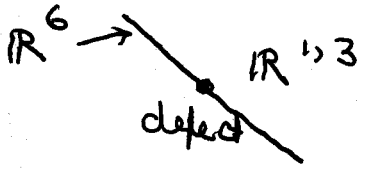
Dave Morrison:

"Supergravity approach to [illegible] theory. Connection between 4d and 2d gauge theories"

AdS/CFT

Gravity theories in dim $d+1$ $\longleftrightarrow$ Gauge theories in dim d

IIB string theory in $\mathbb{R}^{1,9}$ w/ a "defect D3-brane, N units of flux $\longleftrightarrow$ $\mathcal{N}=4$ SYM in $\mathbb{R}^{1,3}$, $G = U(N)$ (as $N \to \infty$)



$(S^5 \times \mathbb{R}^+) \times \mathbb{R}^{1,3}$ w/ flux

$$\xrightarrow{\text{IR}} \boxed{S^5 \times AdS_5}$$

IIB reduces on S^5
$\rightsquigarrow$ effective 4+1 dim'l theory

generalization:

Dp-brane in IIA or IIB

$\mathbb{R}^{1,p}$ $\mathbb{R}^{1,9}$

or

N M2-branes in M-theory

N M5-branes $\mathbb{R}^{1,5}$ ⇝ "mysterious" 6d theory associated to A_{N-1}

Metric solution (basic sugra solution)

$\in M^{1,p}$ = (internal) world volume of brane

$\in N^{1+k}$ = transverse space

$p+k+2 = D \in \{10; 11\}$

$$ds^2 = f(y)^{\frac{3-k}{k-1}} dx_M^2 + f(y)^{\frac{2}{k-1}} dy_N^2$$

w/ k-form field strength

$$\left[d\left(\frac{1}{f(y)}\right) \wedge dvol_M + *\left(d\left(\frac{1}{f(y)}\right) \wedge dvol_M \right) \right]_{k\text{-form}}$$

warped products: we allow certain zeros in $f(y)$

eqn of motion for $f(y) \Rightarrow \Delta f(y) = 0$

(away from defect

$$f(y) = 1 + \frac{c_k \cdot g^{11-D} N}{(r/\ell)^{k-1}}$$

$$\Delta f = \tilde{c}_k \; g^{11-D} \; N \; \delta(y - y_0)$$

↑ location of brane

$\tilde{c}_k \sim \in \mathbb{Z}$ (quantization cond.)

$\tilde{c}_k / (k-1) c_k$ = volume of unit S^k

appropriate limit:

$\frac{r}{\ell} = (u\ell)^{3/k-3}$ $\ell \to \infty$, u fixed

$$\frac{ds^2}{\ell^2} \to (c_k g^{11-D} N)^{\frac{3-k}{k-1}} u^2 dx_M^2 + \frac{4 (c_k g^{11-D} N)^{\frac{2}{k-1}}}{(k-3)^2} \frac{du^2}{u^2} + (c_k g^{11-D} N)^{\frac{2}{k-1}} d\Omega_{S^k}^2$$

thus is either:

warped product $u^2 dx_M^2 + u^{-2} dy^2_{C(S^k)}$

or $\cong$ product metric $\mathbb{R}^+_u \times \mathbb{R}^{1,10}$ on $AdS_{p+2} \times S^k$

$\partial(AdS_5)$ is a sphere of dim 4.

N M5 - branes $\longleftrightarrow$ mysterious 6d theory w/ (2,0) SUSY, A_{N-1}

$\underbrace{\mathbb{R}^{1,5}}_{\text{worldvol.}} \times \mathbb{R}^5$

or

$AdS_7 \times S^4$

reduced on $C \times \mathbb{R}^{1,3}$

$\rightsquigarrow$ effective 4d theor

2 solutions:

$\mathcal{N}=2$ in 4d } AGT

$\mathcal{N}=1$ in 4d "Sicilian theories"

- $\mathcal{N}=2$ $C \subseteq \cancel{X} = K3$ surface locally

M-theory on $X \times \mathbb{R}^{1,6}$ — effective 7-d theory

$\cup$ — $\cup$

M5-brane in $C \times \mathbb{R}^{1,3}$ — 3-brane

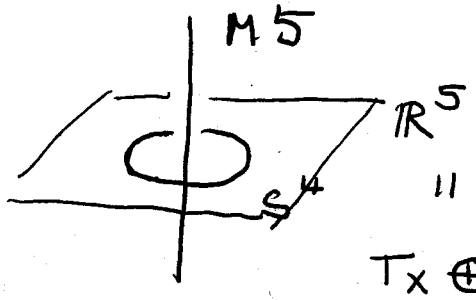
replace $\cancel{X}$ by unit disc bundle Y in T^*C

give C a const curvature metric $=-2$

$(g(C) \geq 2)$

Thm: Y has a hyperKähler metric (incomplete)

geometry of soln:



$T_X \oplus \mathbb{R}^3$

AdS_7

$$S^4 \longrightarrow \mathbb{Z}$$
$$\downarrow$$
$$C \times \mathbb{R}^{1,3}$$

$$S^1 \times S^2 \longrightarrow S^4$$
$$\downarrow$$
$$\longmapsto \text{ interval}$$

$$ds_{11}^2 = (\pi N l_p)^{2/3} \frac{(1+\cos^2\theta)^{1/3}}{2}$$

$$\times \Big[4\, ds^2_{AdS_5} + 2\, ds_C^2 + 2\, d\theta^2$$

$$+ \frac{2\cos^2\theta}{1+\cos^2\theta}\, ds^2_{S^2} + \frac{4\sin^2\theta}{1+\cos^2\theta} (d\chi + v)^2 \Big]$$

χ in S^1

$v = 1\text{-form}$ on C

Thus metric satisfies sugra equation of motion

looks like

$$0 \longmapsto \pi/2$$

$AdS_7 \times S^4$ $\qquad$ $AdS_5 \times C \times S^4$

Supersymmetry ?

- M on $X \times \mathbb{R}^{1,6}$ is supersymmetric provided that X has covariantly const. spinors.

- M5 brane on $C \times \mathbb{R}^{1,3}$ $g(C)=1$ to get susy
- BSV: if C does not have cov. const. spinors you can still get a twisted susy theory on world volume by combining normal bundle w/ R-symmet

Normal bundle of $C \subset X$ is a $u(1)$-bundle
modify susy gens. by $u(1) = SO(2) \subset SO(5)$

(Maldacena-Nuñez)

$\rightsquigarrow$ twisted $N=2$ SYM theory on dim 4.

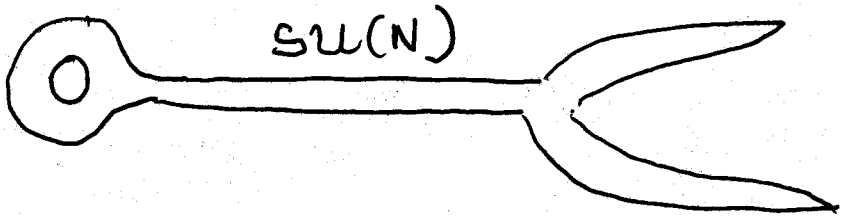
Gaiotto – Maldacena (we should puncture C for full story)

$\mathcal{T}_g$: Teichmüller space

↓

$\mathcal{M}_g$ = moduli space of C w/ g(C)

make choices of metrics which look like



long thin tube ↕ weakly coupled gauge grp

joining ↕ matter in gauge grp

then they investigate effects of S – duality