …A
[2]Mathematical Sciences, University of Southampton, Southampto… SO1… 1BJ, …ited Ki…m
(Received 28 August 2013; published 21 …ch 201…

We study the collective response of a dense atomic sample to light… sentially … …ing …ssical-electrodynamics simulations. In a homogeneously broadened atomic s…ple there … …overt Lorentz-Lorenz local field shift of the resonance, nor a collective Lamb sh… However, the addition of inhomogeneous broadening restores the usual mean-field phenomenology.



right…

no shift!



Cooperative Lamb Shift in an Atomic Vapor Layer of Nanometer Thickness

J. Keaveney,[1] A. Sargsyan,[2] U. Krohn,[1] I. G. Hughes,[1] D. Sarkisyan,[2] and C. S. Adams[1,*]

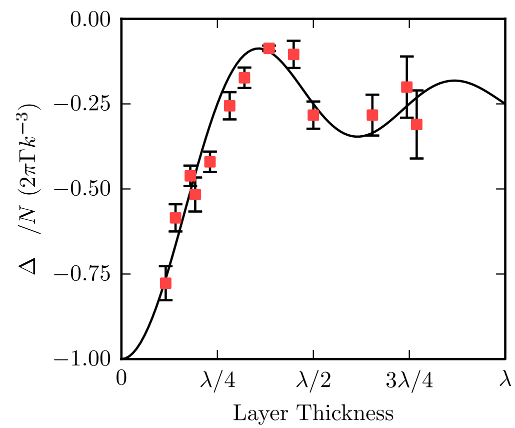
[1]*Department of Physics, Rochester Building, Durham University, South Road, Durham DH1 3LE, United Kingdom*

[2]*Institute for Physical Research, National Academy of Sciences—Ashtarak 2, 0203, Armenia*

(Received 25 January 2012; published 23 April 2012)

We present an experimental measurement of the cooperative Lamb shift and the Lorentz shift using a nanothickness atomic vapor layer with tunable thickness and atomic density. The cooperative Lamb shift

- ideal (Di
- classical
- single-ph
- linear (Ja
- dilute (Adams, Ye,...)

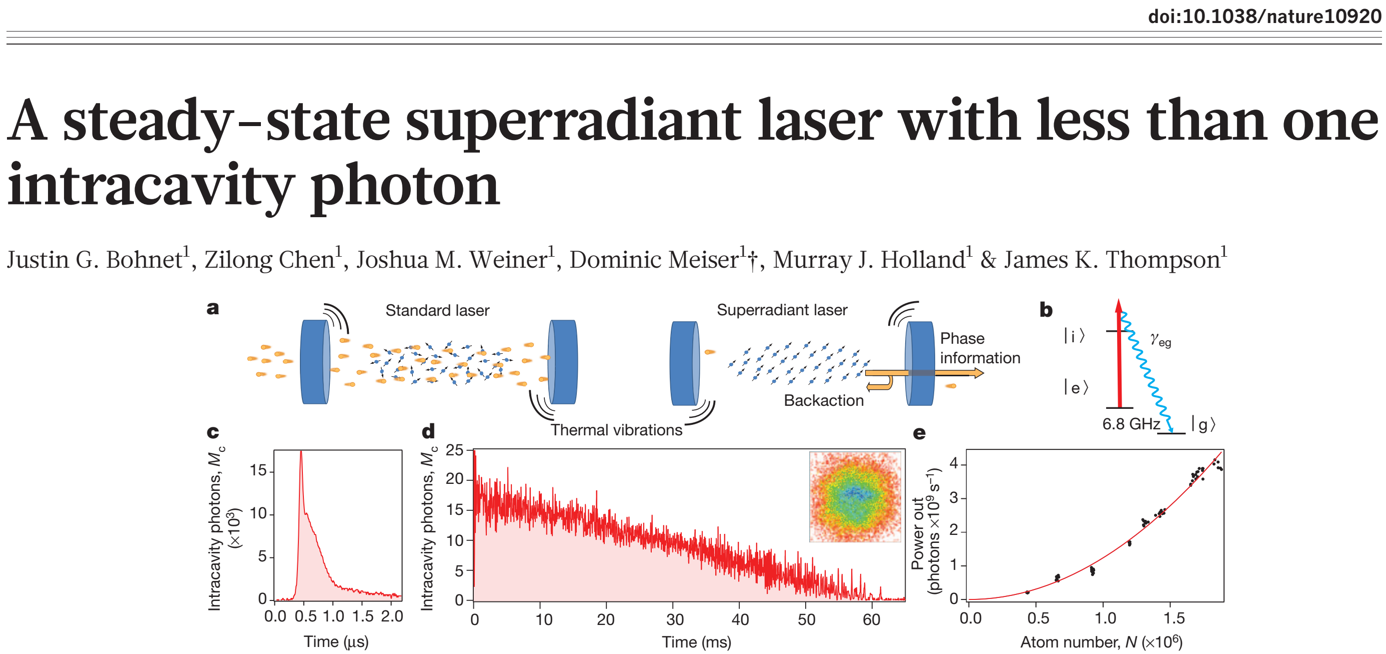


so far…

- ideal (Dicke)
- classical (Friedberg et al., Lemberg,…)
- single-photon (Svidzinsky et al)
- linear (Javanainen)
- dilute

Collective atomic emission and motional effects in a dense coherent medium

S. L. Bromley[1], B. Zhu[1], M. Bishof[1], X. Zhang[1], T. Bothwell[1], J. Schachenmayer[1], T. L. Nicholson[1], R. Kaiser[2], S. F. Yelin[3], M. D. Lukin[4] , A.M. Rey[1], & J. Ye[1]


doi:10.1038/nature10920

A steady-state superradiant laser with less than one intracavity photon

Justin G. Bohnet[1], Zilong Chen[1], Joshua M. Weiner[1], Dominic Meiser[1]†, Murray J. Holland[1] & James K. Thompson[1]

- ideal (
- classic
- single
- linear
- dilute (Adams, Ye,...)
- very low excitation (Holland/Thompson...)

What is "superradiance"?

1. Everything that involves Dicke states
   - (e.g., collective √N effects,
   - bad-cavity limit,
   - …)
2. Only systems involving cooperative (and nonlinear) effects
   - i.e., effect of exchange int
   - more than single excitation

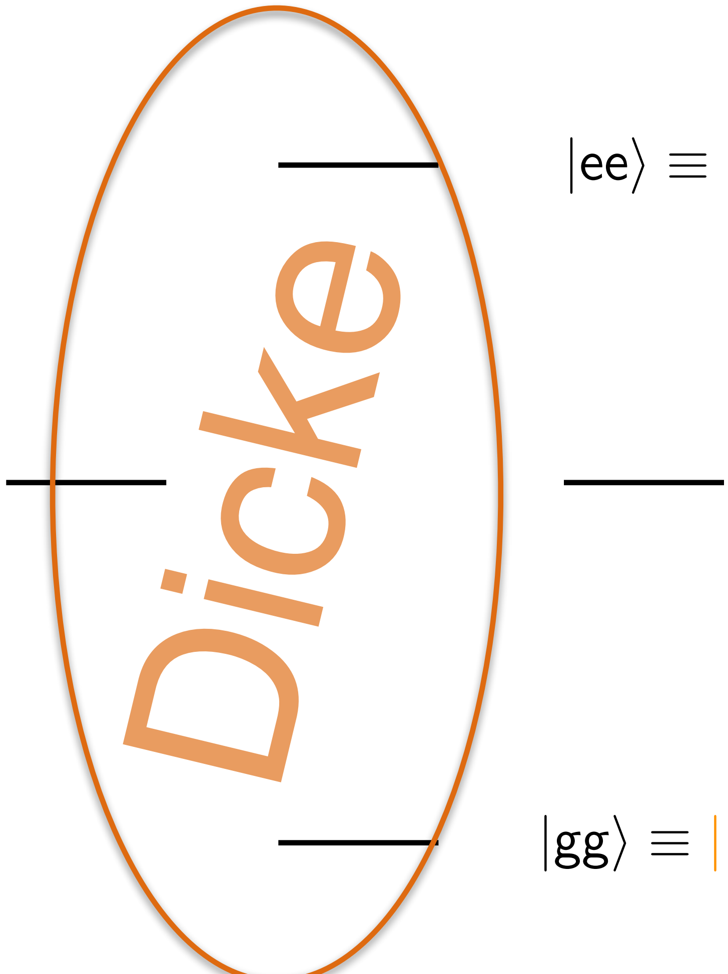
only for purists

Cooperativity

Example: two atoms

$$|ee\rangle \equiv |1,1\rangle$$

$$\frac{1}{\sqrt{2}}(|eg\rangle + |ge\rangle) \equiv |1,0\rangle \qquad \frac{1}{\sqrt{2}}(|eg\rangle - |ge\rangle) \equiv |0,0\rangle$$



$$|gg\rangle \equiv |1,-1\rangle$$

Cooperativity

Example: two atoms

$$|ee\rangle \equiv |1,1\rangle$$

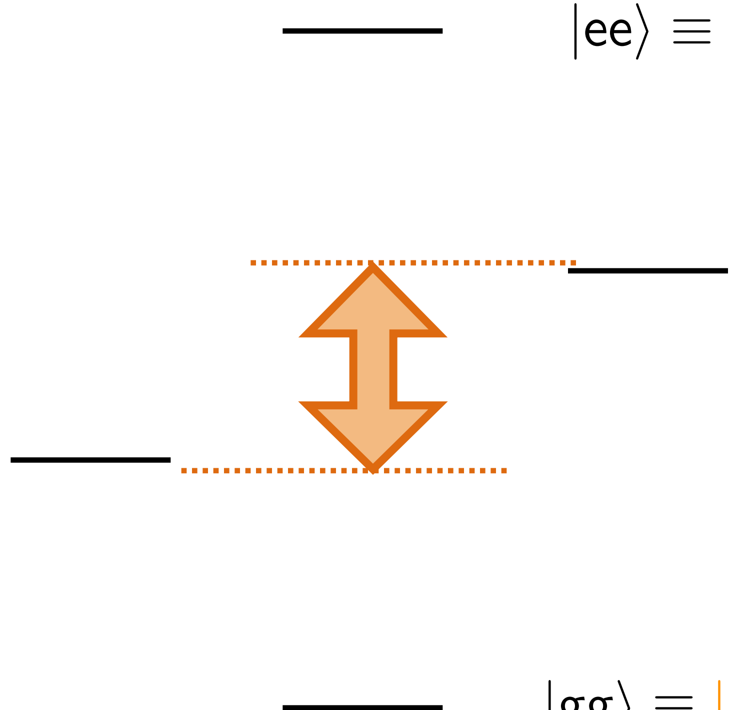
$$\frac{1}{\sqrt{2}}\left(|eg\rangle + |ge\rangle\right) \equiv |1,0\rangle \qquad \frac{1}{\sqrt{2}}\left(|eg\rangle - |ge\rangle\right) \equiv |0,0\rangle$$

$$|gg\rangle \equiv |1,-1\rangle$$

(anti)symmetry costs energy: “exchange interaction”

Cooperativity

Example: two atoms



$$|ee\rangle \equiv |1,1\rangle$$

$$\frac{1}{\sqrt{2}}\left(|eg\rangle - |ge\rangle\right) \equiv |0,0\rangle$$

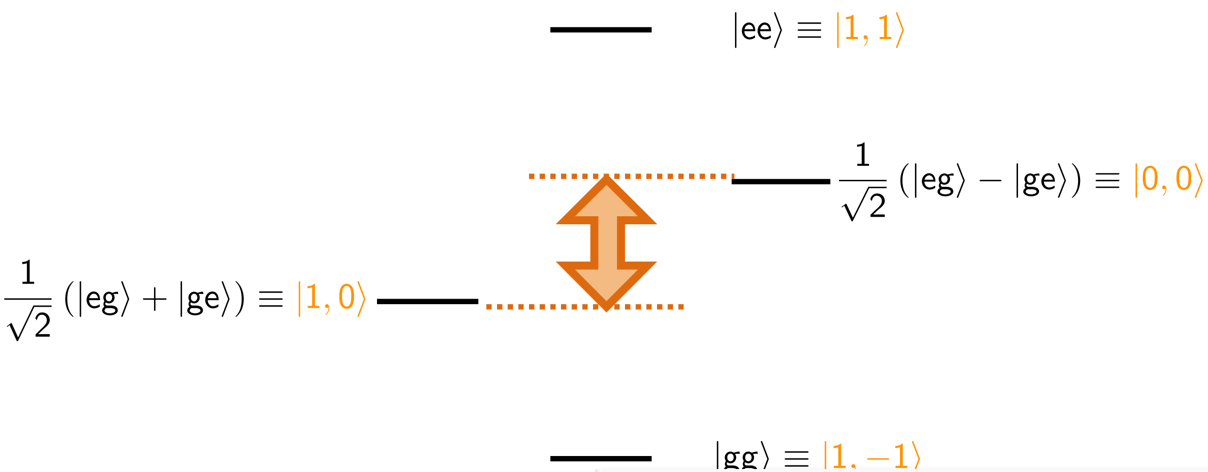
$$\frac{1}{\sqrt{2}}\left(|eg\rangle + |ge\rangle\right) \equiv |1,0\rangle$$

$$|gg\rangle \equiv |1,-1\rangle$$

(anti)symmetry costs energy: “exchange interaction”

Cooperativity

Example: two atoms

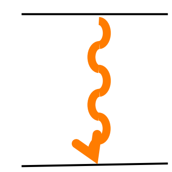


exchange interaction:

- usually dipole-dipole mediated
- creates shift and broadening (Kramers-Kronig)

Dicke model

Dicke states: $|J, M_J\rangle = \text{Sym}\big|\underbrace{e\ldots e}_{J+M_J}\underbrace{g\ldots g}_{J-M_J}\big\rangle$



angular momentum formulation $\Rightarrow$

$$J \leq \frac{N}{2} \quad \text{(= max. excitation)}$$

$$M_J = -J\ldots J \quad \text{(= actual excitation)}$$

Radiation couples only states with equal J

Example: two atoms

$$|ee\rangle \equiv |1,1\rangle$$

$$\frac{1}{\sqrt{2}}(|eg\rangle + |ge\rangle) \equiv |1,0\rangle \qquad \frac{1}{\sqrt{2}}(|eg\rangle - |ge\rangle) \equiv |0,0\rangle$$

$$|gg\rangle \equiv |1,-1\rangle$$

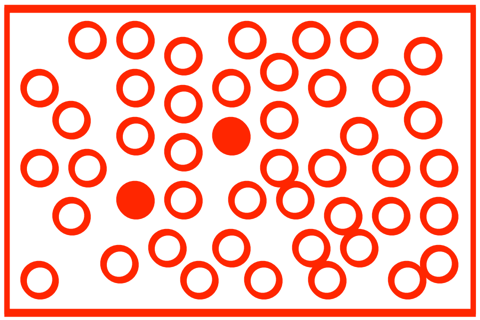
Questions - guideline

- Superradiance - What? Why?
- How do we calculate it (better)?

Dynamics of atoms in dense media - Schwinger-Keldysh & Dyson Eq.

Full dynamics (all degrees of freedom of atoms, fields)

$$H = H_{atoms} + H_{field} - \sum \mathbf{p}_i \mathbf{E}_i$$



two probe atoms

+

surrounding atoms

Two atoms + field

$$V_{probe} = \sum_{i=1,2} \mathbf{p}_i \mathbf{E}_{iH} \Rightarrow S = T e^{-\frac{i}{\hbar}\int d\tau V_{probe}(\tau)}$$

- $\langle e^{sX} \rangle = e^{\sum \frac{s^m}{m!}\langle\langle X^m \rangle\rangle}$
- Gauss: $m \leq 2$

$$\langle \ldots \rangle$$

field degrees of freedom

effective two-atom description

Two atom Master equation

Fleischhauer, Yelin, *PRA 59, 2427 (99)*; Lin, Yelin, *Adv. Atom. Mol. Opt. Phys. 61, 295 (2012)*

Equations of motion

$$
\begin{aligned}
\dot{a} &= \Gamma - (\gamma + 2\Gamma)\, a - \bar{\gamma}\, x - i\,\Omega\,(\rho_{eg} - \rho_{ge}) \\
\dot{n} &= 2\gamma - 2(\gamma + 2\Gamma)\, n - 4\gamma\, a + 4(\bar{\gamma} + 2\bar{\Gamma})\, x - 4\, i\,\Omega\,(m_{eg} - m_{ge}) - 4\, i\,\mathcal{C}\gamma(\rho_{ge} m_{eg} - \rho_{eg} m_{ge}) \\
\dot{x} &= -(\gamma + 2\Gamma)\, x + \frac{\bar{\gamma} + 2\bar{\Gamma}}{2}\, n + \bar{\gamma}\, a - \frac{\bar{\gamma}}{2} + i\,\Omega\,(m_{eg} - m_{ge}) + i\,\mathcal{C}\gamma(\rho_{ge} m_{eg} - \rho_{eg} m_{ge}) \\
\dot{\rho}_{eg} &= -\left(\frac{\gamma + 2\Gamma}{2} + i\,(\delta + 2\Delta - \Delta_{\Omega})\right) \rho_{eg} + \frac{\bar{\gamma} - 2\, i\,\bar{\delta}}{2}\, m_{eg} - i\,(\Omega + \mathcal{C}\gamma\rho_{eg})\,(2\, a - 1) \\
\dot{m}_{eg} &= -\left(3\frac{\gamma + 2\Gamma}{2} + \bar{\gamma} + 2\bar{\Gamma} + i\,(\delta + 2\Delta - \Delta_{\Omega})\right) m_{eg} - (\gamma + \frac{\bar{\gamma}}{2} + i\,\bar{\delta})\, \rho_{eg} \\
&\quad -i\,(\Omega + \mathcal{C}\gamma\rho_{eg})\,(n - 2\, x) - 2\, i\,(\Omega + \mathcal{C}\gamma\rho_{ge})\rho_{eg,eg} \\
\dot{\rho}_{eg,eg} &= -\left((\gamma + 2\Gamma) + 2\, i\,(\delta + 2\Delta - \Delta_{\Omega})\right)\, \rho_{eg,eg} - 2\, i\,(\Omega + \mathcal{C}\gamma\rho_{eg})\, m_{eg}
\end{aligned}
$$

$$\dot{a} = \Gamma - (\gamma + 2\Gamma)\, a - \bar{\gamma}\, x - i\,\Omega\,(\rho_{eg} - \ldots$$

$$\dot{n} = \ldots + 2\Gamma)\, n - 4\gamma\, a + 4(\bar{\gamma} + \ldots$$

$$\dot{x} = \ldots + \frac{\bar{\gamma} + 2\bar{\Gamma}}{2}\, n + \bar{\gamma}\, a - \ldots$$

$$\dot{\rho}_{eg} \ldots 2\Gamma \ldots + i\,(\delta + 2\Delta - \Delta_{\Omega})\Big) \ldots$$

$$\dot{m}_{eg} = -\Bigg(3\frac{\gamma + 2\Gamma}{2} + \bar{\gamma} + 2\bar{\Gamma} + i\,(\delta + 2\Delta \ldots$$

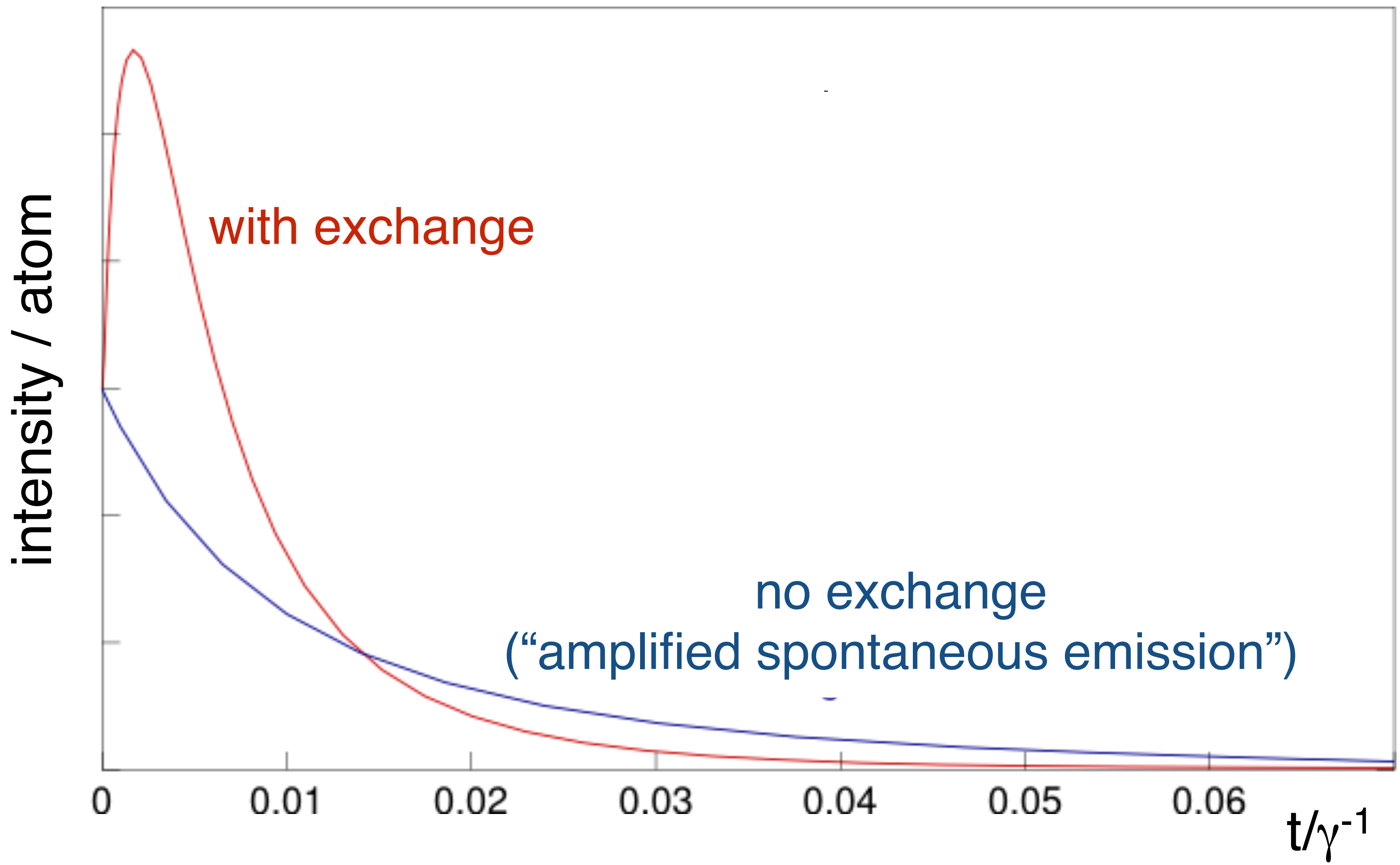
$$-i\,(\Omega + \mathcal{C}\gamma\rho_{eg})\,(n - 2\,x) - 2\,i\,(\Omega \ldots$$

$$\dot{\rho}_{eg,eg} = -\left((\gamma + 2\Gamma) + 2\,i\,(\delta + 2\Delta - \Delta_{\Omega})\right) \ldots$$

average excited state

effective inversion

exchange term



Fleischhauer, Yelin, *PRA 59, 2427 (99)*; Lin, Yelin, *Adv. Atom. Mol. Opt. Phys. 61, 295 (2012)*

$$\dot{a} = \Gamma - (\gamma + 2\Gamma)\, a - \bar{\gamma}\, x - i\,\Omega\,(\rho_{eg} - \ldots$$

$$\dot{n} = 2\gamma - 2(\gamma + 2\Gamma)\, n - 4\gamma\, a + 4(\bar{\gamma} + \ldots$$

$$\dot{x} = \ldots 2\Gamma)\, x + \frac{\bar{\gamma} + 2\bar{\Gamma}}{2}\, n + \bar{\gamma}\, a - \ldots$$

coherence

$$\dot{\rho}_{eg} = - \Big(\ldots + 2\Gamma}{2} + i\,(\delta + 2\Delta - \Delta_{\Omega}) \Big) \ldots$$

$$\dot{m}_{eg} = - \Big(3\,\frac{\gamma + 2\Gamma}{2} + \bar{\gamma} + 2\bar{\Gamma} + i\,(\delta + 2 \ldots$$

$$-i\,(\Omega + \mathcal{C}\gamma\rho_{eg})\,(n - 2\,x) - 2\,i\,(\Omega \ldots$$

$$\dot{\rho}_{eg,eg} = -((\gamma + 2\Gamma) + 2\,i\,(\delta + 2\Delta - \Delta_{\Omega})) \ldots$$

$$- 2\bar{\Gamma}) \, x - 4\, i\, \Omega \,(m_{eg} - m_{ge}) - 4\, i\, \mathcal{C}\gamma(\rho_{ge} m_{eg} - \rho$$

$$- \frac{\bar{\gamma}}{2} + i\, \Omega \,(m_{eg} - m_{ge}) + i\, \mathcal{C}\gamma(\rho_{ge} m_{eg} - \rho_{eg} m_{ge})$$

$$\rho_{eg} + \frac{\bar{\gamma} - 2\, i\, \bar{\delta}}{2} \, m_{eg} - i\,(\Omega + \mathcal{C}\gamma\rho_{eg})\,(2\,a - 1)$$

$$2\Delta - \Delta \quad (\gamma + \frac{\bar{\gamma}}{2} + i\,\bar{\delta})\,\rho$$

$$\Omega + C$$

$$)) \; \rho_{eg,eg} - 2\, i\,(\Omega + \mathcal{C}\gamma\rho_{eg})\, m_{eg}$$

driving with
local field effect
(first order!)

$\langle\langle \hat{E}(t) \rangle\rangle$

$$\gamma - 2(\gamma + 2\Gamma)\, n - 4\gamma\, a + 4(\bar\gamma + 2\bar\Gamma)\, x - 4\, i\, \Omega\, (m_{e} \ldots$$

$$(\gamma + 2\Gamma)\, x + \frac{\bar\gamma + 2\bar\Gamma}{2}\, n + \bar\gamma\, a - \frac{\bar\gamma}{2} + i\, \Omega\, (m_{eg} - m \ldots$$

$$\left(\frac{\gamma + 2\Gamma}{2} + i\,(\delta + 2\Delta - \Delta_\Omega)\right) \rho_{eg} + \frac{\bar\gamma - 2\, i\, \bar\delta}{2}\, m \ldots$$

$$\left(3\,\frac{\gamma + 2\Gamma}{[\ldots]} \ldots 2\bar\Gamma + i\,(\delta + 2\Delta - \Delta_\Omega)\right) m_{eg} - \ldots$$

$$i\,(\Omega \ldots) - \ldots {}_{,eg}$$

$$((\gamma + 2\Gamma) \ldots + 2\Delta - \Delta_\Omega))\, \rho_{eg,eg} - 2\, i\, (\Omega - \ldots$$

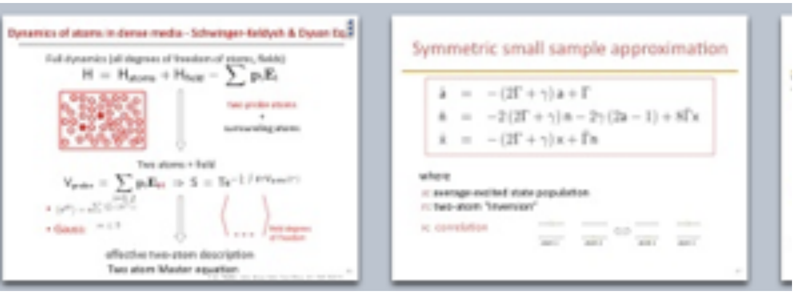
superradiance rate and shifts (second order!)

$$\langle\langle \hat{E}_1(t_1)\hat{E}_2(t_2) \rangle\rangle$$

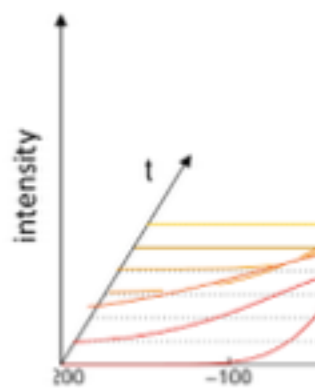
Can one expect superradiance?

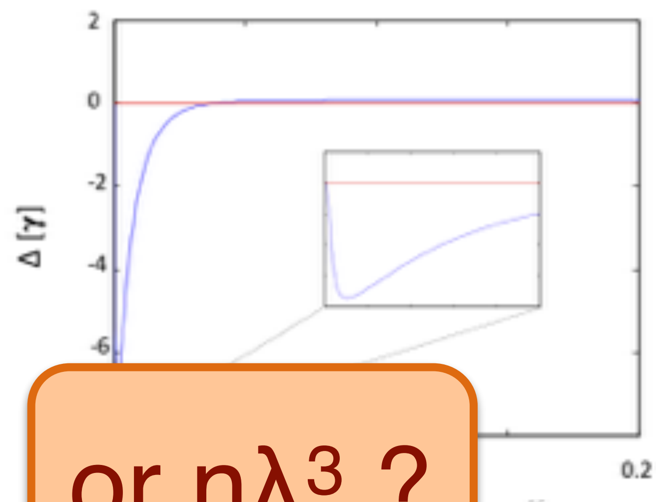
B

... and Chirping



$S(\omega) = \mathrm{FT}$





The impo

or $n\lambda^3$?

$n\lambda^2 r$

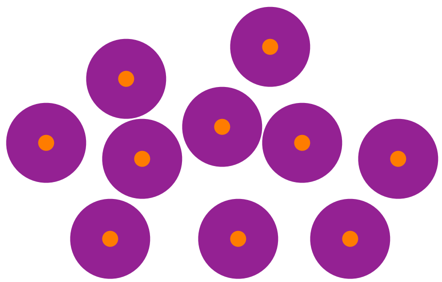
optical depth

n: density, λ: wavelength, r: system size

Lin, Yelin, Adv. Atom. Mol. Opt. Phys. 61, 295 (2012)

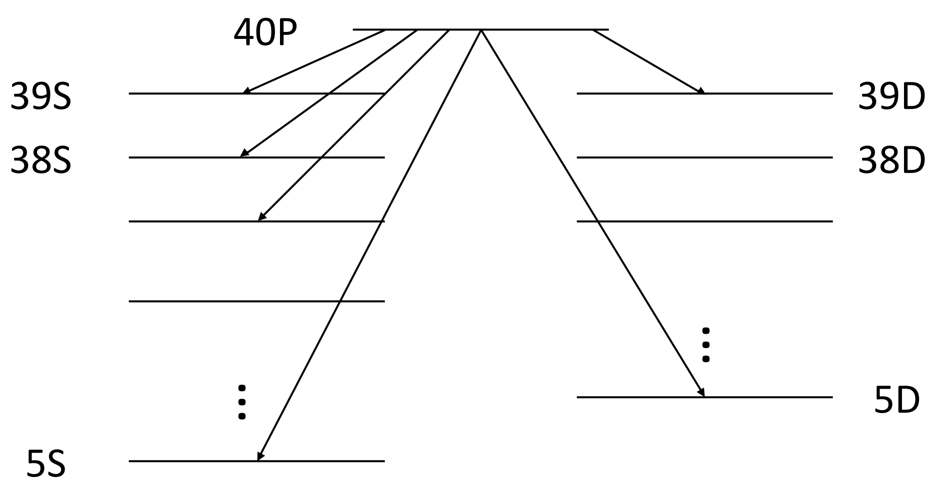
New experimental systems: example

- Ultracold Rydberg atoms

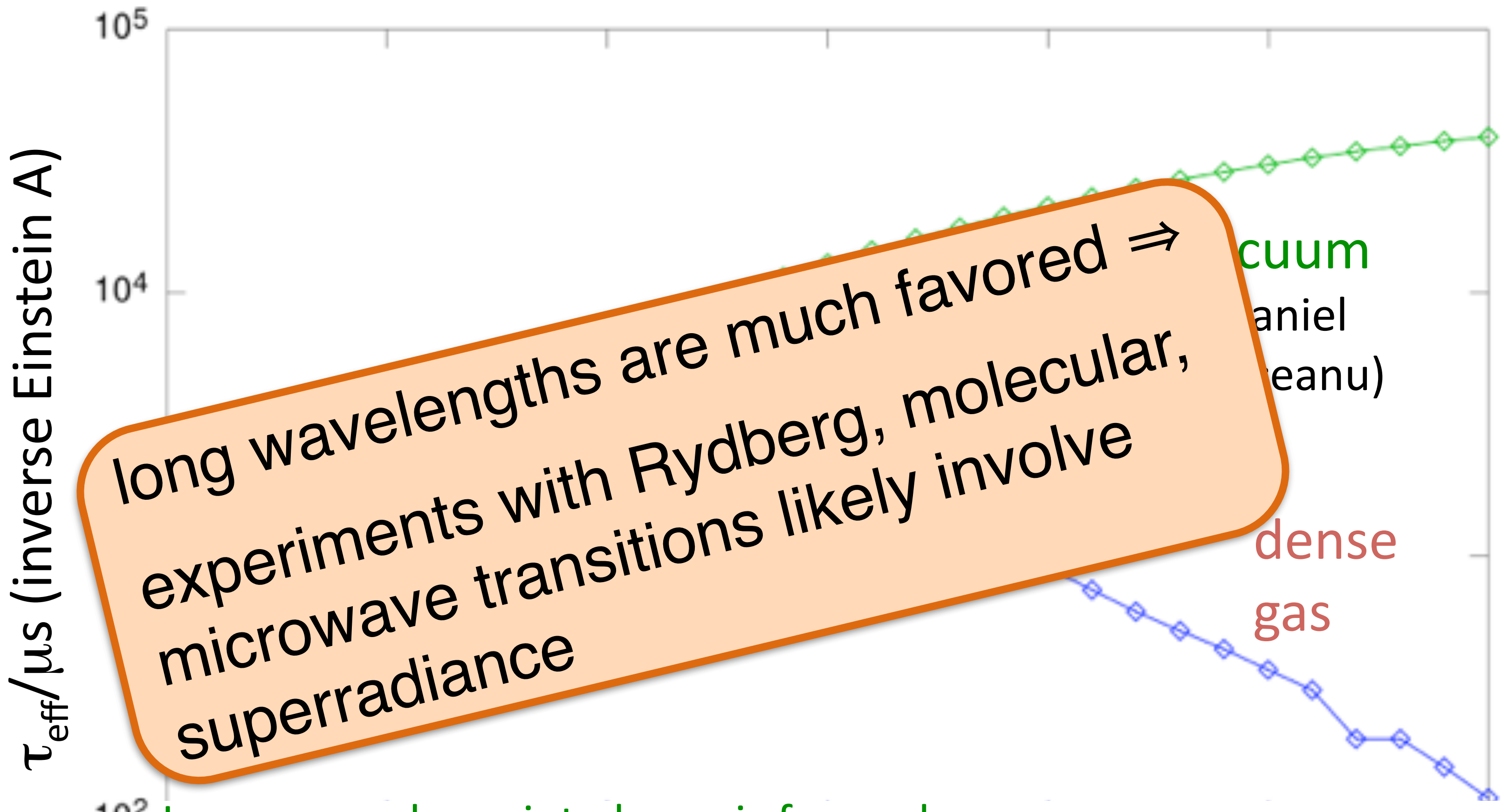


(Phil Gould, Ed Eyler, Uconn)

Rb



Effective decay times from 40P into nS



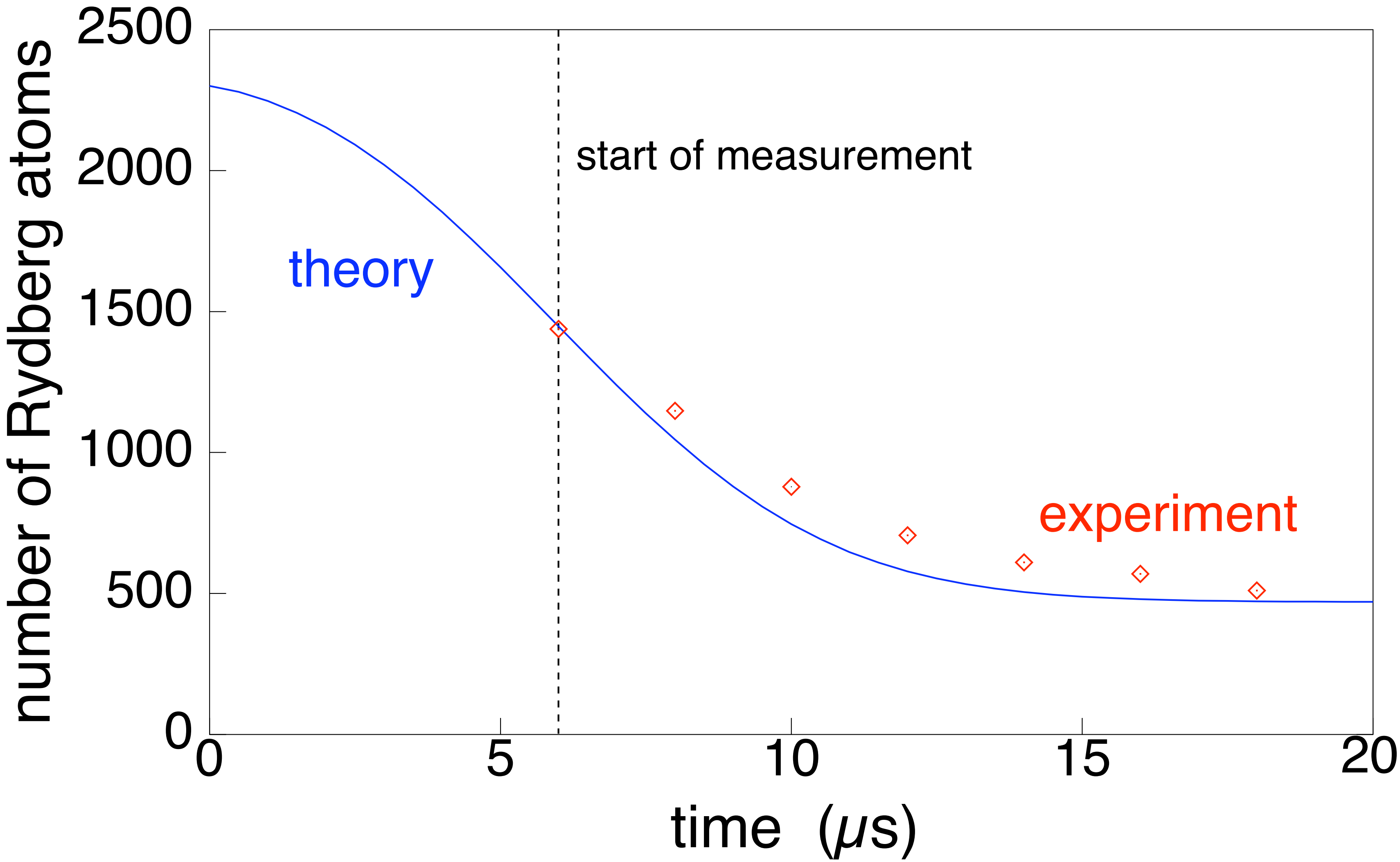
In vacuum: decay into low n is favored

In dense gas: decay into high n is favored ⇨ λ large, n λ^2 r large!

superradiant decay!

Wang, et al.,PRA 75, 033802 (2007); Lin & Yelin Mol. Phys. 111, 1917 (2013).

Superradiance in Rydberg systems



Questions - guideline

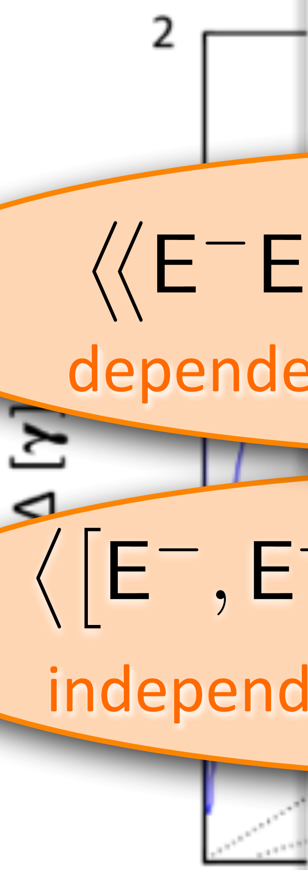
- Superradiance - What? Why?
- How do we calculate it (better)?
- Is there a collective (Lamb) shift?

Collective Lamb shift

- "Lamb shift" is the result of interaction with the vacuum fluctuations
- In the case of altered density of states of the "vacuum" (i.e., the surrounding space), the value of the shift changes
- With a high (superradiant) density of radiators, the density of states inside the medium can be considerably altered

"Collective Lamb shift"

Collective Shift



has spontaneous part....

$$\wp^2 \int d\tau \left\langle \left[E_i^-(t), E_j^+(t+\tau) \right] \right\rangle e^{i\omega\tau}$$

ω'

$$\langle\langle E^- E^+ \rangle\rangle \overset{\propto}{\sim} \langle\langle a^\dagger a \rangle\rangle = n$$

dependent on number of photons

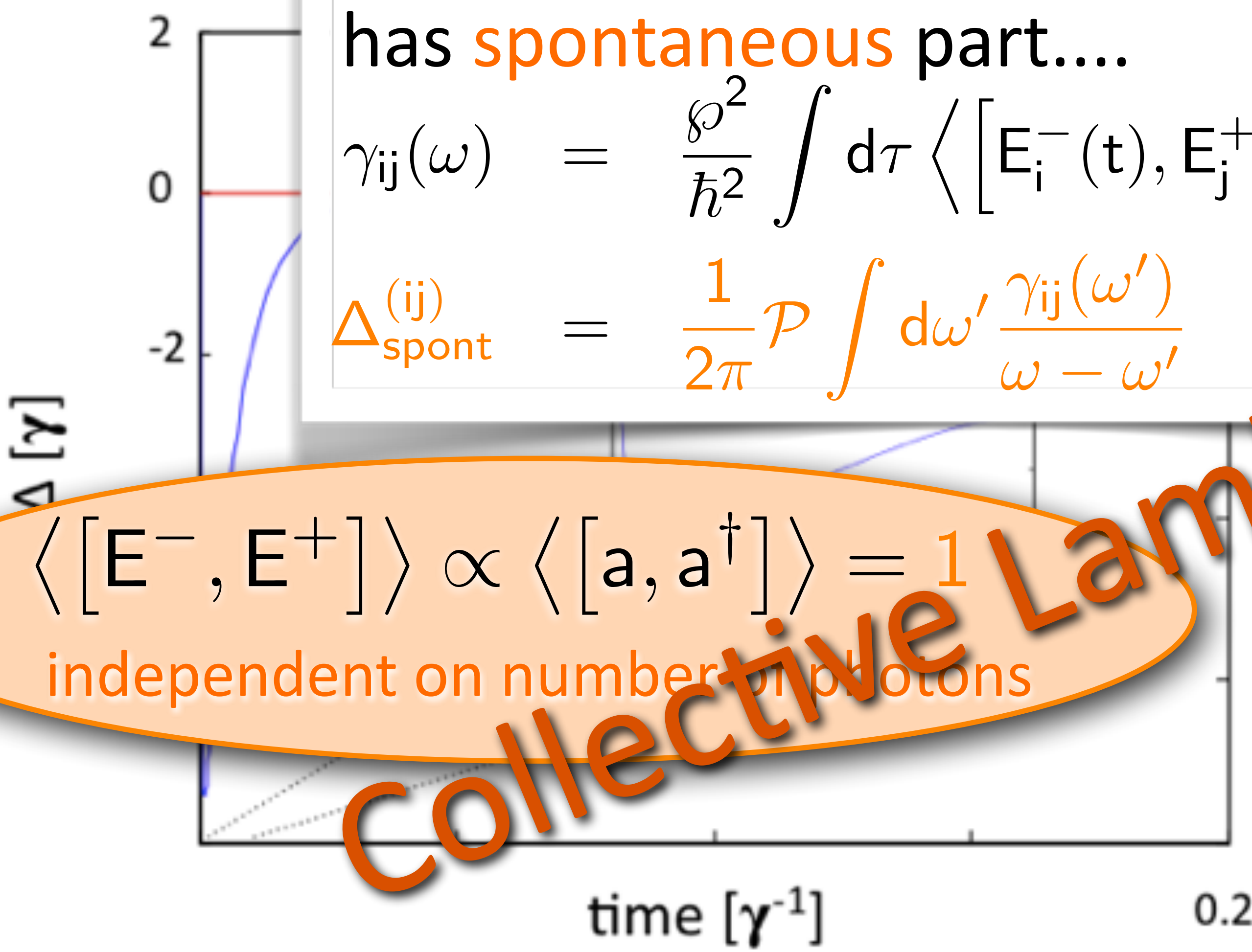
$$\left\langle \left[E^-, E^+ \right] \right\rangle \propto \left\langle \left[a, a^\dagger \right] \right\rangle = 1$$

independent on number of photons

$$(t) E_j^+(t+\tau) \rangle\rangle e^{i\omega\tau}$$

$$\Delta^{(ij)}_{\mathrm{stim}} = \frac{1}{2\pi} \mathcal{P} \int d\omega' \frac{\Gamma_{ij}(\omega')}{\omega - \omega'}$$

Collective Shift



has spontaneous part....

$$\gamma_{ij}(\omega) = \frac{\wp^2}{\hbar^2} \int d\tau \left\langle \left[E_i^-(t), E_j^+(t+\tau) \right] \right\rangle e^{i\omega\tau}$$

$$\Delta^{(ij)}_{spont} = \frac{1}{2\pi} \mathcal{P} \int d\omega' \frac{\gamma_{ij}(\omega')}{\omega - \omega'}$$

$$\left\langle \left[E^-, E^+ \right] \right\rangle \propto \left\langle \left[a, a^\dagger \right] \right\rangle = 1$$

independent on number of photons

Collective Lamb Shift

Collective Lamb Shift in the low-excitation limit ($\propto \Omega^2$)

$$\delta_{\text{coll. Lamb}} = \frac{\cos k_0 r - e^{-\tilde{\gamma} k_0 r} \cos(k_0 r + \tilde{\delta} k_0 r)}{r}$$

with

$$k_0 = \frac{\omega}{c}$$

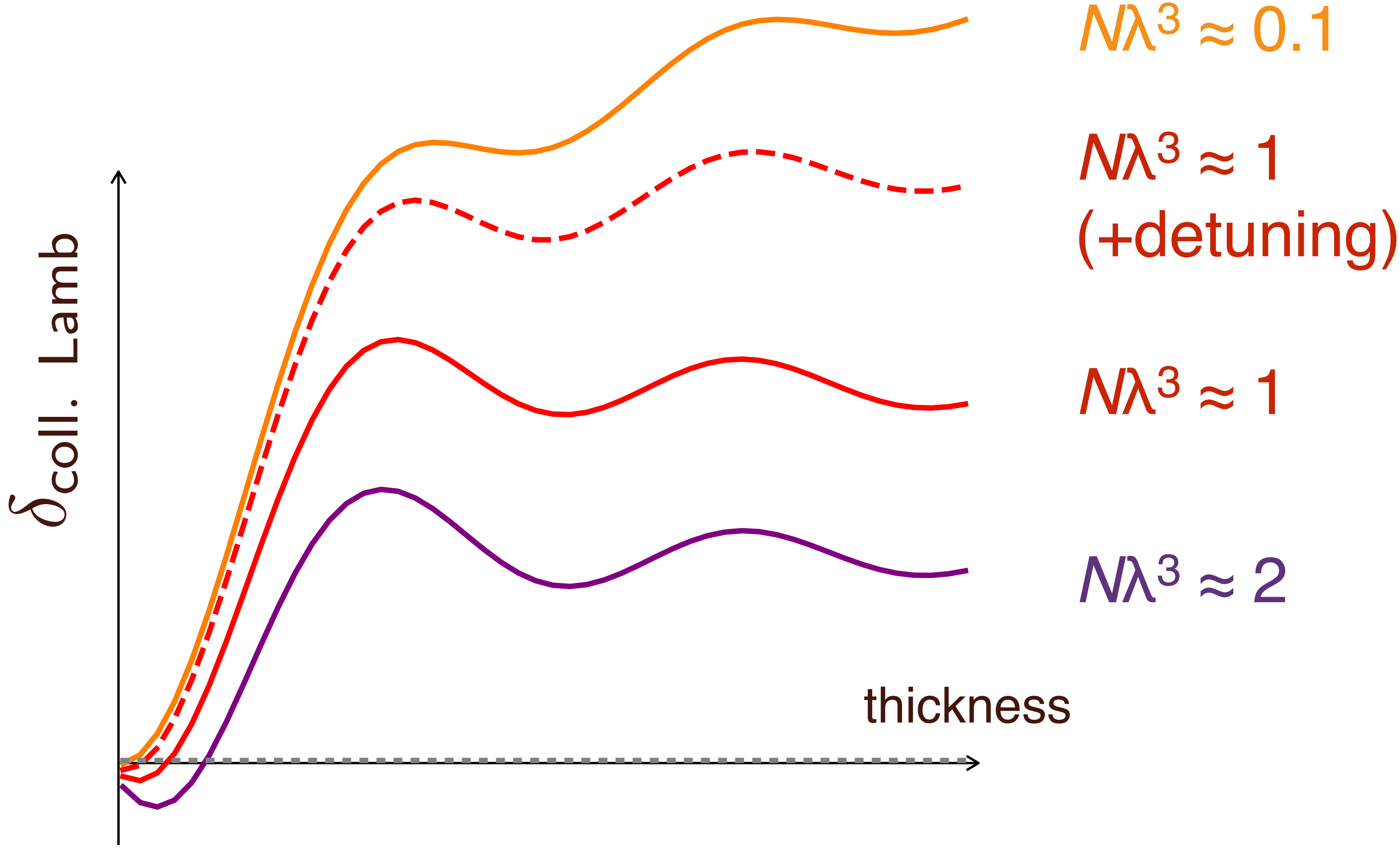
$$\tilde{\gamma} = \mathcal{C} \frac{\frac{\gamma}{2}}{\left(\frac{\gamma}{2}\right)^2 + (\Delta + \mathcal{C}\gamma - \delta_{\text{coll. Lamb}})^2}$$

$$\tilde{\delta} = \mathcal{C} \frac{\Delta + \mathcal{C}\gamma - \delta_{\text{coll. Lamb}}}{\left(\frac{\gamma}{2}\right)^2 + (\Delta + \mathcal{C}\gamma - \delta_{\text{coll. Lamb}})^2}$$

$\mathcal{C}$ = volume per cubic wavelength

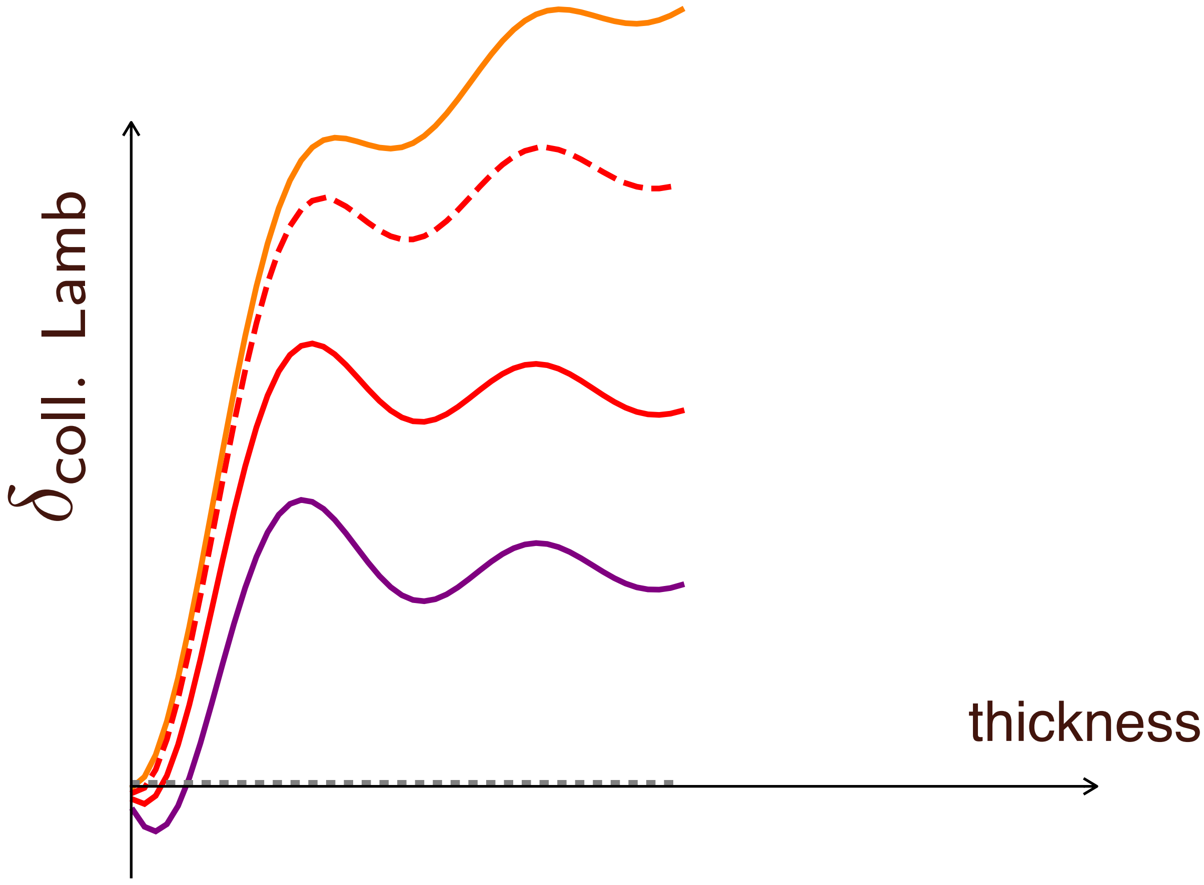
solve self-consistently, normalized vs. (vacuum) Lamb shift

Collective Lamb Shift

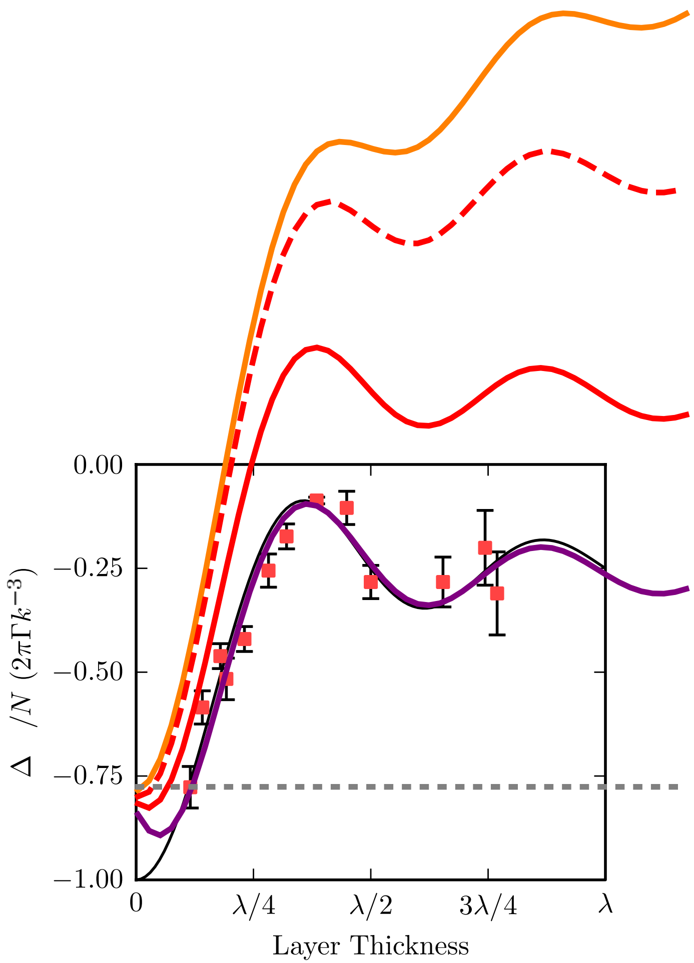


(integrated over thickness)

Collective Lamb Shift



Collective Lamb Shift



Collective Shift

has spontaneous part....

$$\frac{\wp^2}{\ldots}\int d\tau \left\langle \left[E_i^-(t), E_j^+(t+\tau) \right] \right\rangle e^{i\omega\tau}$$

$$\ldots \omega'$$

$$\langle\langle E^- E^+ \rangle\rangle \overset{\propto}{\sim} \langle\langle a^\dagger a \rangle\rangle = n$$

dependent on number of photons

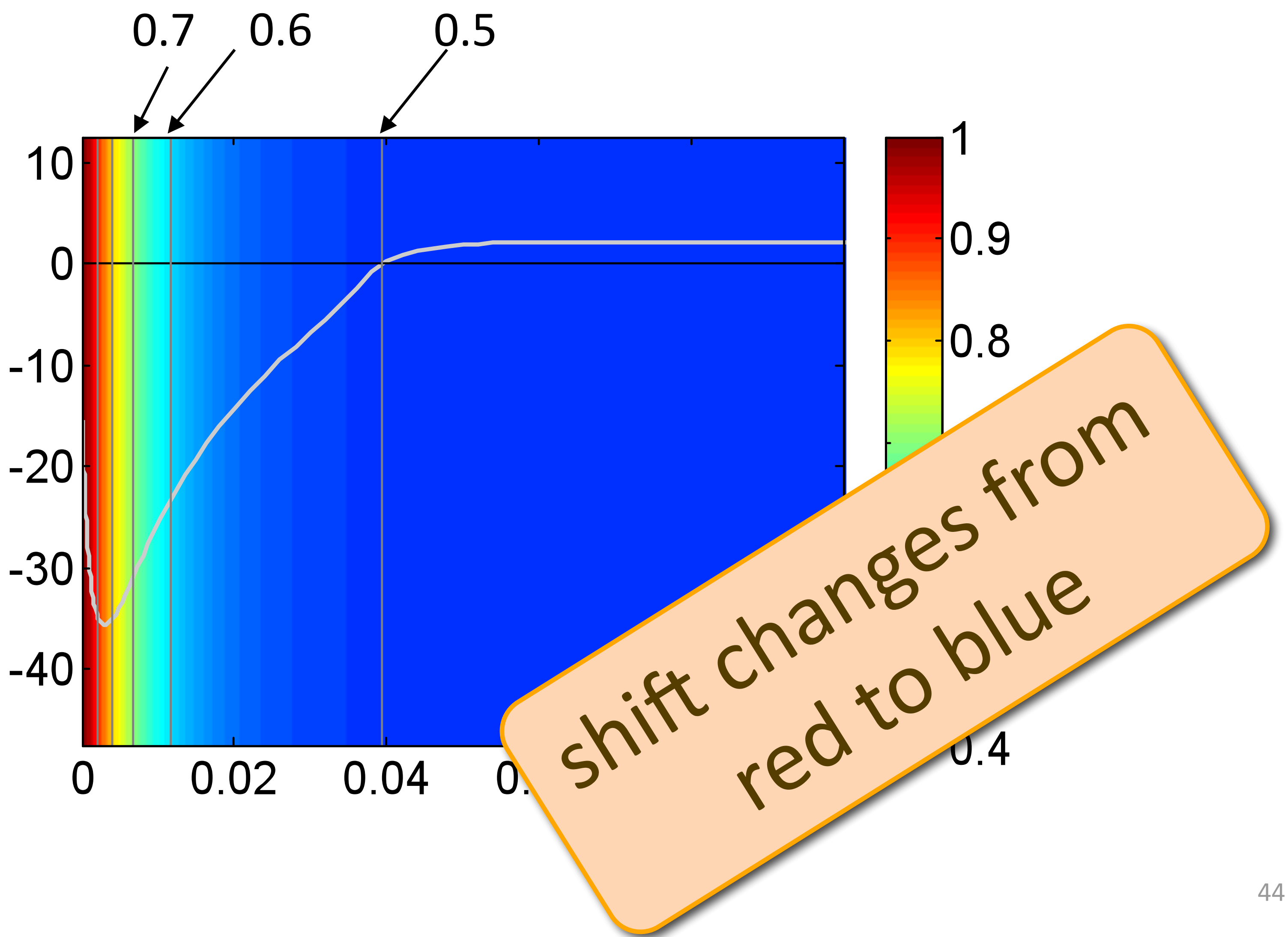
Induced shift

...and "stimulated" part

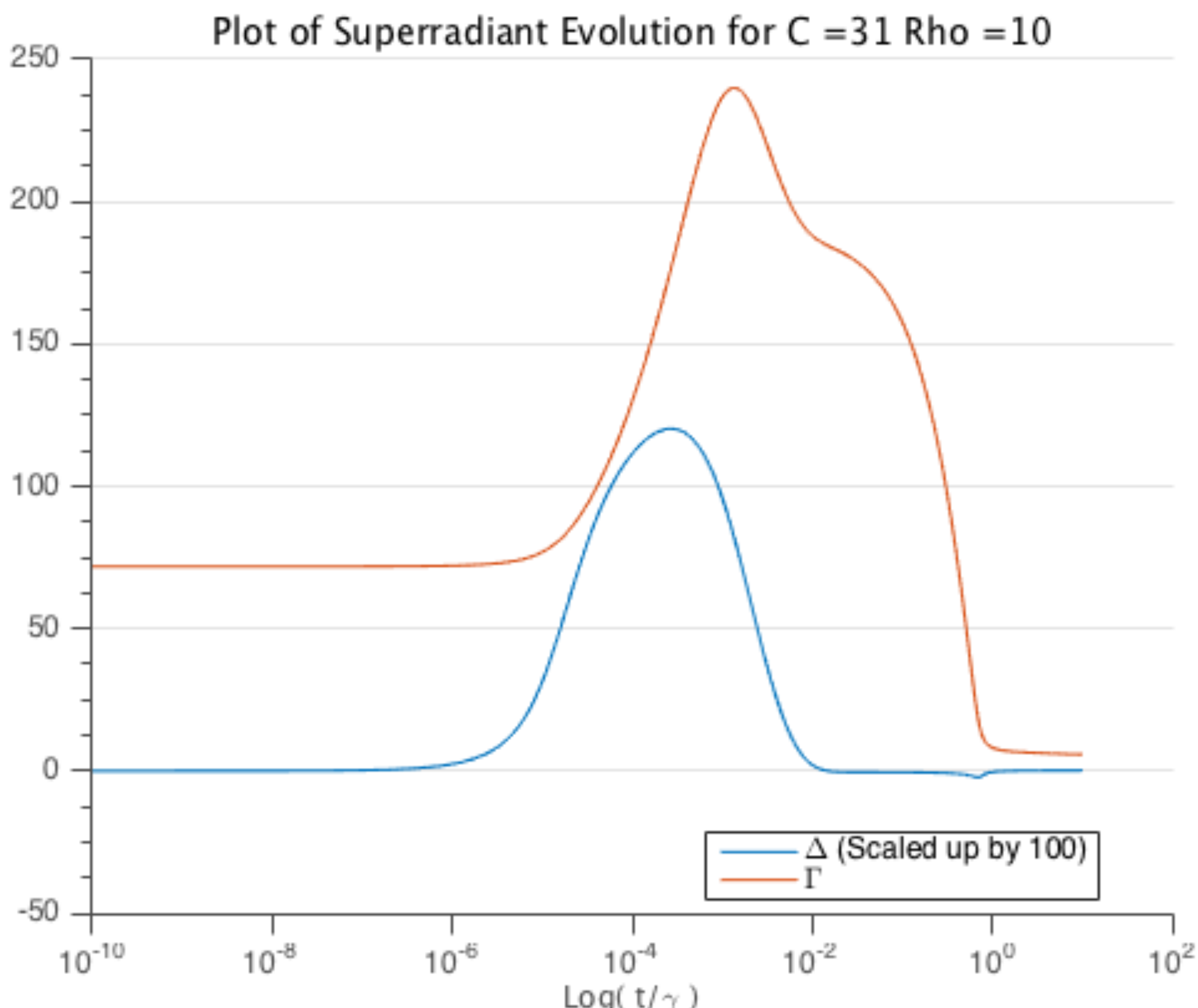
$$\Gamma_{ij}(\omega) = \frac{\wp^2}{\hbar^2}\int d\tau \left\langle\left\langle E_i^-(t) E_j^+(t+\tau) \right\rangle\right\rangle e^{i\omega\tau}$$

$$\Delta^{(ij)}_{\text{stim}} = \frac{1}{2\pi}\mathcal{P}\int d\omega' \frac{\Gamma_{ij}(\omega')}{\omega - \omega'}$$

Induced Shift: Decay of inverted system



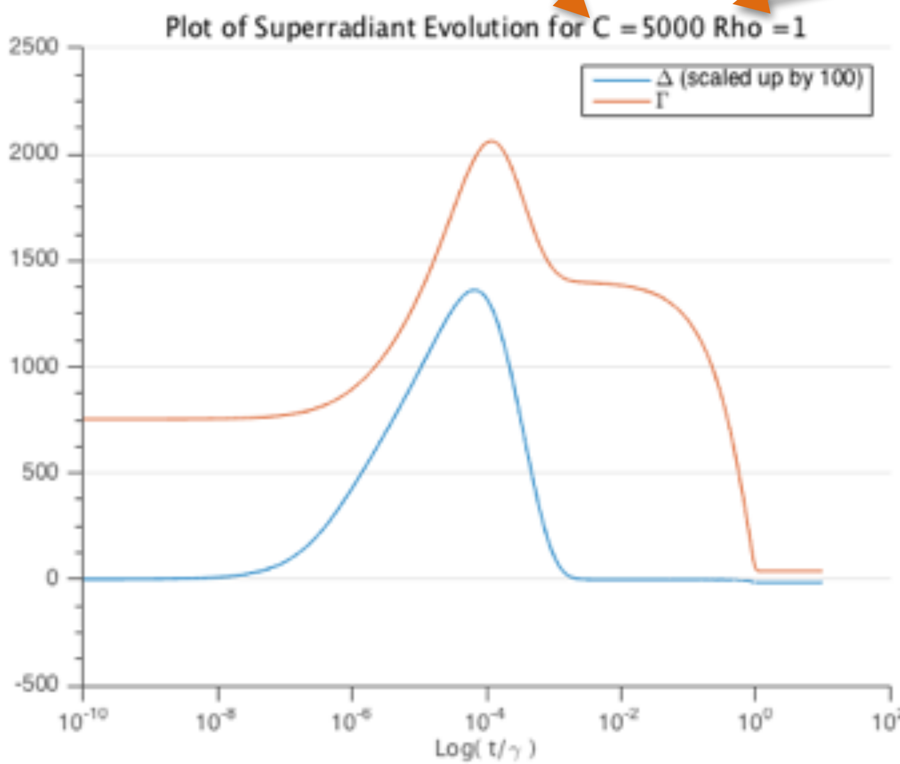
Collective Shift: decay of inverted TLS

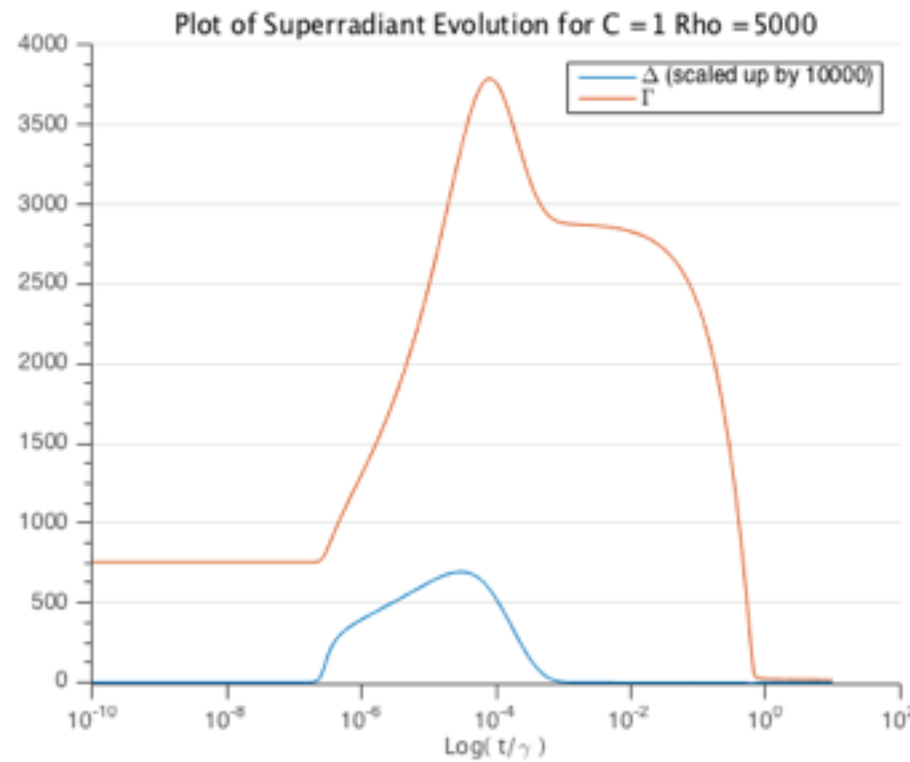


Collective Shift: decay of inverted TLS

$C \propto N\lambda^3$

$C\rho \propto N\lambda^2 L$

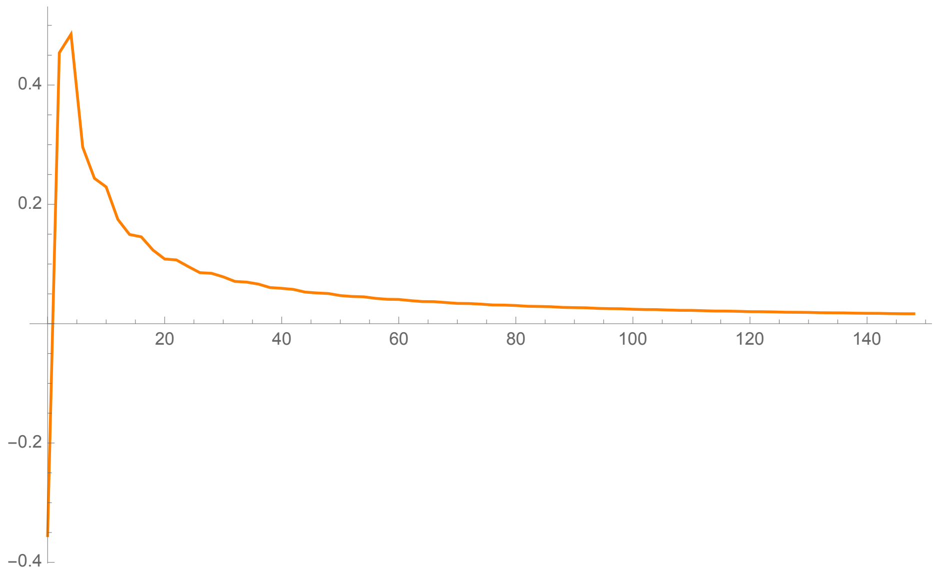


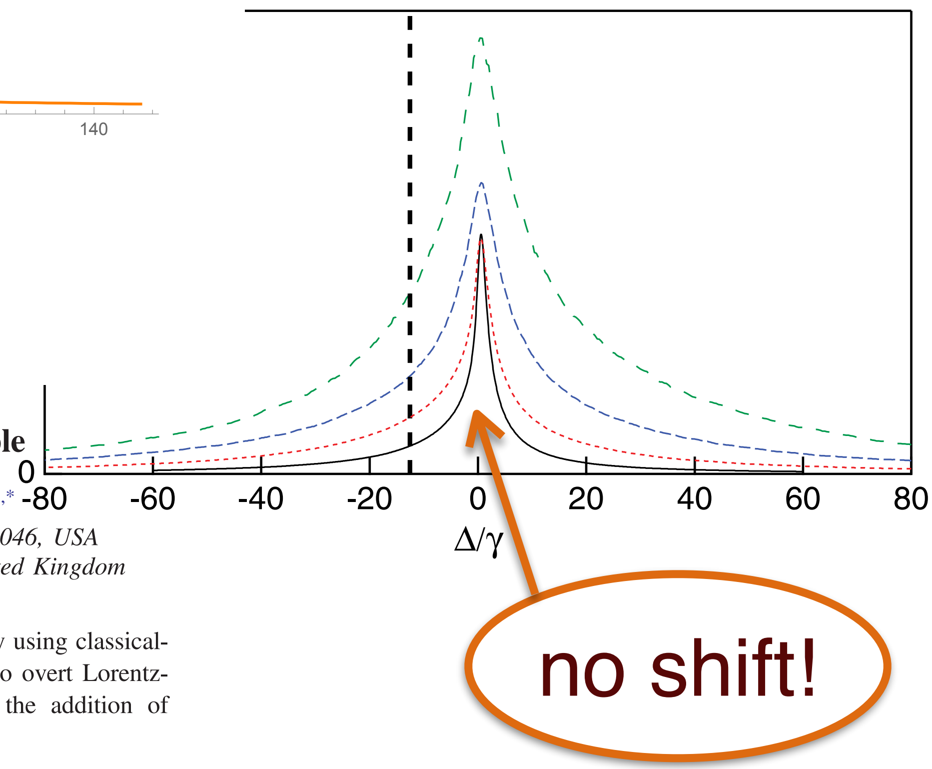


Collective Shift: low-excitation limit ($\propto \Omega^2$)

0

Measurable shift?





Shifts of a Resonance Line in a Dense Atomic Sample

Juha Javanainen,[1] Janne Ruostekoski,[2] Yi Li,[1] and Sung-Mi Yoo[1,*]
[1]*Department of Physics, University of Connecticut, Storrs, Connecticut 06269-3046, USA*
[2]*Mathematical Sciences, University of Southampton, Southampton SO17 1BJ, United Kingdom*
(Received 28 August 2013; published 21 March 2014)

We study the collective response of a dense atomic sample to light essentially exactly using classical-electrodynamics simulations. In a homogeneously broadened atomic sample there is no overt Lorentz-Lorenz local field shift of the resonance, nor a collective Lamb shift. However, the addition of inhomogeneous broadening restores the usual mean-field phenomenology.

Questions - guideline

- Superradiance - What? Why?
- How do we calculate it (better)?
- Is there a collective (Lamb) shift?
- How does entanglement come into the picture?

Superradiance and Entanglement

Does (Dicke) superradiance need/create entanglement?

NO

Wolfe, Yelin, PRL 112, 140402 ('14)

Superradiance and Entanglement

How to define/calculate many-particle entanglement?

Spin Squeezing Inequalities and Entanglement of N Qubit States J. K. Korbicz, J. I. Cirac, M. Lewenstein

Separability in 2×N composite quantum systems B. Kraus, J. I. Cirac, S. Karnas, M. Lewenstein

Entangled symmetric states of N qubits with all positive partial transpositions R. Augusiak, J. Tura, J. Samsonowicz, M. Lewenstein

Four-qubit entangled symmetric states with positive partial transpositions J. Tura, R. Augusiak, P. Hyllus, M. Kuś, J. Samsonowicz, M. Lewenstein

Separability criteria and entanglement witnesses for symmetric

Superradiance and Entanglement

Does (Dicke) superradiance need/create entanglement? (Initial state: no entanglement)

Dicke superradiant time evolution = separable states

↑

constructive proof

Wolfe, Yelin, PRL 112, 140402 ('14)

Superradiance and Entanglement

Does (Dicke) superradiance need/create entanglement? (Initial state: no entanglement)

our system: mixed state of N-atom Dicke states with N+1 known independent coefficients p_i

=

compare to mixture of symmetric product states of N (two-level) atoms (needs N+1 coefficients y_i)

(N+1) - dim. equation system

Wolfe, Yelin, Wolfe, Yelin, PRL 112, 140402 ('14)

Superradiance and Entanglement

Does (Dicke) superradiance need/create entanglement? (Initial state: no entanglement)

our system: mixed state of N-atom Dicke states with N+1 known indepen… coefficients p_i

=

compare to mixture of symmetric product states of N (two-level) atoms (needs N+1 coefficients y_i)

condition: all coefficients $0 \le p_i \le 1$

…+1) - dim. equation system

Wolfe, Yelin, PRL 112, 140402 ('14)

Superradiance and Entanglement

Does (Dicke) superradiance need/create entanglement? (Initial state: no entanglement)

our system: mixed state of N-atom Dicke states with N+1 known indepen... coefficients p_i

=

com... mixture of symm... states of N (t... ms (needs N... ents y_i)

(works!)

condition: all coefficients $0 \le p_i \le 1$

...+1) - dim. equation system

Wolfe, Yelin, PRL 112, 140402 ('14)

Superradiance and Entanglement

Driven superradiant system:

Fuzzy Bunny?

Spin Squeezing

- Correlated (“squeezed”) spins could improve resolution in one direction (“quadrature”).

(Spin) Squeezing

- How to measure squeezing/measurement improvement?

$$\xi^2 \equiv \frac{\text{optimal variance}}{\text{unsqueezed optimal variance}}$$

Kitagawa, Ueda, PRA 47, 5138 (93)

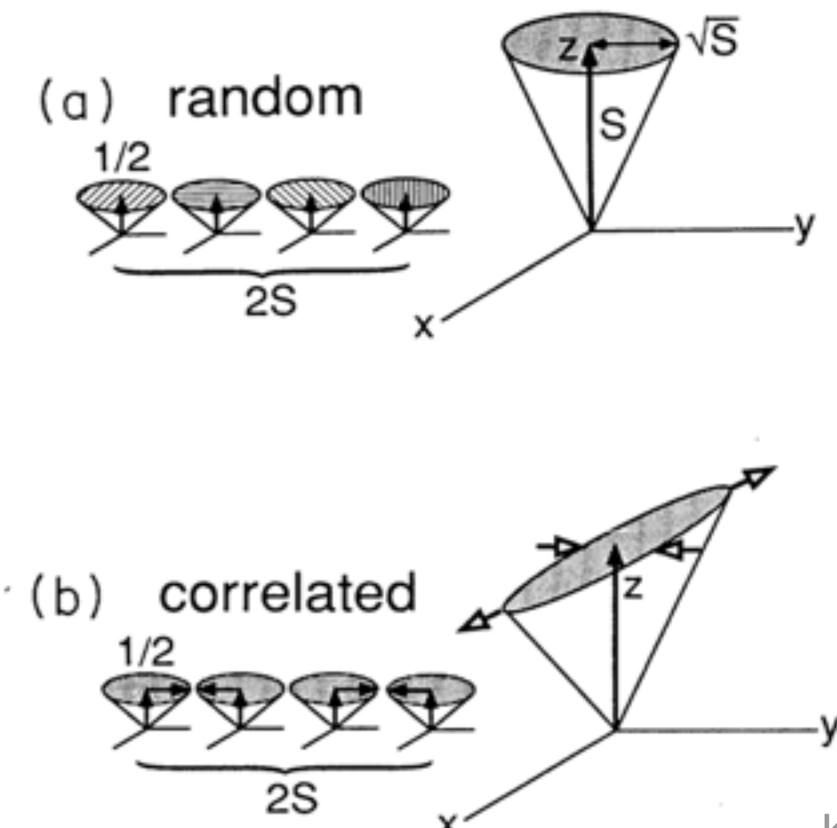
Spin squeezing

Old problem: How to improve metrology by spin squeezing ensembles

➡ Groups of Bigelow, Kuzmich, Lewenstein, Mølmer, Polzik, Sanders, Sørensen, Vuletic, Wineland,…

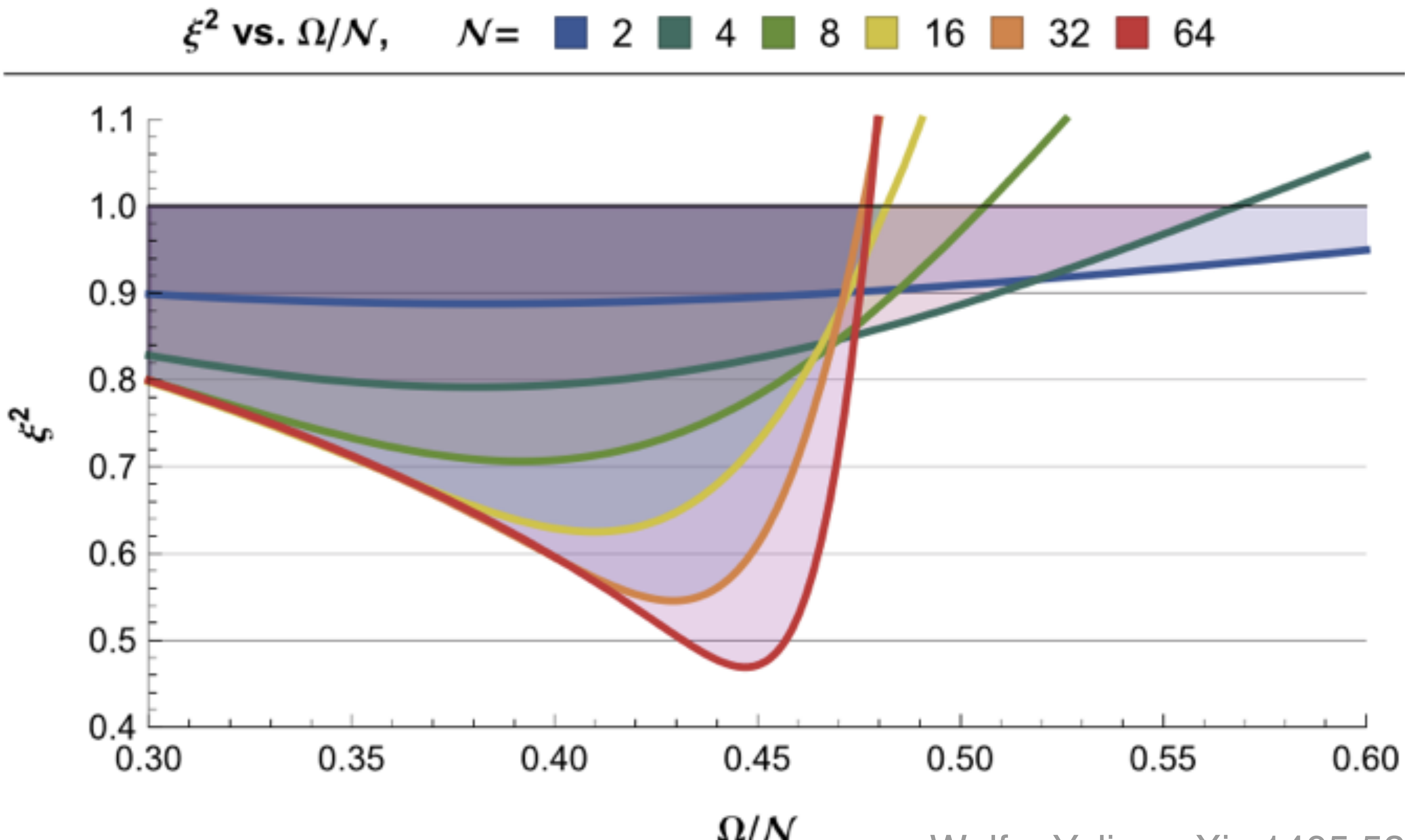
Spin Squeezing

- Correlated ("squeezed") spins could improve resolution in one direction ("quadrature").



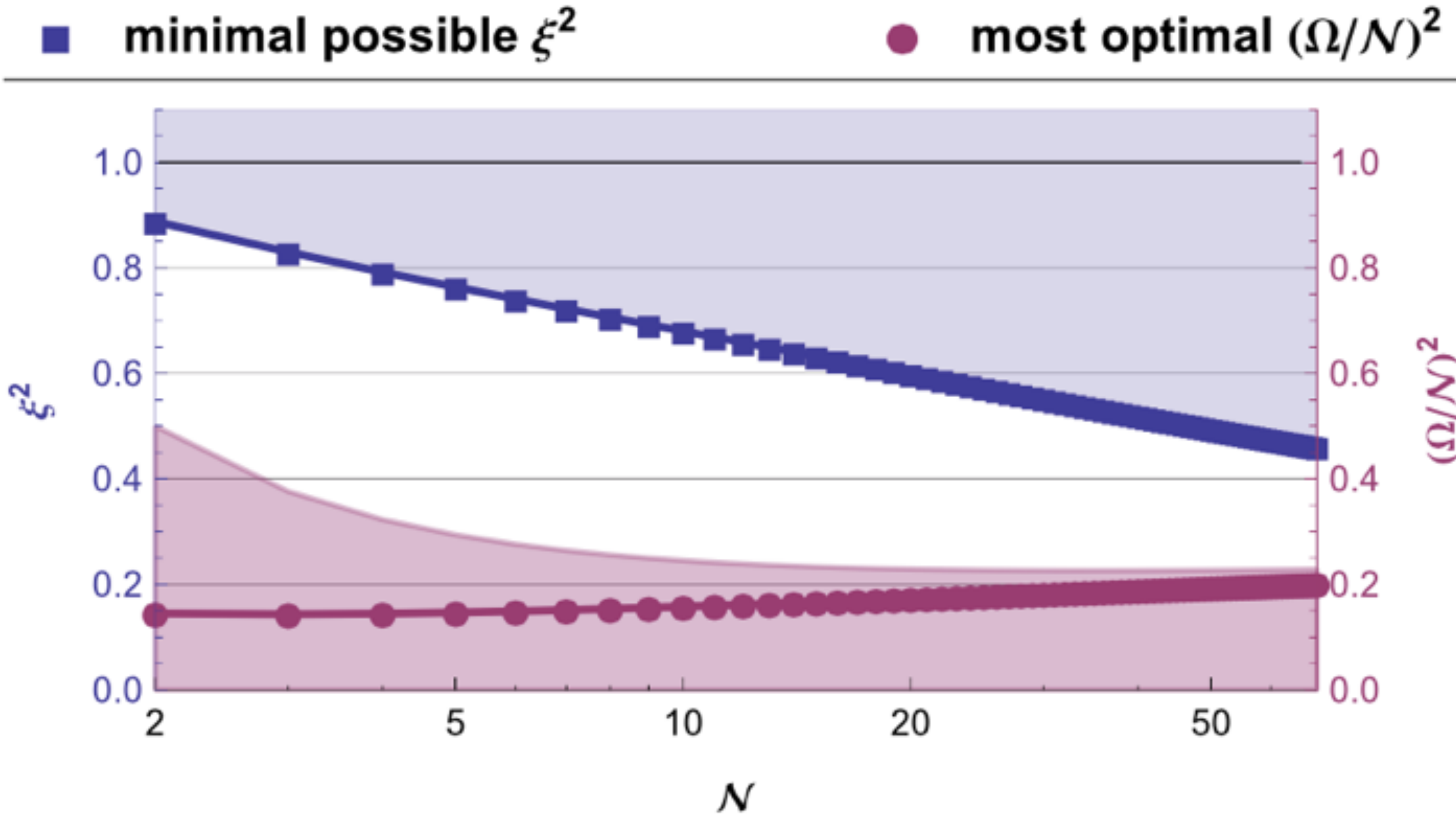
Kitagawa, Ueda, PRA 47, 5138 (93)

Superradiant Spin Squeezing



Wolfe, Yelin, arXiv:1405.5288

Best case for Dicke ensemble



Wolfe, Yelin, arXiv:1405.5288, González Tudela, Porras, PRL 110, 080502 ('13)

Conclusions, Applications and Outlook

- Superradiance - What? Why?
  - ✴Collective effect + exchange
- How do we calculate it (better)?
  - ✴large, homogeneous, self-consistently
  - ➡ small, ordered? higher correlations?
- Is there a collective (Lamb) shift?
  - ✴Yes, and yes.
  - ➡ Find schemes to measure!
- How does entanglement come into the picture?
  - ✴Cooperativity alone does not create entanglement, but cooperative + driving interaction squeeze
  - ➡ Find in more realistic systems + squeeze THz light fields

Dynamics of atoms in dense media - Schwinger-Keldysh & Dyson Eq.

Full dynamics (all degrees of freedom of atoms, fields)

$$H = H_{atoms} + H_{field} - \sum \mathbf{p}_i \mathbf{E}_i$$

two probe atoms

atoms

Thank you!