Drukker: „A super-matrix model for super Chern-Simons theory"

3d theory: Chern-Simons + matter
susy theory ⇝ non-topol.

CFT's	branes
① 4d - $\mathcal{N}=4$ SYM	D3 branes on $\mathbb{R}^{3,1}\times$ 1
② 6d - $\mathcal{N}=(2,0)$ („mysterious theory")	M5 branes $\mathbb{R}^{5,1}\times\mathbb{R}$
↓ dim. reduction
③ 4d - $\mathcal{N}=2$	M5 branes on Riem. Surf C
④ 3d - $\mathcal{N}=8$ („mysterious")	M2 branes on $\mathbb{R}^{2,1}\times$ 1
⑤ - $\mathcal{N}=6$	M2 branes on $\mathbb{R}^{2,1}\times\mathbb{R}$ / $\mathbb{Z}$
⑥ - $\mathcal{N}=2,4$	M5 branes on M_2

AdS-like geometries

① IIB $AdS_5\times S^5$

② M theory on $AdS_7\times S^4$

③ M theory on $AdS_5\times\ldots$

④ M theory on $AdS\times S^7$

⑤ IIA on $AdS_4\times CP^3$

⑥ –

- 4d $\mathcal{N}=2$ theories: Calculation of BPS observables, part function on S^4, Wilson loops on S^4

$$Z_{S^4} = \int d\mu(\alpha)\, |F(\alpha, m)|^2$$

$$\langle W \rangle = \int d\mu(\alpha)\, |F(\alpha, m)|^2 \; T_R e^{\alpha}$$

$\mathcal{N}=4$ SYM $\qquad F = e^{-\frac{1}{g^2} \mathrm{Tr}\, \alpha^2}$

$$Z_{S^4} = \int d\alpha \; \Delta(\alpha)^2 \, e^{-\frac{2}{g^2} \mathrm{Tr}\, \alpha^2}$$

$$\langle W \rangle = \int \ldots \qquad T e^{\alpha}$$

large N limit:

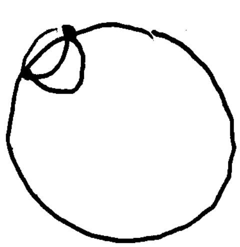
$$\rho(\alpha) = \frac{2}{\pi \lambda} \sqrt{\lambda - \alpha^2}$$

$$\langle W_{\circ} \rangle_{\text{planar}} = \int_{-\sqrt{\lambda}}^{\sqrt{\lambda}} d\alpha \; \rho(\alpha)\, e^{\alpha} = \frac{2}{\sqrt{\lambda}} I_1(\sqrt{\lambda})$$

$$\lambda \to \infty \qquad \langle W \rangle \longrightarrow e^{\sqrt{\lambda}}$$

in geometry pic:

$\mathbb{H}_5$



$$S_{cl} = \frac{\text{Area}}{2\pi\alpha'} = \frac{R^2}{2\pi\alpha'}\left(\frac{\text{circ}}{\epsilon} - 2\pi\right)$$

$$\longrightarrow -\frac{R^2}{\alpha'} = -\sqrt{\lambda}$$

$$\langle W \rangle_{AdS} \sim e^{-S_{cl}} = e^{\sqrt{\lambda}} \leftarrow \text{agrees}$$

- now calculation in 3d theories: ③

vector multiplet

$$S = \frac{k}{4\pi} \int d^3x \ \mathrm{Tr} \left[A \wedge dA + \frac{2}{3} A^3 + \lambda^+ \lambda + 2 D \sigma \right]$$

$$S_{matter} = \int d^3x \sqrt{g} \left(D_\mu \phi^+ D^\mu \phi + \frac{3}{4} \phi^+ \phi + F^+ F - \phi^+ \sigma^2 \phi + \dots \right)$$

Q w/ $Q^2 = J$ ← rotation w/o fixpts on S^3

$$S \longrightarrow S + t \, Q \left((Q\lambda)^+ \lambda \right)$$

saddle points:

$$|Q\lambda|^2 = 0 \implies A = 0 \ , \ D = -\sigma = \text{const}$$

result on S^3: (stationary phase around saddle pt)

$$Z_{S^3} = \int \prod \frac{d\mu_i}{2\pi} \, e^{-\frac{k}{4\pi} 2 \sum \mu^2} \prod_{i<j} \left(2 \sinh \frac{\mu_i - \mu_j}{2} \right)^2$$

$$\times \frac{1}{\pi \det_S \cosh \sigma}$$ ← matter in $S \oplus S^*$

$$\langle W \rangle = \int \dots \mathrm{Tr}_R \left(e^{\mu} \right)$$

↑

$$W = \mathrm{Tr}_R \, P \exp \oint \left(i A^\mu \dot{x}^\mu + |\dot{x}| \sigma \right) ds$$

ABJM model $\quad U(N_1)_k \times U(N_2)_{-k}$

matter $(N_1, \bar{N}_2)$, $(\bar{N}_1, N_2)$

$$\lambda_1 = \frac{N_1}{k} \qquad \lambda_2 = \frac{N_2}{k}$$

$$\sim \; -\frac{R^2}{\alpha'} = -\pi\sqrt{2\lambda}$$

$$\langle W \rangle_{AdS} \sim e^{-S_{cl}} = e^{\pi\sqrt{2\lambda}}$$

from gravity:

$$Z_{S^3} = e^{-S_{SUGRA}}$$

$$S = \frac{1}{k\pi G_N}\int d^4x \sqrt{G}\,(R - 2\Lambda)$$

$$= -\frac{\sqrt{2}\,\pi}{3}\sqrt{k}\; N^{3/2}$$

Wilson loops param. by two reps

W_{R_1, R_2} $\quad$ (1/6 BPS)

W_R $\quad$ (1/2 BPS)

rep of supergroup $U(N_1|N_2)$

$$Z_{ABJM} = \frac{1}{N_1!\, N_2!} \int \prod \frac{d\mu_i}{2\pi} \prod \frac{d\nu_1}{2\pi}$$

$$\times \frac{\prod_{i<1} \left(2\sinh\frac{\mu_i-\mu_j}{2}\right)^2 \prod \left(2\sinh\frac{\nu_i-\nu_j}{2}\right)^2}{\prod\left(2\cosh\frac{\mu_i-\nu_1}{2}\right)^2}$$

$$\times\ e^{-\frac{1}{2g_s}\left(\Sigma\mu_i^2 - \Sigma\nu_i^2\right)}$$

$$g_s = \frac{2\pi}{K}$$

$u(N_1+N_2)$ on S^3

$$= \int d\mu_i\, d\nu_i\, \sinh^2\left(\frac{\mu_i-\mu_j}{2}\right)\sinh^2\left(\frac{\nu_i-\nu_j}{2}\right)\sinh^2\left(\frac{\mu_i}{2}\right.$$

$$\times\ e^{-\mu^2-\nu^2}$$

instead on lens space $S^3/\mathbb{Z}_2$ $]$ $N_2 \to -N$

$u(N_1+N_2) \to u(N_1) \times u(N_2)$

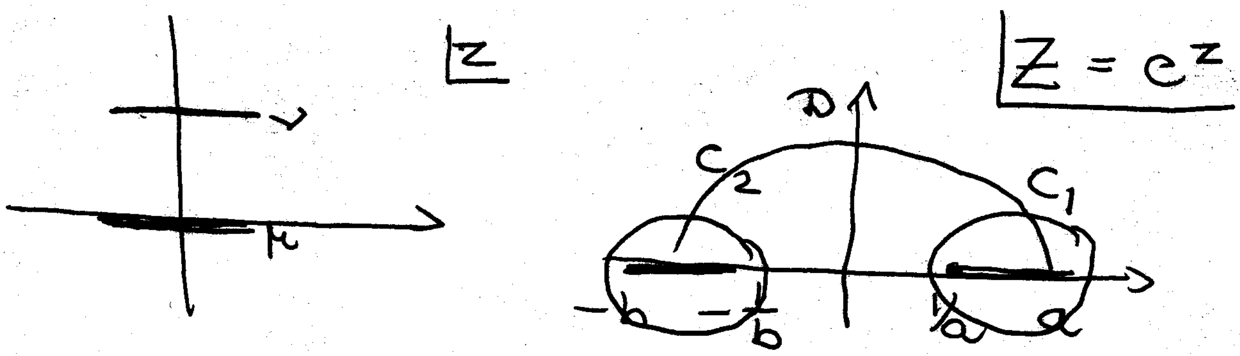
$$\nu_j \to \nu_j + \pi i$$

$u(N_1|N_2) \to u(N_1) \times u(N_2)$

$$\langle W^{1/6}_{R_1,R_2} \rangle = \int \dots \ \mathrm{Tr}_{R_1} e^{\mu} \ \mathrm{Tr}_{R_2} e^{\nu}$$

$$\langle W^{1/2}_{R} \rangle = \int \dots \ \mathrm{STr}_{R} \underbrace{\begin{pmatrix} e^{\mu} & 0 \\ 0 & -e^{\nu} \end{pmatrix}}_{U}$$

$$\omega(z) = g_s \left\langle \mathrm{Tr} \frac{Z+U}{Z-U} \right\rangle$$



$$\omega_0 = \omega(z)_{\text{planar}} = 2 \log \left[\frac{1}{2\sqrt{\beta}} \left(\sqrt{(\zeta+b)(\zeta+1/b)} - \sqrt{(\zeta-a)(\zeta-1/a)} \right) \right.$$

$$\beta = \frac{1}{4}\left(a + \frac{1}{a} + b + \frac{1}{b}\right)$$

$$\left. \zeta = \frac{1}{2}\left(a + \frac{1}{a} - b - \frac{1}{b}\right) \right]$$

$$t_1, t_2 = \frac{1}{4\pi} \oint_{C_1, C_2} \omega_0(z)\, dz \qquad t_1 + t_2 = \log \beta$$

$$\langle W^{1/6}_{\square} \rangle = \frac{1}{4\pi i} \oint \omega_0(z) e^{z}\, dz$$

$$\langle W^{1/2} \rangle = \langle W^{1/6}_{\square_1} - W^{1/6}_{\square_2} \rangle = \frac{\zeta}{2}$$

$$\lambda_1 = \frac{N_1}{K} = 2\pi i \, t_1$$

$$\lambda_2 = \frac{N_2}{K} = -2\pi i \, t_2$$

planar free energy

$$-\frac{1}{2} \oint_D \omega_0 \, dz = \frac{\partial F_0}{\partial t_1} - \frac{\partial F_0}{\partial t_2} = \pi i t$$

$$\frac{\partial t_i}{\partial \zeta} = \kappa \leftarrow \text{elliptic integrals}$$

1

$$\hat{\lambda} = \lambda_1 - \left[\frac{1}{2}\left(B^2 - \frac{1}{4}\right) + \frac{1}{24}\right] \pm \frac{\log^2 \kappa}{2} + \ldots$$

$\kappa = |\zeta|$

B phase of ζ

$$\frac{1}{24}\left(1 + \frac{1}{K^2}\right)$$

non-planar corrections

2 $$\langle W^{1/2} \rangle = \frac{1}{2} |\kappa| \sim e^{\pi \sqrt{2\hat{\lambda}}} \left(1 + e^{-2\pi \sqrt{2\hat{\lambda}}} (\ldots) + \ldots\right)$$

3 $$\partial_\lambda F_0 \sim 2\pi^3 \sqrt{2\lambda} \Rightarrow F = g_s^{-2} F_0$$

$$= \frac{\pi \sqrt{2}}{3} K^2 \lambda^{3/2}$$