Brane Absorption and Conformal Field Theory

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Based mostly on work with

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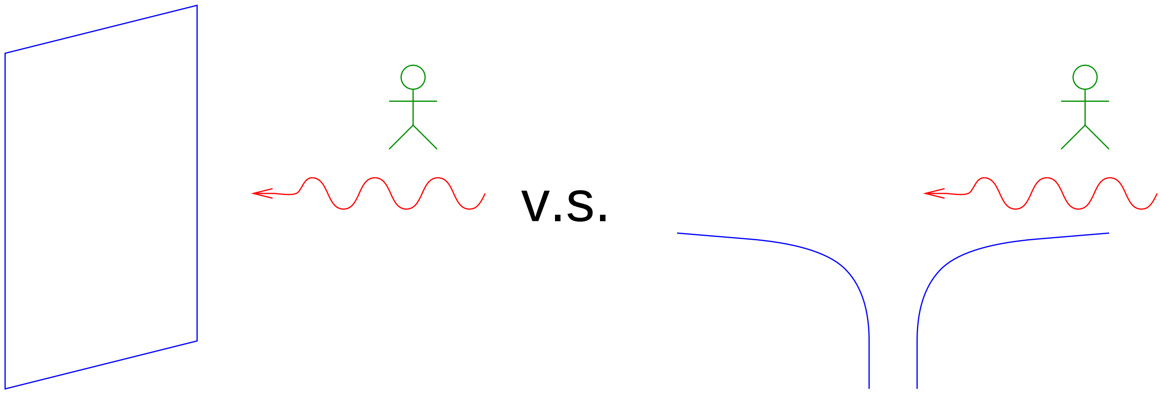
hep-th/9803023

Duality between open string (gauge) theory and closed string (gravity) theory



- Perturbative D-brane description ($gN \ll 1$) v.s. sigma model in a non-trivial background ($gN \gg 1$)
- SYM $\leftrightarrow$ AdS is a limit of this correspondence
- Calculations of Black Hole Entropy and Horizon Area
- Calculations of Absorption cross sections (grey body factors, higher partial waves, etc)

Q: Why does it/doesn't it work?

Try to learn more by doing concrete calculations: D3-brane absorption cross-sections

Review of D3-brane scalar absorption cross-section

SUGRA side:

Metric:

$$ds^2 = H^{-1/2}(-dt^2 + dx_{\|}^2) + H^{1/2} dx_{\perp}^2$$

$$H = 1 + \left(\frac{R}{r}\right)^4, \quad R^4 = gN\alpha'^2$$

Scalar field equation of motion:

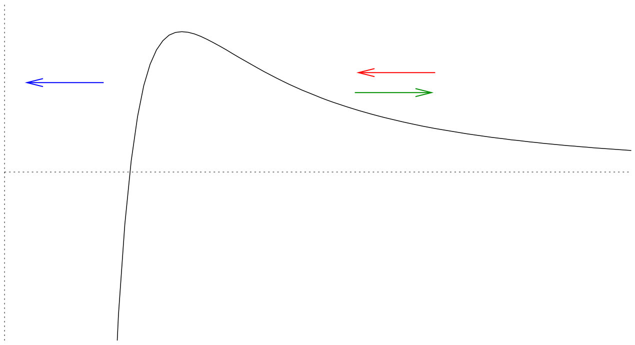
$$\frac{1}{\sqrt{g}} \partial_\mu g^{\mu\nu} \sqrt{g} \partial_\nu \phi = 0$$

$$\rho = \omega r$$

$$\psi = \rho^{5/2} \phi$$

$$\Rightarrow \left[\left(\frac{\partial}{\partial \rho}\right)^2 - \frac{15}{4\rho^2} + 1 + \left(\frac{\omega R}{\rho}\right)^4 \right] \psi = 0$$

$$\left[\left(\frac{\partial}{\partial\rho}\right)^2 - \frac{15}{4\rho^2} + 1 + \left(\frac{\omega R}{\rho}\right)^4\right]\psi = 0$$



$$\left[\left(\frac{\partial}{\partial\rho}\right)^2 - \frac{15}{4\rho^2} + 1\right]\psi = 0 \qquad \rho \gg (\omega R)^2$$

$$\left[\left(\frac{\partial}{\partial\rho}\right)^2 + \frac{15}{4\rho^2} + \left(\frac{\omega R}{\rho}\right)^4\right]\psi = 0 \qquad \rho \ll 1$$

$$\phi = \begin{cases} \frac{32}{\pi}\rho^2 J_2(\rho) \\ i(\omega R)^4\rho^{-2}\left[J_2(\frac{(\omega R)^2}{\rho}) + iN_2\left(\frac{(\omega R)^2}{\rho}\right)\right] \end{cases}$$

$$P = |A|^2 = \frac{\text{Trans}}{\text{Inc}} = \pi^2(\omega R/2)^8 = \frac{\pi^2}{256}\omega^8\alpha'^4(gN)^2$$

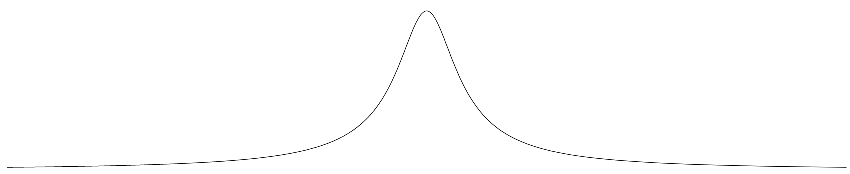
Digression: Tortoise coordinate

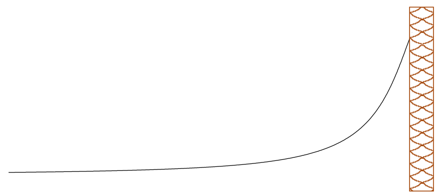
$$u(r) = \int_R^r dr' \sqrt{1 + \frac{R^4}{r'^4}} = \begin{cases} r & r \gg R \\ -\frac{R^2}{r} & r \ll R \end{cases}$$

$$\left[- \left(\frac{\partial}{\partial u} \right)^2 + V(u) - \omega^2 \right] \psi(u) = 0$$

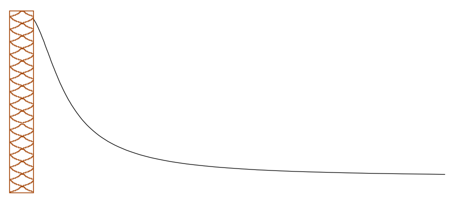
$$V(u(r)) = \frac{5}{4R^2} \frac{(r/R)^2 (3 + (r/R)^4)(1 + 3(r/R)^3)}{(1 + (r/R)^4)^3}$$

$$V(u) = V(-u)$$



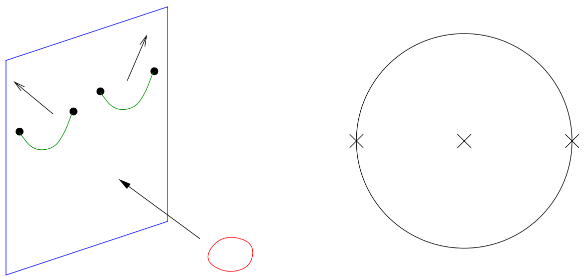


AdS



DBI

Review of absorption cross-section, the D-brane side:



$$
\begin{aligned}
S &= \int d^{10}x\sqrt{g}e^{2\phi}\left(R+(\partial\phi)^2+H^2\right)\\
&\quad+\int d^{p+1}xe^{\phi}\sqrt{-\det(G+F+B)}
\end{aligned}
$$

Expand the DBI action:

$$
\begin{aligned}
L_I &= \frac{p-3}{4}\left(\frac{1}{2}X^M X^N\partial_M\partial_N\phi+\frac{1}{2}\partial_m X_M\partial^m X^M\right)\\
&\quad -\frac{p-7}{4}\left(\frac{1}{4}F_{mn}F^{mn}\phi\right)
\end{aligned}
$$

For D3-brane, ϕ only couples to A.

$$
|A|^2=\sum_{\text{Final}}|\langle\phi(\omega,k)|H_I|\text{Final}\rangle|^2=\pi^2(\omega R/2)^8
$$

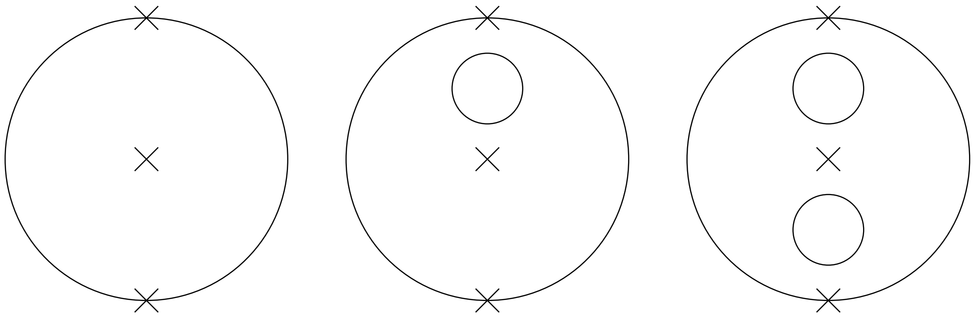
Same as the SUGRA result, including the numerical coefficient!

But! perturbation theory is trustworthy for gN small, whereas SUGRA is trustworthy for gN large. So why should the two agree?

Apparently because gN only appeared in combination $\omega^4\alpha'^2 gN$. Even when gN is large, there is a sensible expansion in terms of this parameter.

The power of gN counts the number of holes in string perturbation theory.

Q: does the fact that only the combination $\omega^4\alpha'^2 gN$ appear in SUGRA imply that for each hole in the world sheet there is a power of $\omega^4\alpha'^2$ in the amplitude?



$\mathcal{O}(\omega^4\alpha'^2)$ $\quad$ $\mathcal{O}(\omega^8\alpha'^4)$ $\quad$ $\mathcal{O}(\omega^{12}\alpha'^6)$ $\quad \ldots$

This would be very nice. It could mean that we can derive the string perturbative expansion supergravity calculation. It could also imply ∞-ly many non-renormalization theorems (if numerical coefficients turns out to match.)

Does this sound too good to be true?

You are darn right it is.

That was a hopeful guess based on computation of <u>one</u> term in a series expansion. We will do some more explicit calculation to show that life is more complicated.

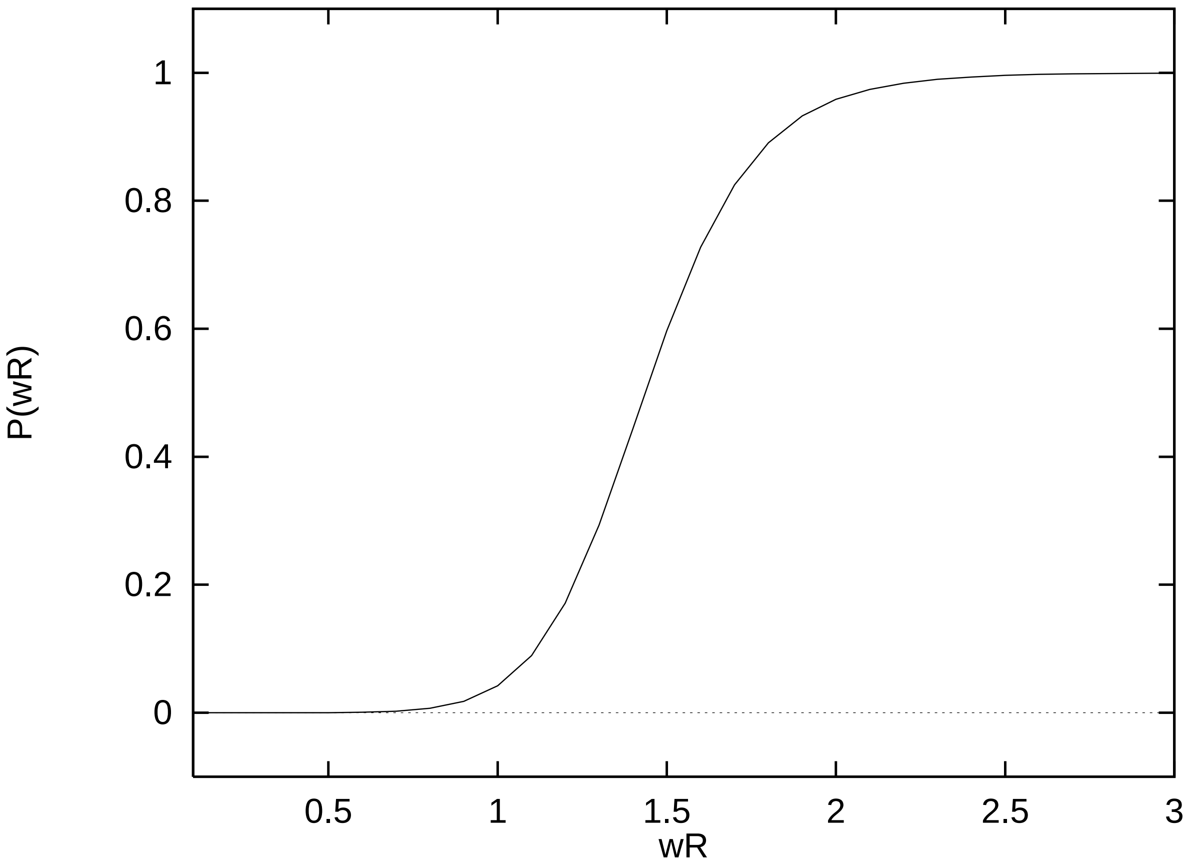
The naive expectation was that SUGRA calculation generates an expansion

$$(\omega R)^8 \left(c_0 + c_1(\omega R)^4 + c_2(\omega R)^8 + \ldots\right)$$

Let's compute subleading terms in this expansion.

One thing we can do is to solve the problem numerically.

$$\left[\left(\frac{\partial}{\partial r}\right)^2+\frac{5}{r}\frac{\partial}{\partial r}-\frac{l(l+2)}{r^2}+\omega^2\left(1+\left(\frac{\omega R}{r}\right)^4\right)\right]\phi=0$$



- $\mathcal{O}((\omega R)^8)$ for small ωR
- Asymptotes to 1

We are actually interested in extracting the subleading terms in the (ωR) expansion analytically. An elegant way to do this is to note that our equation*

$$\left[\left(\frac{\partial}{\partial r}\right)^2 + \frac{5}{r}\frac{\partial}{\partial r} - \frac{l(l+2)}{r^2} + \omega^2\left(1 + \left(\frac{\omega R}{r}\right)^4\right)\right]\phi = 0$$

can be redefined

$$r = Re^z, \qquad \phi(r) = e^{-2z}\psi(z)$$

to read

$$\left[\left(\frac{d}{dz}\right)^2 - \Big((l+2)^2 - 2\omega R \cosh(2z)\Big)\right]\psi(z) = 0$$

This equation is also known as “Mathieu’s modified equation”

Then, look in an old paper titled “The solution of Mathieu’s Differential Equations,” John Dougall (1916).

*Gubser and A.H., forthcomming

The answer:

$$P = \frac{\pi^2}{256}(\omega R)^8 \Big(1 + c_1' \log(\omega R)(\omega R)^4 + c_1(\omega R)^4$$
$$+ c_2'' \log^2(\omega R)(\omega R)^8 + c_2' \log(\omega R)(\omega R)^8$$
$$+ c_2(\omega R)^8\Big)$$

with

$$\begin{aligned}
c_1' &= -\frac{1}{6} = -0.16667 \\
c_1 &= \frac{7}{72} - \frac{\gamma}{6} + \frac{\log(2)}{6} = 0.11654 \\
c_2'' &= \frac{17}{576} = 0.029514 \\
c_2' &= -\frac{161}{4608} + \frac{17\,\gamma}{288} - \frac{17\,\log(2)}{288} = -0.0417824 \\
c_2 &= \frac{297541}{29859840} - \frac{161\,\gamma}{4608} + \frac{17\,\gamma^2}{576} - \frac{13\,\pi^2}{3888} \\
&\quad + \frac{161\,\log(2)}{4608} - \frac{17\,\gamma\,\log(2)}{288} + \frac{17\log(2)^2}{576} \\
&= -0.0185884
\end{aligned}$$

γ is the Euler's constant.

We went looking for expansion of the form

$$(\omega R)^8 \left(c_0 + c_1 (\omega R)^4 + c_2 (\omega R)^8 + \ldots \right)$$

but found instead

$$\begin{aligned}(\omega R)^8 \Big(& 1 + c_1' \log(\omega R)(\omega R)^4 + c_1 (\omega R)^4 \\ & + c_2'' \log^2(\omega R)(\omega R)^8 + c_2' \log(\omega R)(\omega R)^8 \\ & + c_2 (\omega R)^8 \Big)\end{aligned}$$

That is to say, there is a $(\omega R)^4 \log(\omega R)$ correction term which dominates over the $(\omega R)^4$ correction.

Since $R^4 = gN\alpha'^2$,

$$(\omega R)^4 \log(\omega R) \approx \omega^4 \alpha'^2 gN \log(\omega^4 \alpha'^2 gN)$$

This could not possibly come form expansion with respect to holes on the world-sheet.

The explanation is simple.

SUGRA was a limit of the full stringy physics. The limit taken is

$$\begin{array}{rcl} g & \to & 0 \\ N & \to & \infty \\ gN & \to & \infty \\ \omega^2\alpha' & \to & 0 \\ \omega^4\alpha'^2 gN & \to & \text{Fixed} \end{array}$$

The result of this limit is an expansion like

$$(\omega R)^4 \left(1 + c_1' \log(\omega R)(\omega R)^4 + c_1(\omega R)^4 + \ldots\right)$$

which does not admit a sensible gN expansion. But what if the expression prior to taking the SUGRA limit looks something like

$$(\omega R)^4 \Big(1 + c_1' \log((\omega^2\alpha')^\gamma + \omega R)(\omega R)^4 + c_1(\omega R)^4 + \ldots\Big)$$

for some $\gamma > 0$.

<u>This</u> has a sensible gN expansion, as long as we expand before taking the SUGRA limit.

$$(\omega^4\alpha'^2 gN)(1+\gamma\omega^4\alpha'^2 gN \log(\omega^2\alpha') + \ldots)$$

The moral of the story is that although SUGRA appears to depend only on combination ωR, there is no reason for the full string-theory amplitude to behave the same way. SUGRA limit is inaccessible to perturbation theory. We were lucky at leading order in ωR. To reconstruct the next term in (ωR) expansion, we must first sum to all order in gN.

But SUGRA can tell us <u>a little bit</u> about perturbative expansion. The leading term in ωR was an example.

We will now show that SUGRA tells us correctly about the presence of $\omega^8\alpha'^4(gN)^2\log(\omega^2\alpha')$ term at 1-loop.

Review of CFT/Absorption cross-section correspondence: (leading term)

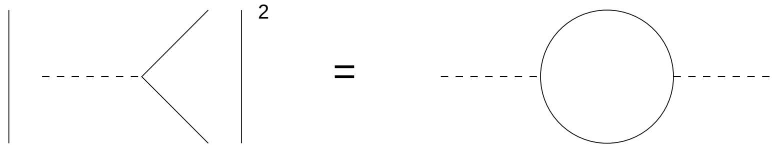
N parallel D3-brane is described by $\mathcal{N} = 4$ SYM in the low-energy limit. This is a conformally invariant theory.

Graviton couple to the stress-energy tensor $T_{\mu\nu}$
Dilaton couples to $\frac{1}{4}F^2$

These are engineering dimension 4 operators

Absorption cross section follows from

$$\sum_f |\langle f|\mathcal{O}|0\rangle|^2 = 2\mathrm{Im}\,[\langle 0|\mathcal{O}^\dagger\mathcal{O}|0\rangle] = \mathrm{Disc}[\langle 0|\mathcal{O}^\dagger\mathcal{O}|0\rangle]$$



One can compute perturbatively:

$$\langle 0|{:}\frac{1}{4}F^2(x){:}{:}\frac{1}{4}F^2(z){:}|0\rangle = \frac{3N^2}{\pi^4}\frac{1}{(x-z)^8}$$

Fourier transform:

$$\int d^4x\, e^{ipx}\, \frac{3N^2}{\pi^4}\frac{1}{x^8} = \frac{p^4}{64\pi^2}\log(p^2/\Lambda^2)$$

$$\begin{aligned}\sigma &= \left(\frac{2\kappa}{2i\omega}\right)\mathrm{Disc}[\langle 0|\mathcal{O}\mathcal{O}|0\rangle] \\ &= |A|^2 \times (\text{phase space factor})\end{aligned}$$

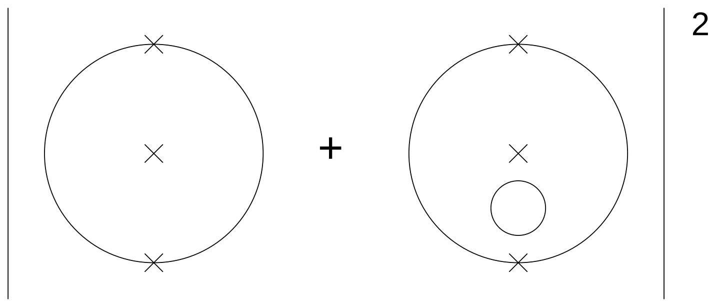
$$|A|^2 = \frac{\pi^2}{256}(\omega R)^8$$

Similarly, $\langle TT\rangle \approx c/x^8$ and $|A|^2 \approx (\omega R)^8$.

We learn from SUGRA that these operators do not acquire anomalous dimensions.

$$(\omega^8\alpha'^4(gN)^2)(1+\omega^4\alpha'^2 gN\log(\omega^2\alpha')+\ldots)$$

$$= |(\omega^4\alpha'^2(gN)+\omega^8\alpha'^4(gN)^2\log(\omega^2\alpha')+\ldots)|^2$$



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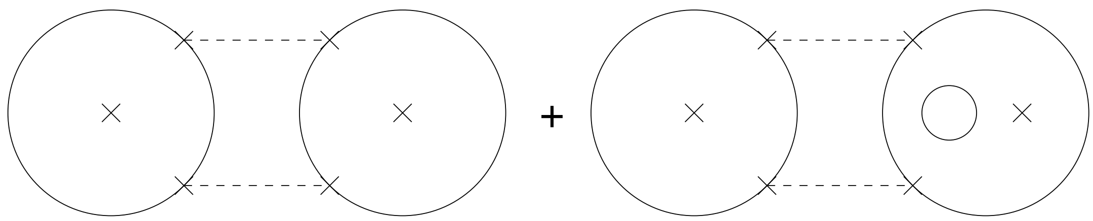

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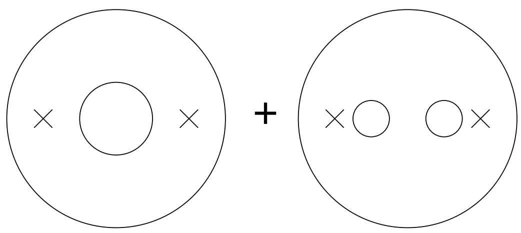

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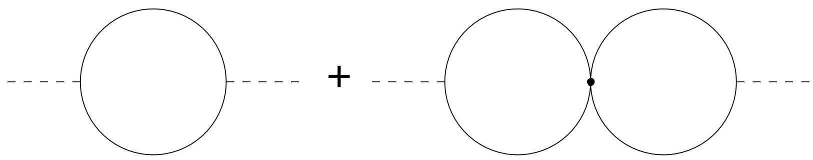


$F^4(z) \sim$ dim 8, $\int d^4z\, F^4(z) \sim$ dim 4

We can compute using Feynman-rules

$$\langle :F^2(x)::F^4(z)::F^2(0): \rangle = \frac{1}{(x-z)^8 z^8}$$

or arrive at the same conclusion using conformal invariance

$$\langle \mathcal{O}_4 \mathcal{O}_8 \mathcal{O}_4 \rangle = \frac{1}{(x-z)^{8+4-4} x^{4+4-8} z^{8+4-4}}$$

The integral over z is a convolution:

$$\tilde{F}(A * B) = \tilde{F}(A)\tilde{F}(B)$$

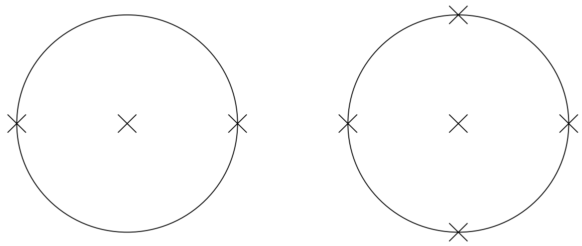
$$\Rightarrow p^8 (\log(p^2/\Lambda^2))^2$$

$$\mathrm{Disc}[p^8(\log(p^2/\Lambda^2))^2] \sim p^8 \log(p^2/\Lambda^2)$$

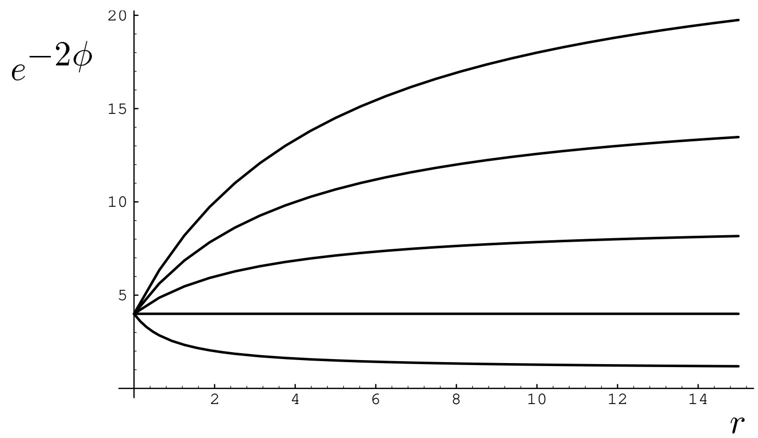
$$\begin{aligned} \sigma &= \left(\frac{2\kappa}{i\omega}\right)(\omega^4 + \omega^8 gN \log(\omega^2/\Lambda^2)) \\ &= \left(\omega^8 \alpha'^4 (gN)^2\right)\left(1 + \omega^4 \alpha'^2 gN \log(\omega^2\alpha')\right) \\ &\quad \times(\text{phase space factor}) \end{aligned}$$

SUGRA tells us that the operator F^4 does not acquire anomalous dimensions:

Coupling to 4-point function have been encountered previously in the context of absorption of fixed scalars by branes



Fixed Scalar*:



For a singly charged RR p-brane wrapped on a p-torus, the volume of the brane in Einstein metric is an example of fixed scalar.

*Ferrara & Kallosh, PRD 54 (1996) 1514

Absorption of fixed scalars by branes*

$$\left[\nabla^2 + \omega^2 \left(1 + \left(\frac{R}{r}\right)^{D-3}\right) - \frac{A}{r^2(B + Cr^{D-3})^2}\right]\phi = 0$$

$$\begin{aligned} A &= 2(D-1)^2(D-3)^2 R^{2(D-3)} \\ B &= (D-1)R^{D-3} \\ C &= 2(D-2) \end{aligned}$$

$D = 7$ for D3-brane This equation is solved by dividing r into regions and matching.

$$|A|^2 \sim \#(\omega R)^\gamma$$

$$\gamma = \frac{(D-3)(D+1)}{D-5} = 16 (\text{for } D = 7)$$

This indicates an existence of an operator of dimension 8 which does not acquire anomalous dimensions.

*Kol & Rajaraman PRD56 (1997) 983-986

This is also consistent with the AdS $\leftrightarrow$ SYM duality conjecture which maps

$$\begin{array}{rcl} \text{SUGRA KK modes} & \leftrightarrow & \text{Chiral Primary Operators} \\ \text{AdS Mass } m^2 & \leftrightarrow & \Delta(\Delta - 4) \end{array}$$

The fixed scalar equation

$$\left[\nabla^2 + \omega^2 \left(1 + \left(\frac{R}{r}\right)^{D-3}\right) - \frac{A}{r^2(B + Cr^{D-3})^2}\right]\phi = 0$$

for r small, reduces to

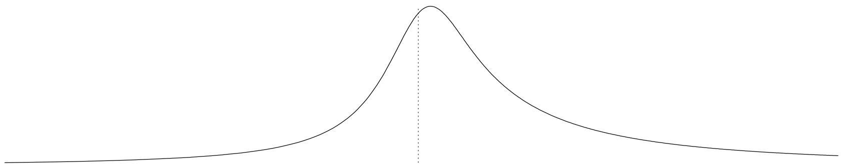
$$\left[\nabla^2 + \omega^2 \left(\frac{R}{r}\right)^{D-3} - \frac{2(D-3)^2}{r^2}\right]\phi = 0$$

$$\Rightarrow m^2 = 32$$

$$\Rightarrow \Delta = 8$$

Brief digression on M2 and M5 branes

- Minimal coupling to dimension 3 and dimension 6 operators
- Fixed scalar coupling to dimension 6 and dimension 12 operators
- Corresponding KK state with the appropriate masses appear in $AdS_{4/7} \times S_{7/4}$ supergravity
- similar $\log(\omega R)$ terms appear at subleading order:
- Tortoise coordinate:



in/out exchange exchanges M2 and M5

Conclusions

SUGRA limit certainly does not know everything there is to know about gN perturbative expansion of string theory. Nonetheless, it appears to know something about the few leading terms:

- $\pi^2/256(\omega R)^8$
- $(\omega R)^{12} \log(\omega^2 \alpha')$

To make further comparisons, we need to either

1. sum to all orders in gN on the D-brane side
2. sum to all orders in $1/gN = \alpha'^2/R^4$ on the SUGRA side

The limit

$$
\begin{aligned}
\omega^2\alpha' &\to 0 \\
g &\to 0 \\
N &\to \infty \\
gN &\to \infty \\
\omega^4\alpha'^2 gN &\to \text{Fixed}
\end{aligned}
$$

is a very non-trivial limit from the D-brane side, summing up some class of diagrams.

Mathieu's differential equation encodes the "non-perturbative data" of this limit.

Reminiscent to double scaling limit of the $c \leq 1$ matrix model and the Painlevé equations.